OPERATIONAL AMPLIFIERS

Mar. 3, 2020

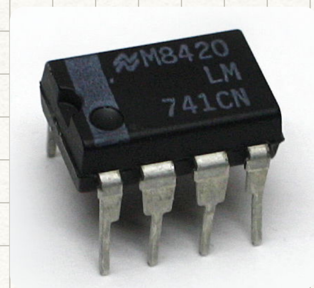
→ amplifying signal

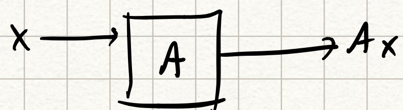
→ building block of analog circuits

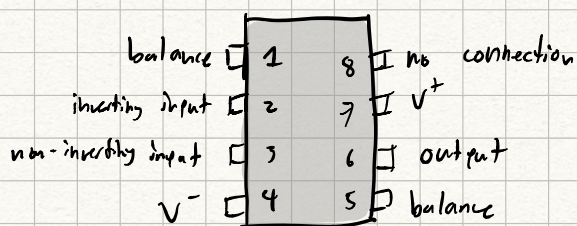
→ behaves like voltage-controlled voltage source

→ designed to perform mathematical operations like

- addition
- subtraction
- multiplication
- division
- differentiation
- integration







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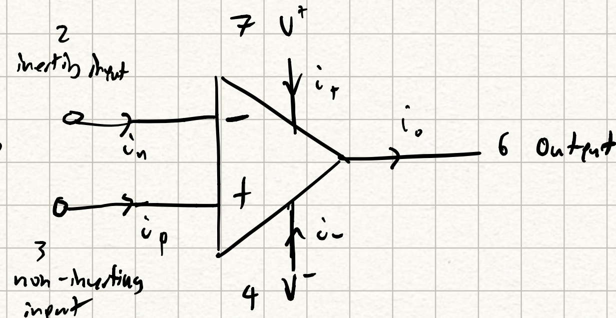


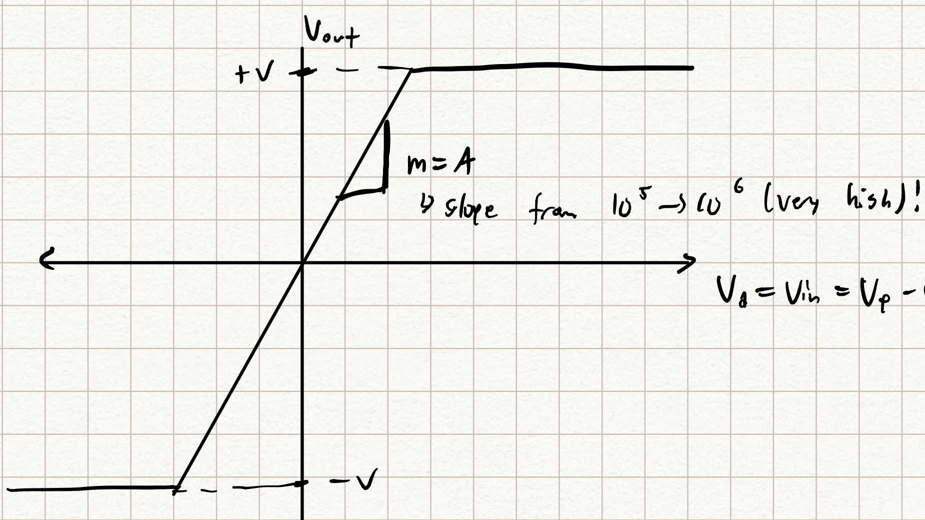
$i_o = i_p + i_n + i_+ + i_-$

has differential input

→ $V_o = A(\underbrace{V_p - V_n}_{V_d})$

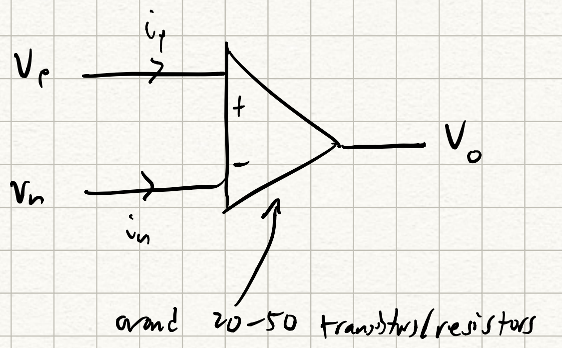
A is known as gain, is a scaling factor

→ usually between $10^5 - 10^6$



$V_d = V_{in} = V_p - V_n$

→ cannot exceed power supply!
→ known as saturation

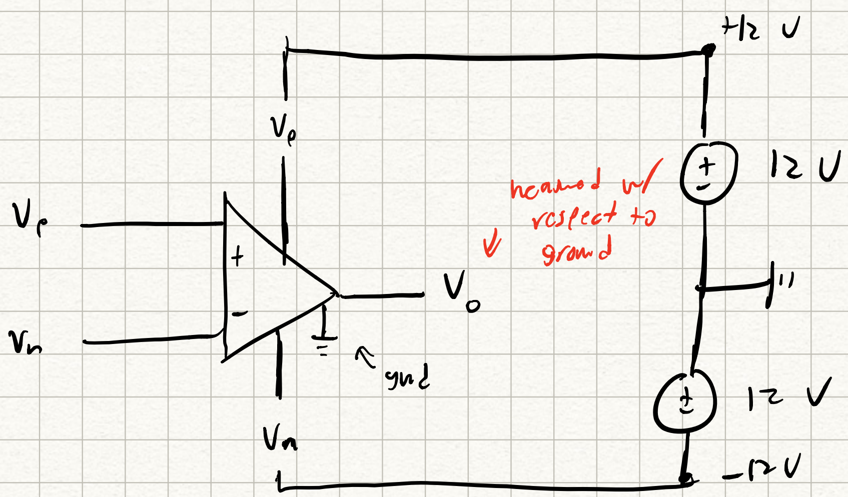


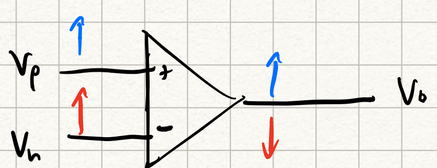
ideally!

$i_p = i_n = 0$

$V_o = A(V_p - V_n)$

$V_o = A V_{in}$

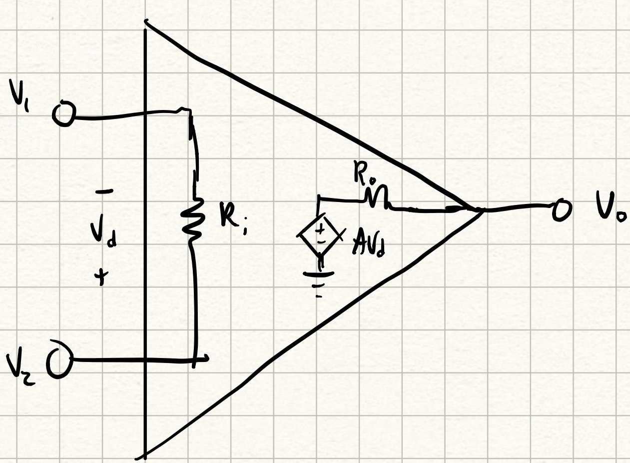




Since $V_o = A(V_p - V_n)$, when

$V_p \uparrow$, $V_o \uparrow$. Thus V_p is non-inverting input

$V_n \uparrow$, $V_o \downarrow$. Thus V_n is inverting input



non-ideal op-amp

R_i is Thevenin equivalent resistance btw. input terminals

R_o is output resistance, or Thevenin equivalent resistance at output

☆ in an ideal op-amp, $R_{out} = 0$.
↳ $V_o = A(V_p - V_n)$
→ why?

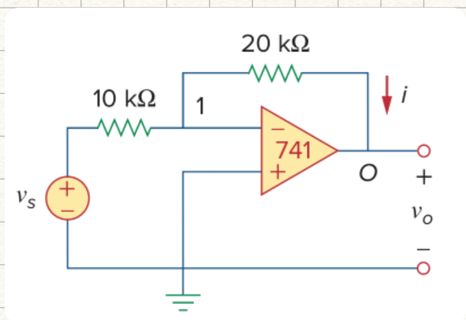
☆ we assume op-amp operates in linear range

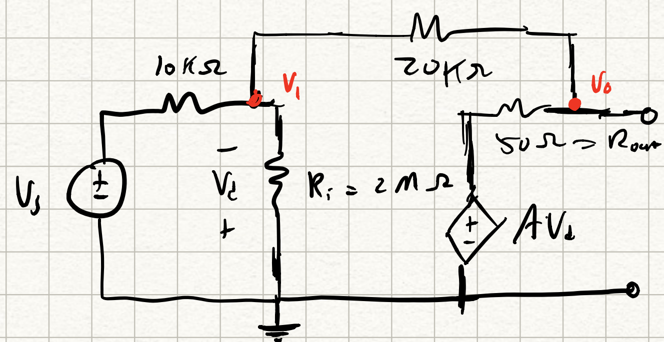
☆ R_{in} is usually really high
→ why?

Ex.

741 op-amp has open loop voltage gain of 2×10^5
input resistance of $2\,M\Omega$
output resistance of $50\,\Omega$.

Find closed loop gain $\frac{V_o}{V_s}$. Find i when $V_s = 2V$.





at node 1:

$$\frac{V_s - V_1}{10\times10^3} = \frac{V_1}{2\times10^6} + \frac{V_1 - V_o}{20\times10^3}$$

V_1:

$$200\,(V_s - V_1) = V_1 + 100(V_1 - V_o)$$

$$200\,V_s = 301\,V_1 - 100\,V_o$$

$$2V_s \simeq 3V_1 - V_o \Rightarrow V_1 = \frac{2V_s + V_o}{3}$$

V_o:

$$\frac{V_o - AV_d}{50} = \frac{V_1 - V_o}{20\times10^3}$$

$$V_d = -V_1, \quad A = 2\times10^5$$

$$\Rightarrow \frac{V_o + (2\times10^5)(V_1)}{50} = \frac{V_1 - V_o}{20\times10^3}$$

$$400V_o + 8\times10^7\,V_1 = V_1 - V_o$$

$$400\,V_o + 8\times10^7\left(\frac{2V_s + V_o}{3}\right) = \left(\frac{2V_s + V_o}{3}\right) - V_o$$

$$0 \simeq 26{,}667{,}067\,V_o + 53{,}333{,}333\,V_s$$

$$\frac{V_o}{V_s} = -1.9999699$$

→ closed loop gain, since 20kΩ connects input & output

when $V_s = 2$, $V_o = -3.9999398$ V.

$$V_1 = \frac{2(2) - 3.9999398}{3} = 20.066667 \text{ mV.}$$

$$i = \frac{V_1 - V_o}{20\times10^3} = 0.199994 \text{ mA.}$$

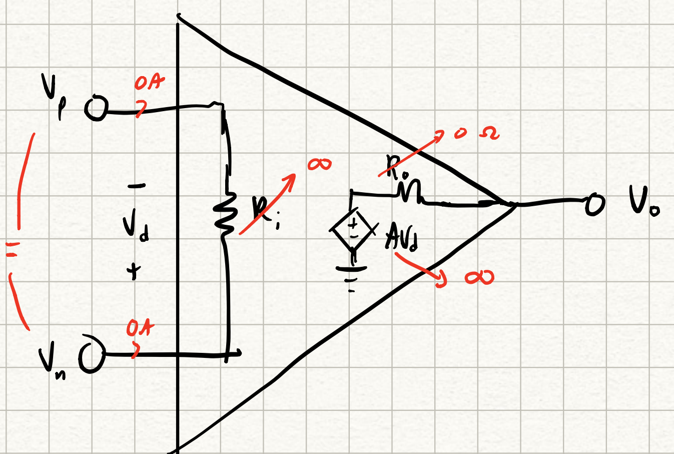
→ dealing w/ non-ideal op-amp are tedious since numbers are really large & need to be precise.

IDEAL OP-AMP

Mar. 02, 2020

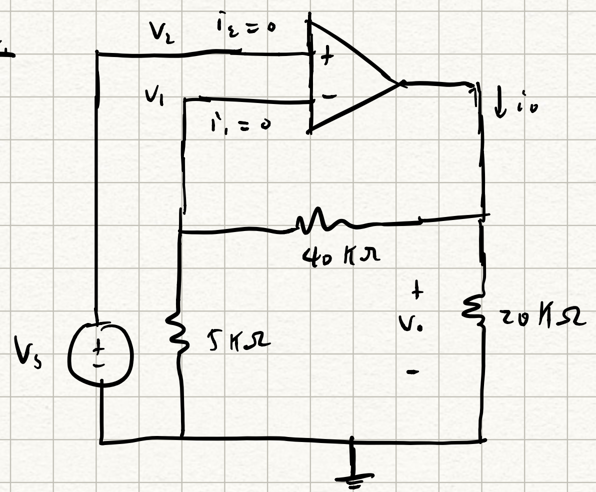
* ideal op-amps are...

1. ∞ open loop gain $A \approx \infty$
2. ∞ input resistance, $R_i \approx \infty$
3. 0 output resistance, $R_o \approx 0$

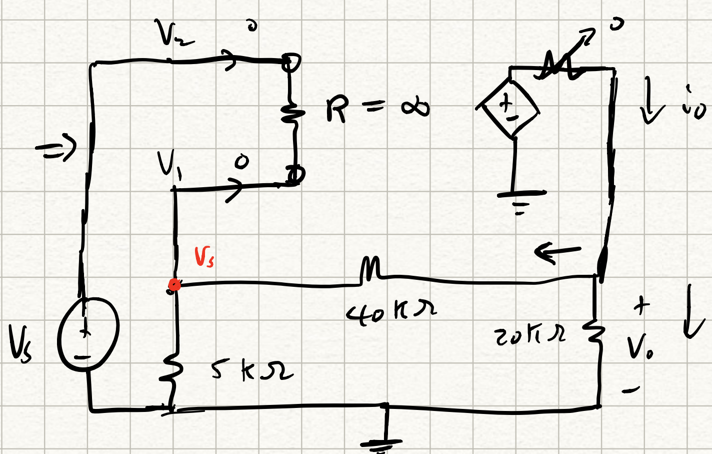


ideal op amp

Ex.



Calculate $\frac{V_o}{V_s}$ and i_o when $V_s = 1$ V.



$V = V_1 = V$

$R_{eq} = (40+5)$

$V_1 = \frac{5}{5+40} V_0 = \frac{V_0}{9}$

$V_2 - V_1 = V_s$ and $V_2 = V_1$, so $V_1 = V_s$.

so $V_s = \frac{V_0}{9}$

so $\frac{V_0}{V_s} = 9$

When $V_s = 1$, $V_0 = 9$

$\Rightarrow V_1 = \frac{1}{9}$

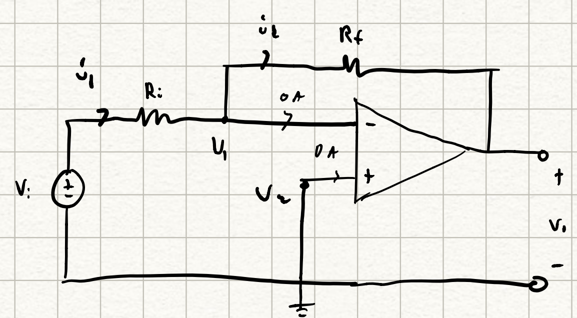
$i_0 = \frac{V_0}{20 \times 10^3} + \frac{V_0 - V_s}{40 \times 10^3}$

$i_0 = \frac{9}{20 \times 10^3} + \frac{9}{45 \times 10^3}$

$i_0 = 0.65 \text{ mA}$

INVERTING AMPLIFIER

→ we want to invert an input voltage.



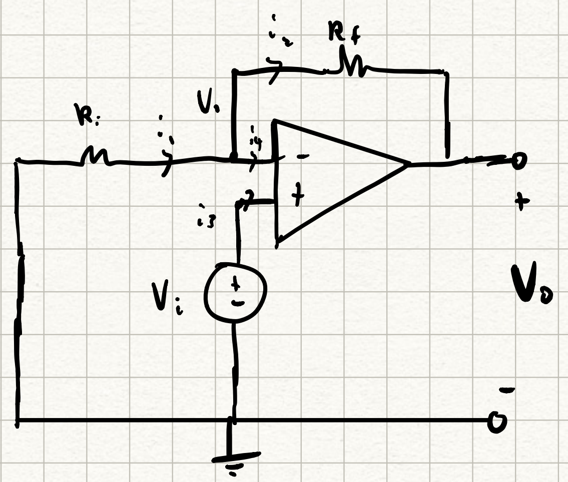
$i_1 = i_2 = \frac{V_i - V_o}{R_i + R_f}$ → this isn't helpful b.c. we want relationship btw. V_i & V_o.

$\Rightarrow \frac{V_i - V_1}{R_1} = \frac{V_1 - V_o}{R_f}$

Since $V_1 = V_2 = 0$ (ideal op amp),

$$\boxed{V_i = -\frac{R_1}{R_f} V_o}$$

NON-INVERTING AMPLIFIER



$i_3 = i_4 = 0$

$i_1 = i_2 = \frac{V_1 - V_o}{R_f} = \frac{0 - V_1}{R_i}$

$V_1 = V_i$

$\rightarrow \frac{V_i - V_o}{R_f} = \frac{-V_i}{R_i}$

$\frac{V_i}{R_f} + \frac{V_i}{R_i} = \frac{V_o}{R_f}$

$V_i R_f \left(\frac{1}{R_f} + \frac{1}{R_i} \right) = V_o$

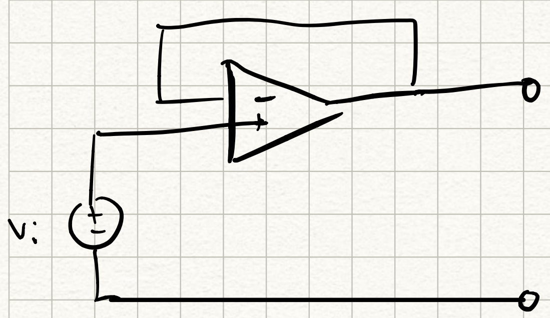
$$\boxed{V_o = \left(1 + \frac{R_f}{R_i}\right) V_i}$$

if $R_i = \infty$ (o.c), $V_o = V_i$

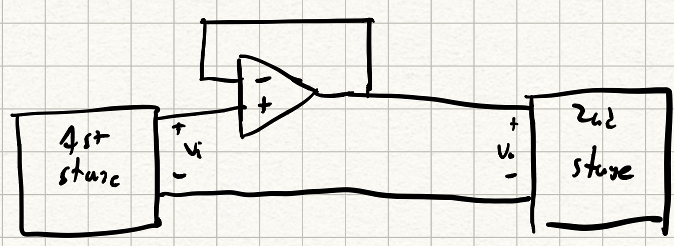
if $R_f = 0$ (s.c), $V_o = V_i$

when $V_o = V_i$, we call it a voltage follower or unity gain amplifier

→ used to isolate two stages of cascading circuit.

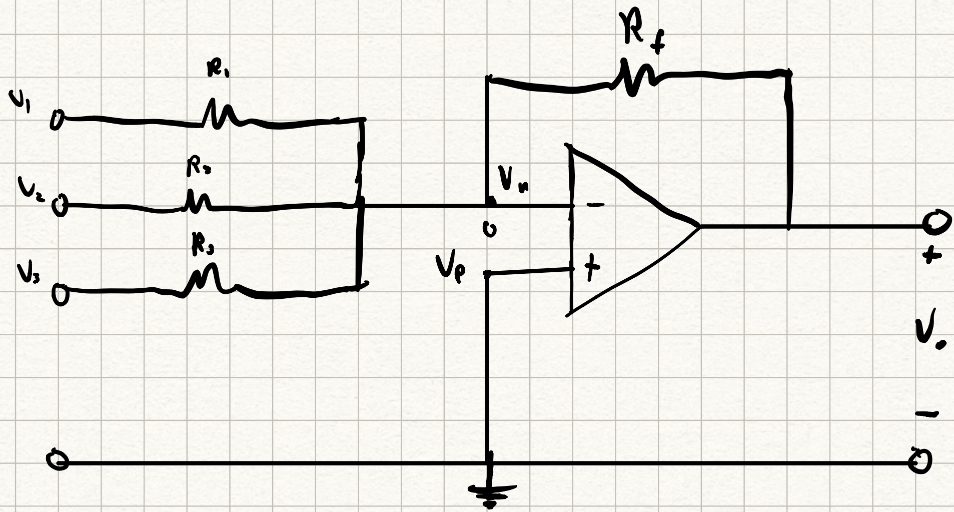


voltage follower



SUMMING AMP.

→ takes multiple inputs and produces output that is weighted sum of inputs

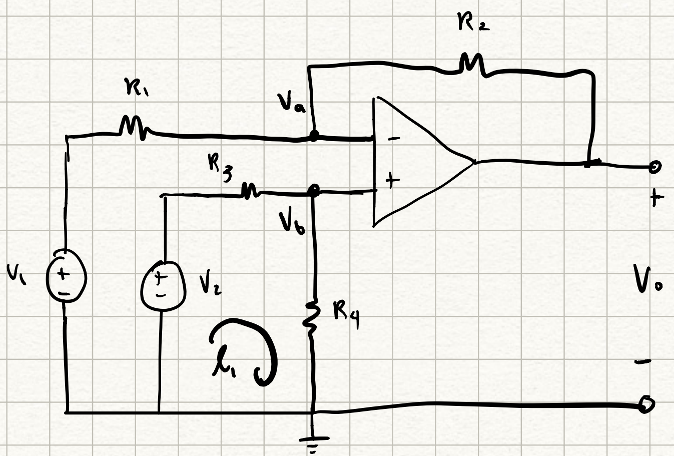


$$V_p = V_n = 0$$

$$\frac{V_1 - \cancel{V_n}}{R_1} + \frac{V_2 \cancel{- V_n}}{R_2} + \frac{V_3 \cancel{- V_n}}{R_3} = \frac{\cancel{V_n} - V_o}{R_f}$$

$$\boxed{V_o = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)}$$

DIFF. AMP.



$$\frac{V_1}{R_1} = \frac{V_a - V_o}{R_2}$$

KVL i_1 :

$$V_2 = i(R_3 + R_4)$$

$$V_b = \frac{R_4}{R_3 + R_4} V_2$$

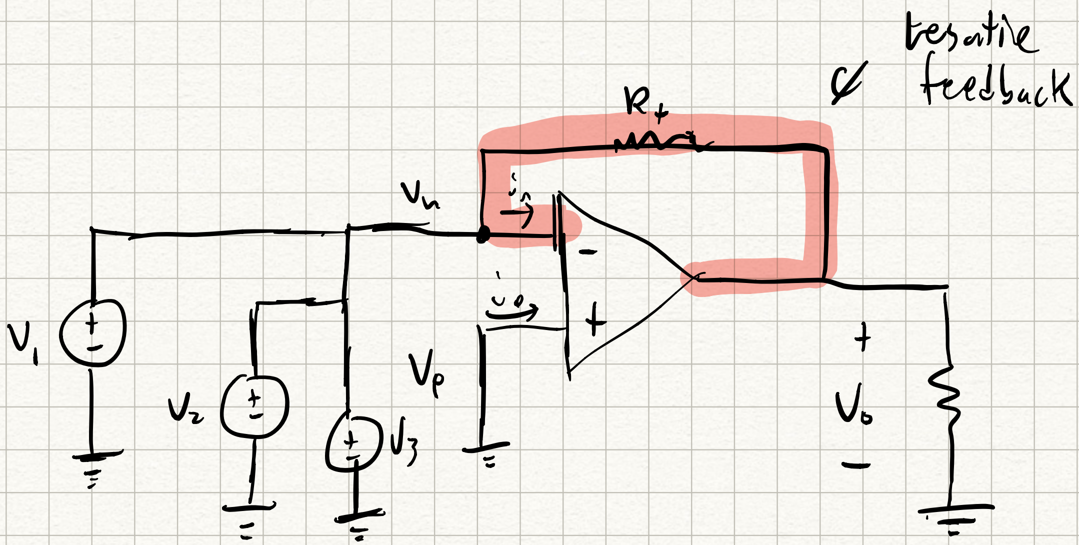
$$V_b = V_a$$

$$\Rightarrow \frac{V_1}{R_1} = \frac{\frac{R_4}{R_3+R_4} V_2 - V_o}{R_2}$$

$$\frac{R_2}{R_1} V_1 = \frac{R_4}{R_3 + R_4} V_2 - V_o$$

$$V_o = \frac{R_4}{R_3 + R_4} V_2 - \frac{R_2}{R_1} V_1$$

ADDER AMPLIFIER

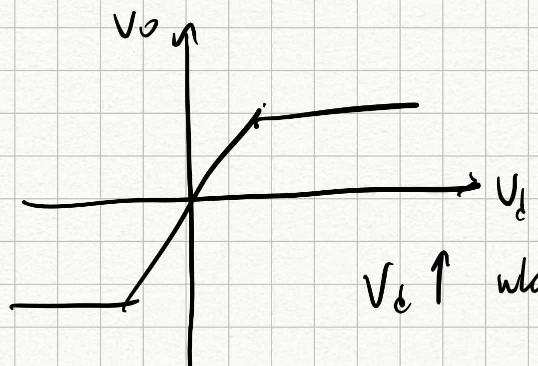


<u>KCL V_n</u>

$$\frac{\cancel{V_n}^{0} - V_1}{R_1} + \frac{\cancel{V_n}^{0} - V_2}{R_2} + \frac{\cancel{V_n}^{0} - V_3}{R_3} + \frac{\cancel{V_n}^{0} - V_o}{R_f} = 0$$

$$V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)$$

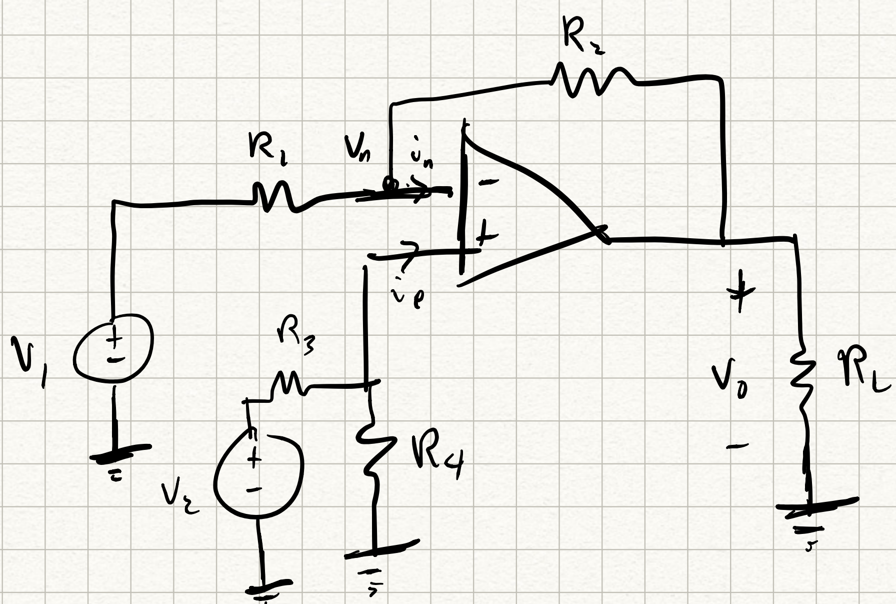
→ Swapping (+) and (−) has no effect
  → becomes positive feedback / drives into saturation
→ negative feedback <u>always</u>



V_d ↑ when positive feedback loop

→ V_d initially large, neg. feedback brings into linear region

DIFFERENCE AMPLIFIER



KCL @ V_n:

$$\frac{V_n}{R_1} - \frac{V_1}{R_1} + \frac{V_n}{R_2} - \frac{V_0}{R_2} = 0$$

$$\frac{V_n - V_1}{R_1} + \frac{V_n - V_0}{R_2} + \cancelto{0}{i_n} = 0$$

$$V_0 = \left(\frac{R_2}{R_1} + 1\right) V_n - \left(\frac{R_2}{R_1}\right) V_1$$

$$V_p = V_n = \frac{R_4}{R_3 + R_4} \cdot V_2$$

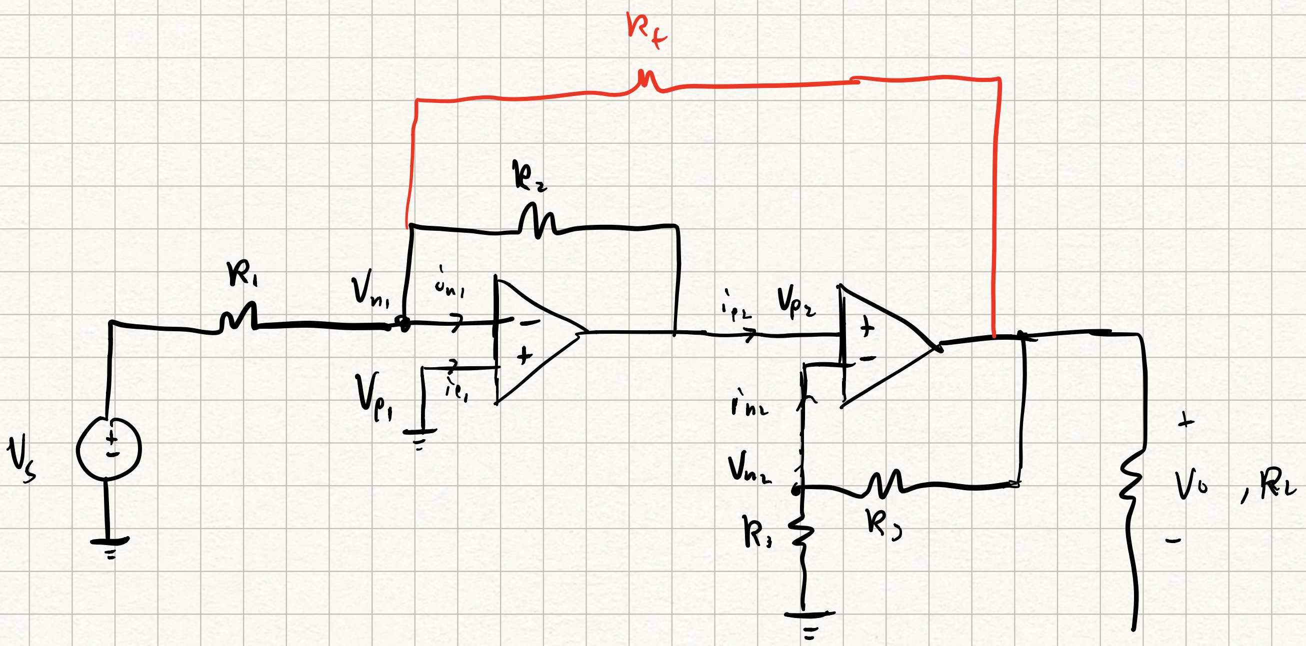
$$= \frac{1}{\left(\frac{R_3}{R_4} + 1\right)} \cdot V_2$$

$$V_0 = \frac{\left(\frac{R_2}{R_1} + 1\right)}{\left(\frac{R_3}{R_4} + 1\right)} V_2 - \left(\frac{R_2}{R_1}\right) V_1$$

$$= \frac{R_2}{R_1} \left[\frac{\frac{R_1}{R_2} + 1}{\frac{R_3}{R_4} + 1} V_2 - V_1 \right]$$

★ when $\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow V_o = \frac{R_2}{R_1}(V_2 - V_1)$

Ex:



$V_{p1} = 0$
$V_{n1} = 0$

KCL @ V_{n1}

$$\frac{\cancel{V_{n1}}^{0} - V_s}{R_1} + \frac{\cancel{V_{n1}}^{0} - V_{p2}}{R_2} + \cancel{i_{n1}}^{0} = 0$$

$$V_s = -\frac{R_1}{R_2} V_{p2} \quad \text{or} \quad \boxed{V_{p2} = -\frac{R_2}{R_1} V_s}$$

$V_{n2} = V_{p2}$

KCL @ V_{n2}

$$+ \frac{V_{n1} - V_o}{R_f}$$

$$\frac{V_{n2}}{R_4} + \frac{V_{n2} - V_o}{R_3} + \cancel{i_{n2}}^{0} = 0$$

$$= -\frac{R_2}{R_1 R_4} V_s - \frac{R_2}{R_1 R_3} V_s - \frac{V_o}{R_3} \Rightarrow 0$$

$$V_o = -\left(\frac{R_2}{R_1}\right)\left(1 + \frac{R_3}{R_4}\right) V_s$$

$R_1 = R_4 = 2K\Omega$

$R_2 = R_3 =$

$$G_v = \frac{V_o}{V_i} = -\underbrace{\left(1 + \frac{20}{2}\right)}_{11}\underbrace{\left(\frac{20}{2}\right)}_{10}$$

$G_v = -116$ V.