SEQUENCES

We can treat sequences as functions.

↳ this allows us to apply limits!

Given sequence $\{a_n\}$, if we have $f(x)$ s.t. $f(n) = a_n$
if $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} a_n = L$

Thus, the following limit properties hold if a_n, b_n are convergent.

1. $\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$
2. $\lim_{n\to\infty} (c \cdot a_n) = c \cdot \lim_{n\to\infty} a_n$
3. $\lim_{n\to\infty} (a_n \cdot b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$
4. $\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$
5. $\lim_{n\to\infty} (a_n)^p = \left(\lim_{n\to\infty} a_n\right)^p$, provided $a_n \geq 0$
6. if $a_n \leq c_n \leq b_n$ for all $n > N$ and $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = L$,

   $\lim_{n\to\infty} c_n = L$. (squeeze thm)
7. if $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$
8. $\{r^n\}_{n=0}^{\infty}$ converges if $-1 < r \leq 1$
   diverges otherwise

   $$\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \end{cases}$$
9. if $\lim_{n\to\infty} a_{2n} = L$ and $\lim_{n\to\infty} a_{2n+1} = L$, $\{a_n\}$ converges and $\lim_{n\to\infty} a_n = L$.

terminology:

- increasing: $a_n < a_{n+1}$ for all n.
- decreasing: $a_n > a_{n+1}$ for all n.
- monotonic: increasing or decreasing
- bounded below: if $\exists m$, $m \leq a_n$ for every n.
- bounded above: if $\exists M$, $M \geq a_n$ for every n,
- bounded: bounded above AND below

☆ if $\{a_n\}$ is bounded and monotonic, it is convergent.
↳ however, can be bounded and not convergent