

Pr. 1 $f(x) = \frac{x-4}{x^2-16}$

možná si:

$$f(x) = \frac{x-4}{(x-4)(x+4)} = \frac{1}{x+4}$$

$$S = [-4, 0]$$

$$f(x) = \frac{x-4}{(x-4)(x+4)}$$

ODSTRANITELNÁ
NESPOJITOST

$$D_f = \mathbb{R} \setminus \{-4\}$$

$$= (-\infty, -4) \cup (-4, 4) \cup (4, +\infty)$$

$$\lim_{x \rightarrow +\infty} \frac{x-4}{(x-4)(x+4)} \stackrel{(F)}{=} \lim_{x \rightarrow +\infty} \frac{x(1-\frac{4}{x})}{x(x+\frac{16}{x})}$$

$$\lim_{x \rightarrow +\infty} \frac{0}{0} \stackrel{(F)}{=} \lim_{x \rightarrow +\infty} \frac{1-\frac{4}{x}}{x+\frac{16}{x}}$$

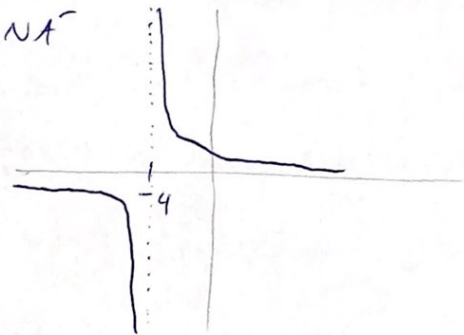
$$\lim_{x \rightarrow +\infty} \frac{0}{0}$$

$$\lim_{x \rightarrow -4^-} \frac{x-4}{x^2-16} = \lim_{x \rightarrow -4^-} \frac{x-4}{(x+4)(x-4)}$$

$$= \lim_{x \rightarrow -4^-} \frac{1}{x+4} = \frac{1}{0^-} = -\infty //$$

$$\lim_{x \rightarrow -4^+} \frac{x-4}{x^2-16} = \lim_{x \rightarrow -4^+} \frac{x-4}{(x+4)(x-4)}$$

$$= \lim_{x \rightarrow -4^+} \frac{1}{x+4} = \frac{1}{0^+} = +\infty //$$



$$\lim_{x \rightarrow +4^-} \frac{(x-4)}{(x-4)(x+4)} =$$

$$= \lim_{x \rightarrow +4^-} \frac{1}{x+4} = \frac{1}{8} //$$

$$\lim_{x \rightarrow +4^+} \frac{x-4}{(x-4)(x+4)} = \lim_{x \rightarrow +4^+} \frac{1}{x+4} =$$

$$= \frac{1}{8} \text{ dodávamej limitu}$$

$$\text{Protože } \lim_{x \rightarrow +4^-} = \lim_{x \rightarrow +4^+}$$

Př. 2 $f(x) = \frac{\sqrt{x^2+2x+5}}{2-2x}$

Funkce signum:

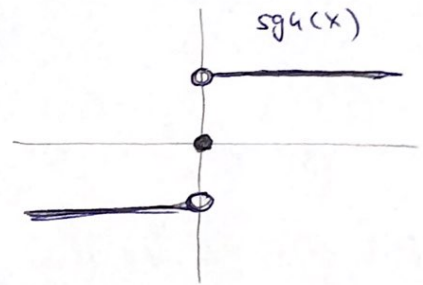
$$\text{sgn}(x) = \begin{cases} 0 & x=0 \\ 1 & x>0 \\ -1 & x<0 \end{cases}$$

Discriminace výrazu x^2+2x+5 :

$$D = b^2 - 4ac = (2)^2 - 4 \cdot 5 \cdot 1 = -16 < 0$$

výraz pod odmocninou je kladný.

$$D_f = \mathbb{R} \setminus \{1\} \rightarrow \begin{matrix} \lim_{x \rightarrow +\infty} f(x) & \lim_{x \rightarrow 1^+} f(x) \\ \lim_{x \rightarrow -\infty} f(x) & \lim_{x \rightarrow 1^-} f(x) \end{matrix}$$



$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+2x+5}}{2-2x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2(1+\frac{2}{x}+\frac{5}{x^2})}}{2-2x} = \lim_{x \rightarrow +\infty} \frac{x \sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{x(\frac{2}{x}-2)} =$$

$$= \lim_{x \rightarrow +\infty} \frac{\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{-2+\frac{2}{x}} \stackrel{\text{Vojt. L.}}{=} \frac{\lim_{x \rightarrow +\infty} \sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{\lim_{x \rightarrow +\infty} (-2+\frac{2}{x})} = \frac{1}{-2} = -\frac{1}{2}$$

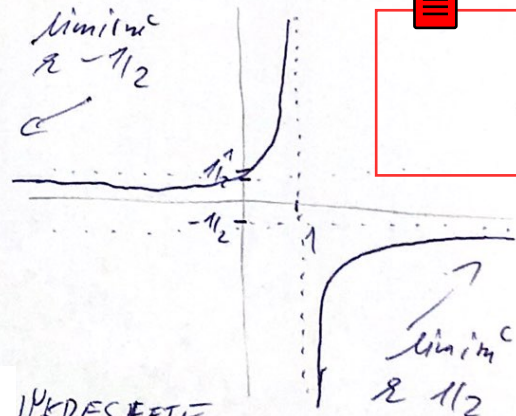
$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+2x+5}}{2-2x} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2(1+\frac{2}{x}+\frac{5}{x^2})}}{2-2x} = \lim_{x \rightarrow -\infty} \frac{|x| \sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{x(-2+\frac{2}{x})}$$

$$= \lim_{x \rightarrow -\infty} \frac{1}{\text{sgn}(x)} \frac{\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{-2+\frac{2}{x}} \stackrel{\text{Vojt. L.}}{=} \lim_{x \rightarrow -\infty} \frac{1}{\text{sgn}(x)} \lim_{x \rightarrow -\infty} \frac{\sqrt{1+\frac{2}{x}+\frac{5}{x^2}}}{-2+\frac{2}{x}} =$$

$$= (-1) \cdot (-\frac{1}{2}) = \frac{1}{2}$$

$$\lim_{x \rightarrow 1^+} \frac{\sqrt{x^2+2x+5}}{2-2x} = \frac{\sqrt{1^2+2 \cdot 1+5}}{0^+} = +\infty$$

$$\lim_{x \rightarrow 1^-} \frac{\sqrt{x^2+2x+5}}{2-2x} = \frac{\sqrt{1^2+2 \cdot 1+5}}{0^-} = -\infty$$



Musíte být opatrnější při práci s nekonečny. Zejména pak, když vytýkáte z odmocniny, zodpovězte si otázku, proč platí následující (připomeňte si definici absolutní hodnoty):

$$\lim_{x \rightarrow -\infty} \sqrt{x^2} = \lim_{x \rightarrow -\infty} |x| = +\infty.$$

INKRESEETE
SI N DESMOSU