SEMINAR ON SHEAF THEORY

HEIDELBERG UNIVERSITY, WINTER SEMESTER 2022-2023

Target audience: Master students and advanced Bachelor students.
Instructor: Florent Schaffhauser.
Language of instruction: English.

Objectives

The purpose of this seminar is to cover the basics of sheaf theory, with a view towards the cohomology of real and complex manifolds. The expositions will be given by the participants, in coordination with the professor in charge of running the seminar.

This seminar can be recommended as a complement to the course on *Real Algebraic Varieties* taught in Heidelberg during the Winter Semester 2022-2023. It will be run in English.

Contents

Topics marked with an * are optional and will depend on our progress in the seminar.

(1) Presheaves and sheaves
(2) Morphisms, kernels and images
(3) Ringed spaces, varieties and manifolds
(4) Derived functors
(5) Sheaf cohomology
(6) Čech cohomology
(7) Invertible sheaves and the Picard group
(8) *Fine sheaves
(9) *Cohomology of real and complex manifolds
(10) *The Hodge decomposition theorem

Evaluation

Everybody is welcome to attend (and give talks). However, to get academic credits for the seminar, you will need to give an exposition and hand in a typed version of your notes. If the number of participants exceeds the number of talks, we will split the sessions in half, so everybody interested can give a talk.

Methodology

For Topics (1)–(7), we will mainly follow [Ten75]. And later in the semester, we will follow [Voi07]. Participants giving an exposition will convene beforehand with the professor, to review the material and prepare the talk. An organisation meeting will be held at least a month before the beginning of the semester (see information on Müsli), after which we will divide the talks between confirmed participants. The seminar will convene weekly for 2h. There will be three Q&A sessions after Talks 4, 8 and 12, to make sure we are all on the same page.

Plan for the talks

Talk n^o	**Topic**	**Material**
1	Presheaves and sheaves: definition and examples	[Ten75] §1, 2.1, 2.2.
2	The sheafification of a presheaf and applications	[Ten75] §2.4
3	Morphisms of sheaves and presheaves	[Ten75] §3.1-3.4, 3.6
4	Ringed spaces and manifolds	[Ten75] §4.3, 4.4
5	Locally free sheaves, invertible sheaves	[Ten75] §4.5
6	Sheaf cohomology I	[Ten75] §5.1-5.2
7	Sheaf cohomology II	[Ten75] §5.3
8	Čech cohomology	[Ten75] §5.4
9	The Picard group	[Ten75] §5.4
10	*Fine sheaves	[Voi07] §I.4
11	*Cohomology of real and complex manifolds	[Voi07] §I.4
12	*The Hodge decomposition theorem	[Voi07] §II.5, II.6

References

[Ten75] B. R. Tennison. *Sheaf theory*. London Mathematical Society Lecture Note Series, No. 20. Cambridge University Press, Cambridge, England-New York-Melbourne, 1975.

[Voi07] C. Voisin. *Hodge theory and complex algebraic geometry. I*, volume 76 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, english edition, 2007. Translated from the French by Leila Schneps.