9:15

a) $f(x) = \ln(x^2 - 2x + 5)$

The goal is to examine the curvature of the function; we compute the first and second derivatives:

$$f'(x) = \frac{1}{x^2 - 2x + 5} \cdot (2x - 2) = \frac{2x - 2}{x^2 - 2x + 5}\,.$$

$$\begin{aligned} f''(x) &= \frac{2 \cdot (x^2 - 2x + 5) - (2x - 2) \cdot (2x - 2)}{(x^2 - 2x + 5)^2} \\ &= \frac{2x^2 - 4x + 10 - (4x^2 - 8x + 4)}{(x^2 - 2x + 5)^2} \\ &= \frac{-2x^2 + 4x + 6}{(x^2 - 2x + 5)^2}\,. \end{aligned}$$

Inflection points are obtained by solving $f''(x) = 0$, i.e.

$$-2x^2 + 4x + 6 = 0 \quad \to \quad (x + 1)(x - 3) = 0\,.$$

Thus the inflection points are $x_1 = -1$ and $x_2 = 3$. To investigate curvature, we need all zeros of the second derivative, so we look at the denominator— the quadratic function $x^2 - 2x + 5$ has discriminant $D = 4 - 20 = -16$, therefore the parabola never intersects the x–axis and we have no other zeros.

We check the sign of f'' on the intervals $(-\infty, -1)$, $(-1, 3)$, and $(3, \infty)$. The analysis gives:

$$\begin{aligned} (-\infty, -1) &: -\,, \\ (-1, 3) &: +\,, \\ (3, \infty) &: -\,. \end{aligned}$$

On the intervals $(-\infty, -1)$ and $(3, \infty)$ the function is concave (second derivative negative), and on $(-1, 3)$ it is convex (second derivative positive). Finally we find the y–coordinates of the inflection points:

$$\begin{aligned} x_1 &= -1 \quad f(-1) = \ln 8 \quad [-1, \ln 8]\,, \\ x_2 &= 3 \quad f(3) = \ln 8 \quad [3, \ln 8]\,. \end{aligned}$$

11:00

a) $f(x) = (x^2 + 3x + 2)\mathrm{e}^{-x}$

The goal is to examine the curvature of the function; we compute the first and second derivatives:

$$\begin{aligned} f'(x) &= (2x + 3)\mathrm{e}^{-x} + (x^2 + 3x + 2)\mathrm{e}^{-x}(-1) \\ &= \mathrm{e}^{-x}(-x^2 - x + 1)\,. \end{aligned}$$

$$\begin{aligned} f'' &= (-2x - 1)\mathrm{e}^{-x} + (-x^2 - x + 1)\mathrm{e}^{-x}(-1) \\ &= \mathrm{e}^{-x}(x^2 - x - 2)\,. \end{aligned}$$

Inflection points are obtained by solving $f''(x) = 0$, i.e.

$$\mathrm{e}^{-x}(x^2 - x - 2) = 0 \qquad x^2 - x - 2 = 0\,,$$

whose solutions we immediately see using Vieta's formulas: $x_1 = -1$ and $x_2 = 2$. We also used the fact that the exponential function is always non-negative. From this we also see that the second derivative has no other zeros, and we check the signs:

$$\begin{aligned} (-\infty, -1) &: +\,, \\ (-1, 2) &: -\,, \\ (2, \infty) &: +\,. \end{aligned}$$

On the intervals $(-\infty, -1)$ and $(2, \infty)$ the function is convex (second derivative positive), and on $(-1, 2)$ it is concave (second derivative negative). Finally we find the y–coordinates of the inflection points:

$$x_1 = -1 \quad f(-1) = 0 \quad [-1,\ 0]\,,$$
$$x_2 = 2 \quad f(2) = 12\mathrm{e}^{-2} \quad [2, 12\mathrm{e}^{-2}]\,.$$