Ollscoil na Gaillimhe
University of Galway

Semester 2 Examinations 2023

Exam Codes	3BME1, 4BCT1, 4BDS1, 4BME1, 4BMS2, 4BS2, 4FM2
Exam	4th Science
Module	Networks
Module Code	CS4423
External Examiner(s)	Prof. Colva Roney–Dougal
Internal Examiner(s)	Prof. G. Pfeiffer Dr A. Carnevale*
Instructions	Answer all **four** questions.
Duration	2 hours
No. of Pages	3 pages (including this cover page)
Discipline	Mathematics

Requirements:

Release to Library	Yes
Release in Exam Venue	Yes
Statistical Tables/ Log Tables	Yes
Non-programmable calculators	Yes

Q1. [25 Marks]

(a) [10 Marks]

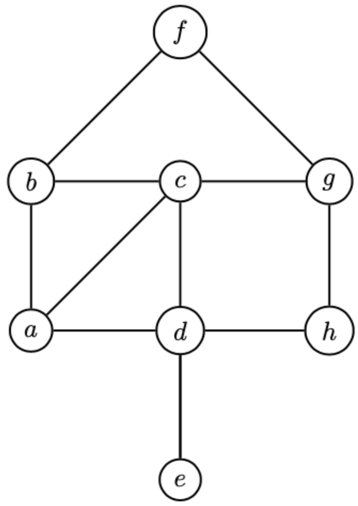
(i) What is a *graph*?

(ii) What are the *order* and the *size* of a graph?

(iii) What is the *adjacency matrix* of a graph?

(iv) How can one compute the degree of a vertex from the adjacency matrix?

(v) How can one compute the size of a graph from its adjacency matrix?



(b) [7 Marks] Describe *Breadth First Search* as an algorithm for computing distances between vertices in a graph. What is its input, what is its output, and what sequence of steps is taken to produce the output from the input?

(c) [4 Marks] Show how the Breadth First Search algorithm for distances proceeds when the inputs are the graph on the right and its node a.

(d) [4 Marks] In the graph on the right, use the Breadth First Search algorithm to also determine a *spanning tree* with root a.

Q2. [25 Marks]

(a) [5 Marks] Define the concept of *normalised degree centrality* for a graph G. Let G be the graph on the vertex set $\{1, 2, 3, 4, 5\}$ with edges $1-2$, $1-3$, $2-4$, $3-4$, $3-5$, and $4-5$. Draw the graph G and determine the normalised degree centrality of all its vertices.

(b) [5 Marks] Define the concept of *normalised closeness centrality for a connected graph* G. Compute the normalised closeness centrality of nodes 1 and 2 of G in (a).

(c) [5 Marks] Define the concept *normalised betweenness centrality* for a connected graph G. Compute the normalised betweenness centrality of nodes 1 and 2 of G in (a).

(d) [10 Marks] Let H be a path graph on 3 vertices. The eigenvalues of the adjacency matrix of A are $-\sqrt{2}$, 0 and $\sqrt{2}$. Use this information to compute the normalised eigenvector centrality of each node in H.

Q3. [25 Marks]

(a) [8 Marks] Describe how to generate a graph from the Erdős–Rényi models $G(n, m)$ and $G(n, p)$. In each model, what is the probability of a randomly chosen graph G to have *exactly* m edges? Justify your answer.

(b) [5 Marks] Suppose one constructed a graph G on 100 vertices by tossing a (fair, 6-sided) die once for each possible edge, adding the edge only if the die shows 6. Then pick a vertex at random in this graph. What is the probability that this vertex has degree 50? (You don't need to return a numerical value. It is enough to give an explicit formula in terms of the given data.)

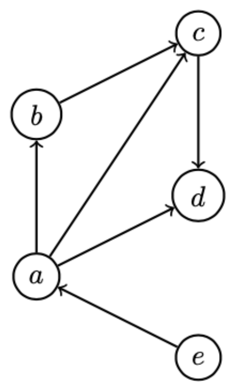
(c) [5 Marks] What is a *giant component* in a graph G? State the *Erdős–Rényi Theorem* on the appearance of a giant component in a graph.

(d) [7 Marks] Describe how to generate an (n, d, p)-WS graph in the *Watts–Strogatz small-world model.* What properties does a random graph sampled from the WS model have, that one would not find in a random graph sampled from the $G(n, p)$ model, or in an (n, d)-circle graph?

Q4. [25 Marks]

(a) [8 Marks] What is the *Prüfer code* of a labelled tree T on n vertices, and how can it be computed from T? How can the degree sequence of a labelled tree be determined from its Prüfer code? Compute the Prüfer code of the tree on the vertex set $X = \{1, 2, 3, 4, 5, 6\}$ with edges 1–2, 2–3, 3–4, 4–5, 4–6.

(b) [5 Marks] What is a *directed graph*? What is the in-degree and what is the out-degree of vertex c in the directed graph on the right?



(c) [7 Marks] For a directed graph G on a vertex set X, define two equivalence relations on X: one that has the *strongly connected components* of G as its equivalence classes, and one that has the *weakly connected components* of G as its equivalence classes.

(d) [5 Marks] Many directed networks, like the World Wide Web, have a giant strongly connected component which covers a substantial part of the network. Name the other parts of the so-called *Bow-Tie diagram* of the components in a directed network, and describe each in terms of a suitable relation on the set of strongly connected components.