NUI Galway
OÉ Gaillimh

Summer Examinations 2017/18

Exam Code(s)	4BS2, 4FM2, 4BCT1, 4BMS2, 4BME1, 1BMG1
Exam(s)	4th Science
Module(s)	Networks
Module Code(s)	CS4423
External Examiner(s)	Prof. T. Brady
Internal Examiner(s)	Prof. G. Ellis
	*Prof. G. Pfeiffer
Instructions	Answer *all* questions.
Duration	2 hours
No. of Pages	4 pages (including this cover page)
Discipline	Mathematics

Requirements:

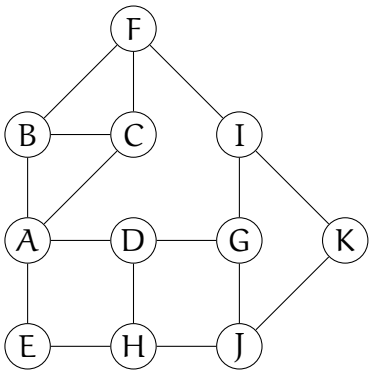
Release to Library	Yes ☑	No ☐
Release in Exam Venue	Yes ☑	No ☐
MCQ	Yes ☐	No ☑
Statistical/Log Tables	Yes ☐	No ☑

1.

(a) [**10** marks]

(i) Describe Breadth First Search as an algorithm for undirected graphs. What is its input, what is its output, and what sequence of steps is taken to produce the output from the input?

(ii) In the network on the side, apply Breadth First Search to determine a spanning tree with root D and the shortest distances from node D to each of the other nodes in the graph.



(b) [**10** marks] A graph, whose edges are all labelled either '+' or '−', satisfies the *Structural Balance Property*, if any triangle in the graph (3 nodes joined by 3 edges) has an odd number of edges (i.e., either 1 or all 3 edges) labelled '+' .

(i) What is a *complete* graph?

(ii) What does *Harary's Balance Theorem* say about a complete graph whose edges are all labelled either '+' or '−'?

(iii) Sketch a proof of Harary's Balance Theorem.

2.

(a) [**10** marks]

(i) Provide a formal definition of a finite n-player game.

(ii) What is a *best response* and what is a *Nash equilibrium* in a game?

(iii) Consider the following 2-player game, where the rows correspond to player A's strategies, and the columns to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

	L	R
U	3, 3	1, 2
D	2, 1	3, 0

Find all (pure) Nash equilibria of this game. Then change one but not both of the payoffs in the (U, L) entry in the above game to some nonnegative integer value, so that the resulting game has no pure Nash equilibrium. Compute a mixed-strategy equilibrium for the resulting game.

(b) [**10** marks] Two cities, A and B, are joined by two routes through distinct cities C and D, and 80 cars need to travel from A to B. The first route consists of a highway from A to a city C and a local road from C into B. The second route consists of a local road from A to a city D, and a highway from D to B. The travel time along a highway is one hour, regardless of the number of cars using it. The travel time along a local road is 10 plus the number of cars using it. Drivers simultaneously choose which route to use.

(i) Draw the transport network described above and label each edge with the time needed to travel along that edge. Let x be the number of cars travelling along the first route. Drivers simultaneously choose which route to use. Find the Nash equilibrium value for x.

(ii) The government builds a new two-way road connecting cities C and D. Describe the new routes from A to B which are added by this connection.

(iii) The new road is very short and takes no travel time at all. Find the new Nash equilibrium. What happens to the total travel time as a result of the availability of the new road?

3.

(a) [**10** marks]

(i) Define the term *bipartite graph*. How can a bipartite graph be characterized in terms of its cycles?

(ii) What does the Matching Theorem say about certain bipartite graphs?

(iii) Sketch a proof of the Matching Theorem.

(b) [**10** marks]

(i) Describe the concept of market clearing prices between a set of sellers and a set of buyers.

(ii) Suppose there are three sellers, A, B and C, and three buyers, X, Y and Z. Each seller is offering a house for sale, and the valuations of the buyers for the houses are as listed in the following matrix.

	A	B	C
X	3	6	4
Y	2	8	1
Z	1	2	3

Describe briefly the bipartite graph auction procedure. Apply this procedure to the above valuations in order to obtain a set of market clearing prices.

4.

(a) [**10** marks] Suppose that $\mathsf{G} = (\mathsf{X}, \mathsf{E})$ is a directed network with node set X and edges $\mathsf{E} \subseteq \mathsf{X}^2$.

(i) Define what it means for G to be *weakly connected*, and what it means to be *strongly connected*.

(ii) In general, a directed network is partitioned into strongly connected components. Describe the equivalence relation that yields strongly connected components as its equivalence classes in terms of E, regarded as a relation on X.

(iii) Many directed networks, like the World Wide Web, have a giant strongly connected component which covers a substantial part of the network. Name the other parts of the so-called Bow-Tie diagram of the components in a directed network, and describe each in terms of a suitable relation on the set of strongly connected components.

(b) [**10** marks]

(i) Describe the PageRank algorithm. What is its input, what is its output, and what sequence of steps is used to compute the output from the input?

(ii) In this context, what is an equilibrium for PageRank?

(iii) In the network below, does the assignment of numbers to the nodes form an equilibrium? Justify your answer.