

## Review Problems II]

1. Two people are involved in a dispute. Person 1 does not know whether person 2 is strong or weak; she assigns probability  $\alpha$  to person 2's being strong. Person 2 is fully informed. Each person can either fight or yield. Each person get the payoff of 0 if she yields (regardless of the other person's action) and a payoff of 1 if she fights and her opponent yields; if both people fight, then their payoffs are  $(-1,1)$  if person 2 is strong and  $(1,-1)$  if person 2 is weak. Formulate this situation as a Bayesian game and find its Nash equilibria if  $\alpha < 1/2$  and if  $\alpha > 1/2$ .
  
2. Whether candidate 1 or candidate 2 is elected depends on the votes of two citizens. The economy may be in one of two states,  $A$  and  $B$ . The citizens agree that candidate 1 is best if the state is  $A$  and candidate 2 is best if the state is  $B$ . Each citizen gets a payoff of 1 if the best candidate for the state wins (obtains more votes than the other candidate), a payoff of 0 if the other candidate wins, and payoff of  $1/2$  if the candidates tie; both citizens are expected value maximizers. Citizen 1 is informed of the state, whereas citizen 2 believes it is  $A$  with probability 0.9 and  $B$  with probability 0.1. Each citizen may either vote for candidate 1, vote for candidate 2, or not vote.
  - (a) Formulate this situation as a Bayesian game. (Construct the table of payoffs for each state.)
  - (b) Show that the game has exactly two pure Nash equilibria, in one of which citizen 2 does not vote and in the other of which she votes for 1.
  - (c) Show that an action of one of the players in the second equilibrium is weakly dominated.
  - (d) Why is "swing voter's curse" an appropriate name for the determinant of citizen 2's decision in the first equilibrium?
  
3. (Double auction) A single seller and a single buyer may trade 0 or 1 unit of a good. The seller (player 1) has cost  $c$ , and the buyer (player 2) has valuation  $v$ , where  $v$  and  $c$  both are private information and are uniformly distributed in  $[0, 1]$ . The seller and the buyer simultaneously and independently choose bids  $b_1, b_2 \in [0, 1]$ . If  $b_1 \leq b_2$ , the two parties trade at price  $t = (b_1 + b_2)/2$ . If  $b_1 > b_2$ , the parties do not trade. The seller's payoff is thus  $u_1 = (b_1 + b_2)/2 - c$  if  $b_1 \leq b_2$ , and 0 if  $b_1 > b_2$ ; the buyer's payoff is  $u_2 = v - (b_1 + b_2)/2$  if  $b_1 \leq b_2$ , and 0 if  $b_1 > b_2$ . Suppose each player's strategy is a strictly increasing, linear function of his type. Solve for the equilibrium.

#### 4. Judicial Procedures

There are three alternative procedures to determine the outcome of a criminal court case. Each has its merits, and you might want to choose among them based on some underlying principles.

1. Status Quo: First determine innocence or guilt, then if guilty consider the appropriate punishment.
2. Roman Tradition: After hearing the evidence, start with the most serious punishment and work down the list. First decide if the death penalty should be imposed for this case. If not, then decide whether a life sentence is justified. If, after proceeding down the list, no sentence is imposed, then the defendant is acquitted.
3. Mandatory Sentencing: First specify the sentence for the crime. Then determine whether the defendant should be convicted.

The difference between these systems is only one of agenda: what gets decided first. To illustrate how important this can be, we consider a case with only three possible outcomes: the death penalty, life imprisonment, and acquittal.

The defendant's fate rests in the hands of three judges. Their decision is determined by a majority vote. This is particularly useful since the three judges are deeply divided.

Judge A holds that the defendant is guilty and should be given the maximum possible sentence. This judge seeks to impose the death penalty. Life imprisonment is her second choice and acquittal is her worst outcome.

Judge B also believes that the defendant is guilty. However, this judge unyieldingly opposes the death penalty. Her most preferred outcome is life imprisonment. The precedent of imposing a death sentence is sufficiently troublesome that she would prefer to see the defendant acquitted rather than executed by the state.

Judge C holds that the defendant is innocent, and thus seeks acquittal. She is on the other side of the fence from the second judge, believing that life in prison is a fate worse than death. (On this the defendant concurs.) Consequently, if acquittal fails, her second-best outcome would be to see the defendant sentenced to death. Life in prison would be the worst outcome.

	Judge A's ranking	Judge B's ranking	Judge C's ranking
Best	Death Sentence	Life in Prison	Acquittal
Middle	Life in Prison	Acquittal	Death Sentence
Worst	Acquittal	Death Sentence	Life in Prison

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What will the judges do under each procedure?