NUI Galway
OÉ Gaillimh

Summer Examinations 2016/17

Exam Code(s)	4BS2, 4FM2, 4BCT1, 4BMS2, 3BME1, 4BME1
Exam(s)	4th Science
Module(s)	Networks
Module Code(s)	CS423
External Examiner(s)	Prof. M. Lawson
Internal Examiner(s)	Prof. G. Ellis
	*Prof. G. Pfeiffer
Instructions	Answer *all* questions.
Duration	2 hours
No. of Pages	4 pages (including this cover page)
Discipline	Mathematics

Requirements:

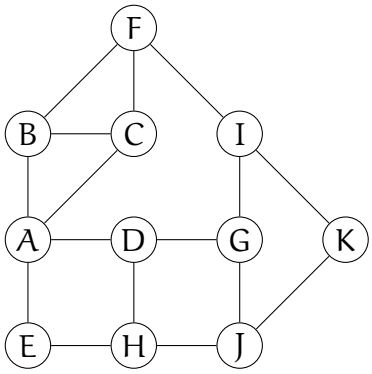
Release to Library	Yes ☑	No ☐
Release in Exam Venue	Yes ☑	No ☐
MCQ	Yes ☐	No ☑
Statistical/Log Tables	Yes ☑	No ☐

1.

(a) [**10** marks]

(i) Describe Breadth First Search as an algorithm for undirected graphs. What is its input, what is its output, and what sequence of steps is taken to produce the output from the input?

(ii) In the network below, apply Breadth First Search to determine a spanning tree with root G and the shortest distances from node G to each of the other nodes in the graph.



(b) [**10** marks]

(i) An edge AB in a graph is called a *bridge* if deleting this edge would cause the nodes A and B to lie in different connected components of the graph. What, in contrast, is a *local bridge*?

(ii) Suppose that a graph has two types of edges: strong ties and weak ties. A node A of the graph is said to satisfy the *Strong Triadic Closure Property* if, for any two nodes B and C, there is an edge (weak or strong) between B and C whenever A has strong ties with both A and B.

Show that, if a node A in the graph satisfies the Strong Triadic Closure Property and is involved in at least two strong ties, then any local bridge that A is involved in is necessarily a weak tie.

2.

(a) [**10** marks]

(i) Provide a formal definition of a finite n-player game.

(ii) What is a *best response* and what is a *Nash equilibrium* in a game?

(iii) For the following 2-player game, find all pure strategy Nash equilibria. The rows here correspond to player A's strategies, and the columns to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

	L	M	R
U	1, 1	2, 3	1, 6
M	3, 4	5, 5	2, 2
D	1, 10	4, 7	0, 4

(b) [**10** marks]

(i) Describe a sealed-bid second price auction as a game.

(ii) What is a dominating strategy for a bidder in a sealed-bid second price auction? Justify your answer.

(iii) A seller wishes to sell an item, which she values at s, in a sealed-bid second price auction. There are two buyers, who value the item at v_1 and v_2, respectively. Both buyers know that the seller will submit her own sealed bid of s, but they do not know the value of s. What is an optimal strategy for a bidder in this auction? Explain your answer.

3.

(a) [**10** marks]

(i) What is a bipartite graph by definition? And how can bipartite graphs be characterized in terms of cycles?

(ii) What does the Matching Theorem say about a bipartite graph?

(iii) Sketch a proof of the Matching Theorem.

(b) [**10** marks]

(i) Describe the concept of market clearing prices between a set of sellers and a set of buyers.

(ii) Suppose there are three sellers, A, B and C, and three buyers, X, Y and Z. Each seller is offering a house for sale, and the valuations of the buyers for the houses are as listed in the following matrix.

	A	B	C
X	3	6	4
Y	2	8	1
Z	1	2	3

Describe briefly the bipartite graph auction procedure. Apply this procedure to the above valuations in order to obtain a set of market clearing prices.

PTO.

4.

(a) [**10** marks] Suppose that $\mathsf{G} = (\mathsf{X}, \mathsf{E})$ is a directed network with node set X and edges $\mathsf{E} \subseteq \mathsf{X}^2$.

(i) Define what it means for G to be *weakly connected*, and what it means to be *strongly connected*.

(ii) In general, a directed network is partitioned into strongly connected components. Describe the equivalence relation that yields strongly connected components as its equivalence classes in terms of E, regarded as a relation on X.

(iii) Many directed networks, like the World Wide Web, have a giant strongly connected component which covers a substantial part of the network. Name the other parts of the so-called Bow-Tie diagram of the components in a directed network, and describe each in terms of a suitable relation on the set of strongly connected components.

(b) [**10** marks]

(i) Describe the Hubs and Authorities procedure for the ranking of web pages. What is its input, what is its output, and what sequence of steps is used to compute the output from the input?

(ii) Why does the Hubs and Authorities ranking procedure converge?

(iii) Show the values obtained by applying two rounds of the Hubs and Authorities ranking procedure to the following network of web pages.