

a) $f(x) = \frac{3x-1}{x-1}$ at the point $x_0 = 2$

We will find the tangent to the function $f(x)$ at the point $x_0 = 2$ as follows. We know that the tangent line is generally given by

$$\begin{aligned} y(x) &= f(x_0) + f'(x_0)(x - x_0) \quad \text{or, equivalently,} \\ y(x) &= f'(x_0)x + f(x_0) - f'(x_0)x_0 \quad (\text{in the form } y(x) = ax + b). \end{aligned}$$

Let's compute the derivative of the function $f(x)$. Since it is in a fractional form, we use the quotient rule:

$$f'(x) = \frac{3 \cdot (x-1) - (3x-1) \cdot 1}{(x-1)^2} = -\frac{2}{(x-1)^2}.$$

Now substitute the point $x_0 = 2$ into the formulas for $f(x)$ and $f'(x)$:

$$f(2) = 5 \quad f'(2) = -2.$$

Therefore, the equation of the tangent line (using the second form) is

$$y(x) = -2x + 5 - (-2) \cdot 2 = -2x + 9.$$

The point of tangency is $T = [x_0, f(x_0)] = [2, 5]$. Our given function $f(x)$ is a rational linear function, so we can rewrite $f(x)$ into the center form to be able to sketch it:

$$(3x-1) : (x-1) = 3 + \frac{2}{x-1},$$

hence the center is at the point $S = [1, 3]$, located in the 1st and 3rd quadrants.

b) $f(x) = \frac{2x-1}{x+2}$ at the point $x_0 = -1$

We will find the tangent to the function $f(x)$ at the point $x_0 = -1$ as follows. We know that the tangent line is generally given by

$$\begin{aligned} y(x) &= f(x_0) + f'(x_0)(x - x_0) \quad \text{or, equivalently,} \\ y(x) &= f'(x_0)x + f(x_0) - f'(x_0)x_0 \quad (\text{in the form } y(x) = ax + b). \end{aligned}$$

Let's compute the derivative of the function $f(x)$. Since it is in a fractional form, we use the quotient rule:

$$f'(x) = \frac{2 \cdot (x+2) - (2x-1) \cdot 1}{(x+2)^2} = \frac{5}{(x+2)^2}.$$

Now substitute the point $x_0 = -1$ into the formulas for $f(x)$ and $f'(x)$:

$$f(-1) = -3 \quad f'(-1) = 5.$$

Therefore, the equation of the tangent line (using the second form) is

$$y(x) = 5x - 3 - 5 \cdot (-1) = 5x + 2.$$

The point of tangency is $T = [x_0, f(x_0)] = [-1, -3]$. Our given function $f(x)$ is a rational linear function, so we can rewrite $f(x)$ into the center form to be able to sketch it:

$$(2x-1) : (x+2) = 2 + \frac{-5}{x-(-2)},$$

hence the center is at the point $S = [-2, 2]$, located in the 2nd and 4th quadrants.

You can plot the graphs of both functions and their tangents, for example, in Desmos:

<https://www.desmos.com/calculator?lang=en>

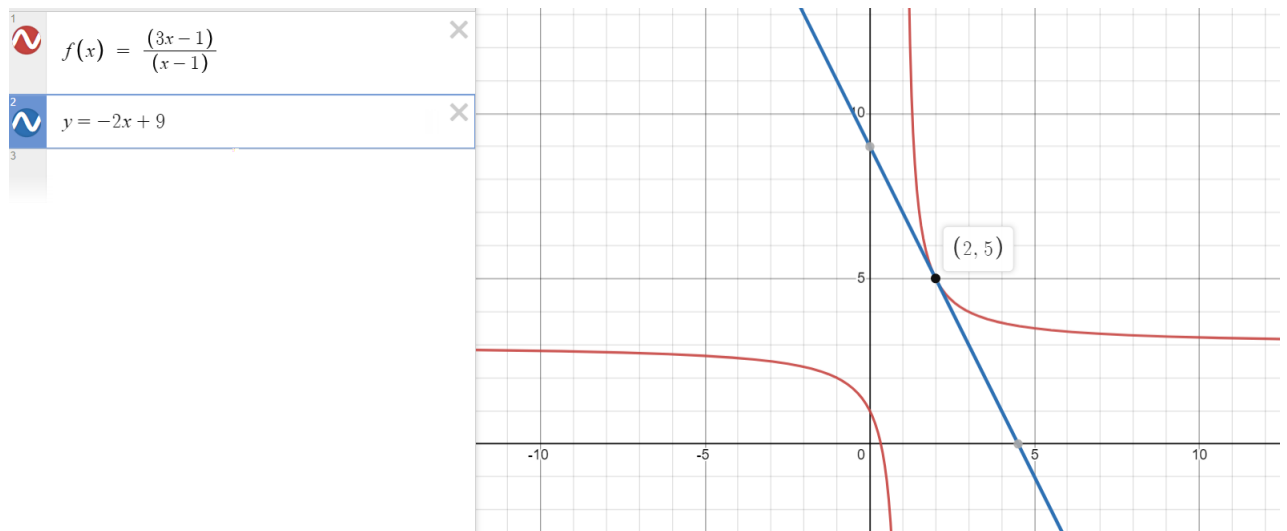


Figure 1: Figure for a).

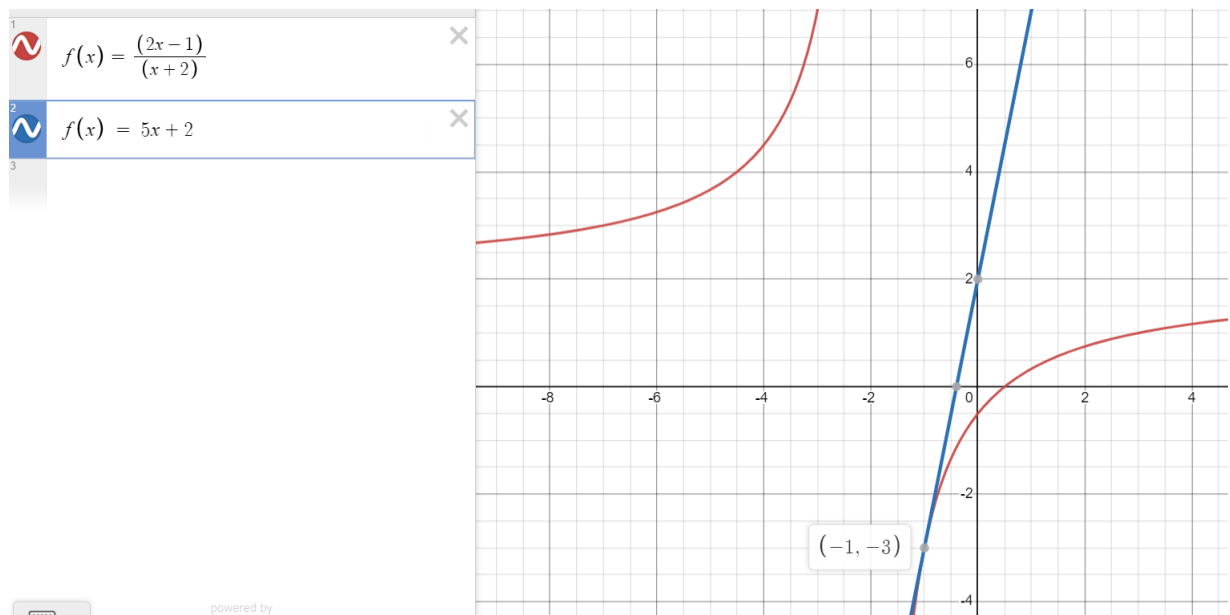


Figure 2: Figure for b).