9:15
a) $f(x) = \sqrt{x^2 - 6x + 10}$

Our goal is to investigate monotonicity; we compute the first derivative

$$f'(x) = \frac{1}{2\sqrt{x^2 - 6x + 10}} \cdot (2x - 6) = \frac{x - 3}{\sqrt{x^2 - 6x + 10}}\,.$$

Solving the equation $f'(x) = 0$ we obtain $x = 3$, which is a critical point (extremum or inflection point). To study monotonicity we need all zeros of the derivative. The discriminant of the expression under the square root is always negative ($b^2 - 4ac = 36 - 40 = -4$), so the expression is always positive. Therefore the only zero is $x = 3$. This point splits the real line into the intervals $(-\infty, 3)$ and $(3, +\infty)$. We examine the sign of f' on these intervals:

$$\begin{aligned} &(-\infty, 3) \quad \text{take e.g. } x = 2 \quad f' < 0\,,\\ &(3, +\infty) \quad \text{take e.g. } x = 4 \quad f' > 0\,. \end{aligned}$$

We see that $x = 3$ is clearly a local minimum (decreasing, then increasing). To determine whether it is also a global minimum, we compute limits at $\pm\infty$, namely

$$\lim_{x\to\pm\infty} \sqrt{x^2 - 6x + 10} = \lim_{x\to\pm\infty} |x|\sqrt{1 - \frac{6}{x} + \frac{10}{x^2}} = +\infty\,.$$

where, when factoring out x^2 from under the square root, we used $\sqrt{x^2} = |x|$. We see that both limits go to plus infinity; the function never approaches minus infinity, and therefore $x = 3$ is also a global minimum. Finally, we find the y-coordinate of this minimum, that is, $f(3) = 1$.

Summary: the function $f(x) = \sqrt{x^2 - 6x + 10}$ has a global minimum at the point $[3, 1]$, is decreasing on $(-\infty, 3)$ and increasing on $(3, +\infty)$.

11:00
a) $f(x) = (x - 1)\mathrm{e}^{3-x}$

Our goal is to investigate monotonicity; we compute the first derivative (note, we differentiate a product of functions):

$$f'(x) = \mathrm{e}^{3-x}(-1) \cdot (x - 1) + \mathrm{e}^{3-x} \cdot 1 = \mathrm{e}^{3-x}\,(-x + 1 + 1) = \mathrm{e}^{3-x}(2 - x)\,.$$

Solving $f'(x) = 0$ gives $x = 2$ (the exponential is always positive and never zero), which is a critical point. We need all zeros of the derivative. As we have argued, the exponential is always positive, so there are no other zeros. Thus the only zero is $x = 2$, dividing the real line into $(-\infty, 2)$ and $(2, +\infty)$. We examine the sign of f':

$$\begin{aligned} &(-\infty, 2) \quad \text{take e.g. } x = 1 \quad f' > 0\,,\\ &(2, +\infty) \quad \text{take e.g. } x = 3 \quad f' < 0\,. \end{aligned}$$

We see that $x = 2$ is a local maximum (increasing, then decreasing). To see whether it is global, we compute limits at $\pm\infty$:

$$\lim_{x\to+\infty} (x - 1)\mathrm{e}^{3-x} = +\infty \cdot 0\,.$$

This is an underined expression ($\mathrm{e}^{-\infty}$ goes to zero!). We apply L'Hôpital's rule, but we first rewrite the expression into a usable form (L'Hôpital applies only to limits of type $0/0$ or ∞/∞ or a/∞ with $a \in \mathbb{R}$):

$$\lim_{x\to+\infty} (x - 1)\mathrm{e}^{3-x} = \lim_{x\to+\infty} \frac{x - 1}{\mathrm{e}^{x-3}} \overset{\text{L'H}}{=} \lim_{x\to+\infty} \frac{1}{\mathrm{e}^{x-3}} = 0\,.$$

And then the limit at minus infinity:

$$\lim_{x\to-\infty} (x - 1)\mathrm{e}^{3-x} = -\infty \cdot \infty = -\infty\,.$$

Since toward $+\infty$ the function approaches zero asymptotically, and toward $-\infty$ it diverges to $-\infty$, the point $x = 2$ is also a global maximum. Finally, we find the y-coordinate of this maximum: $f(2) = \mathrm{e}$.

Summary: the function $f(x) = (x - 1)\mathrm{e}^{3-x}$ has a global maximum at the point $[2, \mathrm{e}]$, is increasing on $(-\infty, 2)$ and decreasing on $(2, +\infty)$.