Lecture 6
Word vectors
Recurrent Neural Networks

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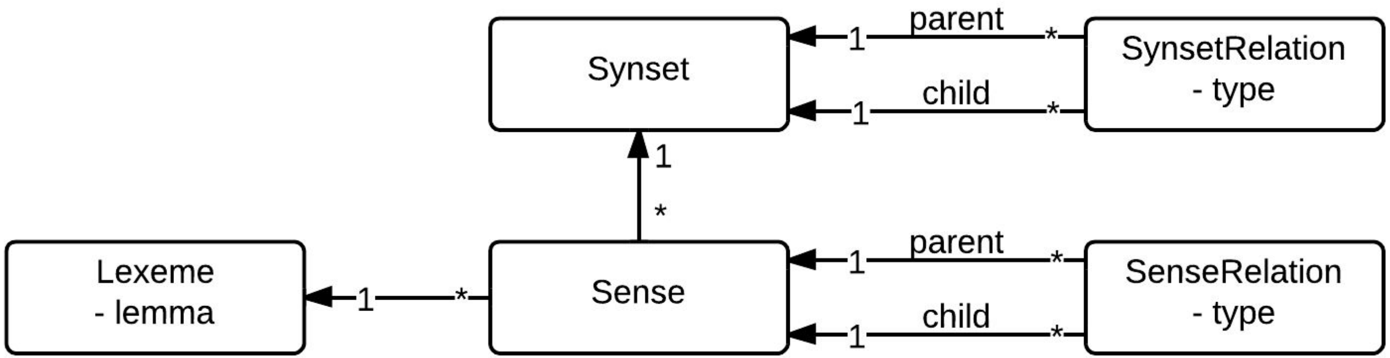
Outline

1. Discrete representations.
2. Matrix of co-occurrence.
3. Embeddings (GloVe, word2vec).
4. Examples.
5. Recurrent Neural Networks

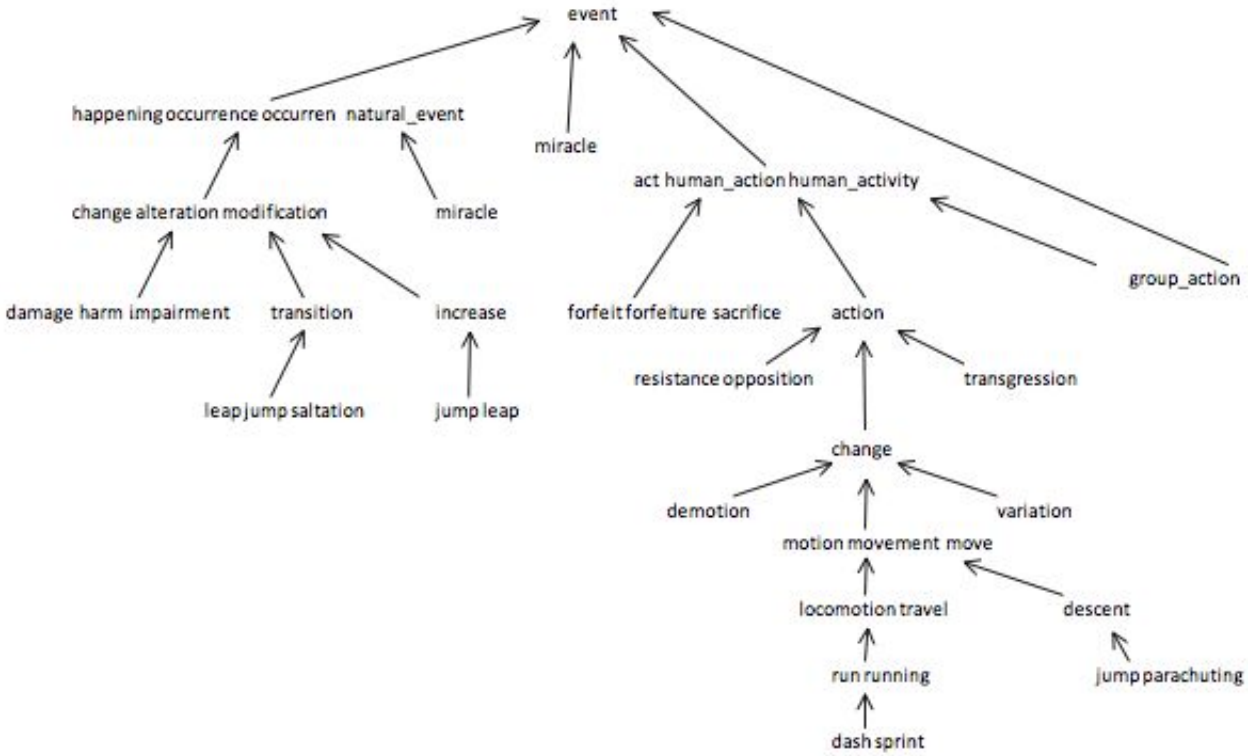
Source: [Stanford CS224d course materials]

How to represent text in a computer?

Use a taxonomy like WordNet that has hypernyms (is-a) relationships and synonym sets



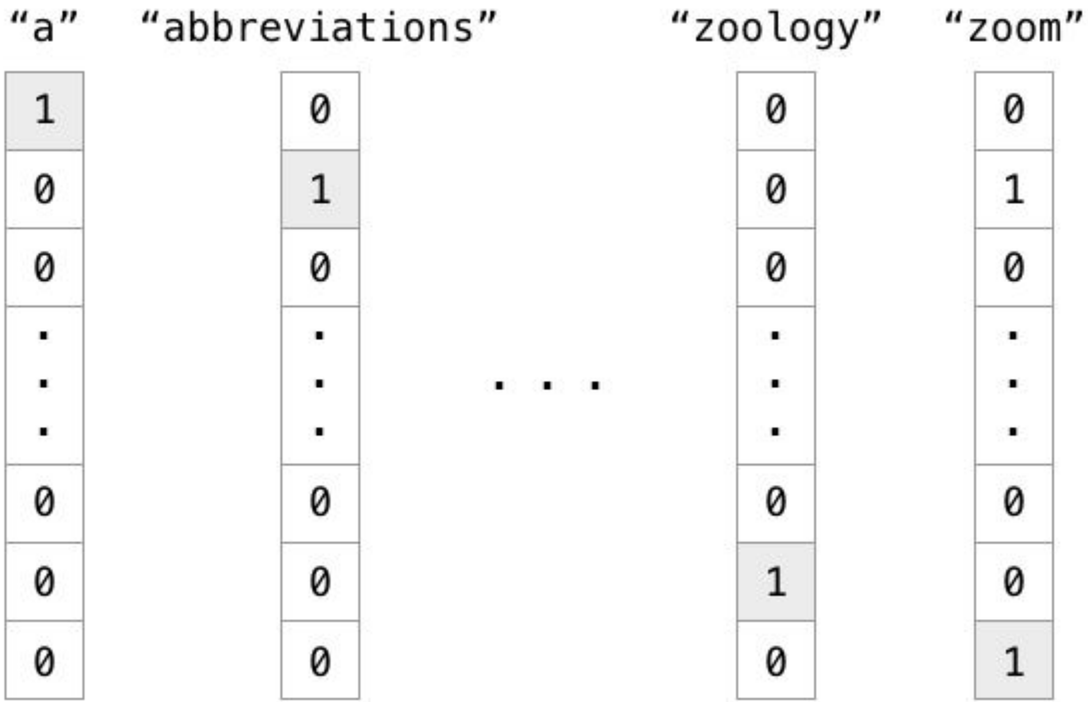
How to represent text in a computer: WordNet



Discrete representations: problems

- Missing new words
- Subjective
- Requires human labor to create and adapt
- Hard to compute accurate word similarity

Discrete representations: one-hot encoding



TF-IDF

$$\mathbf{tf}(t, d) = \frac{f_d(t)}{\max\limits_{w \in d} f_d(w)}$$

$$\mathbf{idf}(t, D) = \ln\left(\frac{|D|}{|\{d \in D : t \in d\}|}\right)$$

$$\mathbf{tfidf}(t, d, D) = \mathbf{tf}(t, d) \cdot \mathbf{idf}(t, D)$$

$$\mathbf{tfidf}'(t, d, D) = \frac{\mathbf{idf}(t, D)}{|D|} + \mathbf{tfidf}(t, d, D)$$

$f_d(t) :=$ frequency of term t in document d

$D :=$ corpus of documents

TF-IDF

If word is repeated in the query, it's probably important

Repetitions of query words in the document ➔ good

Rare words more important

$$s(Q,D) = \sum_{w} tf_{w,Q} \cdot \frac{tf_{w,D}}{tf_{w,D} + \frac{k|D|}{avg|D|}} \cdot \log \frac{|C|}{df_{w}}$$

The more query words we match, the better. Σ over the vocabulary

Repetitions of same word less important than different words. Except in very long documents

TF - term frequency

IDF - Inverse Document Frequency

TF-IDF: make it simple

$$\mathrm{tf}(''\mathsf{this}'', d_1) = \frac{1}{5} = 0.2$$
$$\mathrm{tf}(''\mathsf{this}'', d_2) = \frac{1}{7} \approx 0.14$$

$$\mathrm{idf}(''\mathsf{this}'', D) = \log\left(\frac{2}{2}\right) = 0$$

$$\mathrm{tfidf}(''\mathsf{this}'', d_1, D) = 0.2 \times 0 = 0$$
$$\mathrm{tfidf}(''\mathsf{this}'', d_2, D) = 0.14 \times 0 = 0$$

Document 1

Term	Term Count
this	1
is	1
a	2
sample	1

Document 2

Term	Term Count
this	1
is	1
another	2
example	3

Word ‘this’ is not very informative

Words cooccurrences

One of the most successful ideas of statistical NLP:

“You shall know a word by the company it keeps”

(J. R. Firth 1957: 11)

Words cooccurrences

Finding N-grams in a text

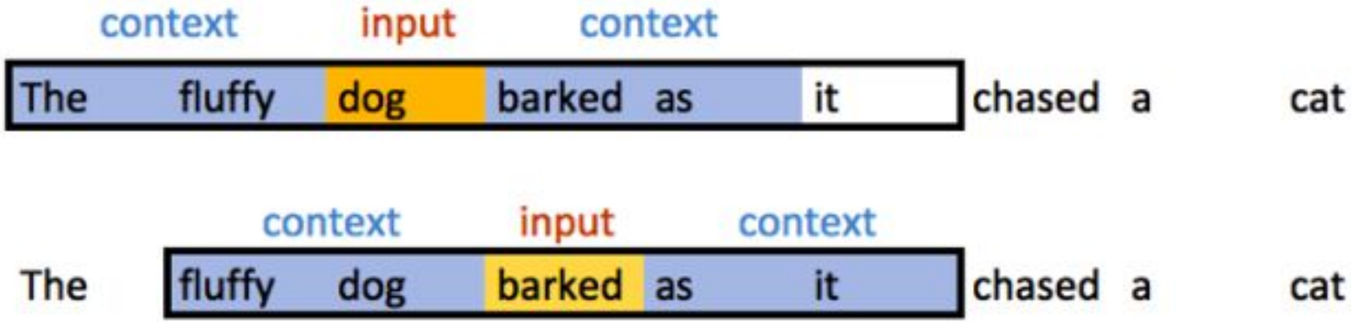
Word-document cooccurrence matrix

Window around each word

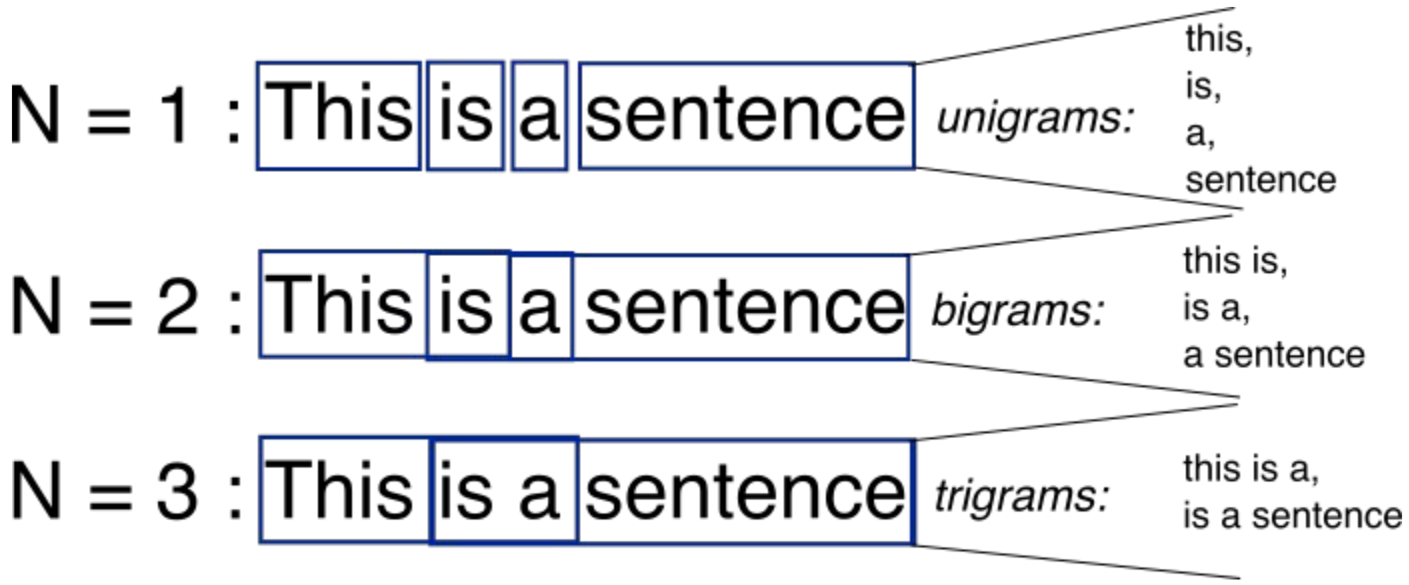
Word-document cooccurrence matrix

$$X = \begin{array}{c} \\ I \\ like \\ enjoy \\ deep \\ learning \\ NLP \\ flying \\ . \end{array} \begin{array}{c} \begin{array}{cccccccc} I & like & enjoy & deep & learning & NLP & flying & . \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Words cooccurrences: sliding window



Words cooccurrences: n-grams



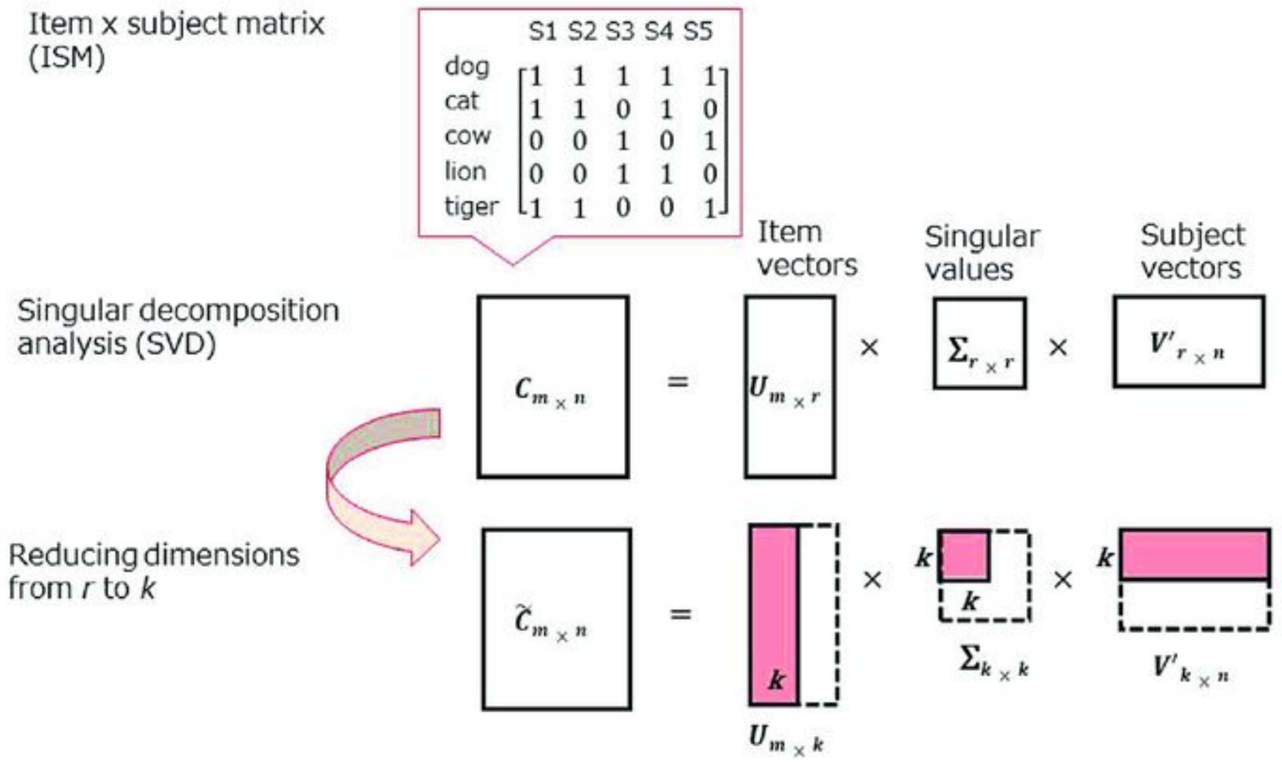
Co-occurrence vectors: problems

- Increase in size with vocabulary
- Very high dimensional: require a lot of storage
- Subsequent classification models have sparsity issues

↓

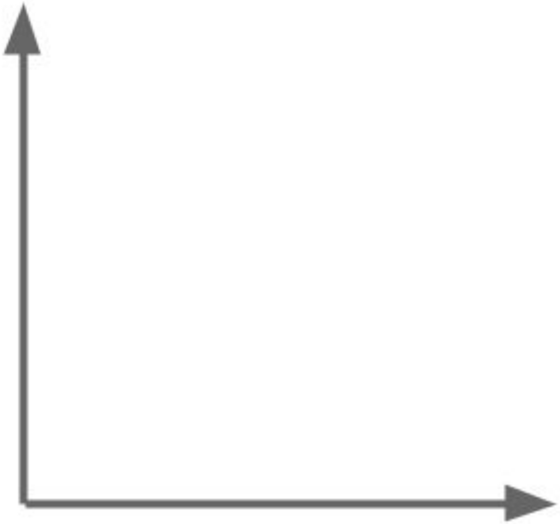
Models are less robust

Reducing dimensionality: SVD of cooccurrence matrix

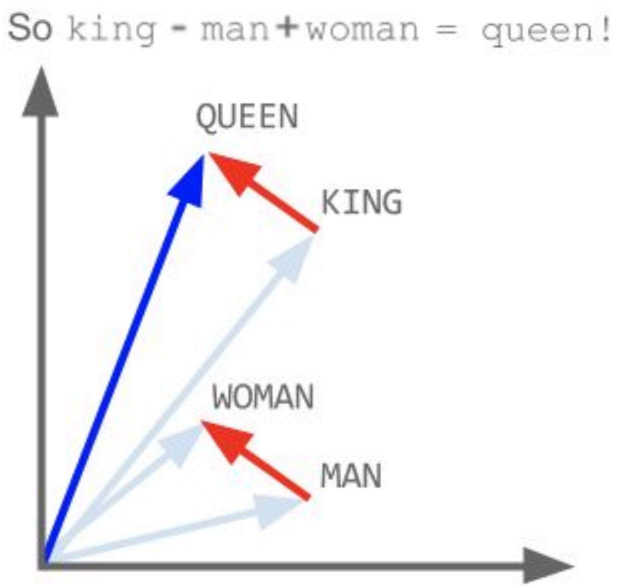


Embeddings: intuition

What is `king` - `man` + `woman`?



Embeddings: intuition

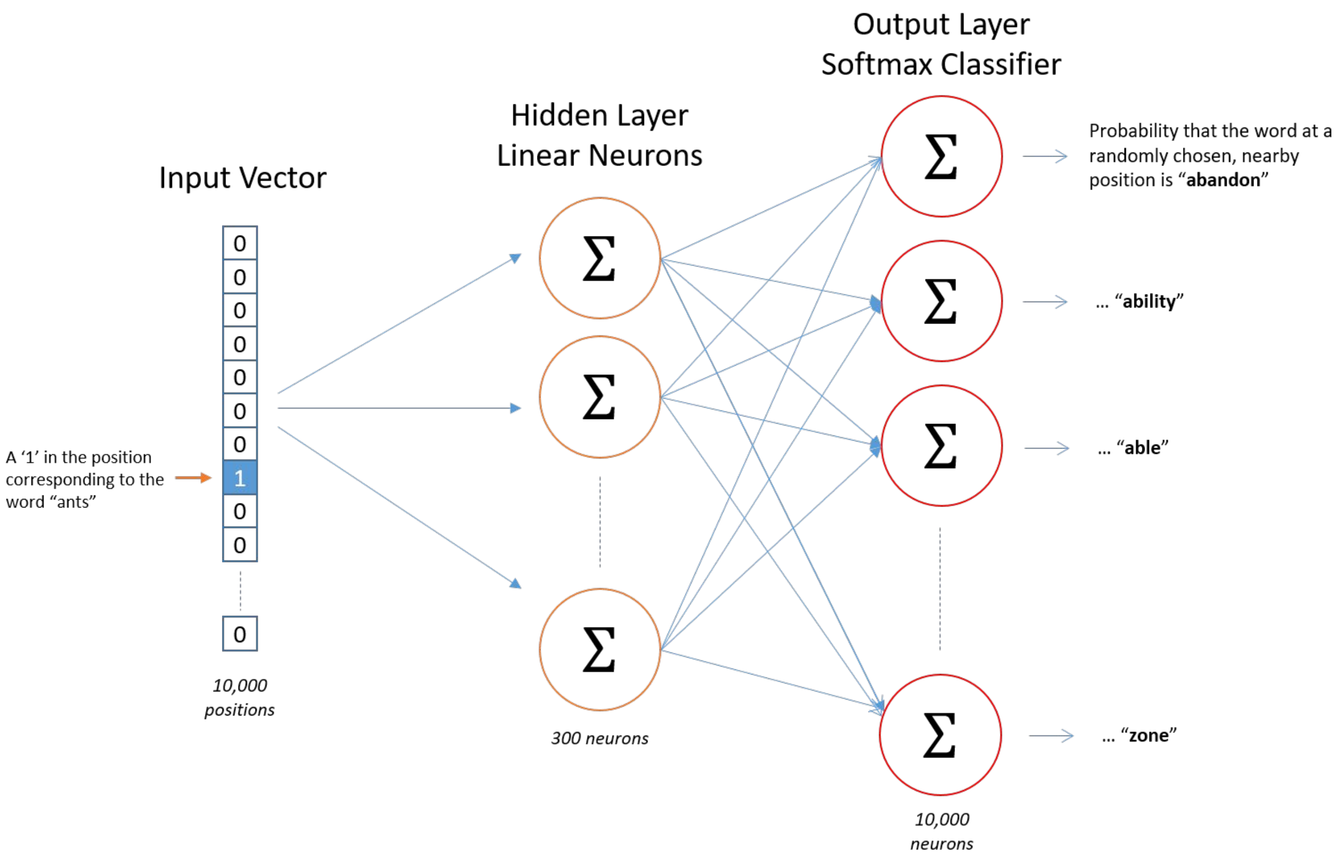


Embeddings: word2vec

Source Text	Training Samples
The quick brown fox jumps over the lazy dog.	(the, quick) (the, brown)
The quick brown fox jumps over the lazy dog.	(quick, the) (quick, brown) (quick, fox)
The quick brown fox jumps over the lazy dog.	(brown, the) (brown, quick) (brown, fox) (brown, jumps)
The quick brown fox jumps over the lazy dog.	(fox, quick) (fox, brown) (fox, jumps) (fox, over)

Source: http://mccormickml.com/2016/04/19/word2vec-tutorial-the-skip-gram-model/

Embeddings: word2vec



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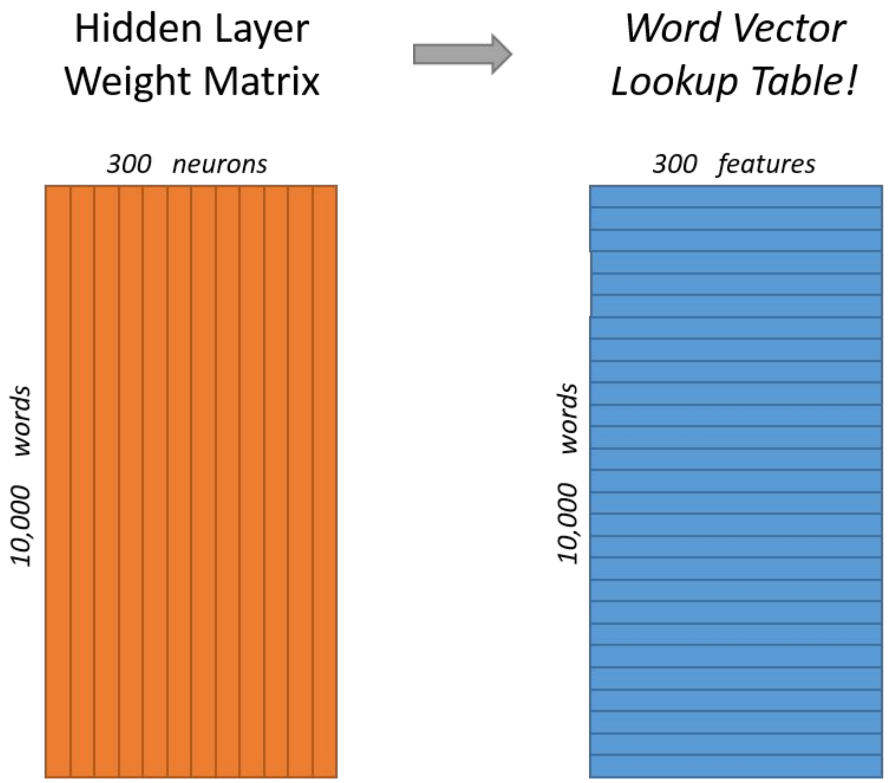
Embeddings: word2vec



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Embeddings: word2vec

- Word vectors with 300 components
- Vocabulary of 10,000 words.
- Weight matrix with 300 x 10,000 = 3 million weights each!

Training is too long and computationally expensive

How to fix this?

Embeddings: word2vec

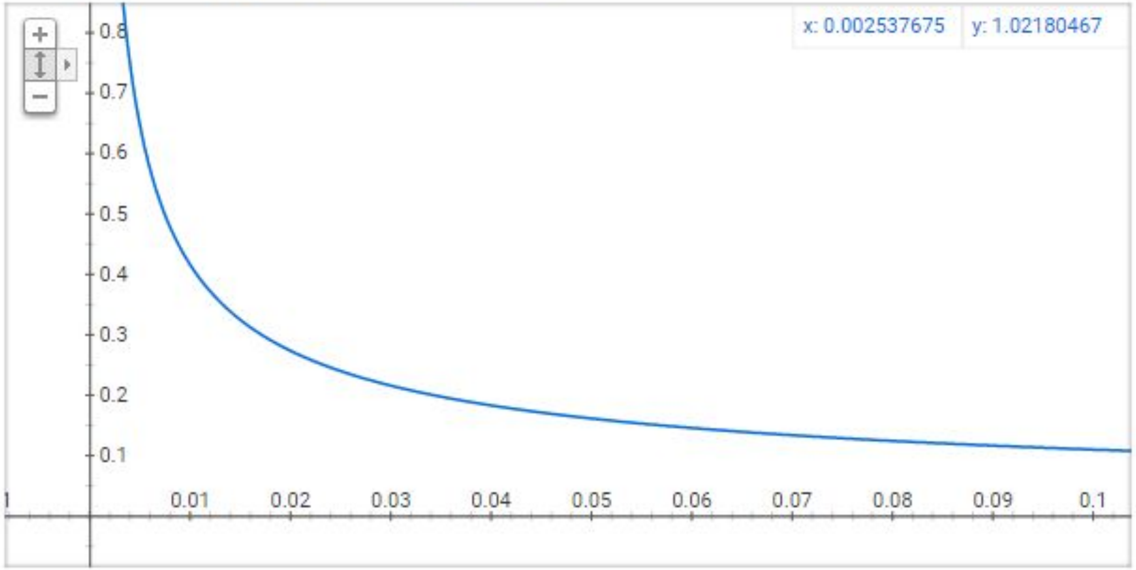
Basic approaches:

1. Treating common word pairs or phrases as single “words” in their model.
2. Subsampling frequent words to decrease the number of training examples.
3. Modifying the optimization objective with a technique they called “Negative Sampling”, which causes each training sample to update only a small percentage of the model’s weights.

Subsampling frequent words.

w_i is the word, $z(w_i)$ is the fraction of this word in the whole

Graph for (sqrt(x/0.001)+1)*0.001/x



$P(w_i)$ is the probability of *keeping* the word:

$$P(w_i) = (\sqrt{\frac{z(w_i)}{0.001}} + 1) \cdot \frac{0.001}{z(w_i)}$$

Source: http://mccormickml.com/2017/01/11/word2vec-tutorial-part-2-negative-sampling/

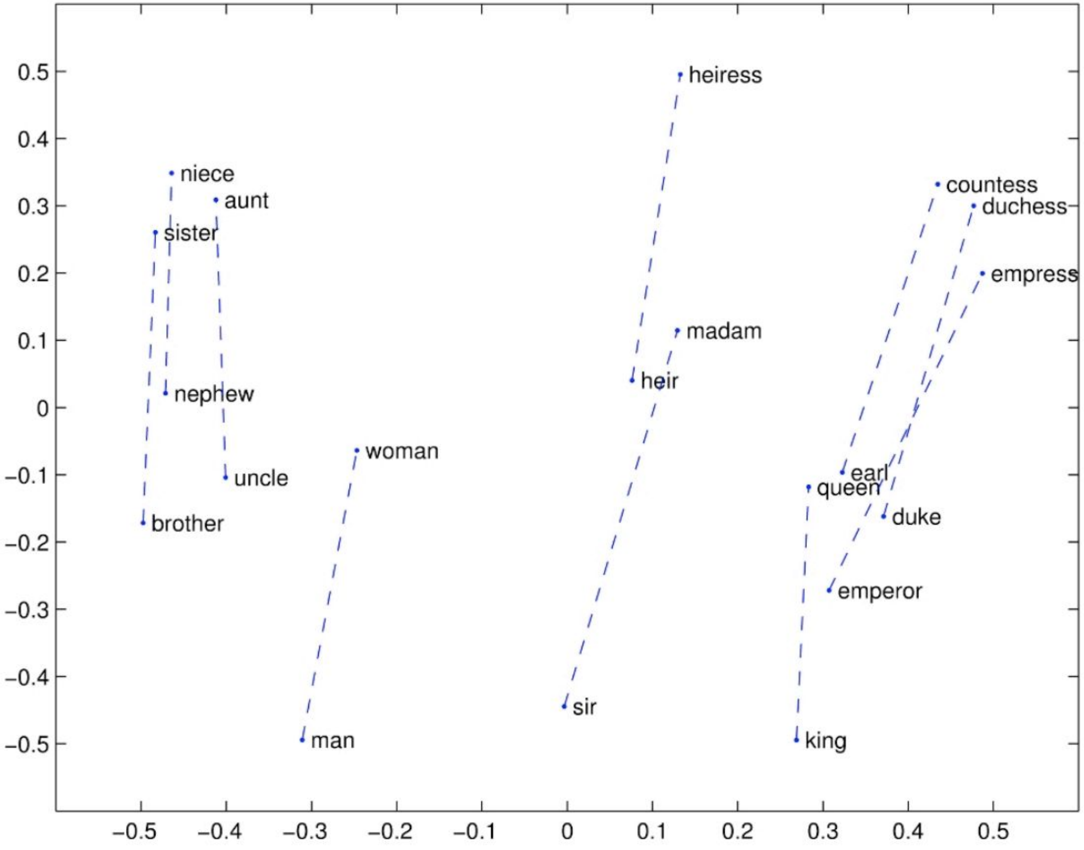
Embeddings: negative sampling

Negative Sampling idea: only few words error is computed. All other words has zero error, so no updates by the backprop mechanism.

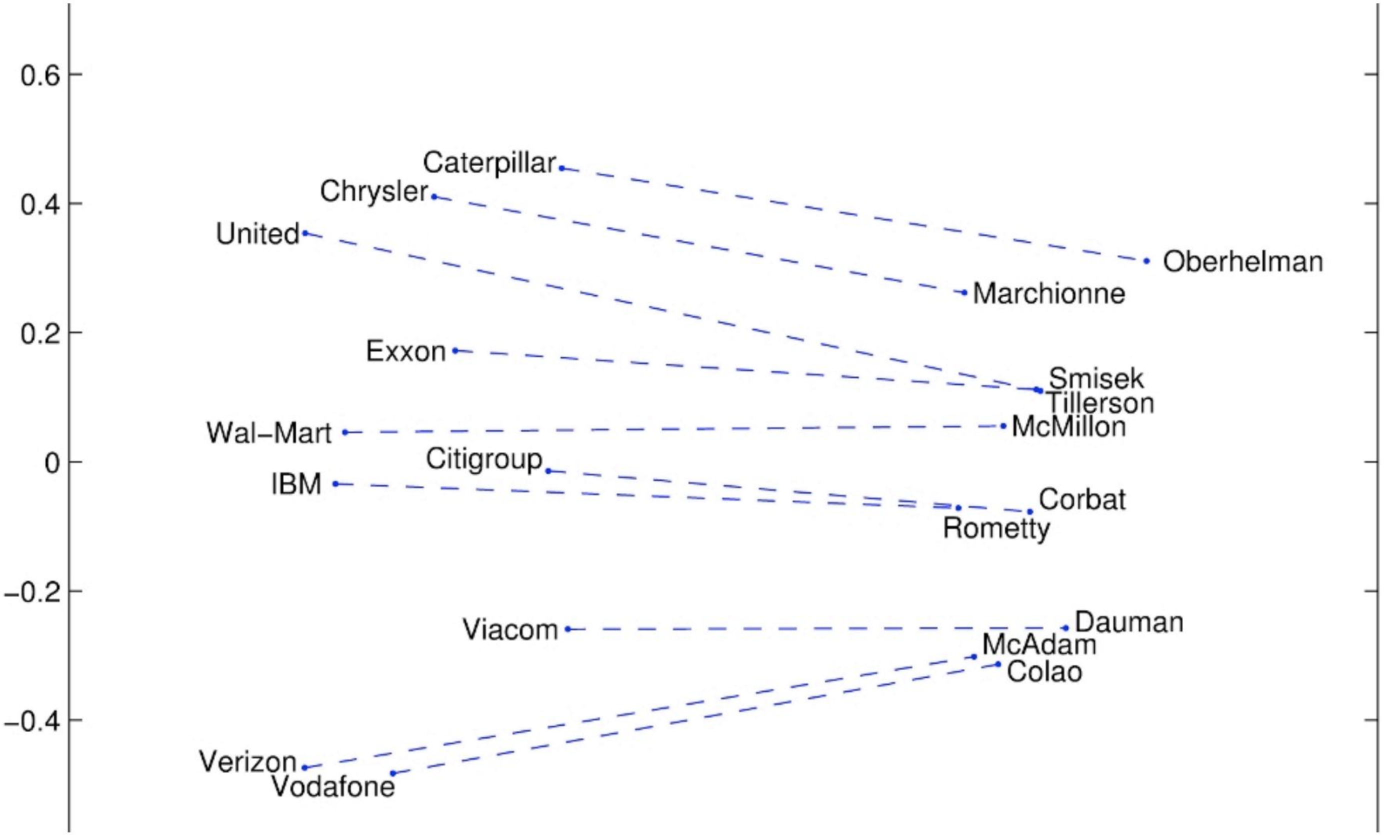
More frequent words are selected to be negative samples more often.The probability for a selecting a word is just it’s weight divided by the sum of weights for all words.

$$P(w_i) = \frac{f(w_i)^{3/4}}{\sum_{j=0}^{n} \left(f(w_j)^{3/4}\right)}$$

GloVe Visualizations



GloVe Visualizations: Company - CEO



Conclusion

Word vectors are simply vectors of numbers that represent the meaning of a word

Approaches:

- One-hot encoding
- Bag-of-words models
- Counts of word / context co-occurrences
- TF-IDF
- Predictions of context given word (skip-gram neural network models, e.g. word2vec)

RNNs generating...

Shakespeare

```
PANDARUS:
Alas, I think he shall be come approached and the day
When little srain would be attain'd into being never fed,
And who is but a chain and subjects of his death,
I should not sleep.

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Breaking and strongly should be buried, when I perish
The earth and thoughts of many states.

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Well, your wit is in the care of side and that.

Second Lord:
They would be ruled after this chamber, and
my fair nues begun out of the fact, to be conveyed,
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Clown:
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VIOLA:
I'll drink it.
```

Algebraic Geometry (Latex)

Proof. Omitted. □

Lemma 0.1. *Let $\mathcal{C}$ be a set of the construction.*

Let $\mathcal{C}$ be a gerber covering. Let $\mathcal{F}$ be a quasi-coherent sheaves of $\mathcal{O}$-modules. We have to show that

$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

.

Proof. This is an algebraic space with the composition of sheaves $\mathcal{F}$ on $X_{\acute{e}tale}$ we have

$$\mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$$

where $\mathcal{G}$ defines an isomorphism $\mathcal{F} \to \mathcal{F}$ of $\mathcal{O}$-modules. □

Lemma 0.2. *This is an integer $\mathcal{Z}$ is injective.*

Proof. See Spaces, Lemma ??. □

Lemma 0.3. *Let S be a scheme. Let X be a scheme and X is an affine open covering. Let $\mathcal{U} \subset \mathcal{X}$ be a canonical and locally of finite type. Let X be a scheme. Let X be a scheme which is equal to the formal complex.*

The following to the construction of the lemma follows.

Let X be a scheme. Let X be a scheme covering. Let

$$b : X \to Y' \to Y \to Y \to Y' \times_X Y \to X.$$

be a morphism of algebraic spaces over S and Y.

Proof. Let X be a nonzero scheme of X. Let X be an algebraic space. Let $\mathcal{F}$ be a quasi-coherent sheaf of $\mathcal{O}_X$-modules. The following are equivalent

1. (1) $\mathcal{F}$ is an algebraic space over S.
2. (2) If X is an affine open covering.

Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of finite type. □

Linux kernel (source code)

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static void action_new_function(struct s_stat_info *wb)
{
  unsigned long flags;
  int lel_idx_bit = e->edd, *sys & ~((unsigned long) *FIRST_COMPAT);
  buf[0] = 0xFFFFFFFF & (bit << 4);
  min(inc, slist->bytes);
  printk(KERN_WARNING "Memory allocated %02x/%02x, "
    "original MLL instead\n"),
    min(min(multi_run - s->len, max) * num_data_in),
    frame_pos, sz + first_seg);
  div_u64_w(val, inb_p);
  spin_unlock(&disk->queue_lock);
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http://karpathy.github.io/2015/05/21/rnn-effectiveness/

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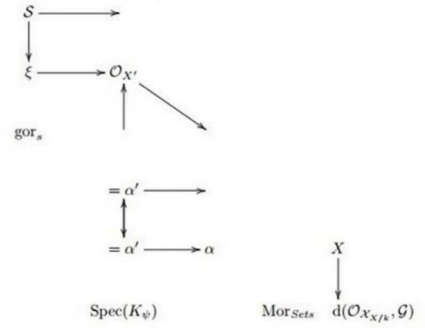
This since $\mathcal{F} \in \mathcal{F}$ and $x \in \mathcal{G}$ the diagram



is a limit. Then $\mathcal{G}$ is a finite type and assume S is a flat and $\mathcal{F}$ and $\mathcal{G}$ is a finite type f_*. This is of finite type diagrams, and

- the composition of $\mathcal{G}$ is a regular sequence,
- $\mathcal{O}_{X'}$ is a sheaf of rings.

□

Proof. We have see that $X = \mathrm{Spec}(R)$ and $\mathcal{F}$ is a finite type representable by algebraic space. The property $\mathcal{F}$ is a finite morphism of algebraic stacks. Then the cohomology of X is an open neighbourhood of U. □

Proof. This is clear that $\mathcal{G}$ is a finite presentation, see Lemmas ??.
A *reduced above* we conclude that U is an open covering of $\mathcal{C}$. The functor $\mathcal{F}$ is a "field

$$\mathcal{O}_{X,x} \longrightarrow \mathcal{F}_{\overline{x}} \ \text{-1}(\mathcal{O}_{X_{\acute{e}tale}}) \longrightarrow \mathcal{O}_{X_\ell}^{-1}\mathcal{O}_{X_\lambda}(\mathcal{O}_{X_\eta}^{\overline{v}})$$

is an isomorphism of covering of $\mathcal{O}_{X_i}$. If $\mathcal{F}$ is the unique element of $\mathcal{F}$ such that X is an isomorphism.
The property $\mathcal{F}$ is a disjoint union of Proposition ?? and we can filtered set of presentations of a scheme $\mathcal{O}_X$-algebra with $\mathcal{F}$ are opens of finite type over S.
If $\mathcal{F}$ is a scheme theoretic image points. □

If $\mathcal{F}$ is a finite direct sum $\mathcal{O}_{X_\lambda}$ is a closed immersion, see Lemma ??. This is a sequence of $\mathcal{F}$ is a similar morphism.

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

```
#include <asm/io.h>
#include <asm/prom.h>
#include <asm/e820.h>
#include <asm/system_info.h>
#include <asm/setew.h>
#include <asm/pgproto.h>

#define REG_PG    vesa_slot_addr_pack
#define PFM_NOCOMP  AFSR(0, load)
#define STACK_DDR(type)     (func)

#define SWAP_ALLOCATE(nr)     (e)
#define emulate_sigs()  arch_get_unaligned_child()
#define access_rw(TST)  asm volatile("movd %%esp, %0, %3" : : "r" (0));   \
  if (__type & DO_READ)

static void stat_PC_SEC __read_mostly offsetof(struct seq_argsqueue, \
          pC>[1]);

static void
os_prefix(unsigned long sys)
{
#ifdef CONFIG_PREEMPT
  PUT_PARAM_RAID(2, sel) = get_state_state();
  set_pid_sum((unsigned long)state, current_state_str(),
           (unsigned long)-1->lr_full; low;
}
```

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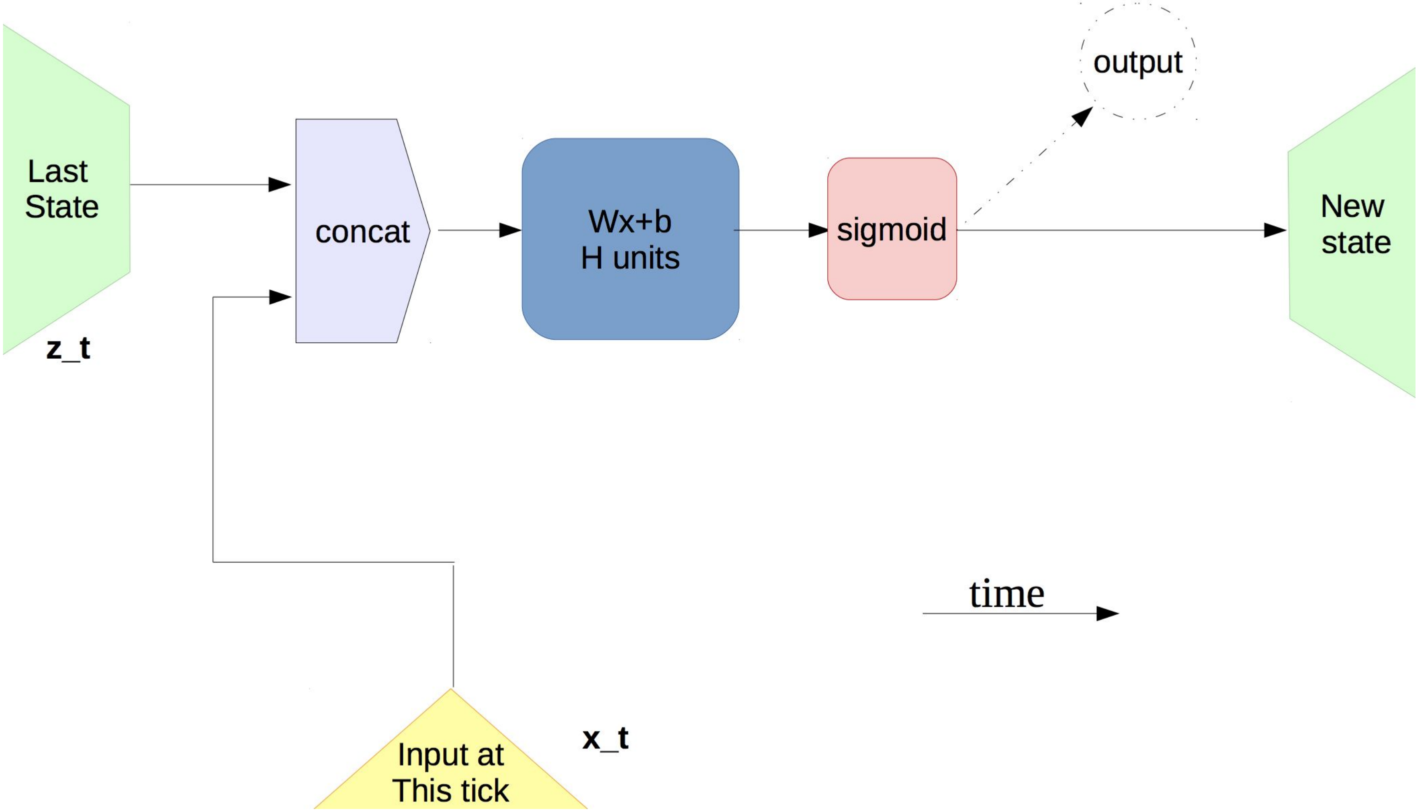
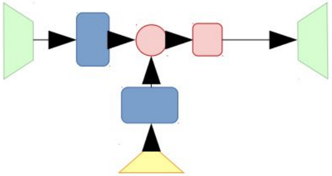
Recurrent neural network

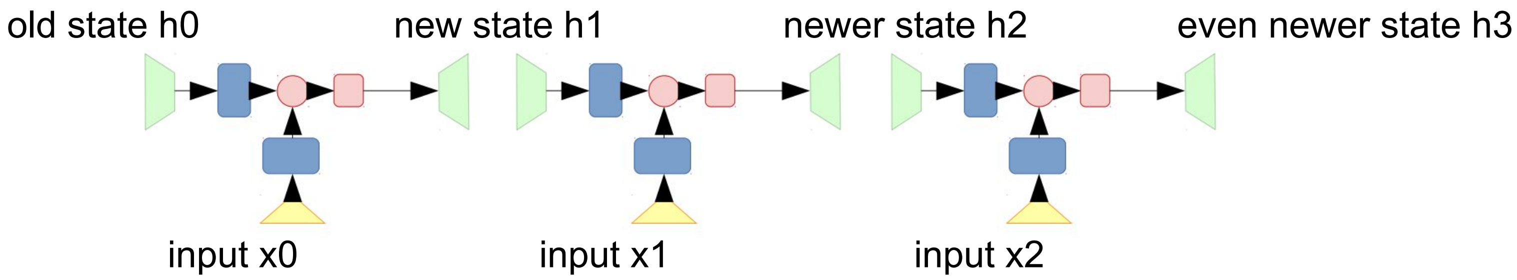


Recurrent neural network



Recurrent neural network

We use same weight matrices for all steps



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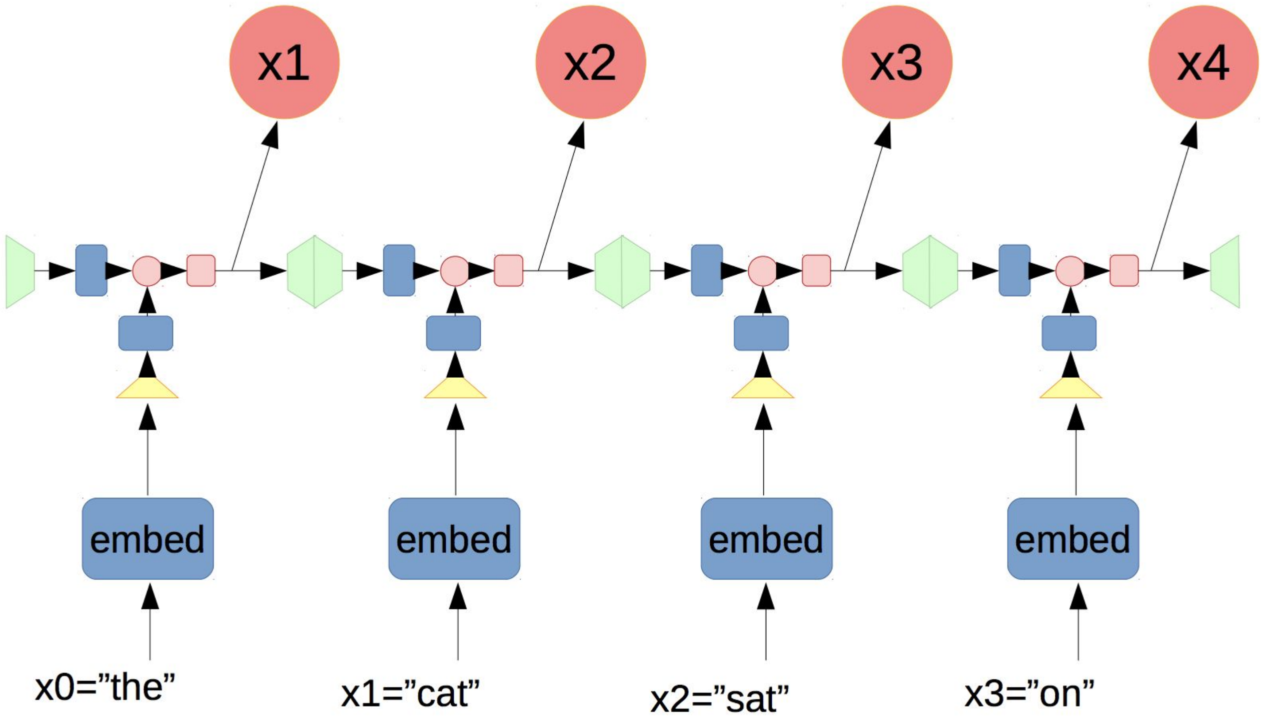

Recurrent neural network

Now with formulas

$$h_0 = \bar{0}$$

$$h_1 = \sigma\big(\langle W_{\text{hid}}[h_0, x_0]\rangle + b\big)$$

$$h_2 = \sigma\big(\langle W_{\text{hid}}[h_1, x_1]\rangle + b\big) = \sigma\big(\langle W_{\text{hid}}[\sigma\big(\langle W_{\text{hid}}[h_0, x_0]\rangle + b, x_1]\rangle + b\big)$$

$$h_{i+1} = \sigma\big(\langle W_{\text{hid}}[h_i, x_i]\rangle + b\big)$$

$$P(x_{i+1}) = \text{softmax}\big(\langle W_{\text{out}}, h_i\rangle + b_{\text{out}}\big)$$

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And who is but a chain and subjects of his death,
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Second Senator:
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Breaking and strongly should be buried, when I perish
The earth and thoughts of many states.

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  div_u64_w(val, inb_p);
  spin_unlock(&disk->queue_lock);
  mutex_unlock(&s->sock->mutex);
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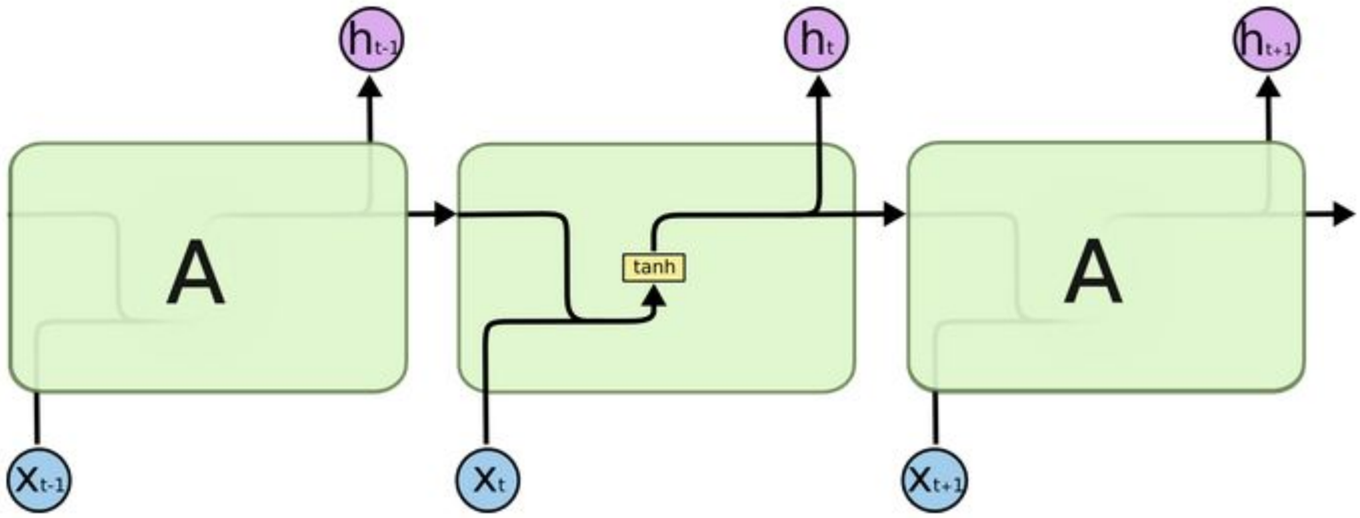
Vanilla RNN



Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

Q & A

That’s all. Feel free to ask any questions.

Remember: *a well-defined problem is halfway to being solved*