Ex.2 Problems

- Problem set 1: 14, 15, 16
- Problem set 2: 11, 12
- $f(x) = \frac{5x-2}{1-x}$
- Find roots of: $f(x) = x^3 + x^2 - 17x + 15$ and find intervals, in which the function $f(x)$ is negative.

a) $f(x) = \frac{x+3}{4-2x}$

Find: asymptotes of the function, intersection points with the system axes, and sketch the graph.

The domain of the function is $D_f = \mathbb{R} \setminus \{2\}$. Next, we can find the asymptotes (and thus the coordinates of the center) as follows:

$$\begin{aligned} &\text{HA: } y = 1/(-2) = -1/2\,, \\ &\text{VA: } x = 2\,, \end{aligned}$$

where we invoked the rules that the horizontal asymptote is given as a ratio of the leading coefficients (that is, the coefficients are next to x, so 1 and -2 in this case) and the vertical asymptote corresponds to the hole in the domain of the function. So, to conclude, the center (that is, the shifted origin) reads $S[2, -1/2]$.

Alternatively, we can convert the function into its center form (division of polynomials):

$$(x+3) : (-2x+4) = -\frac{1}{2} + \frac{5}{-2x+4} = -\frac{1}{2} + \frac{5}{-2(x-2)} = -\frac{1}{2} + \frac{-2.5}{x-2}\,,$$

where we had to factor -2 into the denominator. Comparing to the general formula for the center form,

$$f(x) = y_s + \frac{k}{x - x_s}\,,$$

we obtain $S = [2, -1/2]$. Since $k = -2.5$ and $-2.5 < 0$, the hyperbole branches must be in the second and fourth quadrants.

Ultimately, we will find the intersection points with the $x-$ and $y-$ axes as:

$$P_y = [0, 3/4] \quad \text{and} \quad P_x = [-3, 0]\,.$$