8-多元函数微分学-3

隐函数

双元/多元隐函数

$$F(x,y) = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

隐函数组 u,v

$$\begin{cases} F(x,y,u(x,y),v(x,y)) = 0\\ G(x,y,u(x,y),v(x,y)) = 0 \end{cases}$$

分别对 x(y) 偏导

$$egin{cases} F_x+F_uu_x+F_vv_x=0\ G_x+G_uu_x+G_vv_x=0 \end{cases}$$

方向导数与梯度

$$egin{aligned} rac{\partial f}{\partial ec{l}} &= f_x \cos lpha + f_y \sin lpha \ rac{\partial f}{\partial ec{l}} &=
abla u \cdot ec{l^0} \end{aligned}$$

多元微分学在几何学中的应用

切线

曲线方程:

$$egin{aligned} & F(x,y,z) = 0 \ & G(x,y,z) = 0 \ \end{aligned} \ egin{aligned} & \vec{n_1} = (F_x,F_y,F_z) \ & \vec{n_2} = (G_x,G_y,G_z) \end{aligned} \ \ ec{ au} = ec{n_1} imes ec{n_2} \end{aligned}$$

法向量

$$ec{n}=(F_x,F_y,F_z)$$

Taylor 公式与极值

$$F(t) = f(x_0 + t\Delta x, y_0 + t\Delta y) \ (\Delta x rac{\partial}{\partial x} + \Delta y rac{\partial}{\partial y}) f = \Delta x rac{\partial f}{\partial x} + \Delta y rac{\partial f}{\partial y}$$

极值的判断

$$H = f_{xx}f_{yy} - f_{xy}^2$$

条件极值:拉格朗日乘数法

在约束条件 $\varphi(x,y,z)=0$ 下,求 f(x,y,z) 的极值

$$L(x,y,z,\lambda) = f(x,y,z) + \lambda \varphi(x,y,z)$$