4. 기초수식

- 문제 2: $T(n) = T(n-1) + n$

$(T(0) = 1)$

$$T(n) = T(n-1) + n-1$$

$$T(n) = T(n-2) + 2n-1$$

$$= T(n-3) + 3n-3$$

$$= T(n-4) + 4n-6$$

$$\vdots$$

$$T(n) = T(n-k) + \frac{k(n+n-k+1)}{2}$$

$$= T(n-k) + \frac{k(2n-k+1)}{2}$$

$$T(n) = T(0) + \frac{n(n+1)}{2}$$

$$= 1 + \frac{n(n+1)}{2}$$

$$= \frac{n^2}{2} + \frac{n}{2} + 1 \rightarrow O(n^2)$$

- 문제 4: $T(n) = T\left(\frac{n}{2}\right) + 1$

$T(1) = 1$

$$T(n) = T\left(\frac{n}{2}\right) + 1$$

$$= T\left(\frac{n}{4}\right) + 2$$

$$= T\left(\frac{n}{8}\right) + 3$$

$$\vdots$$

$$T(n) = T\left(\frac{n}{2^k}\right) + k$$

let $k = \log_2 n$

$$T(n) = T\left(\frac{n}{n}\right) + \log_2 n$$

$$= T(1) + \log_2 n$$

$$= \log_2 n + 1 \Rightarrow O(\log n)$$

- 문제 6: $T(n) = 2T\left(\frac{n}{2}\right) + n$

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$= 4T\left(\frac{n}{4}\right) + 2n$$

$$= 8T\left(\frac{n}{8}\right) + 3n$$

$$\vdots$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + k \times n$$

let $k = \log_2 n$ $\Rightarrow$ $T(n) = n\,T\left(\frac{n}{n}\right) + n\log_2 n$

$$= n\,T(1) + n\log_2 n$$

$$\Rightarrow O(n \log n)$$

- 문제 8: $T(n) = T(n-1) + \frac{1}{n}$

$$T(n) = T(n-1) + \frac{1}{n}$$

$$= T(n-2) + \frac{1}{n-1} + \frac{1}{n}$$

$$= T(n-3) + \frac{1}{n-2} + \frac{1}{n-1} + \frac{1}{n}$$

$$\vdots$$

$$T(n) = T(n-k) + \sum_{i=0}^{k-1} \frac{1}{n-i}$$

let $k=n$ $\hookrightarrow$ $T(n) = T(0) + \sum_{i=0}^{n-1} \frac{1}{n-i}$

$$T(n) = \sum_{k=1}^{n} \frac{1}{k} \quad \left(\leq \int_1^n \frac{1}{k}\, dk = \log n \right)$$

$$\Rightarrow O(\log n)$$