a) $f(x) = -3x^2 - 12x + 15$

Identify the coefficients of the given quadratic function as $a = -3$, $b = -12$ a $c = 15$. The intersection point with the axis y reads $P_y = [0, c] = [0, 15]$. The intersection points with the axis x can be found as the roots of the function:

$$-3x^2 - 12x + 15 = 0\,,$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot (-3) \cdot 15}}{2 \cdot (-3)} = \begin{cases} x_1 = \frac{12+18}{-6} = -5 \\ x_2 = \frac{12-18}{-6} = 1 \end{cases}$$

Thus, the intersections with the axis x read $P_{x_1} = [-5, 0]$ a $P_{x_2} = [1, 0]$ and the quadratic function can be written as $-3x^2 - 12x + 15 = -3(x+5)(x-1)$.
The coordinates of the vertex $[x_v, y_v]$ can be obtained as follows: the symmetry of each quadratic function asserts that the coordinate x_v must be equal to the aritmetic average of the roots, that is, $x_v = (-5+1)/2 = -2$, and this value can be plugged into the prescription $f(x)$. Ultimately, we obtain:

$$y_v = f(x_v) = -3(-2)^2 - 12 \cdot (-2) + 15 = 27\,.$$

The vertex is $V = [-2, 27]$.

You can use the Desmos online tool to draw the graph of the function.
`https://www.desmos.com/calculator?lang=en`

Note that if $b^2 - 4ac < 0$ and the quadratic function does not have any real-valued roots, the vertex must be found in a different way:

- The vertex form $f(x) = a(x+m)^2 + n$. In order to transform the classic prescription of a quadratic function into its vertex form, you must perform the completing square procedure.
- (The easiest) Just remember that $x_v = b/2a$ always holds and plug it into the function to retrieve the y_v coordinate.
- Using derivatives.