Dynamic Soaring: Assignment 7

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Rayleigh Cycle

Parameters of Aircraft

The following are the parameters of Aircraft to be considered for estimating the minimum wind speed required for sustaining dynamic soaring (Rayleigh Cycle)

Table 1: Parameters of Aircraft

AR	15
$CD0$	0.01
ρ	1.225 kg/m^3
$C_{L\text{max}}$	1.2
g	9.81
m	8 kg
μ	$\pi/3$

Typical Rayleigh Cycle

A typical Rayleigh cycle is indicated in the figure below It consists of infinitesimal head wind climb (increasing

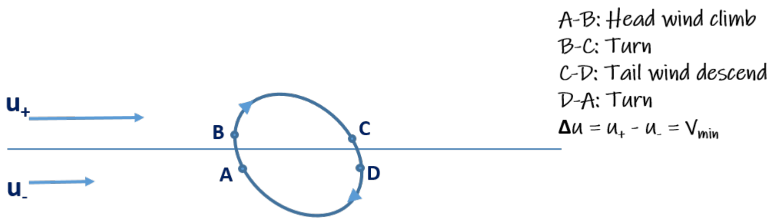

Figure 1: Indicative Rayleigh Cycle

head wind) followed by turn to the tail wind and thereafter infinitesimal descend in the tail wind (decreasing tail wind) followed by turn towards the head wind.

Stall Speed and Estimation of Speed prior to Turn

The stall speed of the aircraft for $m/S = 14\ kg/m^2$ is estimated as follows:

$$V_{\text{stall}} = \sqrt{\frac{2(m/S)g}{\rho C_{L_{max}}}}$$
$$= 13.67\ m/s$$

The above estimated stall speed is the minimum speed at which the aircraft / bird can fly steady and level. However, during turns, the stall speed increases by a factor of $\frac{1}{\sqrt{\cos\mu}}$, where μ is the bank angle (assumed constant) during the turn. Considering the stall speed as the minimum speed after a turn of 180 degrees (π radians) the speed prior to the turn can be estimated using the algebraic expression. The algebraic expression is derived considering a steady level turn with constant bank angle and variation of speed due to drag. In this case, a parabolic drag polar is assumed ($C_D = C_{D_0} + KC_L^2$). The speed prior to the turn is given by the expression:

$$V_i^2 = \sqrt{\frac{b}{a}} \tan\left[-\pi\frac{\sqrt{ab}}{c} + \tan^{-1}\left(V_{\text{stall}}^2\sqrt{\frac{a}{b}}\right)\right] \tag{1}$$

where $c = -\frac{1}{2}\rho g \sin 2\mu$, $a = C_{D0}\rho^2(S/m)\cos^2\mu$ and $b = 4K(m/S)g^2$ For $m/S = 14$ kg/m^2, the stall speed and the corresponding initial speed prior to the turn variation with bank angle (obtained using the algebraic expression mentioned above) is given in the figure 2.

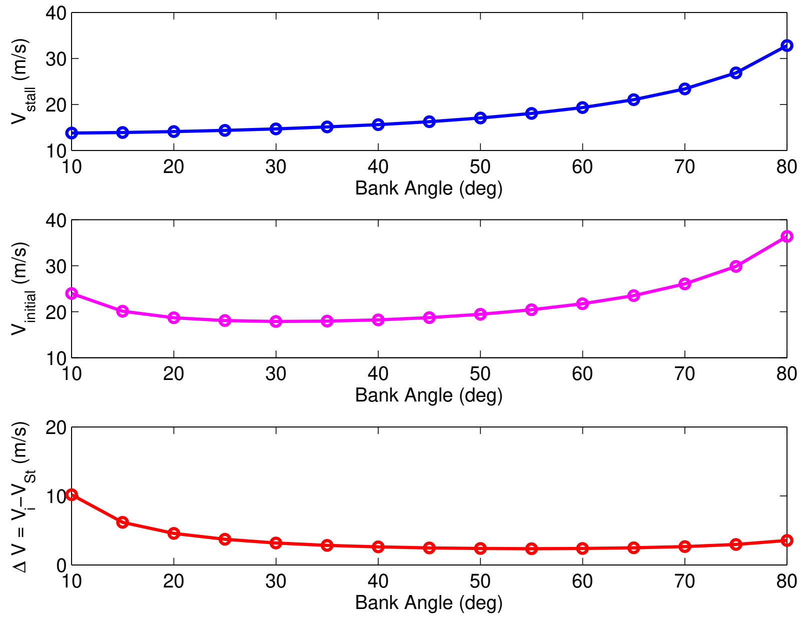

Figure 2: Speed Variation with Bank Angle for $m/S = 14$ kg/m^2

From the plot of ΔV vs Bank Angle, it may be noticed that the difference between the speeds prior and after the turn is much higher at low bank angles than at higher bank angles and very slowly increases at higher bank angles. This is due to the fact, that at low bank angles the speed drops due to very high induced drag.

Minimum Wind Speed Required for Supporting Dynamic Soaring

(a) In view of Rayleigh cycle being an idealized cycle for dynamic soaring and because of the symmetry (refer figure 1), during the energy neutral flight / equilibrium, the following equation holds,

$$f(V_C + \Delta u) = V_C \tag{2}$$

where V_C is the speed after performing an 180 degree turn. By using the 'fsolve' function of Matlab, the minimum wind speed Δu is obtained. The stall speed at point C, $V_C = V_{\text{stall}}$ corresponding to a bank angle of $\pi/3$ is 19.3317 m/s and the obtained minimum wind speed is $\mathbf{\Delta u = 2.3856}$ **m/s** for wing loading $m/S = 14.0$ kg/m^2.

(b) The variation of Δu with bank angle for the wing loading $m/S = 14.0$ kg/m^2 is given in the figure 3

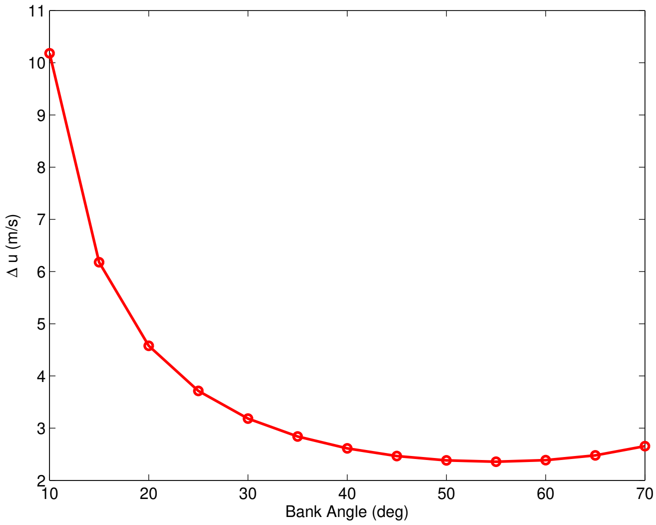

Figure 3: Variation of Δu required for different Bank angle

The table 2 indicates the required Δu required for supporting the dynamic soaring for the wing loading $m/S = 14.0$ kg/m^2:

Table 2: Minimum Wind Speed required for Different Bank Angles

Bank Angle (μ deg)	Δu (m/s)
10	10.1805
20	4.5770
30	3.1834
40	2.6112
50	2.3831
60	2.3856
70	2.6555

Effect of m/S on Required Minimum Wind Speed Δu

The effect of variation of wing loading m/S from 10 kg/m^2 to 30 kg/m^2 on the required minimum wind speed for supporting dynamic soaring is estimated. The variation is given in the figure 4

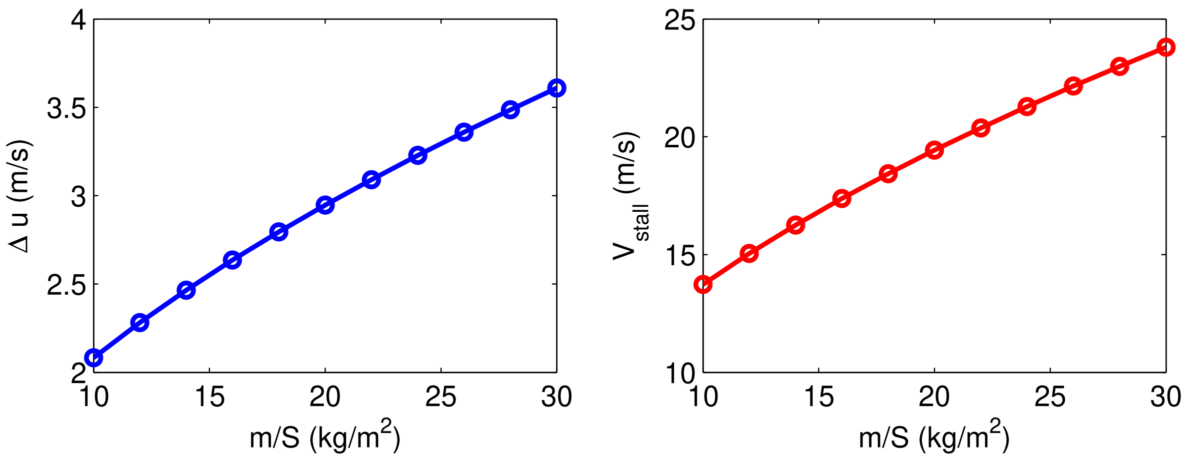

Figure 4: wing loading m/S Vs Δu

It is evident that as the stall speed is increasing due to the increase in wing loading for a fixed bank angle, the minimum required wind speed for supporting the dynamic soaring is also increasing.

The table 3 gives stall speed and the required Δu for different wing loading.

Table 3: Minimum Wind Speed for Different m/S (kg/m^2)

m/S (kg/m^2)	V_{stall} (m/s)	Δu (m/s)
10	13.7388	2.0836
14	16.2559	2.4653
18	18.4325	2.7954
22	20.3779	3.0904
26	22.1531	3.3596
30	23.7963	3.6088

Energy Analysis

The above analysis is performed on the premise that already the equilibrium condition is reached. However to verify the net energy is zero, the following is considered. Since Rayleigh cycle is an idealized cycle with the following maneuvers (refer figure 1):

- During A-B instantaneous climb - the relative wind speed increases with constant ground speed as the aircraft climbs headwind.
- During B-C, level turn from head wind to tail wind, relative wind speed reduces due to drag, however at the end of the turn the aircraft faces tail wind, the ground speed increases with the same relative wind speed.
- During C-D instantaneous descend, the air speed increases and the ground speed remains constant.
- During D-A, level turn from tail wind to head wind, relative wind speed reduces due to drag, however at the end of the turn the aircraft faces head wind.

For $\mu = \pi/4$, $m/S = 14$ kg/m^2, the stall speed is $V_C = 16.2559$ m/s and the minimum wind speed required for dynamic soaring obtained is $\Delta u = 2.4653$ m/s. For this condition, when the energy analysis is performed, the net energy is zero (almost zero due to numerical precision) as is evident from the figure 5 below and hence for the obtained minimum wind, the Rayleigh cycle is energy neutral.

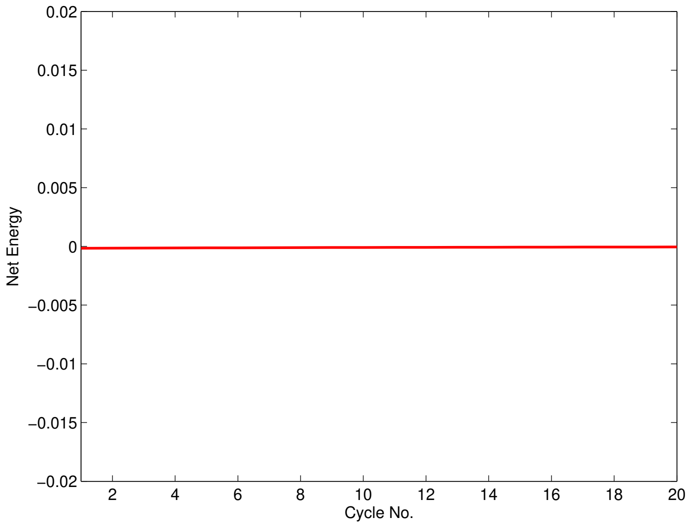

Figure 5: Net Energy for Different Cycles

Matlab Code Listing

The matlab code consists of the following:

- main.m - Initialization, performing function calls and plotting the results. It has got multiple sections which are commented. The section for execution may be uncommented.
- f_turn.m - The function which takes the input of the speed, wing loading and the bank angle and outputs the speed value after the turn of 180 degrees.

Main Code

```
%---------------------------------------------------------------------------
%%
clear all; close all; clc;

global CD0  g rho K

AR = 15;    CD0 = 0.01;     CLmax= 1.2;     g = 9.81;
m = 8;      rho = 1.225;    K = 1/(pi*AR);

%---------------------------------------------------------------------------
%% Stall Speed and the Initial Speed
%
% mu1 = [10:5:80]*pi/180; mbys=14.0;
% for i=1:length(mu1)
%     Vs(i) = sqrt(2*mbys*g/(rho*CLmax*cos(mu1(i))));
%     Vi(i) = f_turn(Vs(i),mbys,mu1(i));
% end
% figure(1); subplot (3,1,1);
% plot (mu1*180/pi,Vs,'bo-','LineWidth',2.0);
% fs='fontsize'; set (gca,fs,12);
% xlabel ('Bank Angle (deg)'); ylabel ('V_{stall} (m/s)');
% subplot(3,1,2);
% plot (mu1*180/pi,Vi,'mo-','LineWidth',2.0);
% fs='fontsize'; set (gca,fs,12);
% xlabel ('Bank Angle (deg)'); ylabel ('V_{initial} (m/s)');
% subplot(3,1,3);
% plot (mu1*180/pi,Vi-Vs,'ro-','LineWidth',2.0);
% fs='fontsize'; set (gca,fs,12); ylim([0 20]);
% xlabel ('Bank Angle (deg)'); ylabel ('\Delta V = V_{i}-V_{St} (m/s)');
% print -depsc 'stall.eps'
% bank_angle = mu1*180/pi;
% [bank_angle' Vi' Vs' (Vi-Vs)']
%---------------------------------------------------------------------------

%% 1(a) Minimum Speed for supporting Dynamic Soaring
%
% mu1 = 60*pi/180; mbys=14.0;
% Vstall = sqrt(2*mbys*g/(rho*CLmax*cos(mu1)));
% Vc = Vstall;
% Vmin = fsolve(@(delu) f_turn(Vc+delu,mbys,mu1)-Vc, [5])
%---------------------------------------------------------------------------

%% 1(b) Minimum Speed variation for supporting Dynamic Soaring with bank angle

% mu1 = [10:5:70]*pi/180; mbys = 14.0;
% for i=1:length(mu1)
%     Vstall = sqrt(2*mbys*g/(rho*CLmax*cos(mu1(i))));
```

```
%     Vc = Vstall;
%     delu(i)=fsolve(@(u) f_turn(Vc+u,mbys,mu1(i))-Vc, [5]);
% end
% plot (mu1*180/pi,delu,'ro-','LineWidth',2); fs='FontSize';
% set (gca,fs,12);
% xlabel ('Bank Angle (deg)'); ylabel('\Delta u (m/s)');
% print -depsc 'delu_phi.eps'
% [(mu1*180/pi)' delu']
%-------------------------------------------------------------------------

%% 2 For bank anlge = pi/4; variation of m/S on Vmin
%
% mu1 = pi/4; mbys = [10:2:30];
%
% for i=1:length(mbys)
%     Vstall = sqrt(2*mbys(i)*g/(rho*CLmax*cos(mu1)));
%     Vc(i) = Vstall;
%     delu(i) = fsolve(@(u) f_turn(Vc(i)+u,mbys(i),mu1)-Vc(i), [5]);
% end
% subplot (2,2,1);
% plot (mbys,delu,'bo-','LineWidth',2);
% fs = 'fontsize'; set(gca,fs,12);
% xlabel ('m/S (kg/m^2)'); ylabel('\Delta u (m/s)');
% subplot (2,2,2);
% plot (mbys,Vc,'ro-','LineWidth',2);
% set (gca,fs,12);
% xlabel ('m/S (kg/m^2)'); ylabel('V_{stall} (m/s)');
% print -depsc 'mbys.eps'
% [mbys' Vc' delu']
%---------------------------------------------------------------------------

%% Energy Analysis

mu1 = pi/4; mbys = 14; delu = 2.4653;
va = 16.2559;
for i=1:20
    iter(i) = i;
    vga = -va;
    vgb = va*sign(vga);
    vb  = abs(-vgb+delu);
    vc  = f_turn(vb,mbys,mu1);
    vgc = vc+delu;
    vgd = vgc;
    vd  = abs(vgd);
    e1  = 0.5*vga^2;
    va  = f_turn(vd,mbys,mu1);
    e2  = 0.5*va^2;
    netE(i) = e1 - e2;
%     plot ([vga vgb vgc vgd -va],[0 delu delu 0 0],'r+-');
%     drawnow
%     pause(1)
%     hold on;
%     xlim([-25 25]); ylim([-1 3]);
end
plot (iter,netE,'r-','LineWidth',2);
fs = 'fontsize'; set (gca,fs,12);
xlabel ('Cycle No.'); ylabel('Net Energy'); xlim([1 20]); ylim([-0.02 0.02]);
print -depsc 'energy1.eps';
%---------------------------------------------------------------------------
```

f_turn.m

```
function [V] = f_turn(V1,mbys,phi)
global CD0 g rho K

%--------------------------------------------------------------------------
% For obtaining the initial speed if Final speed after the turn is provided
% %--------------------------------------------------------------------------
% c= -0.5*rho*g*sin(2*phi);
% a= CD0*rho^2*(1/mbys)*(cos(phi))^2;
% b= 4*K*(mbys)*g^2;
% fac1 = sqrt(b/a);
% fac2 = sqrt(a/b);
% fac3 = sqrt(a*b);
% arg1 = -pi*fac3/c + atan(V1.^2*fac2);
% V = sqrt( fac1*tan(arg1));

%--------------------------------------------------------------------------
% For obtaining the final speed if initial speed prior to turn is provided
%--------------------------------------------------------------------------

c= -0.5*rho*g*sin(2*phi);
a= CD0*rho^2*(1/mbys)*(cos(phi))^2;
b= 4*K*(mbys)*g^2;
fac1 = sqrt(b/a);
fac2 = sqrt(a/b);
fac3 = sqrt(a*b);
arg1 = pi*fac3/c + atan(V1.^2*fac2);
V = sqrt( fac1*tan(arg1));

end
```