Ex.3 Problems

- $2^{3x-4} = 8^{2x+1}$
- $8^{2x+1} = \left(\frac{1}{16}\right)^{3-2x}$
- $4^{2x} - 6 \cdot 4^x + 8 = 0$
- $\sqrt[2x+4]{4^{x+8}} = \sqrt[6]{128}$
- $\log_2(x+1) - \log_2(x) = 1$
- $3\log_5 2 - \log_5(x-1) = \log_5(x+1) - \log_5(x-2)$
- Compound interest formula:
$$s_n = s_0 \left(1 + \frac{p}{100}\right)^n .$$
Q: How long do we have to wait ($n = ?$) for our deposit s_0 to double ($s_n = 2s_0$)? Fix $p = 1\%$.

a) $\log_{10}(x+2) + \log_{10}(x-7) = 2\log_{10}(x-4)$

Our goal is to simplify the equation so that both sides of the equation look like

$$\log_a(\dots) = \log_a(\dots) .$$

Once we succeed, we can compare the arguments.
Right-hand side of the equation: invoke the formula $\alpha \log_a x = \log_a x^\alpha$, in our case we obtain:

$$2\log_{10}(x-4) = \log_{10}(x-4)^2 .$$

Left-hand side of the equation: invoke the formula $\log_a(x \cdot y) = \log_a x + \log_a y$, in our case we obtain:

$$\log_{10}(x+2) + \log_{10}(x-7) = \log_{10}(x+2)(x-7) .$$

If we put it together, the equation simplifies to:

$$\begin{aligned} \log_{10}(x+2)(x-7) &= \log_{10}(x-4)^2 , \\ (x+2)(x-7) &= (x-4)^2 \quad \to \quad x^2 - 7x + 2x - 14 = x^2 - 8x + 16 \\ 3x &= 30 \quad \to \quad x = 10 . \end{aligned}$$

The three logarithms give us three constraints:

$$\begin{aligned} x + 2 &> 0 , \\ x - 7 &> 0 , \\ x - 4 &> 0 . \end{aligned}$$

The intersection of these three intervals is $D_f = (7, \infty)$. The solution we had found, $x = 10$, belongs to this interval.

b) $\sqrt[2x+4]{4^{x+8}} = \sqrt[4]{64}$

In order to solve this exponential equation, both sides of the equation must have the same base. If we recall the formula $\sqrt[c]{a^b} = a^{b/c}$, we can simplify

$$4^{\frac{x+8}{2x+4}} = 64^{1/4} .$$

Next, $4 = 2^2$, and we will use the formula $(a^b)^c = a^{b \cdot c}$. Furthermore, $64 = 8 \cdot 8 = 2^3 \cdot 2^3 = 2^6$. The simplified equation now reads

$$2^{2\frac{x+8}{2x+4}} = 2^{6/4} .$$

The base is the same (two) on both sides of the equation, we can compare the exponents/arguments and we obtain:

$$\begin{aligned} 2\frac{x+8}{2x+4} &= \frac{6}{4}\,, \\ \frac{x+8}{x+2} &= \frac{3}{2}\,, \\ x+8 &= \frac{3}{2}(x+2) \quad \rightarrow \quad 5 = \frac{1}{2}x \quad \rightarrow \quad x = 10\,. \end{aligned}$$

Note that the equation can not be solved for $x = -2$ (the zeroth root is undefined).