Summer Examinations 2010

Exam Code(s) 4BS3, 1PAH1, 1HA1	
Exam(s)	Bachelor of Science (Hons. Maths) Higher Diploma in Applied Science Postgraduate Diploma in Arts
Module(s)	FUNCTIONAL ANALYSIS ADVANCED ANALYSIS II
Module Code(s)	MA482, MA541
Paper No	1
Repeat Paper	
External Examiner(s)	Professor D. Armitage
Internal Examiner(s)	Professor T. Hurley and Professor D. O'Regan
Instructions:	**Full marks for THREE complete answers.**
Duration	2 Hours
No. of Pages	3 Pages

Requirements:

MCQ	Release to Library: Yes
Handout	
Statistical Tables/ Log Tables	Yes
Cambridge Tables	
Graph paper	
Log Graph Paper	
Other Materials	

1. (a) Show $l^{10} \subseteq l^{11}$ and $\|x\|_{l^{11}} \leqq \|x\|_{l^{10}}$ for all $x \in l^{10}$.

(b) Let X and Y be normed spaces with $T : X \to Y$ a linear mapping. If T is continuous at 0 show there exists a $M > 0$ with $||T\,x|| \leq M\,||x||$ for all $x \in X$.

(c) Let X and Y be normed spaces, let $\{T_n\}$ be a sequence in $B(X, Y)$ such that $\lim_{n\to\infty} T_n\,x = T\,x$ for each $x \in X$, and suppose there exists a constant $M > 0$ with $\sup_n \|T_n\| \leq M$. Show

$$T \in B(X, Y) \quad \text{and} \quad \|T\| \leq \liminf_{n\to\infty} \|T_n\|.$$

2. (a) Let X be a normed space and Y a Banach space. Show $B(X, Y)$ is a Banach space.

(b) Let K be a nonempty closed convex subset of a Hilbert space H. Show for every $x \in H$ there is a unique $y \in K$ with

$$\|x - y\| = \inf_{z \in K} \|x - z\|.$$

(c) Show c_0 (with the usual sup norm) is not a Hilbert Space.

3. (a) Show that the dual space $(c_0)'$ is isometrically isomorphic to l^1.

(b) A linear functional is defined on the Banach space l^{10} by

$$\phi(x) = -3\,x_1 + 4\,x_2 + 6\,x_5 - 8\,x_7$$

where $x = (x_1,\, x_2 \ldots) \in l^{10}$. Find the norm of ϕ.

(c) State the Hahn-Banach Theorem. Let X and Y be normed spaces with $X \neq \{0\}$. If $B(X, Y)$ is complete, show Y is complete.

4. (a) Let $\{e_k\}$ be an orthonormal sequence in a Hilbert space H. Show $\sum \alpha_k \, e_k$ converges if (α_k) belongs to l^2.

(b) Suppose H_1 and H_2 are Hilbert spaces and $T : H_1 \to H_2$ a bounded linear operator. Define the Hilbert adjoint T^* of T. Show T^* exists, is unique, and is a bounded linear operator with norm $\|T^*\| = \|T\|$.

(c) Let $S : l^2 \to l^2$ be given by

$$S(x) = (0,\, x_1,\, x_2,\, ...) \text{ when } x = (x_1, x_2, ...)$$

For $n \in \{1, 2, ...\}$ show $S^*(e_{n+1}) = e_n$.