Modern portfolio theory

INTRODUCTION TO PORTFOLIO ANALYSIS IN PYTHON

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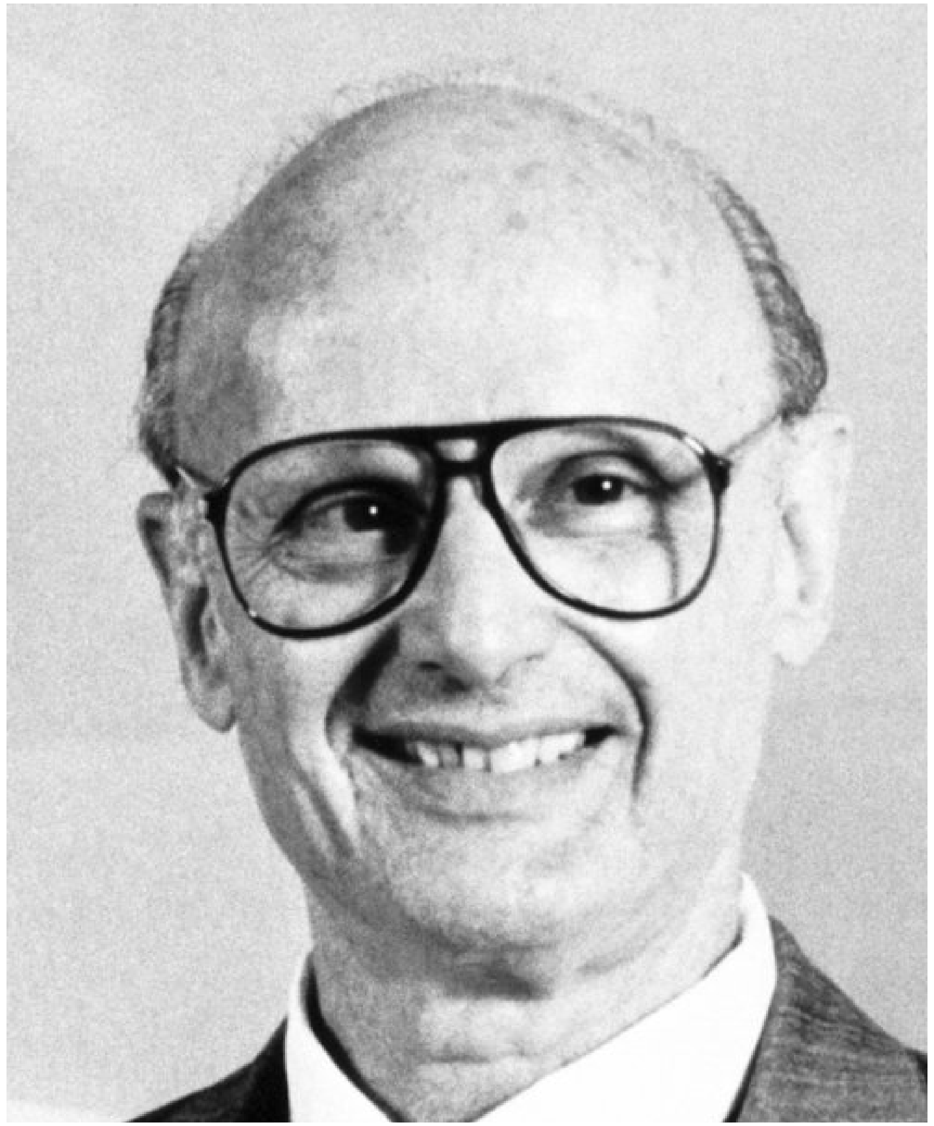
Creating optimal portfolios

What is Portfolio Optimization?

Meet Harry Markowitz



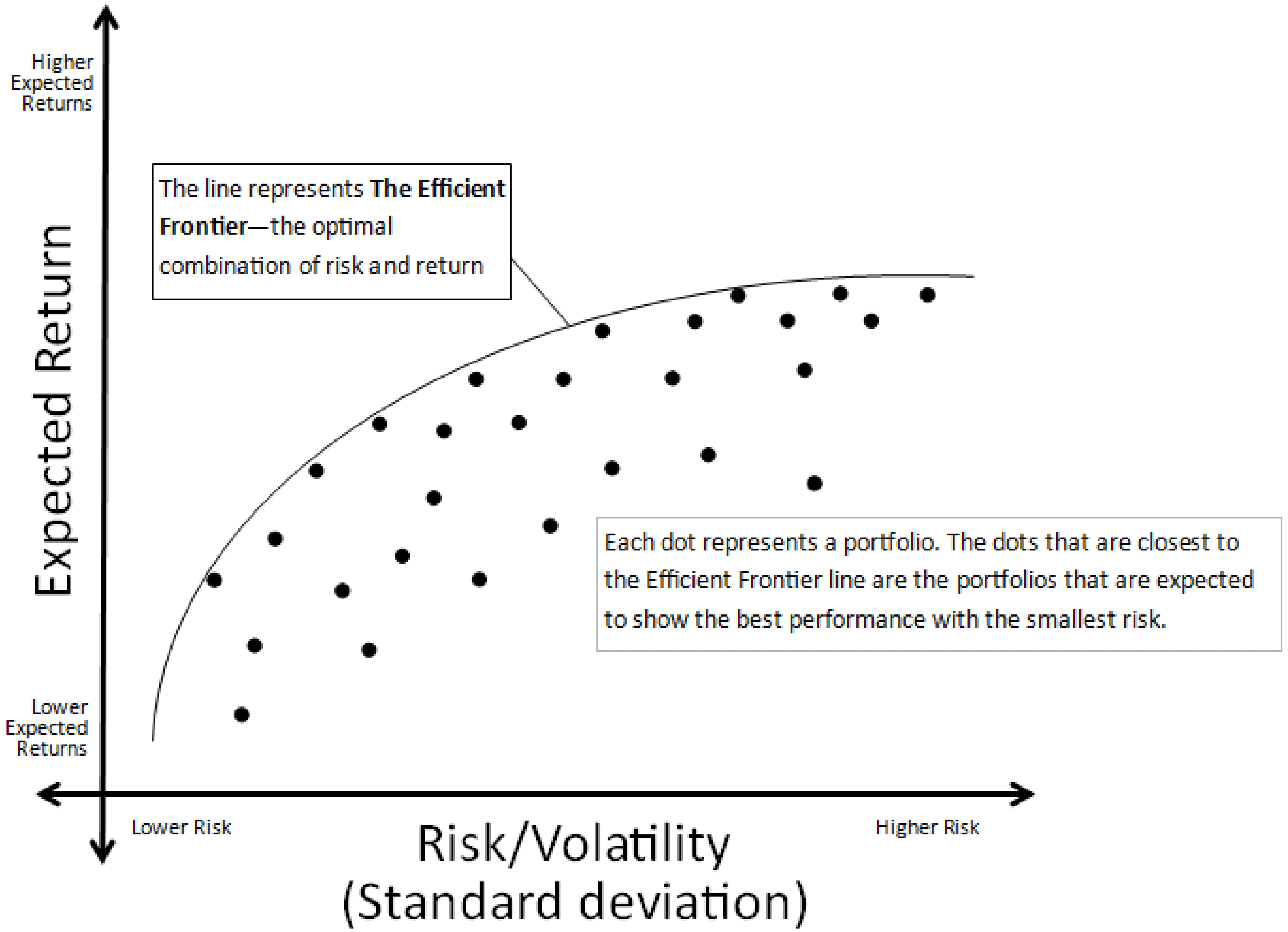
The optimization problem: finding optimal weights

$$
\begin{aligned}
\underset{\omega}{\text{minimise}} \quad & \omega^T \Sigma \omega \\
\text{subject to} \quad & \omega^T \mu \geq \mu^* \\
& \omega^T \mathbf{1} = 1 \\
& \omega_i \geq 0
\end{aligned}
$$

In words:

- Minimize the portfolio variance, subject to:
- The expected mean return is at least some target return
- The weights sum up to 100%
- At least some weights are positive

Varying target returns leads to the Efficient Frontier



PyPortfolioOpt for portfolio optimization

```python
from pypfopt.efficient_frontier import EfficientFrontier
from pypfopt import risk_models
from pypfopt import expected_returns
```

```python
df=pd.read_csv('portfolio.csv')
df.head(2)
                  XOM        RRC        BBY         MA        PFE
date
2010-01-04  54.068794  51.300568  32.524055  22.062426  13.940202
2010-01-05  54.279907  51.993038  33.349487  21.997149  13.741367
```

```python
# Calculate expected annualized returns and sample covariance
mu = expected_returns.mean_historical_return(df)
Sigma = risk_models.sample_cov(df)
```

Get the Efficient Frontier and portfolio weights

```
# Calculate expected annualized returns and risk
mu = expected_returns.mean_historical_return(df)
Sigma = risk_models.sample_cov(df)
```

```
# Obtain the EfficientFrontier
ef = EfficientFrontier(mu, Sigma)
```

```
# Select a chosen optimal portfolio
ef.max_sharpe()
```

Different optimizations

```
# Select the maximum Sharpe portfolio
ef.max_sharpe()
```

```
# Select an optimal return for a target risk
ef.efficient_risk(2.3)
```

```
# Select a minimal risk for a target return
ef.efficient_return(1.5)
```

Calculate portfolio risk and performance

```
# Obtain the performance numbers
ef.portfolio_performance(verbose=True, risk_free_rate = 0.01)
```

```
Expected annual return: 21.3%
Annual volatility: 19.5%
Sharpe Ratio: 0.98
```

Let's optimize a portfolio!

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Maximum Sharpe vs. minimum volatility

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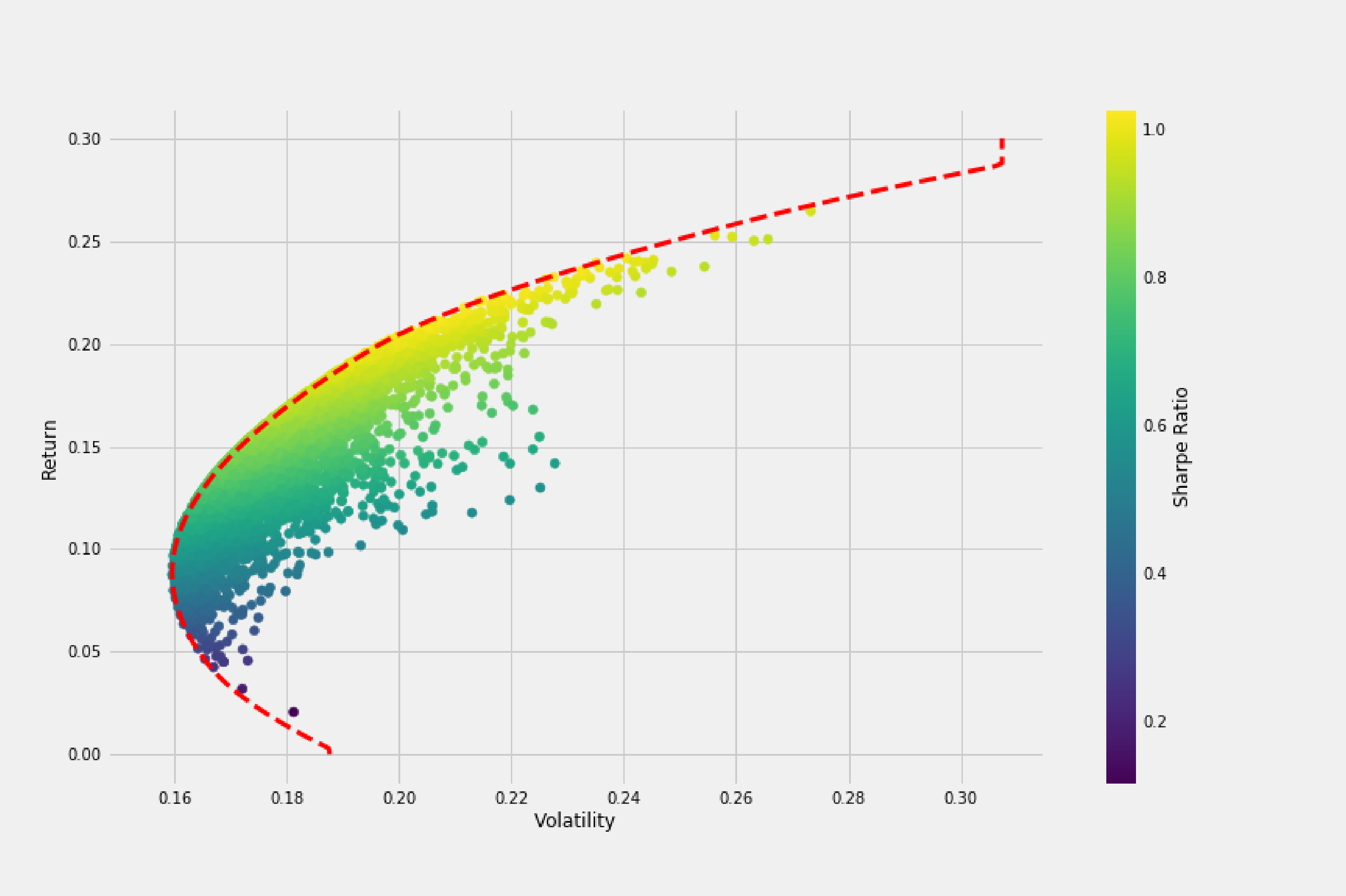
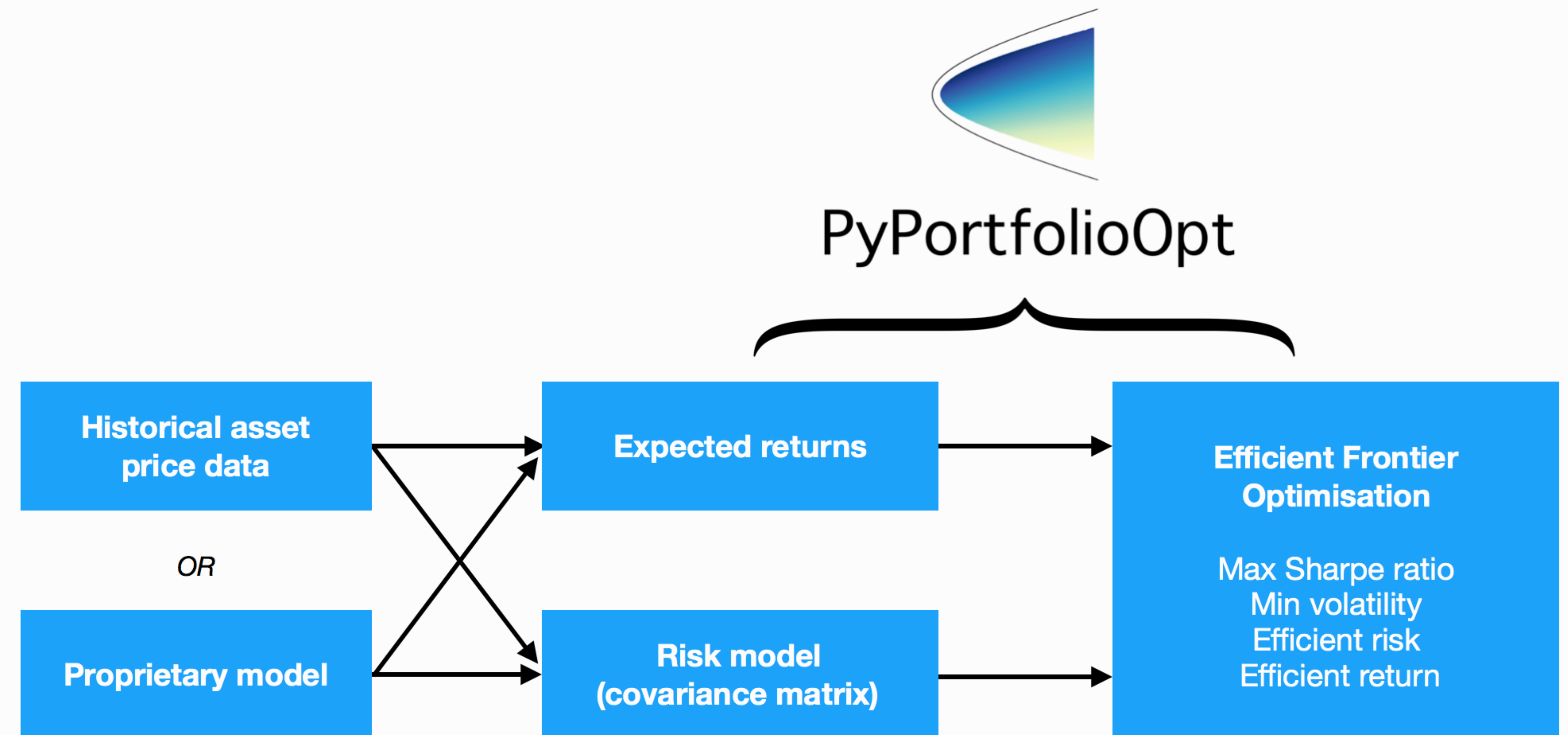
Remember the Efficient Frontier?

- Efficient frontier: all portfolios with an optimal risk and return trade-off
- Maximum Sharpe portfolio: the highest Sharpe ratio on the EF
- Minimum volatility portfolio: the lowest level of risk on the EF



Adjusting PyPortfolioOpt optimization



Maximum Sharpe portfolio

Maximum Sharpe portfolio: the **highest Sharpe ratio** on the EF

```
from pypfopt.efficient_frontier import EfficientFrontier
```

```
# Calculate the Efficient Frontier with mu and S
ef = EfficientFrontier(mu, Sigma)
raw_weights = ef.max_sharpe()
```

```
# Get interpretable weights
cleaned_weights = ef.clean_weights()
```

```
{'GOOG': 0.01269,'AAPL': 0.09202,'FB': 0.19856,
 'BABA': 0.09642,'AMZN': 0.07158,'GE': 0.02456,...}
```

Maximum Sharpe portfolio

```
# Get performance numbers
ef.portfolio_performance(verbose=True)
```

```
Expected annual return: 33.0%
Annual volatility: 21.7%
Sharpe Ratio: 1.43
```

Minimum Volatility Portfolio

Minimum volatility portfolio: the **lowest level of risk** on the EF

```
# Calculate the Efficient Frontier with mu and S
ef = EfficientFrontier(mu, Sigma)
```

```
raw_weights = ef.min_volatility()
```

```
# Get interpretable weights and performance numbers
cleaned_weights = ef.clean_weights()
```

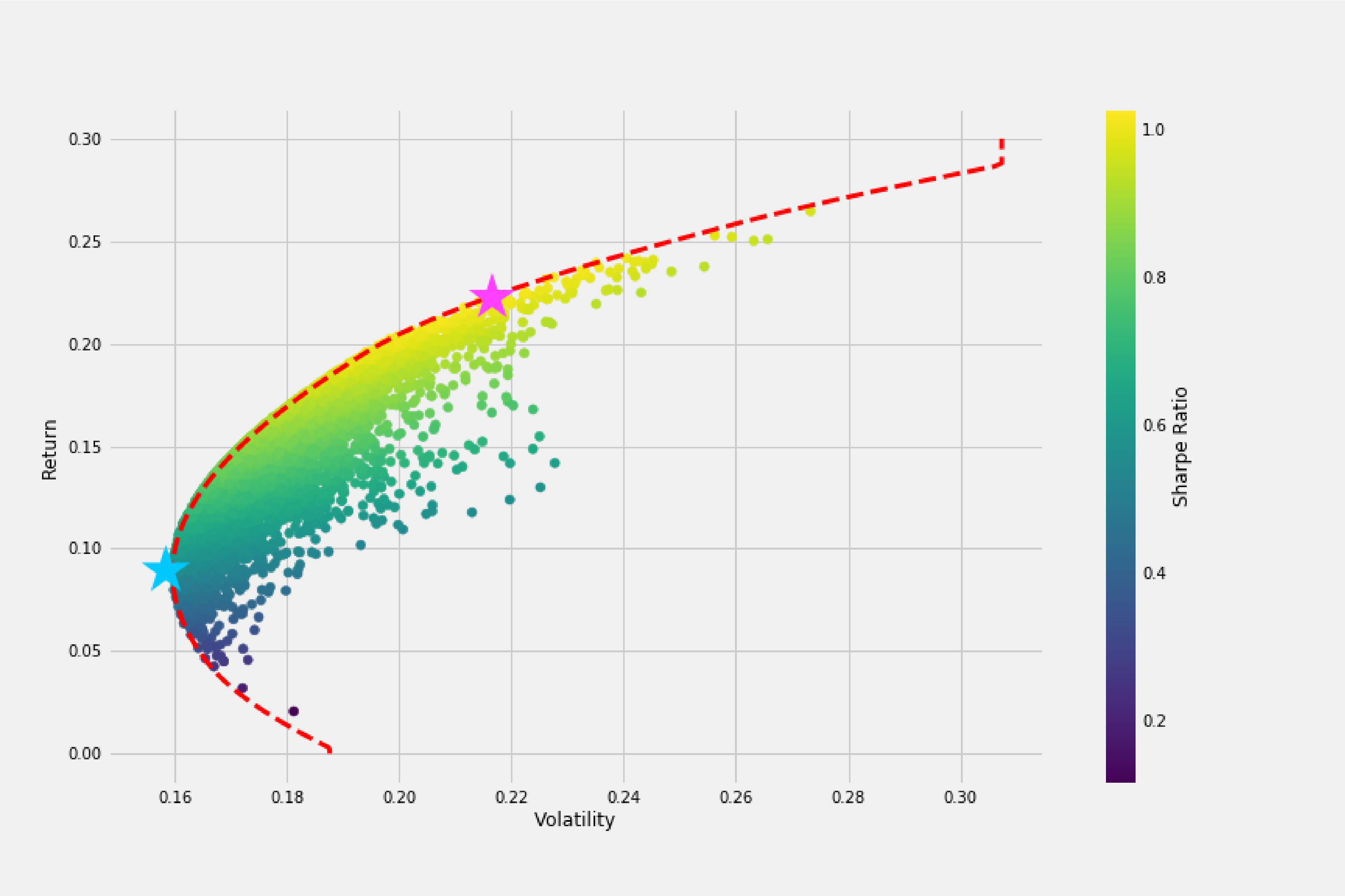
```
{'GOOG': 0.05664, 'AAPL': 0.087, 'FB': 0.1591,
'BABA': 0.09784, 'AMZN': 0.06986, 'GE': 0.0123,...}
```

Minimum Volatility Portfolio

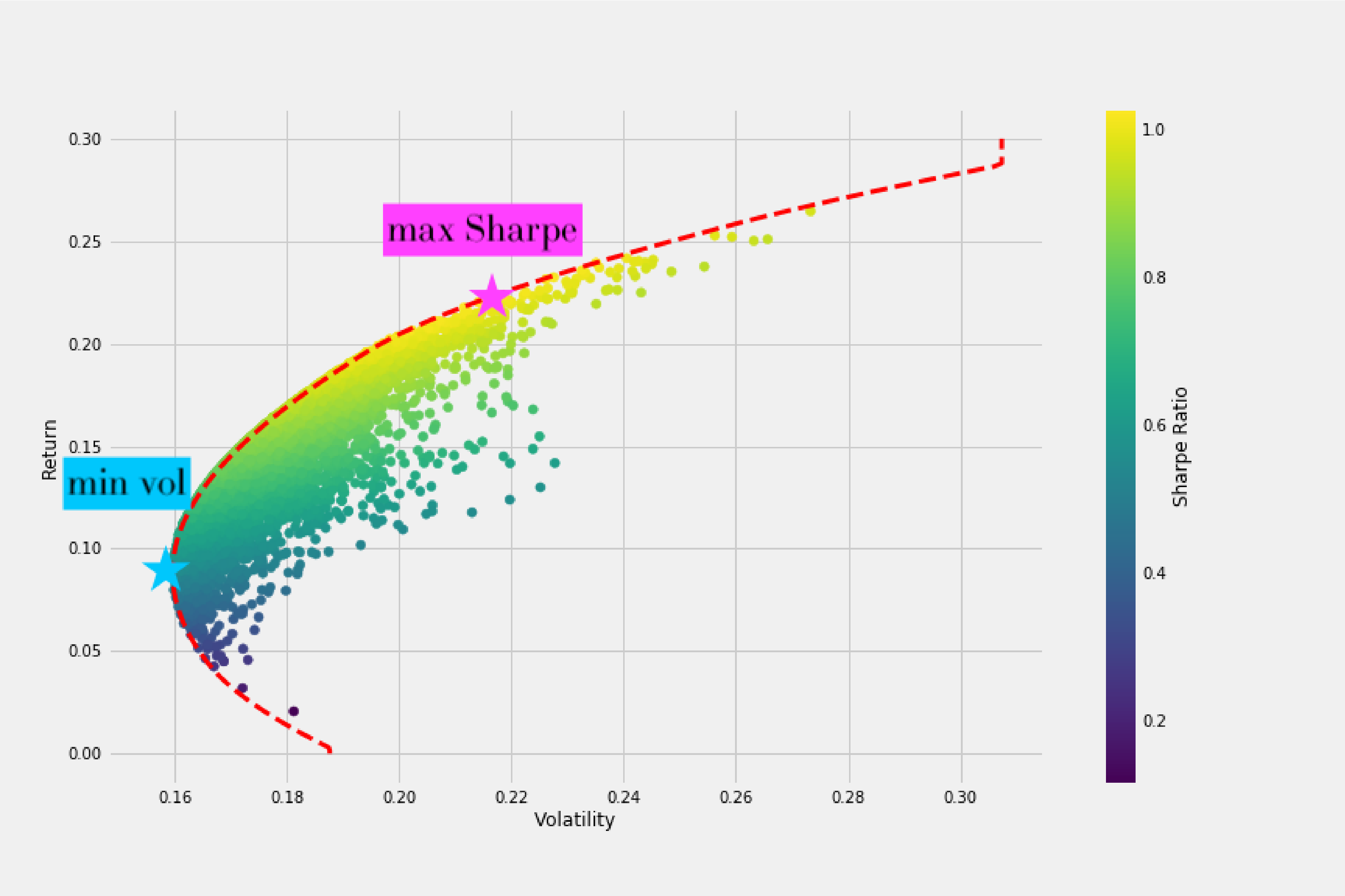
```
ef.portfolio_performance(verbose=True)
```

```
Expected annual return: 17.4%
Annual volatility: 13.2%
Sharpe Ratio: 1.28
```

Let's have another look at the Efficient Frontier



Maximum Sharpe versus Minimum Volatility



Let's practice!

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Alternative portfolio optimization

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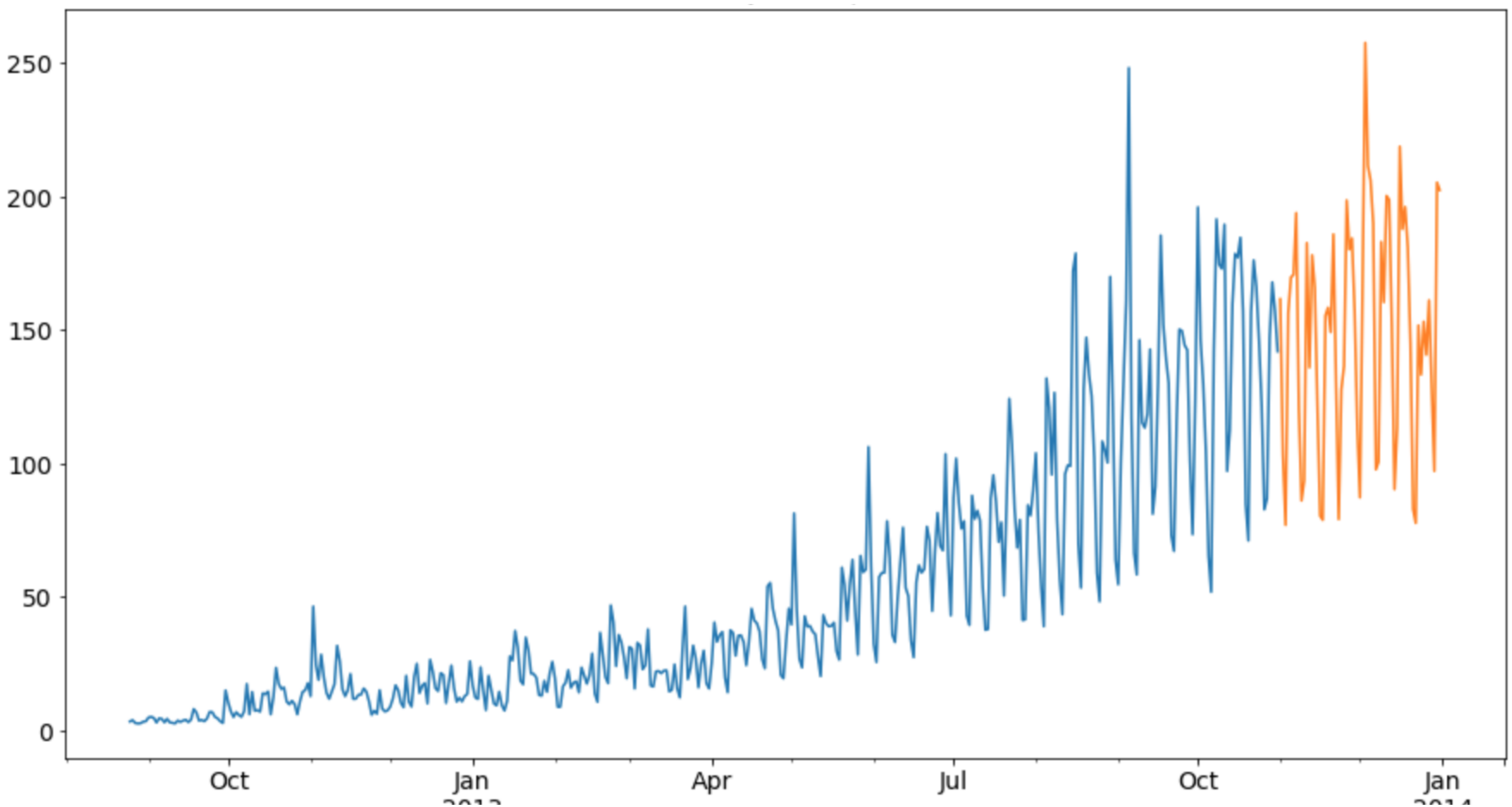
Expected risk and return based on historic data

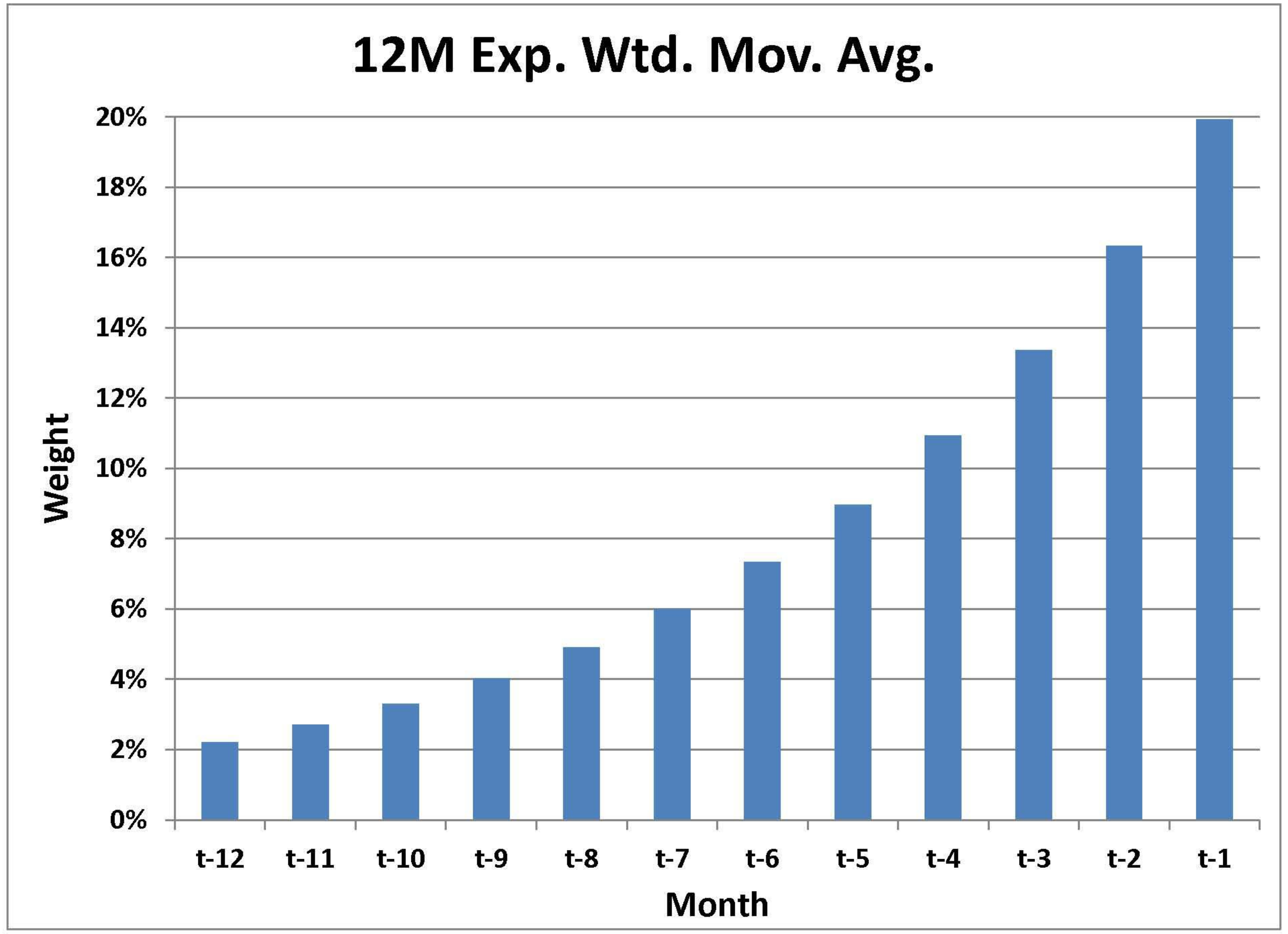
- Mean historic returns, or the historic portfolio variance are **not perfect estimates** of mu and Sigma
- Weights from portfolio optimization therefore **not guaranteed to work well** on future data

Historic data



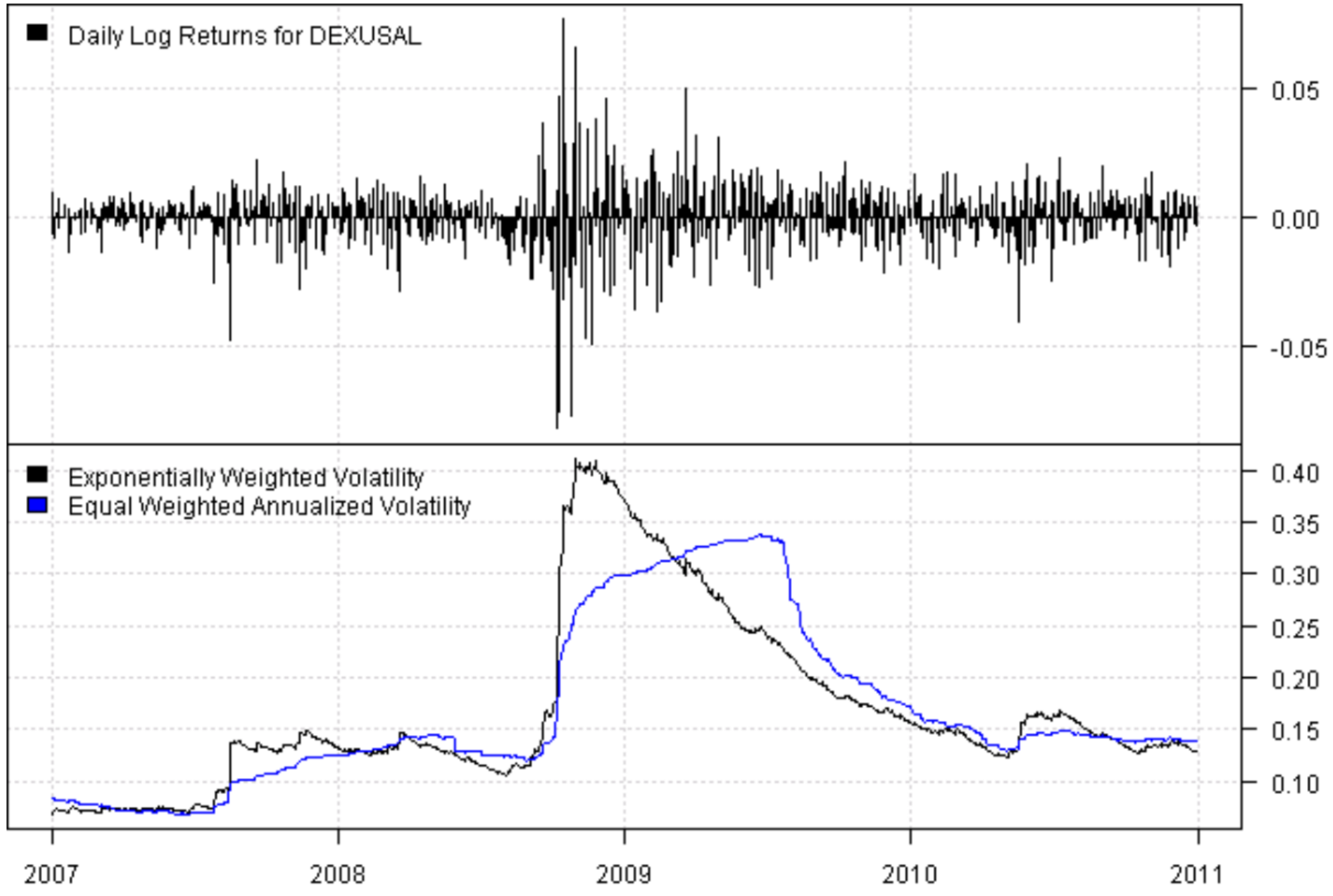
Exponentially weighted returns

- Need better measures for risk and return
- Exponentially weighted risk and return assigns more importance to the most recent data
- Exponential moving average in the graph: most weight on t-1 observation



Exponentially weighted covariance

- The exponential covariance matrix: gives more weight to recent data
- In the graph: exponential weighted volatility in black, follows real volatility better than standard volatility in blue



[1] Source: http://systematicinvestor.github.io/Exponentially [2] Weighted [3] Volatility [4] RCPP

Exponentially weighted returns

```
from pypfopt import expected_returns
```

```
# Exponentially weighted moving average
mu_ema = expected_returns.ema_historical_return(df,
                                span=252, frequency=252)
print(mu_ema)
```

```
symbol
XOM     0.103030
BBY     0.394629
PFE     0.186058
```

Exponentially weighted covariance

```
from pypfopt import risk_models
```

```
# Exponentially weighted covariance
Sigma_ew = risk_models.exp_cov(df, span=180, frequency=252)
```

Using downside risk in the optimization

- Remember the Sortino ratio: it uses the variance of negative returns only
- PyPortfolioOpt allows you to use **semicovariance** in the optimization, this is a measure for downside risk:

$$\text{Downside risk} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\text{return} - \text{target return})^2\, f(t)}$$

$$f(t) = 1 \text{ if return} < \text{target return}$$

Semicovariance in PyPortfolioOpt

```
Sigma_semi = risk_models.semicovariance(df,
                        benchmark=0, frequency=252)


print(Sigma_semi)
```

```
          XOM        BBY        MA         PFE
XOM       0.018939   0.008505   0.006568   0.004058
BBY       0.008505   0.016797   0.009133   0.004404
MA        0.006568   0.009133   0.018711   0.005373
PFE       0.004058   0.004404   0.005373   0.008349
```

Let's practice!

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Recap

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Chapter 1: Calculating risk and return

- A portfolio as a collection of weight and assets
- Diversification
- Mean returns versus cumulative returns
- Variance, standard deviation, correlations and the covariance matrix
- Calculating portfolio variance

Chapter 2: Diving deep into risk measures

- Annualizing returns and risk to compare over different periods
- Sharpe ratio as a measured of risk adjusted returns
- Skewness and Kurtosis: looking beyond mean and variance of a distribution
- Maximum draw-down, downside risk and the Sortino ratio

Chapter 3: Breaking down performance

- Compare to benchmark with active weights and active returns
- Investment factors: explain returns and sources of risk
- Fama French 3 factor model to breakdown performance into explainable factors and alpha
- Pyfolio as a portfolio analysis tool

Chapter 4: Finding the optimal portfolio

- Markowitz' portfolio optimization: efficient frontier, maximum Sharpe and minimum volatility portfolios
- Exponentially weighted risk and return, semicovariance

Continued learning

- Datacamp course on Portfolio Risk Management in Python
- Quantopian's lecture series: **https://www.quantopian.com/lectures**
- Learning by doing: Pyfolio and PyPortfolioOpt

End of this course

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