SUPERPOSITION OF E. FIELDS

Since force follows superposition,

$$\sum F = F_1 + F_2 + F_3 + \dots = q_0 \vec{E}_1 + q_0 \vec{E}_2 + \dots$$

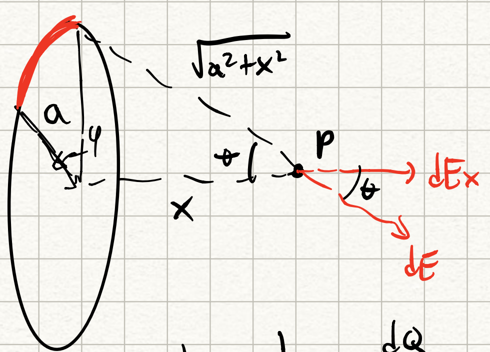
$$\sum E = \frac{\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots}{q_0} = \frac{\vec{F}_1}{q_0} + \frac{\vec{F}_2}{q_0} + \dots = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$

λ = linear charge density $\left(\frac{C}{m}\right)$

σ = surface charge density $\left(\frac{C}{m^2}\right)$

ρ = volume charge density $\left(\frac{C}{m^3}\right)$

ex. field across ring of charge



ah

$$\cos\theta = \frac{x}{\sqrt{a^2+x^2}}$$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{x^2+a^2}$$

$$dQ = \lambda\, ds = \lambda (a\, d\varphi) \Rightarrow Q = \int_0^{2\pi} \lambda a\, d\varphi = 2\pi\lambda a$$

$$dE_x = \frac{1}{4\pi\varepsilon_0} \left(\frac{\lambda a\, d\varphi}{x^2+a^2} \right) \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \left(\frac{\lambda a x}{(x^2+a^2)^{3/2}} \right) d\varphi$$

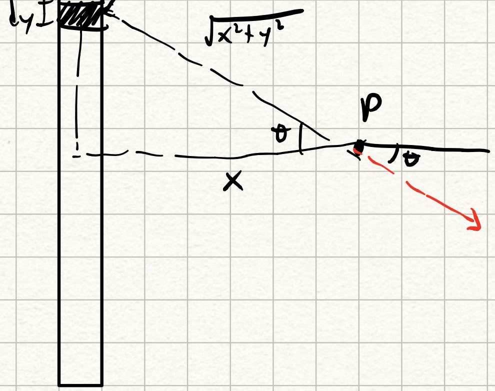
$$E = \int dE_x = \int_0^{2\pi} \frac{\lambda a x}{4\pi\varepsilon_0 (x^2+a^2)^{3/2}}\, d\varphi = \frac{\lambda a x}{4\pi\varepsilon_0 (x^2+a^2)^{3/2}} (2\pi) = \frac{Qx}{4\pi\varepsilon_0 (x^2+a^2)^{3/2}}$$

alternatively

$$dE_x = \frac{1}{4\pi\varepsilon_0} \left(\frac{\lambda x}{(x^2+a^2)^{3/2}} \right) ds \quad , \quad E = \int_0^{2\pi a} \frac{\lambda x}{4\pi\varepsilon_0 (x^2+a^2)^{3/2}}\, ds$$

ex. charged line segment

$$\lambda = \frac{Q}{2a} \quad , \quad dQ = \lambda\, dy = \frac{Q}{2a}\, dy$$



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2a}\frac{1}{(x^2+y^2)}\,dy$$

$$dE_x = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2a}\right)\left(\frac{1}{x^2+y^2}\right)\left(\frac{x}{\sqrt{x^2+y^2}}\right)dy$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2a}\right)\int_{-a}^{a}\frac{x}{(x^2+y^2)^{3/2}}\,dy$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2a}\right)\left(\frac{y}{x\sqrt{x^2+y^2}}\Bigg|_{-a}^{a}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\cancel{2a}}\right)\left(\frac{\cancel{2a}}{x\sqrt{x^2+a^2}}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0\, x\sqrt{x^2+a^2}}$$

$$\int\frac{1}{(a^2+x^2)^{3/2}}\,dx$$

$$x = a\tan u \Rightarrow dx = a\sec^2 u$$

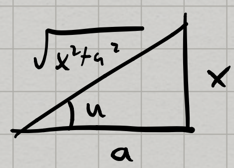
$$= \int\frac{1}{(a^2+a^2\tan^2 u)^{3/2}}\,dx$$

$$= \frac{1}{a^2}\int\frac{1}{\sec^3 u}(a\sec^2 u)\,du$$

$$= \frac{1}{a^2}\int\cos u\,du$$

$$= \frac{\sin u}{a^2} = \frac{1}{a^2}\left(\frac{x}{\sqrt{x^2+a^2}}\right) + C$$

$$\frac{x}{a} = \tan u$$



$$\sin u = \frac{x}{\sqrt{x^2+a^2}}$$

if segment is <u>very</u> short $x \gg a$,

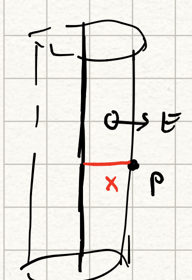
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 x^2}$$

if segment is <u>very</u> long, $a \gg x$,

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0\, x\sqrt{x^2+a^2}} = \frac{1}{2\pi\varepsilon_0}\left(\frac{Q}{2a}\right)\left(\frac{1}{x\sqrt{\left(\frac{x^2}{a^2}\right)+1}}\right) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x} = \frac{\lambda}{2\pi\varepsilon_0 x}$$

so $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 x}$ (infinite line of charge)

- using Gauss's Law

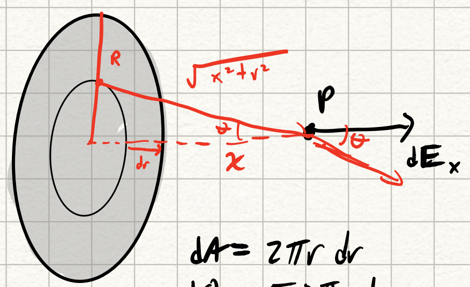


$$\phi = \frac{Q_{enc}}{\varepsilon_0} = \oint \vec{E}\,d\vec{A} = \vec{E}A$$

$$\frac{Q}{\varepsilon_0} = \vec{E}(2\pi x h)$$

$$\lambda = \frac{Q}{h} \Rightarrow \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 x}$$

Ex: field of uniformly charged disk



$$dA = 2\pi r\, dr$$
$$dQ = \sigma\, 2\pi r\, dr$$

$$dE_{ring} = \frac{1}{4\pi\varepsilon_0}\left(\frac{\lambda r\, d\varphi}{x^2+r^2}\right)\left(\frac{x}{\sqrt{x^2+r^2}}\right)$$

$$= \frac{(2\pi r \lambda)\, x}{4\pi\varepsilon_0 (x^2+r^2)^{3/2}}$$

$$= \frac{Qx}{4\pi\varepsilon_0 (x^2+r^2)^{3/2}}$$

$$dE_{disk} = \frac{x\, dQ}{4\pi\varepsilon_0 (x^2+r^2)^{3/2}} = \frac{x}{4\pi\varepsilon_0}\left(\frac{\sigma 2\pi r}{(x^2+r^2)^{3/2}}\right) dr$$

$$E_{disk} = \frac{\cancel{2\pi}\,\sigma x}{\underset{2}{\cancel{4\pi}}\,\varepsilon_0}\int_0^R \frac{r}{(x^2+r^2)^{3/2}}\, dr \qquad \begin{aligned} u &= r^2+x^2 \\ du &= 2r \end{aligned}$$

$$= \frac{\sigma x}{4\varepsilon_0}\int_0^R u^{-3/2}\, du$$

$$= \frac{\sigma x}{4\varepsilon_0}\left(-2u^{-\frac{1}{2}}\Big|_?^?\right)$$

$$= -\frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{r^2+x^2}}\right)\Bigg|_0^R$$

$$= \frac{\sigma x}{2\varepsilon_0}\cdot -\left(\frac{1}{\sqrt{R^2+x^2}} - \frac{1}{x}\right)$$

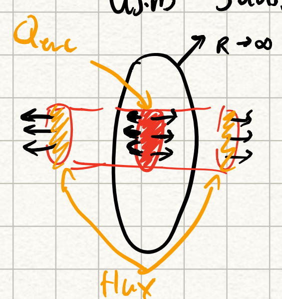
$$= \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{x} - \frac{1}{\sqrt{R^2+x^2}}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{\frac{R^2}{x^2}+1}}\right)$$

if disk is really large,

$$E_{disk} = \frac{\sigma}{2\varepsilon_0}$$

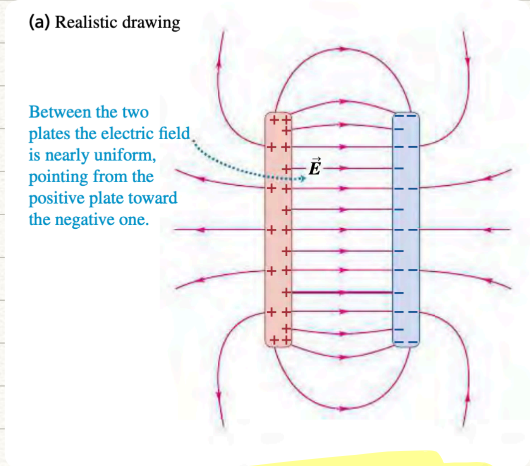
Using Gauss's Law!



$$\Phi = \frac{Q_{enc}}{\varepsilon_0} = \oint \vec{E}\cdot d\vec{A}$$

$$\Rightarrow \frac{Q_{enc}}{\varepsilon_0} = \vec{E}\cdot(2\pi r^2)$$

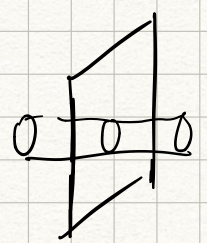
$$Q_{enc} = \sigma \pi r^2$$



$$\Rightarrow \frac{\sigma \cancel{\pi r^2}}{\varepsilon_0} = \vec{E}(2\cancel{\pi r^2})$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}$$

infinite plane



$$\Phi = \frac{Q_{enc}}{\varepsilon_0} = \oint \vec{E}\cdot d\vec{A}$$

$$\Rightarrow \frac{Q_{enc}}{\varepsilon_0} = \vec{E}\cdot(2\pi r^2)$$

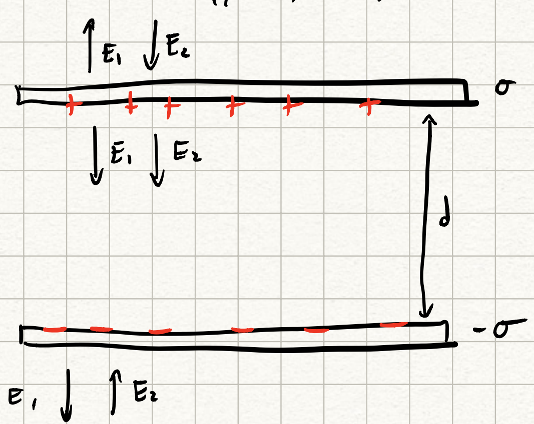
$$Q_{enc} = \sigma\pi r^2$$

$$\Rightarrow \frac{\sigma\pi r^2}{\varepsilon_0} = \vec{E}(2\pi r^2)$$

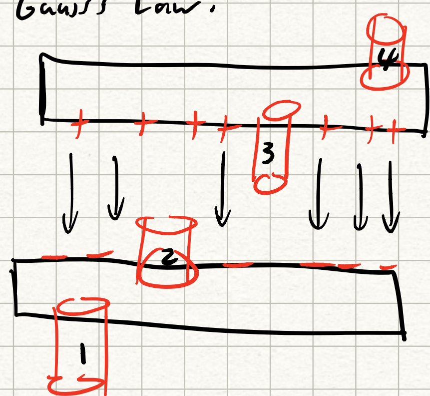
$$\Rightarrow \vec{E} = \frac{\sigma}{2\varepsilon_0}$$

★ electric field for all 2D planes are $E = \frac{\sigma}{2\varepsilon_0}$

ex. field of two oppositely charged conducting sheets, ∞ in length



Using Gauss's Law:



The electric field from a singular plane is

$\vec{E} = \frac{\sigma}{2\varepsilon_0}$, $|\vec{E}_1| = |\vec{E}_2|$

$$\Sigma\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{when above } \sigma \\ \frac{\sigma}{\varepsilon_0} & \text{in between} \\ 0 & \text{when below } -\sigma \end{cases}$$

1: $\Phi = \frac{Q_{enc}}{\varepsilon_0} = \oint \vec{E}\cdot d\vec{A}$

$\frac{0}{\varepsilon_0} = E\cdot(2\pi r^2)$

$E = 0$

2: $\Phi = \frac{-\sigma(\pi r^2)}{\varepsilon_0} = -E(\pi r^2)$ ← only top

$E = \frac{\sigma}{\varepsilon_0}$

3: $\Phi = \frac{\sigma\pi r^2}{\varepsilon_0} = E(\pi r^2)$

$E = \frac{\sigma}{\varepsilon_0}$

4: $E = 0$

☿ Electric field lines are not trajectories

- $\vec{E}$ is tangent to the field line at that point
- $\vec{F} = q\vec{E}$ is exerted on the particle
- however, since moving along curved path, acceleration cannot be tangent to that path.

☿ electric field != electric field lines

- magnitude of $\vec{E}$ is usually different along the same line
- direction of $\vec{E}$ is tangent to line