QuTiP lecture: Jaynes-Cummings-like model in the ultrastrong coupling regime

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Latest version of this ipython notebook lecture is available at: http://github.com/jrjohansson/qutip-lectures

In [1]:
```
# setup the matplotlib graphics library and configure it to show figures inline in the notebook
%pylab inline
```

```
Welcome to pylab, a matplotlib-based Python environment [backend: module://IPython.zmq.pylab.backend_inline].
For more information, type 'help(pylab)'.
```

In [2]:
```
# make qutip available in the rest of the notebook
from qutip import *
```

Introduction

In the Jaynes-Cumming model, the dipole-interaction term between the atom and the cavity field is assumed to be weak, so that a rotating-wave approximation can be performed. For large coupling strengths between the atom and the cavity field the RWA is not justified, and for very large coupling strength interesting properties of the atom-cavity ground state is observed.

To explore this using QuTiP, consider the Hamiltonian

$$H = \hbar \omega_c a^\dagger a + \frac{1}{2}\hbar\omega_a\sigma_z + \hbar g(a^\dagger + a)(\sigma_- + \sigma_+).$$

Note that here we have not transformed the interaction part of the Hamiltonian using the RWA, for which the Hamiltonian would have been

$$H_{\rm RWA} = \hbar \omega_c a^\dagger a + \frac{1}{2}\hbar\omega_a\sigma_z + \hbar g(a^\dagger\sigma_- + a\sigma_+).$$

In this notebook we will calculate the ground state of the Hamiltonian H as a function of the interaction strength g (try to set `use_rwa = True` to use $H_{\rm RWA}$ instead).

The regime g is large compared with all other energy scales in the Hamiltonian H is called the ultrastrong coupling regime, and has been an active topic of research in recent years. See references below.

References:

- P. Nataf et al., Phys. Rev. Lett. 104, 023601 (2010)
- J. Casanova et al., Phys. Rev. Lett. 105, 26360 (2010).
- S. Ashhab et al., Phys. Rev. A 81, 042311 (2010)

Problem parameters

Here we use units where $\hbar = 1$:

In [3]:
```
wc = 1.0  * 2 * pi  # cavity frequency
wa = 1.0  * 2 * pi  # atom frequency

N = 15              # number of cavity fock states
use_rwa = False
```

Setup the operators and the Hamiltonian

In [4]:
```
# operators
a  = tensor(destroy(N), qeye(2))
sm = tensor(qeye(N), destroy(2))

na = sm.dag() * sm  # atom
nc = a.dag() * a    # cavity


# decoupled Hamiltonian
H0 = wc * a.dag() * a + wa * sm.dag() * sm

# interaction Hamiltonian
if use_rwa:
    H1 = (a.dag() * sm + a * sm.dag())
else:
    H1 = (a.dag() + a) * (sm + sm.dag())
```

Find ground state as a function of coupling strength

In [5]:
```
g_vec = linspace(0, 2.0, 101) * 2 * pi # coupling strength vector

psi_list = []
```

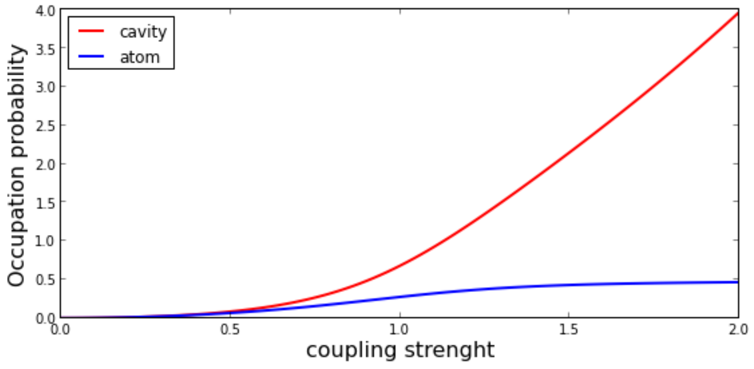
```
for g in g_vec:

    H = H0 + g * H1

    # find the groundstate and its energy
    gnd_energy, gnd_state = H.groundstate()

    # store the ground state
    psi_list.append(gnd_state)
```

Calculate the cavity and atom excitation probabilities as for the calculated ground states:

In [6]:
```
na_expt = expect(na, psi_list) # qubit  occupation probability
nc_expt = expect(nc, psi_list) # cavity occupation probability
```

Plot the ground state occupation probabilities of the cavity and the atom as a function of coupling strenght. Note that for large coupling strength (the ultrastrong coupling regime, where $g > \omega_a, \omega_c$), the ground state has both photonic and atomic excitations.

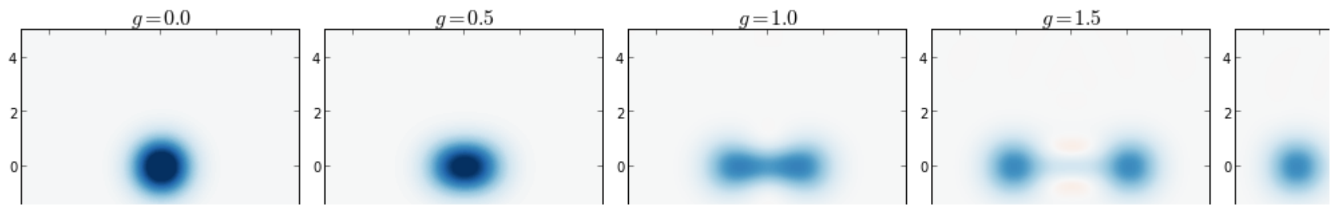
In [7]:
```
fig, axes = subplots(1, 1, sharex=True, figsize=(8,4))

axes.plot(g_vec/(2*pi), nc_expt, 'r', linewidth=2, label="cavity")
axes.plot(g_vec/(2*pi), na_expt, 'b', linewidth=2, label="atom")
axes.set_ylabel("Occupation probability", fontsize=16)
axes.set_xlabel("coupling strenght", fontsize=16)
axes.legend(loc=0)

fig.tight_layout()
```

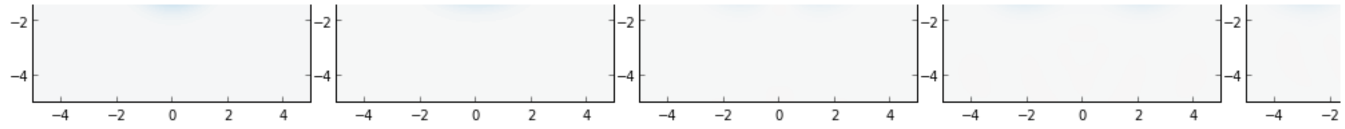


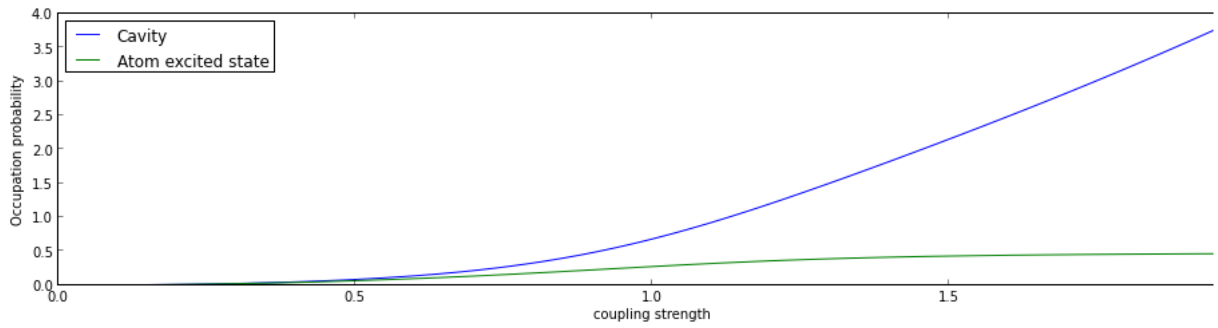
Plot the wigner functions of the cavity as a function of coupling strength

In [8]:
```
g_idx = where([g_vec == 2*pi*g for g in [0.0, 0.5, 1.0, 1.5, 2.0]])[1]
psi_sublist = array(psi_list)[g_idx]

xvec = linspace(-5,5,200)

fig_grid = (2, len(psi_sublist)*2)
fig = figure(figsize=(4*len(psi_sublist),8))

for idx, psi in enumerate(psi_sublist):
    rho_cavity = ptrace(psi, 0)
    W = wigner(rho_cavity, xvec, xvec)
    ax = subplot2grid(fig_grid, (0, 2*idx), colspan=2)
    ax.contourf(xvec, xvec, W, 100, norm=mpl.colors.Normalize(-.25,.25), cmap=get_cmap('RdBu'))
    ax.set_title(r"$g = %.1f$" % (g_vec[g_idx][idx]/(2*pi)), fontsize=16)

# plot the cavity occupation probability in the ground state
ax = subplot2grid(fig_grid, (1, 1), colspan=(fig_grid[1]-2))
ax.plot(g_vec/(2*pi), nc_expt, label="Cavity")
ax.plot(g_vec/(2*pi), na_expt, label="Atom excited state")
ax.legend(loc=0)
ax.set_xlabel('coupling strength')
ax.set_ylabel('Occupation probability')
```

Out [8]:
```
<matplotlib.text.Text at 0x43147d0>
```





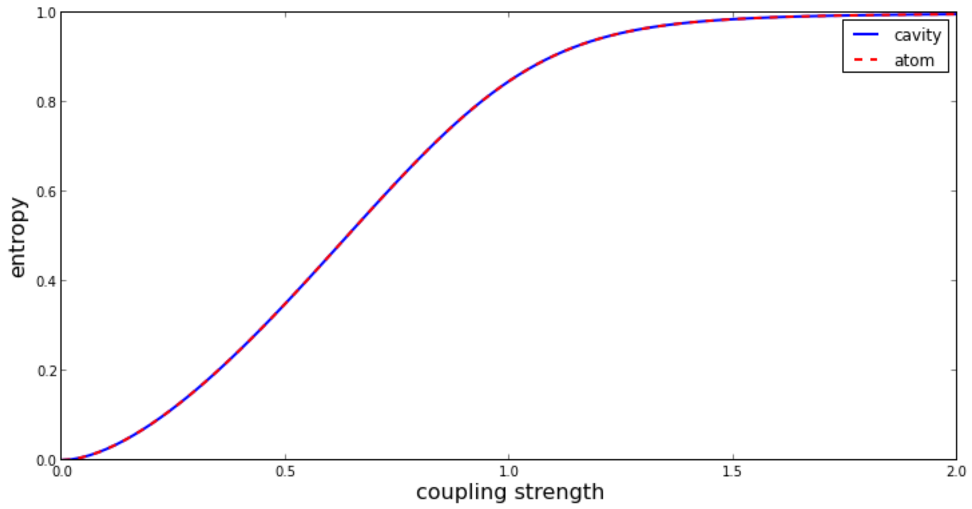


Entropy of atom/cavity as a measure of entanglement

In [9]:
```
entropy_cavity = zeros(shape(g_vec))
entropy_atom   = zeros(shape(g_vec))

for idx, psi in enumerate(psi_list):

    rho_cavity = ptrace(psi, 0)
    entropy_cavity[idx] = entropy_vn(rho_cavity, 2)

    rho_atom = ptrace(psi, 1)
    entropy_atom[idx]   = entropy_vn(rho_atom, 2)
```

In [10]:
```
fig, axes = subplots(1, 1, figsize=(12,6))
axes.plot(g_vec/(2*pi), entropy_cavity, 'b', label="cavity", linewidth=2)
axes.plot(g_vec/(2*pi), entropy_atom, 'r--', label="atom", linewidth=2)
axes.set_ylim(0,1)
axes.set_ylabel("entropy", fontsize=16)
axes.set_xlabel("coupling strength", fontsize=16)
axes.legend(loc=0)
```

Out [10]:
```
<matplotlib.legend.Legend at 0x5073690>
```



Dynamics of an initially excited cavity

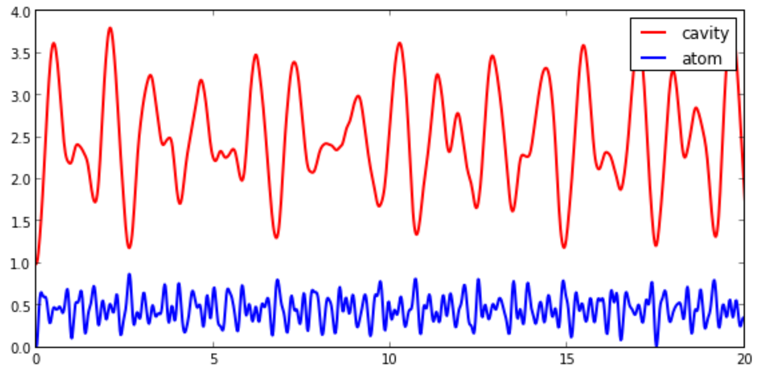
In [11]:
```
H = H0 + 1.0 * 2 * pi * H1

psi0 = tensor(basis(N,1), basis(2,0))
```

In [12]:
```
tlist = linspace(0, 20, 1000)
output = mesolve(H, psi0, tlist, [], [a.dag() * a, sm.dag() * sm])
```

In [13]:
```
fig, axes = subplots(1, 1, sharex=True, figsize=(8,4))
```

```
axes.plot(tlist, output.expect[0], 'r', linewidth=2, label="cavity")
axes.plot(tlist, output.expect[1], 'b', linewidth=2, label="atom")
axes.legend(loc=0)

fig.tight_layout()
```

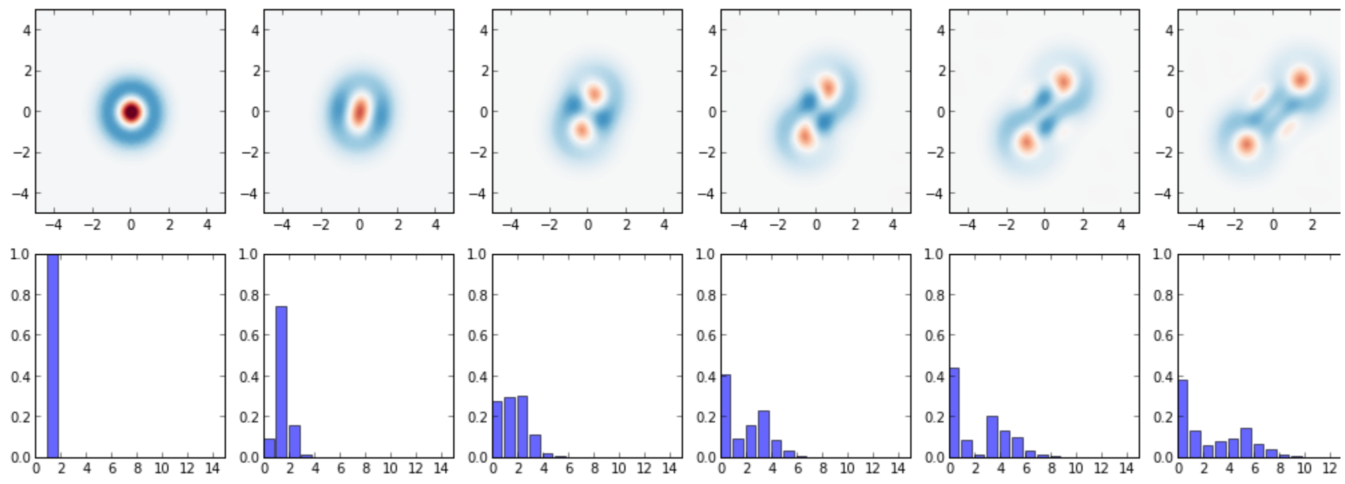
```
/usr/local/lib/python2.7/dist-packages/numpy/core/numeric.py:334: ComplexWarning: Casting complex values to real discards the
part
  maskna=maskna, ownmaskna=ownmaskna)
```



Fock-state distribution and Wigner function for the cavity as a function of time

In [14]:
```
tlist = linspace(0, 0.35, 8)
output = mesolve(H, psi0, tlist, [], [])
```

In [15]:
```
rho_ss_sublist = output.states #[::4]

xvec = linspace(-5,5,200)

fig, axes = subplots(2, len(rho_ss_sublist), figsize=(3*len(rho_ss_sublist), 6))

for idx, rho_ss in enumerate(rho_ss_sublist):

    # trace out the cavity density matrix
    rho_ss_cavity = ptrace(rho_ss, 0)

    # calculate its wigner function
    W = wigner(rho_ss_cavity, xvec, xvec)

    # plot its wigner function
    axes[0,idx].contourf(xvec, xvec, W, 100, norm=mpl.colors.Normalize(-.25,.25), cmap=get_cmap('RdBu'))

    # plot its fock-state distribution
    axes[1,idx].bar(arange(0, N), real(rho_ss_cavity.diag()), color="blue", alpha=0.6)
    axes[1,idx].set_ylim(0, 1)
    axes[1,idx].set_xlim(0, N)
```



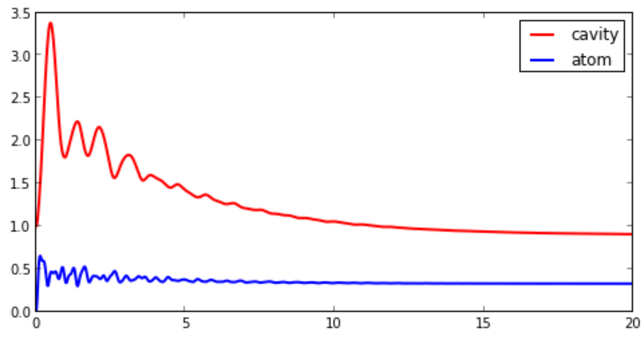
Same thing with a little bit of dissipation

In [16]:
```
kappa = 0.25
```

In [17]:
```
tlist = linspace(0, 20, 1000)
output = mesolve(H, psi0, tlist, [sqrt(kappa) * a], [a.dag() * a, sm.dag() * sm])
```

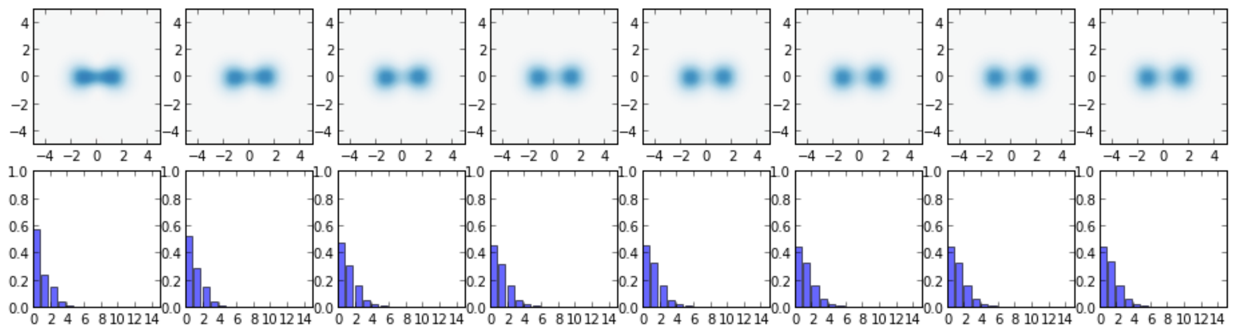
In [18]:
```
fig, axes = subplots(1, 1, sharex=True, figsize=(8,4))
axes.plot(tlist, output.expect[0], 'r', linewidth=2, label="cavity")
axes.plot(tlist, output.expect[1], 'b', linewidth=2, label="atom")
axes.legend(loc=0)
```

Out [18]:
```
<matplotlib.legend.Legend at 0x78ef650>
```



In [26]:
```
tlist = linspace(0, 10, 8)
output = mesolve(H, psi0, tlist, [sqrt(kappa) * a], [])
```

In [33]:
```
xvec = linspace(-5,5,200)

fig, axes = subplots(2, len(output.states), figsize=(2*len(output.states), 4))

for idx, rho_ss in enumerate(output.states):

    # trace out the cavity density matrix
    rho_ss_cavity = ptrace(rho_ss, 0)

    # calculate its wigner function
    W = wigner(rho_ss_cavity, xvec, xvec)

    # plot its wigner function
    axes[0,idx].contourf(xvec, xvec, W, 100, norm=mpl.colors.Normalize(-.25,.25), cmap=get_cmap('RdBu'))

    # plot its fock-state distribution
    axes[1,idx].bar(arange(0, N), real(rho_ss_cavity.diag()), color="blue", alpha=0.6)
    axes[1,idx].set_ylim(0, 1)
    axes[1,idx].set_xlim(0, N)
```



Entropy as a function of time in presence of dissipation and starting in the ideal ground state

In [21]:
```
tlist = linspace(0, 30, 50)

psi0 = H.groundstate()[1]

output = mesolve(H, psi0, tlist, [sqrt(kappa) * a], [])
```

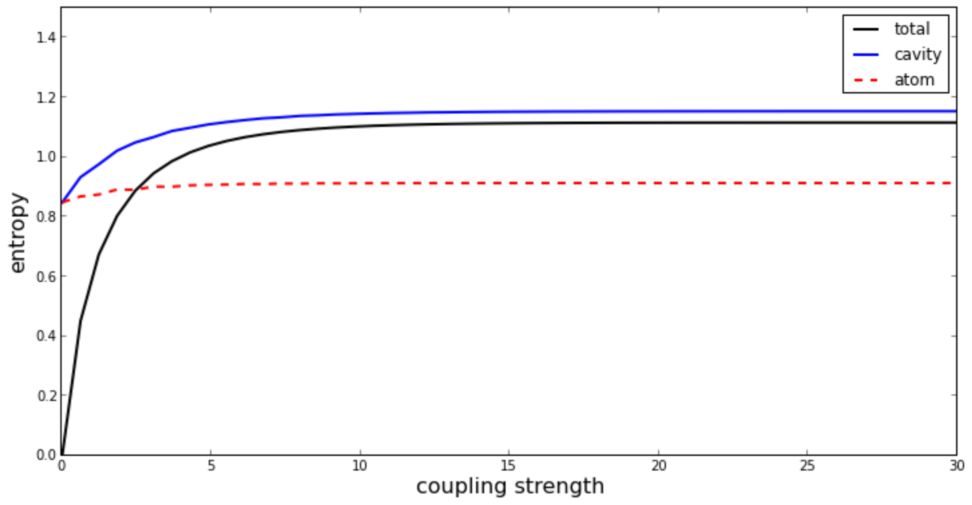
In [22]:
```
entropy_tot    = zeros(shape(tlist))
entropy_cavity = zeros(shape(tlist))
entropy_atom   = zeros(shape(tlist))

for idx, rho in enumerate(output.states):

    entropy_tot[idx] = entropy_vn(rho, 2)
    rho_cavity = ptrace(rho, 0)
    entropy_cavity[idx] = entropy_vn(rho_cavity, 2)

    rho_atom = ptrace(rho, 1)
    entropy_atom[idx]   = entropy_vn(rho_atom, 2)
```

In [25]:

```
fig, axes = subplots(1, 1, figsize=(12,6))
axes.plot(tlist, entropy_tot, 'k', label="total", linewidth=2)
axes.plot(tlist, entropy_cavity, 'b', label="cavity", linewidth=2)
axes.plot(tlist, entropy_atom, 'r--', label="atom", linewidth=2)
axes.set_ylabel("entropy", fontsize=16)
axes.set_xlabel("coupling strength", fontsize=16)
axes.set_ylim(0, 1.5)
axes.legend(loc=0)
```

Out [25]:

```
<matplotlib.legend.Legend at 0x15772110>
```


total
cavity
atom
entropy
coupling strength