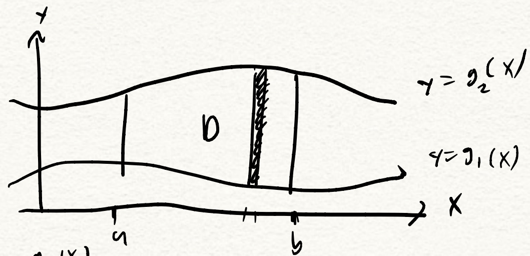
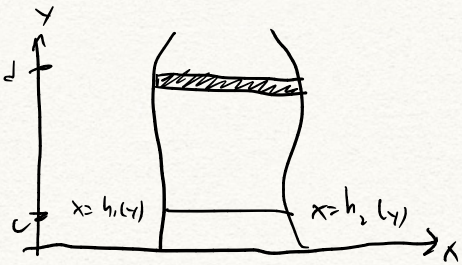
DOUBLE INTEGRALS

If D is a type I region



then $\displaystyle\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$

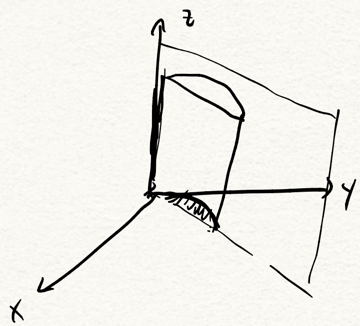
If D is type II region

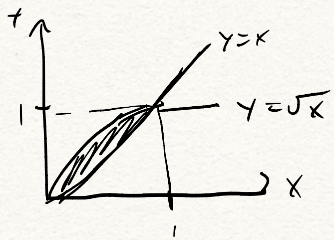


then $\displaystyle\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$

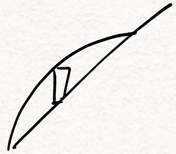
examples!

① find volume of solid bounded above surface $z = xy$, below xy-plane, and on the sides by plane $y = x$ and parabolic cylinder $y = \sqrt{x}$

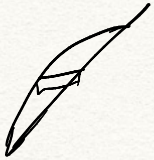




$$V = \int_0^1 \int_{y=x}^{y=\sqrt{x}} xy \, dy \, dx$$



$$V = \int_0^1 \int_{y^2}^{y} xy \, dx \, dy$$



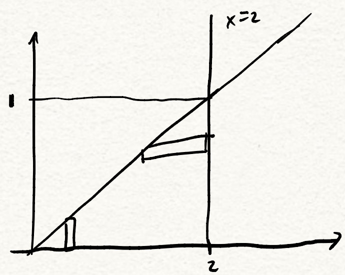
$$= \int_0^1 \left(\frac{1}{2} x^2 y \Big|_{y^2}^{y} \right) dy$$

$$= -\frac{1}{2} \int_0^1 y \, dy$$

$$= -\frac{1}{2} \left(\frac{1}{2} y^2 \right) \Big|_0^1$$

$$=$$

③ consider $\int_0^1 \int_{2y}^{2} e^{x^2} \, dx \, dy$ $\nearrow x=2$ $\quad y \;\; y = \frac{1}{2}x$



Reverse the order:

$$= \int_0^2 \int_0^{\frac{1}{2}x} e^{x^2} \, dy \, dx$$

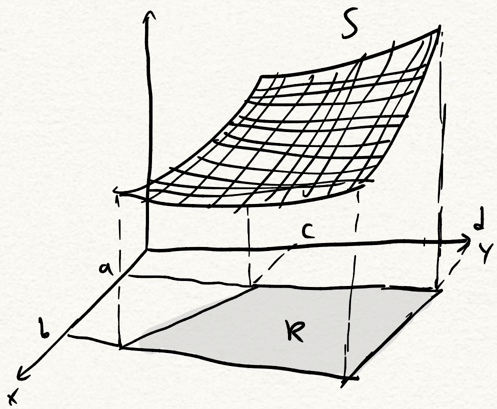
$$= \int_0^2 y e^{x^2} \Big|_0^{\frac{1}{2}x} dx$$

$$= \int_0^2 \frac{1}{2} x e^{x} \, dx$$

$$= \frac{1}{4} x e^{x} \Big|_0^2$$

$$= \frac{1}{4}(e^4 - 1)$$

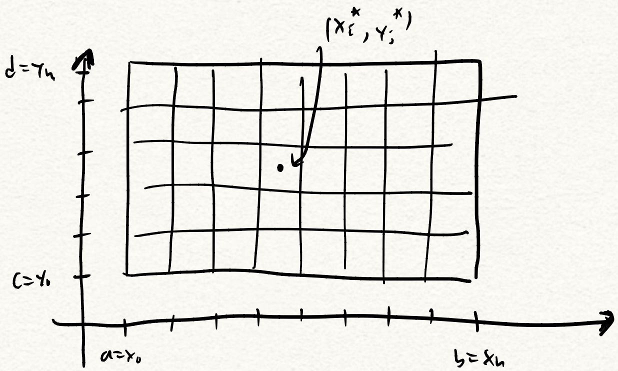
DOUBLE INTEGRALS



- Whereas in single integrals, we integrated over a 1D region, in double integrals we integrate over a 2D region.

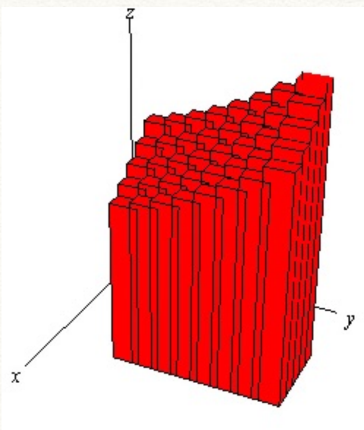
$R = [a, b] \times [c, d]$

To find volume under S, divide R into series of sq



divide $a \le x \le b$ into n subintervals

divide $c \le y \le d$ into m subintervals



volume of each box is

$$f(x_i^*, y_j^*)\,\Delta A$$

$$V \approx \sum_{i=1}^{n}\sum_{j=1}^{m} f(x_i^*, y_j^*)\,\Delta A$$

$$= \lim_{(i,j)\to(\infty,\infty)} \sum_{i=1}^{n}\sum_{j=1}^{m} f(x_i^*, y_j^*)\,\Delta A$$

$$\boxed{\iint_R f(x,y)\,dA = \lim_{(i,j)\to(\infty,\infty)} \sum_{i=1}^{n}\sum_{j=1}^{m} f(x_i^*, y_j^*)\,\Delta A}$$