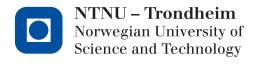# Linear Systems TTK4115 - Helicopter lab

Alex Danielsen – XXXXXX   Sindre Hansen – 732719
Daniel Nakken – 740939

October 2016
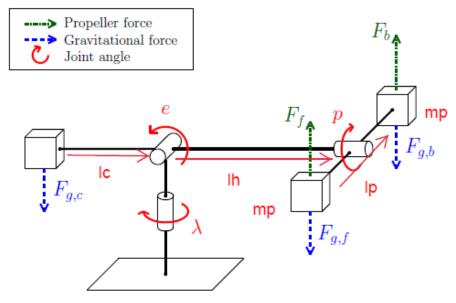
Contents

1 Part 1 - Mathematical modeling

1.1 Problem 1

We use the helicopter model fig. 1 as our starting point for deriving the equations of motion.

Figure 1: the helicopter model figure 7 from the assignment [2, p.12] with relevant distances drawn in.

The equations of motion for the pitch is the momentum around the point p in the clockwise direction as shown in fig. 1:

$$J_p\ddot{p} = l_p(F_{g,b} - F_b - F_{g,f} + F_f)$$
$$= l_p(m_pg - mp_g + K_fV_f - V_b)$$
$$= l_pK_f(V_f - V_b)$$

Since $V_d = V_f - V_b$, we can write this as:

$$J_p\ddot{p} = l_pK_fVd \tag{1}$$

Here, we can see that $L_1 = l_pK_f$.

Figure 2: the elevation model

1.2 Problem 2

1.3 Problem 3

1.4 Problem 4

2 Part 2 – Monovariable control

2.1 Problem 1

We are given the controller shown in eq. (2).

$$\tilde{V}_d = K_{pp}(\tilde{p}_c - \tilde{p}) - K_{pd}\dot{\tilde{p}} \tag{2}$$

We take this controller and substitute it in the equation for pitch angle (??).

$$\ddot{\tilde{p}} = K_1 K_{pp}(\tilde{p}_c - \tilde{p}) - K_1 K_{pd}\dot{\tilde{p}} \tag{3}$$

Now we Laplace transform eq. (3) to find the transfer function $\frac{\tilde{p}(s)}{\tilde{p}_c(s)}$.

$$\ddot{\tilde{p}} + K_1 K_{pd}\dot{\tilde{p}} + K_1 K_{pp}\tilde{p} = K_1 K_{pp}\tilde{p}_c$$
$$\mathcal{L} \rightarrow$$
$$s^2\tilde{p}(s) + sK_1 K_{pd}\tilde{p}(s) + K_1 K_{pp}\tilde{p}(s) = K_1 K_{pp}\tilde{p}_c(s)$$

Which gives us our transfer function

$$\frac{\tilde{p}(s)}{\tilde{p}_c(s)} = \frac{K_1 K_{pp}}{s^2 + K_1 K_{pd}s + K_1 K_{pp}} \tag{4}$$

The linearized pitch dynamics can be regarded as a second-order linear system, which means that if we place eq. (4) on the form shown in eq. (5) we can determine K_{pp} and K_{pd} from ω and ζ.

$$h(s) = \frac{\omega^2}{s^2 + 2\zeta\omega^2 s + \omega^2} \tag{5}$$

This gives us the following relations

$$\omega = \sqrt{K_1 K_{pp}} \tag{6}$$
$$2\zeta\omega^2 = K_1 K_{pd}$$
$$\zeta = \frac{K_1 K_{pd}}{2\omega^2} = \frac{K_{pd}}{2K_{pp}} \tag{7}$$

We know that for a critically damped system $\zeta = 1$, which gives us

$$K_{pd} = 2K_{pp} \tag{8}$$

We chose a $K_{pp} = 3$ and then from the relation in eq. (8) we get $K_{pd} = 6$. With these values the response of the pitch angle to the input was slower than desired. Therefore, K_{pp} was increased to $K_{pp} = 12.5$ and K_{pd} was lowered to underdamp

3

the system, until it was sufficiently responsive at $K_{pd} = 0.7K_{pp} = 8.75$. At these values the system responded faster with only minor oscillations. It was observed that larger values of K_{pp} gave rise to larger oscillations.

Figure for that shows poles on Im/Re axis?

At the critically damped point, the poles lie on the same point on the x-axis. When K_{pd} is increased in relation to K_{pp} the system is over-damped the poles move away from each other along the x-axis. When K_{pd} decreased in relation to K_{pp}, the system is under damped and the poles move away from each other vertically from the critically damped point.

With the PD controller, it was significantly easier to control the helicopter than with just feed forward joystick control.

2.2 Problem 2

References

[1] Chi-Tsong Chen, *Linear System Theory and Design*, Oxford University Press, 4th edition, 2014

[2] Kristoffer Gryte, *Helicopter lab assignment*, Department of Engineering Cybernetics, NTNU, Version 4.5, 2015