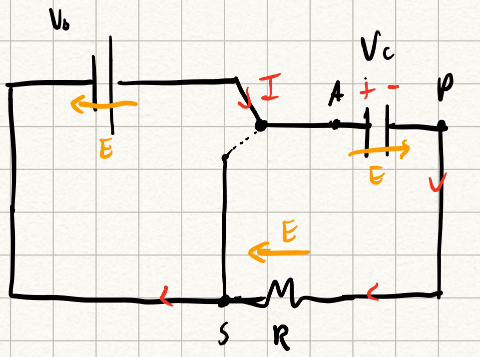
RC CIRCUITS



$V_c = V_A - V_P$

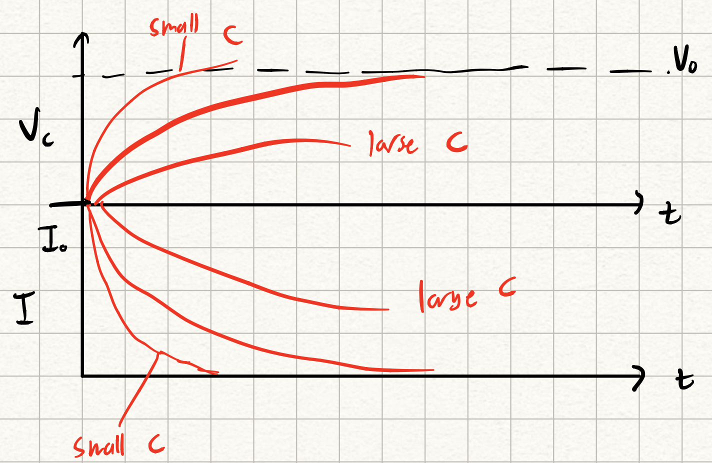
$V_E - IR = V_P - V_C$

$t = 0, \quad V_c = 0$

$t > 0, \quad V_c \uparrow, \ I \downarrow$

$t \to \infty, \quad V_c = V_0 \quad I = 0$

charging a capacitor makes positive charge accumulate on one side, which will oppose the Emf until $V_c = V_0$



$V_c = V_0 + (\cancel{V_i} - V_\infty) e^{-t/RC}$

$\Rightarrow V_c = V_0 (1 - e^{-t/RC})$

$I = \cancel{I_\infty} + (I_0 - \cancel{I_\infty}) e^{-t/RC}$

$\Rightarrow I = I_0 e^{-t/RC} = \frac{V_0}{R} e^{-t/RC}$

$\oint \vec{E} \cdot d\vec{l} = 0 \qquad I = \frac{dQ}{dt}, \quad V_c = \frac{Q}{C}$

$\Rightarrow V_c + IR - V_0 = 0$

$\frac{Q}{C} + R\frac{dQ}{dt} - V_0 = 0$

$R\,dQ = (V_0 - \frac{Q}{C})\,dt$

$RC\,dQ = (V_0 C - Q)\,dt$

$\frac{1}{V_0 C - Q}\,dQ = \frac{1}{RC}\,dt$

$\ln(V_0 C - Q) = \frac{-t}{RC} + K$

$V_0 C - Q = e^{-t/RC + K}$

$Q = V_0 C - e^{-t/RC} e^{K}$

$Q(t=0) = V_0 C - e^{K} = 0$

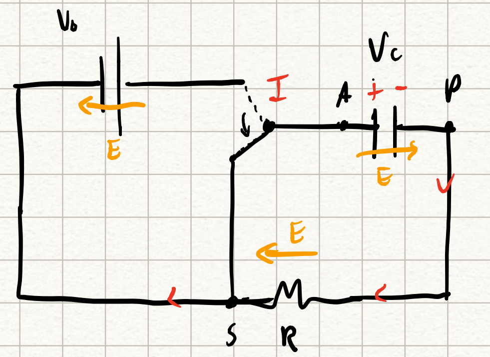
$e^{K} = V_0 C$

$\boxed{Q = V_0 C (1 - e^{-t/RC})} \quad \xRightarrow{\frac{d}{dt}} \quad \boxed{I = \frac{V_0}{R} e^{-t/RC}}$

rearranging,

$\boxed{V = V_0 (1 - e^{-t/RC})}$

if we now flip the switch,



$$V_c + IR = 0$$

$$\frac{Q}{C} + R\frac{dQ}{dt} = 0$$

$$\frac{Q}{C} = -R\frac{dQ}{dt}$$

$$Q + RC\frac{dQ}{dt} = 0$$

$$V = 0 + (V_0)e^{-t/RC}$$

$$RC\frac{dQ}{dt} = -Q$$

$$I = \frac{V_0}{R}e^{-t/RC}$$

$$-\frac{1}{Q}\,dQ = \frac{1}{RC}\,dt$$

$$\ln(-Q) = -\frac{t}{RC} + K$$

$$-Q = e^{-t/RC + K}$$

$$-Q(t) = e^{K} = 0$$

$$-Q = e^{-t/RC}$$