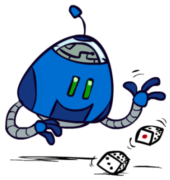
# CSE 473: Artificial Intelligence

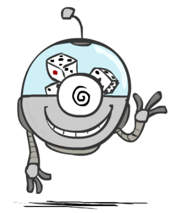
Uncertainty, Utilities



Dieter Fox

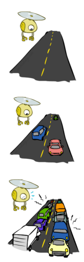


## Probabilities



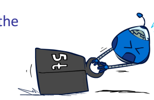
## Reminder: Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes
- Example: Traffic on freeway
  - Random variable: T = whether there's traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25
- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one
- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.25,
  - P(T=heavy | Hour=8am) = 0.60
  - We'll talk about methods for reasoning and updating probabilities later

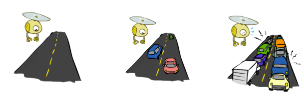



## Reminder: Expectations

- The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes
- Example: How long to get to the airport?

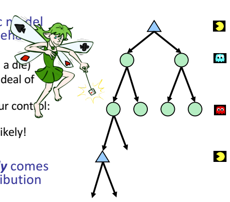


| Time: | 20 min | | 30 min | | 60 min | | |
|---|---|---|---|---|---|---|---|
| | x | + | x | + | x | ⇒ | 35 min |
| Probability: | 0.25 | | 0.50 | | 0.25 | | |



## What Probabilities to Use?

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a chance node for any outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!
- For now, assume each chance node ***magically*** comes along with probabilities that specify the distribution over its outcomes

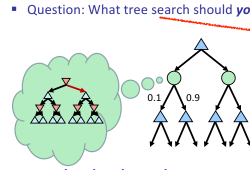


## Informed Probabilities

- Let's say you know that your opponent is sometimes lazy. 20% of the time, she moves randomly, but usually (80%) she runs a depth 2 minimax to decide her move

sophisticated case

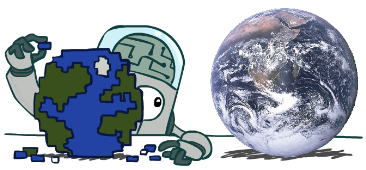
- Question: What tree search should ***you*** use?




two-step look ahead

- Answer: Expectimax!
  - To figure out EACH chance node's probabilities, you have to run a simulation of your opponent
  - This kind of thing gets very slow very quickly
  - Even worse if you have to simulate your opponent simulating you…
  - … except for minimax, which has the nice property that it all collapses into one game tree

## Modeling Assumptions



## The Dangers of Optimism and Pessimism

Dangerous Optimism

Assuming chance when the world is adversarial

Dangerous Pessimism

Assuming the worst case when it's not likely

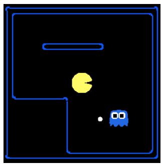
## Video of Demo World Assumptions
## Random Ghost – Expectimax Pacman

## Video of Demo World Assumptions
## Adversarial Ghost – Minimax Pacman

## Video of Demo World Assumptions
## Adversarial Ghost – Expectimax Pacman

## Video of Demo World Assumptions
## Random Ghost – Minimax Pacman

## Assumptions vs. Reality

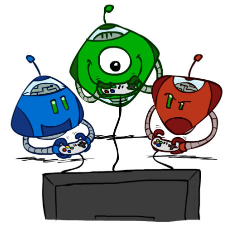


| | Adversarial Ghost | Random Ghost |
|---|---|---|
| Minimax Pacman | Won 5/5<br>Avg. Score: 483 | Won 5/5<br>Avg. Score: 493 |
| Expectimax Pacman | Won 1/5<br>Avg. Score: -303 | Won 5/5<br>Avg. Score: 503 |

Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

## Other Game Types

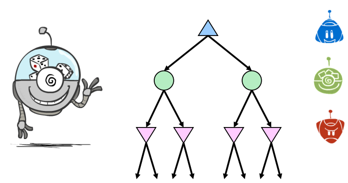


## Example: Backgammon

Image: Wikipedia

## Mixed Layer Types

- E.g. Backgammon
- Expectiminimax
  - Environment is an extra "random agent" player that moves after each min/max agent
  - Each node computes the appropriate combination of its children

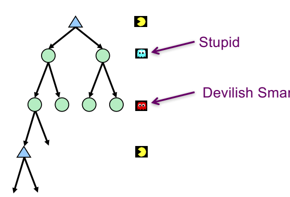


## Example: Backgammon

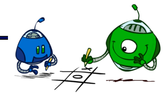
- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ~ 20 legal moves
  - Depth 2 = 20 x (21 x 20)$^3$ = 1.2 x 10$^9$
- As depth increases, probability of reaching a given search node shrinks
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier…
- Historic AI (1992): TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play
- 1$^{st}$ AI world champion in any game!

Image: Wikipedia

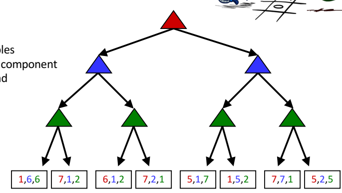
## Different Types of Ghosts?

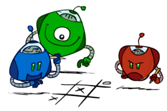


## Multi-Agent Utilities

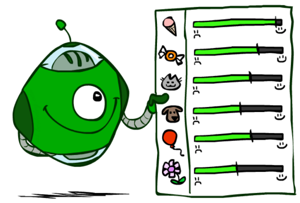


- What if the game is not zero-sum, or has multiple players?
- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically…



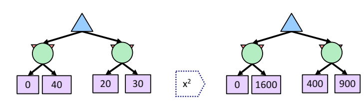
## Utilities



## Maximum Expected Utility

- Why should we average utilities? utilities with weight
- Principle of maximum expected utility:
  - A rational agent should chose the action that maximizes its expected utility, given its knowledge
- Questions:
  - Where do utilities come from?
  - How do we know such utilities even exist?
  - How do we know that averaging even makes sense?
  - What if our behavior (preferences) can't be described by utilities?

## What Utilities to Use?



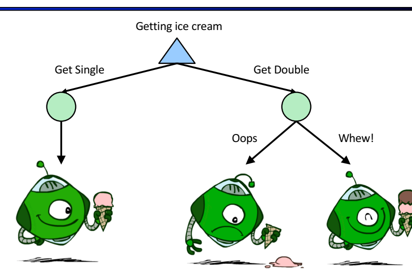

minimax considers the ordering

- For worst-case minimax reasoning, terminal function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this insensitivity to monotonic transformations
- For average-case expectimax reasoning, we need *magnitudes* to be meaningful

## Utilities

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences
- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any "rational" preferences can be summarized as a utility function
- We hard-wire utilities and let behaviors emerge
  - Why don't we let agents pick utilities?
  - Why don't we prescribe behaviors?

## Utilities: Uncertain Outcomes



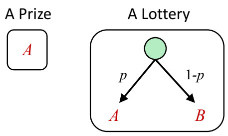
# Preferences

- An agent must have preferences among:
  - Prizes: $A$, $B$, etc.
  - Lotteries: situations with uncertain prizes

$$L = [p, A;\ \ (1-p), B]$$

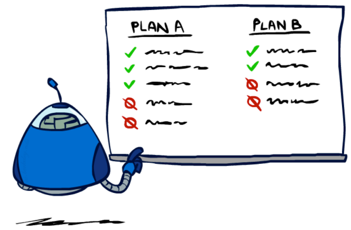
what do you prefer

- Notation:
  - Preference: $A \succ B$
  - Indifference: $A \sim B$





# Rationality

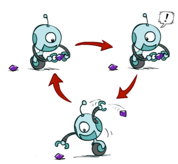



# Rational Preferences

- We want some constraints on preferences before we call them rational, such as:

Axiom of Transitivity: $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$

- For example: an agent with intransitive preferences can be induced to give away all of its money
  - If B > C, then an agent with C would pay (say) 1 cent to get B
  - If A > B, then an agent with B would pay (say) 1 cent to get A
  - If C > A, then an agent with A would pay (say) 1 cent to get C



# Rational Preferences

The Axioms of Rationality

Orderability
$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$
Transitivity
$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$
Continuity
$$A \succ B \succ C \Rightarrow \exists p\ [p, A;\ 1-p, C] \sim B$$
Substitutability
$$A \sim B \Rightarrow [p, A;\ 1-p, C] \sim [p, B; 1-p, C]$$
Monotonicity
$$A \succ B \Rightarrow (p \geq q \Leftrightarrow [p, A;\ 1-p, B] \succeq [q, A;\ 1-q, B])$$

Theorem: Rational preferences imply behavior describable as maximization of expected utility

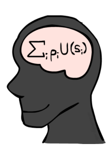
# MEU Principle

- Theorem [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:

$$U(A) \geq U(B) \ \Leftrightarrow\ A \succeq B$$

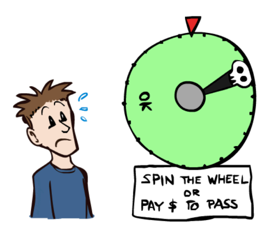
$$U([p_1, S_1;\ \ldots\ ;\ p_n, S_n]) = \sum_i p_i U(S_i)$$

  - I.e. values assigned by U preserve preferences of both prizes and lotteries!




- Maximum expected utility (MEU) principle:
  - Choose the action that maximizes expected utility
  - Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tic-tac-toe, a reflex vacuum cleaner

# Human Utilities



## Utility Scales

insurance

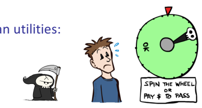
- Normalized utilities: $u_+ = 1.0$, $u_- = 0.0$
- Micromorts: one-millionth chance of death, useful for paying to reduce product risks, etc.
- QALYs: quality-adjusted life years, useful for medical decisions involving substantial risk
- Note: behavior is invariant under positive linear transformation

$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

- With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes

## Human Utilities

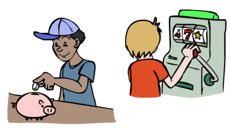
- Utilities map states to real numbers. Which numbers?
- Standard approach to assessment (elicitation) of human utilities:
  - Compare a prize A to a standard lottery $L_p$ between
    - "best possible prize" $u_+$ with probability p
    - "worst possible catastrophe" $u_-$ with probability 1-p
  - Adjust lottery probability p until indifference: A ~ $L_p$
  - Resulting p is a utility in [0,1]

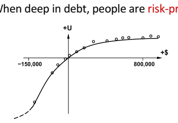



Pay $30 ~ [0.999999: No change; 0.000001: Instant death]

## Money

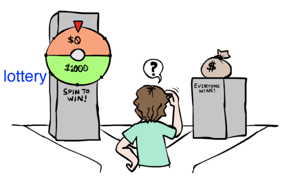
- Money does not behave as a utility function, but we can talk about the utility of having money (or being in debt)
- Given a lottery L = [p, $X; (1-p), $Y]
  - The expected monetary value EMV(L) is p*X + (1-p)*Y
  - U(L) = p*U($X) + (1-p)*U($Y)
  - Typically, U(L) < U( EMV(L) )
  - In this sense, people are risk-averse   convex
  - When deep in debt, people are risk-prone





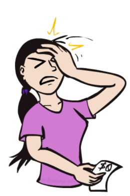

## Example: Insurance

- Consider the lottery [0.5, $1000;  0.5, $0]
  - What is its expected monetary value?  ($500)
  - What is its certainty equivalent?
    - Monetary value acceptable in lieu of lottery
    - $400 for most people prefer 400 than lottery
  - Difference of $100 is the insurance premium
    - There's an insurance industry because people will pay to reduce their risk
    - If everyone were risk-neutral, no insurance needed!
  - It's win-win: you'd rather have the $400 and the insurance company would rather have the lottery (their utility curve is flat and they have many lotteries)




## Example: Human Rationality?

- Famous example of Allais (1953)
  - A: [0.8, $4k;    0.2, $0]
  - B: [1.0, $3k;    0.0, $0]
  - C: [0.2, $4k;    0.8, $0]
  - D: [0.25, $3k;    0.75, $0]
- Most people prefer B > A, C > D
- But if U($0) = 0, then
  - B > A → U($3k) > 0.8 U($4k)
  - C > D → 0.8 U($4k) > U($3k)



## Kahneman & Tversky

**Choose between**

| Option A | | Option B | |
|---|---|---|---|
| 33% | $2500 | 100% | $2400 |
| 66% | $2400 | | |
| 01% | 0 | | |
| [18] | | [82]* | |

## Kahneman & Tversky

**Choose between**

Option C

- 33% $2500
- 67% 0

[83]*

Option D

- 34% $2400
- 66% 0

[17]

## Kahneman & Tversky

**Choose between**

Option A

- 33% $2500
- ~~66%~~ $2400
- 01% 0

[18]

Option B

- ~~100%~~ $2400

-66% chance of $2400 from both options

[82]*

Option C

- 33% $2500
- 67% 0

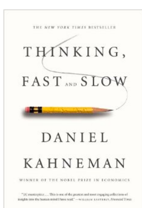
[83]*

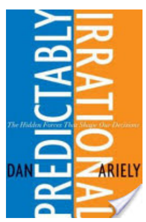
Option D

- 34% $2400
- 66% 0

[17]

## Recommended

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