AC POWER

instantaneous power

$$p(t) = \frac{v^2(t)}{R} = v(t) \cdot i(t)$$

average power

$$P_{av} = \frac{1}{T} \int_0^T p(t)\, dt$$

$$= \frac{1}{T} \int_0^T \frac{v^2(t)}{R}\, dt$$

$$= \frac{1}{R}\left(\sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}\right)^2$$

define V_{RMS} as $\sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}$

so $\boxed{P_{av} = \frac{V_{rms}^2}{R}}$

for sinusoidals, $\boxed{V_{RMS} = \frac{V_p}{\sqrt{2}}}$

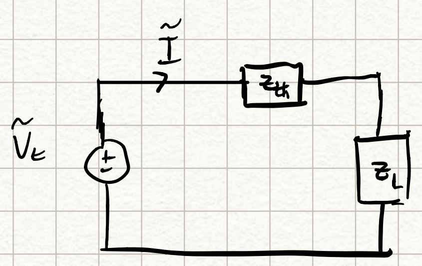
from this, $\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$

maximum power

Similar to DC, we first Thevenin-ize the circuit:

$Z_{th} = R_{th} + X_{th}\, j$

$Z_L = R_L + X_L\, j$



We know that maximum power is transferred when

$R_{th} = R_L$ and $X_L = -X_{th}$

$Z_L = R_{th} - X_{th} j = Z^*_{th}$

$$\boxed{Z_L = Z^*_{th}}$$

thus, if Z_{th} is inductive, Z_L is capacitive
if Z_{th} is capacitive, Z_L is inductive

$$\boxed{P_{max} = \frac{|V_{th}|^2}{8R_{th}}}$$

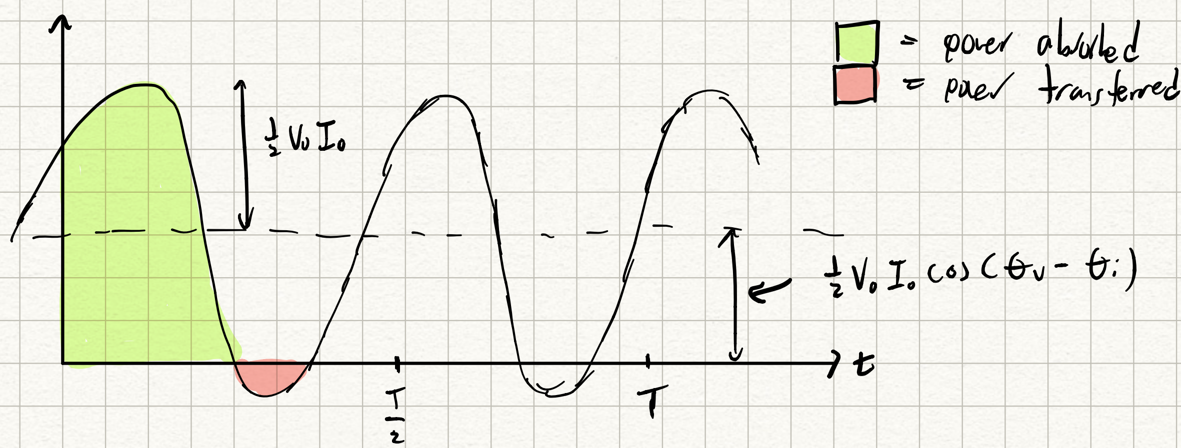
real power

if $V(t) = V_m \cos(\omega t + \theta_v)$ and
$i(t) = I_m \cos(\omega t + \theta_i)$,

$$p(t) = V(t) \cdot i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$\because \cos(a)\cos(b) = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$,

$$\underbrace{p(t)}_{\text{instant power} =} = \frac{1}{2} V_m I_m \Big[\underbrace{\cos(\theta_v - \theta_i)}_{\text{constant}} + \underbrace{\cos(2\omega t + \theta_v + \theta_i)}_{\text{time dependent w/ double angular freq.}}\Big]$$



let;

$$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \quad \text{(real/active power)}}$$

and

$$\boxed{Q = \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i) \quad \text{(reactive power)}}$$

$$p(t) = \frac{1}{2} V_m I_m [\cos(\theta_v - \theta_i) + \cos(2wt + \theta_v - \theta_i)]$$

$\because \cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta,$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\cos(2wt) + \frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)\sin(2wt)$$

$$= P + P\cos(2wt) + Q\sin(2wt)$$

power factor

$pf = \cos(\theta_v - \theta_i)$

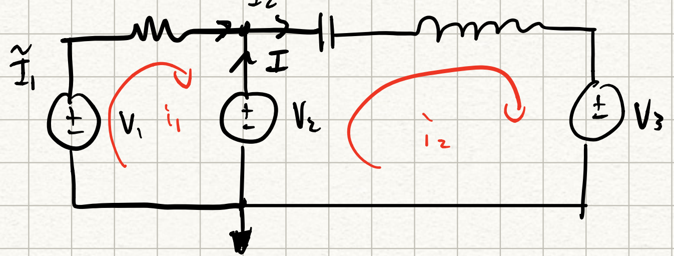
$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} V_m \cdot I_m \cdot pf$

$Q = CV$

$-(62.5j) \quad (7j)$

$-V_2 + \frac{Q_c}{C} + L\frac{di}{dt} + V_3 = 0$

ex.



$+V_2 - V_3 - \frac{Q_c}{C} = +L\frac{di}{dt}$

$I = \int 10\cos(1000t + \frac{\pi}{2}) - 240\cos(1000 - \frac{\pi}{2}) -$

$W = 1000$

$\tilde{V}_1 = 15 I_1 + \tilde{V}_2$

$Z_R = 15\,\Omega$

$Z_C = -j\frac{1}{wC} = -j\frac{1}{1000(16\times10^{-6})}$

$Z_L = jwL = j(1000)(7\times10^{-3})$

$V_2 = I_2(Z_1 + Z_2) + V_3$

$I_1 = \frac{V_1 - V_2}{15}$

$\tilde{V}_1 = 20 \angle 0°$

$V_2 = 10 \angle 90°$

$-\tilde{V}_2 = 10 \angle 90°$

$-\tilde{V}_3 = 240 \angle -90°$

$\Sigma I = i_2 - i_1$

$I_1 = \frac{4}{3} - \frac{2}{3}j$

$= 26.565°$

$I_2 = \frac{V_2 - V_3}{7 - 62.5j}$

$= -4.5045\ A = 4.5045 \angle$

$pf = \cos(0 - 26.565°) = 0.8944$

$\Sigma I = -4.5045 - \left(\frac{4}{3} - \frac{2}{3}j\right) = -5.17 - \frac{1}{3}i$

$\Rightarrow -176.31°$