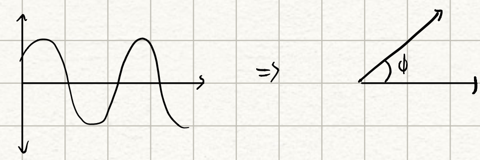
AC ANALYSIS

In DC, we had $V = IR$, where V, I, R are scalars.

In AC, we treat them as vectors:

$$\vec{V} = \vec{I}\vec{Z}$$

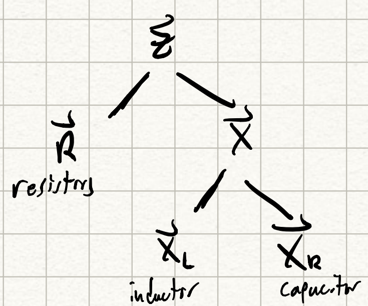
$\vec{V}$ and $\vec{I}$ are phase vectors (phasors)



We write this in the form,

$$\boxed{\tilde{V} = V_m e^{j\theta} = V_m \angle\theta \quad \text{(complex \#)}}$$

$\vec{Z}$ is impedance, which is made up of resistance and reactance



$$\boxed{\begin{aligned} X_L &= -\frac{1}{\omega C} \\ X_C &= \omega L \end{aligned}}$$

$\vec{Z}$ is expressed as a complex #

$$\boxed{\vec{Z} = R + Xj}$$

↳ j is used since i represents current.

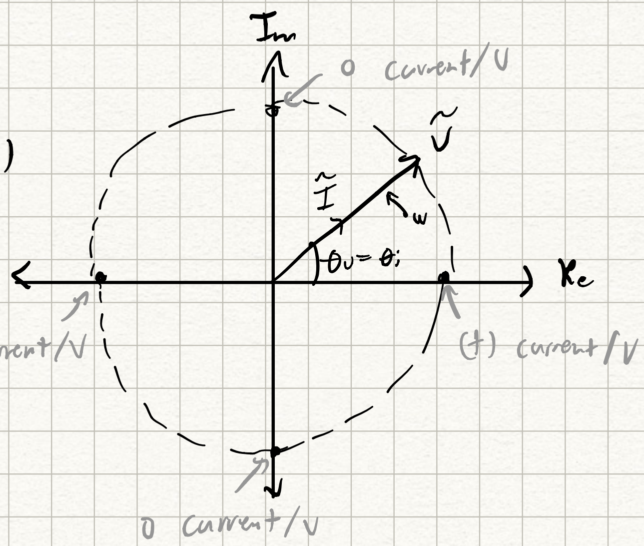
- in DC, capacitors = open
  inductors = short
- in AC, capacitors slow ΔV, so V lags behind I by $90°$
  inductors slow ΔI, so I lags behind V by $90°$

CIRCUIT ELEMENTS

resistor

impedance:

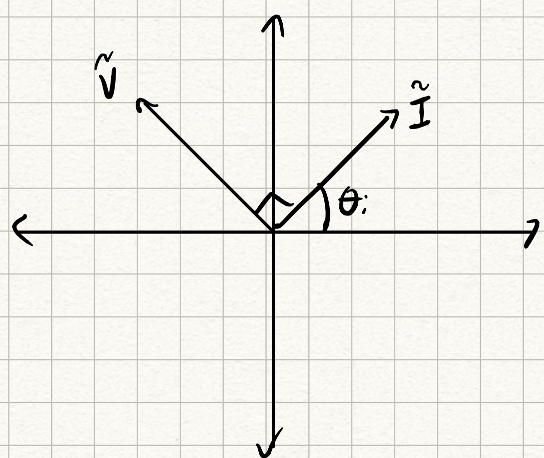
$$\boxed{Z_R = R}$$ (purely real)



V, I are in-phase

inductor

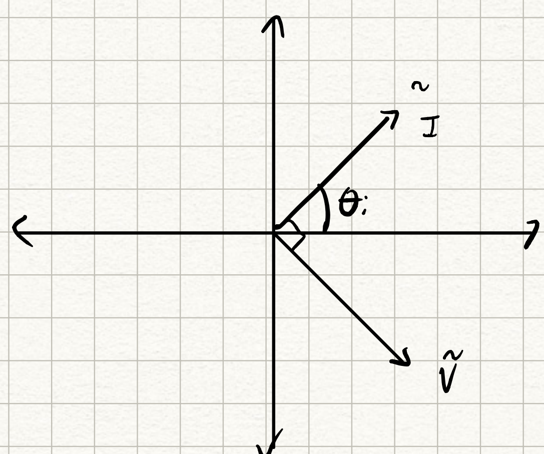
$$\boxed{Z_L = j\omega L}$$



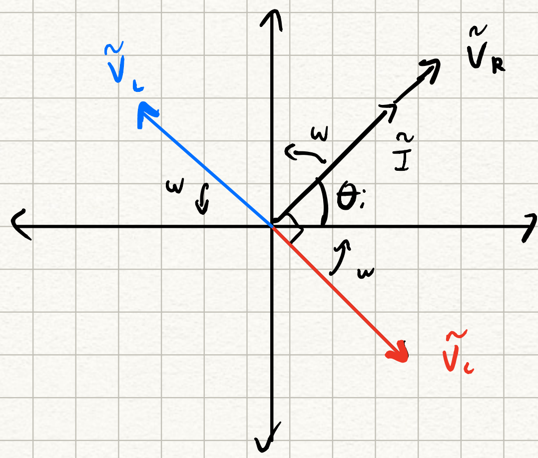
- Voltage leads by 90°

capacitor

$$\boxed{Z_c = \frac{1}{j\omega C} = -j\frac{1}{\omega C}}$$



we can combine these like spokes on a wheel:



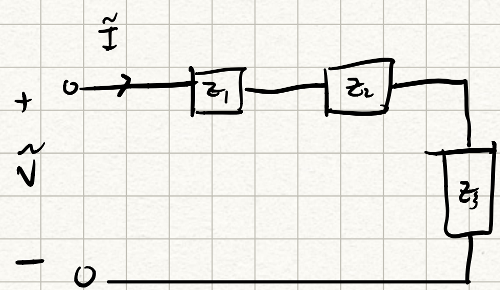
- all of these spin around at ang. freq ω.
- Since x-axis is Re axis,

↳ I, V (+) when $\theta = 0$
↳ I, V 0 when $\theta = \pi/2, \frac{3\pi}{2}$
↳ I, V (−) when $\theta = -\pi$

IMPEDANCE IN SERIES

KVL:

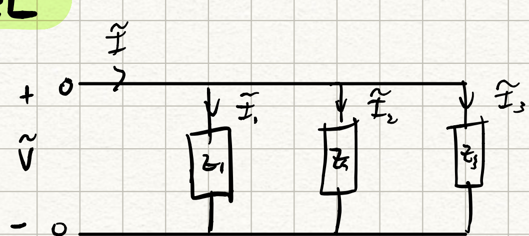
$$\tilde{V} = Z_1\tilde{I} + Z_2\tilde{I} + Z_3\tilde{I}$$
$$= \tilde{I}(Z_1 + Z_2 + Z_3)$$



$$\boxed{\tilde{V} = Z\tilde{I} \text{ where } Z = Z_1 + Z_2 + Z_3}$$

IMPEDANCE IN PARALLEL

$$\tilde{I} = \tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3$$
$$= \frac{\tilde{V}}{Z_1} + \frac{\tilde{V}}{Z_2} + \frac{\tilde{V}}{Z_3}$$



$$I = \frac{\tilde{V}}{Z} \quad \text{where}$$

$$\boxed{\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

ex1

$$Z_L = j\omega L = j(58)(16\times10^{-3})$$

$$Z_R = 4$$

$$Z_C = -j\frac{1}{\omega C} = -j\frac{1}{(58)(225\times10^{-6})}$$

$$Z = \left(\frac{1}{Z_C} + \frac{1}{Z_R} + \frac{1}{Z_C}\right)^{-1}$$

$$= 0.209 + 0.89i$$

$$= 0.9144 \angle 76.78^\circ$$

if positive, inductive
if negative, capacitive
if zero, resistive

ADMITTANCE

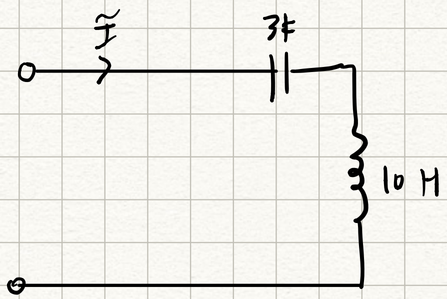
$$\boxed{Y = \frac{1}{Z} = \frac{1}{R + X_i} = G + jB}$$

(Siemens, S or Ω^{-1})

G = conductance

B = Susceptance

Ex:



for which values of w does this behave like a short circuit?

$$Z = jwL - j\frac{1}{wC} = 0$$

$$jwL = j\frac{1}{wC}$$

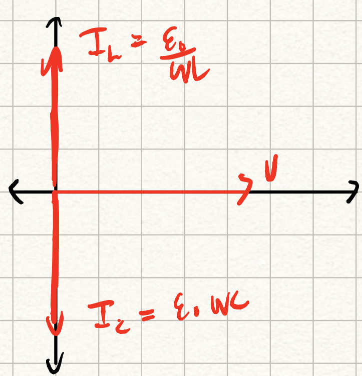
$$w = \sqrt{\frac{1}{LC}}$$

$$\cancel{\mathcal{E}_0}\, wL = \frac{\mathcal{E}_0}{wC}$$

$$w^2 = \frac{1}{LC}$$

$$w = \sqrt{\frac{1}{LC}}$$

open circuit?

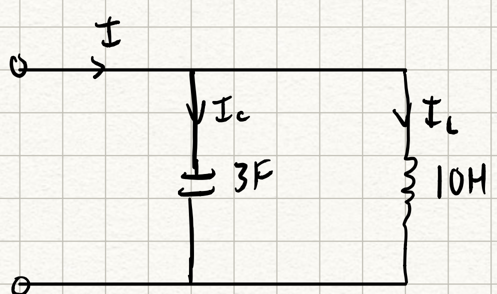


$$I = \mathcal{E}_0 wC = \frac{\mathcal{E}_0}{wL}$$

when $w \to 0$, C makes $I \to 0$.

$$I_L = \frac{\mathcal{E}_0}{Z_L}, \quad Z_L = jwL$$

$$I_C = \frac{\mathcal{E}_0}{Z_C}, \quad Z_C = -j\frac{1}{wC}$$



i) short circuit?

$w = 0$, L acts like s.c.
C acts like o.c.

i) open circuit

$$I = I_C + I_L = \mathcal{E}_0\left(\frac{1}{wL} - wC\right)$$

$$\frac{1}{wL} = wC$$

$$w^2 = \sqrt{\frac{1}{LC}}$$