TTK4115 Project
Report

Helicopter lab

Master of Technology in Engineering Cybernetics

Submitted by

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1 Part I - Mathematical modelling

1.1 Problem 1

$$J_p\ddot{p} = L_1 V_d \tag{1}$$

$$J_e\ddot{e} = L_2\cos(e) + L_3 V_s \cos(p) \tag{2}$$

$$J_\lambda\ddot{\lambda} = L_4 V_s \cos(e)\sin(p) \tag{3}$$

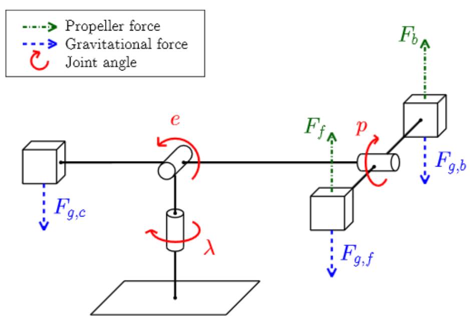

Figure 1: Helicopter model with forces

To show that the equations of motion can be stated as in equations (1), (2) and (3), we use the Newton's 2nd law of rotation.
Firstly, we use the Newton's 2nd law of rotation about the pitch angle:

$$J_p\ddot{p} = \Sigma T_p = l_p m_p - l_p m_p + l_p F_f - l_p F_b \tag{4}$$

$$J_p\ddot{p} = l_p(F_f - F_b) = l_p K_f(V_f - V_b) = l_p K_f V_d \tag{5}$$

Comparing equation (1) and (5), we can see that:

$$L_1 = l_p K_f \tag{6}$$

We now use Newton's 2nd law of rotation about the elevation angle:

$$J_e\ddot{e} = \Sigma T_e = F_{g_c}\cos(e) l_c - F_{g_h}\cos(e) l_h + V_s K_f \cos(p) l_h \tag{7}$$

Where $F_{g_h} = 2m_p g$ og $F_{g_c} = m_c g$.

$$J_e \ddot{e} = (F_{g_c} l_c - F_{g_h}) \cos(e) + l_h V_s K_f \cos(p) \tag{8}$$

Comparing equation (2) and (8), we can see that:

$$L_2 = F_{g_c} l_c - F_{g_h} \tag{9}$$

$$L_3 = l_h K_f \tag{10}$$

Finally, we use the Newton's 2nd law of rotation about the travel angle:

$$J_\lambda \ddot{\lambda} = \Sigma T_\lambda = F_s \cos(e) \sin(p) = K_f V_s \cos(e) \sin(p) \tag{11}$$

Where $F_s = K_f V_f + K_f V_b = K_f V_s$
Comparing equation (3) and (11), we can see that:

$$L_4 = K_f \tag{12}$$

1.2 Problem 2

The equations of motion is wanted to be linearized around the point $\left[p^*, e^*, \lambda^*\right]^T$. We find the values of V_d^* and V_s^* such that $\left[p^*, e^*, \lambda^*\right]^T$ is an equlibrium point of the system.

$$J_p \ddot{p}^* = L_1 V_d^* \tag{13}$$

When $\ddot{p}^* = \ddot{p} = 0$, V_d^* must be equal to zero.

$$V_d^* = 0 \tag{14}$$

$$J_e \ddot{e}^* = L_2 \cos(e^*) + L_3 V_s^* \cos(p^*) \tag{15}$$

When $\ddot{e^*} = \dot{e^*} = p^* = 0$, V_s^* must be equal to:

$$V_s^* = -\frac{L_2}{L_3} \tag{16}$$

We do a transformation of the coordinates to simplify further analysis by finding the corresponding matrices to the states $\left[\tilde{p}, \tilde{e}, \tilde{\lambda}\right]^T$ and the inputs $\left[\tilde{V_s}, \tilde{V_d},\right]^T$. From (1), (2) and (3) we have:

$$\dot{\mathbf{x}} = \begin{bmatrix} \ddot{p} \\ \ddot{e} \\ \ddot{\lambda} \end{bmatrix} = \begin{bmatrix} \frac{L_1 V_d}{J_p} \\ \frac{L_2 \cos(e) + L_3 V_s \cos(p)}{J_e} \\ \frac{L_4 V_s \cos(e) \sin(p)}{J_\lambda} \end{bmatrix} \tag{17}$$

With linearization in the point $\mathbf{x} = \mathbf{0}$, the matrices $\mathbf{A}$ and $\mathbf{B}$ will be:

$$\mathbf{A} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\bigg|_{\mathbf{x}=\mathbf{0}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{L_4 V_s^*}{J_\lambda} & 0 & 0 \end{bmatrix} \tag{18}$$

$$\mathbf{B} = \frac{\partial \mathbf{f}}{\partial \mathbf{u}}\bigg|_{\mathbf{x}=\mathbf{0}} = \begin{bmatrix} 0 & \frac{L_1}{J_p} \\ \frac{L_3}{J_e} & 0 \\ 0 & 0 \end{bmatrix} \tag{19}$$

Now, we have that:

$$\Delta\dot{\tilde{\mathbf{x}}} = \mathbf{A}\Delta\tilde{\mathbf{x}} + \mathbf{B}\Delta\tilde{\mathbf{u}} \tag{20}$$

$$\Delta\dot{\tilde{\mathbf{x}}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{L_4 V_s^*}{J_\lambda} & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \tilde{e} \\ \tilde{\lambda} \end{bmatrix} + \begin{bmatrix} 0 & \frac{L_1}{J_p} \\ \frac{L_3}{J_e} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V_s} \\ \tilde{V_d} \end{bmatrix} = \begin{bmatrix} \frac{L_1}{J_p}\tilde{V_d} \\ \frac{L_3}{J_e}\tilde{V_s} \\ \frac{L_4 V_s^*}{J_\lambda}\tilde{p} \end{bmatrix} \tag{21}$$

Together with equation (16), this gives us the values for K_1, K_2 and K_3:

$$K_1 = \frac{L_1}{J_p} \tag{22}$$

$$K_2 = \frac{L_3}{J_e} \tag{23}$$

$$K_3 = -\frac{L_2 L_4}{L_3 J_\lambda} \tag{24}$$

1.3 Problem 3

After several tests, we found out that the best values were $x = -2$ and $y = 7$.
The sum of the voltages, V_s, responded to y, and worked as an input for the elevation. The more voltage sum, the higher input for the elevation control.

The difference in the voltages, V_d, respond to x, and worked as an input for the pitch. The more voltage difference, the higher input for the pitch control.
We also noticed that we had to use a lot more input and force to elevate the helicopter, than to maneuver it sideways. This made it difficult to control the pitch our self.
The behaviour of the physical model, did not function as the theoretical. If we tried to just use the y input, the helicopter started to change the pitch angle. This is probably because of other factors involving peripheral forces, such as weight difference between the motors etc.

1.4 Problem 4

We found out that the angle between table position and equilibrium position is 30° vertically. By substracting $\frac{30\pi}{180}$ from the elevation encoder, we get the desired output of the elevation.
The voltage sum, V_s, was measured to be 7V when the helicopter was at the equilibrium position. We tested the helicopter with the constant y input, which was set to V_s.

$$V_s^* = 7 = -\frac{L_2}{L_3} = -\frac{m_c l_c g - 2m_p l_h g}{l_h K_f} \tag{25}$$

If we solve for K_f, we get:

$$K_f = \frac{2m_p l_h g - m_c l_c g}{7l_h} = 0.1427 \tag{26}$$

The resulting system is shown in figure 2.

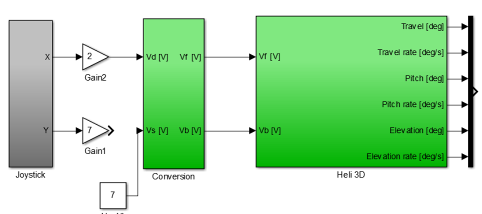

Figure 2: System - Part I

2 Part II - Monovaraible control

2.1 Problem 1

$$\tilde{V_d} = K_{pp}(\tilde{p_c} - \tilde{p}) - K_{pd}\dot{\tilde{p}} \quad (27)$$

$$\ddot{\tilde{p}} = K_1 \tilde{V_d} \quad (28)$$

We can then get:

$$\ddot{\tilde{p}} = K_1(K_{pp}(\tilde{p_c} - \tilde{p}) - K_{pd}\dot{\tilde{p}}) \quad (29)$$

In the s-plane:

$$s^2\tilde{p} = K_1(K_{pp}(\tilde{p_c} - \tilde{p}) - sK_{pd}\tilde{p}) \quad (30)$$

Solve for $\frac{\tilde{p}}{\tilde{p_c}}$ and get:

$$\frac{\tilde{p}}{\tilde{p_c}} = \frac{1}{\frac{s^2}{K_1K_{pp}} + \frac{K_{pd}}{K_{pp}}s + 1} \quad (31)$$

From the transfer function (31), we now have:

$$\omega_0 = \sqrt{K_1K_{pp}} \quad (32)$$

$$2\zeta\omega_0 = K_1K_{pd} \quad (33)$$

We want $\zeta = 1$ for critical damping.

$$K_{pp} = \frac{\omega_0^2}{K_1} \quad (34)$$

$$K_{pd} = \frac{2\omega_0}{K_1} \quad (35)$$

This gives us a relationship between K_{pp} and K_{pd}:

$$\frac{K_{pd}}{\sqrt{K_{pp}}} = \frac{2}{\sqrt{K_1}} \quad (36)$$

From (22) we have that:

$$K_1 = \frac{L_1}{J_p} = \frac{K_f}{2m_pl_p} = 0.5663 \quad (37)$$

The relationship from (36) is now:

$$\frac{K_{pd}}{\sqrt{K_{pp}}} = 2.6578 \tag{38}$$

We tried with different values for Kpp in the formula:

$$K_{pd} = 2.6578\sqrt{K_{pp}} \tag{39}$$

After several attempts we ended up with values:

$$K_{pp} = 11 \tag{40}$$

$$K_{pd} = 8.815 \tag{41}$$

We are able to change the eigenvalues with different values of K_{pp} and K_{pd}, but we desire $\zeta = 1$ for a critically damped system. If we choose a $0 \leq \zeta < 1$ (underdamped), we will get complex conjugated poles. If we choose $\zeta > 1$(overdamped), we will get two different real poles. Since the system is not linear, it is not possible to increase K_{pp} too much. If we do that, the system (pitch) becomes unstable.
By implementing the PD-controller in our system, it is much easier to control the pitch angle. Results of the simulink diagram of the PD controller is shown in figure 3.

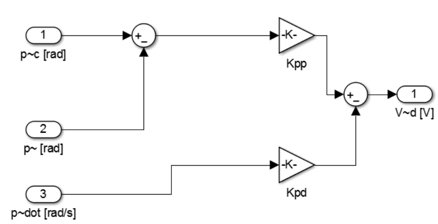

Figure 3: PD controller

2.2 Problem 2

We have that:

$$\tilde{p_c} = K_{rp}(\dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}}) \tag{42}$$

Where $K_{rp} < 0$ and $\tilde{p} = \tilde{p_c}$

$$\ddot{\tilde{\lambda}} = \frac{L_4 V_s^*}{J_\lambda}\tilde{p} = -\frac{L_4 L_2}{L_3 J_\lambda}\tilde{p} \tag{43}$$

Then we have that:

$$\tilde{p} = -\frac{L_3 J_\lambda}{L_2 L_4}\ddot{\tilde{\lambda}} \tag{44}$$

Where we define a constant

$$K_{temp} = -\frac{L_3 J_\lambda}{L_2 L_4} = \frac{1}{K_3} \tag{45}$$

Now we have:

$$\ddot{\tilde{\lambda}} = K_3 K_{rp}(\dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}}) \tag{46}$$

If we laplace transform this equation to the s-plane, we get:

$$s^2\tilde{\lambda} = K_3 K_{rp}(s\tilde{\lambda}_c - s\tilde{\lambda}) \tag{47}$$

If we now solve this for $\dfrac{\tilde{\lambda}}{\tilde{\lambda}_c}$ we get:

$$\frac{\tilde{\lambda}}{\tilde{\lambda}_c} = \frac{K_3 K_{rp}}{s + K_3 K_{rp}} \tag{48}$$

We can now see that:

$$K_3 K_{rp} = \rho \tag{49}$$

The helicopter now uses the x value to control the travel velocity. The travel responds correctly. We tuned K_{rp} to be -1.
The result of the simulink diagram of the P controller is shown in figure 4.

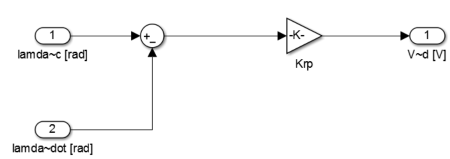

Figure 4: P controller

3 Part III - Multivariable control

3.1 Problem 1

To formulate the state space of the system, we take a look at (6a) and (6b) from the task sheet:

$$\ddot{\tilde{p}} = K_1 \tilde{V}_d \tag{50}$$

$$\ddot{\tilde{e}} = K_2 \tilde{V}_s \tag{51}$$

We want the equation $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ with:

$$\mathbf{x} = \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \dot{\tilde{e}} \end{bmatrix} \tag{52}$$

And

$$\mathbf{u} = \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix} \tag{53}$$

This responds to the given system below:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\tilde{p}} \\ \ddot{\tilde{p}} \\ \ddot{\tilde{e}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \dot{\tilde{e}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ K_2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix} \tag{54}$$

Where $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ K_2 & 0 \end{bmatrix}$.

3.2 Problem 2

Firstly, we examine the controllability of the system:

$$\boldsymbol{\zeta} = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & K_1 & 0 & 0 \\ 0 & K_1 & 0 & 0 & 0 & 0 \\ K_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{55}$$

$$\text{Rank}(\boldsymbol{\zeta}) = 3 = n \tag{56}$$

This means that $\boldsymbol{\zeta}$ has a full rank, hence the system is controllable.

We now use Bryson's rule as a rule of thumb to find start values for our **Q** and **R** matrices. The rule goes like this:

$$\mathbf{Q}_{ii} = \frac{1}{(\text{maximum acceptable value of } z_i)^2} \tag{57}$$

$$\mathbf{R}_{jj} = \frac{1}{(\text{maximum acceptable value of } u_j)^2} \tag{58}$$

Since this didn't gave the results we wanted, we later tuned the values for our prioritized states and ended up with:

$$\mathbf{Q} = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 70 \end{bmatrix} \tag{59}$$

We used the identity matrix for **R** and tuned **Q** from there.

$$\mathbf{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{60}$$

We then found the **K** matrix by using the matlab function $\mathbf{K} = lqr(\mathbf{A}, \mathbf{B}, \mathbf{Q}, \mathbf{R})$

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 8.36 \\ 10 & 6.73 & 0 \end{bmatrix} \tag{61}$$

With the limit $\mathbf{y}(t) \to \mathbf{r}$ when $t \to \infty$, the equation $\mathbf{u} = \mathbf{Pr} - \mathbf{Kx}$ and the state space equation, we can calculate the matrix **P**.

$$\mathbf{Ax} + \mathbf{B}(\mathbf{Pr} - \mathbf{Kx}) = 0 \tag{62}$$

$$\mathbf{x} = (\mathbf{BK} - \mathbf{A})^{-1}\mathbf{BPr} \tag{63}$$

$$\mathbf{y} = \mathbf{C}(\mathbf{BK} - \mathbf{A})^{-1}\mathbf{BPr} \tag{64}$$

$$1 = \mathbf{C}(\mathbf{BK} - \mathbf{A})^{-1}\mathbf{BP} \tag{65}$$

$$\mathbf{P} = (\mathbf{C}(\mathbf{BK} - \mathbf{A})^{-1})^{-1} \tag{66}$$

Calculating the values of matrix **P**, we get:

$$\mathbf{P} = \begin{bmatrix} 0 & 8.36 \\ 10 & 0 \end{bmatrix} \tag{67}$$

We chose to prioritize the pitch and elevation rate. This gave the most stable system. The pitch was most prioritized because it is important to be able to maneuver the helicopter as best as possible in the horizontal direction. The elevation rate was prioritized such that there were low velocity in vertical direction. Pitch rate is not that important to the cost function, because the velocity is not important in the pitch directions.

3.3 Problem 3

With the two new states ζ and γ, the matrices become:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{68}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ K_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{69}$$

$$\mathbf{Q} = \begin{bmatrix} 40 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 50 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix} \tag{70}$$

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 7.1 & 0 & 0.7 \\ 8.05 & 6.2 & 0 & 2 & 0 \end{bmatrix} \tag{71}$$

$$\mathbf{P} = \begin{bmatrix} 0 & 7.1 \\ 8.05 & 0 \end{bmatrix} \tag{72}$$

The integral effect made both the pitch and the elevation reach the desired reference faster and with the right tuning made the helicopter behave well. We know this from theory because the integral effect sums up the error from the desired states and actual states over time. This makes the controller reach its equilibrium faster. The resulting system diagram in simulink is shown in figure 5.

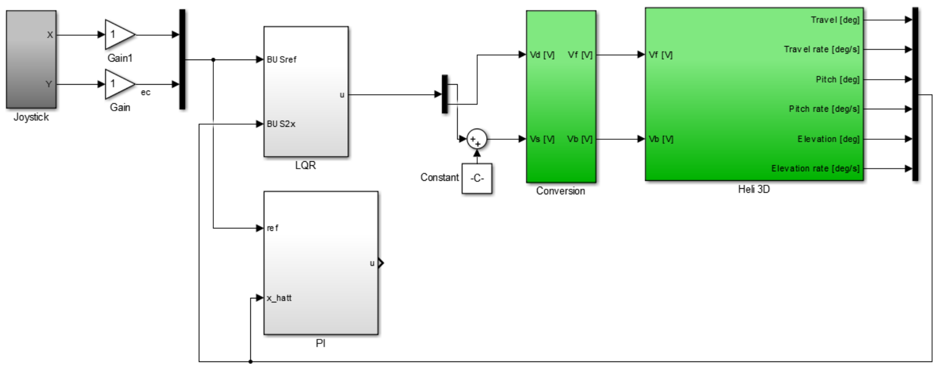

Figure 5: The system after part 3

4 Part IV - State estimation

4.1 Problem 1

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{73}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\tilde{p}} \\ \ddot{\tilde{p}} \\ \dot{\tilde{e}} \\ \ddot{\tilde{e}} \\ \dot{\tilde{\lambda}} \\ \ddot{\tilde{\lambda}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ K_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \\ \dot{\tilde{\lambda}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ 0 & 0 \\ K_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix} \tag{74}$$

$$\mathbf{y} = \begin{bmatrix} \tilde{p} \\ \tilde{e} \\ \tilde{\lambda} \end{bmatrix} = \mathbf{C}\mathbf{x} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \\ \dot{\tilde{\lambda}} \end{bmatrix} \tag{75}$$

4.2 Problem 2

We examined the observability by solving the observability matrix and check if it has full rank.

$$\mathbf{O} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \mathbf{CA}^3 \\ \mathbf{CA}^4 \\ \mathbf{CA}^5 \end{bmatrix} \tag{76}$$

By using the matlab function $obsv(\mathbf{A}, \mathbf{C})$, the function returns a matrix with rank 6, which is a full rank, hence the system is observable.
To create a linear observer, we had to find $\mathbf{L}$. We did this by using the place function in matlab.

$$\mathbf{L} = place(\mathbf{A}', \mathbf{C}', poles).' \tag{77}$$

To use this function, we needed to find some poles for the observer in the left half plane which gave the system stability. We found the poles by using the poles of the controller $\mathbf{A} - \mathbf{BK}$, and placed the new poles on a half circle just about 6 times the radius of the most left pole in the controller. The pole of the controller furthest to the left in the plane has the value $-17 + 0i$. We stored the poles in a vector, and used the place function. The placement of the poles in the closed-loop observer is essential to how the helicopter behaves. Making sure that all the poles of the observer is further to the left in the plane than the poles of the controller, is a crucial part of the pole placement. The result of placing the poles like this is that the observer reacts faster than the controller. This is desired. We ended up with a half circle of radius 100 and a angle of 10° between the poles.

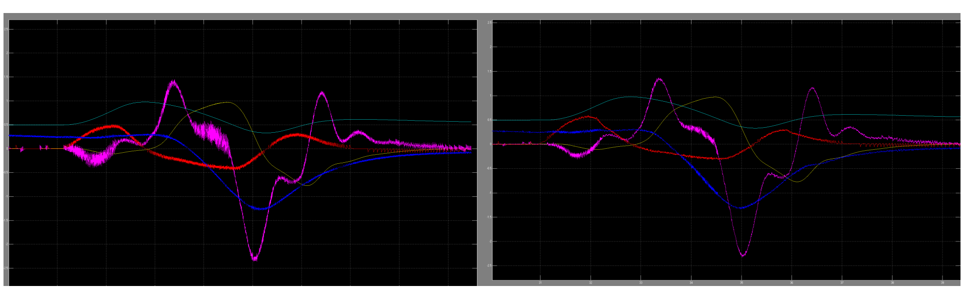

Figure 6: Measured and estimated with estimator and LQR

The results of adding an estimator to the system is shown in figure 6. The left window shows the plot of the measured states and the right window shows the estimated states. The intention of adding an estimator is to eliminate the white noise from the measured states. As we can see from the plots the estimated states are smoother than the measured.

4.3 Problem 3

We want to see if the system is observable if one only measures $\tilde{e}$ and $\tilde{\lambda}$.

$$\mathbf{y}_1 = \begin{bmatrix} \tilde{e} \\ \tilde{\lambda} \end{bmatrix} \rightarrow \mathbf{C}_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{78}$$

We calculate the observability matrix with (76) as we did earlier. We then get a matrix $\mathbf{O}_1$ with full rank, hence the system is observable.

$$\mathbf{y}_2 = \begin{bmatrix} \tilde{p} \\ \tilde{e} \end{bmatrix} \rightarrow \mathbf{C}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{79}$$

By calculating the observability matrix $\mathbf{O}_2$, we get a matrix with rank 4, which is not a full rank matrix. Hence the system is not observable.

Now we tested the observable system, which in theory should be stable. However the helicopter preformed rather badly. The figures 7 and 8 shows the plots of the measured and estimated states when one only measures $\tilde{e}$ and $\tilde{\lambda}$. The corresponding color and states is explained in table 1. The estimated state of $\dot{\tilde{p}}$ is shown in figure 7 to be the worst case of the estimated states. Its value is much larger than the other states. Figure 8 shows the estimate state of $\tilde{p}$. The state is clearly not good. The reason for that the estimated pitch and pitch rate values are such poorly is that the system has to calculate the pitch based on the travel rate.

Table 1:

$\tilde{p}$	yellow
$\dot{\tilde{p}}$	magentha
$\tilde{e}$	cyan
$\dot{\tilde{e}}$	red
$\tilde{\lambda}$	green
$\dot{\tilde{\lambda}}$	blue

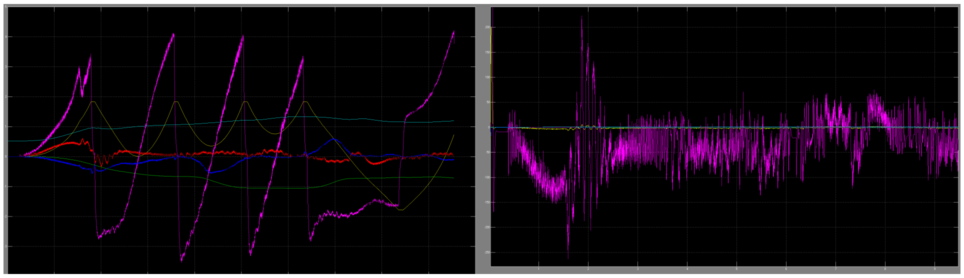

Figure 7: Measured and estimated state plots

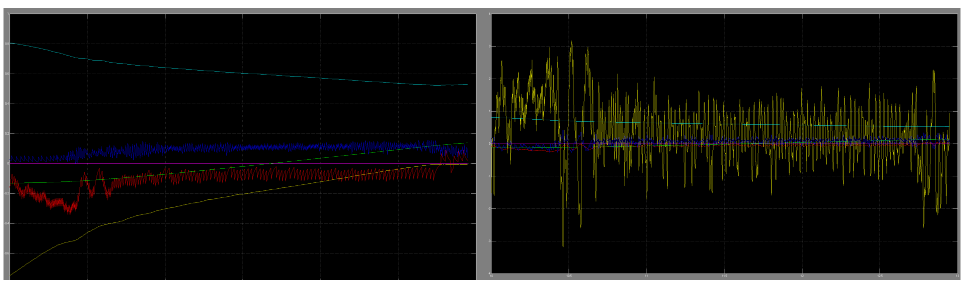

Figure 8: Measured and estimated state plots (without pitch-rate plotted)

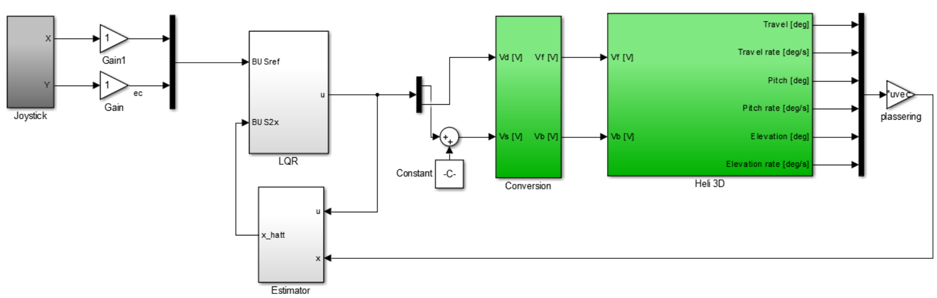

Figure 9: The final system with estimator