

1. A monopolist can produce at a constant average (and marginal) cost $AC = MC = 5$. The market demand curve is $Q = 53 - P$.
 - (1) Calculate the monopolist's profit-maximizing price and output, and calculate its profit.
 - (2) Assume that a second vendor joins the market. The output of the first manufacturer is Q_1 , and the output of the second manufacturer is Q_2 . Market demand function is now $Q_1 + Q_2 = 53 - P$. The second manufacturer has the same cost as the first manufacturer. Calculate the profit of each manufacturer (as a function of Q_1 and Q_2).
 - (3) Find the "response curve" of each manufacturer. (Assume that each firm chooses its profit-maximizing production level, when its competitor's output is given).
 - (4) Calculate Cournot equilibrium. What is the market price? What is the profit of each manufacturer? (That is, calculate the value of Q_1 , Q_2 , P , and the profit of each manufacturer based on question (2) and (3))
 - (5) Now there are N manufacturers in this industry, each has the same cost $AC = MC = 5$. Find Cournot equilibrium. How many products each manufacturer will produce? What will the market price be? And how much profits each manufacturer will make? Also, prove that when N becomes larger, the market price is close to the price under perfect competition.

2. Consider the Bertrand duopoly model with homogeneous products. Suppose that the quantity that consumers demand from i is $a - p_i$ when $p_i < p_j$, 0 when $p_i > p_j$, and $(a - p_i)/2$ when $p_i = p_j$. Suppose also that there are no fixed costs and that marginal costs are constant at c , where $c < a$. Show that if the firms choose prices simultaneously, then the unique Nash equilibrium is that both firms charge the price c .

3. The game being played is the one shown below; call this the "true" game

		2	
		L	R
1	T	6, 3	0, 9
	B	3, 3	3, 0

Player 1 knows that she is playing this game, while Player 2 is uncertain as to whether she is playing this game or a different game, shown below, where the payoffs of Player 1 are different:

		2	
		L	R
1	T	0, 3	3, 9
	B	3, 3	0, 0

For convenience, refer to the “true game” as the game where Player 1 is of type b and the “different game” as the game where Player 1 is of type a .

Suppose that Player 2 assigns probability $\frac{2}{3}$ to Player 1 being of type a and probability $\frac{1}{3}$ to Player 1 being of type b . Find a Nash equilibrium.

4. Consider a Cournot duopoly operating in a market with inverse demand $P(Q) = a - Q$, where $Q = q_1 + q_2$ is the aggregate quantity on the market. Both firms have total costs $c_i(q_i) = cq_i$, but demand is uncertain: it is high ($a = a_H$) with probability θ and low ($a = a_L$) with probability $1 - \theta$. Furthermore, information is asymmetric: firm 1 knows whether demand is high or low, but firm 2 does not. All of this is common knowledge. The two firms simultaneously choose quantities. What are the strategy spaces for the two firms? Make assumptions concerning a_H , a_L , θ , and c such that all equilibrium quantities are positive. What is the Bayesian Nash equilibrium of this game?

5. The following static game of complete information (Matching Pennies) has no pure-strategy Nash equilibrium but has one mixed-strategy Nash equilibrium: each player plays H with probability $\frac{1}{2}$.

		2	
		H	T
1	H	1, -1	-1, 1
	T	-1, 1	1, -1

Provide a pure-strategy Bayesian Nash equilibrium of a corresponding game of incomplete information such that as incomplete information disappears, the players' behavior in the Bayesian Nash equilibrium approaches their behavior in the mixed-strategy Nash equilibrium in the original game of complete information.

Hint: Consider the following Bayesian game in which t_1 and t_2 are independently and uniformly distributed on $[0, x]$.

		2	
		H	T
1	H	$1+t_1$, -1	-1, $1-t_2$
	T	-1, 1	1, -1