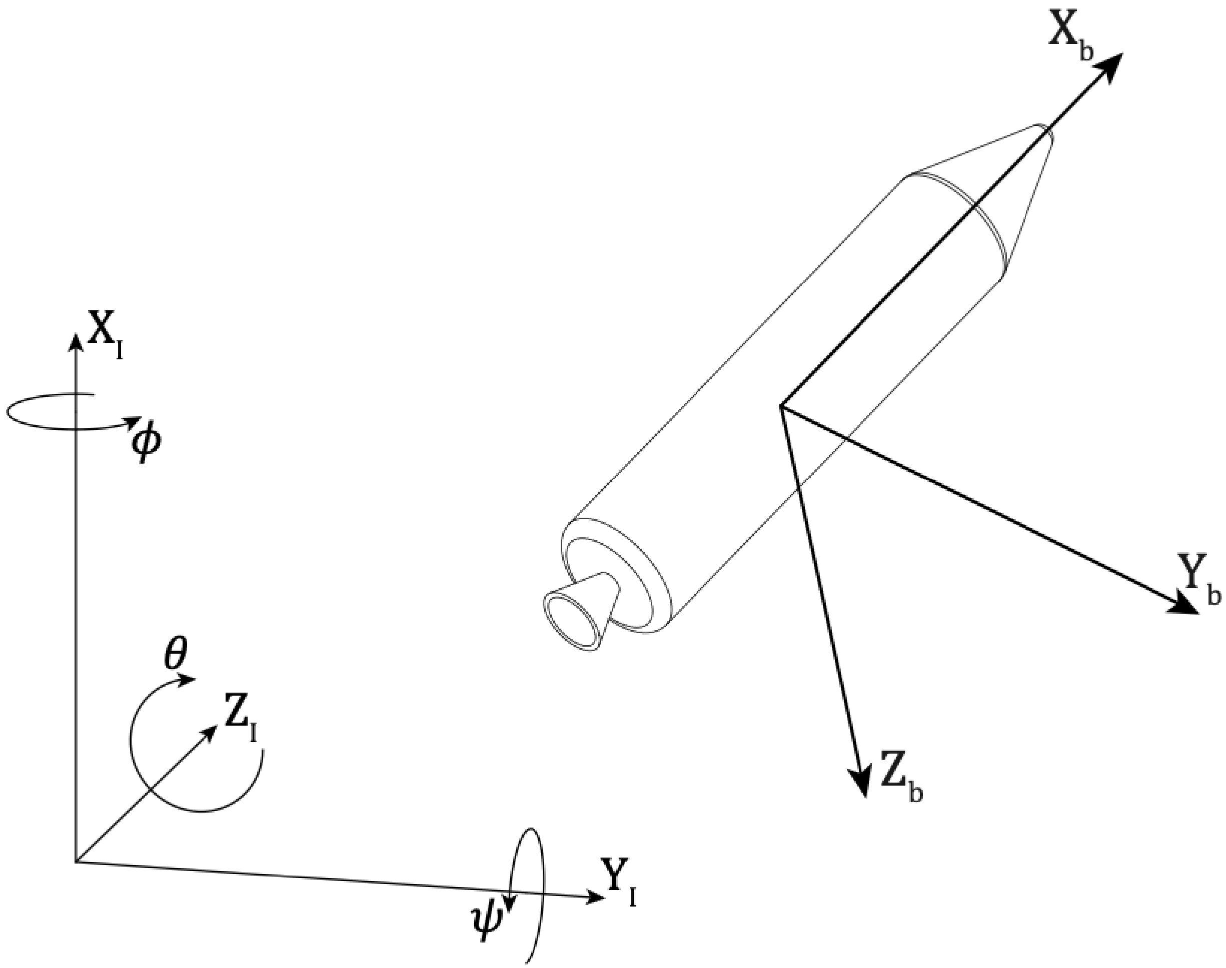
Model Predictive Control for Powered Descent Guidance and Control

In this project, we explore the real-time implementation of Model Predictive Control (MPC) for spacecraft guidance, with a focus on thrust vector control during ascent and descent.

The model

The vehicle in the study is modeled as a rigid body with six degrees of freedom (6-DoF). Forces and torques from aerodynamics, gravity, and the propulsion system act on the spacecraft's center of gravity (CoG), influencing its motion.
The study introduces inertial and body reference frames (depicted in Figure 1), where the former is fixed in space, and the latter is moving, linked to the rocket



Body Kinematics and Kinetics

It is possible to define the equations of motion (EoM) for the translational and rotational dynamics by means of Newton's second law:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{1}{m} \cdot C_I^b \cdot \begin{bmatrix} F_{bx} \\ F_{by} \\ F_{bz} \end{bmatrix} + \begin{bmatrix} g_{I_x} \\ g_{I_y} \\ g_{I_z} \end{bmatrix}$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yx} & I_{zz} \end{bmatrix}^{-1} \left(\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} - \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yx} & I_{zz} \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \right)$$

- **Linear Accelerations:**
  $\ddot{x}_I$, $\ddot{y}_I$, $\ddot{z}_I$ represent the body's linear accelerations along the X_I, Y_I, and Z_I axes of the inertial reference frame.
- **Forces:**
  F_{bx}, F_{by}, F_{bz} represent the forces applied to the body along the X_I, Y_I, and Z_I axes, respectively.
- **Gravity:**
  g_x, g_y, g_z represent the components of the gravitational force acting on the body along the X_I, Y_I, and Z_I axes, respectively.
- **Angular Accelerations:**
  $\ddot{\phi}$, $\ddot{\theta}$, $\ddot{\psi}$ represent the body's angular accelerations about the roll (ϕ), pitch (θ), and yaw (ψ) axes, respectively.
- **Moments:**
  M_x, M_y, M_z represent the moments acting on the body about the center of gravity (CG) along the X_I, Y_I, and Z_I axes, respectively.
- **Inertia Tensor:**
  I_{xx}, I_{yy}, I_{zz} represent the diagonal components of the inertia tensor, which describe the body's resistance to rotational acceleration around the CG.

Simplifications

We assume the following model simplifications:

- no planet rotation;
- flat planet surface;
- uniform gravity field;
- negligible aerodynamic effects;
- diagonal inertia matrix.

Control

And we consider four virtual control inputs to pilot the spacecraft:

- **U1** is the rolling moment;
- **U2** is the pitching moment;
- **U3** is the yawing moment;
- **U4** is the thrust force.

Now the control design can be written as:

$$\begin{cases} \ddot{x} = \dfrac{U_4}{m} \cdot \cos(\theta) \cdot \cos(\psi) - g \\ \ddot{y} = \dfrac{U_4}{m} \cdot \cos(\theta) \cdot \sin(\psi) \\ \ddot{z} = -\dfrac{U_4}{m} \cdot \sin(\theta) \\ \ddot{\phi} = \dfrac{U_1}{I_{xx}} \\ \ddot{\theta} = \dfrac{U_2}{I_{yy}} \\ \ddot{\psi} = \dfrac{U_3}{I_{zz}} \end{cases}$$

Creating the model

Now, in order to use the MPC method to this system we need some additional equation in order to treat this second order ODE as a one order expression.
We reformulate the second order ODEs by introducing the following states:

$$
\begin{aligned}
x_1 &= x \\
x_2 &= y \\
x_3 &= z \\
x_4 &= \phi \\
x_5 &= \theta \\
x_6 &= \psi \\
x_7 &= \dot{x} \\
x_8 &= \dot{y} \\
x_9 &= \dot{z} \\
x_{10} &= \dot{\phi} \\
x_{11} &= \dot{\theta} \\
x_{12} &= \dot{\psi}
\end{aligned}
$$

In this way the model can be rewritten as a first-order ODE:

$$
\begin{cases}
\dot{x}_1 = x_7 \\
\dot{x}_2 = x_8 \\
\dot{x}_3 = x_9 \\
\dot{x}_4 = x_{10} \\
\dot{x}_5 = x_{11} \\
\dot{x}_6 = x_{12} \\
\dot{x}_7 = \dfrac{U_4}{m} \cdot \cos(x_5) \cdot \cos(x_6) - g \\
\dot{x}_8 = \dfrac{U_4}{m} \cdot \cos(x_5) \cdot \sin(x_6) \\
\dot{x}_9 = -\dfrac{U_4}{m} \cdot \sin(x_5) \\
\dot{x}_{10} = \dfrac{U_1}{I_{xx}} \\
\dot{x}_{11} = \dfrac{U_2}{I_{yy}} \\
\dot{x}_{12} = \dfrac{U_3}{I_{zz}}
\end{cases}
$$