ID2090: Assignment 4

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Apollonius Theorem

Introdution

Apollonius's Theorem relates the length of a median with the length of it's sides.

Say for a triangle \triangle ABC. And let a median be drawn from A to the opposite side, at M. The sum of the squares of the two sides, i.e. AB and AC are equal to twice the sum of the squares AM and BM.

So,

$$|AB|^2 + |AC|^2 = 2(|AD|^2 + |BM|^2)$$
(1)

It can also be restated as

$$2(|AB|^2 + |AC|^2) = |BC|^2 + 4|AM|^2$$
(2)

It is a special form of Stewart's theorem, and is also called as Extended Pythagoras Theorem. For an isosceles triangle, the theorem reduces to Pythagorean theorem for the smaller triangle.

The theorem is named for the ancient Greek mathematician Apollonius of Perga.

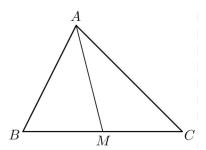


Figure 1: Figure of the above explanation

How I found this equation

My coaching was very math oriented and I went to a place where they trained for math Olympiads. So, I was introduced to this, Basic Proportionality Theorem, Ceva's and Menelaus' theorems in 8th standard. I found the entire thing very interesting and took a liking to pure geometry. I like the cat and mouse approach to be used in geometry which made me choose this equation.

¹Wikipedia article on Apollonius Theorem