Dynamic Soaring: Assignment 6

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Characteristics of UAV

The characteristics of the UAV model are the following:

- $C_{L\max} = 1$
- $C_{D0} = 0.01$
- $m/S = 14\ kg/m^2$
- $AR = 15$
- Lift Induced Drag Factor $K = 1/(\pi AR)$ (Corresponding to Elliptic Wing)

From the above, the stall speed is given as

$$V_{\text{stall}} = \sqrt{\frac{2m/Sg}{\rho C_{L\max}}} = 14.9743\ \ ms^{-1} \tag{1}$$

As per the given problem at point C (after turning), the velocity is $V_{\text{stall}} = V_C = 14.9743\ ms^{-1}$

Simplified Version of Dynamic Soaring

The figure below gives the simplified version of dynamic soaring.

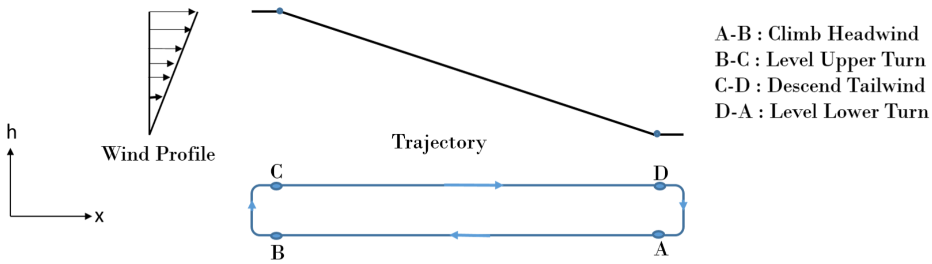

Figure 1: Indicative Single Loop Dynamic Soaring Trajectory

Climb

Considering the bird is climbing headwind the equations of motion in the relative wind frame is given as follows:

$$\frac{dV}{dt} = -g\sin\gamma - \cos\gamma(-\beta V\sin\gamma) - \frac{D}{m} \tag{2}$$

$$L = mg\cos\gamma \tag{3}$$

In the above the $\dot{\gamma}$ is considered zero and hence the second equation in the above set. From this the C_L is obtained which is used to find the drag coefficient (using parabolic drag polar) to estimate the drag. When the bird is climbing in the increasing head wind with an initial speed of V_i with a flight path angle γ, at certain height the air speed reduces to a value (V_B) which after performing level turn reduces further to Stall speed (V_C).

Turn

The heading angle (χ) change at constant altitude is governed by,

$$mV\frac{d\chi}{dt} = L\sin\mu \tag{4}$$

where L is the Lift, μ is the bank angle Also during the level turn

$$m\frac{dV}{dt} = -D \tag{5}$$

Considering a constant bank angle turn (at level altitude),

$$L\cos\mu = mg \tag{6}$$

With the help of the above three equations,

$$\frac{dV}{d\chi} = \frac{-DV}{mg\tan\mu} \tag{7}$$

For the UAV to turn to an angle of π radians (180 degrees), the equation can be written as,

$$\int_{V_i}^{V_f} \frac{-mg\tan\mu}{DV}dV = \int_0^{\pi} d\chi \tag{8}$$

where V_i and V_f are respectively velocity prior and after the turn. The integrand of the LHS can be written as

$$\frac{-mg\tan\mu}{DV} = \frac{-g\sin\mu}{(C_{D0}+KC_L^2)\frac{1}{2}\rho\frac{S}{m}\cos\mu V^3}$$

The denominator can be written simplified after substituting for $C_L = \frac{2g(m/S)}{\rho\cos\mu V^2}$

$$\begin{aligned}\text{Den} &= C_{D0}\frac{1}{2}\rho\frac{S}{m}\cos\mu V^3 + K\frac{2(m/S)g^2}{\rho\cos\mu V} \\ &= \frac{1}{2\rho\cos\mu V}\left[\left(C_{D0}\rho^2\frac{S}{m}\cos^2\mu\right)V^4 + \left(4K(m/S)g^2\right)\right]\end{aligned}$$

Therefore the integrand of LHS of equation 8 can be written as

$$\frac{-mg\tan\mu}{DV} = \frac{-\rho g\sin 2\mu V}{[(C_{D0}\rho^2(S/m)\cos^2\mu)\,V^4 + (4K(m/S)g^2)]}$$

Therefore the LHS of equation can be written as,

$$\begin{aligned}\int_{V_i}^{V_f} \frac{-mg\tan\mu}{DV}dV &= \int_{V_i}^{V_f} \frac{-\rho g\sin 2\mu}{[(C_{D0}\rho^2(S/m)\cos^2\mu)\,V^4 + (4K(m/S)g^2)]}VdV \\ &= \int_{V_i}^{V_f} \frac{c}{aV^4+b}d(V^2) \\ &= \int_{V_i^2}^{V_f^2} \frac{c}{au^2+b}du \\ &= \frac{c}{a}\int_{V_i^2}^{V_f^2} \frac{1}{u^2+(\sqrt{b/a})^2}du \\ &= \frac{c}{a}\sqrt{\frac{a}{b}}\tan^{-1}\left(\sqrt{\frac{a}{b}}u\right)\Bigg|_{V_i^2}^{V_f^2} \\ &= \frac{c}{\sqrt{ab}}\left[\tan^{-1}\left(V_f^2\sqrt{\frac{a}{b}}\right) - \tan^{-1}\left(V_i^2\sqrt{\frac{a}{b}}\right)\right]\end{aligned}$$

where, $c = -\frac{1}{2}\rho g \sin 2\mu$, $a = C_{D0}\rho^2(S/m)\cos^2\mu$ and $b = 4K(m/S)g^2$ Therefore the equation 8 can be simplified as

$$\frac{c}{\sqrt{ab}}\left[\tan^{-1}\left(V_f^2\sqrt{\frac{a}{b}}\right) - \tan^{-1}\left(V_i^2\sqrt{\frac{a}{b}}\right)\right] = \pi \tag{9}$$

The above equation may be further simplified as

$$V_f^2 = \sqrt{\frac{b}{a}}\tan\left[\pi\frac{\sqrt{ab}}{c} + \tan^{-1}\left(V_i^2\sqrt{\frac{a}{b}}\right)\right] \tag{10}$$

or else

$$V_i^2 = \sqrt{\frac{b}{a}}\tan\left[-\pi\frac{\sqrt{ab}}{c} + \tan^{-1}\left(V_f^2\sqrt{\frac{a}{b}}\right)\right] \tag{11}$$

Since V_f is known apriori i.e., at the point C which is equal to 14.9743 ms^{-1}, V_B can be obtained using the above equation. Evaluating the above considering the bank angle $\mu = 60^0$, we get,

$$V_B = 17.4535 \;\; ms^{-1} \tag{12}$$

Height Determination

For an assumption of initial velocity $V_i = 25\ m/s$ on ground, and the slope of wind profile $\beta = 0.25s^{-1}$, the velocity of the UAV at different height can be estimated by performing the integration of the equation 3 for the constant flight path angle of 45 degrees. The height at which the aircraft attains a speed corresponding to $V_B = 17.4535\ m/s$ can be estimated. The speed attained by climbing at different height is given in the plot below:

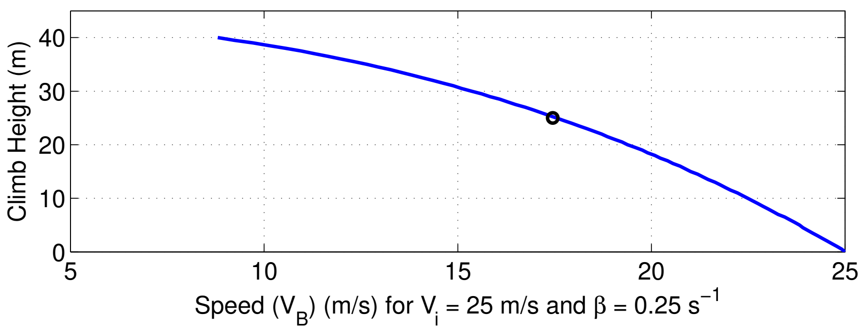

Figure 2: Speed at point B at different Height

From the figure, the height at which the speed attains a speed of $V_B = 17.4535\ \ m/s$ is found to be at $24.95\ m \approx 25\ m$. At this height if the bird / aircraft performs a level turn of 180 degrees with bank angle $\mu = 60^0$, the speed drops to the stall speed corresponding to $V_C = 14.97\ m/s$.

Descend

Subsequent to the turn, the aircraft is assumed to instantaneously change the flight path angle and start the descend in the tail wind. The equation of motion in the relative wind frame is given by equation 3 (only difference here is γ to be mentioned as negative value):

$$\frac{dV}{dt} = -g\sin\gamma - \cos\gamma(\beta)(V\sin\gamma) - \frac{D}{m} \tag{13}$$

$$L = mg\cos\gamma \tag{14}$$

The above equation along with

$$\frac{dh}{dt} = V \sin\gamma$$

can be used to find the speed when it reaches the ground where $h = 0$. At this place, the aircraft will be performing a 180 degree turn towards the head wind. The same algebraic equations obtained for the turn in the previous turn section may be utilized for the finding the speed loss due to the turn. After one cycle, the bird will be repeating the cycles.

Energy Lost During Turns with Different Bank Angles

During one turn, the mechanical energy lost is given by

$$E_{\text{lost}} = \frac{1}{2}m(V_B^2 - V_C^2) \tag{15}$$

where V_C is the velocity after turn and V_B is the velocity before the turn. For a fixed value of V_B at an altitude of 25 m from ground corresponding to $V_A = 25\ m/s$, the velocity corresponding to different bank angles, the mechanical energy lost is obtained. For the conditions specified ($m = 1$), the energy lost variation with bank angle is given in the figure 3.

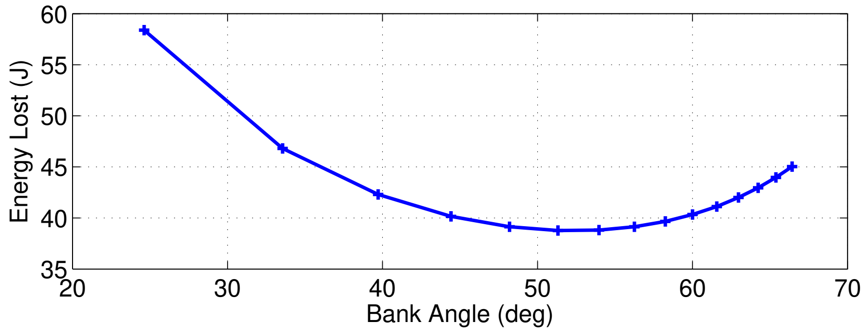

Figure 3: Energy Lost during 180 degree turn for different bank angle (μ)

From the above variation it was observed that at bank angle of approximately 52 degrees, the energy lost is minimum for the conditions specified. This corresponds to a load factor of 1.62 approximately which the bird can sustain comfortably.

Simplified Energy Analysis

In a simplified energy analysis, during climb in the increasing head wind and during descend in tail wind, the energy is gained with a part of it expended due to drag. During level turns energy is lost. It is required to identify the height to which the bird has to climb such that the energy lost is equal to the energy gained. In this analysis energy loss comprises of three components. By varying the heights, the energy lost and energy gain can be found by using these expressions:

$$\text{Energy Lost} = \frac{1}{2}m(V_A^2 - V_B^2) + \frac{1}{2}m(V_B^2 - V_C^2) + \frac{1}{2}m(V_D^2 - V_C^2) \tag{16}$$

$$\text{Energy Gain} = \frac{1}{2}m(V_D^2 - V_A^2) \tag{17}$$

At equilibrium both will be equal. The same is plotted for different heights for the conditions of $V_A = 25\ m/s$ $u_w = \beta h$ ($\beta = 0.25 s^{-1}$), flight path angle $\gamma = 45^0$, turn with bank angle of $\mu = 60^0$. The figure 4 gives the

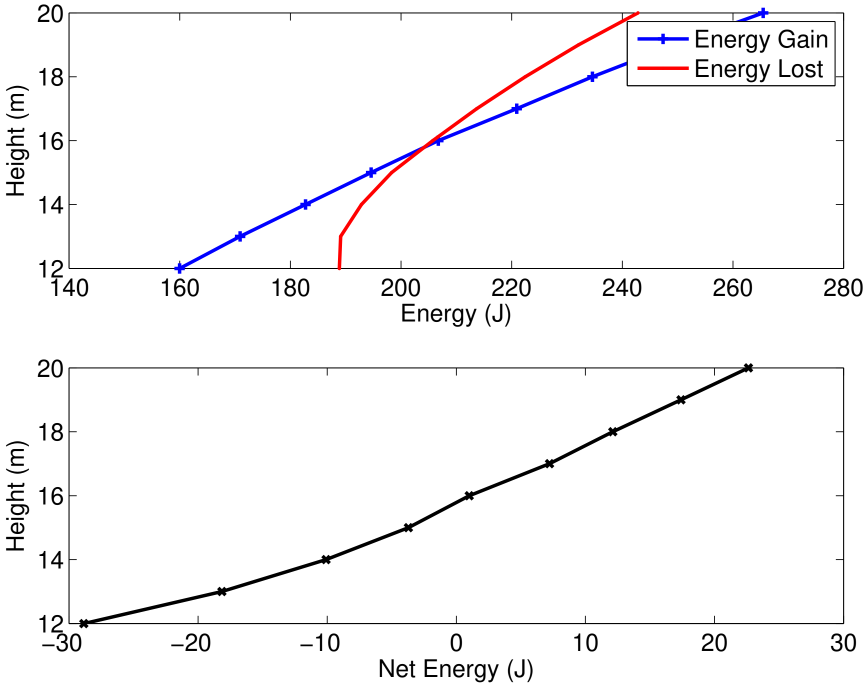

Figure 4: Energy Analysis

variation of energy gain and energy lost at various heights and also net energy with various heights. From the figure, it is found that the height to which the bird / aircraft have to climb such that energy gained is energy lost and energy neutral trajectory is achieved. It is found to be 15.8 m approximately.

Program Listing

The program consists of the following:

1. Main.m - Initialization and performing function calls and plot the outputs
2. f_climb.m - Climb function which performs the integration of the acceleration (deceleration actually) during the climb using ode45 and calls the sub function xot_climb.m
3. f_upperturn.m - The function which estimates the speed after the turn at a height using algebraic equation.
4. f_descend.m - The function which performs the integration of the acceleration during the descend using ode45 and calls the sub function xdot_descend.m
5. f_lowerturn.m - The function which estimates the speed after the turn in the ground using algebraic equation.

Main Program

```
clear all; close all; clc;

global AR mbys CD0 g rho K delchi beta

AR = 15; CD0 = 0.01; mbys = 14.0; g=9.81;
K = 1/(pi*AR); rho = 1.225;
delchi= pi; Clmax = 1.0; beta = 0.25;

% Initial velocity = Vi
hmin = 0.0;
gamma = 45*pi/180;

Vi = 25;
% VA = Vi;
mu = pi/3;

%%
% %-----------------------------------------------------------------------
% % This portion of the code is to evaluates the height corresponding to the
% % given initial speed at A, reduces to a particular value of VB such that
% % after turn exactly equals the stall speed of the bird VC = Vstall.
% %-----------------------------------------------------------------------
% %
% Vstall = sqrt(2*mbys*g/(rho*Clmax));
% hmax1=[0:0.5:40];
% VB = 17.4535;
% for i=1:length(hmax1)
%      VB1(i) = f_climb(VA,hmin,hmax1(i),gamma);
% %    VC1 = f_upperturn(VB1,mu);
% end
% index1 = find(VB1>=VB);
% n1=index1(end); hmax1(n1);
% % str=num2str(hmax(n));
% figure(1), subplot(2,1,1), plot (VB1,hmax1,'b-',VB,hmax1(n1),'ok','LineWidth',2.0); grid on;
% set (gca,'Fontsize',14); % gtext('Height=11.5 m','FontSize',14)
% xlim([5 25]); ylim([0 45]);
% xlabel ('Speed (V_{B}) (m/s) for V_i = 25 m/s and \beta = 0.25 s^{-1}','FontSize',14);
% ylabel ('Climb Height (m)','FontSize',14);
% %-----------------------------------------------------------------------
```

```
%%
% %------------------------------------------------------------------------
% % This portion of the code evaluates the energy lost due to one 180 degree
% % turn for a given initial speed.
% %------------------------------------------------------------------------
%
% VB = 17.5450;   % speed corresponding to VA = 25 m/s
% n1=[1.1:0.1:2.5];
% for i=1:length(n)
%     mu1(i) = acos(1./n1(i));
%     VC(i) = f_upperturn(VB,mu1(i));
%     delE(i) = 0.5*(VB^2 - VC(i)^2)
% end
% figure(2); subplot (2,1,1), plot (mu1*180/pi,delE,'-+b','LineWidth',2.0); grid on;
% set (gca,'FontSize',14);
% xlabel('Bank Angle (deg)'); ylabel ('Energy Lost (J)');
%
% %------------------------------------------------------------------------

%%
%------------------------------------------------------------------------
% This portion of the code evaluates the energy gain and energy lost
% Energy lost   - during the turns
% Energy gained - during climb phase (head wind) and descend phase (tail
% wind)
% All for the initial same initial conditions.
%
% hmax = hmax1(n1);
% hmax = 16.515;
hmax = [12:1:20]; VA(1) = Vi;
for i=1:length(hmax)
    VB(i) = f_climb(VA(i),hmin,hmax(i),gamma);
    Elost1 = 0.5*(VA(i)^2 - VB(i)^2); % during climb energy gain due to wind;
    VC(i) = f_upperturn(VB(i),mu);
    Elost2 = 0.5*(VB(i)^2 - VC(i)^2); % during turn energy lost
    VD(i) = f_descend(VC(i),hmax(i),hmin,gamma);
    Egain2 = 0.5*(VD(i)^2 - VC(i)^2); % during descend energy gain due to wind;
    VA(i+1) = f_lowerturn(VD(i),mu);
    Elost3 = 0.5*(VD(i)^2 - VA(i+1)^2); % during turn energy lost
    Egain(i) = Egain2;
    Elost(i) = Elost1 + Elost2 + Elost3;
end
figure(1);
subplot (2,1,1); plot (Egain,hmax,'b+-',Elost,hmax,'r.-','LineWidth',2.0);
set (gca,'Fontsize',14); xlabel ('Energy (J)','FontSize',14);
ylabel('Height (m)','FontSize',14); legend('Energy Gain','Energy Lost');
subplot (2,1,2); plot (Egain-Elost,hmax,'kx-','LineWidth',2.0);
set (gca,'Fontsize',14);
xlabel('Net Energy (J)','FontSize',14); ylabel('Height (m)','FontSize',14);
%------------------------------------------------------------------------
```

f_climb

```
function [V] = f_climb(Vi,hmin,hmax,gamma)
global g
% V = sqrt(Vi^2-2*g*(hmax-hmin));
% t = hmax/(Vi*sin(gamma));

% tmax = Vi/(sqrt(2*g*hmax));
```

```
tmax = 4;
tspan = [0:0.01:tmax];
x0=[hmin;Vi];

[t,x] = ode45(@(t,x) xdot_climb(t,x),tspan,x0);
index = find((x(:,1)<=hmax));
n=index(end); t(n);
V = x(n,2);
% hf = x(n,1);
end

function [xd] = xdot_climb(t,x)
global mbys CD0 g rho K beta
gamma = 45*pi/180;

% % x(1) = height; x(2) = velocity;

xd = zeros(2,1);

CL = 2*mbys*g*cos(gamma)/(rho*x(2)^2);
CD = CD0+ K*CL^2;
Dbym = CD*0.5*rho*x(2)^2/(mbys);

xd(1)= x(2)*sin(gamma);
xd(2) = -g*sin(gamma)-(-beta)*cos(gamma)*x(2)*sin(gamma)- Dbym;
end
```

f_upperturn.m

```
function [V] = f_upperturn(V1,mu)
global AR mbys CD0 g rho K delchi beta

c= -0.5*rho*g*sin(2*mu);
a= CD0*rho^2*(1/mbys)*(cos(mu))^2;
b= 4*K*(mbys)*g^2;
fac1 = sqrt(b/a);
fac2 = sqrt(a/b);
fac3 = sqrt(a*b);
% Vf = sqrt(2*mbys*g/(rho*Clmax));
arg1 = pi*fac3/c + atan(V1^2*fac2);
V = sqrt( fac1*tan(arg1));
end
```

f_descend

```
function [V] = f_descend(V1,hmax,hmin,gamma)

% global g
% V = sqrt(Vi^2+2*g*(hmax-hmin));

t1=5;
tspan = [0:0.01:t1];
x0=[hmax;V1];

[t,x] = ode45(@xdot_descend,tspan,x0);
index = find((x(:,1)>=hmin));
n=index(end); x(n,1);
V = x(n,2);
end


function [xd] = xdot_descend(t,x)
global mbys CD0 g rho K beta
gamma = -45*pi/180;

% % x(1) = height; x(2) = velocity;
xd = zeros(2,1);

CL = 2*mbys*g*cos(gamma)/(rho*x(2)^2);
CD = CD0+ K*CL^2;
Dbym = CD*0.5*rho*x(2)^2/(mbys);

xd(1)= x(2)*sin(gamma);
xd(2) = -g*sin(gamma)-beta*cos(gamma)*x(2)*sin(gamma)- Dbym;
end
```

f_lowerturn

```
function [V] = f_lowerturn(V1,mu)
global AR mbys CD0 g rho K delchi beta

c= -0.5*rho*g*sin(2*mu);
a= CD0*rho^2*(1/mbys)*(cos(mu))^2;
b= 4*K*(mbys)*g^2;
fac1 = sqrt(b/a);
fac2 = sqrt(a/b);
fac3 = sqrt(a*b);
% Vf = sqrt(2*mbys*g/(rho*Clmax));
arg1 = pi*fac3/c + atan(V1^2*fac2);
V = sqrt( fac1*tan(arg1));
end
```