OPTIMIZATION

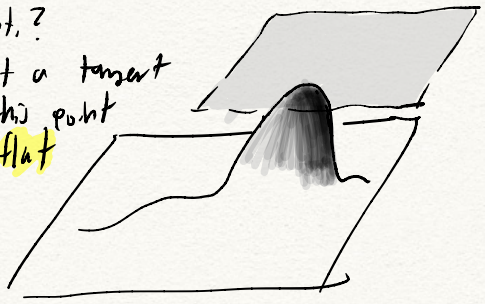
$f(x, y, \dots)$

→ ex. profits; dependent on multiple variables and want to maximize

→ cost function: minimize, useful for ML/AI.

- how to find this pt.?

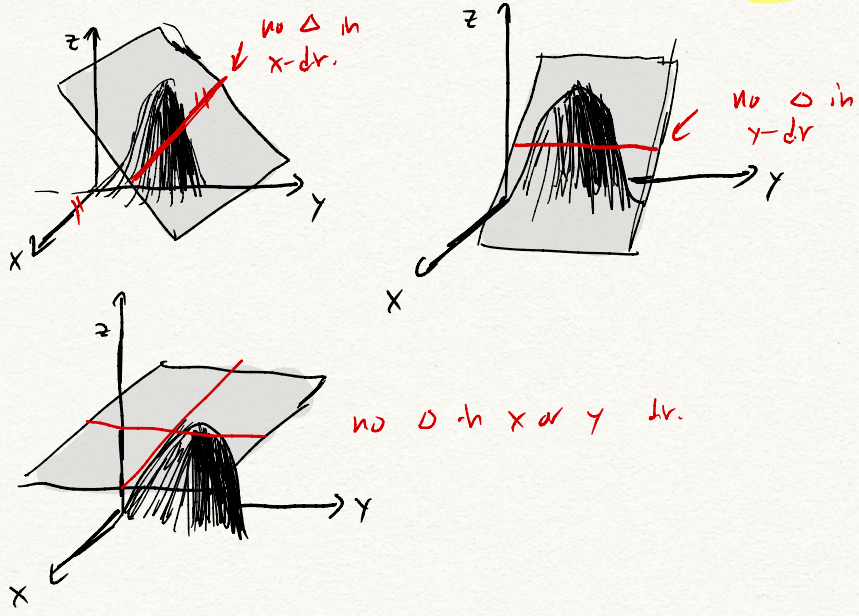
  ↳ notice that a tangent plane at this point will be flat



↳ if there's any slope to the plane, you can "climb" up it.

$$\Rightarrow \frac{\partial f}{\partial x}(x_0, y_0) = 0 \quad \text{and} \quad \frac{\partial f}{\partial y}(x_0, y_0) = 0 \Rightarrow \nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix}$$

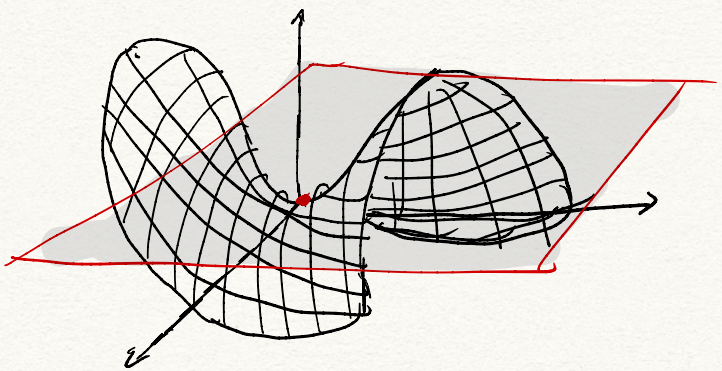
(gradient)



- any peak is a "local maxima"
  any trough is a "local minima"

HOWEVER: just because $\nabla f(x_0, y_0) = 0 \not\Rightarrow (x_0, y_0)$ is a local maxima or minima.

consider $f(x,y) = x^2 - y^2$



$$\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial f}{\partial y} = -2y$$

$$\Rightarrow \frac{\partial f}{\partial x}(0,0) = 0 \qquad \frac{\partial f}{\partial y} = (0,0) = 0$$

a saddle point is when one cross section is a local maxima and another cross section is a local minima

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a point (a,b) is a critical point if

- $\nabla f(a,b) = 0$ [i.e. $f_x = 0$ and $f_y = 0$]
  - if any one is zero, not a critical point!
- $f_x(a,b)$ AND/OR $f_y(a,b)$ doesn't exist.

- all saddle points are critical points
- all relative extrema are critical points
- not all critical points are relative extrema