VECTOR BUNDLES ON ALGEBRAIC CURVES

HEIDELBERG UNIVERSITY, SUMMER SEMESTER 2023

Target audience: Master students and PhD candidates.
Instructor: Florent Schaffhauser.
Language of instruction: English.

Objectives

The purpose of this course is to give an introduction to moduli theory, through the study of vector bundles on (complex, smooth, projective) algebraic curves.

The fist part of the course will be devoted to learning the basis of **Geometric Invariant Theory**, used to construct quotients of algebraic varieties for actions of reductive groups. In the second part, we will apply this to the study vector bundles on algebraic curves and construct **Moduli Spaces of Semistable Vector Bundles**, following Le Potier's exposition of Simpson's method in [LP97].

Contents

- Part 1: Geometric Invariant Theory.
  (1) Actions of algebraic groups.
  (2) Invariant functions (Hilbert's 14th problem).
  (3) The construction of quotient varieties.
  (4) Projective quotients.
  (5) The Hilbert-Mumford criterion.
- Part 2: Moduli Spaces of Semistable Vector Bundles.
  (1) Vector bundles on algebraic curves. Stability and semi-stability.
  (2) The Harder-Narasimhan filtration. Jordan-Hölder filtrations and polystable bundles.
  (3) Bounded families and the construction of the moduli space.
  (4) *Optional:* Topological properties of moduli spaces of vector bundles.
  (5) *Optional:* The stacky point of view.

Pre-requisites

- Algebraic varieties (over algebraically closed fields).
- Sheaves.
- *Optional:* Algebraic curves and the Riemann-Roch formula.

Depending on the needs of the audience, a review of the optional pre-requisites might be conducted upon request at the beginning of the course.

Evaluation

Depending on the number of students, the exam may be either written or oral (this will be announced within the first month of the course). To be admitted to the exam, students will need to collect at least 50% of the points from the graded problem sets.

Textbooks and supplementary reading

For the first part (Geometric Invariant Theory), we will follow [Muk03] and [Sch08]. For the second part (Moduli spaces of Vector Bundles on Curves), we will mostly follow [LP97].

References

[LP97] Joseph Le Potier. *Lectures on vector bundles*, volume 54 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1997. Translated by A. Maciocia.

[Muk03] Shigeru Mukai. *An introduction to invariants and moduli*, volume 81 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2003. Translated from the 1998 and 2000 Japanese editions by W. M. Oxbury.

[New78] Peter E. Newstead. *Introduction to moduli problems and orbit spaces*, volume 51 of *Tata Institute of Fundamental Research Lectures on Mathematics and Physics*. Tata Institute of Fundamental Research, Bombay; Narosa Publishing House, New Delhi, 1978.

[Sch08] Alexander H. W. Schmitt. *Geometric invariant theory and decorated principal bundles*. Zurich Lectures in Advanced Mathematics. European Mathematical Society (EMS), Zürich, 2008.