LINEAR CIRCUITS

linearity is property describing linear relation btw. cause/effect

- has to satisfy proportionality & additivity
- proportionality (homogeneity)
  - → $V = iR$
  - ⇒ $KiR = KV$
- additivity
  - → $V_1 = i_1 R$, $V_2 = i_2 R$

    applying $i_1 + i_2$ gives

    $V = (i_1 + i_2)R = i_1 R + i_2 R = V_1 + V_2$

- $p = i^2 R$ is non linear but quadratic

SUPERPOSITION

if circuit has 2+ independent sources, can

1. use nodal/mesh analysis
2. use **superposition**
   - → determine contribution of each source and add.

method

1. turn off all independent sources but one. Find voltage/current due to that source
2. repeat for other sources
3. add them up.

★ current sources become open; voltage sources are connected.

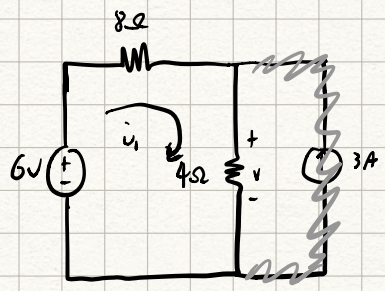
- usually takes lots of work; 3 sources = 3 circuits

ex.

contribution by 6V

let $V = V_1 + V_2$

to find V_1, set $V_2 = 0$



KVL:

$6 = (8+4)\, i_1$

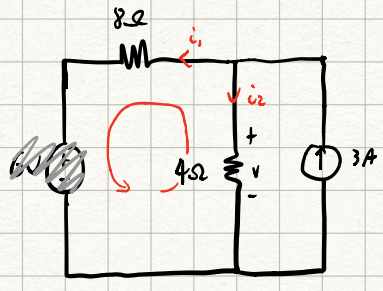
$i_1 = \frac{1}{2}\,A$

$V_1 = 4(i_1) = 2V.$

We can also use voltage division:

$V_1 = \frac{4}{4+8}(6) = 2V$

to find V_2, set $V_1 = 0$



KCL:

$3 = i_1 + i_2$

KVL:

$8 i_1 = 4 i_2$

$i_2 = 2 i_1$

We can use current division:

$i_2 = \frac{8}{8+4}(3) = 2A$

$V_2 = 4 i_2 = 8V$

$\Rightarrow i_1 = 1A$

$i_2 = 2A$

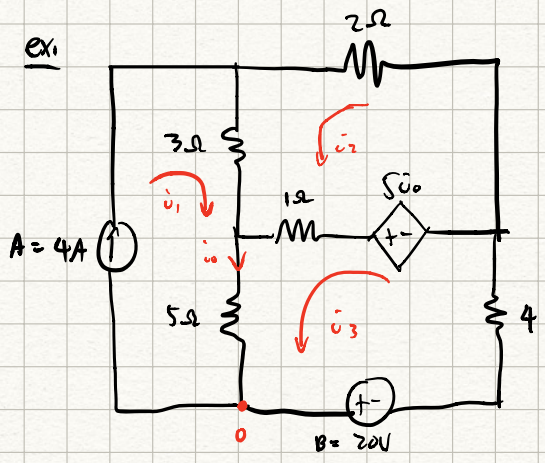
$V_2 = 2(4) = 8V$

$V = V_1 + V_2 = 2 + 8 = 10V$

also... $i_{4\Omega} = \frac{1}{2} + 2 = 2.5\,A$ down

$i_{8\Omega} = \frac{1}{2} - 1 = 0.5\,A$ left

ex.



find i_0 using superposition

$i_0 = i_{0A} + i_{0B}$

turn off 20V!

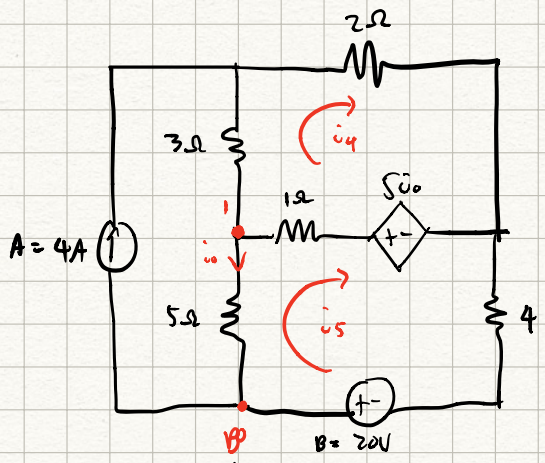
loop 1: $i_1 = 4A$

loop 2: $3i_1 + 6i_2 - i_3 + 5i_{0A} = 0$

loop 3: $-5i_0 + 5i_1 - i_2 + 10 i_3 = 0$

node 0: $i_0 = i_1 + i_3$

$i_0 - i_1 - i_3 = 0$



$i_0 = \frac{52}{17}\,A$

turn off 4A!

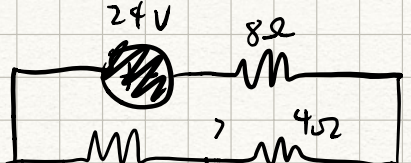
loop 4: $5i_{0B} + 6i_4 + i_5 = 0$

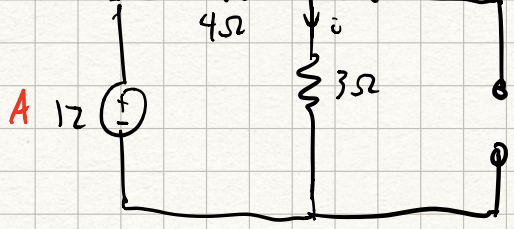
loop 5: $5i_{0B} + i_4 + 10 i_5 = 20$

node 1: $i_{0B} + i_5 = 0$

$i_{0B} = -\frac{60}{17}$

$i_0 = i_{0A} + i_{0B} = \frac{52}{17} - \frac{60}{17} = \frac{-8}{17}\,A$

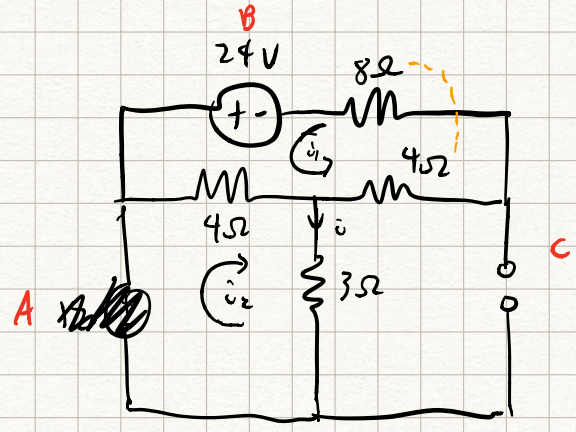




A'

$\left(\frac{1}{12}+\frac{1}{4}\right)^{-1} = 3\Omega + 3\Omega = 6\Omega$

$i_A = \frac{12}{6} = 2A$



B':

$24 = 16\, i_1 + 4 i_2$

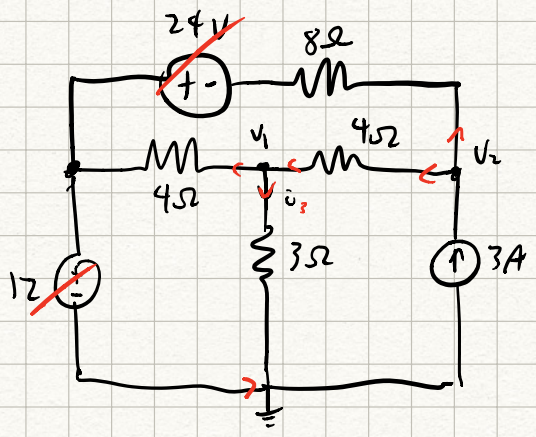
$6 = 9\, i_1 + i_2$

$0 = 4 i_1 + 7 i_2$

$i_1 = \frac{7}{4}$

$i_2 = -1$

C':



@ V_2: $3 = \frac{V_2}{8} + \frac{V_2 - V_1}{4}$

$\Rightarrow 24 = -2V_1 + 3V_2$

@ V_1: $\frac{V_1}{3} + \frac{V_1}{4} = \frac{V_2 - V_1}{4}$

$\Rightarrow 7V_1 = 3V_2 - 3V_1$

$\Rightarrow 10V_1 = 3V_2$

$\Rightarrow 10V_1 - 3V_2 = 0$

$V_1 = 3$

$V_2 = 10$

$i_3 = \frac{V_1}{3} = 1\,A.$

So $i = 2 - 1 + 1 = 2\,A.$