Dynamic Soaring: Assignment 1

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The assignment is to plot the trajectory of the displacement when a sphere of radius $r = 0.15m$ is falling under gravity vertically whose drag coefficient of $C_D = 1.0$ in a wind field which is considered constant acting vertically upward $\vec{w_z} = -w_z\hat{k}$

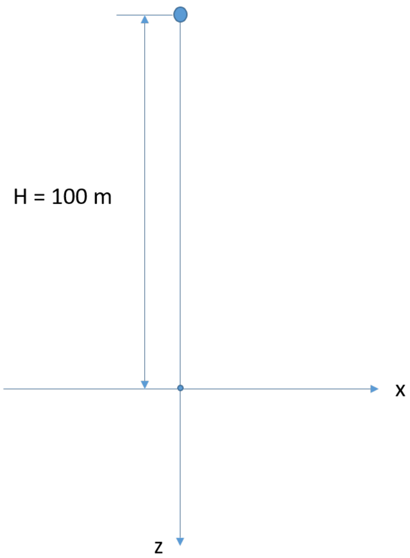

Figure 1: Indicative Figure of the Problem

Equation of Motion in Inertial Frame

$$\ddot{z} = g - C_D\frac{1}{2}\rho(\dot{z} + w_z)^2\frac{S}{m} \tag{1}$$

where g is the acceleration due to gravity $(9.8ms^{-2})$, C_D is the drag coefficient with respect to the frontal area, ρ is the density of air assumed constant (kg/m^3), S is the frontal area of the sphere $= \pi r^2$ and m is the mass of the sphere in kg

The above equation may be written in state-space form here under.

Assume $x_1 = z$ and $x_2 = \dot{z}$ then

$$\left\{\begin{array}{c} \dot{x_1} \\ \dot{x_2} \end{array}\right\} = \left\{\begin{array}{c} \dot{z} \\ \ddot{z} \end{array}\right\} = \left\{\begin{array}{c} x_2 \\ g - C_D\frac{1}{2}\rho(x_2 + w_z)^2\frac{S}{m} \end{array}\right\} \tag{2}$$

The initial conditions in this frame of reference are $z = x_1 =$ -100.0 and $\dot{z} = x_2 = 0.0$ m/s.

Equation of Motion in Relative Wind Frame

The speed of the sphere with respect to air is given by $V = \dot{z} + w_z$ The equation of motion is given by:

$$m\frac{d(V - w_z)}{dt} = mg - C_D\frac{1}{2}\rho V^2 S \tag{3}$$

$$\frac{dV}{dt} = g - C_D \frac{1}{2}\rho V^2 \frac{S}{m} + \frac{dw_z}{dt}$$
$$= g - C_D \frac{1}{2}\rho V^2 \frac{S}{m} + \frac{dw_z}{dz}\dot{z}$$

where $\dot{z} = V - w_z$ In this case,the equation can be written as follows:

$$\left\{ \begin{array}{c} \dot{z} \\ \dot{V} \end{array} \right\} = \left\{ \begin{array}{c} V - w_z \\ g - C_D \frac{1}{2}\rho V^2 \frac{S}{m} + \frac{dw_z}{dz}\frac{dz}{dt} \end{array} \right\} = \left\{ \begin{array}{c} V - w_z \\ g - C_D \frac{1}{2}\rho V^2 \frac{S}{m} + \frac{dw_z}{dz}(V - w_z) \end{array} \right\} \tag{4}$$

The initial conditions are $z(0)$ = -100.0 and due to upward wind velocity $V(0) = 3.0$ m/s.

Results of the Simulation

The various cases for the simulation in both Inertial frame and Relative Wind Frame are the following:

- Case 1: No Drag and No Vertically Upward Wind
- Case 2: Drag with C_D=1.0 and No Vertically Upward Wind
- Case 3: Drag with C_D=1.0 and Constant Vertically Upward Wind of 3.0 m/s
- Case 4: Drag with C_D=1.0 and Constant Vertically Upward Wind of 3.0 m/s till $z > 30$ and below that $0.1z$

Case 1: No Drag and No Vertically Upward Wind

The Inertial frame vertical distance Z Vs time computed through Inertial frame EOM and Relative Wind EOM for the Case 1 are shown in figure 2.

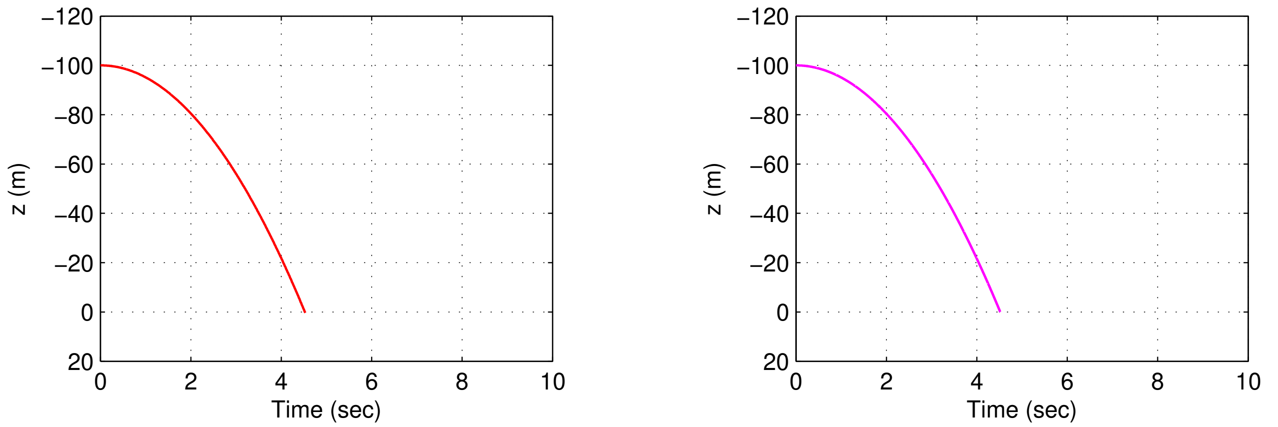

Figure 2: Inertial Vertical Distance Vs time obtained from Inertial and Relative Wind EOM

The Inertial frame vertical speed $\dot{Z}$ Vs time computed through Inertial frame EOM and Relative Vertical velocity V for the Case 1 are shown in figure 3. It is observed that the both $\dot{z}$ and V is increasing linearly with time as there is no drag and no relative wind and is only due to the acceleration due to gravity. Figure 4 gives the Kinetic Energy variation with time for this case.

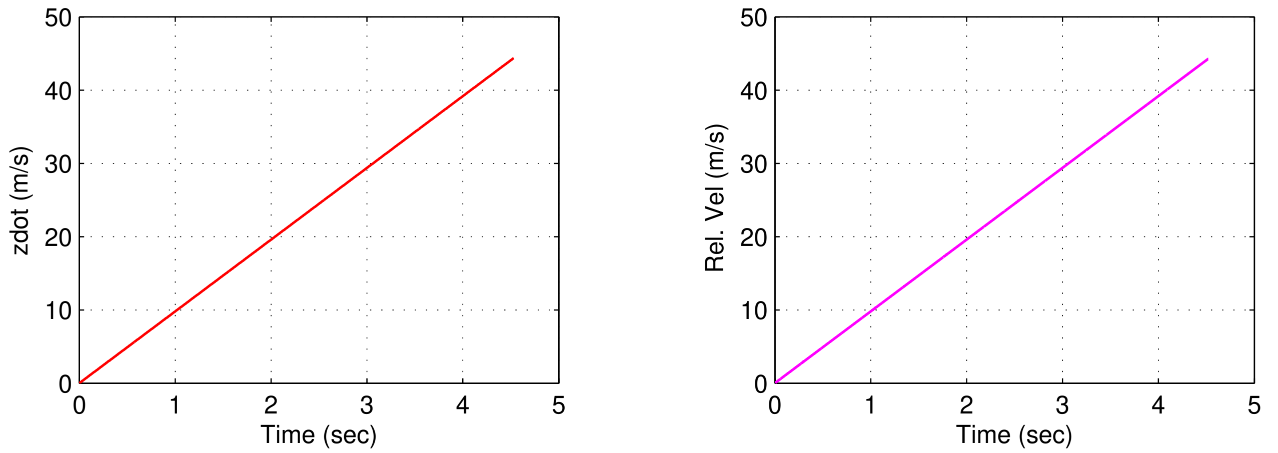

Figure 3: Vertical Speed $\dot{Z}$ Vs time and V Vs time

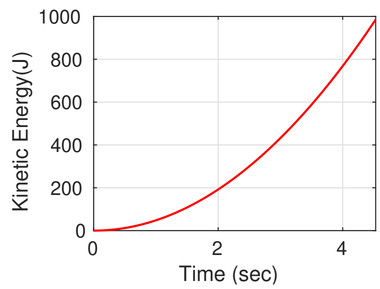

Figure 4: Case 1 - Kinetic Energy Vs Time

Case 2: Drag with C_D=1.0 and No Vertically Upward Wind

The Inertial frame vertical distance Z Vs time computed through Inertial frame EOM and Relative Wind EOM for the Case 2 are shown in figure 5.

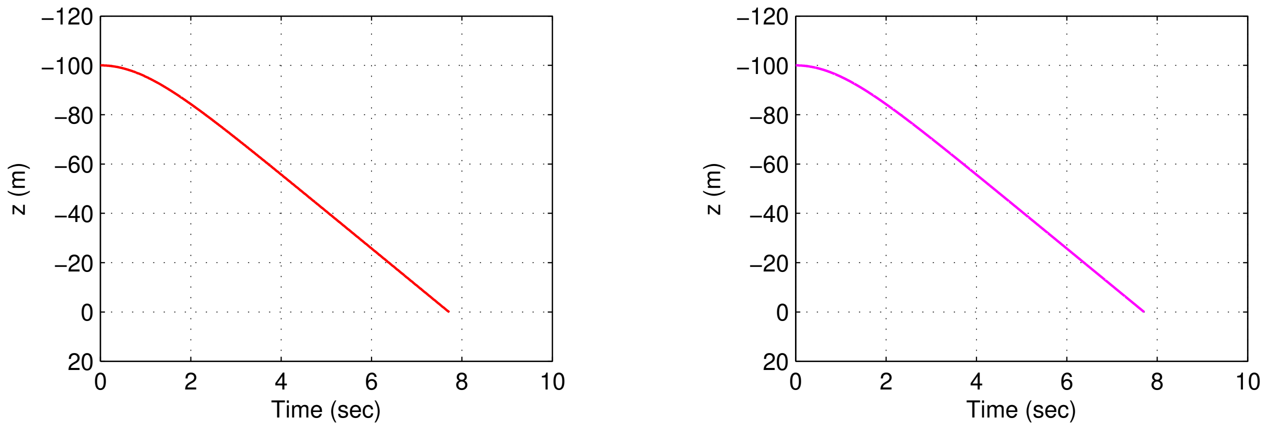

Figure 5: Inertial Vertical Distance Vs time obtained from Inertial and Relative Wind EOM

The Inertial frame vertical speed $\dot{Z}$ Vs time computed through Inertial frame EOM and Relative Vertical velocity V for the Case 2 are shown in figure 6. It is observed that in both the cases the velocity reaches a constant value (terminal velocity) due to the force balance between the gravity and drag. Figure 7 gives the Kinetic Energy variation with time for this case.

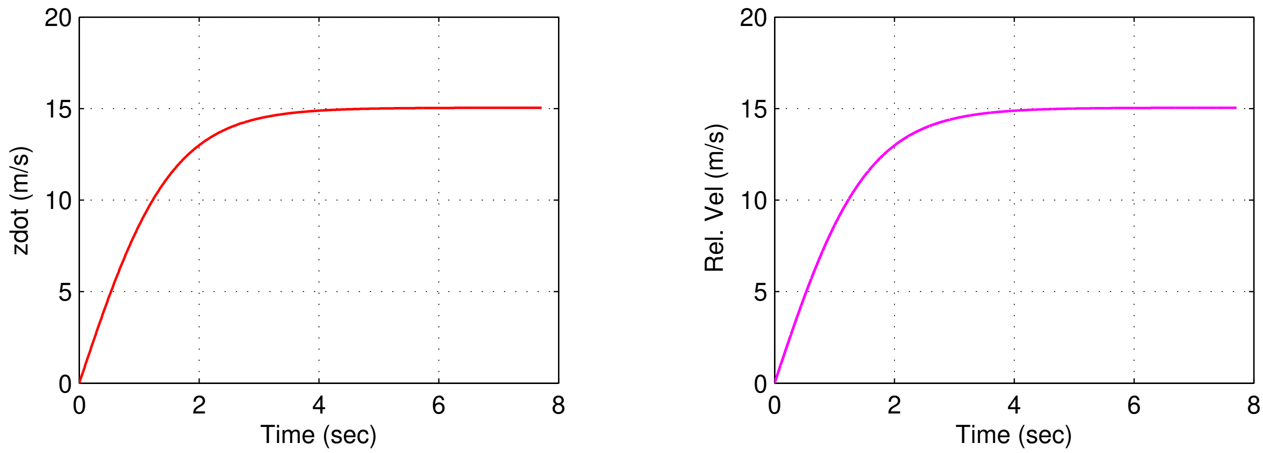

Figure 6: Vertical Speed $\dot{Z}$ Vs time and V Vs time

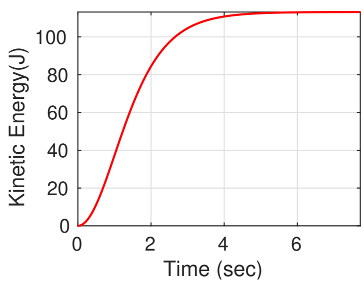

Figure 7: Case 2 - Kinetic Energy Vs Time

Case 3: Drag with C_D=1.0 and Constant Vertically Upward Wind of 3.0 m/s

The Inertial frame vertical distance Z Vs time computed through Inertial frame EOM and Relative Wind EOM for the Case 3 are shown in figure 8.

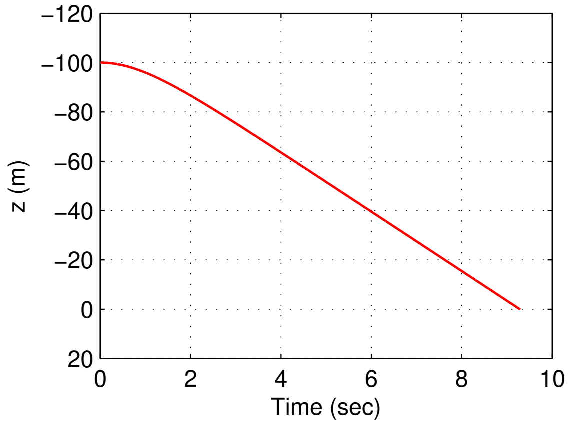

Figure 8: Inertial Vertical Distance Vs time obtained from Inertial and Relative Wind EOM

The Inertial frame vertical speed $\dot{Z}$ Vs time computed through Inertial frame EOM and Relative Vertical velocity V for the Case 3 are shown in figure 9. In this case, it is observed that the velocity reaches a constant value (terminal velocity) due to the force balance between the gravity and drag. However, in the wind frame, the constant upward wind of 3.0 m/s is seen with respect to the inertial vertical speed ($\dot{Z}$). Figure 10 gives the Kinetic Energy variation with time for this case.

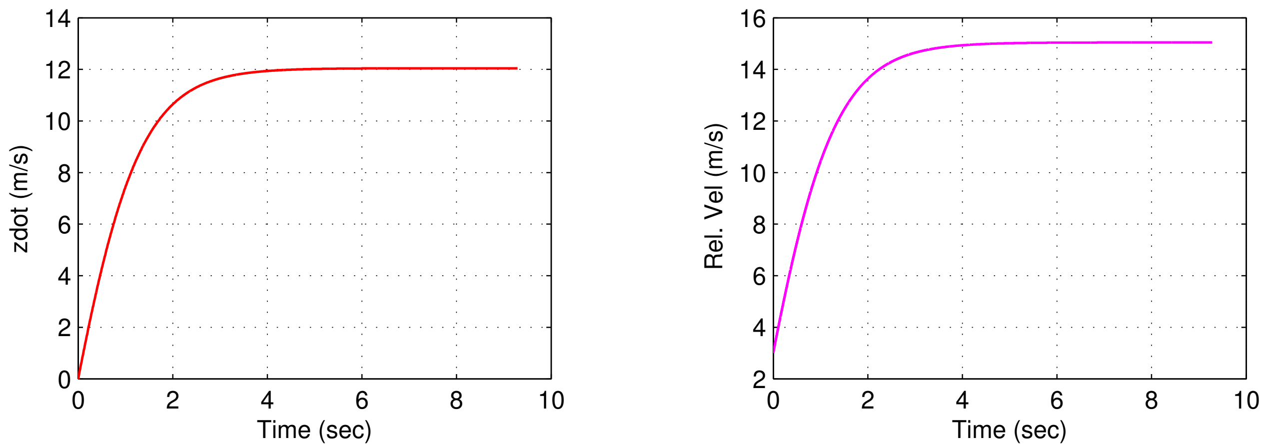

Figure 9: Vertical Speed $\dot{Z}$ Vs time and V Vs time

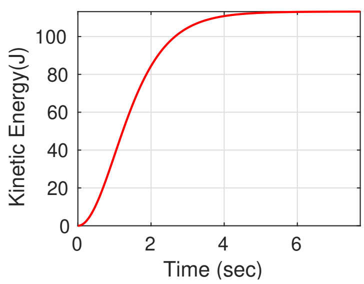

Figure 10: Case 3 - Kinetic Energy Vs Time

Case 4: Drag with C_D=1.0 and Constant Vertically Upward Wind of 3.0 m/s till $z > 30$ and $0.1z$

The Inertial frame vertical distance Z Vs time computed through Inertial frame EOM and Relative Wind EOM for the Case 4 are shown in figure 11.

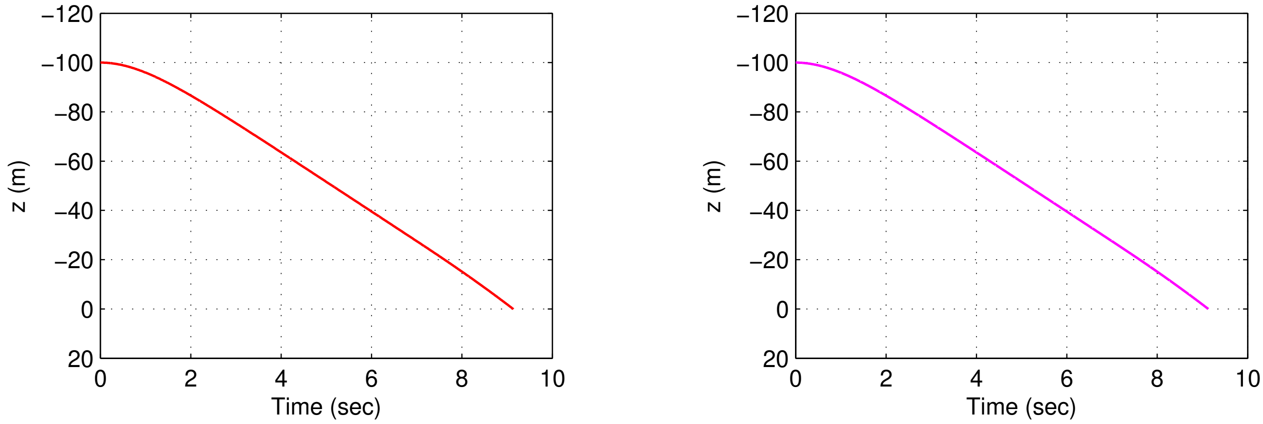

Figure 11: Inertial Vertical Distance Vs time obtained from Inertial and Relative Wind EOM

The Inertial frame vertical speed $\dot{Z}$ Vs time computed through Inertial frame EOM and Relative Vertical velocity V for the Case 4 are shown in figure 12. In this case, due to the gradient in the wind speed the force balance is disturbed after the sphere reaches a distance of 30 m from the ground and the gravity is accelerating further which is shown in the inertial velocity $\dot{Z}$ increase. However, in the wind frame, the vertical speed V is reduced due to the gradient in the wind field. Figure 13 gives the Kinetic Energy variation with time for this case.

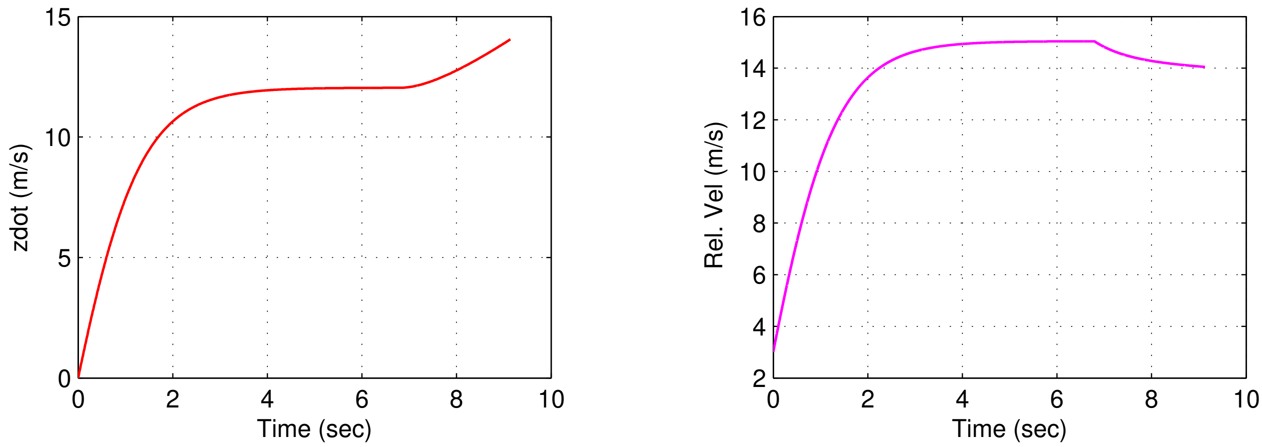

Figure 12: Vertical Speed $\dot{Z}$ Vs time and V Vs time

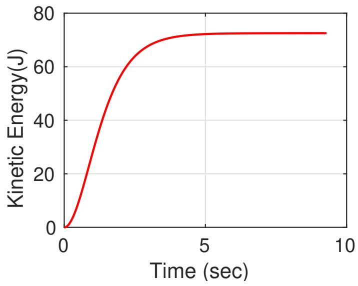

Figure 13: Case 4 - Kinetic Energy Vs Time

Simulated Values at the Impact Point

The time of impact T, Inertial Velocity $\dot{z}$, the Speed in the Relative Wind frame V and the Kinetic Energy at the point of impact are tabulated in table 1.

Cases	Time (sec)	$\dot{z}(m/s)$	V(m/s)	Energy (J)
Case 1	4.519	44.394	44.286	985.414
Case 2	7.711	15.044	15.044	113.159
Case 3	9.283	12.045	15.045	72.540
Case 4	9.131	14.055	14.047	98.776

Table 1: Final Values Tabulated. For Energy Inertial Velocity is considered

Program Listing

```
clear all; close all; clc;

%---------------------------------------------------------------------------
% Inertial Reference Frame
% Program written by Izza (AE18B006) and RV Ramkumar (AE20D025)
%---------------------------------------------------------------------------
% Case 1: CD = 0; Ww = 0.0;
% Case 2: CD = 1.0; Ww = 0.0;
% Case 3: CD = 1.0; Ww = 3.0;
% Case 4: CD = 1.0; Ww = 0.1*z till z <30 m and Ww = 3.0 when z > 30m;
%--------------------------------------------------------------------------
% Integration using simple Euler Equation
%--------------------------------------------------------------------------
% Initial condition & other constants
%--------------------------------------------------------------------------
g = 9.8;  rho = 1.225;
rad = 0.15;  S = pi*rad^2;
mass = 1.0; CD = 0.0;
Sbym = S/mass;

%--------------------------------------------------------------------------
CD(1) = 0.0;    CD(2) = 1.0;    CD(3) = 1.0;    CD(4) = 1.0;
windv(1) = 0.0; windv(2) = 0.0; windv(3) = 3.0; windv(4) = 3.0;
dwdz1(1) = 0.0; dwdz1(2) = 0.0; dwdz1(3) = 0.0; dwdz1(4) = 0.1;

for j=1:4
    z(1,j) = -100.0;
    zdot(1,j) = 0.0;
    t(1,j) = 0.0;
    KE(1,j)= 0.5 * mass* zdot(1,j)^2;
end
%-------------------------------------------------------------------------
delt = 0.001;
t1 = [0:delt:12];
imax = size(t1,2);

for j=1:4
    for i=1:imax-1
        K(j) = 0.5*rho*Sbym*CD(j);
```

```
        if j<=3
            Ww = windv(j);
        else
            if abs(z(i,j))>30
                Ww = windv(j);
            else
               dwdz = -dwdz1(4);
               Ww = abs(dwdz)*abs(z(i,4)); % since Ww is still positive upwards
            end
        end
        zdot(i+1,j) = zdot(i,j) + (g-K(j)*(zdot(i,j)+Ww).^2)*delt;
        z(i+1,j) = z(i,j)+zdot(i,j)*delt;
        t(i+1,j) = t(i,j)+delt;
        KE(i,j) = 0.5*mass*zdot(i,j)^2;
    end
end

index_1 = find(z(:,1)>=0); index_2 = find(z(:,2)>=0);
index_3 = find(z(:,3)>=0); index_4 = find(z(:,4)>=0);

n1 = index_1(1); n2 = index_2(1); n3 = index_3(1); n4 = index_4(1);
%----------------------------------------------------------------------------
% Plotting of the Results
%----------------------------------------------------------------------------
figure(1); subplot(2,2,1);
plot (t([1:n1],1),z([1:n1],1),'r-','LineWidth',1.0); grid on;
ylabel ('z (m)'); xlabel ('Time (sec)');
axis([0 10 -120 20]);
set(gca,'YDir','reverse');
print -depsc case1_inertial_z.eps

figure(2); subplot(2,2,1);
plot (t([1:n2],2),z([1:n2],2),'r-','LineWidth',1.0); grid on;
ylabel ('z (m)'); xlabel ('Time (sec)');
axis([0 10 -120 20]);
set(gca,'YDir','reverse');
print -depsc case2_inertial_z.eps

figure(3); subplot (2,2,1);
plot (t([1:n3],3),z([1:n3],3),'r-', 'LineWidth',1.0); grid on;
```

```
ylabel ('z (m)'); xlabel ('Time (sec)');
axis([0 10 -120 20]);
set(gca,'YDir','reverse');
print -depsc case3_inertial_z.eps

figure(4); subplot(2,2,1);
plot (t([1:n4],4),z([1:n4],4),'r-', 'LineWidth',1.0); grid on;
ylabel ('z (m)'); xlabel ('Time (sec)');
axis([0 10 -120 20]);
set(gca,'YDir','reverse');
print -depsc case4_inertial_z.eps

figure(5);subplot(2,2,1);
plot (t([1:n1],1),zdot([1:n1],1),'r-','LineWidth',1.0); grid on;
ylabel ('zdot (m/s)'); xlabel ('Time (sec)');
print -depsc case1_inertial_zdot.eps

figure(6); subplot(2,2,1);
plot (t([1:n2],2),zdot([1:n2],2),'r-','LineWidth',1.0); grid on;
ylabel ('zdot (m/s)'); xlabel ('Time (sec)');
print -depsc case2_inertial_zdot.eps

figure(7); subplot(2,2,1);
plot (t([1:n3],3),zdot([1:n3],3),'r-','LineWidth',1.0); grid on;
ylabel ('zdot (m/s)'); xlabel ('Time (sec)');
print -depsc case3_inertial_zdot.eps

figure(8); subplot(2,2,1);
plot (t([1:n4],4),zdot([1:n4],4),'r-', 'LineWidth',1.0); grid on;
ylabel ('zdot (m/s)'); xlabel ('Time (sec)');
print -depsc case4_inertial_zdot.eps

figure(9);subplot(2,2,1);
plot (t([1:n1],1),KE([1:n1],1),'r-','LineWidth',1.0); grid on;
ylabel ('Kinetic Energy(J)'); xlabel ('Time (sec)');
print -depsc case1_inertial_KE.eps

figure(10); subplot(2,2,1);
plot (t([1:n2],2),KE([1:n2],2),'r-','LineWidth',1.0); grid on;
ylabel ('Kinetic Energy(J)'); xlabel ('Time (sec)');
```

```
print -depsc case2_inertial_KE.eps

figure(11); subplot(2,2,1);
plot (t([1:n3],3),KE([1:n3],3),'r-','LineWidth',1.0); grid on;
ylabel ('Kinetic Energy(J)'); xlabel ('Time (sec)');
print -depsc case3_inertial_KE.eps

figure(12); subplot(2,2,1);
plot (t([1:n4],4),KE([1:n4],4),'r-', 'LineWidth',1.0); grid on;
ylabel ('Kinetic Energy(J)'); xlabel ('Time (sec)');
print -depsc case4_inertial_KE.eps


%---------------------------------------------------------------------------
% End of Program
%---------------------------------------------------------------------------
```

```
clear all; close all; clc;

%---------------------------------------------------------------------------
% Relative Wind Frame of Reference
% Program written by Izza (AE18B006) and RV Ramkumar (AE20D025)
%---------------------------------------------------------------------------
% Case 1: CD = 0; Ww = 0.0;
% Case 2: CD = 1.0; Ww = 0.0;
% Case 3: CD = 1.0; Ww = 3.0;
% Case 4: CD = 1.0; Ww = 0.1*z till z <30 m and Ww = 3.0 when z > 30m
%---------------------------------------------------------------------------
% Integration using simple Euler Equation
%---------------------------------------------------------------------------
% Initial condition & other constants

g = 9.8;  rho = 1.225;
rad = 0.15;  S = pi*rad^2;
mass = 1.0; CD = 1.0;
Sbym = S/mass;

%---------------------------------------------------------------------------
CD(1) = 0.0;    CD(2) = 1.0;    CD(3) = 1.0;    CD(4) = 1.0;
windv(1) = 0.0; windv(2) = 0.0; windv(3) = 3.0; windv(4) = 3.0;
dwdz1(1) = 0.0; dwdz1(2) = 0.0; dwdz1(3) = 0.0; dwdz1(4) = 0.1;

% Upward direction of the wind is considered positive for wind
%---------------------------------------------------------------------------

for j=1:4
    z(1,j) = -100.0;
    vel(1,j) = windv(j);
    t(1,j) = 0.0;
end

delt = 0.001;
t1 = [0:delt:16];
imax = size(t1,2);

for j=1:4
    for i=1:imax-1
```

```
        K = 0.5*rho*Sbym*CD(j);
       if j<=3
            Ww = windv(j);
            dwdz = 0.0;
        else
            if abs(z(i,j))>30
                Ww = windv(j);
                dwdz = 0.0;
            else
               dwdz = -dwdz1(4);
               Ww = abs(dwdz)*abs(z(i,4)); % since Ww is still positive upwards
            end
       end
        vel(i+1,j) = vel(i,j) + (g - K*vel(i,j)^2 + dwdz*(vel(i,j)-Ww))*delt;
        z(i+1,j) = z(i,j) + (vel(i,j)-Ww)*delt;
        t(i+1,j) = t(i,j)+delt;
    end
end
index_1 = find(z(:,1)>=0); index_2 = find(z(:,2)>=0);
index_3 = find(z(:,3)>=0); index_4 = find(z(:,4)>=0);

n1 = index_1(1); n2 = index_2(1); n3 = index_3(1); n4 = index_4(1);

%-----------------------------------------------------------------------------
% Plotting of the Results
%-----------------------------------------------------------------------------
figure(1); subplot(2,2,1);
plot (t([1:n1],1),z([1:n1],1),'m-','LineWidth',1.0); grid on;
ylabel ('z (m)'); xlabel ('Time (sec)');
axis([0 10 -120 20]);
set(gca,'YDir','reverse');
print -depsc case1_rel_z.eps

figure(2); subplot (2,2,1);
plot (t([1:n2],2),z([1:n2],2),'m-','LineWidth',1.0); grid on;
ylabel ('z (m)'); xlabel ('Time (sec)');
axis([0 10 -120 20]);
set(gca,'YDir','reverse');
print -depsc case2_rel_z.eps
```

```
figure(3); subplot (2,2,1);
plot (t([1:n3],3),z([1:n3],3),'m-', 'LineWidth',1.0); grid on;
ylabel ('z (m)'); xlabel ('Time (sec)');
axis([0 10 -120 20]);
set(gca,'YDir','reverse');
print -depsc case3_rel_z.eps

figure(4); subplot (2,2,1);
plot (t([1:n4],4),z([1:n4],4),'m-', 'LineWidth',1.0); grid on;
ylabel ('z (m)'); xlabel ('Time (sec)');
axis([0 10 -120 20]);
set(gca,'YDir','reverse');
print -depsc case4_rel_z.eps

figure(5); subplot(2,2,1)
plot (t([1:n1],1),vel([1:n1],1),'m-','LineWidth',1.0); grid on;
ylabel ('Rel. Vel (m/s)'); xlabel ('Time (sec)');
print -depsc case1_rel_vel.eps

figure(6); subplot (2,2,1);
plot (t([1:n2],2),vel([1:n2],2),'m-','LineWidth',1.0); grid on;
ylabel ('Rel. Vel (m/s)'); xlabel ('Time (sec)');
print -depsc case2_rel_vel.eps

figure(7); subplot (2,2,1);
plot (t([1:n3],3),vel([1:n3],3),'m-','LineWidth',1.0); grid on;
ylabel ('Rel. Vel (m/s)'); xlabel ('Time (sec)');
print -depsc case3_rel_vel.eps

figure(8); subplot (2,2,1);
plot (t([1:n4],4),vel([1:n4],4),'m-', 'LineWidth',1.0); grid on;
ylabel ('Rel. Vel (m/s)'); xlabel ('Time (sec)');
print -depsc case4_rel_vel.eps

%--------------------------------------------------------------------------
% End of Program
%--------------------------------------------------------------------------
```