Lecture 7
Vanishing gradient
Advanced RNN layers
CNNs for texts

Vladislav Goncharenko

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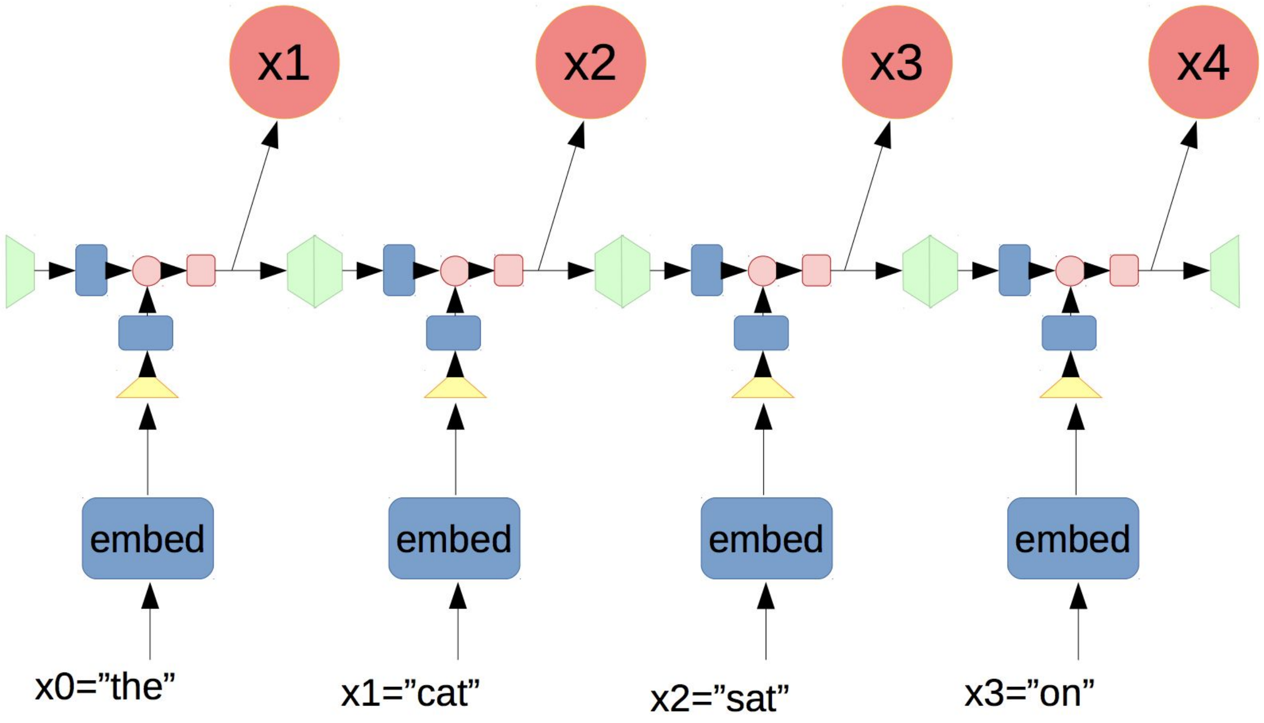
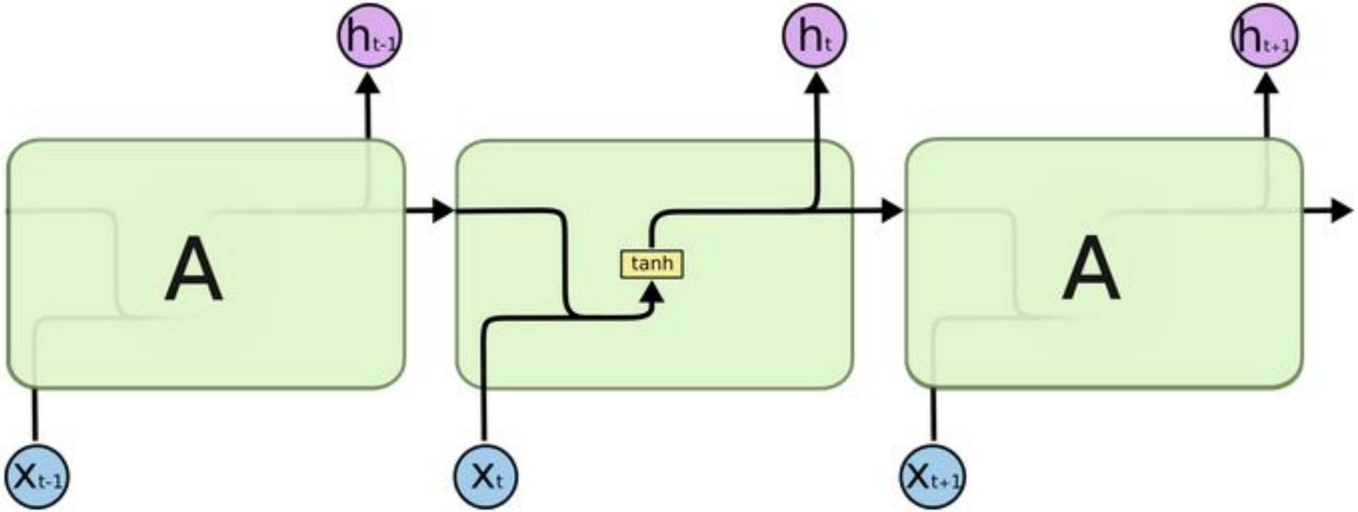
Outline

- Simple RNN recap
- Complex RNN:
    - Vanishing gradient
    - Exploding gradient
    - LSTM/GRU
    - Gradient clipping
    - Skip connections
    - Residual networks as ensembles
- CNNs for text

Recap: RNN

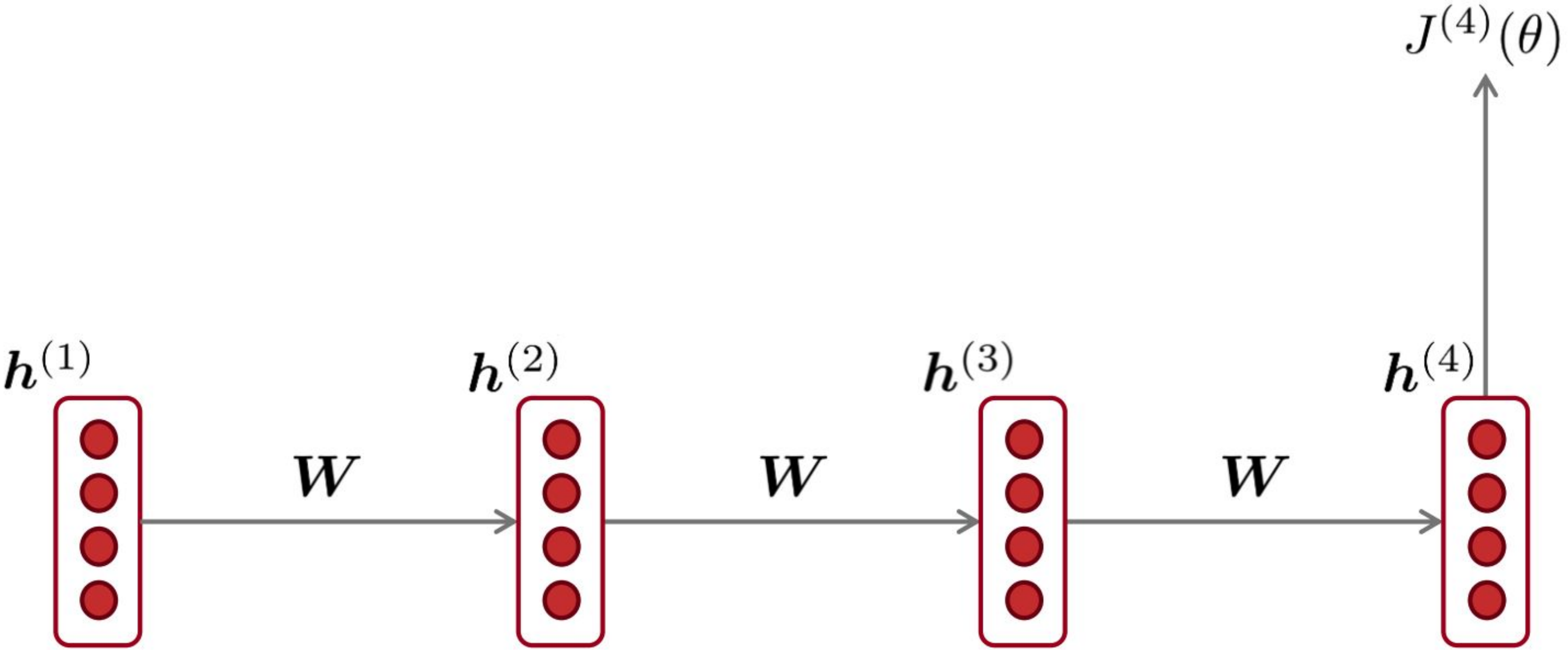


Recap: Vanilla RNN



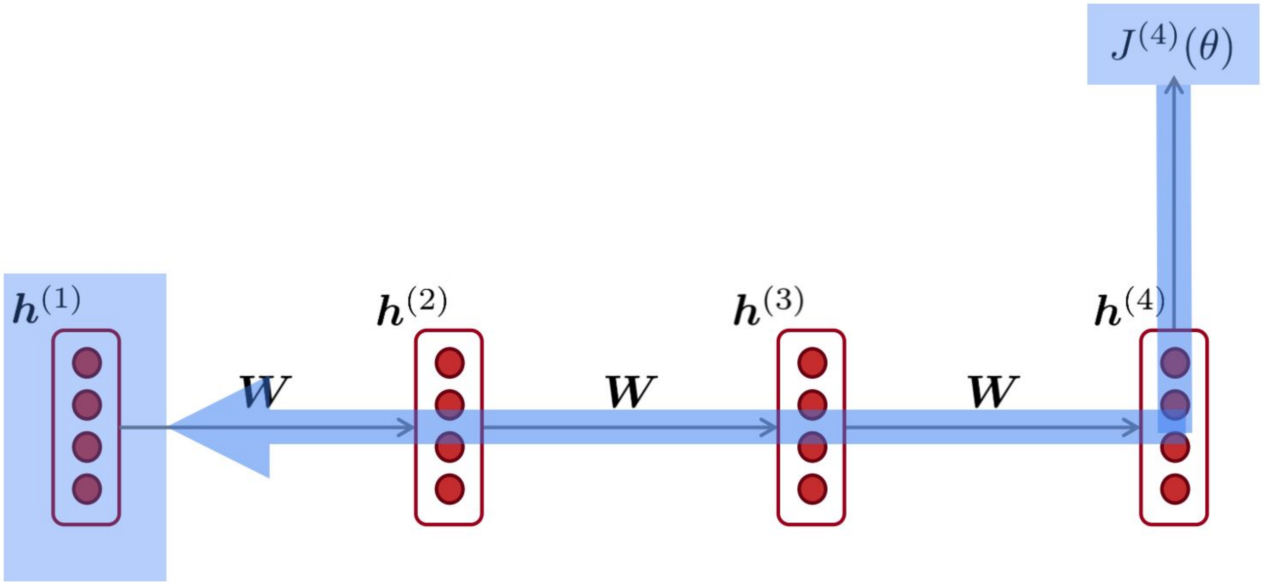
Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

Vanishing gradient problem



Based on: Lecture by Abigail See, [CS224n Lecture 7]

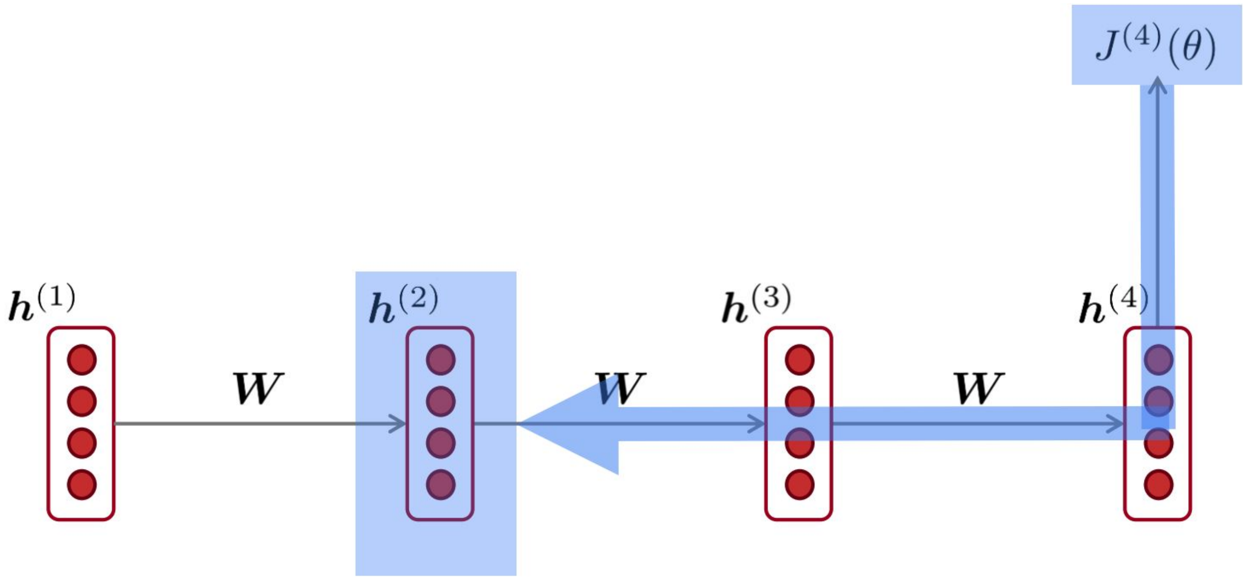
Vanishing gradient problem



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \ ?$$

Based on: Lecture by Abigail See, [CS224n Lecture 7]

Vanishing gradient problem

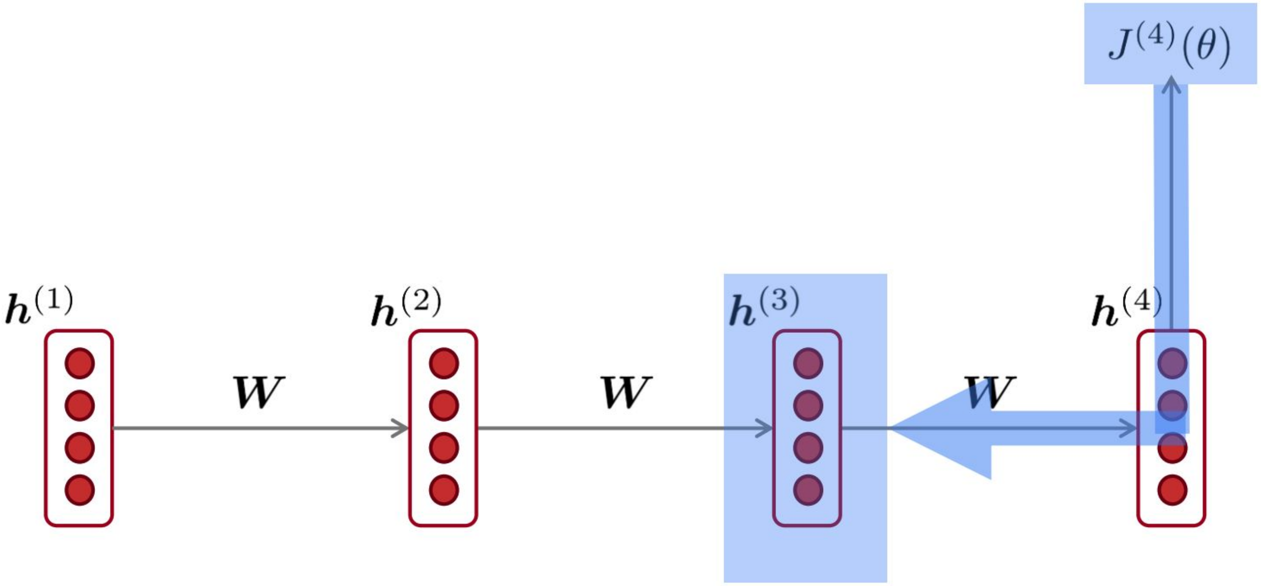


$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \frac{\partial \boldsymbol{h}^{(2)}}{\partial \boldsymbol{h}^{(1)}} \times \frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(2)}}$$

chain rule!

Based on: Lecture by Abigail See, [CS224n Lecture 7]

Vanishing gradient problem



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \frac{\partial \boldsymbol{h}^{(2)}}{\partial \boldsymbol{h}^{(1)}} \times \qquad \frac{\partial \boldsymbol{h}^{(3)}}{\partial \boldsymbol{h}^{(2)}} \times \frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(3)}}$$

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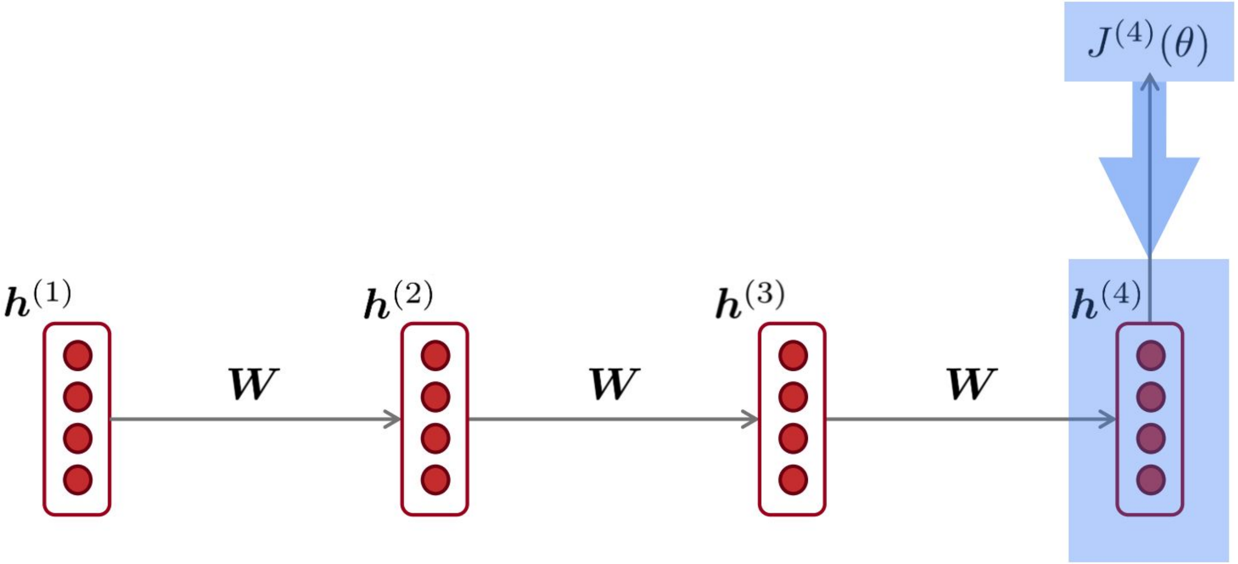

$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \frac{\partial \boldsymbol{h}^{(2)}}{\partial \boldsymbol{h}^{(1)}} \times \frac{\partial \boldsymbol{h}^{(3)}}{\partial \boldsymbol{h}^{(2)}} \times \frac{\partial \boldsymbol{h}^{(4)}}{\partial \boldsymbol{h}^{(3)}} \times \frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(4)}}$$

chain rule!

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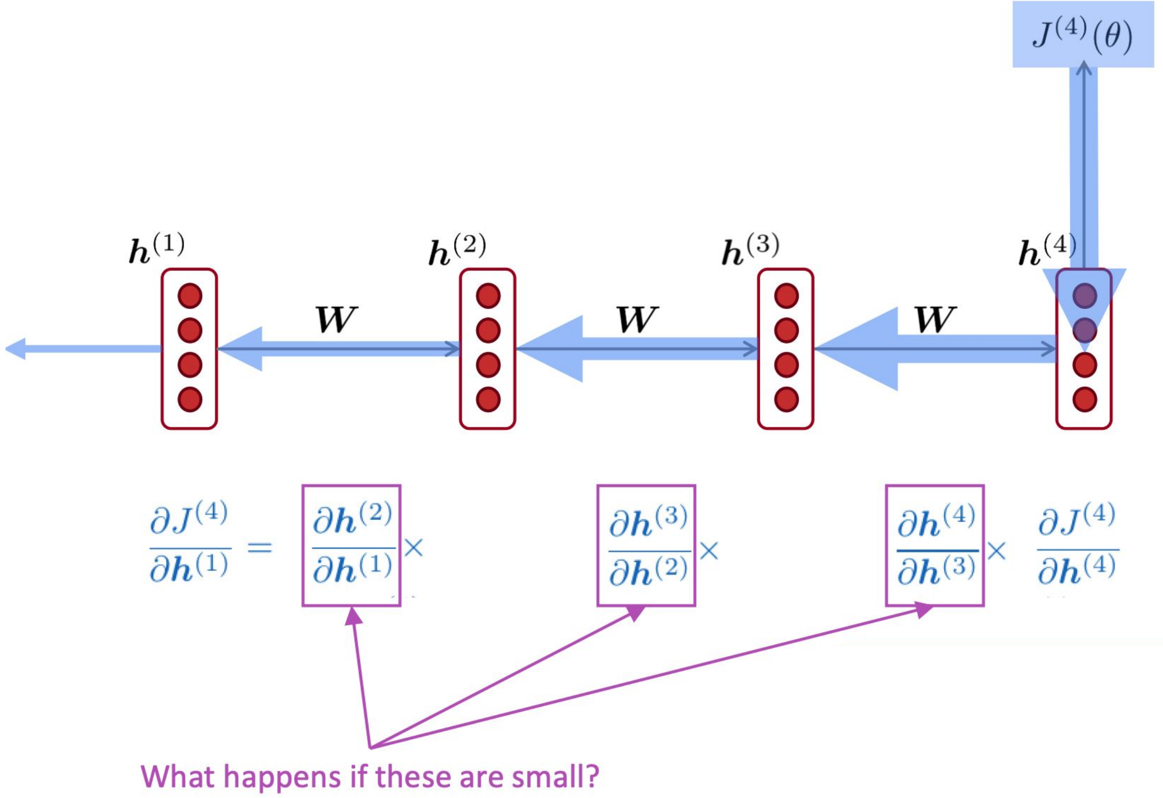
Vanishing gradient problem

Vanishing gradient problem:

When the derivatives are small, the gradient signal gets smaller and smaller as it backpropagates further

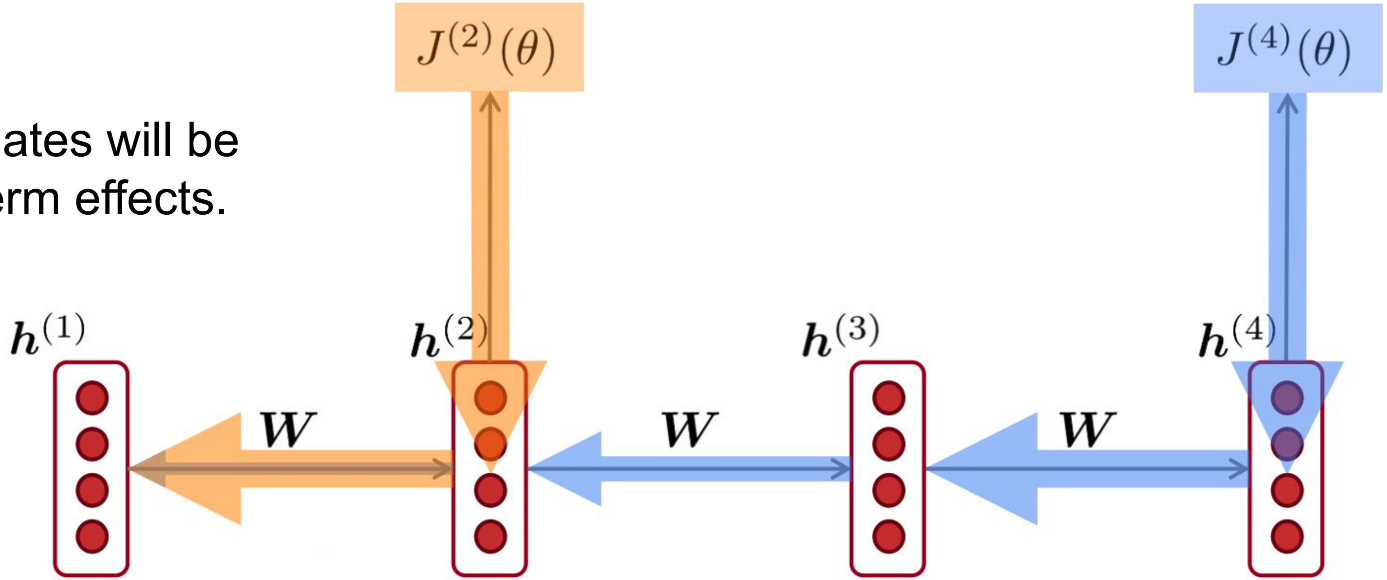


More info: "On the difficulty of training recurrent neural networks", Pascanu et al, 2013
http://proceedings.mlr.press/v28/pascanu13.pdf

Vanishing gradient problem

Gradient signal from far away is lost because it's much smaller than from close-by.

So model weights updates will be based only on short-term effects.



Based on: Lecture by Abigail See, [CS224n Lecture 7]

Exploding gradient problem

- If the gradient becomes too big, then the SGD update step becomes too big:

$$\theta^{new} = \theta^{old} - \overbrace{\alpha}^{\text{learning rate}} \underbrace{\nabla_\theta J(\theta)}_{\text{gradient}}$$

- This can cause bad updates: we take too large a step and reach a bad parameter configuration (with large loss)

- In the worst case, this will result in Inf or NaN in your network (then you have to restart training from an earlier checkpoint)

Based on: Lecture by Abigail See, CS224n Lecture 7

Exploding gradient solution

- Gradient clipping: if the norm of the gradient is greater than some threshold, scale it down before applying SGD update

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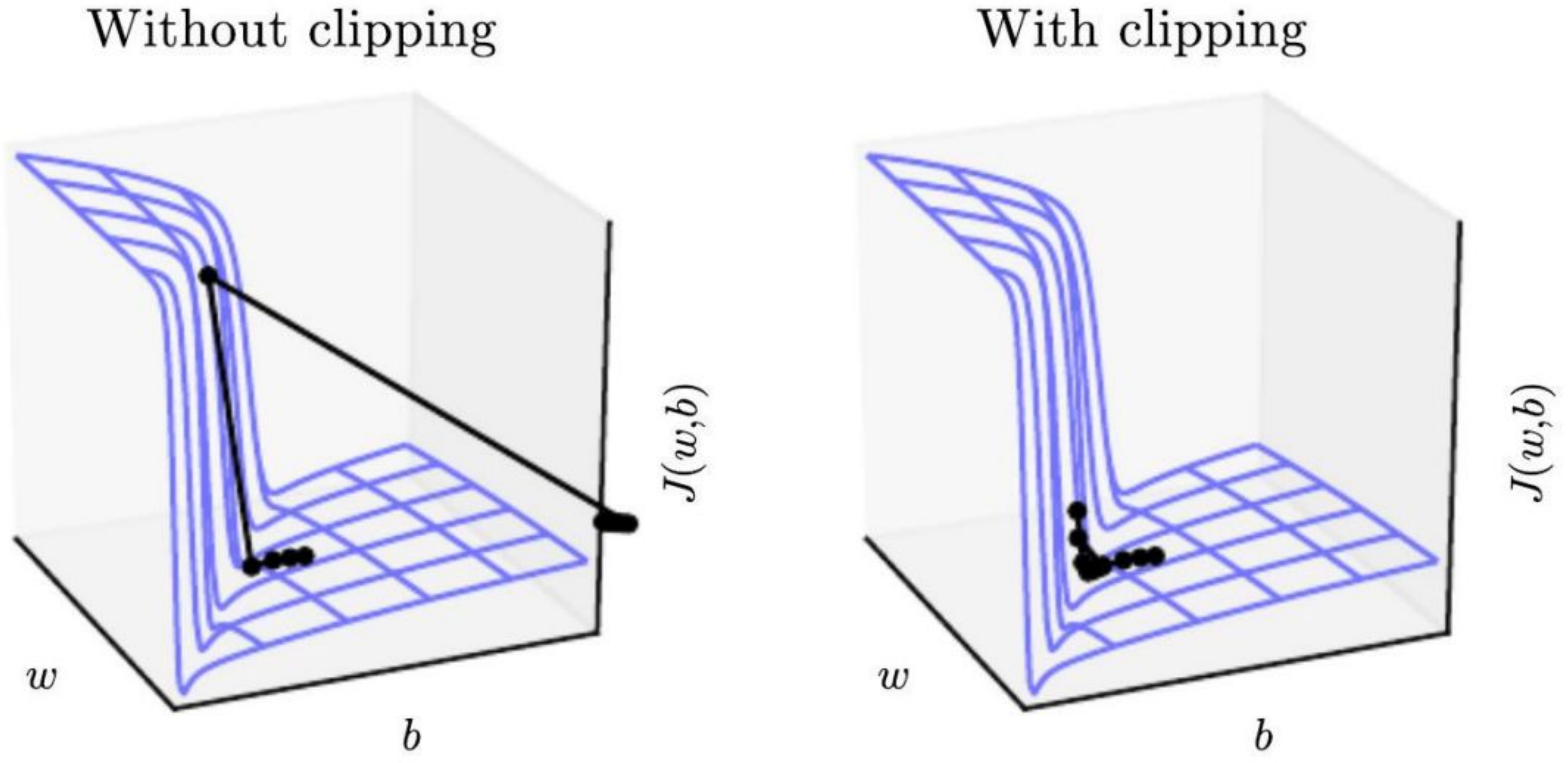
Algorithm 1 Pseudo-code for norm clipping

$\hat{\mathbf{g}} \leftarrow \frac{\partial \mathcal{E}}{\partial \theta}$
if $\|\hat{\mathbf{g}}\| \geq threshold$ **then**
  $\hat{\mathbf{g}} \leftarrow \frac{threshold}{\|\hat{\mathbf{g}}\|} \hat{\mathbf{g}}$
end if

- Intuition: take a step in the same direction, but a smaller step

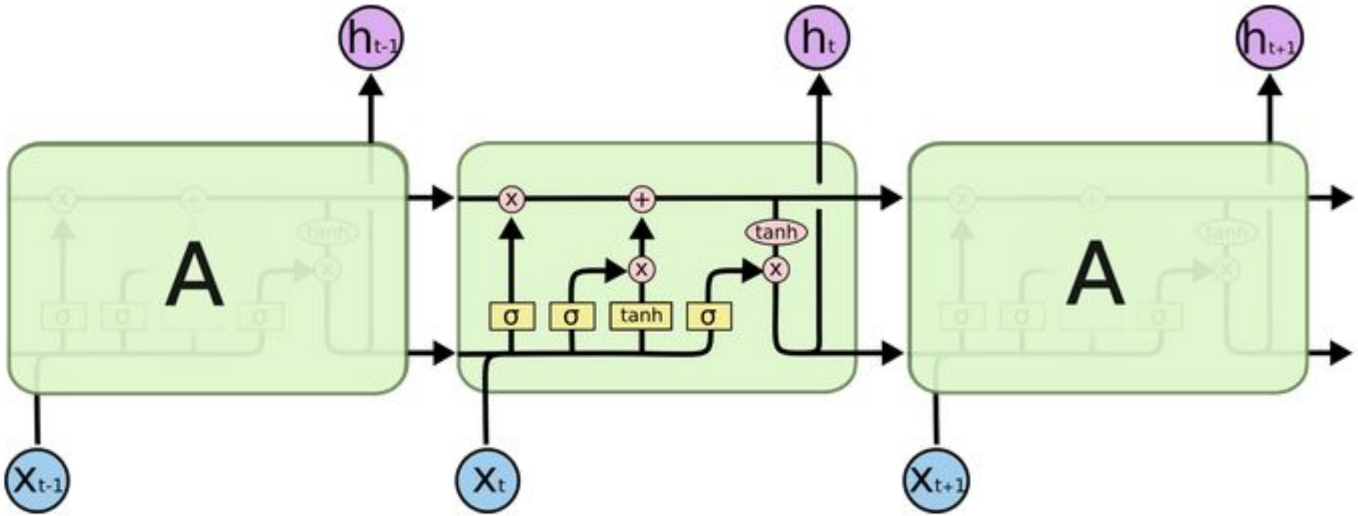
Based on: Lecture by Abigail See, [CS224n Lecture 7]

Exploding gradient solution



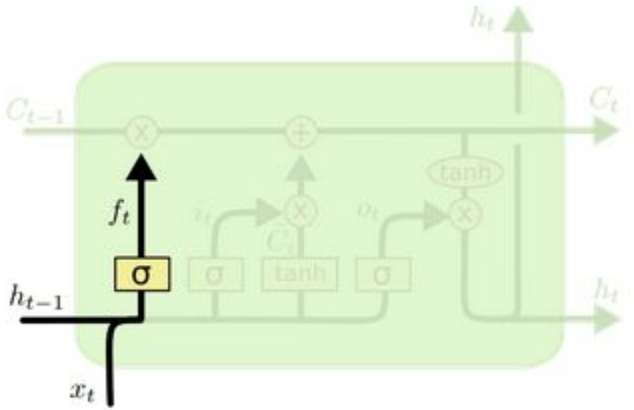
Based on: Lecture by Abigail See, [CS224n Lecture 7]

LSTM



Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

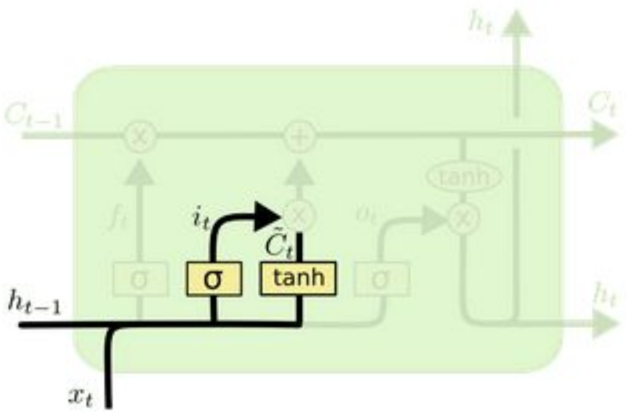
LSTM: quick overview



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$

Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTM: quick overview

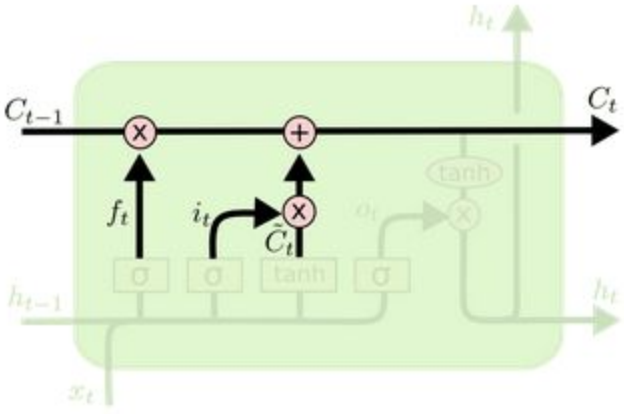


$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

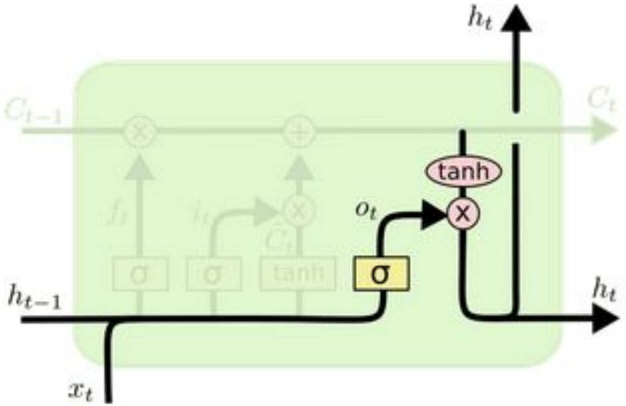
LSTM: quick overview



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTM: quick overview

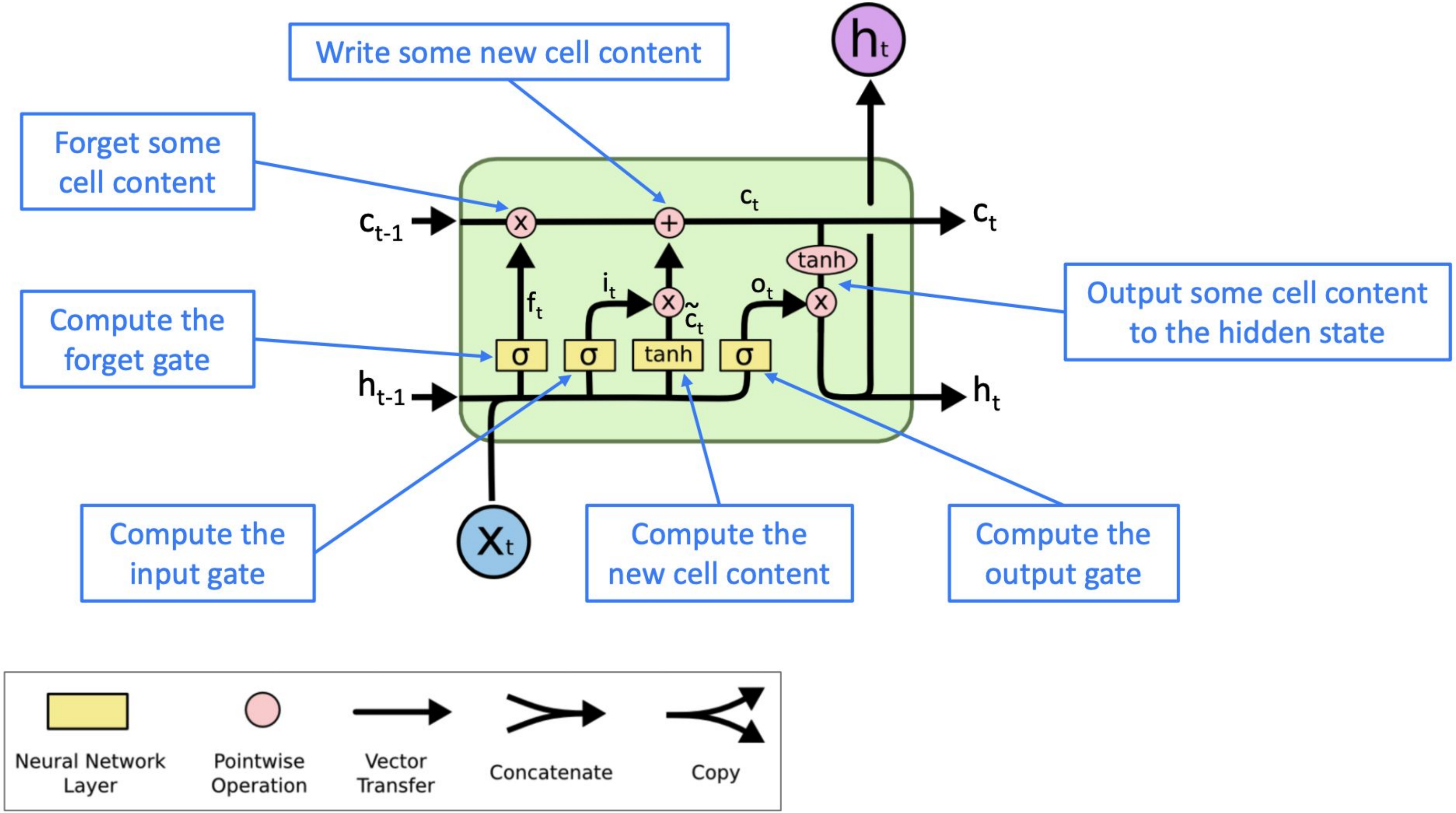


$$o_t = \sigma\left(W_o \left[h_{t-1}, x_t\right] + b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$

Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

Vanishing gradient: LSTM



Based on: Lecture by Abigail See, [CS224n Lecture 7]

Vanishing gradient: LSTM

Sigmoid function: all gate values are between 0 and 1

Forget gate: controls what is kept vs forgotten, from previous cell state

Input gate: controls what parts of the new cell content are written to cell

Output gate: controls what parts of cell are output to hidden state

$$\boldsymbol{f}^{(t)} = \sigma\left(\boldsymbol{W}_f \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_f \boldsymbol{x}^{(t)} + \boldsymbol{b}_f\right)$$

$$\boldsymbol{i}^{(t)} = \sigma\left(\boldsymbol{W}_i \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_i \boldsymbol{x}^{(t)} + \boldsymbol{b}_i\right)$$

$$\boldsymbol{o}^{(t)} = \sigma\left(\boldsymbol{W}_o \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_o \boldsymbol{x}^{(t)} + \boldsymbol{b}_o\right)$$

New cell content: this is the new content to be written to the cell

Cell state: erase ("forget") some content from last cell state, and write ("input") some new cell content

Hidden state: read ("output") some content from the cell

$$\tilde{\boldsymbol{c}}^{(t)} = \tanh\left(\boldsymbol{W}_c \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_c \boldsymbol{x}^{(t)} + \boldsymbol{b}_c\right)$$

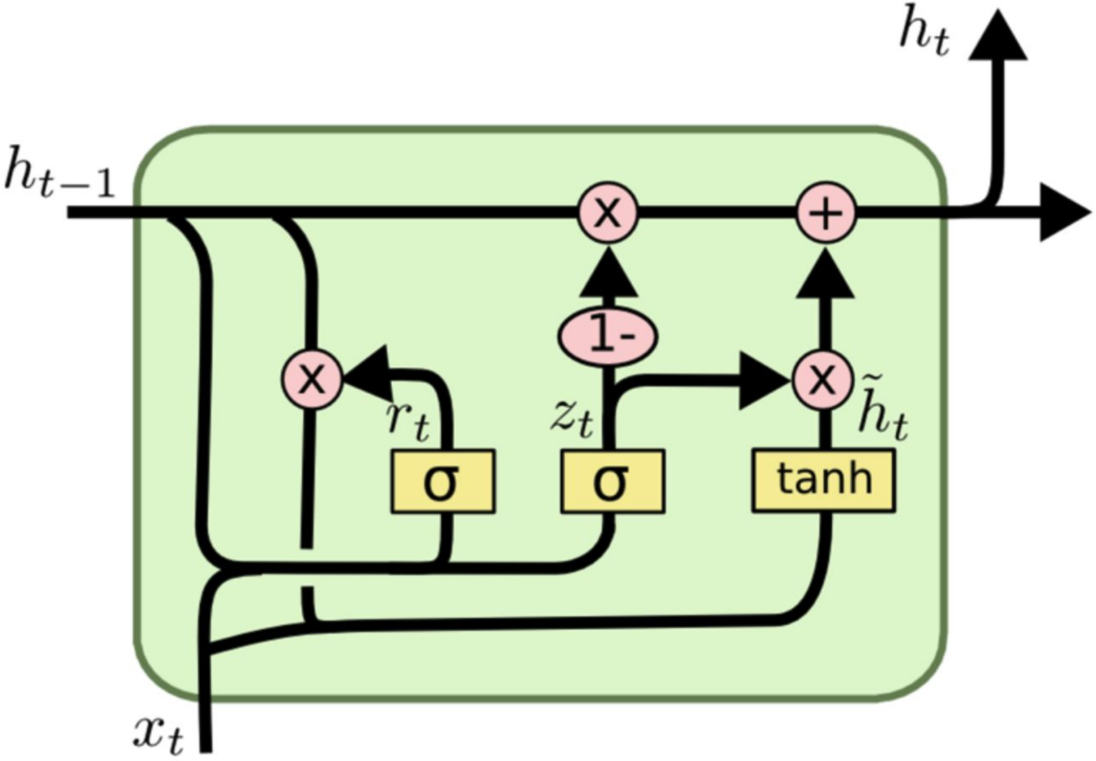
$$\boldsymbol{c}^{(t)} = \boldsymbol{f}^{(t)} \circ \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \circ \tilde{\boldsymbol{c}}^{(t)}$$

$$\boldsymbol{h}^{(t)} = \boldsymbol{o}^{(t)} \circ \tanh \boldsymbol{c}^{(t)}$$

All these are vectors of same length n

Gates are applied using element-wise product

Vanishing gradient: GRU



Vanishing gradient: GRU

Update gate: controls what parts of hidden state are updated vs preserved

Reset gate: controls what parts of previous hidden state are used to compute new content

$$\boldsymbol{u}^{(t)} = \sigma\left(\boldsymbol{W}_u \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_u \boldsymbol{x}^{(t)} + \boldsymbol{b}_u\right)$$

$$\boldsymbol{r}^{(t)} = \sigma\left(\boldsymbol{W}_r \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_r \boldsymbol{x}^{(t)} + \boldsymbol{b}_r\right)$$

New hidden state content: reset gate selects useful parts of prev hidden state. Use this and current input to compute new hidden content.

Hidden state: update gate simultaneously controls what is kept from previous hidden state, and what is updated to new hidden state content

$$\tilde{\boldsymbol{h}}^{(t)} = \tanh\left(\boldsymbol{W}_h(\boldsymbol{r}^{(t)} \circ \boldsymbol{h}^{(t-1)}) + \boldsymbol{U}_h \boldsymbol{x}^{(t)} + \boldsymbol{b}_h\right)$$

$$\boldsymbol{h}^{(t)} = (1 - \boldsymbol{u}^{(t)}) \circ \boldsymbol{h}^{(t-1)} + \boldsymbol{u}^{(t)} \circ \tilde{\boldsymbol{h}}^{(t)}$$

How does this solve vanishing gradient?
Like LSTM, GRU makes it easier to retain info long-term (e.g. by setting update gate to 0)

Vanishing gradient: LSTM vs GRU

- LSTM and GRU are both great
  - GRU is quicker to compute and has fewer parameters than LSTM
  - There is no conclusive evidence that one consistently performs better than the other
  - LSTM is a good default choice (especially if your data has particularly long dependencies, or you have lots of training data)

Rule of thumb: start with LSTM, but switch to GRU if you want something more efficient

Vanishing gradient in non-RNN

Vanishing gradient is present in **all** deep neural network architectures.

- Due to chain rule / choice of nonlinearity function, gradient can become vanishingly small during backpropagation
- Lower levels are hard to train and are trained slower
- **Potential solution**: direct (or skip-) connections (just like in ResNet)

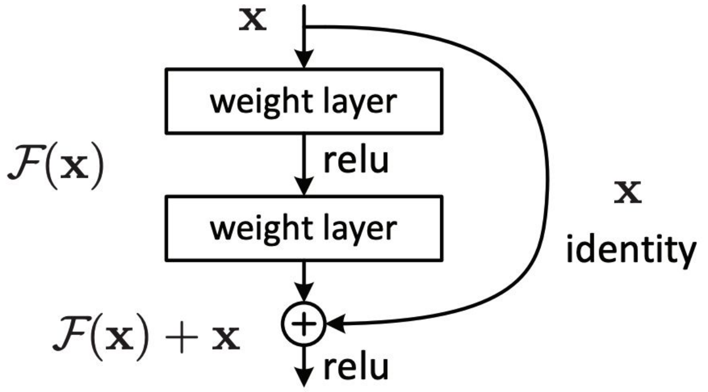


Figure 2. Residual learning: a building block.

Source: "Deep Residual Learning for Image Recognition", He et al, 2015. https://arxiv.org/pdf/1512.03385.pdf

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Vanishing gradient is present in **all** deep neural network architectures.

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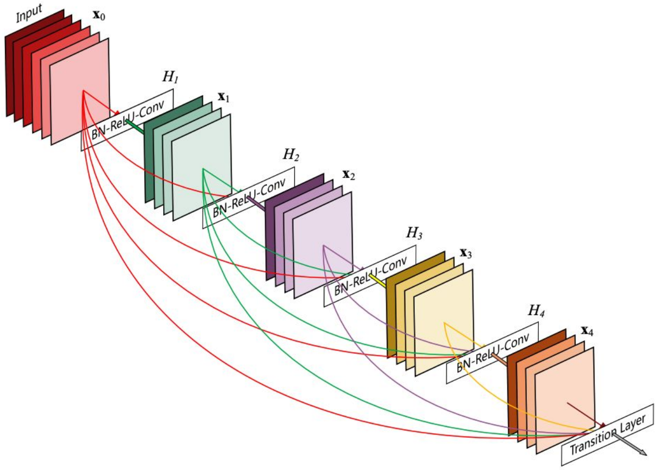
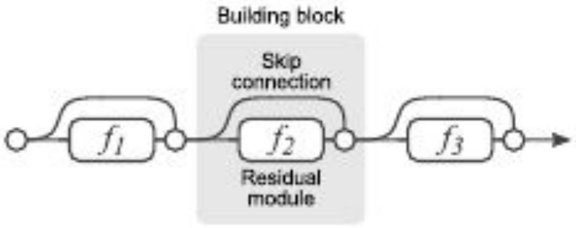
- **Potential solution**: dense connections (just like in DenseNet)



Source: "Densely Connected Convolutional Networks", Huang et al, 2017 https://arxiv.org/pdf/1608.06993.pdf

Another view on ResNets and vanishing gradient

"**Residual Networks Behave Like Ensembles of Relatively Shallow Networks**"



(a) Conventional 3-block residual network

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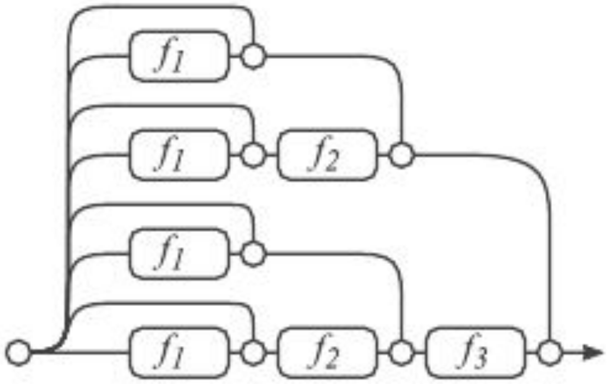

(b) Unraveled view of (a)

Source: https://arxiv.org/pdf/1605.06431.pdf

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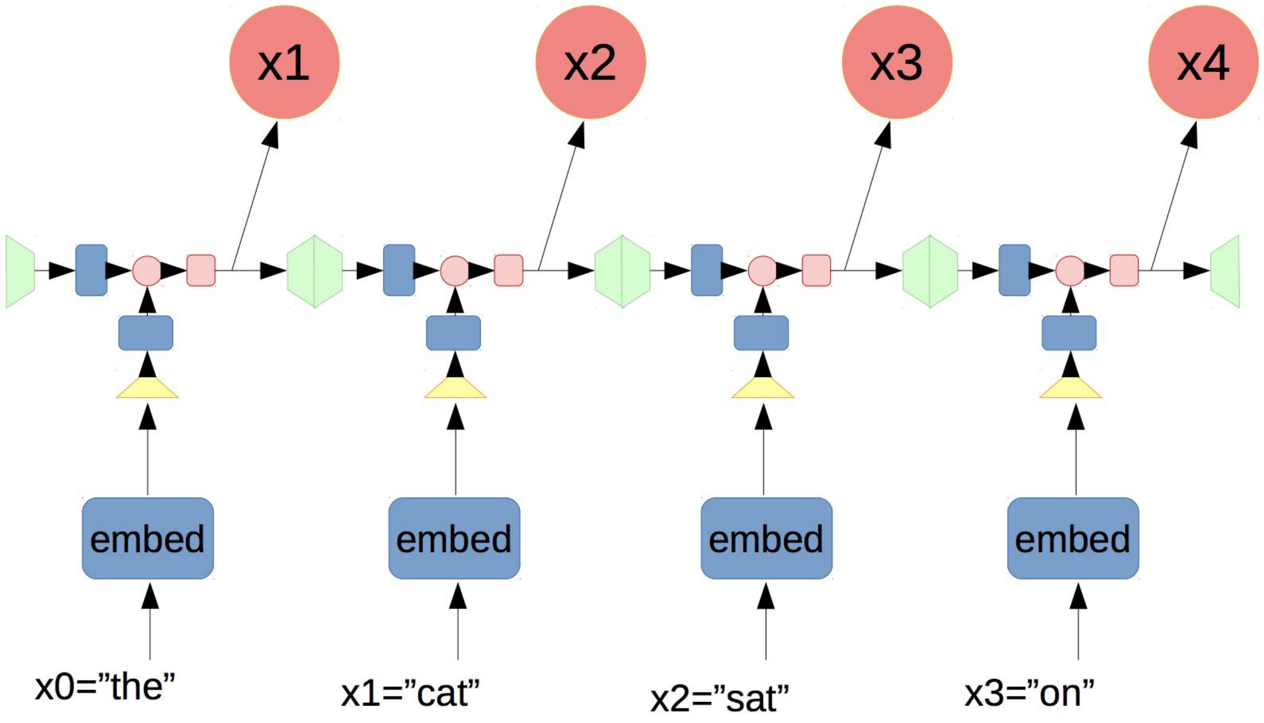
- Due to chain rule / choice of nonlinearity function, gradient can become vanishingly small during backpropagation
- Lower levels are hard to train and are trained slower
- **Potential solution(but not actually for that problem)**: dense connections (just like in DenseNet)

Conclusion:

Though vanishing/exploding gradients are a general problem, RNNs are particularly unstable due to the repeated multiplication by the same weight matrix [Bengio et al, 1994]. Gradients magnitude drops exponentially with connection length.

Source: ”Learning Long-Term Dependencies with Gradient Descent is Difficult", Bengio et al. 1994, http://ai.dinfo.unifi.it/paolo//ps/tnn-94-gradient.pdf

Recap: RNN



From RNN to CNN

- RNN: Get compositional vectors for grammatical phrases only

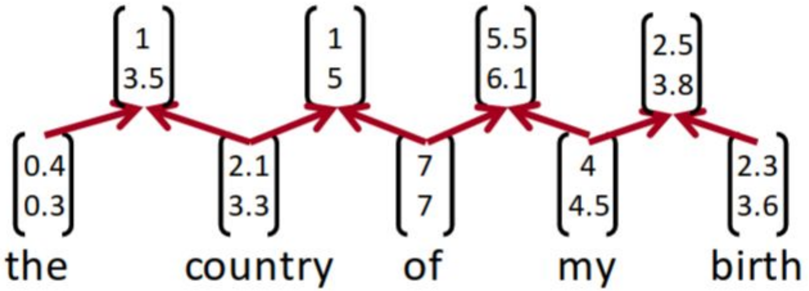
- CNN: What if we compute vectors for every possible phrase?
  - Example: *"the country of my birth"* computes vectors for:
    - *the country, country of, of my, my birth, the country of, country of my, of my birth, the country of my, country of my birth*

- Regardless of whether it is grammatical
- Wouldn't need parser
- Not very linguistically or cognitively plausible

Based on: Lecture by Richard Socher 5/12/16, http://cs224d.stanford.edu

From RNN to CNN

- Imagine using only bigrams



- Same operation as in RNN, but for every pair

$$p = \tanh\left(W \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + b\right)$$

- Can be interpreted as convolution over the word vectors

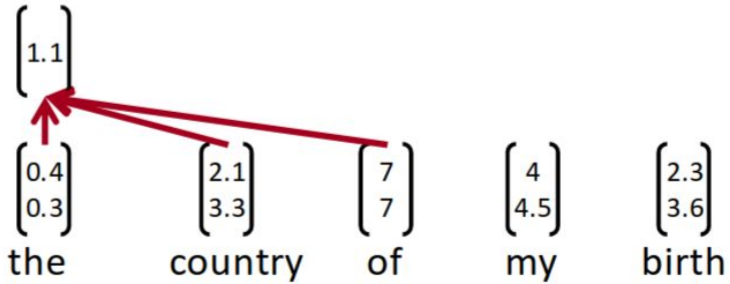
Based on: Lecture by Richard Socher 5/12/16, http://cs224d.stanford.edu

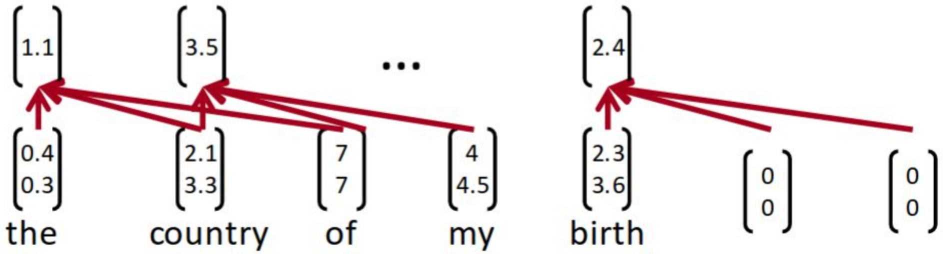
One layer CNN

- Simple convolution + pooling
- Window size may be different (2 or more)
- The feature map based on bigrams:

$$\mathbf{c} = [c_1, c_2, \ldots, c_{n-h+1}] \in \mathbb{R}^{n-h+1}$$

$$c_i = f(\mathbf{w}^T \mathbf{x}_{i:i+h-1} + b)$$





Based on: Lecture by Richard Socher 5/12/16, http://cs224d.stanford.edu

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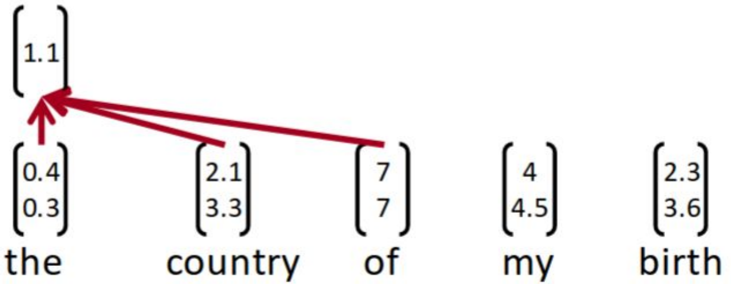
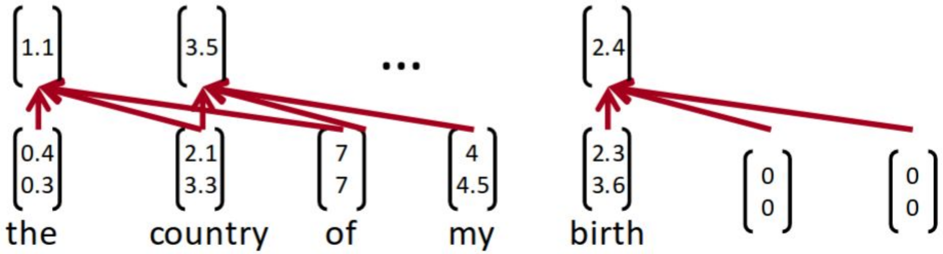
What's next?

$$c_i = f(\mathbf{w}^T \mathbf{x}_{i:i+h-1} + b)$$





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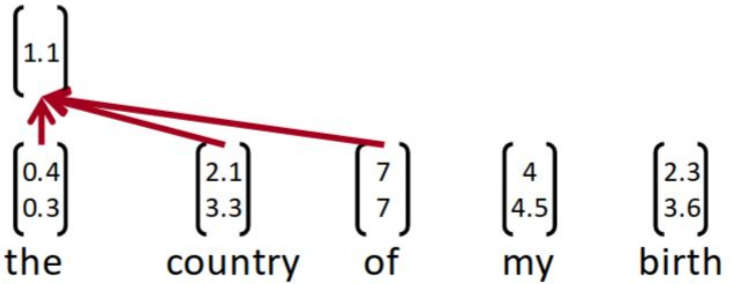
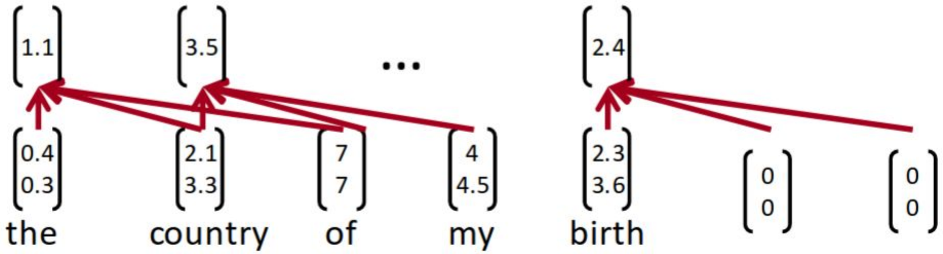


What's next?

We need more features!



Based on: Lecture by Richard Socher 5/12/16, http://cs224d.stanford.edu

One layer CNN

- Feature representation is based on some applied filter:

$$\mathbf{c} = [c_1, c_2, \ldots, c_{n-h+1}] \in \mathbb{R}^{n-h+1}$$

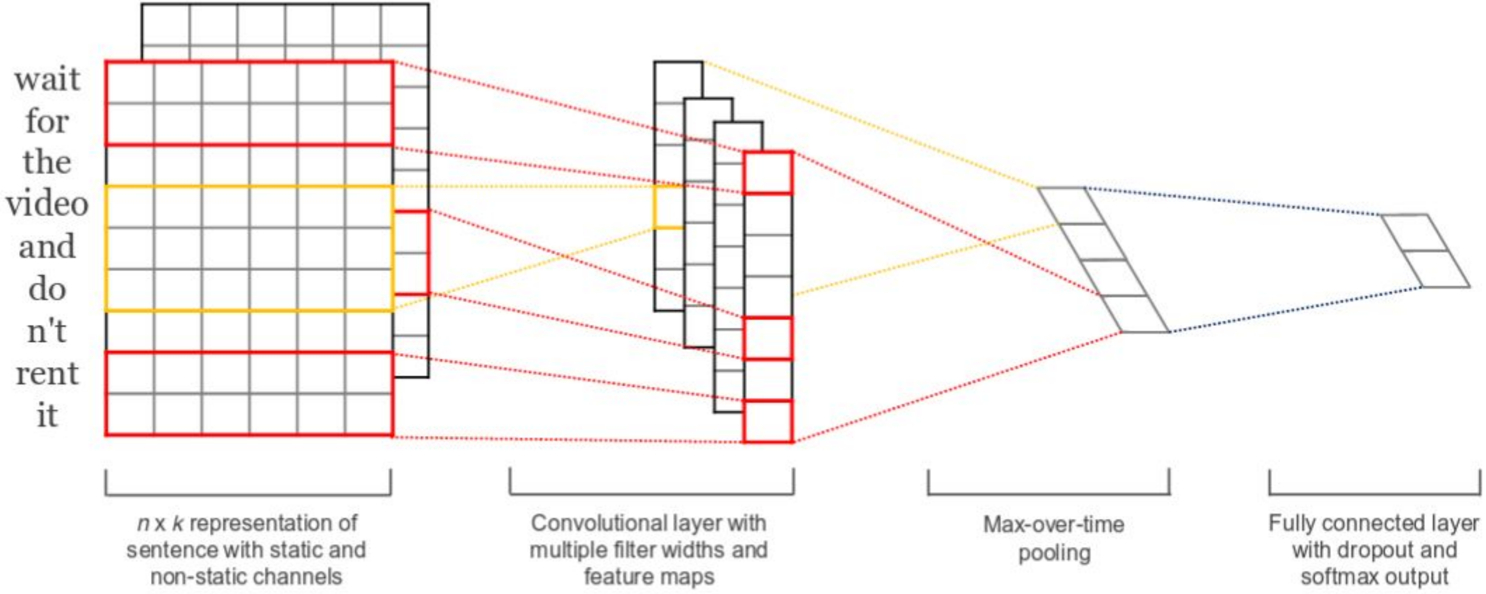
- Let's use pooling over the time axis: $\hat{c} = \max\{\mathbf{c}\}$

- Now the length of **c** is irrelevant!

- So we can use filters based on unigrams, bigrams, tri-grams, 4-grams, etc.

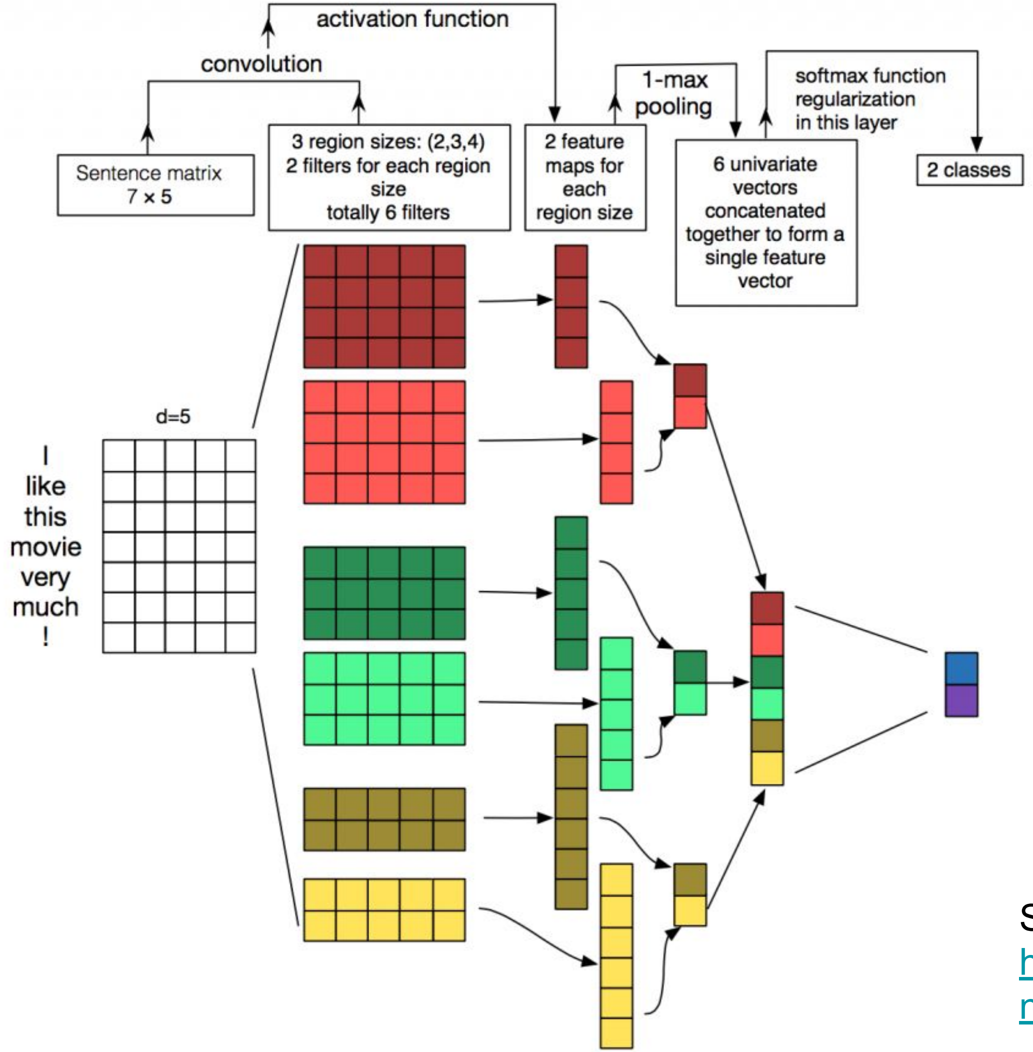
Based on: Lecture by Richard Socher 5/12/16, http://cs224d.stanford.edu

Another example from Kim (2014) paper



Based on: Lecture by Richard Socher 5/12/16, http://cs224d.stanford.edu

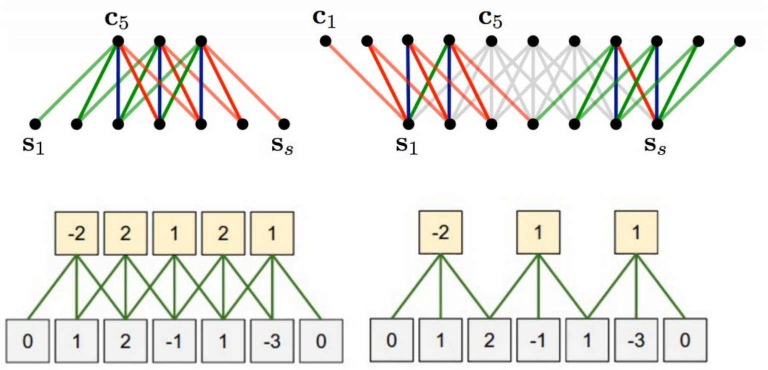
Example CNN structure



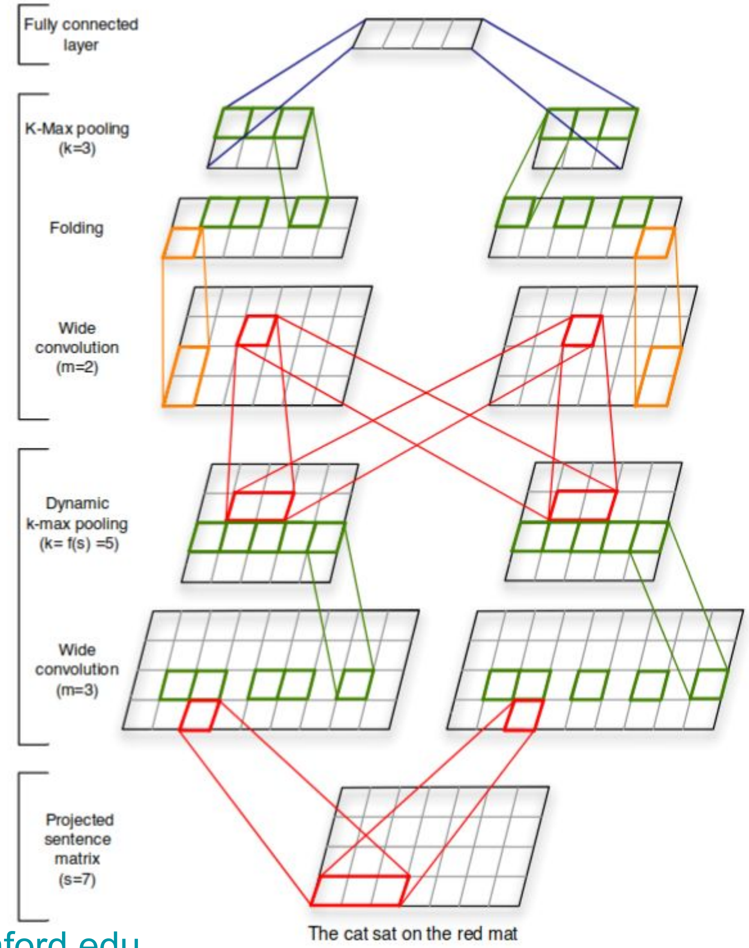
Source: http://www.wildml.com/2015/11/understanding-convolutional-neural-networks-for-nlp/

More about CNN

- Narrow vs wide convolution (stride and zero-padding)



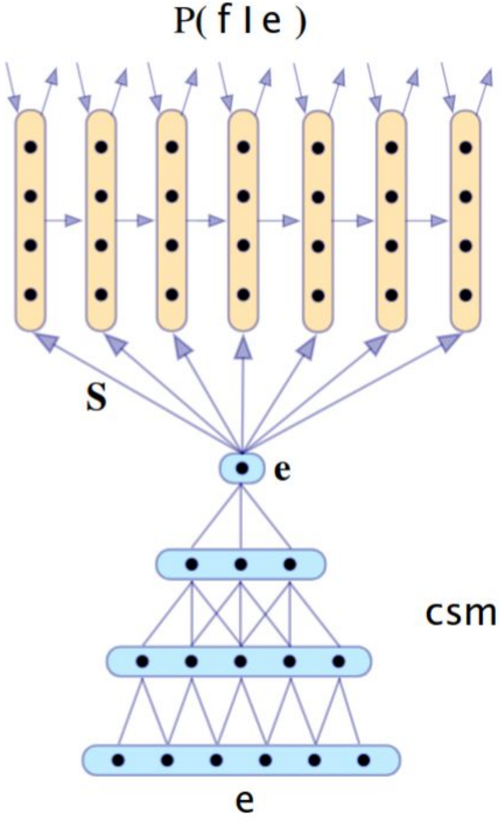
- Complex pooling schemes over sequences
- Great readings (e.g. Kalchbrenner et. al. 2014)



Based on: Lecture by Richard Socher 5/12/16, http://cs224d.stanford.edu

CNN applications

- Neural machine translation: CNN as encoder, RNN as decoder
- Kalchbrenner and Blunsom (2013) "Recurrent Continuous Translation Models"
- One of the first neural machine translation efforts



Based on: Lecture by Richard Socher 5/12/16, http://cs224d.stanford.edu

Outro and Q & A

- Vanishing gradient is present not only in RNNs
  - Use some kind of memory or skip-connections
- LSTM and GRU are both great
  - GRU is quicker, LSTM catch more complex dependencies
- Rule of thumb: start with LSTM, but switch to GRU if you want something more computationally efficient
- Clip your gradients
- Combining RNN and CNN worlds? Why not ;)

That's all. Feel free to ask any questions.