Summer Examinations 2021-2022

Exam Codes	1BMG1, 1GDS1, 3BA1, 4BCT1, 4BDS1, 4BME1, 4BMS2, 4BS2, 4FM2
Exam	4th Science
Module	Networks
Module Code	CS4423
External Examiner(s)	Prof Colva Roney–Dougal
Internal Examiner(s)	Dr N. Madden Dr A. Carnevale*
Instructions	Answer all **four** questions.
Duration	2 hours
No. of Pages	3 pages (including this cover page)
Discipline	Mathematics

Requirements:

Release to Library	Yes
Release in Exam Venue	Yes
Statistical Tables/ Log Tables	Yes

Q1. [15 Marks]

(a) [3 Marks] What is a *graph*? What is a *tree*? Give an example of a graph of *order* 5 and *size* 6. Does a tree of *order* 5 and *size* 6 exist? Justify your answer.

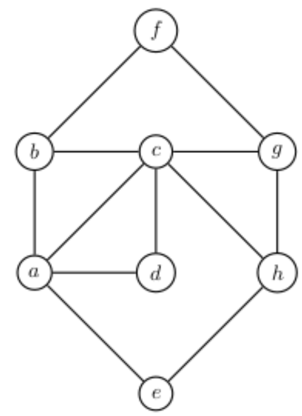
(b) [5 Marks] What is the *Prüfer code* of a labelled tree and how can it be obtained? How can the degree sequence of a labelled tree be obtained from its Prüfer code? Which labelled tree on vertices $\{0, 1, \ldots, 6\}$ has Prüfer code $[1, 1, 1, 1, 1]$? Justify your answer.

(c) [3 Marks] Describe *Breadth First Search* as an algorithm for computing distances between vertices in a graph. What is its input, what is its output, and what sequence of steps is taken to produce the output from the input?

(d) [4 Marks] In the network on the right, apply Breadth First Search to determine

(i) a spanning tree with root a, and

(ii) the shortest distances from vertex a to each of the other vertices in the graph.



Q2. [15 Marks]

(a) [5 Marks] Define the concepts of *degree centrality* and of *normalised degree centrality* for a graph G. Let G be the graph on the vertex set $\{0, 1, 2, 3, 4, 5, 6\}$ with edges $0-3$, $1-2$, $1-3$, $2-3$, $3-4$, $3-5$, $3-6$ and $4-5$. Draw the graph G and determine the normalised degree centrality of all its vertices.

(b) [5 Marks] Define the concepts of *closeness centrality* and *normalised closeness centrality* for a connected graph G. Give an example of a connected graph on 7 vertices with a vertex of normalised closeness centrality 1.

(c) [5 Marks] Define the concepts of *betweenness centrality* and *normalised betweenness centrality* for a connected graph G. Give an example of a connected graph on 7 vertices with a vertex of normalised betweenness centrality 0.

Q3. [15 Marks]

(a) [3 Marks] Describe how to generate a graph from the Erdős–Rényi model $G(n, m)$.

(b) [4 Marks] Describe how to generate a graph from the Erdős–Rényi model $G(n, p)$. Following this model, what is the probability of a randomly chosen graph G to have *exactly* m edges? Justify your answer.

(c) [4 Marks] Construct a graph G on 80 vertices by tossing a fair coin once for each possible edge, adding the edge only if the coin shows heads. Then pick a vertex at random in this graph. What is the probability that this vertex has degree 40?

(d) [4 Marks] What is a *giant component* in a graph G? State the *Erdős–Rényi Theorem* on the appearance of a giant component in a graph.

Q4. [15 Marks]

(a) [3 Marks] Sketch an example of an (n, d)-circle graph for $n = 8$ and $d = 2$. How many edges does this graph have?

(b) [5 Marks] Describe how to generate an (n, d, p)-WS graph in the *Watts–Strogatz small-world model.* What properties does a random graph sampled from the WS model have, that one would not find in a random graph sampled from the $G(n, p)$ model, or in an (n, d)-circle graph?

(c) [4 Marks] For a directed graph G on a vertex set X, define two equivalence relations on X: one that has the *strongly connected components* of G as its equivalence classes, and one that has the *weakly connected components* of G as its equivalence classes.

(d) [3 Marks] Many directed networks, like the World Wide Web, have a giant strongly connected component which covers a substantial part of the network. Name the other parts of the so-called *Bow-Tie diagram* of the components in a directed network, and describe each in terms of a suitable relation on the set of strongly connected components.