2. a

Regular Gradient Descent

Alpha: 0.00820  # of interations: 117  Cost: 11.70771
Alpha: 0.00810  # of interations: 118  Cost: 11.70900
Alpha: 0.00800  # of interations: 120  Cost: 11.70933
Alpha: 0.00790  # of interations: 122  Cost: 11.70968
Alpha: 0.00780  # of interations: 124  Cost: 11.71006
Alpha: 0.00770  # of interations: 126  Cost: 11.71047
Alpha: 0.00760  # of interations: 128  Cost: 11.71090
Alpha: 0.00750  # of interations: 130  Cost: 11.71136
Alpha: 0.00740  # of interations: 132  Cost: 11.71185
Alpha: 0.00730  # of interations: 134  Cost: 11.71237
Alpha: 0.00720  # of interations: 135  Cost: 11.71391
Alpha: 0.00710  # of interations: 137  Cost: 11.71450
Alpha: 0.00700  # of interations: 139  Cost: 11.71514
Alpha: 0.00690  # of interations: 142  Cost: 11.71484
Alpha: 0.00680  # of interations: 144  Cost: 11.71556
Alpha: 0.00670  # of interations: 146  Cost: 11.71632
Alpha: 0.00660  # of interations: 148  Cost: 11.71714
Alpha: 0.00650  # of interations: 150  Cost: 11.71800
Alpha: 0.00640  # of interations: 152  Cost: 11.71892
Alpha: 0.00630  # of interations: 154  Cost: 11.71990
Alpha: 0.00620  # of interations: 157  Cost: 11.71996
Alpha: 0.00610  # of interations: 159  Cost: 11.72106
Alpha: 0.00600  # of interations: 161  Cost: 11.72223
Alpha: 0.00590  # of interations: 164  Cost: 11.72249
Alpha: 0.00580  # of interations: 166  Cost: 11.72381
Alpha: 0.00570  # of interations: 169  Cost: 11.72422
Alpha: 0.00560  # of interations: 171  Cost: 11.72569
Alpha: 0.00550  # of interations: 174  Cost: 11.72628
Alpha: 0.00540  # of interations: 176  Cost: 11.72793
Alpha: 0.00530  # of interations: 179  Cost: 11.72870
Alpha: 0.00520  # of interations: 182  Cost: 11.72957
Alpha: 0.00510  # of interations: 185  Cost: 11.73055
Alpha: 0.00500  # of interations: 187  Cost: 11.73263
Alpha: 0.00490  # of interations: 190  Cost: 11.73384
Alpha: 0.00480  # of interations: 193  Cost: 11.73518
Alpha: 0.00470  # of interations: 197  Cost: 11.73567
Alpha: 0.00460  # of interations: 200  Cost: 11.73729
Alpha: 0.00450  # of interations: 203  Cost: 11.73906
Alpha: 0.00440  # of interations: 207  Cost: 11.74001
Alpha: 0.00430  # of interations: 210  Cost: 11.74212
Alpha: 0.00420  # of interations: 214  Cost: 11.74342
Alpha: 0.00410  # of interations: 218  Cost: 11.74492
Alpha: 0.00400  # of interations: 221  Cost: 11.74761
Alpha: 0.00390  # of interations: 225  Cost: 11.74954
Alpha: 0.00380  # of interations: 230  Cost: 11.75072

Alpha: 0.00370  # of interations: 234  Cost: 11.75315
Alpha: 0.00360  # of interations: 238  Cost: 11.75585
Alpha: 0.00350  # of interations: 243  Cost: 11.75785
Alpha: 0.00340  # of interations: 248  Cost: 11.76017
Alpha: 0.00330  # of interations: 253  Cost: 11.76284
Alpha: 0.00320  # of interations: 258  Cost: 11.76588
Alpha: 0.00310  # of interations: 264  Cost: 11.76833
Alpha: 0.00300  # of interations: 269  Cost: 11.77221
Alpha: 0.00290  # of interations: 275  Cost: 11.77557
Alpha: 0.00280  # of interations: 282  Cost: 11.77847
Alpha: 0.00270  # of interations: 288  Cost: 11.78292
Alpha: 0.00260  # of interations: 295  Cost: 11.78702
Alpha: 0.00250  # of interations: 303  Cost: 11.79081
Alpha: 0.00240  # of interations: 310  Cost: 11.79636
Alpha: 0.00230  # of interations: 318  Cost: 11.80177
Alpha: 0.00220  # of interations: 327  Cost: 11.80713
Alpha: 0.00210  # of interations: 336  Cost: 11.81355
Alpha: 0.00200  # of interations: 346  Cost: 11.82017
Alpha: 0.00190  # of interations: 357  Cost: 11.82715
Alpha: 0.00180  # of interations: 368  Cost: 11.83566
Alpha: 0.00170  # of interations: 380  Cost: 11.84495
Alpha: 0.00160  # of interations: 393  Cost: 11.85527
Alpha: 0.00150  # of interations: 407  Cost: 11.86695
Alpha: 0.00140  # of interations: 423  Cost: 11.87942
Alpha: 0.00130  # of interations: 439  Cost: 11.89517
Alpha: 0.00120  # of interations: 458  Cost: 11.91185
Alpha: 0.00110  # of interations: 478  Cost: 11.93231
Alpha: 0.00100  # of interations: 500  Cost: 11.95663
Alpha: 0.00090  # of interations: 524  Cost: 11.98628
Alpha: 0.00080  # of interations: 551  Cost: 12.02230
Alpha: 0.00070  # of interations: 580  Cost: 12.06852
Alpha: 0.00060  # of interations: 612  Cost: 12.12809
Alpha: 0.00050  # of interations: 643  Cost: 12.21122
Alpha: 0.00040  # of interations: 668  Cost: 12.33250
Alpha: 0.00030  # of interations: 665  Cost: 12.52812
Alpha: 0.00020  # of interations: 561  Cost: 12.88404
Alpha: 0.00010  # of interations: 501  Cost: 13.29143

Newton’s Method Gradient Descent

```
def log_hess(theta, x):
        g = logistic_func(theta,x)
        hess = 0
        for index in range(x.shape[0]-1):
                x_i = x[index,:]
                hess += x_i.T.dot(x_i.T)*g[index]*(1-g[index])
        return hess

def grad_desc(theta, x, y, tol, maxiter):
        nll_vec = []
        nll_vec.append(neg_log_like(theta, x, y))
        nll_delta = 2.0*tol
        iter = 0
        while(nll_delta > tol) and (iter < maxiter):
                alpha = 1/log_hess(theta, x)
                theta = theta -(alpha * log_grad(theta, x, y))
                nll_vec.append(neg_log_like(theta, x, y))
                nll_delta = nll_vec[-2] - nll_vec[-1]
                iter += 1
        return theta, np.array(nll_vec), iter, alpha
```

Stochastic Gradient Descent

```
def grad_desc(theta, x, y, alpha, tol, maxiter):
        nll_vec = []
        data = np.c_[x,y]
        batch = int(floor(data.shape[0]/5))
        nll_vec.append(neg_log_like(theta, x[0:batch,:], y[0:batch]))
        nll_delta = 2.0*tol
        iter = 0
        while(nll_delta > tol) and (iter < maxiter):
                data = np.random.permutation(data)
                theta = theta - (alpha * log_grad(theta, data[0:batch,:-1], data[0:batch,-1]))
                nll_vec.append(neg_log_like(theta, data[0:batch,:-1], data[0:batch,-1]))
                nll_delta = nll_vec[-2] - nll_vec[-1]
                iter += 1
        return theta, np.array(nll_vec), iter
```

Resulting Observation

```
Alpha: 0.50000  # of interations: 2  Cost: 129.62750
Alpha: 0.49900  # of interations: 1  Cost: 203.62524
Alpha: 0.49800  # of interations: 2  Cost: 251.77695
Alpha: 0.49700  # of interations: 1  Cost: 32.00748
Alpha: 0.49600  # of interations: 1  Cost: 167.49938
Alpha: 0.49500  # of interations: 2  Cost: 149.26191
Alpha: 0.49400  # of interations: 1  Cost:   inf
Alpha: 0.49300  # of interations: 1  Cost:   inf
Alpha: 0.49200  # of interations: 1  Cost: 176.44268
Alpha: 0.27300  # of interations: 1  Cost: 52.43674
Alpha: 0.27200  # of interations: 1  Cost: 44.35423
Alpha: 0.27100  # of interations: 2  Cost: 61.14006
Alpha: 0.27000  # of interations: 1  Cost: 169.95902
Alpha: 0.26900  # of interations: 1  Cost: 82.70133
Alpha: 0.26800  # of interations: 1  Cost: 62.11942
Alpha: 0.26700  # of interations: 1  Cost: 197.65449
Alpha: 0.26600  # of interations: 2  Cost: 171.02543
Alpha: 0.26500  # of interations: 1  Cost: 83.40872
Alpha: 0.26400  # of interations: 3  Cost: 58.21649
Alpha: 0.26300  # of interations: 1  Cost: 173.10278
Alpha: 0.26200  # of interations: 1  Cost: 173.59790
Alpha: 0.26100  # of interations: 1  Cost: 29.24289
Alpha: 0.26000  # of interations: 1  Cost: 64.17494
Alpha: 0.25900  # of interations: 1  Cost: 64.95948
Alpha: 0.25800  # of interations: 1  Cost: 58.31495
Alpha: 0.25700  # of interations: 1  Cost: 143.58909
Alpha: 0.25600  # of interations: 1  Cost: 88.13453
Alpha: 0.25500  # of interations: 1  Cost:   inf
Alpha: 0.25400  # of interations: 1  Cost: 56.57836
Alpha: 0.25300  # of interations: 1  Cost: 88.17467
Alpha: 0.25200  # of interations: 1  Cost: 45.47287
```

Alpha: 0.25100  # of interations: 1  Cost: 14.31586
Alpha: 0.25000  # of interations: 1  Cost: 187.48836
Alpha: 0.24900  # of interations: 2  Cost: 31.15759
Alpha: 0.24800  # of interations: 1  Cost: 56.08093
Alpha: 0.24700  # of interations: 1  Cost:   inf
Alpha: 0.24600  # of interations: 1  Cost: 20.57147
Alpha: 0.24500  # of interations: 1  Cost: 68.99551
Alpha: 0.24400  # of interations: 1  Cost: 43.89835
Alpha: 0.24300  # of interations: 1  Cost: 55.58632
Alpha: 0.24200  # of interations: 3  Cost: 37.29635
Alpha: 0.24100  # of interations: 1  Cost: 134.41976
Alpha: 0.24000  # of interations: 1  Cost: 83.45740
Alpha: 0.23900  # of interations: 2  Cost: 3.34446
Alpha: 0.23800  # of interations: 1  Cost: 103.12732
Alpha: 0.23700  # of interations: 1  Cost: 78.81791
Alpha: 0.23600  # of interations: 2  Cost: 183.59118
Alpha: 0.23500  # of interations: 1  Cost: 51.38854
Alpha: 0.23400  # of interations: 1  Cost: 21.50910
Alpha: 0.23300  # of interations: 1  Cost: 220.49460
Alpha: 0.23200  # of interations: 1  Cost: 95.56876
Alpha: 0.23100  # of interations: 1  Cost: 227.26913
Alpha: 0.23000  # of interations: 2  Cost: 120.59612
Alpha: 0.22900  # of interations: 2  Cost: 36.04365
Alpha: 0.22800  # of interations: 1  Cost: 141.07218
Alpha: 0.22700  # of interations: 1  Cost: 34.45223
Alpha: 0.22600  # of interations: 2  Cost: 13.31275
Alpha: 0.22500  # of interations: 1  Cost: 65.70742
Alpha: 0.22400  # of interations: 1  Cost: 112.99532
Alpha: 0.22300  # of interations: 1  Cost: 126.51554
Alpha: 0.22200  # of interations: 1  Cost: 36.41009
Alpha: 0.22100  # of interations: 1  Cost: 116.58514
Alpha: 0.22000  # of interations: 2  Cost: 0.00000
Alpha: 0.21900  # of interations: 2  Cost: 171.01833
Alpha: 0.21800  # of interations: 2  Cost: -0.00000
Alpha: 0.21700  # of interations: 1  Cost: 64.01915
Alpha: 0.21600  # of interations: 1  Cost: 125.86349
Alpha: 0.21500  # of interations: 3  Cost: -0.00000
Alpha: 0.21400  # of interations: 2  Cost:   inf
Alpha: 0.21300  # of interations: 1  Cost: 126.29238
Alpha: 0.21200  # of interations: 1  Cost: 197.40709
Alpha: 0.21100  # of interations: 1  Cost: 55.92166
Alpha: 0.21000  # of interations: 2  Cost: 29.58229
Alpha: 0.20900  # of interations: 2  Cost: 19.18745
Alpha: 0.20800  # of interations: 1  Cost: 27.88601
Alpha: 0.20700  # of interations: 2  Cost: 35.17466
Alpha: 0.20600  # of interations: 2  Cost: 20.53279
Alpha: 0.20500  # of interations: 1  Cost: 14.36742
Alpha: 0.20400  # of interations: 1  Cost: 17.46300
Alpha: 0.20300  # of interations: 2  Cost: 44.44431