Proseminar on computer-assisted mathematics

Session 9 - Final projects

```
def fac : ℕ → ℕ
| 0 := 1
| (n + 1) := (n + 1) * fac n

#check fac     fac : ℕ → ℕ
#eval fac 5     120

def fac_pos : ∀ (n : ℕ) , fac n > 0 := sorry
```

Florent Schaffhauser
Heidelberg University, Summer semester 2023

It is time we started thinking about final projects!

These can be either mathematically-oriented or programming-oriented.

To help you pick a lane, we will discuss examples below.

Example 1 - Well-orderings

An ordered set is said to be well-ordered if the order is total and every non-empty subset contains a minimal element.

Using induction, it is possible to prove that the natural numbers, with their usual ordering, form a well-ordered set.

The goal of this project is to formalize that statement in Lean and prove it.

Example 2 - The fundamental theorem of algebra

A non-constant polynomial with complex coefficients has a complex root.

A proof of this using concepts from linear algebra can be found here:

https://kconrad.math.uconn.edu/blurbs/fundthmalg/fundthmalglinear.pdf

The goal of this project is to formalize fragments from the above paper and prove them (this can be split between several teams).

Example 3 - Gamification

The goal of this project is to design a game (similar to the Natural Number Game).

For the Lean content, you can use the material that we went through in the seminar.

Ideally, the game will be playable in a browser and the template will be editable and remain available in the repository of our seminar.

Example 4 - Online textbook

Lean textbooks are often online-based and interactive:

https://leanprover.github.io/introduction_to_lean/

https://leanprover.github.io/theorem_proving_in_lean/

The goal of this is to build such a resource, using the material that we went through in the seminar.

Example 5 - Documentation

The goal of this project is to solve the last world of the Natural number game (Inequality world) and document the solution.

Other options

Of course, you are free to suggest other projects.

I only ask that you validate them with me before you start investing a lot of time on your project.

A good starting point would be: pick a mathematical result that you like and send it to me!