SERIES

for $\{a_n\}_{n=1}^{\infty}$, define S_n as:

$$S_1 = a_1$$
$$S_2 = a_1 + a_2$$
$$S_3 = a_1 + a_2 + a_3$$
$$\vdots$$
$$S_n = a_1 + \dots + a_n = \sum_{i=1}^{n} a_i$$

S_n is called a partial sum.

$\lim_{n\to\infty} \{S_n\}_{n=1}^{\infty} = \sum_{i=1}^{\infty} a_i$ is referred to as an infinite series.

if $\{S_n\}_{n=1}^{\infty}$ is conversent, $\sum_{i=1}^{\infty} a_i$ is also conversent.
→ similar for divergent.

Properties

If $\sum a_n$ and $\sum b_n$ are both conversant,

1. $\sum c\, a_n = c \cdot \sum a_n$
2. $\sum_{n=k}^{\infty}(a_n \pm b_n) = \sum_{n=k}^{\infty} a_n + \sum_{n=k}^{\infty} b_n$

multiplying two series

$$\left(\sum_{n=0}^{\infty} a_n\right)\left(\sum_{n=0}^{\infty} b_n\right) \neq \sum_{n=0}^{\infty} (a_n b_n)$$

consider $\left(1 - x^2 + \frac{1}{2}x^4 + \dots\right)\left(1 - \frac{x^2}{2} + \frac{x^4}{24} - \dots\right)$

$e^x \cdot \cos(x)$

↳ we have to FOIL out all the terms.

$$\left(\sum_{n=0}^{\infty} a_n\right)\left(\sum_{n=0}^{\infty} b_n\right) = \sum_{n=0}^{\infty} c_n, \text{ where } c_n = \sum_{i=0}^{n} (a_i b_{n-i})$$

re-indexing a series:

$$\sum_{n=2}^{\infty} \frac{n+5}{2^n}$$

let $n = i+2$

$$\sum_{i=0}^{\infty} \frac{i+7}{2^{i+2}}$$

testing for convergence / divergence:

if $\lim_{n\to\infty} S_n = S$, then $\sum_{i=1}^{\infty} a_i = S$ and series is convergent

- $\sum_{n=1}^{\infty} n$ diverges
- $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ converges

$S_1 = 1/3$
$S_2 = 1/3 + 1/8$
$S_3 = 1/3 + 1/8 + 1/15$
$S_4 = 1/3 + 1/8 + 1/15 + 1/24$
$S_n = \frac{3}{4} - \frac{1}{2n} - \frac{1}{2(n+1)}$

- $\sum_{n=1}^{\infty} (-1)^n$ diverges
- $\sum_{n=1}^{\infty} \frac{1}{3^{n-1}}$ converges

$S_1 = 1$
$S_2 = 1 + \frac{1}{3}$
$S_3 = 1 + 1/3 + 1/9$

$S_n = 3/2\left(1 - \frac{1}{3^n}\right)$

☆ if Σa_n is convergent, then $\lim_{n\to\infty} a_n = 0$

☆ if Σa_n and Σb_n are divergent, $\Sigma (a_n + b_n)$ may be convergent