Proseminar on computer-assisted mathematics

Session 9 - Final projects

```
def fac : ℕ → ℕ
| 0 := 1
| (n + 1) := (n + 1) * fac n

#check fac     fac : ℕ → ℕ
#eval fac 5     120

def fac_pos : ∀ (n : ℕ) , fac n > 0 := sorry
```

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It is time we started thinking about final projects!

These can be either mathematically-oriented or interface-oriented.

To help you pick a lane, we will discuss examples below.

Example 1 - Well-orderings

An ordered set is said to be well-ordered if the order is total and every non-empty subset contains a minimal element.

Using induction, it is possible to prove that the natural numbers, with their usual ordering, form a well-ordered set.

The goal of this project is to formalize that statement in Lean and prove it.

Example 2 - The fundamental theorem of algebra

A non-constant polynomial with complex coefficients has a complex root.

A proof of this using concepts from linear algebra can be found here:

https://kconrad.math.uconn.edu/blurbs/fundthmalg/fundthmalglinear.pdf

The goal of this project is to formalize fragments from the above paper and prove them (this can be split between several teams).

Example 3 - Gamification

The goal of this project is to design a game (similar to the Natural Number Game).

For the Lean content, you can use the material that we went through in the seminar.

Ideally, the game will be playable in a browser and the template will be editable and remain available in the repository of our seminar.

Example 4 - Online textbook

Lean textbooks are often online-based and interactive:

https://leanprover.github.io/introduction_to_lean/

https://leanprover.github.io/theorem_proving_in_lean/

The goal of this is to build such a resource, using the material that we went through in the seminar.

Other options

Of course, you are free to suggest other projects.

I only ask that you validate them with me before you start investing a lot of time on your project.

A good starting point would be: pick a mathematical result that you like and send it to me!