Ollscoil na Gaillimhe
University of Galway

CT336/CT404
Graphics & Image Processing

Sem 1 (Autumn) 2024-25,
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Lecture 6: Morphological Image Processing

- Last Lecture:
  - Image Filtering in Spatial Domain (part 2 of 2)
  - Image Filtering in Frequency Domain
    - What is a frequency domain?
    - 2D Fourier Transform
    - Low-pass and high-pass filters

- Today:
  - Binary Image Processing using Morphological Techniques
    - What is morphology?
    - A morphological image processing pipeline
      - Basic morphological operations
      - Extraction of binary regions (connected components)
      - Distance Transform & Skeletonization
      - Properties of Binary Regions

- Acknowledgement: These lecture slides have been prepared using digital materials available with your Text book: ‘Image Processing and Analysis by Stan Birchfield, 1st Edition, Cengage Learning’

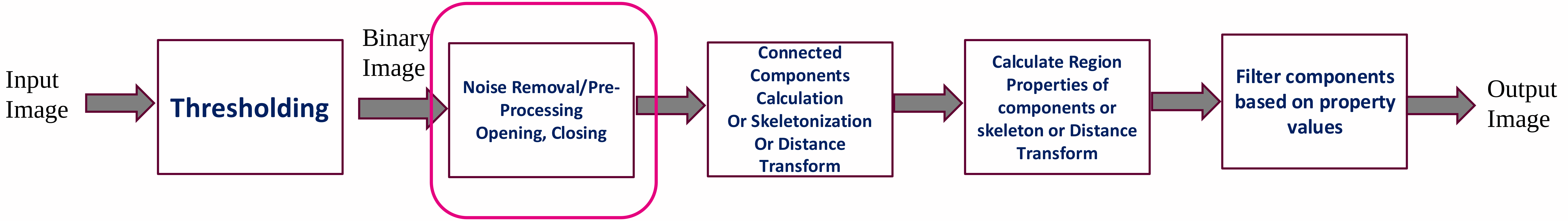
Morphological Image Processing

- The term 'morphology' refers to shape
- Morphological image processing assumes that an image consists of structures that may be handled by mathematical set theory
- Normally applied to binary (B&W) images – e.g. after applying thresholding
- A 'set' is a group of pixels
- Different set operations can be performed on this ‘set’ of pixels

Figure 4.1 Left: A grayscale image of several types of fruit on a dark conveyor belt. Right: A binary image resulting from thresholding. The white pixels are ON and indicate the foreground, while the black pixels are OFF and indicate the background.

Morphological Image Processing

- Applications: Image Analysis at a very small scale (small regions of pixels)
  - Medical Image Processing
  - Scientific Image Processing
  - Industrial Inspection
- A general morphological image processing pipeline



Morphological Image Processing

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- Basic Set Operations:

$$\mathcal{A} \cup \mathcal{B} \equiv \{\mathbf{z} : \mathbf{z} \in \mathcal{A} \text{ or } \mathbf{z} \in \mathcal{B}\} \qquad \text{(union)}$$

$$\mathcal{A} \cap \mathcal{B} \equiv \{\mathbf{z} : \mathbf{z} \in \mathcal{A} \text{ and } \mathbf{z} \in \mathcal{B}\} \qquad \text{(intersection)}$$

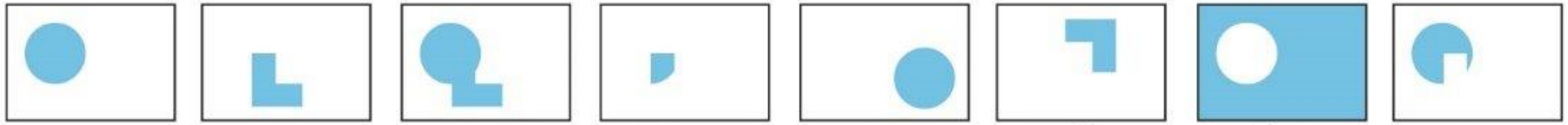
$$\mathcal{A}_{\mathbf{b}} \equiv \{\mathbf{z} : \mathbf{z} = \mathbf{a} + \mathbf{b}, \mathbf{a} \in \mathcal{A}\} \qquad \text{(translation)}$$

$$\check{\mathcal{B}} \equiv \{\mathbf{z} : \mathbf{z} = -\mathbf{b}, \mathbf{b} \in \mathcal{B}\} \qquad \text{(reflection)}$$

$$\ldots \notin \mathcal{A}\} \qquad \text{(complement)}$$

$$\mathcal{A} \setminus \mathcal{B} \equiv \{\mathbf{z} : \mathbf{z} \in \mathcal{A}, \mathbf{z} \notin \mathcal{B}\} = \mathbf{A} \cap \neg\mathcal{B} \qquad \text{(difference)}$$

Figure 4.2 Set operators. The first two columns show two sets $\mathcal{A}$ and $\mathcal{B}$ in blue. Then, from left to right, shown are the union, intersection, shift of $\mathcal{A}$ by some amount $\mathbf{b}$ (not related to $\mathcal{B}$), reflection about the origin (assumed to be at the center), complement, and set difference.



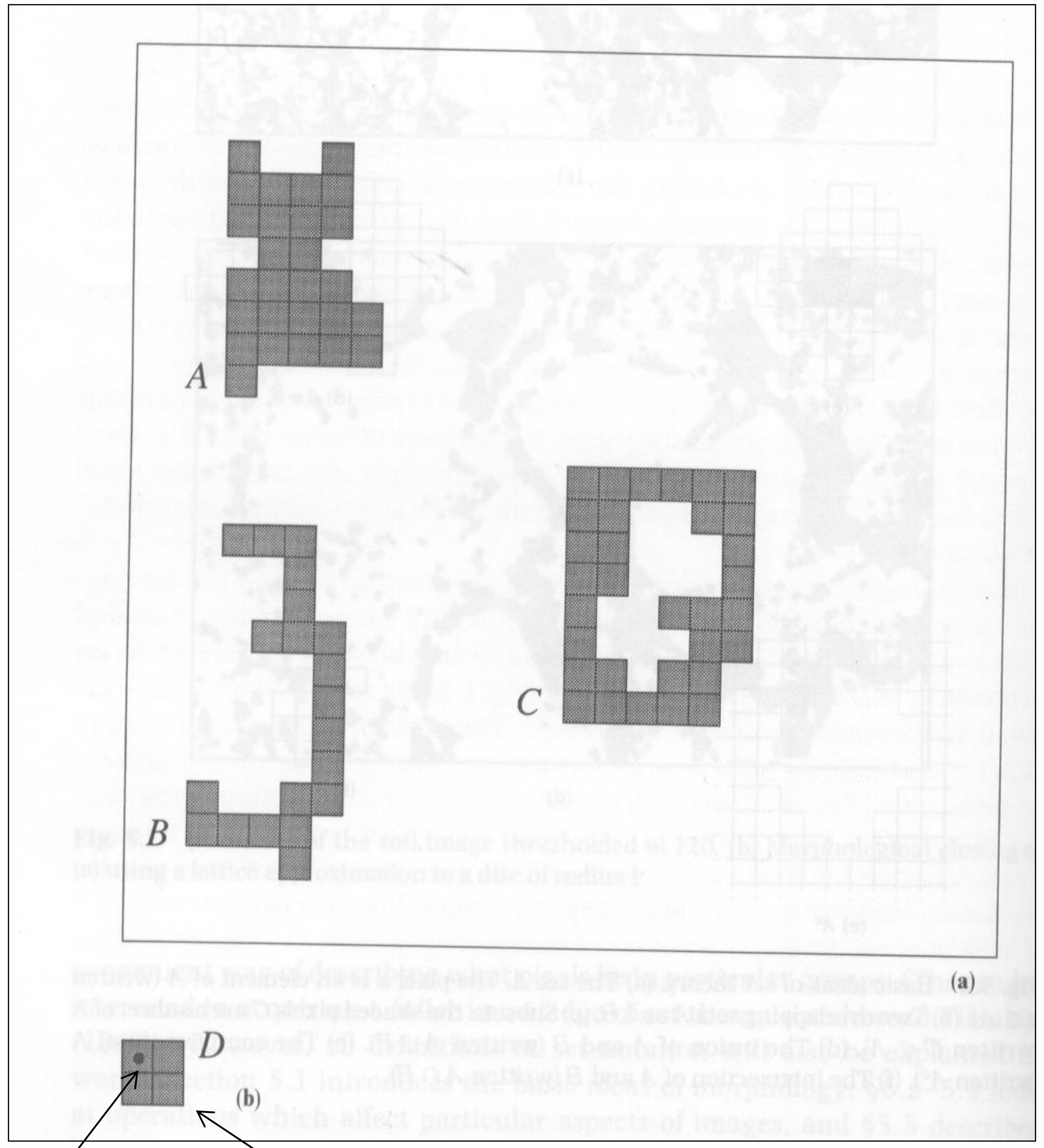
$\mathcal{A}$ | $\mathcal{B}$ | $\mathcal{A} \cup \mathcal{B}$ | $\mathcal{A} \cap \mathcal{B}$ | $\mathcal{A}_{\mathbf{b}}$ | $\check{\mathcal{B}}$ | $\neg\mathcal{A}$ | $\mathcal{A} \setminus \mathcal{B}$

Morphological Image Processing

- **Structuring Element (SE)** :
  - We can describe morphological differences is in terms of intersections with 'test' sets (Structuring Elements)
  - SE’s are of much smaller size than the original image

  - The three black objects A,B,C are very similar in size (i.e. number of pixels) but different in morphology/shape
  - e.g. several possible translations of D will fit into A, but none will fit into B

  - *Morphological operations transform an image => changing a pixel from black to white (or vice versa) if a defined structuring element 'fits' at that point*
  - The shape+size of the structuring element directly affect the information about the image obtained by the operation



Reference pixel

Test sets, or 'structuring elements‘ such as D allow analysis of shape

Basic Morphological Operations

- These operations are performed by set operations between a given image and an SE
- Erosion, Dilation, Opening, Closing

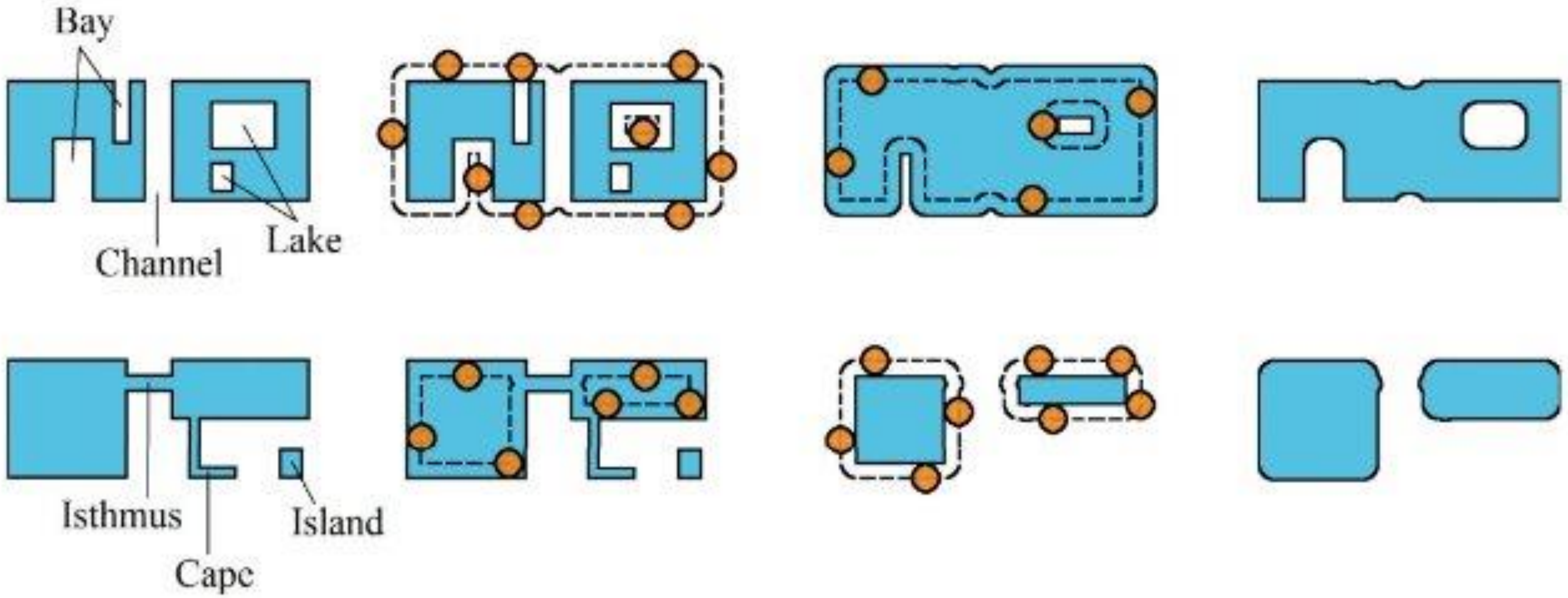
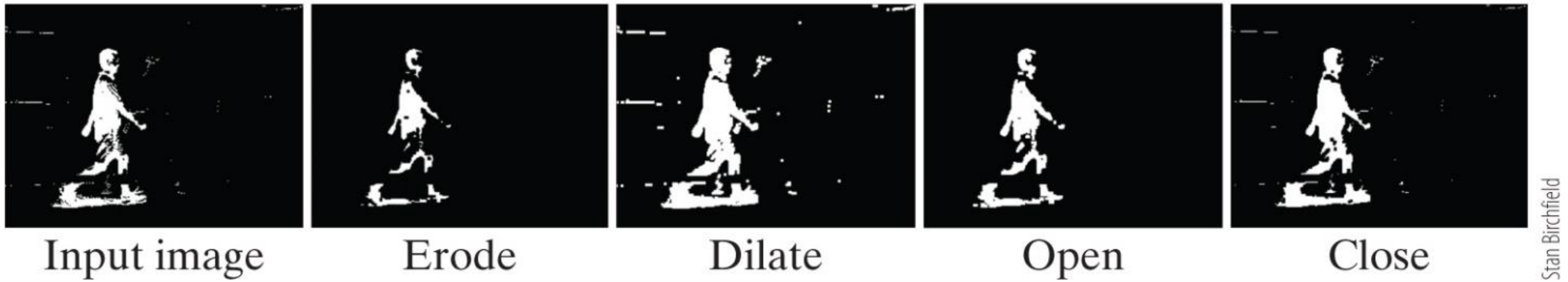


Figure 4.10 TOP: Dilation of a binary region by a circular structuring element can be visualized as rolling the disk along the outside of the region; the result is enclosed by the path of the center of the disk. The right column shows the result of closing (dilation followed by erosion), which fills lakes, bays, and channels. BOTTOM: Erosion can be visualized as rolling the disk along the inside of the region; the result is again enclosed by the path of the center of the disk. The right column shows the result of opening (erosion followed by dilation), which removes capes, isthmuses, and islands.

Basic Morphological Operations

Figure 4.11 A binary image and the result of morphological operations: Erode, dilate, open, and close. Erosion removes salt noise but shrinks the foreground. Dilate fills pepper noise but expands the foreground. Opening and closing removes the respective types of noise while retaining the overall size of the foreground.



Basic Morphological Operations

- Thinning: Clever erosion (removing pixels) by crafted **Ternary** SE's (on, off, don't care X)

Figure 4.14 Structuring elements commonly used for morphological thinning. Colored pixels are ON, white pixels are OFF, and x indicates DONT-CARE. The top row shows the 4 edge SEs, while the bottom row shows the 4 corner SEs.

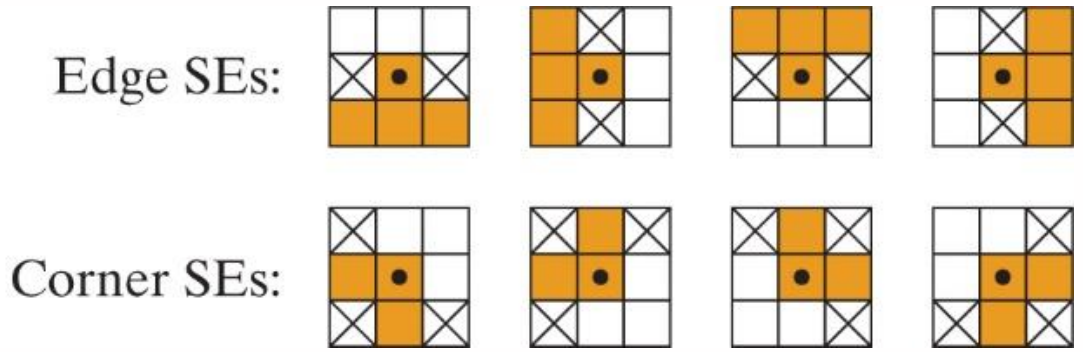
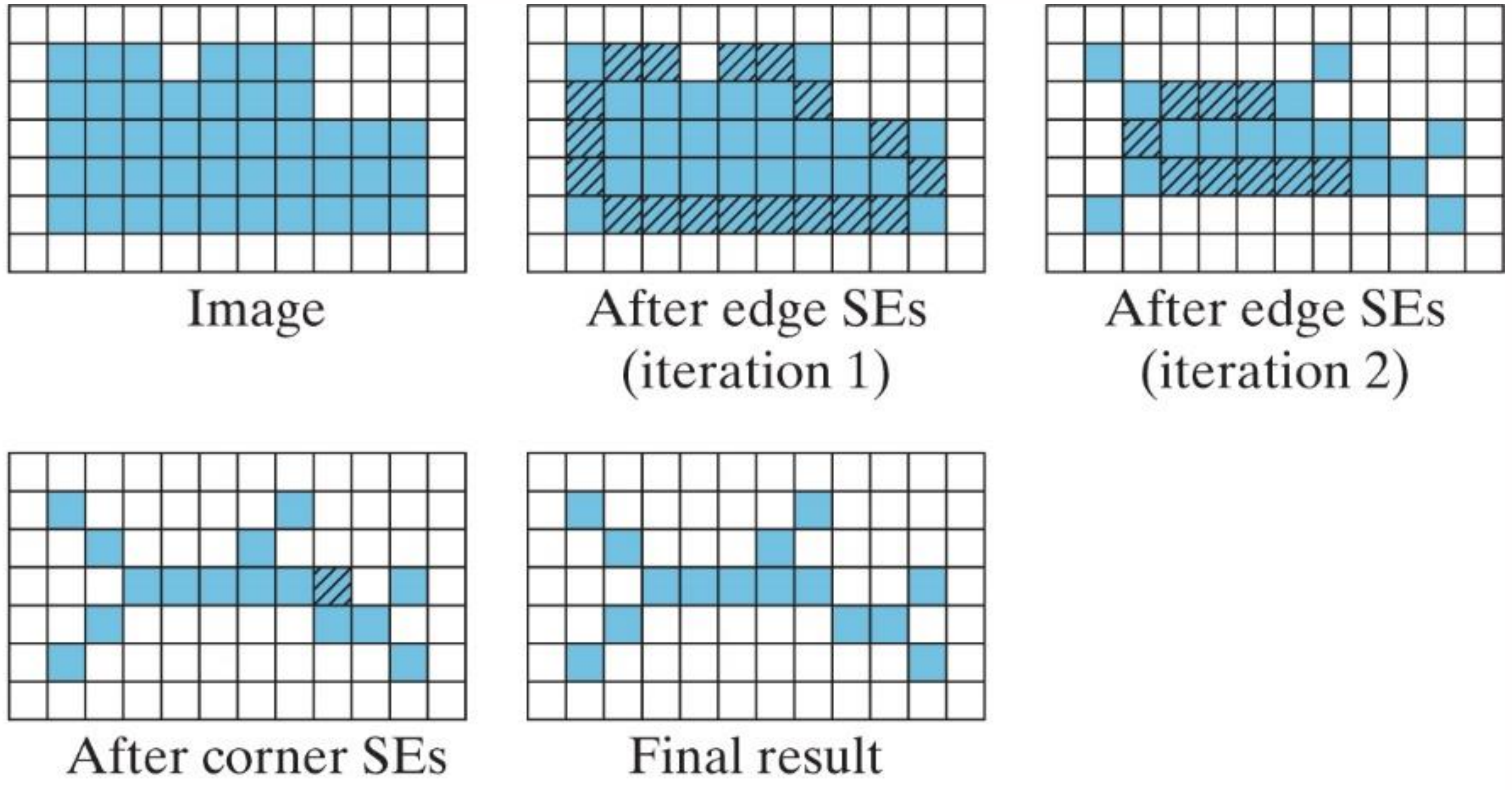


Figure 4.16 Morphological thinning of the same binary image using the same SEs as the previous example. In this case, however, the edge SEs are applied repeatedly as a set until convergence, before applying the corner SEs repeatedly as a sequence. In the first iteration pixels along the top, right, bottom, and left of the region are removed by the edge SEs. In the second iteration, 9 additional pixels are removed by the edge SEs. In the final iteration a single pixel is removed by one of the corner SEs, thus producing a thinner skeleton than in the previous example.



Basic Morphological Operations

- Thickening: Opposite of Thinning (add pixels to foreground)

Figure 4.17 Structuring elements commonly used for morphological thickening.

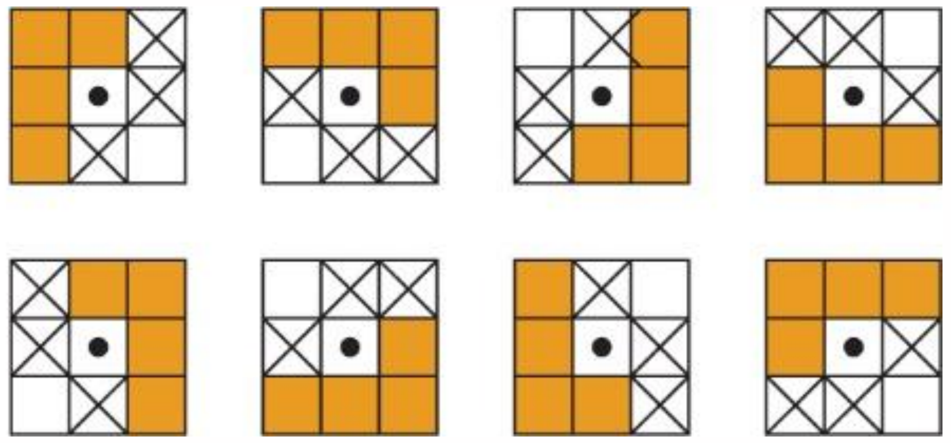
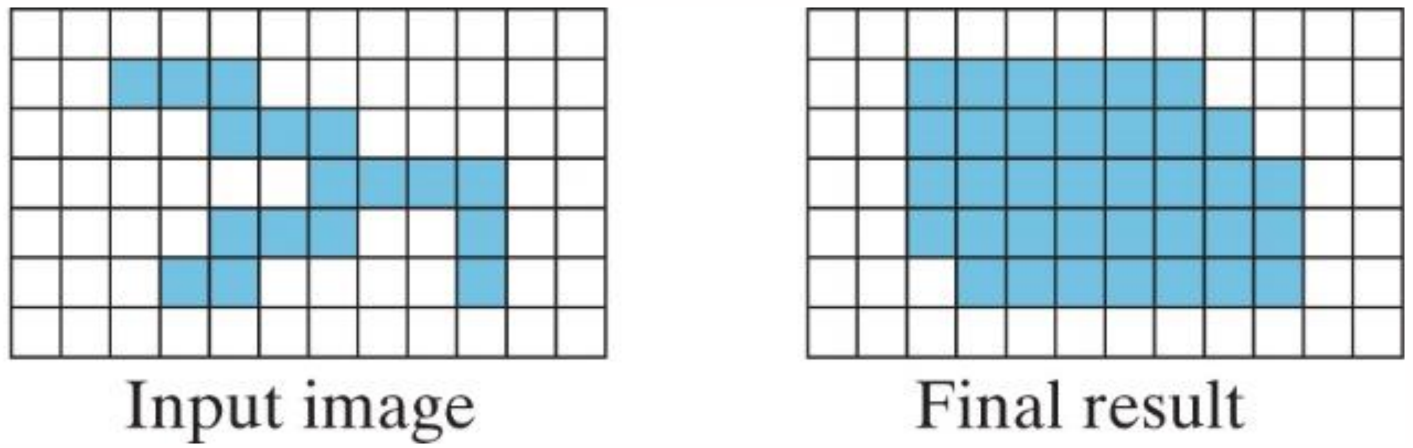
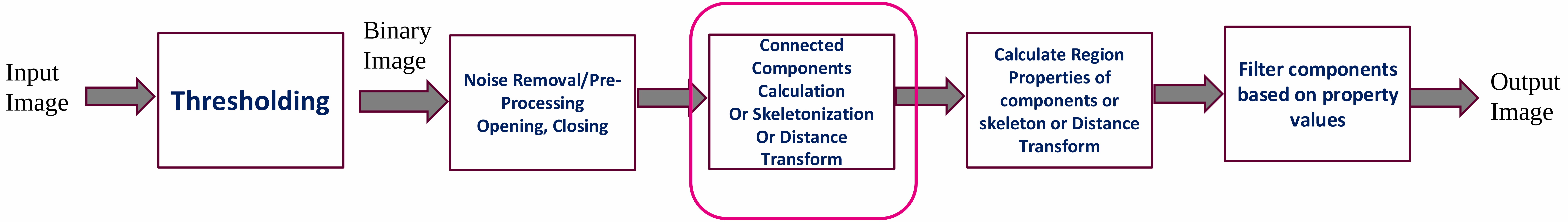


Figure 4.18 Morphological thickening of a binary image using the SEs in Figure 4.17. Shown are the original image (left) and the final result after convergence (right). The thickened result is an approximation to the convex hull.



Morphological Image Processing

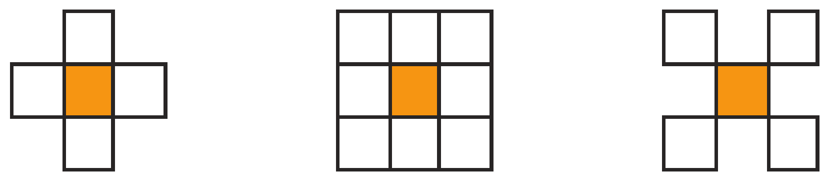
- A general morphological image processing pipeline



Connected Components

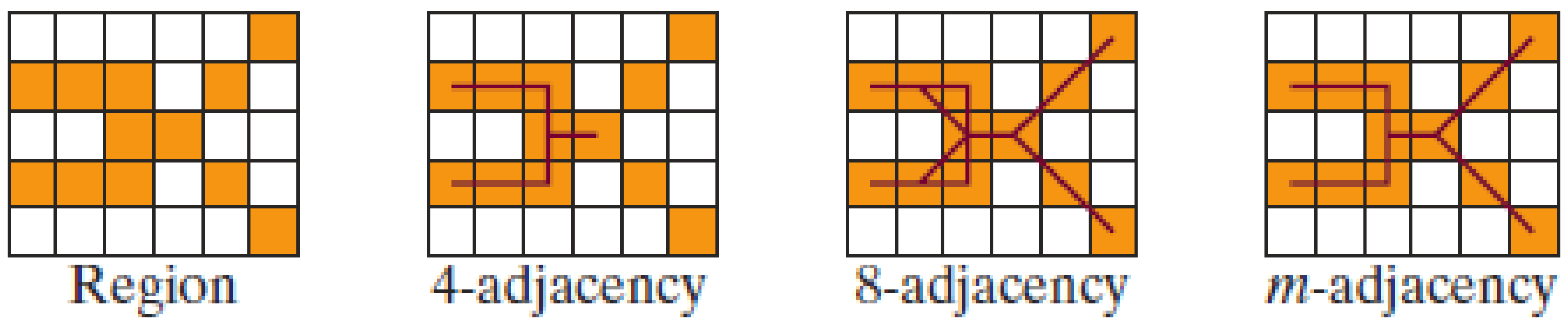
- Neighbors and Connectivity:

Figure 4.20 Commonly used neighborhoods. From left to right: $\mathcal{N}_4$, $\mathcal{N}_8$, and $\mathcal{N}_D$.



  - A pixel q = (qx, qy) is a neighbor of pixel p = (px, py) if q is in the neighborhood of p, denoted $q \in N(p)$ where N is the neighborhood function.
  - Two pixels are said to be **adjacent** if they have the same value and if they are neighbors of each other.
  - Pixels are said to be **connected** (or **contiguous**) if there exists a path between them

Figure 4.21 A binary region and the 4-, 8-, and *m*-adjacency of its pixels. Note that *m*-adjacency removes the loops that sometimes occur with 8-adjacency.

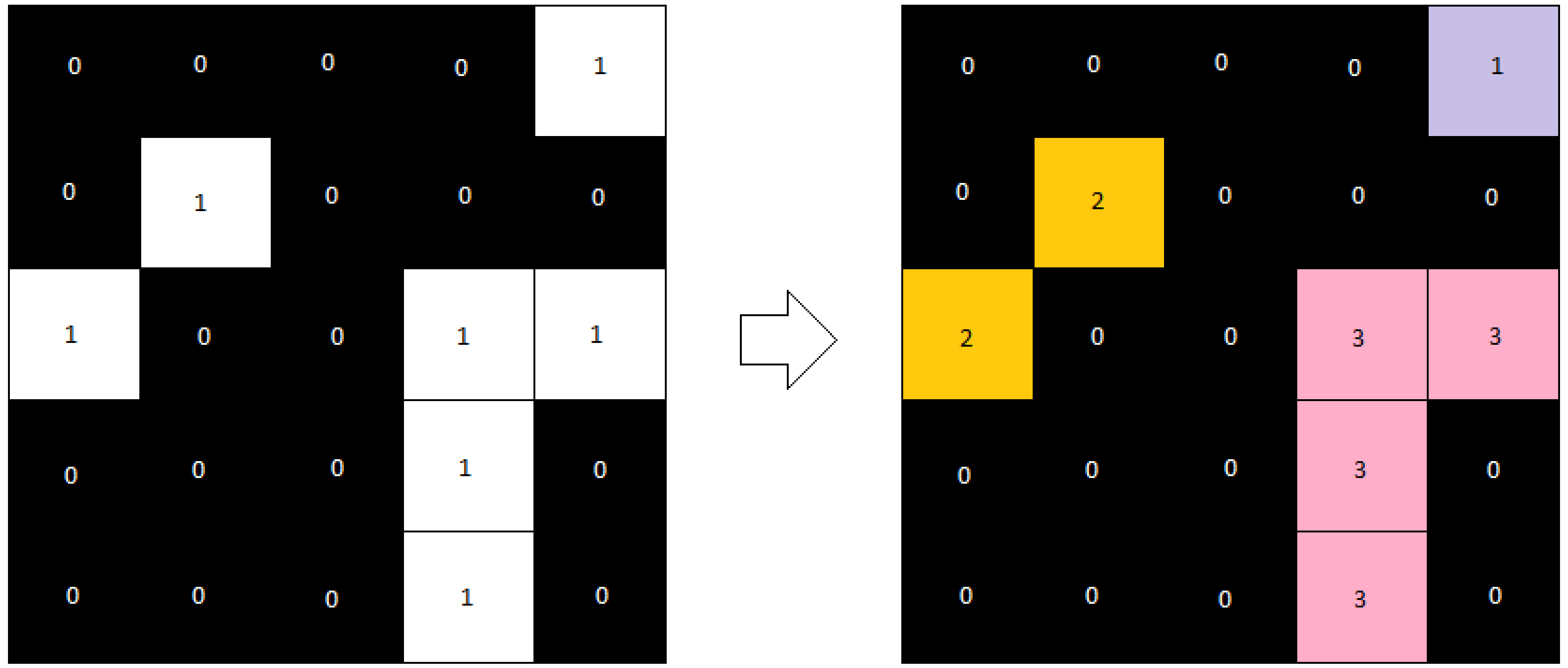


Connected Components

- **Connected component:** defined as a maximal set of pixels that are all connected with one another.

- **Connected component labeling**: the process of assigning a unique identifier to every pixel in the image indicating to which connected component it belongs.

- *How are connected components calculated?*

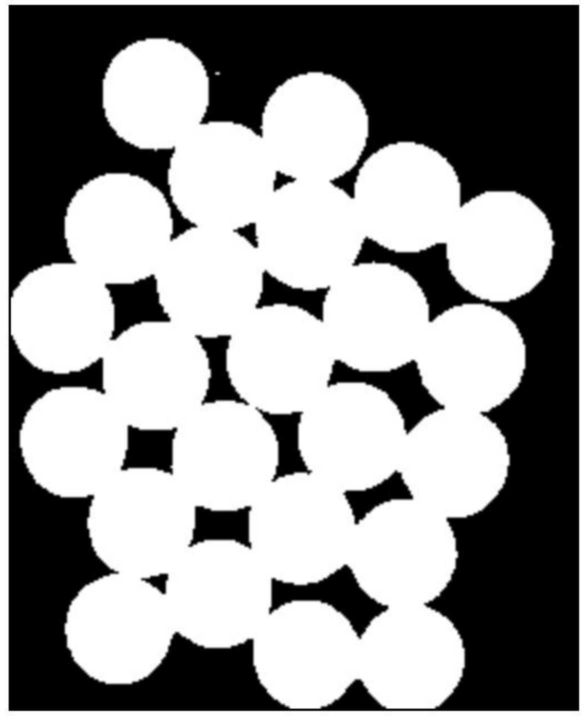


Binary Image
Image source

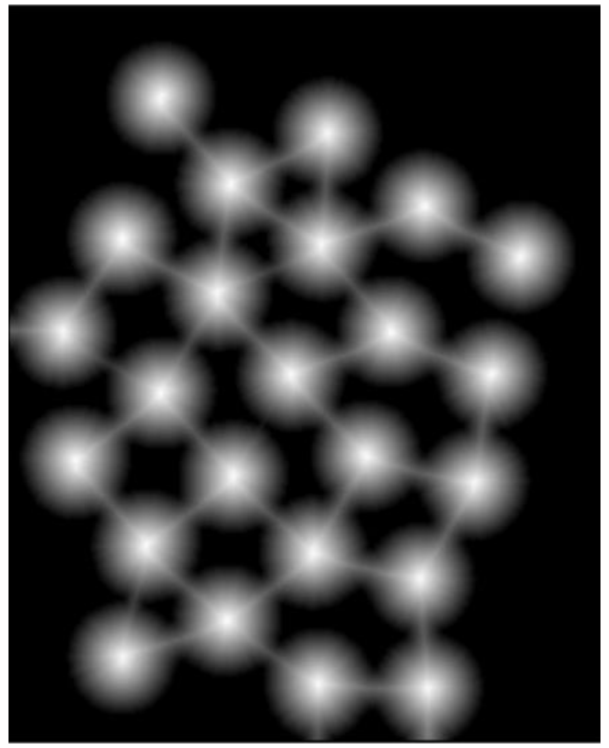
Labelled connected components

Distance Transform

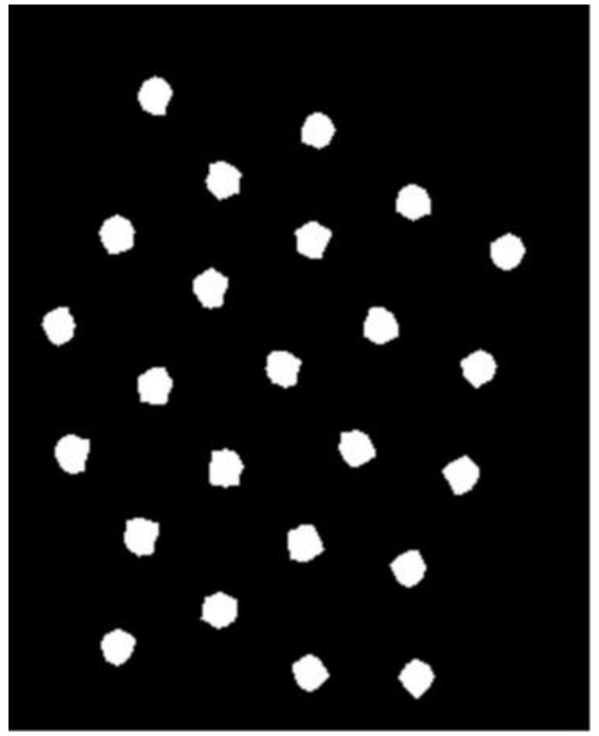
- Replaces pixels of one value (black or white) in a binary image with their distance to the nearest pixel of opposite value (white or black)
- Useful for template matching
- or ‘granulometry’ (=studying the size distribution/properties of objects) – this allows the study of groups of objects:
- How many? What size?
- How are they distributed spatially?
- The assumption is that a local maximum is the centre of a distinct object: use ‘non-maximal suppression’ to remove all other pixels



After threshold



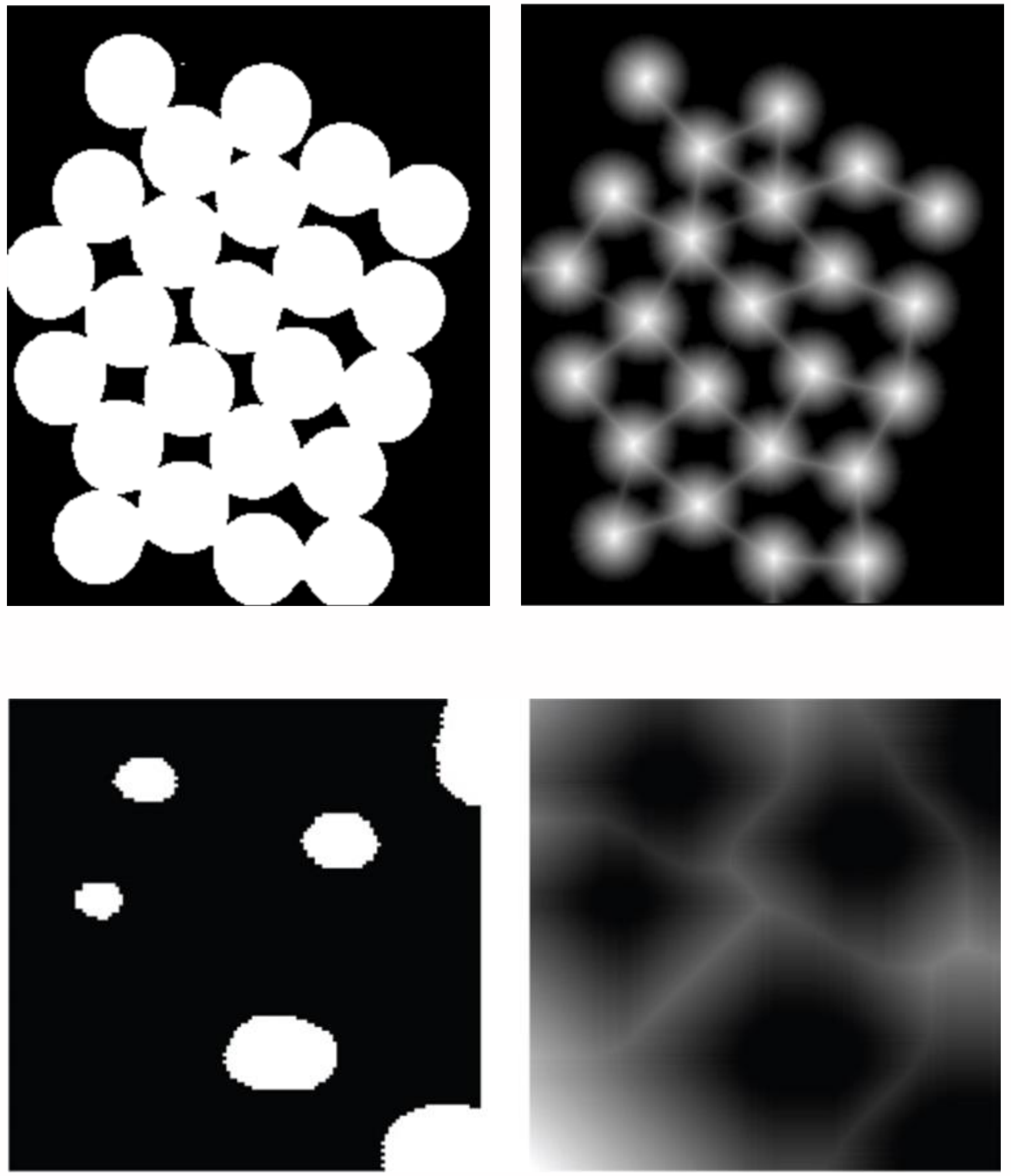
Distance transform of the thresholded image



Result after threshold applied to distance transform

Distance Transform

- What is the difference?



Skeletonization

- A common approach to skeletonization is to repeatedly **thin** the image until the result converges.
- Different skeletonization algorithms are available.

Figure 4.42 The skeleton of a binary region is defined as the locus of points where the wave fronts of fires set to the boundary meet, or equivalently as the locus of the centers of the maximal balls.

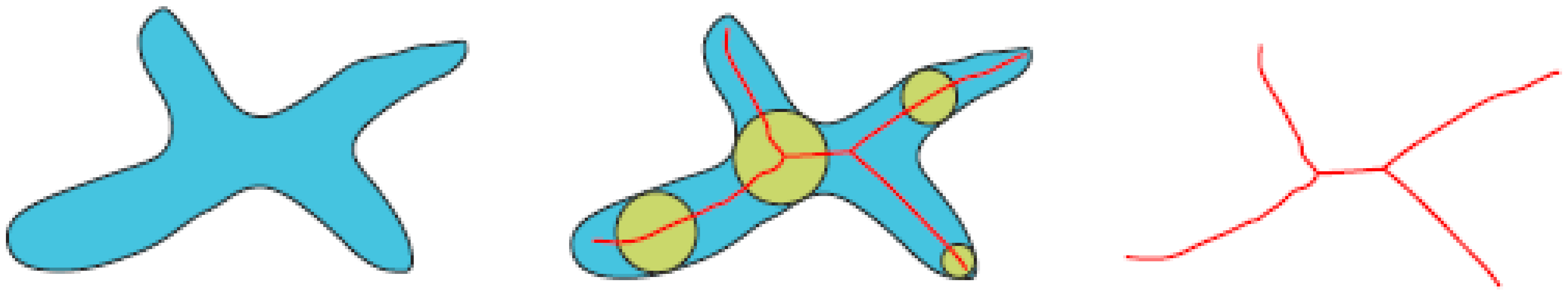
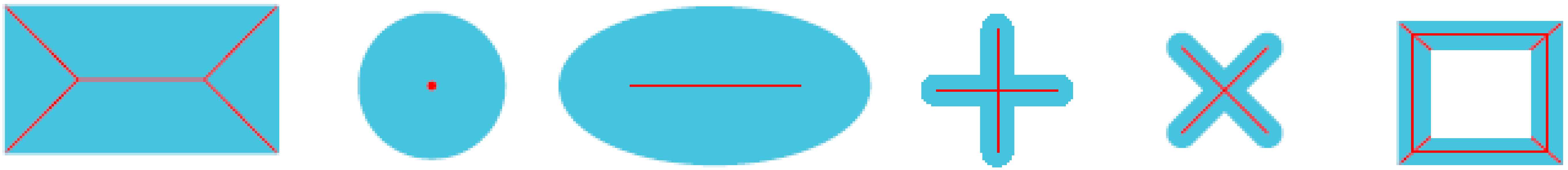


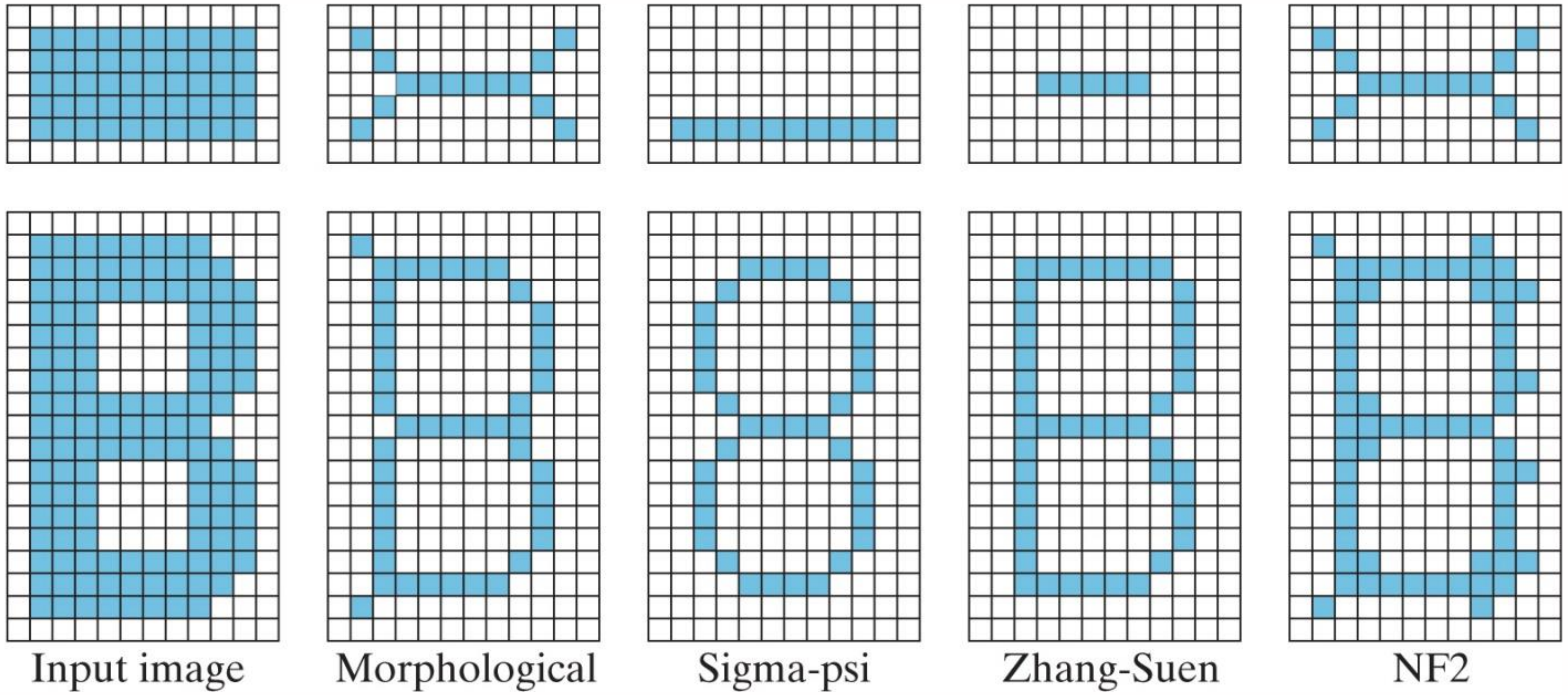
Figure 4.43 Six different continuous shapes (blue) and their skeletons (thin red lines).



Skeletonization

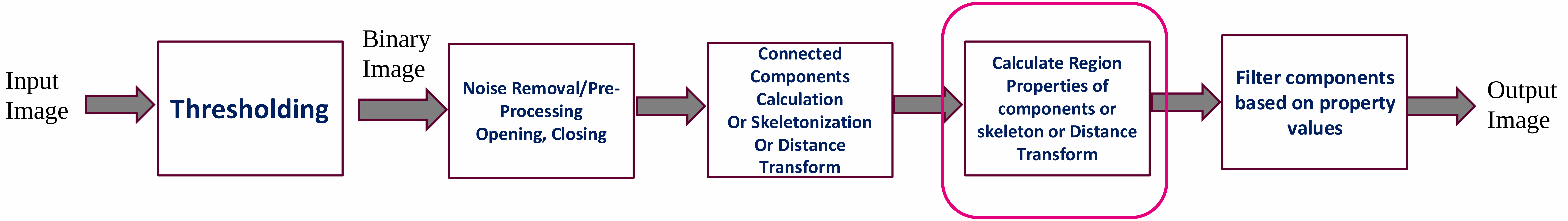
- Different skeletonization algorithms are available.

Figure 4.49 Comparison of the various skeletonization algorithms on two different binary images.



Morphological Image Processing

- A general morphological image processing pipeline



Region Properties

- Regular Moments $$m_{pq} \equiv \sum \sum x^p y^q f(x, y) \qquad (4.101)$$
- Central Moments $$\mu_{pq} \equiv \sum \sum (x - \bar{x})^p (y - \bar{y})^q f(x, y)$$

$$\mu_{00} = m_{00}$$
$$\mu_{10} = 0$$
$$\mu_{01} = 0$$

- *Covariance Matrix* $$\mathbf{C}_{\{2\times2\}} = E[(\mathbf{x} - \bar{\mathbf{x}})(\mathbf{x} - \bar{\mathbf{x}})^{\mathsf{T}} \rho(\mathbf{x})] \qquad (4.126)$$
$$= \frac{1}{\mu_{00}} \begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix} \qquad (4.131)$$

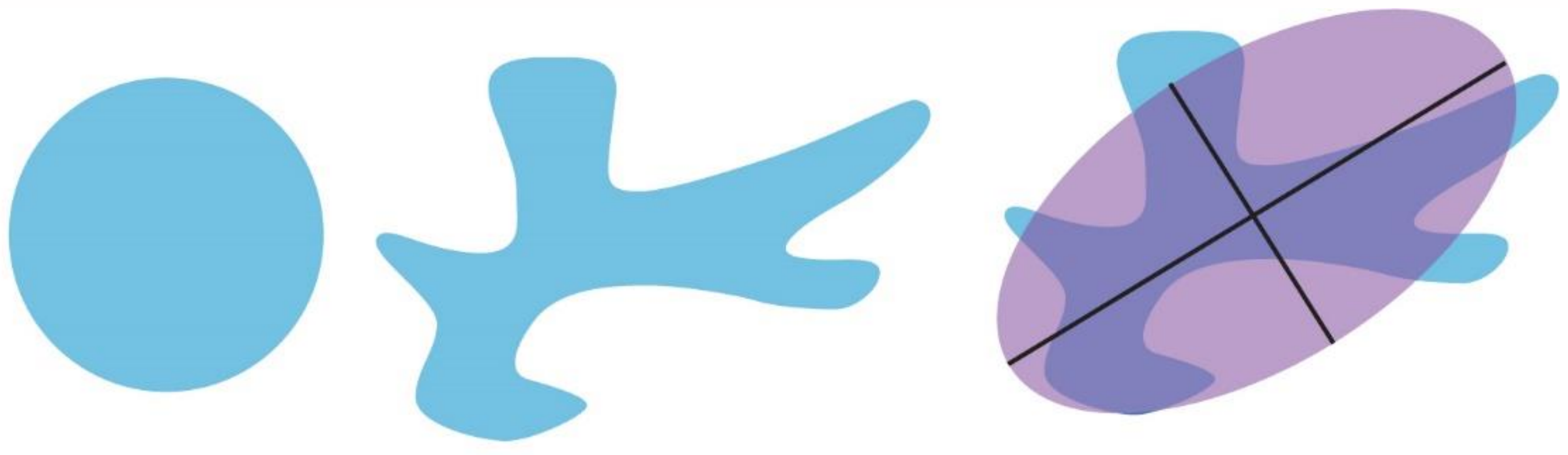
- Area
- Perimeter
- Orientation (From Covariance Matrix)
- **Compactness**
- **Eccentricity**
- **Convex Hull**
- **Euler Number**

Region Properties

- **Compactness**

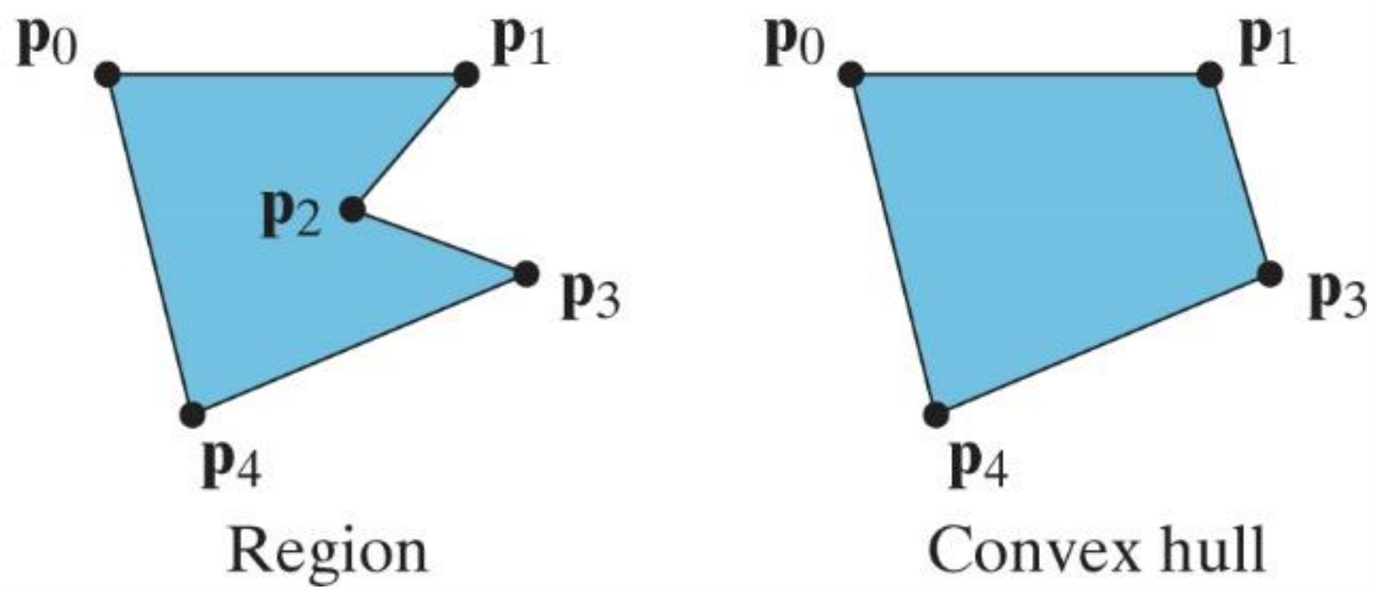
$$\text{compactness} = \frac{4\pi(\text{area})}{(\text{perimeter})^2}$$

Figure 4.34 Left: A circle is the most compact shape, with a compactness of 1. Middle: A shape whose compactness is less than 1. Right: The eccentricity of the shape is computed as the eccentricity of the best-fitting ellipse.



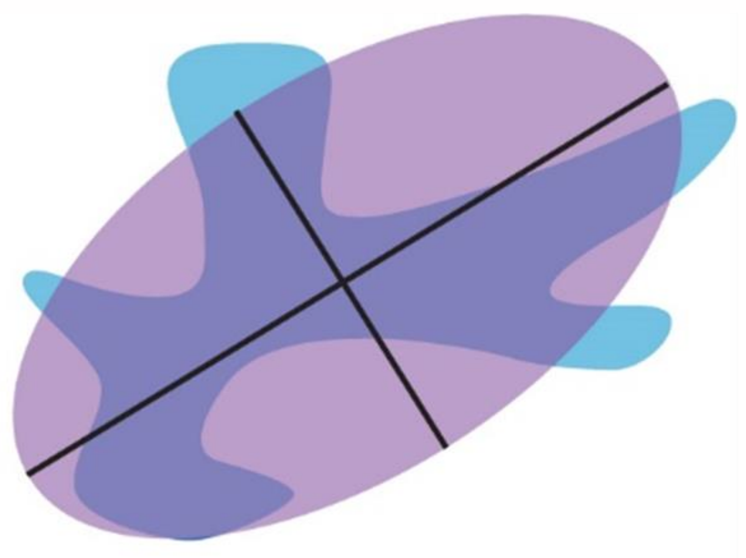
- **Convex Hull:** Find convex hull of a binary region and calculate its properties instead.

Figure 4.35 Left: An arbitrarily-shaped region in the plane. Right: The convex hull of the region is the shape that results from enveloping the region with a rubber band, which removes all concavities. All vertices are locally convex except for $\mathbf{p}_2$.



Region Properties

- Eccentricity
  - Measures the elongatedness of a binary region i.e. how far it is from being rotationally symmetric around its centroid.
  - How to calculate it?
  - Find the best/tightest fitting ellipse around the region
  - Then align this ellipse with the axes
  - Find the Covariance Matrix of this ellipse



$$\mathbf{C} = \mathbf{P} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \mathbf{P}^{\mathsf{T}} \quad (4.151)$$

  - The eccentricity will be a ratio of the eigen values of this matrix

$$\text{eccentricity} = \sqrt{\frac{\lambda_1 - \lambda_2}{\lambda_1}} \quad (4.154)$$

  - NOT surprisingly, this covariance matrix is obtained by using the moments of the binary region encapsulated by the ellipse

$$\mathbf{C} = \frac{1}{\mu_{00}} \begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix} = \eta \begin{bmatrix} c & -\frac{b}{2} \\ -\frac{b}{2} & a \end{bmatrix} \quad (4.145)$$

Region Properties – Euler Number

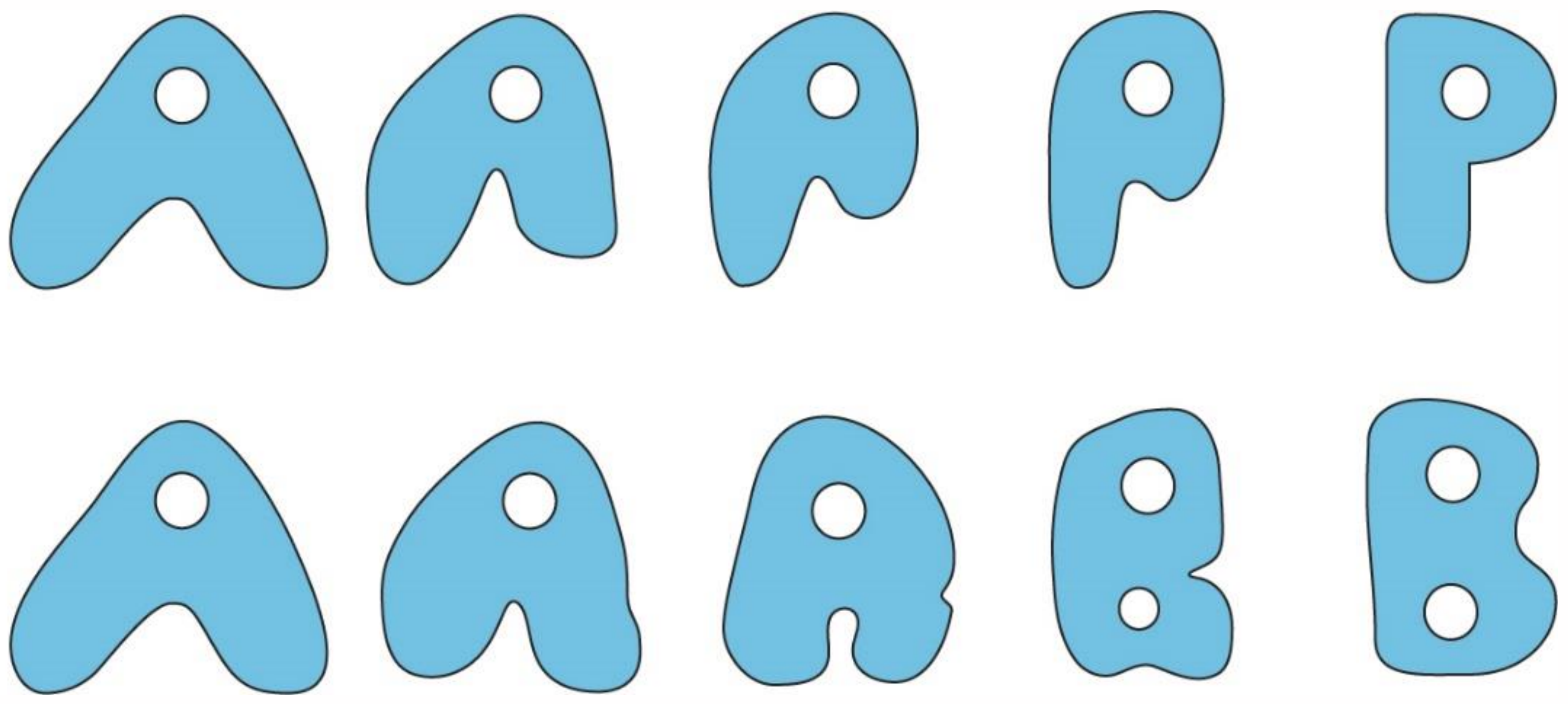
- Mathematical Topology:

  Study of properties of objects that are preserved under continuous deformations of the objects e.g. bending, stretching and compressing but NOT tearing or sewing.

- Homotopy/rubber sheet deformations:

  defined to be the deformation which does NOT change the topology of the object

Figure 4.38 Top: The letters "A" and "P" are related by a homotopy, because there is a continuous deformation that relates the two shapes. Bottom: The letters "A" and "B" are not related by a homotopy, because there is not a continuous deformation that relates the two shapes. Rather, tearing the region to produce the extra hole is necessary (or sewing the hole in the case of the reverse transformation).



Region Properties – Euler Number

- Euler Number/Characteristic: A topological invariant characteristic

$$\text{Euler number} \equiv \text{number of regions} - \text{number of holes} \tag{4.164}$$

$$\text{Euler number} = \text{number of vertices} - \text{number of edges} + \text{number of faces} \tag{4.165}$$

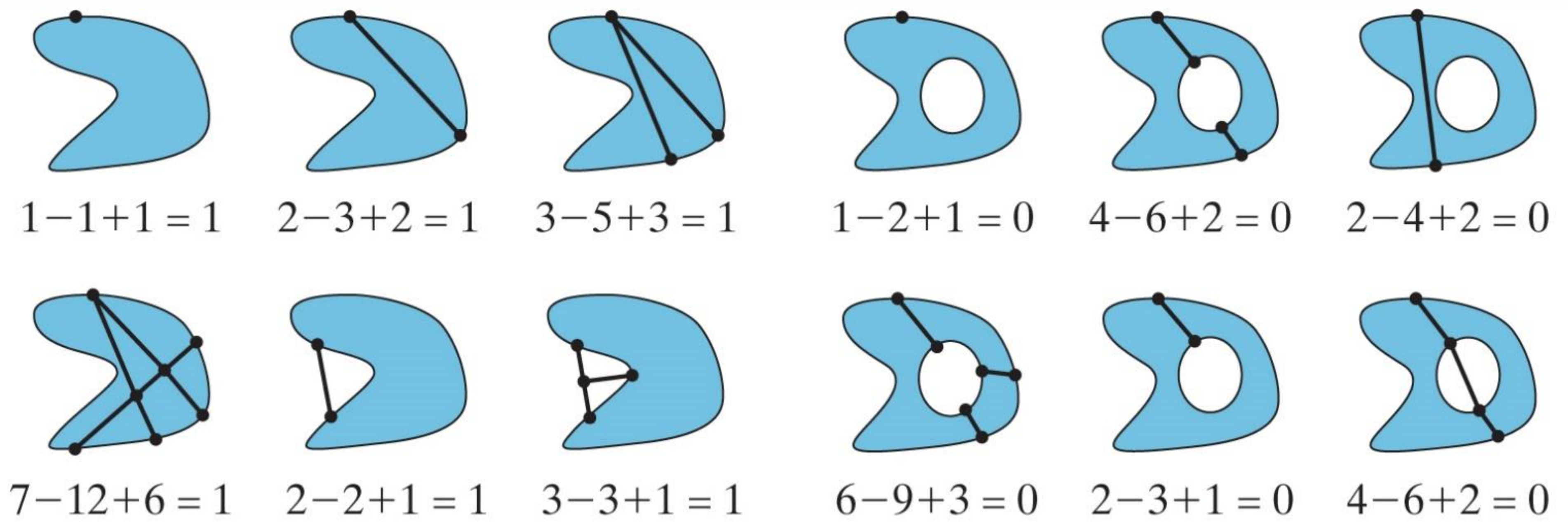


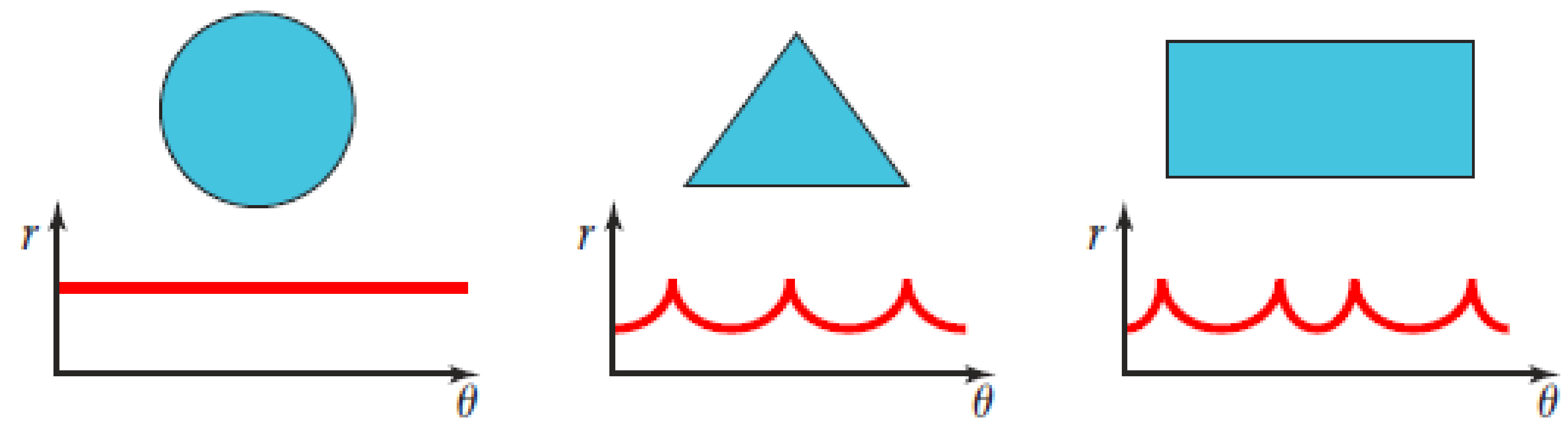
Figure 4.39 Various tesselations of a region whose Euler number is 1 (left) and 0 (right), showing that the Euler number is not dependent upon the particular tesselation chosen. Under each figure is the number of vertices minus edges plus faces according to Equation (4.165). Note there is implicitly at least one vertex, edges intersect at vertices, and external vertices or edges are not counted.

Region Boundary Representations

- Classically used in ‘shape’ analysis or recognition
- For distinguishing the shape of the boundary, generally we want to transform the sequence into a representation that is invariant to translation, rotation, and/or scale changes, as well as to the starting pixel.

- Signature: The most basic type of signature is known as the centroidal profile, or r-θ curve.
- This approach captures the distance r from the center of the region as a function of the angle θ.

Figure 4.52 The centroidal profile (r-θ plot) of several shapes.



Next Time

- Edges & Features
- Image Segmentation (Part 1 of 2)

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Thank *you*