Dynamic Soaring: Assignment 2

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Case 1 - Range without updraft using Analytical Expression

The aircraft shall not be losing the altitude $\frac{dh}{dt} = 0.0$ implying the aircraft can be decelerated upto V_{stall} (by increasing the C_L (α) upto C_{Lmax} (α_{stall}) and sustain the weight. In this case, the velocity in the wind frame is same as the velocity in the inertial frame of reference and hence with an assumption of $L = C_L\ \frac{1}{2}\ \rho V^2 S = mg$, we get

$$\ddot{x} = \dot{V} = \frac{-\text{Drag}}{m} = -\frac{C_D \frac{1}{2}\rho V^2}{m/S} = -\left[\frac{(C_{Do} + KC_L^2)\frac{1}{2}\rho V^2}{m/S}\right] \tag{1}$$

where C_{Do} is the parasitic drag coefficient; K is the Lift induced drag component factor and $C_L = \frac{2(m/S)g}{\rho V^2}$

The above equation (1) can be written as follows in the form of two first order differential equation as follows:

$$\left\{\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \end{array}\right\} = \left\{\begin{array}{c} x_2 \\ -\left[\frac{(C_{Do}+KC_L^2)\frac{1}{2}\rho x_2^2}{m/S}\right] \end{array}\right\} \tag{2}$$

where $x_1 = x =$ distance and $x_2 = \dot{x} =$ velocity;

Further, the same equation (1) can also be written as,

$$\dot{V} = \frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} = V\frac{dV}{dx} = -\left[C_{Do}\frac{\frac{1}{2}\rho V^2}{(m/S)} + Kg^2\frac{(m/S)}{\frac{1}{2}\rho V^2}\right] \tag{3}$$

which can be written as

$$\frac{dV^2}{dx} = -\left[\left(\frac{C_{Do}\rho}{m/S}\right)V^2 + \left(\frac{4Kg^2(m/S)}{\rho}\right)\left(\frac{1}{V^2}\right)\right] \tag{4}$$

Assume $a = \left(\frac{C_{Do}\rho}{m/S}\right)$, $b = \left(\frac{4Kg^2(m/S)}{\rho}\right)$ and $V^2 = V_{sq}$, then

$$\frac{-V_{sq}dV_{sq}}{aV_{sq}^2 + b} = dx \tag{5}$$

$$\int_{V_i^2}^{V_f^2} \frac{-V_{sq}dV_{sq}}{aV_{sq}^2 + b} = \Delta x \tag{6}$$

Putting $V_{sq} = V^2$ and Integrating the above from V_i^2 to V_f^2 , the distance covered is given by,

$$\Delta x = \frac{1}{2a} ln\left(\frac{aV_i^4 + b}{aV_f^4 + b}\right) \tag{7}$$

where V_i corresponds to the initial velocity and the final velocity V_f corresponds to $C_{L_{max}}$, $V_f = \sqrt{\left(\frac{2(m/S)g}{\rho C_{L_{max}}}\right)}$

Analytically, by substituting various values of the parameters viz. $C_{Do} = 0.01$; $K = \frac{1}{\pi AR} = 0.0212$; $g = 9.80ms^{-2}$; $m/S = 14kgm^{-2}$; $V_i = 20ms^{-1}$; $V_f = \sqrt{\frac{2(m/S)g}{\rho C_{Lmax}}} = 14.9743ms^{-1}$, we can obtain the parameters a and b and thereby range Δx.

$$a = \left(\frac{C_{Do}\rho}{m/S}\right) = 8.75\times 10^{-4} \text{ and } b = \left(\frac{4Kg^2(m/S)}{\rho}\right) = 93.0765 \tag{8}$$

The range obtained is given by

$$\Delta x = \frac{1}{2a} ln\left(\frac{aV_i^4 + b}{aV_f^4 + b}\right) = 292.58\ m \tag{9}$$

Equations of Motion with Updraft

This section gives the equations of motion when an aircraft is encountering an updraft. Assuming that the aircraft is flying steady and level with a velocity of V. The figure 1 gives a detailed explanation of the wind reference frame.

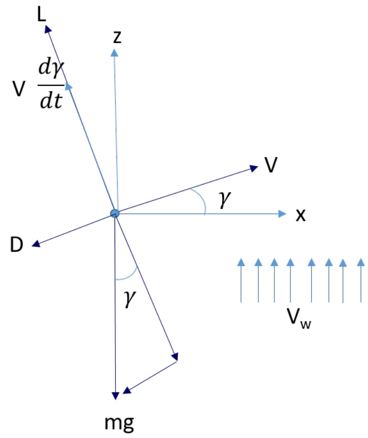

Figure 1: Picture indicating the Reference Frames

Since the wind velocity V_w is considered acting upwards in the direction of z, and is constant, the equations motion with respect to the inertail frame of reference are as follows:

$$\begin{aligned} \frac{dx}{dt} &= V\cos\gamma \\ \frac{dz}{dt} &= V\sin\gamma - V_w \\ \frac{d^2x}{dt^2} &= \frac{dV}{dt}\cos\gamma - V\frac{d\gamma}{dt}\sin\gamma \\ \frac{d^2z}{dt^2} &= \frac{dV}{dt}\sin\gamma + V\frac{d\gamma}{dt}\cos\gamma \end{aligned}$$

The above can also be written as

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} V\cos\gamma \\ V\sin\gamma - V_w \end{bmatrix} \tag{10}$$

and

$$\begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} \dot{V} \\ V\dot{\gamma} \end{bmatrix} \tag{11}$$

Along the wind reference frame, considering the forces of Lift L, Drag D and gravity, the equations of motion are the following:

$$\begin{bmatrix} \dot{V} \\ V\dot{\gamma} \end{bmatrix} = \begin{bmatrix} -\frac{D}{m} - g\sin\gamma \\ \frac{L}{m} - g\cos\gamma \end{bmatrix} \tag{12}$$

where $D = (C_{D_0} + KC_L^2)\frac{1}{2}\rho V^2 S$ and $L = C_L\frac{1}{2}\rho V^2 S$. Further, the relative velocity acting on the point mass is given as $V = \sqrt{V_i^2 + V_w^2}$ where V_i is the initial velocity in the horizontal direction. The navigation equation viz. the equations (10) remains the same in the wind reference frame as well.

Simulation with No Updraft

For the weight to be balanced by lift and to cater for the reduction of speed due to drag, the only possibility is to increase the lift coefficient C_L (increasing the α, the angle of attack) till the stall speed. The stall speed is 14.97 m/s and till this speed, the aircraft can fly without losing altitude without any updraft i.e $V_w = 0.0$. The simulation is performed and variation of speed and the range covered with time till stall is as shown in figure 2. The corresponding control input C_L with time is given in figure 3. The total range obtained by simulation without losing altitude is 292.6 m as and is as estimated using the analytical expression.

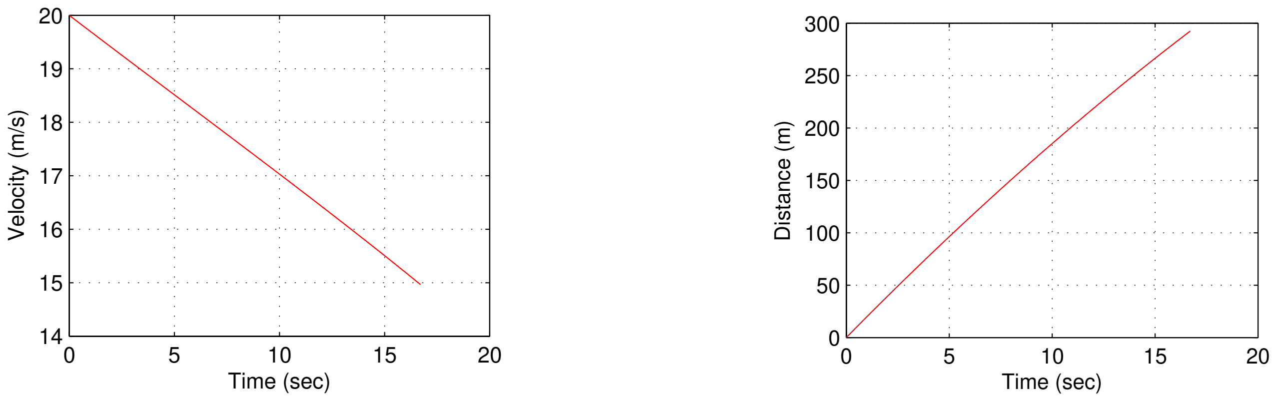

Figure 2: Speed Vs Time and Distance Vs Time

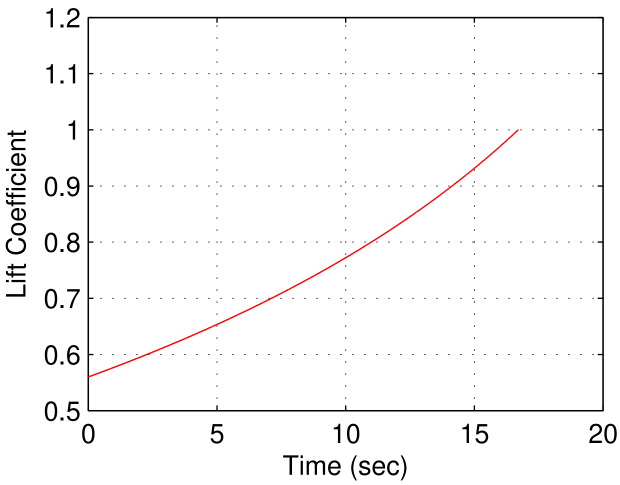

Figure 3: Lift Coefficient C_L Vs Time

Simulation with Updraft

For the given characteristics of the aircraft i.e., $C_{L_{max}} = 1.0$, $C_{D_0} = 1.0$, $AR = 15.0$ with elliptic wing, the glide polar (Speed vs Sink Speed) and the Lift to Drag ratio for different speed is plotted in the figure 4.

When the vertical wind speed is more than or equal to 0.5944 m/s ($V \sin\gamma$) corresponding to the speed of 20.0 m/s $\left[\gamma = \tan^{-1}(\frac{C_D}{C_L})\right]$ the aircraft will not be sinking and hence the aircraft theoretically has infinite range as long as the updraft is available. Further there will be complete force balance without reduction of speed. In

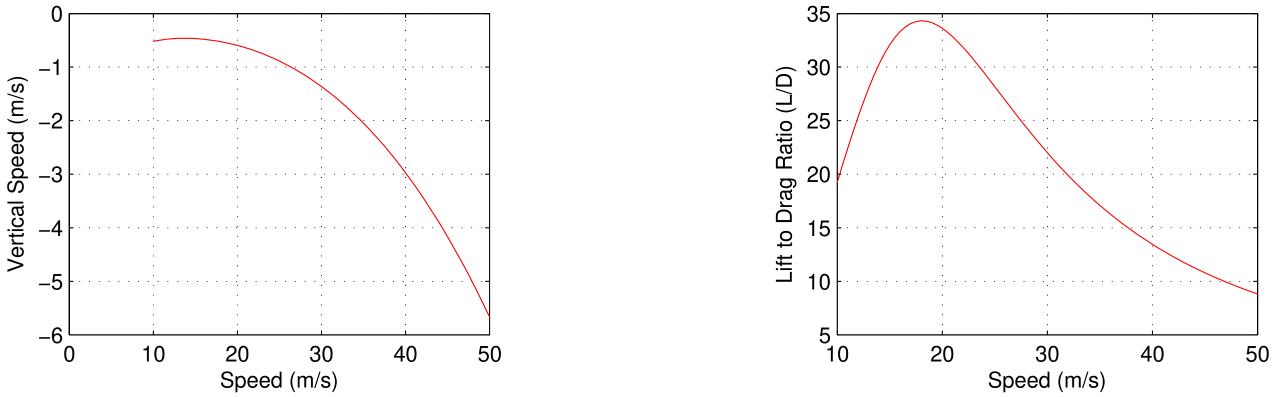

Figure 4: Vertical Speed Vs Speed and Lift to Drag Ratio Vs Speed

addition, when the updraft speed is more than the lowest speed of $V_s = 0.462$ m/s, the aircraft tend to achieve a force balance at a speed much lower than the speed corresponding to $C_{L_{max}}$ and hence the time to cover the distance is higher. However the range is found to be infinite here also.

Simulation is performed with a constraint assumption of $\frac{dz}{dt} = 0.0 \Rightarrow V \sin\gamma = V_w$. Differentiating this equation with time and substituting the value relation of $\frac{dV}{dt}$ and $V\frac{d\gamma}{dt}$, will get an equation in terms of control variable C_L. The resulting quadratic equation on C_L being the control variable is solved and the condition of C_L is imposed to less than 1.0 being the $C_{L_{max}}$.

$$\frac{dV}{dt}\sin\gamma + V\frac{d\gamma}{dt}\cos\gamma = 0.0$$

$$\implies \left(-\frac{(C_{D_0} + KC_L^2)\frac{1}{2}\rho V^2}{m/S} - g\sin\gamma\right)\sin\gamma + \left(\frac{C_L\frac{1}{2}\rho V^2}{m/S} - g\cos\gamma\right)\cos\gamma = 0.0$$

The distance vs time for a vertical updraft velocity of $V_w = 0.4$ is simulated and the figures 5 and 6 indicates the same. The distance obtained for this updraft velocity without losing altitude with an initial condition of $V_i = 20m/s$ is 1417 m before it reaches $C_{L_{max}} = 1.0$.

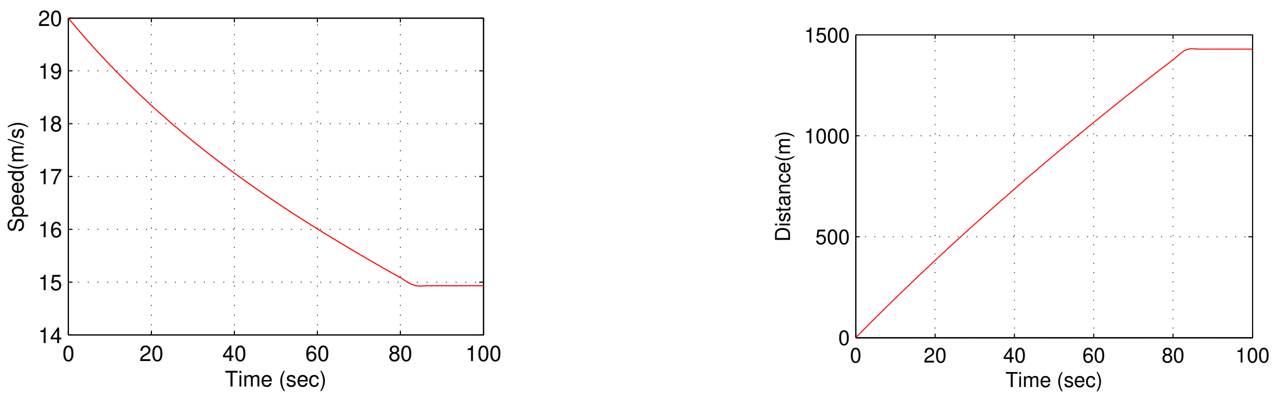

Figure 5: Speed Vs Time and Distance Vs Time for the case of Updraft Speed $V_w = 0.4$ m/s

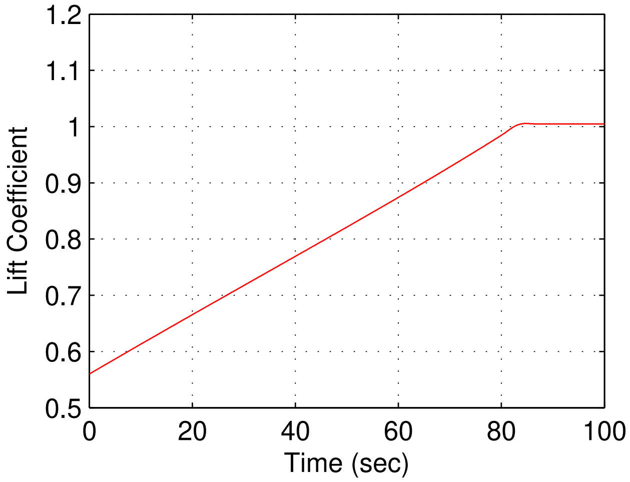

Figure 6: Lift Coefficient C_L Vs Time for the Case of Updraft Speed $V_w = 0.4$ m/s

The range or distance (Δx) for different V_w for the initial velocity of 20 m/s is given below.

Table 1: Range Δx for different updraft Velocity

Updraft (V_w m/s)	Range (Δx)
0.0	293.0
0.1	376.0
0.2	496.0
0.3	729.0
0.4	1417.0

From the glide polar curve [refer figure 4], it is possible to estimate the vertical updraft speed above which the range can be theoretically infinite. The table below 2 indicates the critical updraft speed above which the range is theoretically infinite for different initial speeds.

Table 2: Critical updraft speed for different initial speeds

Initial Speed (V_i m/s)	Critical Updraft Speed (m/s)
20.0	0.59
25.0	0.88
30.0	1.36
35.0	2.05
40.0	2.97

From the simulation, it is also found that at an updraft speed of 0.462 m/s which is the lowest vertical sink speed in the glide polar, much below the critical updraft speed, the aircraft range is very large (infinite). In this case the force balance is achieved at a reduced speed than the initial speed.

The table below 3 gives the range for different initial speeds for different updraft speeds

Table 3: Range (m) for different initial and updraft speeds

V_i (m/s)	Range (m)				
	V_w=0.0	V_w=0.1	V_w=0.2	V_w=0.3	V_w=0.4
20.0	293	376	496	729	1417
25.0	660	777	990	1345	2240
30.0	1010	1176	1419	1830	2795
35.0	1330	1518	1785	2221	3219
40.0	1620	1820	2100	2550	3564

The ratio of range to the range obtained at zero updraft speed for different initial speeds and various updraft is given in the figure 7.

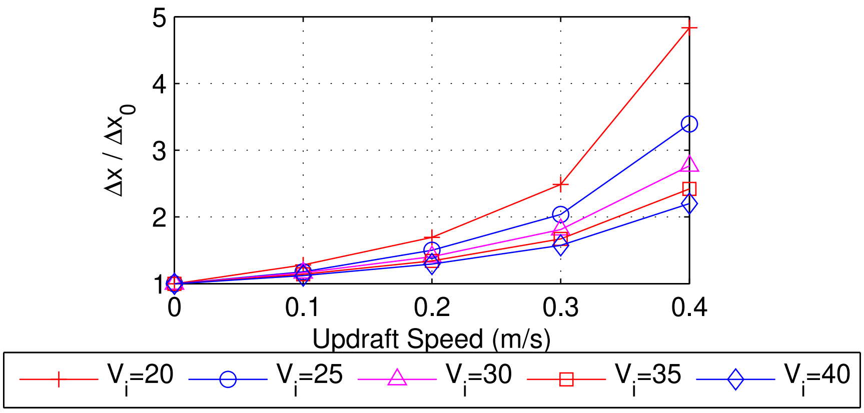

Figure 7: Range Ratio for Different Updraft Speeds

```
%------------------------------------------------------------------------
% Solution of Equations of Motion in Wind Axis Reference Frame
% No updraft and the deceleration of the speed is allowed till Stall speed.
% Initial speed is 20 m/s
%------------------------------------------------------------------------

clear all; close all; clc;

global g rho AR CD0 mbyS Vw

%-----------------------------------------------------------------------
%Initial Conditions and other constants
%-----------------------------------------------------------------------
g  = 9.80; rho = 1.225; AR = 14; CD0 = 0.01; CLmax = 1.0; mbyS = 14;
Vi = 20.0;
Vf = sqrt((2*mbyS*g)/(rho*CLmax));
K = 1/(pi*AR);
Vw = 0.0;

xini = 0.0;
xinit = [xini Vi 100];
tspan = [0:0.01:20];
%----------------------------------------------------------------------
% Solution using ODE45 code
%----------------------------------------------------------------------

[t,x] = ode45(@axwf2,tspan,xinit);

index_if=find((x(:,2)<=Vf));
n=index_if(1);
% imax = size (t,1);
imax = n;

figure(1); subplot (2,2,1);
plot (t([1:n]),CL([1:n]),'r-'); grid on;
xlabel ('Time (sec)');
ylabel ('Lift Coefficient (C_L)');
print -depsc 'case1-time_vs_lift_coeff.eps'

figure(2); subplot (2,2,1);
```

```
% plot (t,x(:,2),'r-'); grid on;
plot (t([1:n]),x([1:n],2), 'r-'); grid on;
xlabel ('Time (sec)');
ylabel ('Velocity (m/s)');
print -depsc 'case1-time_vs_velocity.eps'

figure(3); subplot (2,2,1);
% plot (t,x(:,1),'r-'); grid on;
plot (t([1:n]),x([1:n],1),'r-'); grid on;
xlabel ('Time (sec)');
ylabel ('Range (m)');
print -depsc 'case1-time_vs_distance.eps'

figure(4); subplot (2,2,1);
% plot (t,LbyD,'r-'); grid on;
plot (t([1:n]),LbyD([1:n]),'r-'); grid on;
xlabel ('Time (sec)');
ylabel ('C_L / C_D');

%---------------------------------------------------------------------
% EOF
%---------------------------------------------------------------------

%---------------------------------------------------------------------
% Function axwf2.m called in the main function
%---------------------------------------------------------------------
function dxdt = axwf2(t,x)

global g rho AR CD0 mbyS Vw

K = 1/(pi*AR);
dxdt = zeros(3,1);
gama = tan(Vw / x(2));

CL = (2*mbyS*g*cos(gama))/(rho*(x(2)^2+Vw^2));
CD = CD0 + K*CL^2;
Dbym = (CD*0.5*rho*(x(2)^2+Vw^2))/(mbyS);

% xdot
dxdt(1) = sqrt(x(2)^2+Vw^2)*cos(gama);
```

```
% vdot
dxdt(2) = -Dbym + g*sin(gama);
% zdot
dxdt(3) = sqrt(x(2)^2+Vw^2)*sin(gama);

%--------------------------------------------------------------------
% EOF of Function
%--------------------------------------------------------------------
```

```
%-----------------------------------------------------------------------
% Program to calculate the distance and range when there is a updraft
%-----------------------------------------------------------------------
clear all; close all; clc;

global g rho AR CD0 mbyS Vw
%-----------------------------------------------------------------------
%Initial Conditions and other constants
%-----------------------------------------------------------------------
g  = 9.80; rho = 1.225; AR = 15; CD0 = 0.01; CLmax = 1.0; mbyS = 14;
Vi = 20.0; Vw = 0.40;

Vf = sqrt((2*mbyS*g)/(rho*CLmax));
K = 1/(pi*AR);
gama_ini = asin(-Vw/Vi);
z = 0.0; x = 0.0;
V1 = Vi;
xinit = [V1 gama_ini x z];
tspan = [0:0.1:1000];


% Solution using ODE45

[t,x] = ode45(@axwf3,tspan,xinit);

vel=x(:,1); gama=x(:,2); dist = x(:,3); ht = x(:,4);
n=size(t);
for i=1:n
    CL(i) = 2*mbyS*g/(rho*vel(i)^2);
end

%------------------------------------------------------------------------
%Plotting the results
%------------------------------------------------------------------------
figure(1); subplot (2,2,1);
plot (t,vel,'r-'); grid on;
xlabel ('Time (sec)'); ylabel('Speed(m/s)');
print -depsc 'case2-speed_Vw0_4.eps'

figure(2); subplot (2,2,1);
```

```
plot (t,dist,'r-'); grid on;
xlabel ('Time (sec)'); ylabel ('Distance(m)');
print -depsc 'case2-range_Vw0_4.eps';

figure(3); subplot (2,2,1);
plot (t,CL,'r-'); grid on;
xlabel ('Time (sec)'); ylabel ('Lift Coefficient');
print -depsc 'case2-CL_Vw0_4.eps';



%----------------------------------------------------------------------
% Beginning of Function axwf3.m called by main program
%----------------------------------------------------------------------

function dxdt = axwf3(t,x)

global g rho AR CD0 mbyS Vw

K = 1/(pi*AR);
dxdt = zeros(4,1);

Vel = x(1); gama = x(2); dist = x(3); ht = x(4);

qbar = 0.5*rho*Vel^2;
fac  = qbar/mbyS;
a = K*fac*sin(gama);
b = fac*cos(gama);
c = g + CD0*fac*sin(gama);
p = roots([a -b c]);
if (p(1)<=1 & p(1) >0)
    cL1 = p(1);
elseif (p(2)<=1 & p(2)>0)
    cL1 = p(2);
else
    return
end

CL = cL1;
CD = CD0 + K*CL^2;
Dbym = (CD*0.5*rho*x(1)^2)/mbyS;
```

```
Lbym = (CL*0.5*rho*x(1)^2)/mbyS;

% vdot
dxdt(1) = -Dbym - g*sin(x(2));
% gamadot
dxdt(2) = 1.0/x(1)*(Lbym-g*cos(x(2)));
% xdot
dxdt(3) = x(1)*cos(x(2));
% zdot
dxdt(4) = x(1)*sin(x(2))+Vw;

%--------------------------------------------------------------------
% EOF of Function
%--------------------------------------------------------------------
```

```
%---------------------------------------------------------------------------
% Program to calculate the glide polar of the characteristics
% given in the problem of Assignment No.2
%---------------------------------------------------------------------------
clear all; close all; clc;
cdo = 0.01; clmax = 1.0;
g=9.81; AR=15.0;
mbys = 14.0; k=1/(pi*AR); rho = 1.225;
Vmin=sqrt(2*mbys*g/(rho*clmax));
V = [10:0.5:50]; %forward speed
imax = size(V,2);
for i=1:imax
    cl(i) = (2*mbys*g)/(rho*V(i)^2);
    cd(i) = cdo+k*cl(i)^2;
    DVbyW(i) = (cd(i)*0.5*rho*V(i)^3)/(mbys*g); %sink speed;
    lbyd(i) = V(i)/DVbyW(i);
    gama(i) = asin(1.0/lbyd(i));
end
figure(1); subplot (2,2,1);
plot (V,-DVbyW,'r-'); grid on;
xlabel ('Speed (m/s)');
ylabel ('Vertical Speed (m/s)');
xlim([0 50]);
 print -depsc 'glide1.eps'

figure(2); subplot (2,2,1);
plot (V,lbyd,'r-'); grid on;
xlabel ('Speed (m/s)');
ylabel ('Lift to Drag Ratio (L/D)');
print -depsc 'LbyD.eps';
xlim([0 50]);

figure(3);
plot (V,gama*180/pi,'r-'); grid on;
xlabel ('Speed (m/s');
ylabel ('glide angle (deg)');
xlim([0 50]);
%--------------------------------------------------------------------
% EOF of Function
%--------------------------------------------------------------------
```