

SEMINAR ON LIE ALGEBRAS

HEIDELBERG UNIVERSITY, WINTER SEMESTER 2023-2024

Target audience: Bachelor and Master students.

Instructor: Florent Schaffhauser.

Language: English.

Modality: Block seminar, 8 talks of 1h each (Tentative dates: 08-09.12, 9am–4pm).

Organisation meeting: 11.10, 4pm (online).

OBJECTIVES

The purpose of this seminar is to cover the basics of Lie theory, with a view towards understanding the *structure and classification of semisimple Lie algebras*. The expositions will be given by the students, in coordination with the professor.

TOPICS

- (1) Nilpotent and solvable Lie algebras
- (2) Simple and semisimple Lie algebras
- (3) Reductive Lie algebras, Cartan subalgebras
- (4) Representations of the Lie algebra $\mathfrak{sl}(2; \mathbb{C})$
- (5) Root systems and root space decompositions
- (6) Cartan matrices and Dynkin diagrams
- (7) The Jacobson-Morozov theorem and parabolic subalgebras
- (8) Nilpotent orbits and Kostant's theorem

EVALUATION

Everybody is welcome to attend. However, to get academic credits for the seminar, you will need to give an exposition and hand in a typed version of your notes.

This seminar will be run as a block seminar and there will be only eight spots to give talks, so be sure to register early if you are interested in getting credits!

METHODOLOGY

For Topics (1)–(6), we will follow [Ser87], with the help of [Hum72], while for Topics (7) and (8), we will follow [CM93]. Participants giving an exposition will convene beforehand with the professor, to review the material and prepare the talk.

REFERENCES

- [CM93] David H. Collingwood and William M. McGovern. *Nilpotent orbits in semisimple Lie algebras*. Van Nostrand Reinhold Mathematics Series. Van Nostrand Reinhold Co., New York, 1993.
- [Hum72] James E. Humphreys. *Introduction to Lie algebras and representation theory*. Graduate Texts in Mathematics, Vol. 9. Springer-Verlag, New York-Berlin, 1972.
- [Ser87] Jean-Pierre Serre. *Complex semisimple Lie algebras*. Springer-Verlag, New York, 1987. Translated from the French by G. A. Jones.