OLLSCOIL NA GAILLIMHE
UNIVERSITY OF GALWAY

Semester 1 Examinations 2023/2024

Exam Code(s)	3BA1, 4BMU1, 3BMS2, 4BMS2, 3BS9, 4BS2
Exam(s)	3rd/4th Arts, 3rd/4th Science.
Module(s)	Euclidean and non-Euclidean Geometry
Module Code(s)	MA3101
Repeat Paper	No
External Examiner(s)	Prof. Colva Roney-Dougal
Internal Examiner(s)	Dr. Tobias Rossmann Dr. A Mohajer*
Instructions:	Answer **all four** questions.
Duration	2 hours
No. of Pages	3 pages (including this cover page)
Discipline(s)	Mathematics

Requirements:

MCQ	Yes ☐	No ☑
Release to Library	Yes ☑	No ☐
Handout	Yes ☐	No ☑
Statistical Tables/ Log Tables	Yes ☑	No ☐
Cambridge Tables	Yes ☐	No ☑
Graph paper	Yes ☐	No ☑
Log Graph Paper	Yes ☐	No ☑
Other Materials	Students may use their own electronic calculators which must not be capable of storing text.	

Q1 [25 marks]

Let $S^2 \subset \mathbb{R}^3$ be the sphere of radius one centred at the origin.

(a) Define a *great circle* in S^2 and describe (without proof) the geodesic between two points on S^2. [**8 marks**]

(b) Show that in a spherical geodesic triangle on S^2, the sum of any two sides is greater than the third side. [**8 marks**]

(c) Let T be a triangle in S^2 with all sides of length $\pi/4$. Calculate the area of T. [**9 marks**]

Q2 [25 marks]

(a) Let $\mathbb{U}$ be the upper sheet of the hyperboloid and consider the curve γ in $\mathbb{U}$ defined by

$$\gamma : [2,3] \to \mathbb{U}$$
$$t \mapsto (\sqrt{2t-2}, t-1, t)$$

Calculate the length of γ.

[**7 marks**]

(b) Compute the hyperbolic geodesic length between the two points $x = (4,8,9), y = (2,2,3)$ on $\mathbb{U}$. [**6 marks**]

(c) Prove that the positive y-axis is a hyperbolic geodesic of the upper half plane $\mathbb{H}$. [**6 marks**]

(d) Find the area of the hyperbolic (geodesic) triangle in the upper half plane $\mathbb{H}$ with vertices $(-3,0), (-2,0)$ and $(-1,0)$. [**6 marks**]

Q3 [25 marks]

(a) Let $\mathbb{P}(V)$ be a 3-dimensional projective space. Show that any two planes in $\mathbb{P}(V)$ intersect in a line. [**8 marks**]

(b) If $M \in PGL_2(\mathbb{R})$ is a non-trivial involution ($M \neq \lambda \mathrm{id}, M^2 = \mathrm{id}$) then it has either 0 or 2 fixed points. [**8 marks**]

(c) Show that a bijective map $f : \mathbb{R}^2 \to \mathbb{R}^2$ which maps lines to lines must be affine, i.e., there exists a matrix $A \in GL_2(\mathbb{R})$ and a vector $b \in \mathbb{R}^2$ such that $f(x) = Ax + b$. [**9 marks**]

Q4 [25 marks]

(a) Calculate the first and second fundamental forms as well as the Gaussian and Mean curvatures of the surface $z = x^2 - y^2$. [**10 marks**]

(b) Find the Gaussian curvature K at all points of the surface given by $z = x^2 + y^2$. Evaluate $\int_R K dA$ where R is the region of the surface from $z = 0$ to $z = a^2$ ($a \in \mathbb{R}$). [**10 marks**]

(c) Give an example of a smooth cubic surface and describe all its lines. [**5 marks**]