CE 473: Artificial Intelligence

Autumn 2017

A* Search

first AI robot search algorithm

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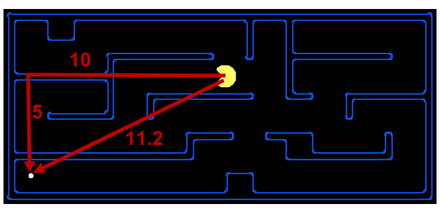
Based on slides from Pieter Abbeel & Dan Klein
Multiple slides from Stuart Russell, Andrew Moore, Luke Zettlemoyer

Today

- A* Search
- Heuristic Design
- Graph search

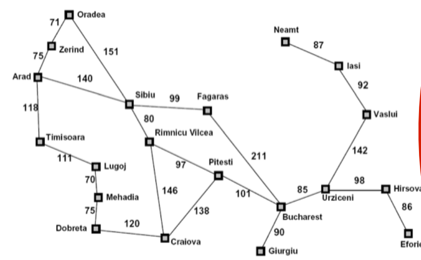
What is a *Heuristic*?

- An ***estimate*** of how close a state is to a goal
- Designed for a particular search problem



- Examples: Manhattan distance: 10+5 = 15
  Euclidean distance: 11.2

Example: Heuristic Function



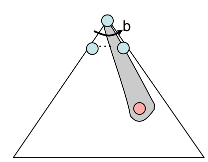
Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

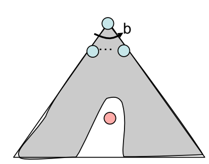
h(x)

Greedy Search

Best First (Greedy)

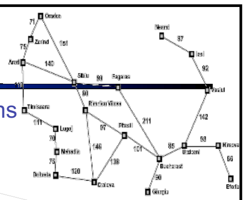
- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state
- A common case:
  - Best-first takes you straight to the (wrong) goal
- Worst-case: like a badly-guided DFS

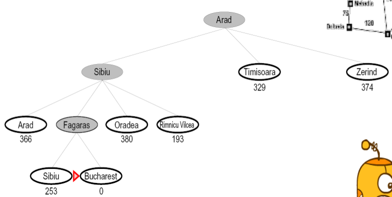


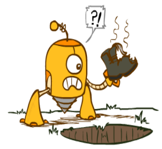


Greedy Search

- Expand the node that seems closest…





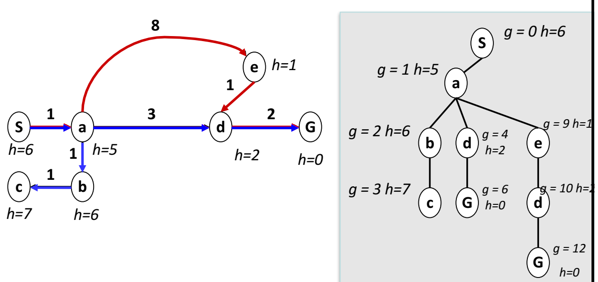


- What can go wrong?

A* Search

Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)

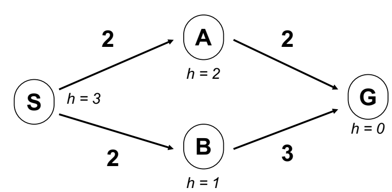


- A* Search orders by the sum: f(n) = g(n) + h(n)

Example: Teg Grenager

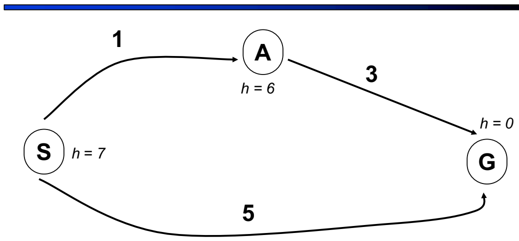
When should A* terminate?

- Should we stop when we enqueue a goal?



- No: only stop when we dequeue a goal

Is A* Optimal?



- What went wrong?
- Actual bad goal cost < estimated good path cost
- We need estimates to be less than or equal to actual costs!

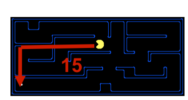
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$ underestimated

where $h^*(n)$ is the true cost to a nearest goal

- Example:

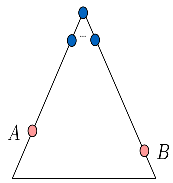


- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Optimality of A* Tree Search

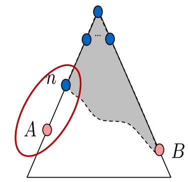
Assume:
- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:
- A will exit the fringe before B



Optimality of A* Tree Search

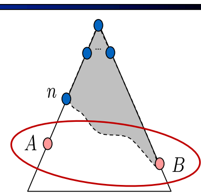
Proof:
- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)



$f(n) = g(n) + h(n)$	Definition of f-cost
$f(n) \leq g(A)$	Admissibility of h
$g(A) = f(A)$	h = 0 at a goal

Optimality of A* Tree Search

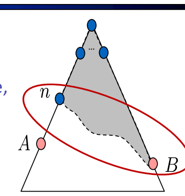
Proof:
- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)



$g(A) < g(B)$	B is suboptimal
$f(A) < f(B)$	h = 0 at a goal

Optimality of A* Tree Search

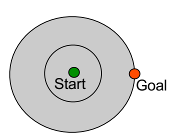
Proof:
- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)
  3. *n* expands before B
- All ancestors of A expand before B
  [since n will put the next node n' toward A on the fringe; repeat claim for n' until you hit A]
- A expands before B
- A* search is optimal



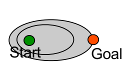
$$f(n) \leq f(A) < f(B)$$

UCS vs A* Contours

- Uniform-cost expanded in all directions

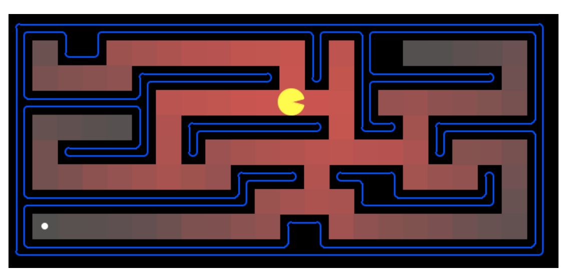


- A* expands mainly toward the goal, but hedges its bets to ensure optimality



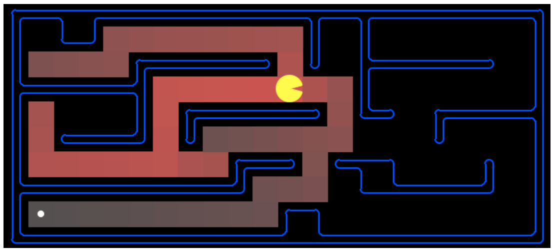
Which Algorithm?

- Uniform cost search (UCS):



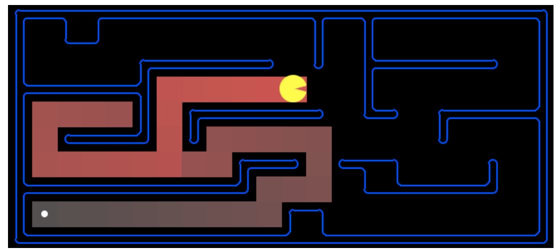
Which Algorithm?

- A*, Manhattan Heuristic:



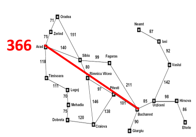
Which Algorithm?

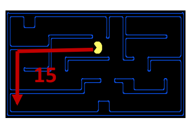
- Best First / Greedy, Manhattan Heuristic:



Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics
- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available

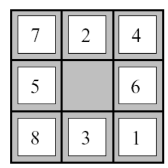




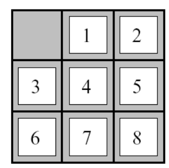
- Inadmissible heuristics are often useful too

Creating Heuristics

8-puzzle:



Start State

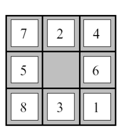


Goal State

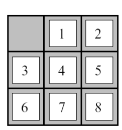
- What are the states?
- How many states? 9!
- What are the actions? 2,3,4
- What states can I reach from the start state?
- What should the costs be?

8 Puzzle I

- Heuristic: Number of tiles misplaced
- h(start) = 8
- Is it admissible?



Start State



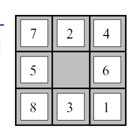
Goal State

	Average nodes expanded when optimal path has length…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

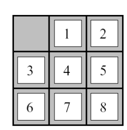
8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- h(start) = 3 + 1 + 2 + …

  = 18
- Admissible?



Start State



Goal State

	Average nodes expanded when optimal path has length…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MANHATTAN	12	25	73

make it more similar to actually cost

8 Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?
- With A*: a trade-off between quality of estimate and work per node!

Trivial Heuristics, Dominance

- Dominance: $h_a \geq h_c$ if

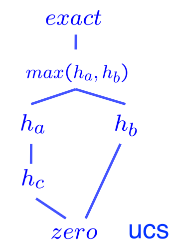
$$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

find the co-dominance

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic

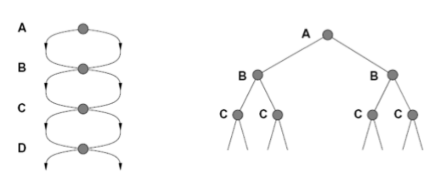


A* Applications

- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …

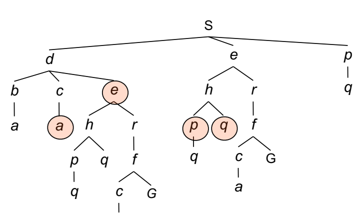
Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work. Why?



Graph Search

- In BFS, for example, we shouldn't bother expanding some nodes (which, and why?)

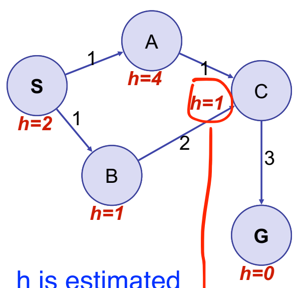


Graph Search

- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states ("closed set")
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Hint: in python, store the closed set as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

A* Graph Search Gone Wrong

State space graph



Search tree

S (0+2)

A (1+4) B (1+1)

C (2+1) C (3+1)

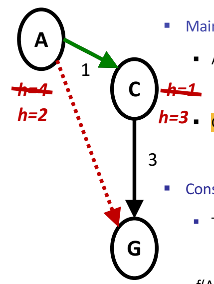
G (5+0) G (6+0)

h is estimated

wrongly estimated

?

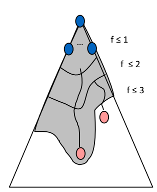
Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal
    - h(A) ≤ actual cost from A to G
  - Consistency: heuristic "arc" cost ≤ actual cost for each arc
    - h(A) – h(C) ≤ cost(A to C)
- Consequences of consistency:
  - The f value along a path never decreases
    - h(A) ≤ cost(A to C) + h(C)
    - $f(A) = g(A) + h(A) \leq \underbrace{g(A) + cost(A\ to\ C)}_{g(C)} + h(C) = f(C)$
  - A* graph search is optimal

Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Nodes are popped with non-decreasing f-scores: for all n, n' with n' popped after n : f(n') ≥ f(n)
    - Proof by induction: (1) always pop the lowest f-score from the fringe, (2) all new nodes have larger (or equal) scores, (3) add them to the fringe, (4) repeat!
  - For every state s, nodes that reach s optimally are expanded before nodes that reach s sub-optimally
  - Result: A* graph search is optimal



by consistency

Optimality

- Tree search:
  - A* optimal if heuristic is admissible (and non-negative)
  - UCS is a special case (h = 0)

typically are same(generally could be not)

- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)
- Consistency implies admissibility
- In general, natural admissible heuristics tend to be consistent, especially if from relaxed problems

Summary: A*

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems