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Temperature-Anisotropy Driven Mirror Waves in Space Plasma

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Abstract

In this paper an analysis of the excitation conditions of mirror waves is done, which propagate parallel to an external magnetic field. There are found analytical expressions for the dispersion relations of the waves in case of different plasma conditions. These relations may be used in future to develop the nonlinear theory of mirror waves. In comparison with former analytical works, in the study the influence of the magnetic field and finite temperatures of the ions parallel to the magnetic field are taken into account. Application is done for the earth's magnetosheath.

1 Introduction

The calculation of relaxation times of plasmas with temperature-anisotropy driven turbulence is very important for the interpretation of recent satellite data obtained for the solar (CORONAS, SOHO) and the magnetosheath (AMPTE, ISEE1, ISEE2) plasmas [1, 2]. It was already shown that under the condition of parallel proton plasma-beta $\beta_{p\|}$ larger than unity, the mirror instability seems to be the fastest growing plasma instability in an electron-proton plasma with $T_{p\perp} > T_{p\|}$ and $T_e \lesssim T_i$ [3, 4] ($\beta_{p\|} = 2\mu_o n_p k_B T_{p\|}/B_o^2$ is the parallel proton plasma-β, T_p and T_e are the mean proton and electron temperatures, $\vec{B}_o$ designates the mean magnetic induction, n_p is the proton density). Such conditions exist in the earth's magnetosheath (see Table 1 and Fig. 1 representing experimental data given in [2]). Thus, here an analysis of the excitation conditions of mirror waves is done, which propagate parallel to the magnetic induction $\vec{B}_o$.

t $[UT]$	x $[km]$	B_o $[\gamma]$	v $[km/s]$	n $[cm^{-3}]$	$T_{p\perp}$ $[10^6K]$	$T_{p\|}$ $[10^6K]$	T_e $[10^6K]$
9.45	0	20	200	13	6.0	4.5	0.94
10.33	6040	21	185	15	6.0	4.5	1.04
11.23	12330	23	175	16	6.0	4.2	0.79
12.13	18620	30	160	15	5.8	4.0	0.80
12.53	23700	60	130	11	5.0	2.1	0.58
13.03	24910	68	125	6	4.5	1.4	0.61

Table 1. Magnetosheath crossing of AMPTE satellite on Oct.24, 1985 [2]. x, $B_o(x)$, $v(x)$, $n(x)$, T_e, $T_{p\perp}$, and $T_{p\|}$ are obtained by the experiment. t is the time of the measurement.

2 Dispersion relation for anisotropic convecting plasmas

Describing the distributions of the convective anisotropic plasma components by a bi-maxwellian function shifted along the mean magnetic field $\vec{B}_o = B_o \vec{n}_z$ by the drift velocities v_{Da}

$$f_o = \frac{1}{\pi^{3/2}} \frac{1}{v_{a||} v_{a\perp}^2} \exp\left\{ -\frac{(v_z - v_{Da})^2}{v_{a||}^2} - \frac{v_\perp^2}{v_{a\perp}^2} \right\}, \quad v_{a\perp,||}^2 = \frac{2k_B T_{a\perp,||}}{m_a}, \tag{1}$$

the dispersion relation of waves propagating parallel to the drift velocities v_{Da} falls into two equations

$$D_{zz} = 1 + \frac{2}{k_z^2} \sum_a \frac{\omega_{pa}^2}{v_{a||}^2} \left(1 + \frac{\omega - k_z v_{Da}}{k_z v_{a||}} Z\left\{ \frac{\omega - k_z v_{Da}}{k_z v_{a||}} \right\} \right) = 0 \tag{2}$$

and

$$D^\pm = D_{xx} \pm i D_{xy} = 1 - \frac{c^2 k_z^2}{\omega^2} - \sum_a \frac{\omega_{pa}^2}{\omega^2} \left(\left[1 - \frac{v_{a\perp}^2}{v_{a||}^2} \right] \right. \tag{3}$$

$$\left. + \left[\frac{(k_z v_{Da} - \omega)}{k_z v_{a||}} + \frac{(\omega - k_z v_{Da} \pm \omega_{ca})}{k_z v_{a||}} \left(1 - \frac{v_{a\perp}^2}{v_{a||}^2} \right) \right] Z\left\{ \frac{\omega - k_z v_{Da} \pm \omega_{ca}}{k_z v_{a||}} \right\} \right) = 0.$$

$Z\{z\} = 2ie^{-z^2} \int_{-\infty}^{iz} e^{-t^2} dt$ is the plasma dispersion function. $Z(z)$ can also be presented by Taylor series,

$$Z\{| z |< 1\} = i\sqrt{\pi} e^{-z^2} - 2z - \frac{4z^2}{3} + ..., \quad Z\{| z |> 1\} = i\sqrt{\pi}\sigma e^{-z^2} - \frac{1}{z} - \frac{1}{z^3} + ..., \tag{4}$$

$$\sigma = \begin{cases} 0 & \text{for Im } z > 1/ \mid \text{Re } z \mid \\ 1 & \text{for } \mid \text{Im } z \mid < 1/ \mid \text{Re } z \mid . \\ 2 & \text{for -Im } z > 1/ \mid \text{Re } z \mid \end{cases} \tag{5}$$

$D^\pm$ depends on the anisotropy of the temperature components of the charges parallel and perpendicular to the magnetic field, and D_{zz} does not depend on it. Thus, while there will be no additional instabilities of linearly polarized waves (Eq. (2)) because of the temperature anisotropy, new types of unstable circularly polarized waves may be excited. Neglecting the particle drifts and looking for mirror waves only, it follows from Eq. (3)

$$-\gamma^2 = c^2 k_z^2 + \sum_a \omega_{pa}^2 \left(1 - \frac{v_{a\perp}^2}{v_{a||}^2} + \left[\pm \frac{\omega_{ca}}{k_z v_{a||}} - \frac{v_{a\perp}^2 (i\gamma \pm \omega_{ca})}{k_z v_{a||}^3} \right] Z\left\{ \frac{i\gamma \pm \omega_{ca}}{k_z v_{a||}} \right\} \right). \tag{6}$$

As $\gamma^2 > 0$ and $k_z^2 > 0$, one has the instability condition

$$\sum_a \omega_{pa}^2 \left(1 - \frac{v_{a\perp}^2}{v_{a||}^2} + \left[\pm \frac{\omega_{ca}}{k_z v_{a||}} - \frac{v_{a\perp}^2 (i\gamma \pm \omega_{ca})}{k_z v_{a||}^3} \right] Z\left\{ \frac{i\gamma \pm \omega_{ca}}{k_z v_{a||}} \right\} \right) < 0. \tag{7}$$

In the case $\gamma < k_z^2 v_{a||}^2/\omega_{ca} < \omega_{ca}$, Eq. (6) may be transformed into

$$-\gamma^2 = c^2 k_z^2 + \sum_a \omega_{pa}^2 \Big(1 - \frac{v_{a\perp}^2}{v_{a||}^2} - \left[\pm \frac{\omega_{ca}}{k_z v_{a||}} - \frac{v_{a\perp}^2 (i\gamma \pm \omega_{ca})}{k_z v_{a||}^3} \right] \cdot \left[\frac{k_z v_{a||}}{i\gamma \pm \omega_{ca}} + \frac{k_z^3 v_{a||}^3}{2(i\gamma \pm \omega_{ca})^3} - i\sqrt{\pi} \exp\left\{ -\frac{\omega_{ca}^2}{k_z^2 v_{a||}^2} \right\} \right] \Big), \tag{8}$$

and thus one finds in second order with respect to $\mid \gamma/\omega_{ca} \mid$

$$-\gamma^2 \left(1 + \frac{c^2}{v_A^2} + \frac{3k_z^2 c^2}{2} \sum_a \frac{(2\beta_{a||} - \beta_{a\perp})}{\omega_{ca}^2} \right) = \frac{c^2 k_z^2}{2} \left(2 + \beta_\perp - \beta_{||} \pm i\gamma \sum_a \frac{(3\beta_{a||} - 2\beta_{a\perp})}{\omega_{ca}} \right) + \sum_a \omega_{pa}^2 \left[\pm\omega_{ca} - \frac{v_{a\perp}^2}{v_{a||}^2} (i\gamma \pm \omega_{ca}) \right] \frac{i\sqrt{\pi}}{k_z v_{a||}} \exp\left\{ -\frac{\omega_{ca}^2}{k_z^2 v_{a||}^2} \right\}, \tag{9}$$

$$\beta_\perp = \sum_{a=e,p} \beta_{a\perp}, \quad \beta_{||} = \sum_{a=e,p} \beta_{a||}.$$

Thus, an instability might occur in plasmas with $\beta_{||} \gg 2 + \beta_\perp$, for instance in solar flares, the solar wind and the bow-shock at the dayside of the earth's magnetosheath. In the contrary case, $\mid \gamma \mid \lesssim \mid \omega_{ca} \mid \ll \mid k_z v_{a||} \mid$, taking the plasma dispersion function in first order with respect to the argument $(i\gamma \pm \omega_{ca})/k_z v_{a||}$ into account, the dispersion relation reads

$$-\gamma^2 \left[1 - \frac{2}{k_z^2} \sum_a \frac{\omega_{pa}^2 v_{a\perp}^2}{v_{a||}^4} \right] = c^2 k_z^2 + \sum_a \omega_{pa}^2 \left[\left(1 - \frac{v_{a\perp}^2}{v_{a||}^2} \right) \left(1 - \frac{2\omega_{ca}^2}{k_z^2 v_{a||}^2} \right) + \frac{\sqrt{\pi}\gamma v_{a\perp}^2}{k_z v_{a||}^3} \pm \frac{i\sqrt{\pi}\omega_{ca}}{k_z v_{a||}} \left(1 - \frac{v_{a\perp}^2}{v_{a||}^2} \right) \mp \frac{2i\gamma\omega_{ca}}{k_z^2 v_{a||}^2} \right]. \tag{10}$$

Thus, in such a case, plasma instability may be excited if $v_{a\perp} > v_{a||}$. This condition is satisfied in the inner of the earth's magnetosheath. Fig. 1 shows numerical results for the growth rate of left-hand (lower sign in Eq. 10) circularly polarized mirror waves along the AMPTE satellite trajectory from the bow shock ($h = 0$) to the magnetopause ($h \approx 2.5 \cdot 10^7$ m) calculated for the magnetosheath parameters presented in Table 1. It is found that the growth rate of the mirror waves in the middle of the magnetosheath is larger than the growth rate near the magnetopause. That means the growth rate of the waves increases with increasing plasma-beta and increasing temperature anisotropy.

3 Conclusions

- The excitation of mirror waves in magnetized collision-free space plasmas with temperature anisotropy with respect to the mean magnetic field is studied.
- Analytical expressions for the wave dispersion are derived, which take into account additional physical effects, but are yet simple enough so that they may be used within future nonlinear wave studies.
- It is found that under the condition $\mid (\gamma - \omega_{ci}) \mid \gg k_z v_{a||}$, parallel to the magnetic

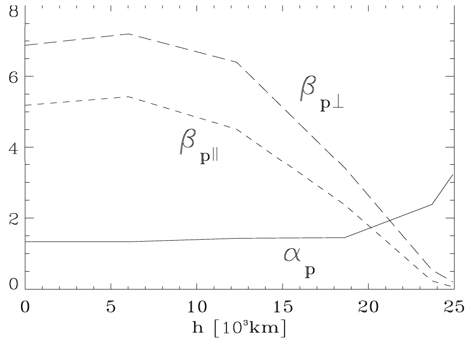

Fig. 1. Proton plasmas beta and proton-anisotropy along the satellite trajectory from the bow shock (BS) to the magnetopause (MP).

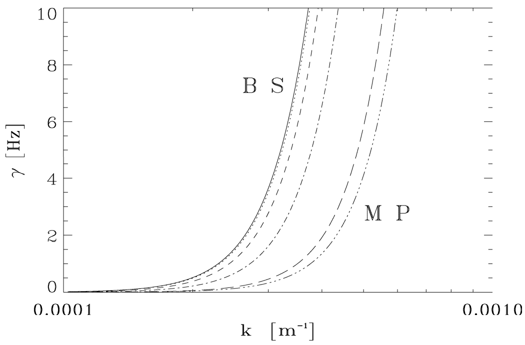

Fig. 2. Growth rate of mirror waves as function of the wave number in the case $\mid \gamma \mid \lesssim \mid \omega_{ca} \mid \ll \mid k_z v_{a||} \mid$.

field propagating circularly-polarized mirror waves may be excited if the parallel temperature components are larger than the perpendicular ones, and, in the case $\mid (\gamma - \omega_{ci}) \mid \ll k_z v_{a||}$, along the magnetic field direction propagating circularly-polarized mirror waves are generated when the perpendicular temperatures are larger than the parallel ones.

• It is shown that under the conditions obtained by the AMPTE satellite on October 24, 1985, quasi-parallel to the mean magnetic field propagating mirror waves might have been excited in the magnetosheath. There the growth rate of the waves increases with increasing plasma-beta and increasing temperature anisotropy.

• The general behaviour of the calculated mirror-wave increment along the AMPTE satellite trajectory coincides with the experimentally found mirror wave threshold $K_{ph} = T_{p\perp}/T_{p||} - (1 + 0.85\beta_{p||}^{0.5})$ given by Phan et al. [5]

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