

Given a basis $B = [b_1 b_2 \dots b_n]$ with Gram Schmidt Orthogonalization $\tilde{B} = [\tilde{b}_1 \tilde{b}_2 \dots \tilde{b}_n]$, where

$$\begin{aligned}\tilde{b}_i &= b_i - u_{i,1} * \tilde{b}_1 - \dots - u_{i,i-1} * \tilde{b}_{i-1}; 1 \leq i \leq n \text{ and} \\ u_{i,j} &= \frac{\langle b_i, \tilde{b}_j \rangle}{\langle \tilde{b}_j, \tilde{b}_j \rangle}; 1 \leq j < i \leq n.\end{aligned}$$

Then the following properties are satisfied by an L3-reduced B

1. $|u_{i,j}| \leq \frac{1}{2}$ for $1 \leq j < i \leq n$
2. $\delta * \|\tilde{b}_i\|^2 \leq \|u_{i+1,i} * \tilde{b}_i + \tilde{b}_{i+1}\|^2$
3. $\det(\Delta) \leq \prod_{i=1}^n \|b_i\| \leq \det(\Delta) * (\prod_{i=1}^n (1 + \frac{\alpha}{4} * (\frac{\alpha^{i-1}-1}{\alpha-1})))^{\frac{1}{2}}$
where $\alpha = \frac{1}{\delta-1/4}$
4. $\|b_j\| \leq \alpha^{\frac{i-j}{2}} * (1 + \frac{\alpha}{4} * (\frac{\alpha^{i-1}-1}{\alpha-1}))^{\frac{1}{2}} * \|\tilde{b}_i\|$, for all $i > j$
5. $\|b_1\| \leq \alpha^{\frac{n-1}{4}} * \det(\Delta)^{\frac{1}{n}}$
6. For $x \in \Delta$ and $x \neq 0$, $\|b_1\| \leq \alpha^{\frac{n-1}{2}} \|x\|$