Ex.4 Problems

- $\lim_{n\to+\infty} \frac{2n}{2n+1}$
- $\lim_{n\to+\infty} \frac{2n^4-n^3+4n^2}{5n^5+n-1}$
- Problem set 4: Problems 4, 14

a) $\lim_{n\to+\infty} \frac{-2(6+4n)}{(n+2)^2-(n-3)^2}$.

First, we simplify the denominator using the formula $(a+b)^2 = a^2+2ab+b^2$ and $(a-b)^2 = a^2-2ab+b^2$, we get

$$\lim_{n\to+\infty} \frac{-2(6+4n)}{(n^2+4n+4)-(n^2-6n+9)} = \lim_{n\to+\infty} \frac{-12-8n}{10n-5}\,.$$

Now we apply Trick 1 and factor out n from both the numerator and the denominator, then

$$\lim_{n\to+\infty} \frac{n}{n}\frac{-8-\frac{12}{n}}{10-\frac{5}{n}} \overset{\text{Arithmetics}}{=} \frac{\lim_{n\to+\infty} -8-12/n}{\lim_{n\to+\infty} 10-5/n} = -8/10 = -4/5\,,$$

where we used that $\lim_{n\to+\infty} 5/n = 0$ and $\lim_{n\to+\infty} 12/n = 0$. We see that the sequence converges, and thus the limit is finite.

b) $\lim_{n\to+\infty} \frac{n^2+4n+7}{(n-2)^2-(n+3)^2}$.

$$\lim_{n\to+\infty} \frac{n^2+4n+7}{(n^2-4n+4)-(n^2+6n+9)} = \lim_{n\to+\infty} \frac{n^2+4n+7}{-10n-5}\,.$$

Now we apply Trick 1 and factor out n^2 from the numerator and n from the denominator, then

$$\begin{aligned}\lim_{n\to+\infty} \frac{n^2}{n}\frac{1+\frac{4}{n}+\frac{7}{n^2}}{-10-\frac{5}{n}} &\overset{\text{Arithmetics}}{=} \lim_{n\to+\infty} n \cdot \lim_{n\to+\infty} \frac{1+\frac{4}{n}+\frac{7}{n^2}}{-10-\frac{5}{n}} \\ &= +\infty\cdot(-1/10) \\ &= -\infty\,,\end{aligned}$$

where we used that $\lim_{n\to+\infty} 4/n = 0$, $\lim_{n\to+\infty} 5/n = 0$, and $\lim_{n\to+\infty} 7/n^2 = 0$. We see that the sequence diverges, and thus the limit is infinite.

Notice how multiplying infinity by a negative number changed the result from $+\infty$ to $-\infty$.