
# MATLAB and Its Application in Engineering

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# Linear Programming

1. Given a range of foods to choose from, what is the diet of lowest cost that meets an individual's nutritional requirements?

2. What is the most profitable schedule an airline can devise given a particular fleet of planes, a certain level of staffing, and expected demands on the various routes?

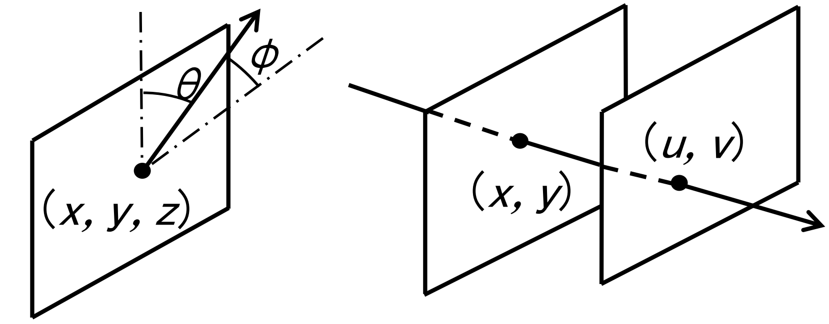
3. Where should a company locate its factories and warehouses so that the costs of transporting raw materials and finished products are minimized?

4. How should the equipment in an oil refinery be operated, so as to maximize rate of production while meeting given standards of quality?

Simple problems of this type can be formulated as optimization problems, in which the goal is to select values that maximize or minimize a given objective function, subject to certain constraints.

# 2.1、光场成像原理

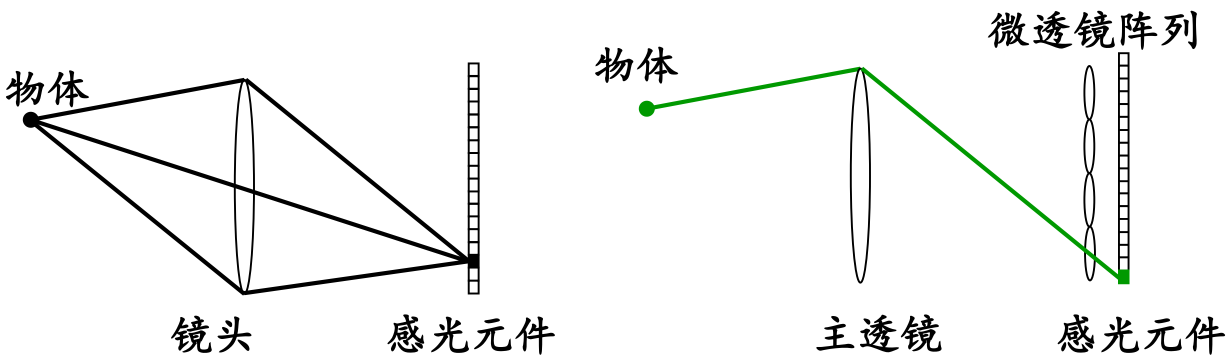
- 光场描述光线空间分布特性$L(x, y, z, \theta, \varphi)$
  - 位置坐标$(x, y, z)$
  - 传播角度$(\theta, \varphi)$




(a) 方向-点参数化　　(b) 两平面参数化

- 均匀介质→四维光场函数$L(x, y, u, v)$

- 光场成像与传统成像




(a) 传统相机成像　　(b) 光场相机成像

# 2.1、光场成像原理

- 光场描述光线空间分布特性$L(x,y,z,\theta,\varphi)$
  - 位置坐标$(x,y,z)$
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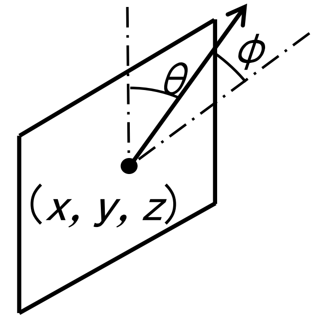
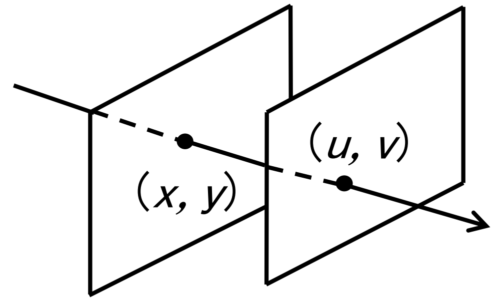


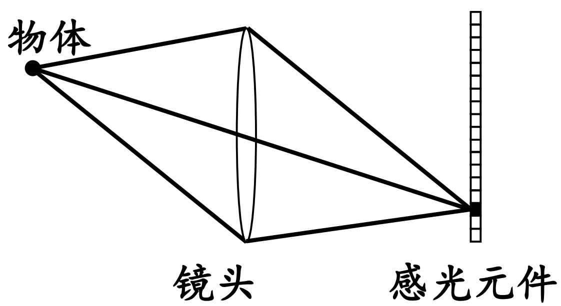

(a) 方向-点参数化



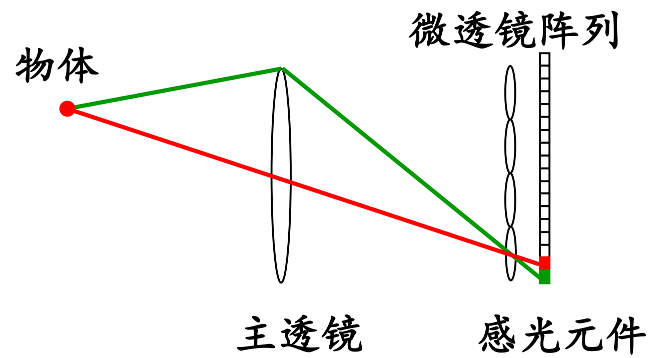

(b) 两平面参数化

- 均匀介质→四维光场函数$L(x,y,u,v)$

- 光场成像与传统成像




(a) 传统相机成像




(b) 光场相机成像

# 2.1、光场成像原理

- 光场描述光线空间分布特性$L(x, y, z, \theta, \varphi)$
  - 位置坐标$(x, y, z)$
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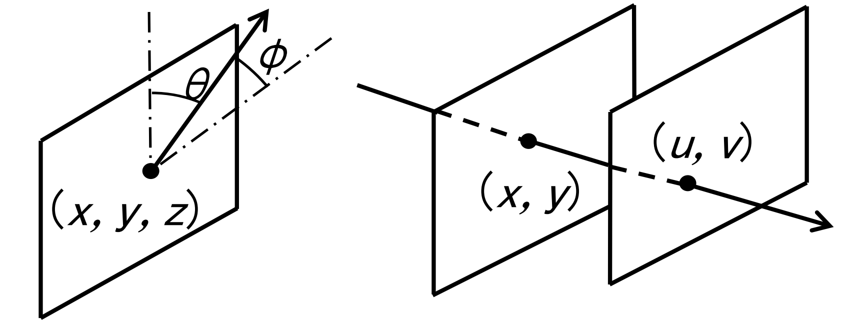
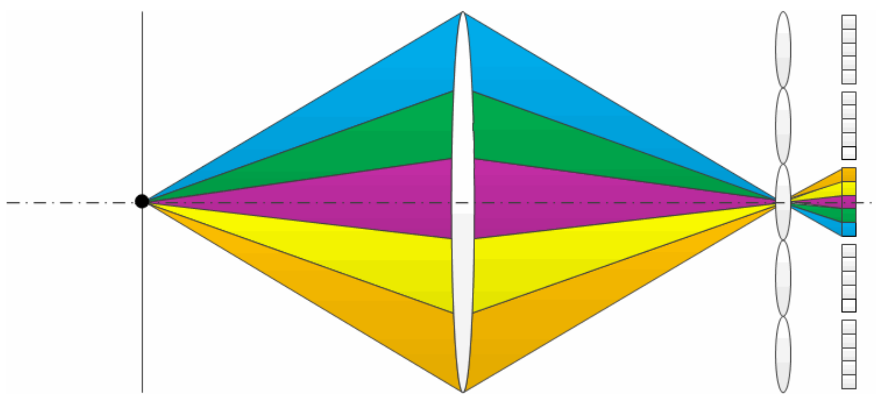


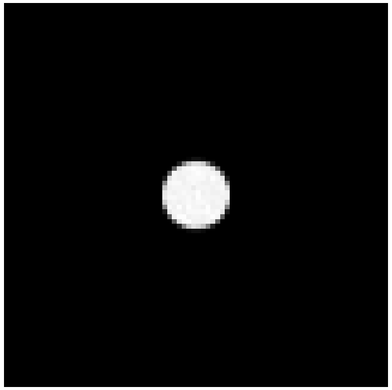

(a) 方向-点参数化　　(b) 两平面参数化

- 均匀介质→四维光场函数$L(x, y, u, v)$

- 光场成像与传统成像




光场图像



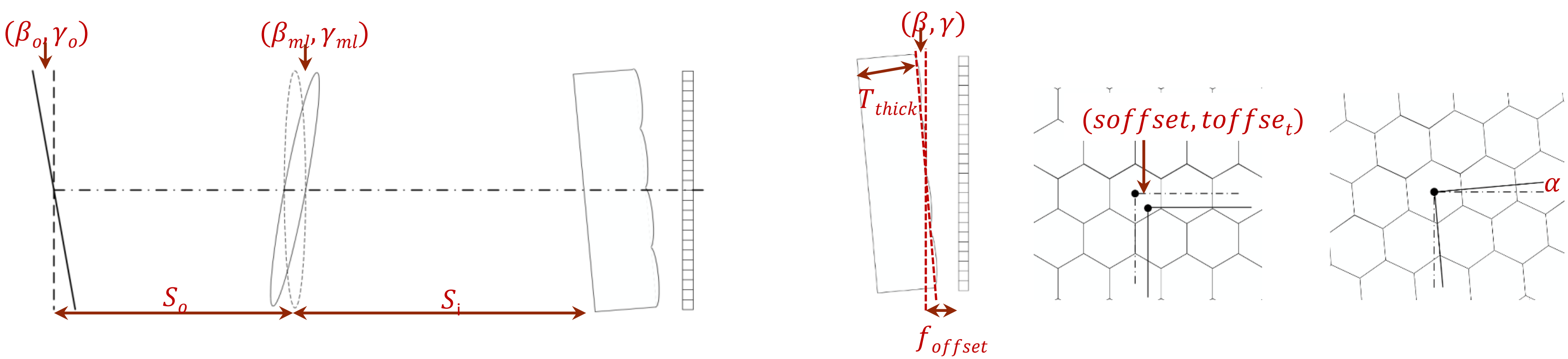
# Plenoptic camera calibration

Background:

- Assembly error and mechanical error in light field camera

Purpose of calibration:

- Get all parameters in raytracing process (15 parameters)

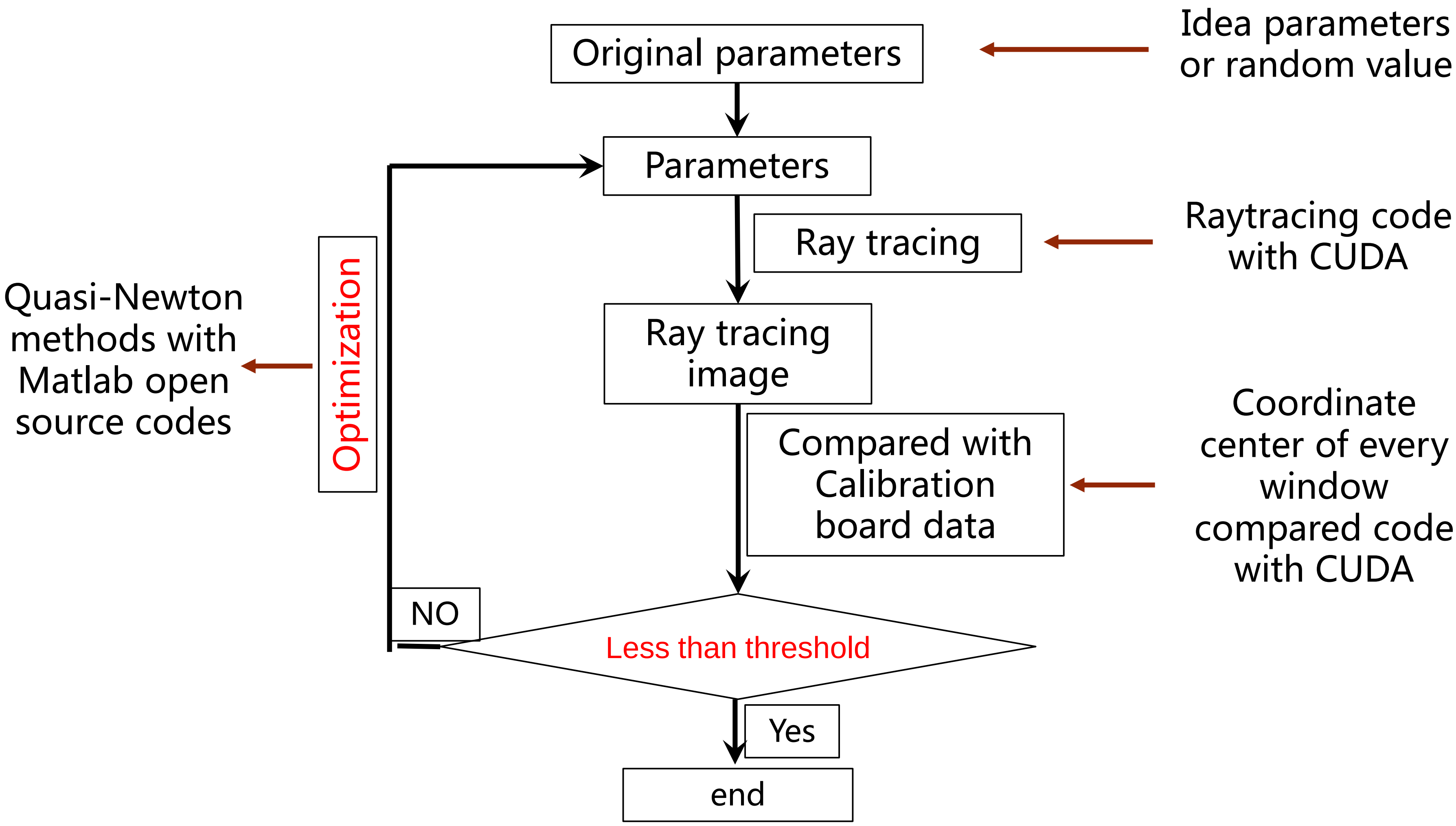



MLA: 3D rotate$(\alpha, \beta, \gamma)$
3D shift $(so_{ffset}, to_{ffset}, fo_{ffset})$
thickness $T_{thick}$

Objective distance $S_o$
Calibration board rotation$(\beta_o, \gamma_o)$
Main lens rotation$(\beta_{ml}, \gamma_{ml})$
Image distance $S_i$
Objective distance liquid and air $S_{hr}$ and $S_{lr}$

# Plenoptic camera calibration

- Optimization to get parameters which achieves minimum difference of raytracing result and captured calibration board image



# Plenoptic camera calibration

- Calibration parameters:

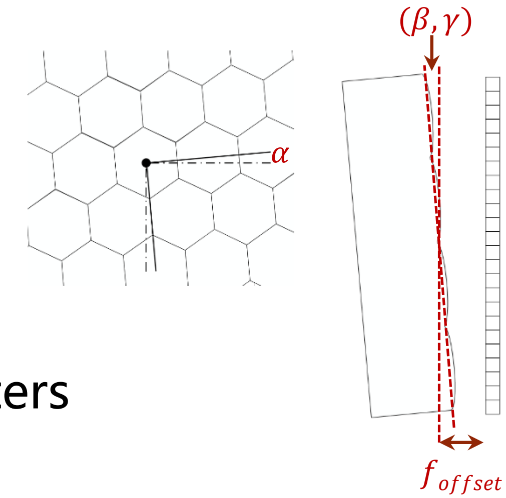
  3D rotate $(\alpha, \beta, \gamma)$ and z direction shift $(f_{offset})$

- Original parameters

  Ideal parameters :$(\alpha, \beta, \gamma, f_{offset}) = (0.308, 0, 0, 0)$

- Synthetic captured calibration board image parameters

  Real parameters : $(0.318, 0.001, 0.01, 0.02)$




- Optimization
  - Algorithm: Quasi-Newton method
  - Codes: optimization: matlab

    Raytracing: CUDA

    Different factor: CUDA

- Calibration result :

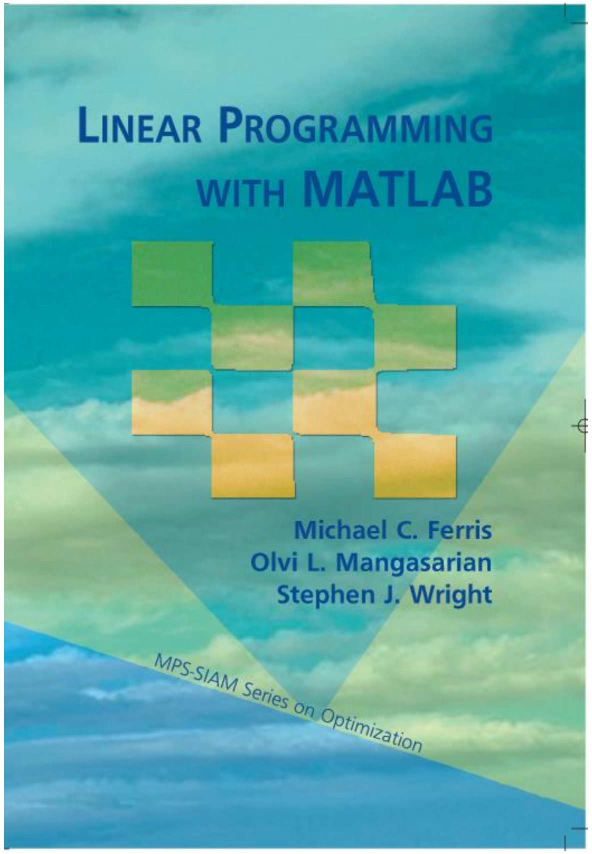
  $(\alpha, \beta, \gamma, foff_{set}) = (0.318312, 0.000983, 0.011669, 0.018553)$

  $$\sum((CO_{rtx\_i} - CO_{cax\_i})^2 + (CO_{rty\_i} - CO_{cay\_i})^2) = 0.4215 pixel^2$$

  Computational time : 916.9 s

# Linear Programming

- Mathematical Model
- Graphic Solution
- The Simplex Method



# An Example: Snape's Dairy

Snape runs a business that produces and sells dairy products from the milk of three cows. The cows produce 22 gallons of milk each week, which are turned into ice cream and butter.

The butter-making process requires 2 gallons of milk to produce one kilogram of butter
The butter-making process requires 3 gallons of milk to make one gallon of ice cream.

Snape owns a huge refrigerator that can store practically unlimited amounts of butter, but his freezer can hold at most 6 gallons of ice cream.

Snape can spend at most 6 hours per week on manufacturing butter and ice cream. One hour of work is needed to produce either 4 gallons of ice cream or one kilogram of butter.

Snape's products have a great reputation, and he always sells everything he produces. He sets the prices to ensure a profit of \$5 per gallon of ice cream and \$4 per kilogram of butter.

He would like to figure out how much ice cream and butter he should produce to maximize his profit.

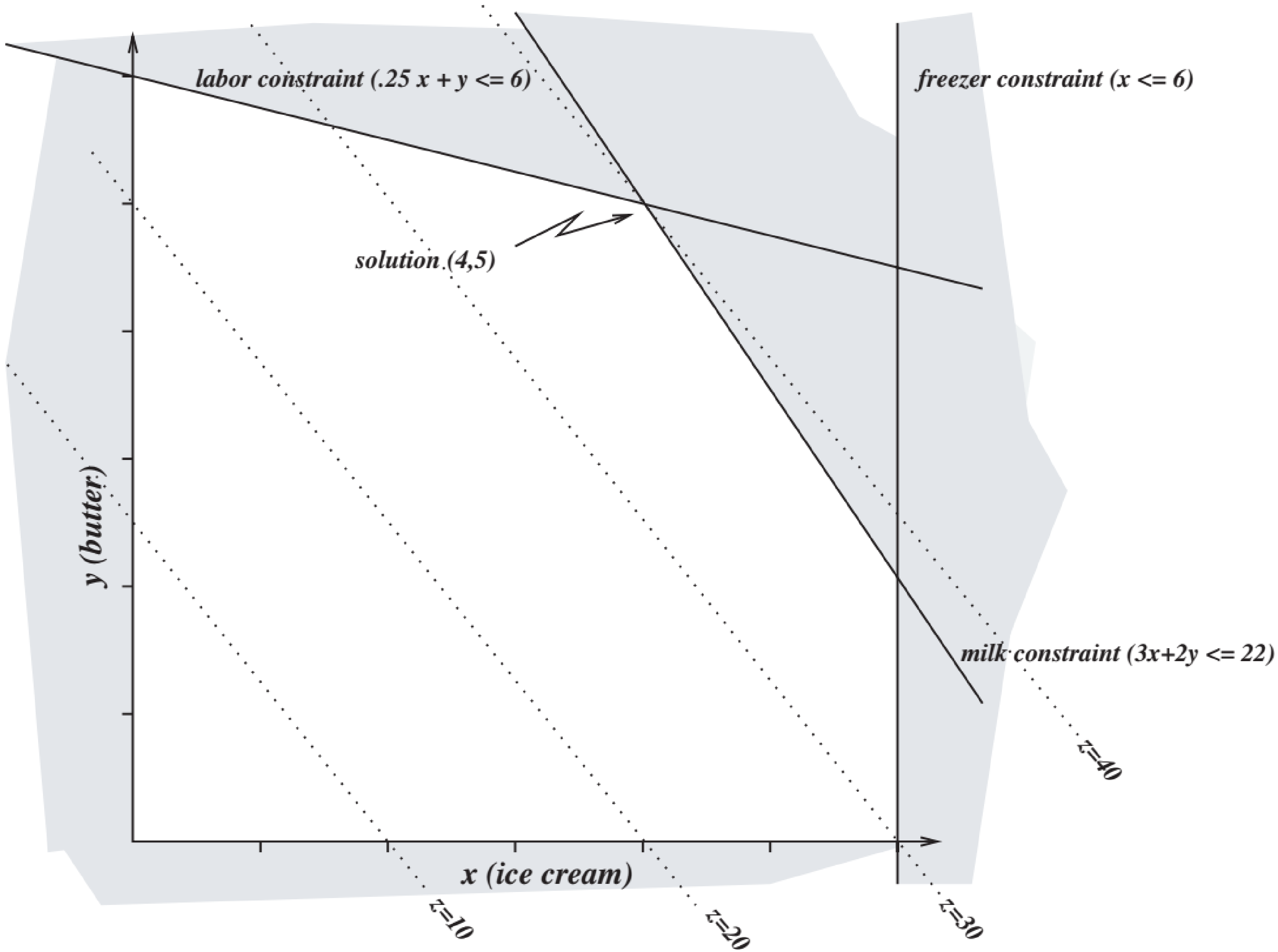
# An Example: Snape's Dairy

Identify the two variables: number of gallons of ice cream *x*
and the number of kilograms of butter *y*

Objective (profit) function: $z = 5x + 4y$ dollars.

$$\begin{array}{lrcrcr}
\max\limits_{x,y} & z = 5x + 4y & & & & \\
\text{subject to} & x & & & \leq & 6, \\
 & .25x & + & y & \leq & 6, \\
 & 3x & + & 2y & \leq & 22, \\
 & & & x, y & \geq & 0.
\end{array}$$

# An Example: Snape's Dairy



# Mathematical Model

Application: **Resource Allocation**

Soft drink company “Coca-Pepsa” (哈哈娃) owns four production lines (A, B, C, D), which can produce many kinds of drinks. A recent technology upgrade releases extra work-hours of 12, 8, 16 and 12 for line A, B, C and D respectively. The company plans to use these work-hours to produce new products “DoubleCool” (多加宝) and “LuckyKing”(老王吉). Profit and time cost of these produces are

| | Time cost（hr） | | | | Profit（Yuan） |
|---|---|---|---|---|---|
| | A | B | C | D | |
| DoubleCool | 2.5 | 1 | 2 | 0 | 20 |
| LuckyKing | 2 | 2 | 0 | 4 | 30 |

# Mathematical Model

Question: Make a production plan to maximise the profit

Assume the production of “DoubleCool” is x1, and “LuckyKing” is x2

Profit equation $z = 20x_1 + 30x_2$

Constrains

$$\begin{cases} 2.5x_1 + 2x_2 \leq 12 \\ x_1 + 2x_2 \leq 8 \\ 4x_1 \leq 16 \\ 4x_2 \leq 12 \\ x_1, x_2 \geq 0 \end{cases}$$

# Mathematical Model

Application: Production plan

Metallurgic plant “铁生馆” plans to produce lead(铅) no less than 30 tons, cooper no less than 35 tons and iron 45 tons. There are four types of minerals available:

| Mineral | lead（%） | cooper（%） | iron（%） | cost（yuan/ton） |
|---|---|---|---|---|
| A | 2 | 4 | 4 | 10 |
| B | 3 | 2 | 2 | 15 |
| C | 1 | 3 | 3 | 30 |
| D | 0.5 | 1 | 5 | 25 |

# Mathematical Model

Question: Decide the purchase plan for the minerals to minimise the cost

Assume to purchase x1, x2, x3 and x4 tons of minerals A, B, C and D

Total cost $z = 10x_1 + 15x_2 + 30x_3 + 25x_4$

constrains

$$\begin{cases} 0.02x_1 + 0.03x_2 + 0.01x_3 + 0.005x_4 \geq 30 \\ 0.04x_1 + 0.02x_2 + 0.03x_3 + 0.01x_4 \geq 35 \\ 0.04x_1 + 0.02x_2 + 0.03x_3 + 0.05x_4 = 45 \\ \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

# Mathematical Model

Maximise/Minimise $z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

Subject to
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le b_m \\ x_1, x_2, \ldots, x_n \ge 0 \end{cases}$$

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

$$b = (b_1, b_2, \ldots, b_m)^T$$

$$c = (c_1, c_2, \ldots, c_n)$$

Max Z =cx

s.t. Ax≤b

# Mathematical Model

Question: Make a production plan to maximise the profit

Assume the production of "DoubleCool" is x1, and "LuckyKing" is x2

Profit equation $z = 20x_1 + 30x_2$

Constrains

$$\begin{cases} 2.5x_1 + 2x_2 \leq 12 \\ x_1 + 2x_2 \leq 8 \\ 4x_1 \leq 16 \\ 4x_2 \leq 12 \\ x_1, x_2 \geq 0 \end{cases}$$

# Graphic Solution

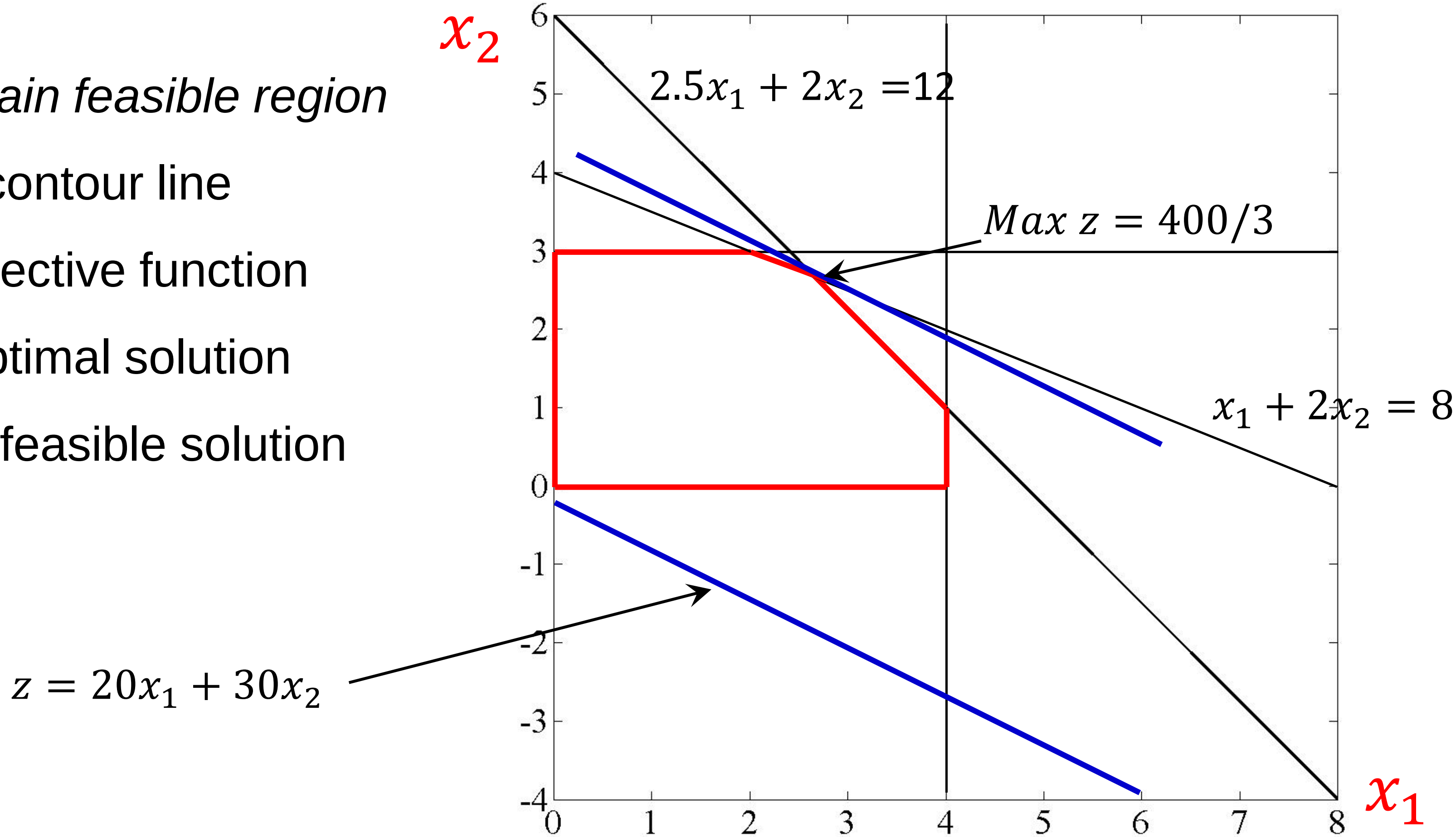
*Feasible Solutions*: The set of points satisfying all of the constraints

*Optimal Solution*: Feasible Solutions that maximise/minimise the objective function

**1**: *ascertain feasible region*

**2**: draw contour line of objective function

**3**: find optimal solution in the feasible solution



# Graphic Solution

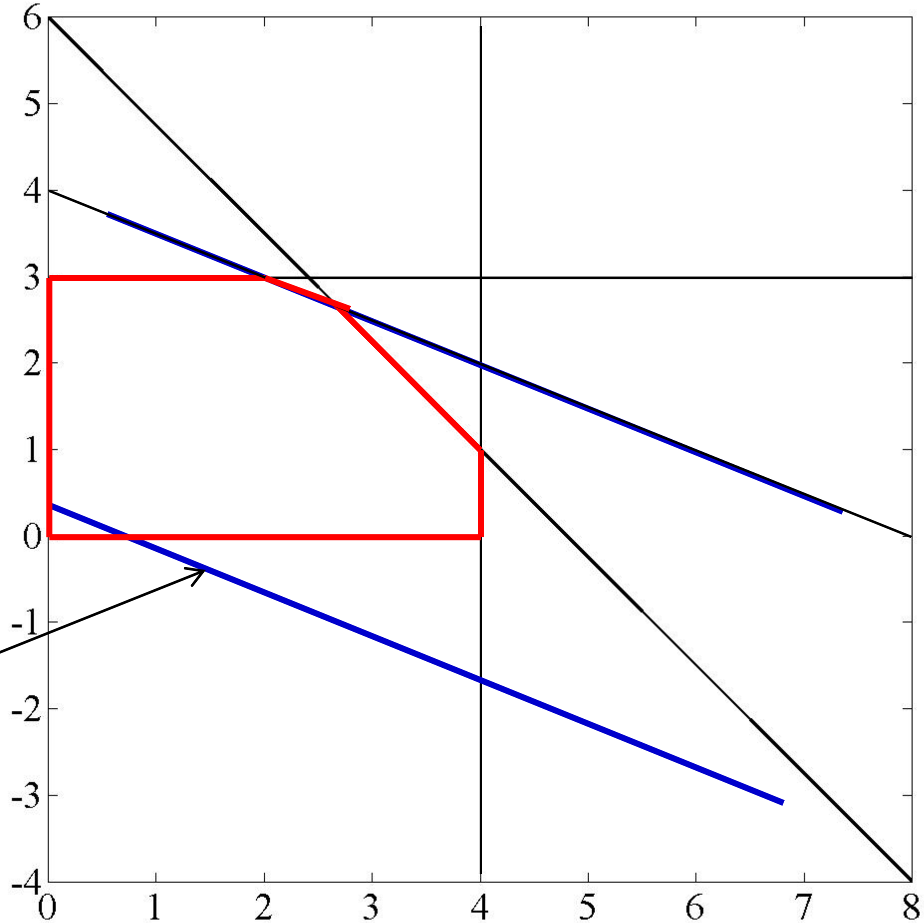
*Feasible Solutions*： The set of points satisfying all of the constraints

*Optimal Solution*：Feasible Solutions that maximise/minimise the objective function

Infinite solutions

$$z = 20x_1 + 40x_2$$

$$z = 20x_1 + 40x_2$$



# Graphic Solution

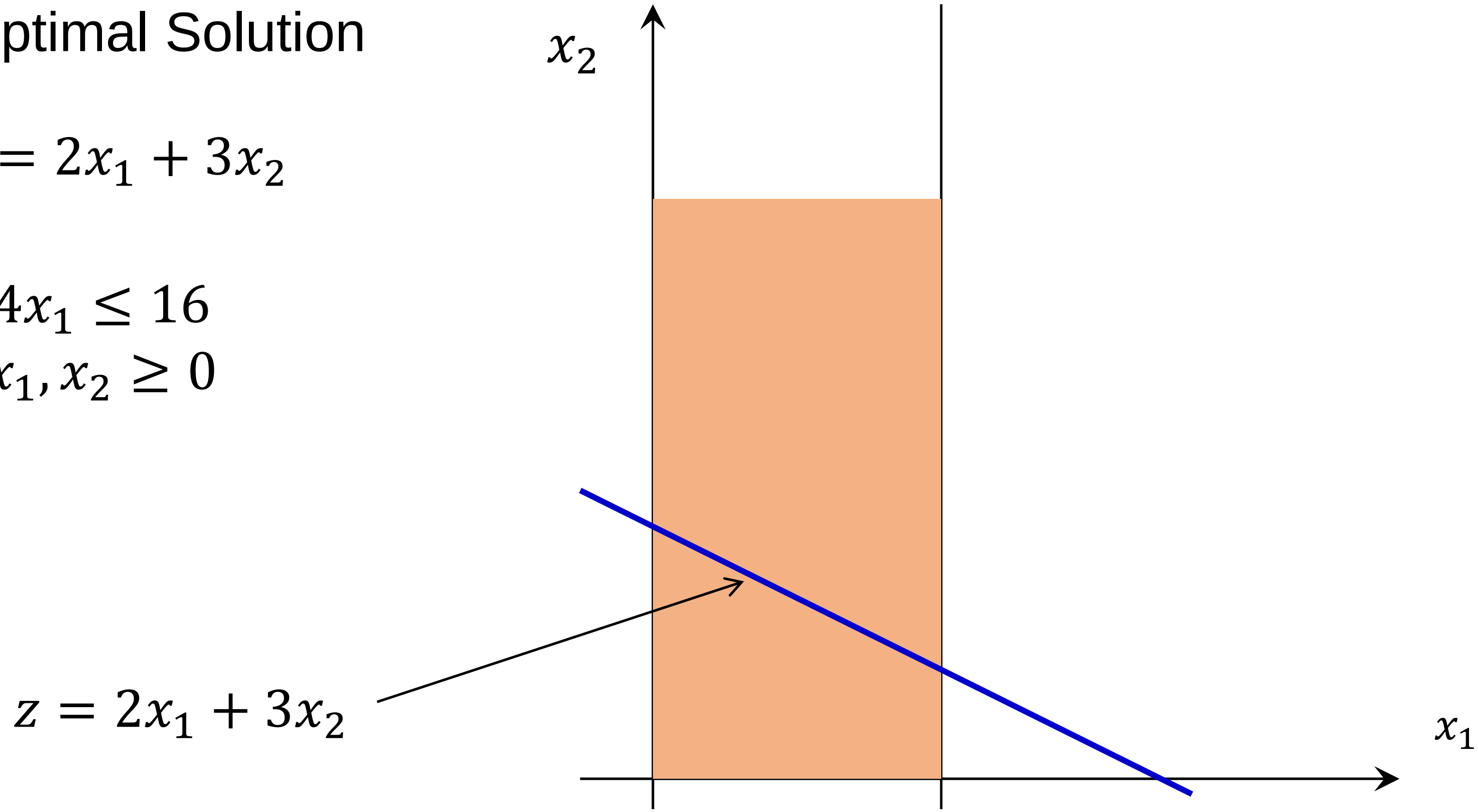
*Feasible Solutions*: The set of points satisfying all of the constraints

*Optimal Solution*: Feasible Solutions that maximise/minimise the objective function

No Optimal Solution

$$z = 2x_1 + 3x_2$$

$$\begin{cases} 4x_1 \le 16 \\ x_1, x_2 \ge 0 \end{cases}$$



# Graphic Solution

*Feasible Solutions*： The set of points satisfying all of the constraints

*Optimal Solution*：Feasible Solutions that maximise/minimise

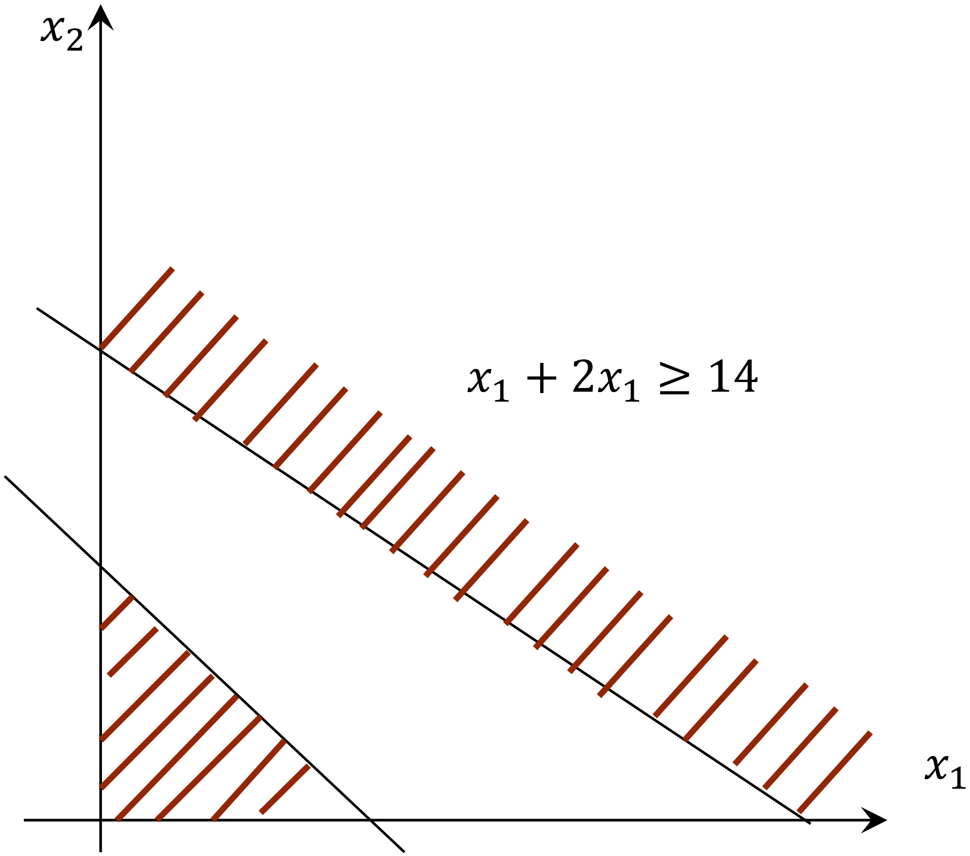
the objective function

No Feasible Solutions

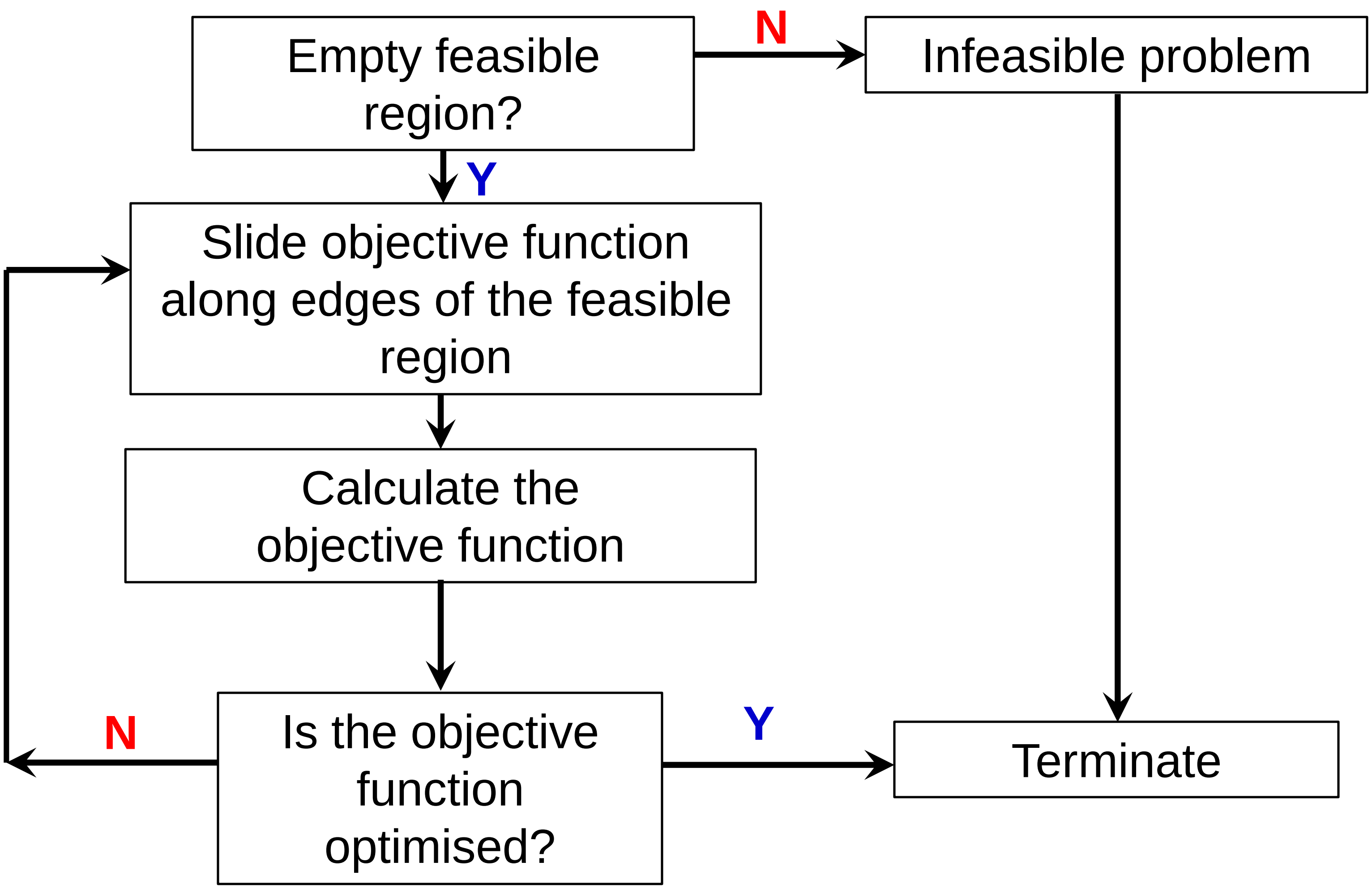
$$z = 2x_1 + 3x_2$$

$$\begin{cases} 2x_1 + 2x_2 \leq 12 \\ x_1 + 2x_1 \geq 14 \\ x_1, x_2 \geq 0 \end{cases}$$

$$2x_1 + 2x_2 \leq 12$$



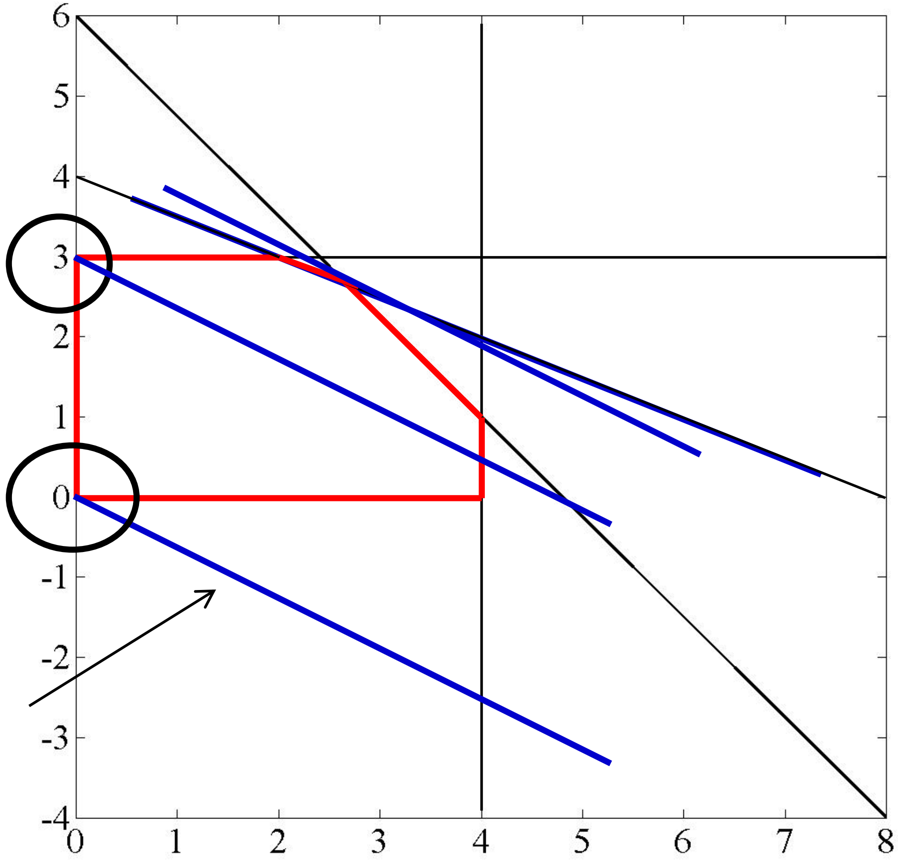
# The Simplex Method-basic concept



# The Simplex Method - principle

1. Finds a vertex (最高点) of the feasible region to use as a starting point
2. Check whether it is optimal solution or not, if not
3. Moves from this vertex to an adjacent vertex for which the value of the objective function is higher; if not only one adjacent vertex make the objective function be higher, chose the vertex with the greatest growth rate
4. Return back step 2, and iterate.

$$z = 20x_1 + 30x_2$$



# The Simplex Method-basic concept

*Original problem*

Max: $z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

s.t.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ \ldots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + \ \ldots + a_{2n}x_n \le b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ \ldots + a_{mn}x_n \le b_m \\ x_1, x_2, \ldots, x_n \ge 0 \end{cases}$$

*Include supplementary variables*

$$\text{s.t.} \begin{cases} a_{11}x_1 + a_{12}x_2 + \ \ldots + a_{1n}x_n + \boxed{x_{n+1}} = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ \ldots + a_{2n}x_n + \boxed{x_{n+2}} = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ \ldots + a_{mn}x_n + \boxed{x_{n+m}} = b_m \\ x_1, x_2, \ldots, x_n, x_{n+1} \ldots, x_{n+m} \ge 0 \end{cases}$$

# The Simplex Method-basic concept

*Original problem*

Max $Z = cx$

s.t. $Ax \le b$ $\qquad$ $\mathbf{x} \ge 0$

*Include supplementary variables* $X_s$

Max $Z = cx$

s.t. $(A\ I)\begin{pmatrix} X \\ X_s \end{pmatrix} = b$ $\qquad$ $\begin{pmatrix} X \\ X_s \end{pmatrix} \ge 0$

*Let* $X = 0$, *find the first feasible solution from* $X_s = b$

*Substitute the* $1^{st}$ *feasible solution to objection to receive the initial guess of Z*

# The Simplex Method - principle

Resource Allocation for “DoubleCool” (多加宝) , “LuckyKing”(老王吉).

$$Max\ z = 20x_1 + 30x_2$$

Include supplementary variables

Constraints

$$\begin{cases} 2.5x_1 + 2x_2 \le 12 \\ x_1 + 2x_2 \le 8 \\ 4x_1 \le 16 \\ 4x_2 \le 12 \\ x_1, x_2 \ge 0 \end{cases} \qquad \begin{cases} 2.5x_1 + 2x_2 + x_3 = 12 \\ x_1 + 2x_2 + x_4 = 8 \\ 4x_1 + x_5 = 16 \\ 4x_2 + x_6 = 12 \\ x_1, x_2, \dots, x_6 \ge 0 \end{cases}$$

$$c = (20\ 30) \qquad A = \begin{bmatrix} 2.5 & 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad b = \begin{pmatrix} 12 \\ 8 \\ 16 \\ 12 \end{pmatrix}$$

# The Simplex Method - principle

Resource Allocation for “DoubleCool” (多加宝) , “LuckyKing”(老王吉).

Supplementary variables: $x_3, x_4, x_5, x_6$

Basic variables: $x_1, x_2$

Assume all basic variables to 0 to find the 1st feasible solutions

$X_1 =$（0, 0, 12, 8, 16, 12）

$Z_1 = 0$

Inspect $Max\, z = 20x_1 + 30x_2$

Let $x_2$ be positive, Z will have the greatest growth rate

# The Simplex Method - principle

Substitute $x_1 = 0$ into constraint equations

$$\begin{cases} x_3 = 12 - 2x_2 \geq 0 \\ x_4 = 8 - 2x_2 \geq 0 \\ x_5 = 16 \geq 0 \\ x_6 = 12 - 4x_2 \geq 0 \end{cases} \Rightarrow \begin{cases} x_2 \leq 6 \\ x_2 \leq 4 \\ \\ x_2 \leq 3 \end{cases}$$

To satisfy all constrains

New supplementary variables: $x_3, x_4, x_5, x_2$

New basic variables: $x_1, x_6$

# The Simplex Method - principle

1st iteration

2nd method， treat the variables on the right side of the equation as numerator, Compare with the positive coefficient of $x_2$.

$$\begin{cases} 2.5x_1 + 2x_2 + x_3 = 12 \\ x_1 + 2x_2 + x_4 = 8 \\ 4x_1 + x_5 = 16 \\ 4x_2 + x_6 = 12 \end{cases} \qquad \begin{array}{c} 12/2 \\ 8/2 \\ - \\ 12/4 \end{array}$$

the smallest ratio corresponding to the 4th equition

Hence, choose $x_6$ as new basic variables

New supplementary variables: $x_3, x_4, x_5, x_2$

New basic variables: $x_1, x_6$

# The Simplex Method - principle

Use elimination method to adjust the coefficients of $x_2$

$(A\ I)\begin{pmatrix} X \\ X_s \end{pmatrix}$=b

$$\begin{cases} 2.5x_1 + 2x_2 + x_3 = 12 \\ x_1 + 2x_2 + x_4 = 8 \\ 4x_1 + x_5 = 16 \\ 4x_2 + x_6 = 12 \\ z - 20x_1 - 30x_2 = 0 \end{cases} \Longrightarrow \begin{cases} 2.5x_1 + \boxed{x_3} - 0.5x_6 = 6 \\ x_1 + \boxed{x_4} - 0.5x_6 = 2 \\ 4x_1 + \boxed{x_5} = 16 \\ \boxed{x_2} + 0.25x_6 = 3 \\ z - 20x_1 + 7.5x_6 = 90 \end{cases}$$

Let basic variables to 0 $x_1 = 0, x_6 = 0$

New feasible solutions

$$x = (0, \quad 3, \quad 6, \quad 2, \quad 16, \quad 0)$$

# The Simplex Method - principle

Using matrix manipulation in Matlab to achieve elimination

$$\begin{cases} 2.5x_1 + 2x_2 + x_3 = 12 \\ x_1 + 2x_2 + x_4 = 8 \\ 4x_1 + x_5 = 16 \\ 4x_2 + x_6 = 12 \\ z - 20x_1 - 30x_2 = 0 \end{cases}
\begin{array}{l} \longrightarrow a_1 = [2.5, 2, 1, 0, 0, 0, 12] \\ \longrightarrow a_2 = [1, 2, 0, 1, 0, 0, 8] \\ \longrightarrow a_3 = [4, 0, 0, 0, 1, 0, 16] \\ \longrightarrow a_4 = [0, 4, 0, 0, 0, 1, 12] \\ \longrightarrow a_5 = [-20, -30, 0, 0, 0, 0, 0] \end{array}$$

$$a_1 = a_4 * (-2) + a_1 \qquad a_1 = [2.5, 0, 1, 0, 0, -0.5, 6]$$

$$a_2 = a_4 * (-2) + a_2 \qquad a_2 = [1, 0, 0, 1, 0, -0.5, 2]$$

$$a_4 = a_4 * (1/4) \qquad a_4 = [0, 1, 0, 0, 0, 0.25, 3]$$

$$a_5 = a_4 * (30) + a_5 \qquad a_5 = [-20, 0, 0, 0, 0, 7.5, 90]$$

# The Simplex Method - principle

**2nd iteration**

Inspect $z - 20x_1 + 7.5x_6 = 90$

Let $x_1$ be positive, Z will continue to increase

$$\begin{cases} 2.5x_1 + x_3 - 0.5x_6 = 6 \\ x_1 + x_4 - 0.5x_6 = 2 \\ 4x_1 + x_5 = 16 \\ 4x_2 + 0.25x_6 = 3 \\ z - 20x_1 + 7.5x_6 = 90 \end{cases} \longleftarrow \begin{matrix} 6/2.5 \\ 2/1 \\ 16/4 \\ - \end{matrix}$$

the smallest ratio corresponding to the 2nd equition

Hence, choose $x_4$ as new basic variables

New supplementary variables: $x_3, x_1, x_5, x_2$

New basic variables: $x_4, x_6$

# The Simplex Method - principle

Use elimination method to adjust the coefficients of $x_1$

$$(A\ I)\begin{pmatrix} X \\ X_s \end{pmatrix} = b$$

$$\begin{cases} 2.5x_1 + x_3 - 0.5x_6 = 6 \\ x_1 + x_4 - 0.5x_6 = 2 \\ 4x_1 + x_5 = 16 \\ 4x_2 + 0.25x_6 = 3 \\ z - 20x_1 + 7.5x_6 = 90 \end{cases} \Rightarrow \begin{cases} \boxed{x_3} - 2.5x_4 + 0.75x_6 = 1 \\ \boxed{x_1} + x_4 - 0.5x_6 = 2 \\ -4x_4 + \boxed{x_5} + 2x_6 = 8 \\ \boxed{x_2} + 0.25x_6 = 3 \\ z + 20x_4 - 2.5x_6 = 130 \end{cases}$$

Let basic variables to 0 $x_4 = 0, x_6 = 0$

New feasible solutions $x = (2,\quad 3,\quad 1,\quad 0,\quad 8,\quad 0)$

# The Simplex Method - principle

3rd iteration

Inspect $z + 20x_4 - 2.5x_6 = 130$

Let $x_6$ be positive, Z will continue to increase

$$\begin{cases} x_3 - 2.5x_4 + 0.75x_6 = 1 \\ x_1 + x_4 - 0.5x_6 = 2 \\ -4x_4 + x_5 + 2x_6 = 8 \\ x_2 + 0.25x_6 = 3 \\ z + 20x_4 - 2.5x_6 = 130 \end{cases} \longleftarrow \quad \begin{matrix} 1/0.75 \\ - \\ 8/2 \\ 3/0.25 \end{matrix}$$

the smallest ratio corresponding to the $1st$ equition

Hence, choose $x_3$ as new basic variables

New supplementary variables: $x_6, x_1, x_5, x_2$

New basic variables: $x_4, x_3$

# The Simplex Method - principle

Use elimination method to adjust the coefficients of $x_6$

$(A\ I)\begin{pmatrix} X \\ X_s \end{pmatrix}=b$

$$\begin{cases} x_3 - 2.5x_4 + 0.75x_6 = 1 \\ x_1 + x_4 - 0.5x_6 = 2 \\ -4x_4 + x_5 + 2x_6 = 8 \\ x_2 + 0.25x_6 = 3 \\ z + 20x_4 - 2.5x_6 = 130 \end{cases} \Rightarrow \begin{cases} 1.333x_3 - 3.333x_4 + \boxed{x_6} = 1.3333 \\ \boxed{x_1} + 0.6667x_3 + 0.6667x_4 = 2.6667 \\ -2.6667x_3 + 2.6667x_4 + \boxed{x_5} = 5.6667 \\ \boxed{x_2} - 0.3333x_3 + 0.8333x_4 = 2.6667 \\ z + 3.3333x_3 + 11.6667x_4 = 133.3333 \end{cases}$$

Let basic variables to 0 $\quad x_3 = 0, x_4 = 0$

New feasible solutions

$x$ = （2.6667, 2.6667, 0, 0, 5.6667, 1.3333）

# The Simplex Method - principle

**1st iteration**

$$\begin{cases} 2.5x_1 + 2x_2 + x_3 = 12 \\ x_1 + 2x_2 + x_4 = 8 \\ 4x_1 + x_5 = 16 \\ 4x_2 + x_6 = 12 \\ z - 20x_1 - 30x_2 = 0 \end{cases}$$

New supplementary variables: $x_3, x_4, x_5, x_2$

New basic variables: $x_1, x_6$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| $x_3$ | 12 | 2.5 | 2 | 1 | 0 | 0 | 0 | 12/2.5=4.8 |
| $x_4$ | 8 | 1 | 2 | 0 | 1 | 0 | 0 | 8/2=4 |
| $x_5$ | 16 | 4 | 0 | 0 | 0 | 1 | 0 | — |
| $x_6$ | 12 | 0 | $\boxed{[4]}$ | 0 | 0 | 0 | 1 | $\boxed{12/4=3}$ |
| Z | 0 | -20 | -30 | 0 | 0 | 0 | 0 | $\min\theta = 3$ |

# The Simplex Method - principle

**2nd iteration**

$$\begin{cases} 2.5x_1 + x_3 - 0.5x_6 = 6 \\ x_1 + x_4 - 0.5x_6 = 2 \\ 4x_1 + x_5 = 16 \\ x_2 + 0.25x_6 = 3 \\ z - 20x_1 + 7.5x_6 = 90 \end{cases}$$

New supplementary variables: $x_3, x_1, x_5, x_2$

New basic variables: $x_4, x_6$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| $x_3$ | 6 | 2.5 | 0 | 1 | 0 | 0 | -0.5 | 6/2.5=2.4 |
| $x_4$ | 2 | $\boxed{[1]}$ | 0 | 0 | 1 | 0 | -0.5 | $\boxed{2/1=2}$ |
| $x_5$ | 16 | 4 | 0 | 0 | 0 | 1 | 0 | 16/4=4 |
| $x_2$ | 3 | 0 | 1 | 0 | 0 | 0 | 0.25 | — |
| Z | 90 | $\boxed{-20}$ | 0 | 0 | 0 | 0 | 7.5 | $\min\theta = 2$ |

# The Simplex Method - principle

**3rd iteration**

$$\begin{cases} x_3 - 2.5x_4 + 0.75x_6 = 1 \\ x_1 + x_4 - 0.5x_6 = 2 \\ -4x_4 + x_5 + 2x_6 = 8 \\ x_2 + 0.25x_6 = 3 \\ z + 20x_4 - 2.5x_6 = 130 \end{cases}$$

New supplementary variables: $x_6, x_1, x_5, x_2$

New basic variables: $x_4, x_3$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| $x_3$ | 1 | 0 | 0 | 1 | -2.5 | 0 | $\boxed{[0.75]}$ | $\boxed{1/0.75=4/3}$ |
| $x_1$ | 2 | 1 | 0 | 0 | 1 | 0 | -0.5 | — |
| $x_5$ | 8 | 0 | 0 | 0 | -4 | 1 | 2 | 8/2=4 |
| $x_2$ | 3 | 0 | 1 | 0 | 0 | 0 | 0.25 | 3/0.25=12 |
| Z | 130 | 0 | 0 | 0 | 2 | 0 | $\boxed{-2.5}$ | $\min\theta = 4/3$ |

# The Simplex Method - principle

**4th iteration**

$$\begin{cases} 1.333x_3 - 3.333x_4 + x_6 = 1.3333 \\ x_1 + 0.6667x_3 + 0.6667x_4 = 2.6667 \\ -2.6667x_3 + 2.6667x_4 + x_5 = 5.6667 \\ x_2 - 0.3333x_3 + 0.8333x_4 = 2.6667 \\ z + 3.3333x_3 + 11.6667x_4 = 133.3333 \end{cases}$$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| $x_6$ | 1.3333 | 0 | 0 | 1.3333 | -3.3333 | 0 | 1 | |
| $x_1$ | 2.6667 | 1 | 0 | 0.6667 | 0.6667 | 0 | 0 | |
| $x_5$ | 5.6667 | 0 | 0 | -2.6667 | 2.6667 | 1 | 0 | |
| $x_2$ | 2.6667 | 0 | 1 | -0.3333 | 0.8333 | 0 | 0 | |
| Z | 133.3333 | 0 | 0 | 3.3333 | 11.6667 | 0 | 0 | |

# Other non-standard forms of mathematical model -big M method

$$\max Z = -3x_1 + x_3$$

$$s.t.\begin{cases} x_1 + x_2 + x_3 \le 4 \\ -2x_1 + x_2 - x_3 \le 1 \\ 3x_2 + x_3 = 9 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

Supplementary variables $x_4, x_5, x_6$

$$\max Z = -3x_1 + x_3 + 0x_4 + 0x_5 - Mx_6$$

$$s.t.\begin{cases} x_1 + x_2 + x_3 + x_4 = 4 \\ -2x_1 + x_2 - x_3 + x_5 = 1 \\ 3x_2 + x_3 + x_6 = 9 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

## Other non-standard forms of mathematical model -big M method

$$\max Z = -3x_1 + x_3$$

$$s.t.\begin{cases} x_1 + x_2 + x_3 \le 4 \\ -2x_1 + x_2 - x_3 \ge 1 \\ 3x_2 + x_3 = 9 \\ x_1, x_2, x_3 \ge 0 \end{cases} \Rightarrow s.t.\begin{cases} x_1 + x_2 + x_3 + x_4 = 4 \\ -2x_1 + x_2 - x_3 - x_5 = 1 \\ 3x_2 + x_3 = 9 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

$$\max Z = -3x_1 + x_3 + 0x_4 + 0x_5 - Mx_6 - Mx_7$$

$$s.t.\begin{cases} x_1 + x_2 + x_3 + x_4 = 4 \\ -2x_1 + x_2 - x_3 - x_5 + x_6 = 1 \\ 3x_2 + x_3 + x_7 = 9 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

$$B=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

# Other non-standard forms of mathematical model -big M method

$$s.t.\begin{cases} x_1+x_2+x_3+x_4=4 \\ -2x_1+x_2-x_3-x_5+x_6=1 \\ 3x_2+x_3+x_7=9 \\ x_1,x_2,x_3\geq 0 \end{cases}$$

Supplementary variables: $x_4, x_6, x_7$

Basic variables $x_1, x_2, x_3, x_5$

$$\max Z=-3x_1+x_3+0x_4+0x_5-\boxed{Mx_6-Mx_7}$$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|---|
| $x_4$ | 4 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | |
| $x_6$ | 1 | -2 | 1 | -1 | 0 | -1 | 1 | 0 | |
| $x_7$ | 9 | 0 | 3 | 1 | 0 | 0 | 0 | 1 | |
| Z | 0 | 3 | 0 | -1 | 0 | 0 | M | M | |

# Other non-standard forms of mathematical model -big M method

$$s.t.\begin{cases} x_1 + x_2 + x_3 + x_4 = 4 \\ -2x_1 + x_2 - x_3 - x_5 + x_6 = 1 \\ 3x_2 + x_3 + x_7 = 9 \\ z + 3x_1 - x_3 + 0x_4 + 0x_5 + Mx_6 + Mx_7 = 0 \end{cases}$$

New supplementary variables:

$x_4, x_2, x_7$

New basic variables:

$x_1, x_6, x_3, x_5$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|---|
| $x_4$ | 4 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 4/1 |
| $x_6$ | 1 | -2 | 1 | -1 | 0 | -1 | 1 | 0 | 1/1 |
| $x_7$ | 9 | 0 | 3 | 1 | 0 | 0 | 0 | 1 | 9/3 |
| Z | -10M | 3+2M | -4M | -1 | 0 | M | 0 | 0 | |

# Other non-standard forms of mathematical model -big M method

**1st iteration**

New supplementary variables: $x_4, x_2, x_1$

New basic variables: $x_7, x_6, x_3, x_5$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|---|
| $x_4$ | 3 | 3 | 0 | 2 | $\boxed{1}$ | 1 | -1 | 0 | 3/3 |
| $x_2$ | 1 | -2 | $\boxed{1}$ | -1 | 0 | -1 | 1 | 0 | — |
| $x_7$ | 6 | [6] | 0 | 4 | 0 | 3 | -3 | $\boxed{1}$ | 9/6 |
| Z | -6M | 3-6M | 0 | -4M-1 | 0 | -3M | 4M | 0 | |

# Other non-standard forms of mathematical model -big M method

**2nd iteration**

New supplementary variables: $x_4, x_2, x_3$

New basic variables: $x_7, x_6, x_1, x_5$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|---|
| $x_4$ | 0 | 0 | 0 | 0 | $\boxed{1}$ | -0.5 | 0.5 | -0.5 | — |
| $x_2$ | 3 | 0 | $\boxed{1}$ | 1/3 | 0 | 0 | 1 | 1/3 | 9 |
| $x_1$ | 1 | $\boxed{1}$ | 0 | [2/3] | 0 | 0.5 | -0.5 | 1/6 | 3/2 |
| Z | -3 | 0 | 0 | -3 | 0 | -1.5 | M+3/2 | M-1/2 | |

# Other non-standard forms of mathematical model -big M method

**3rd iteration**

Maximum Z=3/2

Optimal solution $x_1 = 0, x_2 = \frac{2}{5}, x_3 = \frac{3}{2}, x_4 = x_5 = x_6 = x_7 = 0$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|---|
| $x_4$ | 0 | 0 | 0 | 0 | $\boxed{1}$ | -0.5 | 0.5 | -0.5 | — |
| $x_2$ | 5/2 | -1/2 | $\boxed{1}$ | 0 | 0 | -1/4 | 1/4 | 1/3 | 9 |
| $x_3$ | 3/2 | 3/2 | 0 | $\boxed{[1]}$ | 0 | 3/4 | -3/4 | 1/4 | 3/2 |
| Z | 3/2 | 9/2 | 0 | 0 | 0 | 3/4 | M-3/4 | M+1/4 | |

# The Tabulate Simplex Method to solve infinite solutions

$$\max Z = 20x_1 + 40x_2 \quad s.t.\begin{cases} 2.5x_1 + 2x_2 \le 12 \\ x_1 + 2x_2 \le 8 \\ 4x_1 \le 16 \\ 4x_2 \le 12 \\ x_1, x_2 \ge 0 \end{cases}$$

New supplementary variables: $x_2, x_3, x_4, x_5$

New basic variables: $x_1, x_6$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| $x_3$ | 12 | 2.5 | 2 | $\boxed{1}$ | 0 | 0 | 0 | 12/2.5=4.8 |
| $x_4$ | 8 | 1 | 2 | 0 | $\boxed{1}$ | 0 | 0 | 8/2=4 |
| $x_5$ | 16 | 4 | 0 | 0 | 0 | $\boxed{1}$ | 0 | — |
| $x_6$ | 12 | 0 | [4] | 0 | 0 | 0 | $\boxed{1}$ | 12/4=3 |
| Z | 0 | -20 | -40 | 0 | 0 | 0 | 0 | $\min\theta = 3$ |

# The Tableau Simplex Method to solve infinite solutions

$$\max Z = 20x_1 + 40x_2 \quad s.t.\begin{cases} 2.5x_1 + 2x_2 \le 12 \\ x_1 + 2x_2 \le 8 \\ 4x_1 \le 16 \\ 4x_2 \le 12 \\ x_1, x_2 \ge 0 \end{cases}$$

New supplementary variables: $x_2, x_3, x_1, x_5$

New basic variables: $x_4, x_6$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| $x_3$ | 6 | 2.5 | 0 | $\boxed{1}$ | 0 | 0 | -0.5 | 6/2.5=2.4 |
| $x_4$ | 2 | [1] | 0 | 0 | $\boxed{1}$ | 0 | -0.5 | 2/1=2 |
| $x_5$ | 16 | 4 | 0 | 0 | 0 | $\boxed{1}$ | 0 | 16/4=4 |
| $x_2$ | 3 | 0 | $\boxed{1}$ | 0 | 0 | 0 | 0.25 | — |
| Z | 120 | -20 | 0 | 0 | 0 | 0 | 10 | $\min\theta = 2$ |

# The Tableau Simplex Method to solve infinite solutions

$$\max Z = 160$$

Optimal solution $x_1 = 0, x_2 = 3, x_3 = 1, x_4 = 0, x_5 = 8, x_6 = 0$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| $x_3$ | 1 | 0 | 0 | 1 | -2.5 | 0 | [0.75] | 1/0.75=4/3 |
| $x_1$ | 2 | 1 | 0 | 0 | 1 | 0 | -0.5 | — |
| $x_5$ | 8 | 0 | 0 | 0 | -4 | 1 | 2 | 8/2=4 |
| $x_2$ | 3 | 0 | 1 | 0 | 0 | 0 | 0.25 | 3/0.25=12 |
| Z | 160 | 0 | 0 | 0 | 20 | 0 | 0 | $\min \theta = 4/3$ |

# The Tableau Simplex Method to solve infinite solutions

$$\max Z = 160$$

| base | b | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| $x_6$ | 1.3333 | 0 | 0 | 1.3333 | -3.3333 | 0 | 1 | |
| $x_1$ | 2.6667 | 1 | 0 | 0 | 0.6667 | 0 | 0 | |
| $x_5$ | 5.6667 | 0 | 0 | -2.6667 | 2.6667 | 1 | 0 | |
| $x_2$ | 2.6667 | 0 | 1 | -0.3333 | 0.8333 | 0 | 0 | |
| Z | 160 | 0 | 0 | 0 | 20 | 0 | 0 | |

# The Matrix Form of the Simplex Method

$$\max Z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

$$s.t.\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le b_2 \\ \quad\quad\quad\quad\quad . \\ \quad\quad\quad\quad\quad . \\ \quad\quad\quad\quad\quad . \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le b_m \\ x_1 \ge 0, x_2 \ge 0, \ldots, x_n \ge 0 \end{cases}$$

$$\max Z = cX$$

$$s.t. AX \le b$$

$$X \ge 0$$

# The Matrix Form of the Simplex Method

$$\max Z = cX$$

$$s.t. AX \le b$$

$$X \ge 0$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_m \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{bmatrix}, c = \begin{pmatrix} c_1 & c_2 & . & . & . & c_n \end{pmatrix}$$

# The Matrix Form of the Simplex Method

Transform into standard form

$$\max Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

$$s.t.\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + \boxed{x_{n+1}} = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + x_{n+2} = b_2 \\ \cdot \\ \cdot \\ \cdot \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n + x_{n+m} = b_m \\ x_1 \ge 0, x_2 \ge 0, \ldots, x_{n+m} \ge 0 \end{cases}$$

Supplementary variable

$$X_s = \begin{bmatrix} x_{n+1} \\ x_{n+2} \\ \cdot \\ \cdot \\ \cdot \\ x_{n+m} \end{bmatrix}$$

# The Matrix Form of the Simplex Method

Simplify constraint equation into

$$\begin{pmatrix} A & I \end{pmatrix} \begin{pmatrix} X \\ X_s \end{pmatrix} = b \quad \begin{pmatrix} X \\ X_s \end{pmatrix} \geq 0$$

Let basic variable X equal to 0 to find the basic feasible solution $\binom{0}{b}$

---

Judging whether initial basic feasible solution is the optimal solution

If not, update supplementary variables, and use elimination method

to get new coefficient matrix $(Bx_B) = b$

Let new basic variable x equal to 0 to find new feasible solution

$$x_B = B^{-1}b$$

Calculating objective function $Z = c_B x_B = c_B B^{-1} b$

# Linear programming - Matlab Linprog Function

Standard format

$$\min Z = c_1 x_1 + \ldots + c_n x_n$$

$$c = \begin{pmatrix} c_1 & c_2 & . & . & . & c_n \end{pmatrix}^T$$

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$$

$$A_{eq} = \begin{pmatrix} a_{11}^{eq} & . & . & . & a_{1n}^{eq} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ a_{m1}^{eq} & . & . & . & a_{mn}^{eq} \end{pmatrix}$$

$$s.t.\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + \ldots + a_{2n}x_n \le b_2 \\ . \\ . \\ . \\ a_{m1}x_1 + \ldots + a_{mn}x_n \le b_m \\ a_{11}^{eq}x_1 + \ldots + a_{1n}^{eq}x_n = b_1^{eq} \\ a_{21}^{eq}x_1 + \ldots + a_{2n}^{eq}x_n = b_2^{eq} \\ . \\ . \\ . \\ a_{h1}^{eq}x_1 + \ldots + a_{hn}^{eq}x_n = b_h^{eq} \\ lx_1 \ge x_1 \ge ux_1 \quad lx_2 \ge x_2 \ge ux_2 \ldots lx_n \ge x_n \ge ux_n \end{cases}$$

# Linear programming-Matlab Linprog Function

Standard format

$$\min Z = c_1x_1 + \ldots + c_nx_n$$

$$lb = \begin{pmatrix} lx_1 & lx_2 & . & . & . & lx_n \end{pmatrix}$$

$$ub = \begin{pmatrix} ux_1 & ux_2 & . & . & . & ux_n \end{pmatrix}$$

$$\min c^T X$$

$$s.t.\begin{cases} AX \le b \\ A_{eq}X = b_{eq} \\ lb \le X \le ub \end{cases}$$

$$s.t.\begin{cases} a_{11}x_1 + \ldots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + \ldots + a_{2n}x_n \le b_2 \\ . \\ . \\ . \\ a_{m1}x_1 + \ldots + a_{mn}x_n \le b_m \\ a_{11}^{eq}x_1 + \ldots + a_{1n}^{eq}x_n = b_1^{eq} \\ a_{21}^{eq}x_1 + \ldots + a_{2n}^{eq}x_n = b_2^{eq} \\ . \\ . \\ . \\ a_{h1}^{eq}x_1 + \ldots + a_{hn}^{eq}x_n = b_h^{eq} \\ lx_1 \ge x_1 \ge ux_1 \quad lx_2 \ge x_2 \ge ux_2 \ldots lx_n \ge x_n \ge ux_n \end{cases}$$

# Linear programming-Matlab Linprog Function

$$x = linprog(c, A, b)$$

Solving problems with only inequality constraints

$$\min c^T x$$

s.t. $Ax \le b$

Example

$$\max Z = 2x_1 + 3x_2$$

$$s.t.\begin{cases} 2x_1 + 2x_2 \le 12 \\ x_1 + 2x_2 \le 8 \\ 4x_1 \le 16 \\ 4x_2 \le 12 \end{cases}$$

# Linear programming-Matlab Linprog Function

$$x = linprog(c, A, b, Aeq, beq)$$

Solving problems containing equality constraints

$$\min c^T x$$

$$\text{s.t.} \begin{cases} Ax \leq b \\ A_{eq} = b_{eq} \end{cases}$$

$$x = linprog(c, A, b, Aeq, beq, lb, ub)$$

Solve linear programming problem in standard form

$$\min c^T x$$

$$\text{s.t.} \begin{cases} Ax \leq b \\ A_{eq} = b_{eq} \\ \mathrm{lb} \leq x \leq ub \end{cases}$$

# Linear programming-Matlab Linprog Function

$$x = linprog(c, A, b, [], [], lb, ub)$$

Solving problems without equality constraints

$$\min c^T x$$

$$\text{s.t.} \begin{cases} Ax \leq b \\ \text{lb} \leq x \leq ub \end{cases}$$

$$x = linprog(c, [], [], Aeq, beq, lb, ub)$$

Solving problems without inequality constraints

$$\min c^T x$$

$$\text{s.t.} \begin{cases} Ax = b \\ \text{lb} \leq x \leq ub \end{cases}$$

# Linear programming-Matlab Linprog Function

$$[x, fval, exitflag] = linprog(c, A, b, Aeq, beq, lb, ub)$$

1 linprog converged to a solution X.

0 Maximum number of iterations reached.

-2 No feasible point found.

-3 Problem is unbounded.

-4 NaN value encountered during execution of algorithm.

-5 Both primal and dual problems are infeasible.

-7 Magnitude of search direction became too small; no further progress can be made. The problem is ill-posed or badly conditioned.

Example: A Company produces two types of glass windows, factory A manufactures aluminum window frame, and factory B manufactures wooden window frame, whereas factory C produces glass and provides final assembly for these two types of windows.

Factory A has 4 work-hours available, factory B has 12 work-hours available and factory C has 18 work-hours available. Make a production plan for the two products to maximize the profit.

| | Factory A | Factory B | Factory C | Profit |
|---|---|---|---|---|
| Aluminum window | 1 | -- | 3 | 3 |
| Wooden window | -- | 2 | 2 | 5 |

| | Factory A | Factory B | Factory C | Profit |
|---|---|---|---|---|
| Aluminum window x1 | 1 | -- | 3 | 3 |
| Wooden window x2 | -- | 2 | 2 | 5 |

$$\max Z = 3x_1 + 5x_2 \quad s.t.\begin{cases} x_1 \leq 4 \\ 2x_2 \leq 12 \\ 3x_1 + 2x_2 \leq 18 \\ x_1, x_2 \geq 0 \end{cases}$$

$$\max Z = 3x_1 + 5x_2$$

$$s.t.\begin{cases} x_1 \le 4 \\ 2x_2 \le 12 \\ 3x_1 + 2x_2 \le 18 \\ x_1, x_2 \ge 0 \end{cases}$$

$$\min \quad -Z = -3x_1 + -5x_2$$

$$s.t.\begin{cases} x_1 + 0 \times x_2 \le 4 \\ 0 \times x_1 + 2x_2 \le 12 \\ 3x_1 + 2x_2 \le 18 \\ x_1, x_2 \ge 0 \end{cases}$$

```
>> c=[-3,-5];
>> A=[1,0;0,2;3,2];
>> B=[4;12;18];
>> lb=[0;0];
>>ub=[];
>> [x,fval,exitflag]=linprog(c,A,B, [],[],lb,ub)
```

```
Optimization terminated
x =2.0000
    6.0000
fval =  -36.0000
exitflag =    1
```

# Application: Resource Allocation

Soft drink company “Coca-Pepsa” (哈哈娃) owns four production lines (A, B, C, D), which can produce many kinds of drinks. A recent technology upgrade releases extra work-hours of 12, 8, 16 and 12 for line A, B, C and D respectively. The company plans to use these work-hours to produce new products “DoubleCool” (多加宝) and “LuckyKing”(老王吉). Profit and time cost of these produces are

| | Time cost（hr） | | | | Profit（Yuan） |
|---|---|---|---|---|---|
| | A | B | C | D | |
| DoubleCool | 2.5 | 1 | 2 | 0 | 20 |
| LuckyKing | 2 | 2 | 0 | 4 | 30 |

# Application: Resource Allocation

Question: Make a production plan to maximise the profit

Assume the production of “DoubleCool” is x1, and

“LuckyKing” is x2

Profit equation $z = 20x_1 + 30x_2$

Constrains

$$\begin{cases} 2.5x_1 + 2x_2 \le 12 \\ x_1 + 2x_2 \le 8 \\ 4x_1 \le 16 \\ 4x_2 \le 12 \\ x_1, x_2 \ge 0 \end{cases}$$

# Application: Resource Allocation

$$Max\ z = 20x_1 + 30x_2$$

$$\begin{cases} 2.5x_1 + 2x_2 \le 12 \\ x_1 + 2x_2 \le 8 \\ 4x_1 \le 16 \\ 4x_2 \le 12 \\ x_1, x_2 \ge 0 \end{cases}$$

$$Min\ -z = -20x_1 - 30x_2$$

$$\begin{cases} 2.5x_1 + 2x_2 \le 12 \\ x_1 + 2x_2 \le 8 \\ 4x_1 + 0 \times x_2 \le 16 \\ 0 \times x_1 + 4x_2 \le 12 \\ x_1, x_2 \ge 0 \end{cases}$$

```
>> c=[-20,-30];
>> A=[2.5,2;1,2;4,0;0,4];
>> B=[12;8;16;12];
>> lb=[0;0];
>>ub=[];
>> [x,fval,exitflag]=linprog(c,A,B, [],[],lb,ub)
```

```
Optimization terminated

x =2.6667
   2.6667

fval =  -133.3333

exitflag =    1
```

# Application: Production plan

Metallurgic plant “铁生馆” plans to produce lead(铅) no less than 30 tons, cooper no less than 35 tons and iron 45 tons. There are four types of minerals available:

| Mineral | lead（%） | cooper（%） | iron（%） | cost（yuan/ton） |
|---|---|---|---|---|
| A | 2 | 4 | 4 | 10 |
| B | 3 | 2 | 2 | 15 |
| C | 1 | 3 | 3 | 30 |
| D | 0.5 | 1 | 5 | 25 |

# Application: Production plan

Question: Decide the purchase plan for the minerals to minimise the cost

Assume to purchase x1, x2, x3 and x4 tons of minerals A, B, C and D

Total cost $z = 10x_1 + 15x_2 + 30x_3 + 25x_4$

constrains
$$\begin{cases} 0.02x_1 + 0.03x_2 + 0.01x_3 + 0.005x_4 \geq 30 \\ 0.04x_1 + 0.02x_2 + 0.03x_3 + 0.01x_4 \geq 35 \\ 0.04x_1 + 0.02x_2 + 0.03x_3 + 0.05x_4 = 45 \\ \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

# Application: Production plan

Standard form

$$\min c^T x$$
$$s.t.\begin{cases} Ax \le b \\ A_{eq}x = b_{eq} \\ lb \le x \le ub \end{cases}$$

$$c = \begin{pmatrix} 10 \\ 15 \\ 30 \\ 25 \end{pmatrix} \qquad A = \begin{pmatrix} -0.02 & -0.03 & -0.01 & -0.005 \\ -0.04 & -0.02 & -0.03 & -0.01 \end{pmatrix}$$

$$A_{eq} = (0.04 \;\; 0.02 \;\; 0.03 \;\; 0.05)$$

$$b = \begin{pmatrix} -30 \\ -35 \end{pmatrix} \qquad b_{eq} = 45 \qquad lb = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

# Application: Production plan

```
>> c=[10;15;30;25];
>> b=[-30;-35];
>> A=[-0.02,-0.03,-0.01,-0.005;-0.04,-0.02,-0.03,-0.01];
>> Aeq=[0.04,0.02,0.03,0.05];
>> beq=45;
>> lb=[0;0;0;0];
>> [x,fval,exitflag]=linprog(c,A,b,Aeq,beq,lb,[])

Optimization terminated.
x =
  937.5000
  375.0000
    0.0000
    0.0000
fval =    15000                                   exitflag =    1
```