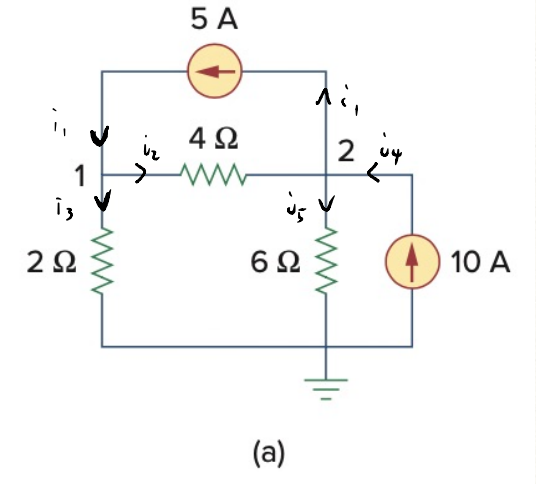
NODAL ANALYSIS

- We can express currents using Ohm's Law to simplify things
- Uses KCL to find voltage

Ex: Calculate node voltages



① $i_1 = i_2 + i_3$

$5 = \frac{V_1 - V_2}{4} + \frac{V_1 - 0}{2}$

$20 = V_1 - V_2 + 2V_1$

$20 = 3V_1 - V_2$

② $i_4 = i_1 + i_5$

$10 = 5 + \frac{V_2 - 0}{6}$

$60 = 30 + V_2$

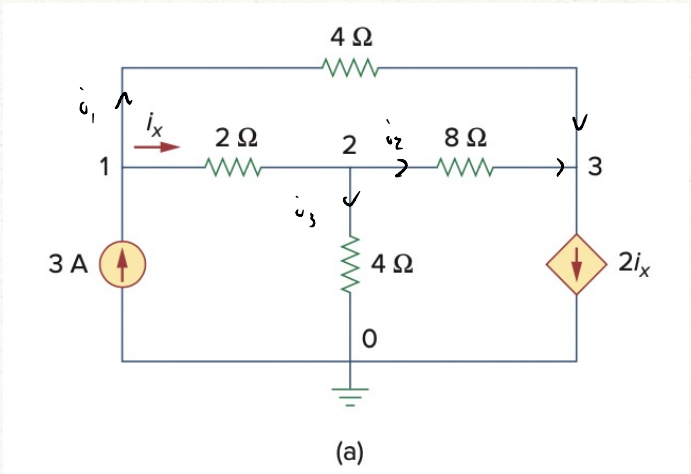
$V_2 = 30\ V$

② → ① $20 = 3V_1 - 30$

$50 = 3V_1$

$V_1 = \frac{50}{3} = 13.\overline{3}\ V$

Ex.



(a)

① $3 = i_1 + i_x$

$3 = \frac{V_1 - V_3}{4} + \frac{V_1 - V_2}{2}$

$12 = V_1 - V_3 + 2V_1 - 2V_2$

$12 = 3V_1 - 2V_2 - V_3$

② $i_x = i_2 + i_3$

$\frac{V_1 - V_2}{2} = \frac{V_2 - V_3}{8} + \frac{V_2}{4}$

$4V_1 - 4V_2 = V_2 - V_3 + 2V_2$

$4V_1 - 7V_2 + V_3 = 0$

③ $2i_x = i_2 + i_1$

$2\left(\frac{V_1 - V_2}{2}\right) = \frac{V_2 - V_3}{8} + \frac{V_1 - V_3}{4}$

$8V_1 - 8V_2 = V_2 - V_3 + 2V_1 - 2V_3$

$6V_1 - 9V_2 + 3V_3 = 0$

$2V_1 - 3V_2 + V_3 = 0$

① $2V_1 - 3V_2 + V_3 = 0$

② $4V_1 - 7V_2 + V_3 = 0$

③ $3V_1 - 2V_2 - V_3 = 12$

① + ② : $5V_1 - 5V_2 = 12$

② − ① : $2V_1 - 4V_2 = 0$

$V_1 - 2V_2 = 0$

$V_1 = 2V_2$

$10V_2 - 5V_2 = 12$

$V_2 = \frac{12}{5} = 2.4\ V$

$V_1 = 4.8\ V$

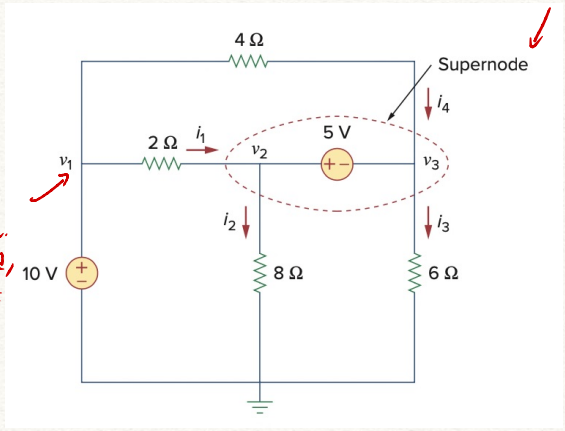
$V_3 = -2.4\ V$

$V_3 = 3(2.4) - 2(4.8)$

$= -2.4\ V$

WITH VOLTAGE SOURCES

if voltage src. is btw two non-ground nodes, can combine into supernode.



if voltage src is btw. node & ground, set voltage = battery.

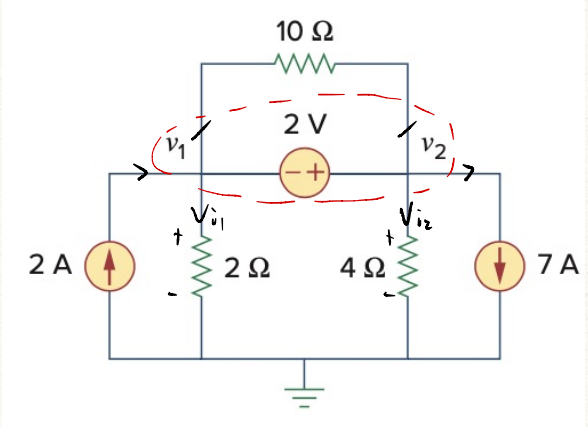
- KCL must still hold over a supernode. i.e. $i_1 + i_4 = i_2 + i_3$

$$\Rightarrow \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = \frac{V_2}{8} + \frac{V_3}{6}$$

- at the supernode, $V_2 - V_3 = 5$ (KVL)

- ✱ Voltage inside supernode provides an eqn required to solve the system.
- ✱ Supernode has no voltage of its own
- ✱ Supernodes require application of KCL/KVL

Ex.



$V_2 - V_1 = 2\text{ V}$

$V_2 = 2 + V_1$

KCL @ SN:

$2 = i_1 + i_2 + 7$

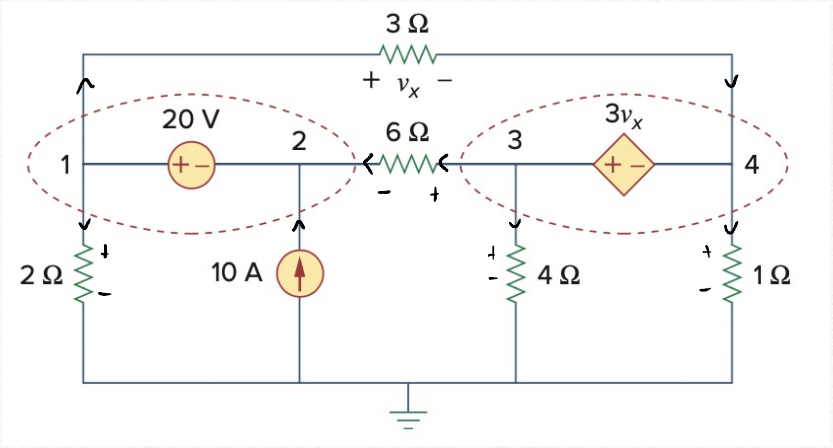
$2 = \frac{V_1}{2} + \frac{V_2}{4} + 7$

$8 = 2V_1 + V_2 + 28$

$-20 = 2V_1 + 2 + V_1$

$-22 = 3V_1$

$V_1 = -\frac{22}{3}\text{ V}$ $\quad V_2 = -\frac{16}{3}$



$V_1 - V_2 = 20$

$V_3 - V_4 = 3V_x$

$V_x = V_1 - V_4$ not correct!

$V_3 - V_4 = 3V_1 - 3V_4$

$\Rightarrow 3V_1 - V_3 - 2V_4 = 0$

KCL @ SN:

$\frac{V_3 - V_2}{6} + 10 = \frac{V_1}{2} + \frac{V_1 - V_4}{3}$

$V_3 - V_2 + 60 = 3V_1 + 2V_1 - 2V_4$

$\frac{V_1 - V_4}{3} = \frac{V_4}{1} + \frac{V_2}{4} + \frac{V_3 - V_2}{6}$

$8V_1 - 8V_4 = 24V_4 + 6V_2 + 4V_3 - 4V_2$

$$\begin{cases} 5V_1 + V_2 - V_3 - 2V_4 = 60 \\ 4V_1 + 2V_2 - 5V_3 - 16V_4 = 0 \\ 3V_1 - V_3 - 2V_4 = 0 \\ V_1 - V_2 = 20 \end{cases}$$

$V_1 = \frac{80}{3}$ $V_2 = \frac{20}{3}$ $V_3 = \frac{520}{3}$ $V_4 = -\frac{140}{3}$

$V_1 = 26.\bar{6}\ V$ $V_2 = 6.\bar{6}\ V$ $V_3 = 173.\bar{3}\ V$ $V_4 = -46.\bar{6}\ V$