Pr. 1

$$\lim_{n\to\infty} \frac{2^n + 3^n + 5\cdot 2^{n-1}}{4^n + 3^n}$$

Trick 2: factor out the exponential term with the highest base.

The largest base here is 4, so factor 4^n from numerator and denominator.

$$= \lim_{n\to\infty} \frac{4^n\left(\frac{2^n}{4^n} + \frac{3^n}{4^n} + 5\cdot\frac{2^{n-1}}{4^n}\right)}{4^n\left(1+\frac{3^n}{4^n}\right)} = \lim_{n\to\infty} \frac{\left(\left(\frac{2}{4}\right)^n + \left(\frac{3}{4}\right)^n + 5\cdot\frac{1}{2}\left(\frac{2}{4}\right)^n\right)}{1+\left(\frac{3}{4}\right)^n}.$$

Since $0 < \dfrac{2}{4} = \dfrac{1}{2} < 1$ and $0 < \dfrac{3}{4} < 1$, both $\left(\dfrac{1}{2}\right)^n$ and $\left(\dfrac{3}{4}\right)^n$ go to 0. Thus

$$= \frac{0+0+0}{1+0} = 0.$$

Pr. 2

$$\lim_{n\to\infty} n\left(\sqrt{n^2+3n+1} - n\right)$$

Trick 3: multiply by the conjugate (same expression with opposite sign).

$$\lim_{n\to\infty} n(\sqrt{n^2+3n+1} - n) = \lim_{n\to\infty} n\cdot\frac{(\sqrt{n^2+3n+1}-n)(\sqrt{n^2+3n+1}+n)}{\sqrt{n^2+3n+1}+n}$$

$$= \lim_{n\to\infty} n\cdot\frac{(n^2+3n+1)-n^2}{\sqrt{n^2+3n+1}+n} = \lim_{n\to\infty} n\cdot\frac{3n+1}{\sqrt{n^2+3n+1}+n}.$$

Let us simplify further by factoring out n^2 from inside the root, we obtain

$$\lim_{n\to\infty} n\frac{3n+1}{n\sqrt{1+\frac{3}{n}+\frac{1}{n^2}}+n} = \lim_{n\to\infty} \frac{n}{n}\frac{3n+1}{\sqrt{1+\frac{3}{n}+\frac{1}{n^2}}+1} = \lim_{n\to\infty} \frac{3n+1}{\sqrt{1+\frac{3}{n}+\frac{1}{n^2}}+1}$$

This limit diverges and goes to infinity because the numerator grows as $3n$ and the denominator tends to 2.