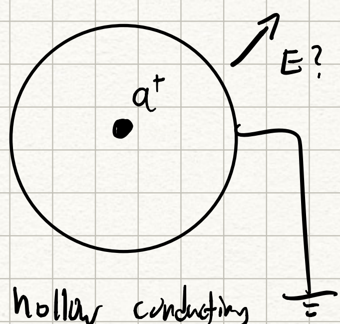
thus, $E = \frac{\sigma}{\varepsilon_0}$.

ex)



hollow conducting sphere

initially, q^- on inner surface
q on outer surface

no, electrons will be drawn from GND such that the sphere will have no charge on outer surface while maintaining q^- charge on inner surface.

ELECTRIC POTENTIAL

- Electrical Potential Energy: measure of work done in $\vec{E}$

$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b F\cos\theta\, dl$, where θ is angle between $\vec{F}$ and $d\vec{l}$ at each point

if $\vec{F}$ is a conservative force, then $\vec{F}$ can always be expressed as potential NRG.

$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$

↳ when $W_{a\to b}$ is positive, $U_a > U_b$, PE↓
(ex. ball falling, F does positive work)

↳ when ball tossed, g does negative work, PE↑.

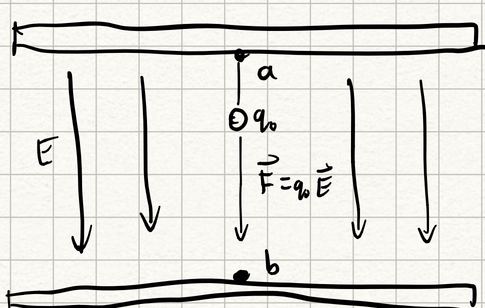
if only conservative forces do work, then,

$\Delta KE = -\Delta PE$

$KE_b - KE_a = -(U_b - U_a)$

$KE_b + U_b = KE_a + U_a$

$KE_i + U_i = KE_f + U_f$ (energy is conserved)



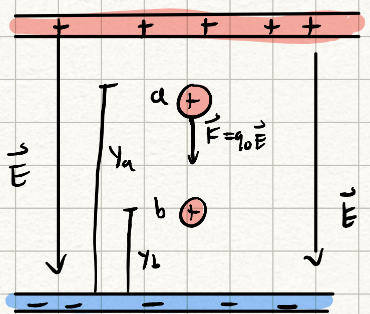
$W_{a\to b} = q_0 E d$

since q is (+), W is (+) since it goes along $\vec{E}$.

$W_{a\to b} = -\Delta U$

$\Rightarrow \Delta U = -q_0 E d$ (potential NRG ↓ from a→b)

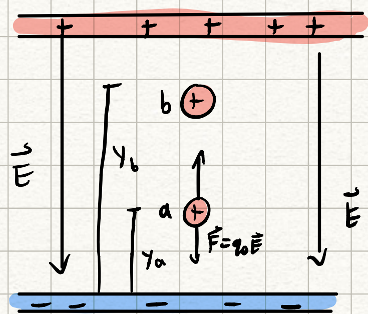
$$W_{a\to b} = -\Delta U = -(U_b - U_a) = -(qEy_b - qEy_a) = qE(y_a - y_b)$$



(+) charge, along $\vec{E}$

→ positive work by $\vec{E}$

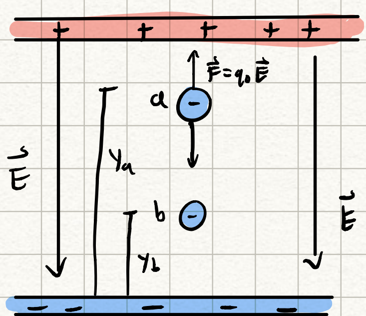
→ $U\downarrow$



(+) charge, against $\vec{E}$

→ negative work by $\vec{E}$

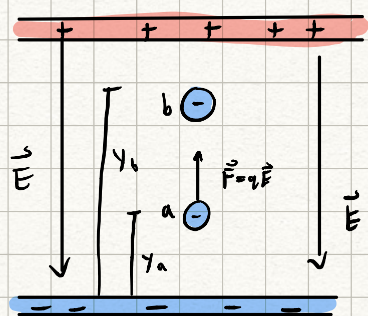
→ $U\uparrow$



(−) charge, along $\vec{E}$

→ negative work by $\vec{E}$

→ $U\uparrow$

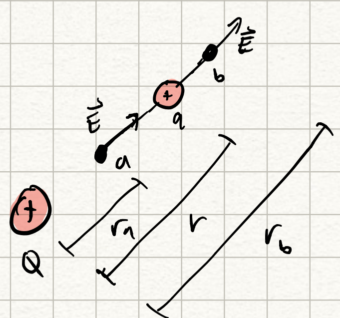


(−) charge, against $\vec{E}$

→ positive work by $\vec{E}$

→ $U\downarrow$

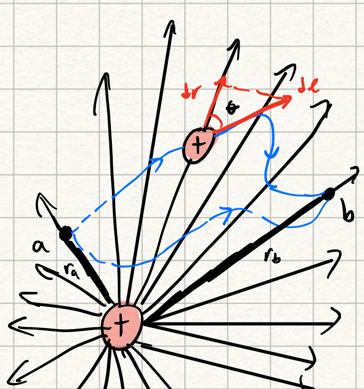
CALCULATING WORK



$$F = k\frac{Qq}{r^2}$$

$$W_{a\to b} = \int_{r_a}^{r_b} k\frac{Qq}{r^2}\,dr = kQq\int_{r_a}^{r_b}\frac{1}{r^2}\,dr = -kQq\left(\frac{1}{r}\Big|_{r_a}^{r_b}\right)$$

$$= \frac{Qq}{4\pi\epsilon_0}\underbrace{\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}_{U\downarrow}$$



$$W_{a\to b} = \int_{r_a}^{r_b} k\frac{Qq}{r^2}\cos\theta\,dl$$

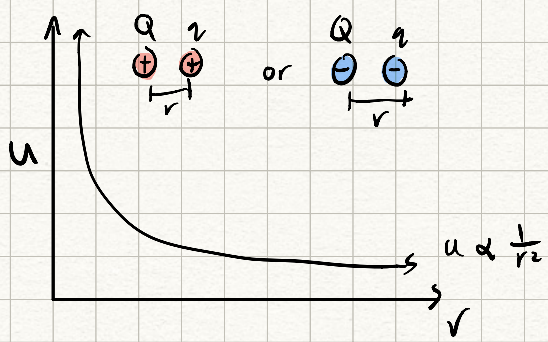
$$= \int_{r_a}^{r_b} \frac{kQq}{r^2}\,dr$$

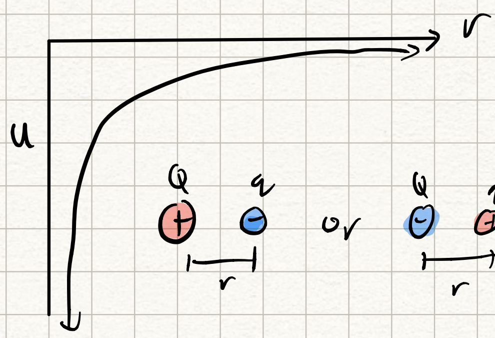
$$= \frac{Qq}{\dots}\left(\frac{1}{\dots} - \frac{1}{\dots}\right)$$ which is the same as before

$\frac{}{4\pi\epsilon_0}\left(\frac{}{r_a} - \frac{}{r_b}\right)$, which is the same as before.

$U = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r}$ EPE w/ 2 point charges

$U \propto -\frac{1}{r^2}$





☆ potential energy is a shared property between the two charges

this also holds if q_0 or q is spherically-symmetrical distribution w/ total charge q.
↳ gauss's law tells us that this is equivalent to charge all concentrated @ ctr.

multiple point charges

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \dots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

ELECTRIC POTENTIAL

- closely related to $\vec{E}$
  ↳ often easier to find potential than field
- defined as potential energy / unit charge

$$V = \frac{U}{q_0} = \frac{J}{C}$$

$$V_{ab} = V_a - V_b = -\frac{\Delta U}{q_0} = \frac{W_{a\to b}}{q_0}$$

V_{ab} is the amount of work required to move a unit charge, q_0, from $a \to b$.

V_{ba} " " " " " " " " " " " " $b \to a$.

point charge: $V = \frac{Kq}{r} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}\right)$

collection of pt. charges: $V = K\sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$

continuous distribution of charge: $V = K\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\underbrace{\int}_{\text{integral over charge dist.}} \frac{dq}{r}$

dq } charge element

all of these are taken w/ respect to ∞.

r: dist from charge elem.

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q\vec{E}\cdot d\vec{l}$$

↳ $\vec{E}$ is electrical force / unit charge

↳ moving w/ $\vec{E}$ = move in direction of decreasing V

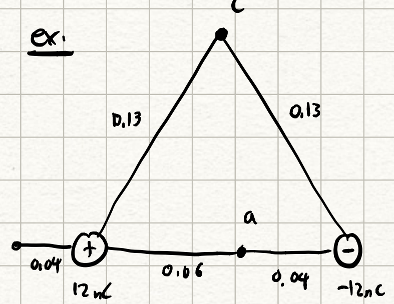
$$E = -\nabla V = \left\langle \frac{\partial V}{\partial x}\hat{i}, \frac{\partial V}{\partial y}\hat{j}, \frac{\partial V}{\partial z}\hat{k}\right\rangle$$

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = -\int_b^a \vec{E}\cdot d\vec{l}$$

When using Gauss's Law you can easily determine $\vec{E}$, which can then be used to calculate V.

otherwise, usually easier to find voltage & determine $\vec{E}$.

ex.



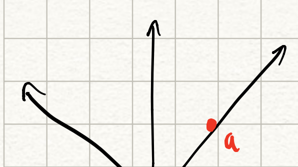
find potential at a, b, c

$$V_a = K\left(\frac{12\,nC}{0.06} - \frac{12\,nC}{0.04}\right) = -9\times10^{11}\ \frac{J}{nC} = -900\ V$$

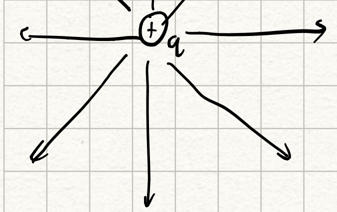
$$V_b = K\left(\frac{12}{0.04} - \frac{12}{0.1}\right) = 1.62\times10^{12}\ \frac{J}{nC} = 1620\ V$$

$$V_c = K\left(\frac{12}{\sqrt{0.13^2+0.05^2}} - \frac{12}{\sqrt{0.13^2+0.05^2}}\right) = 0$$

ex: find V by integrating $\vec{E}$ from pt. charge q.



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\right)$$



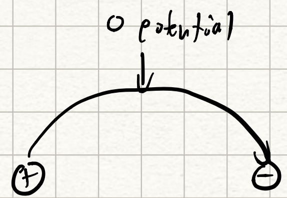
$$V_a - V_\infty = \int_a \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\right) dr = -\frac{1}{4\pi\varepsilon_0}(q)\int_\infty^a \frac{1}{r^2}\,dr$$

$$= -\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\infty} - \frac{1}{r}\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r}$$

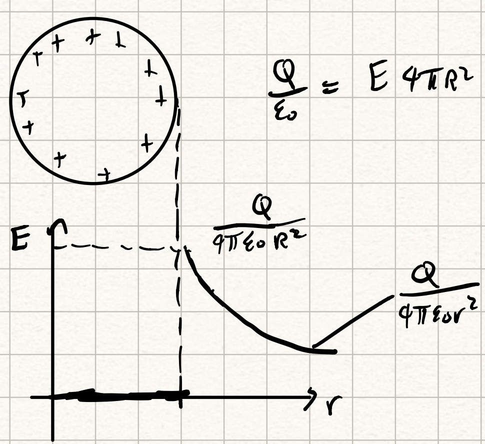
? if electrical potential is zero does that mean electric field is zero?

no!

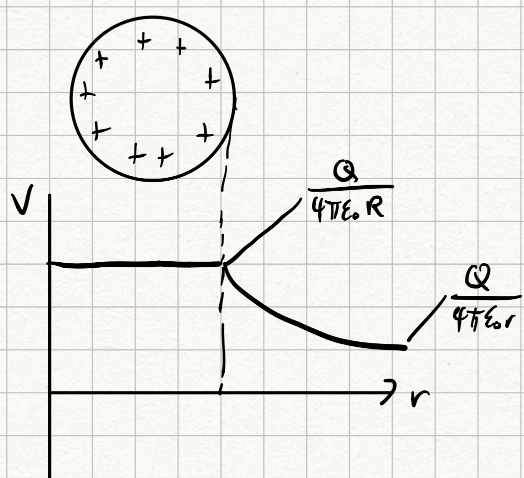


V=0 doesn't mean the point is at equil. & doesn't move. It means that it takes no NRG to move to that pt. from ∞.

V=−∞ means that the point will no longer move.



$$\frac{Q}{\varepsilon_0} = E\,4\pi R^2$$



- there is a maximum potential to which a conductor in air can be raised.
- at $|E| = 3\times10^6$ V/m, the air becomes ionized & becomes a conductor itself

from the previous example,

$V = ER$ outside conductor.

thus, if E_{cr} is max, then

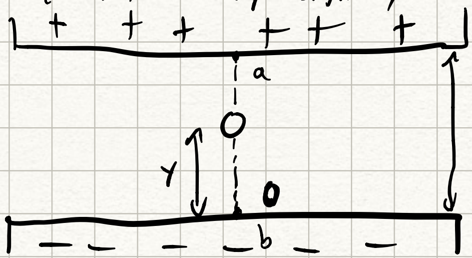
$V_{cr} = E_{cr} R$ is max voltage.

- with a very small radius of curvature, like a sharp point, small potentials can be enough to ionize air around it.
  - ↳ produces ozone & purple hue
  - → Corona Effect

↳ laser printers use corona discharge to spray electrons

- large radius conductors are used in situations where we want to prevent corona discharge.
  - ↳ ex. blunt rod at top of Empire State allows for more charge buildup from excess charge in atmosphere (during thunderstorm) so that lightning strikes the rod instead.

ex. find potential at any height y



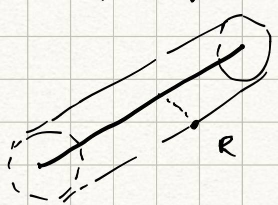
- ~~the field is constant~~ ⇒ ~~equipotential~~ field is zero only.

$$V(y) = \frac{U(y)}{q_0} = \frac{q_0 E y}{q_0} = Ey$$

$$V_a - V_b = \int_a^b E\,d = \vec{E}\Delta d$$

$$\vec{E} = \frac{V_{ab}}{\Delta d}$$

ex. find potential at distance R from ∞ line charge w/ λ.



$$\Phi = \frac{Q_{enc}}{\varepsilon_0} = \oint \vec{E}\,d\vec{A}$$

$$= E(2\pi r h)$$

$$\lambda = \frac{Q_{enc}}{h}$$

$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$V_R - V_\infty = -\int_\infty^R \frac{\lambda}{2\pi\varepsilon_0 r}\,dr$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}(\ln R - \ln \infty)$$

$$= \infty$$

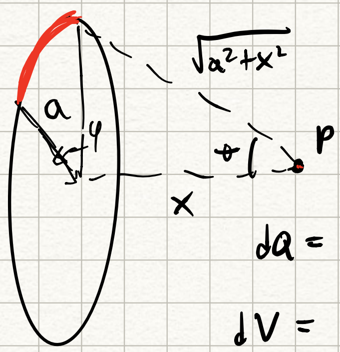
charge distribution extends to ∞.

however, for two radii, r_1 and r_2, where $r_1 < r_2$, we can set one as reference voltage (i.e. $V_2 = 0$).

$$V_1 - \cancel{V_2}^{\,0} = \int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon_0 r}\,dr = \frac{\lambda}{2\pi\varepsilon_0}\left(\ln \frac{r_2}{r_1}\right)$$

V_1 should decrease as r_1 ↑ ✓

ex: ring of charge



$$dQ = \lambda\, ds = \left(\frac{Q}{2\pi a}\right) ds$$

$$dV = k\left(\frac{dQ}{r}\right) = k\frac{Q}{2\pi r a}\, ds$$

$$V = k\frac{Q}{2\pi r a}\int_0^{2\pi a} ds$$

$$= \frac{1}{4\pi\varepsilon_0 a}\left(\frac{Q}{2\pi r}\right)(2\pi a)$$

$$= \frac{Q}{4\pi\varepsilon_0\sqrt{a^2+x^2}}$$

alternatively,

$$dV = \frac{k}{r}\, dQ$$

$$V = \frac{k}{r}\int dQ$$

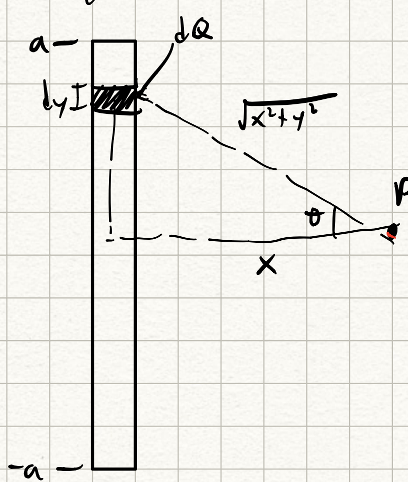
$$= \frac{Q}{4\pi\varepsilon_0\sqrt{a^2+x^2}}$$

check!

$x \gg a$, $V = \dfrac{Q}{4\pi\varepsilon_0 x}$, which is potential from pt. charge

ex.

potential from line of charge



$$V = \frac{kQ}{r}$$

$$dV = k\frac{dQ}{r}, \quad dQ = \left(\frac{Q}{2a}\right) dy$$

$$V = k\int\frac{dQ}{r} = \frac{kQ}{2a}\int\frac{1}{r}\, dy$$

$$= \frac{kQ}{2a}\int\frac{1}{\sqrt{x^2+y^2}}\, dy$$

$$= \frac{kQ}{2a}\ln\left(y+\sqrt{y^2+x^2}\right)\Big|_{-a}^{a}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2a}\right)\ln\left(\frac{\sqrt{a^2+x^2}+a}{\sqrt{a^2+x^2}-a}\right)$$