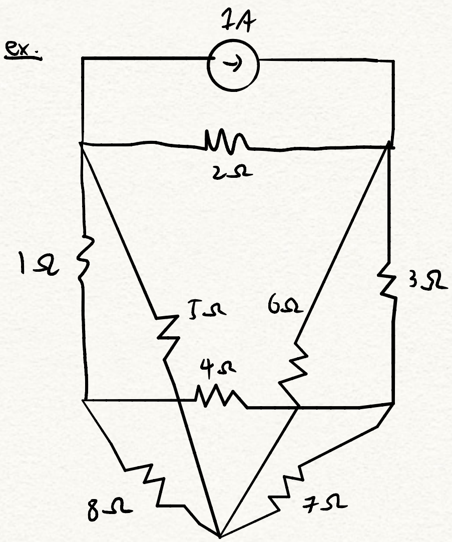
MESH ANALYSIS

- does not use KCL, can solve w/ less variables & less eqns.
- uses KVL to find currents, whereas Nodal Analysis uses KCL to find voltages.
- only works on planar circuits

- planar circuit is one that can be drawn in plane w/ no branches crossing e/o.
  ↳ a circuit that is initially nonplanar might still work, if it can be redrawn as planar.

ex.



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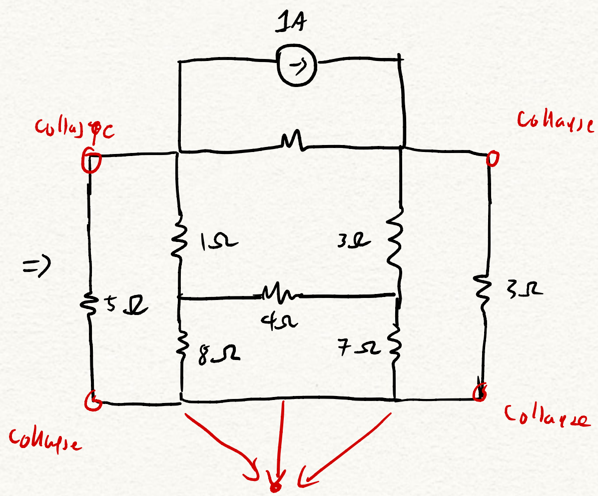
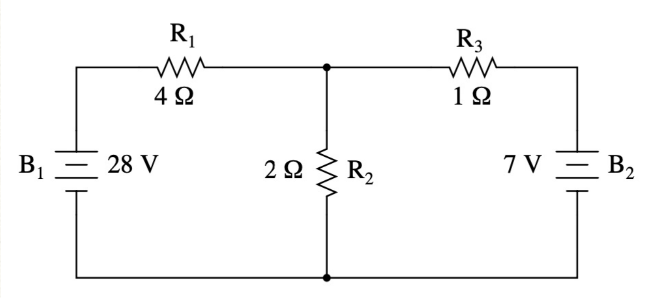
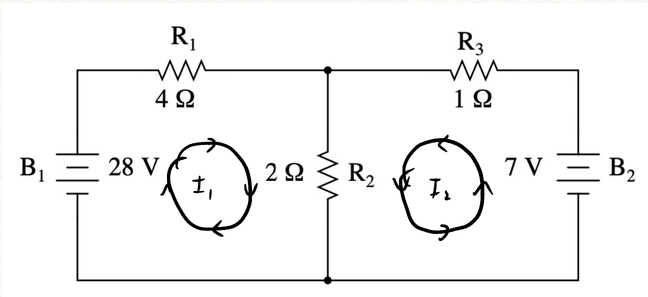


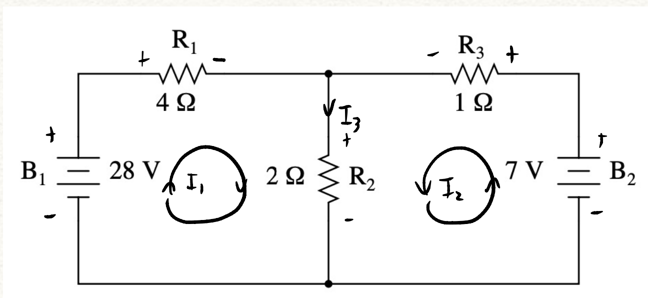


1. Identify loops



- gets its name by thinking of current loops meshing together like spinning gears.
- choice of current is arbitrary, but easier to solve if going thru same direction via intersecting components

2. Label voltage drop polarities



3. write KVL for each loop

$$-28 + 4I_1 + 2(I_1 + I_2) = 0$$
$$-7 + I_2 + 2(I_1 + I_2) = 0$$

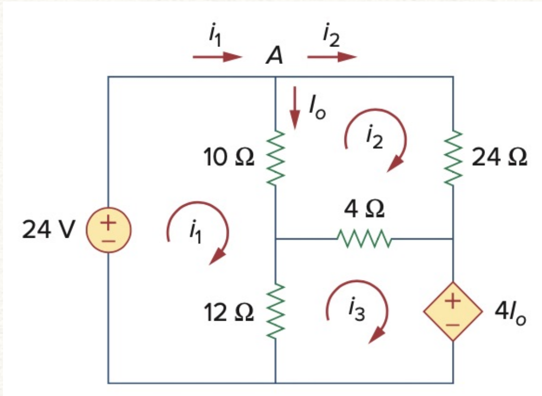
4. Solve for currents

$I_1 = 5$ A

$I_2 = -1$ A

$I_3 = I_1 + I_2 = 4$ A

ex.



☆ don't assign polarities!!

$10 + 4 + 24 =$

$14 \Rightarrow 38$

$$24 = 10(I_1 - I_2) + 12(I_1 - I_3)$$
$$0 = 24I_2 + 4(I_2 - I_3) + 10(I_2 - I_1)$$
$$4I_o + 12(I_3 - I_1) + 4(I_3 - I_2) = 0$$
$$I_o = I_1 - I_2$$
$$\Rightarrow 4(I_1 - I_2) + 12(I_3 - I_1) + 4(I_3 - I_2) = 0$$

$$\Rightarrow \begin{aligned} 22I_1 - 10I_2 - 12I_3 &= 24 \\ -10I_1 + 38I_2 - 4I_3 &= 0 \\ -8I_1 - 8I_2 + 16I_3 &= 0 \end{aligned}$$

$-4 \ -4$

$I_1 = 2.25$ A

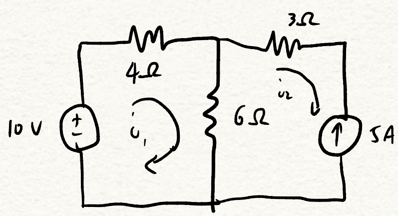
$I_2 = 0.75$ A

$I_3 = 1.5$ A

$I_o = 1.5$ A

WITH CURRENT SOURCES

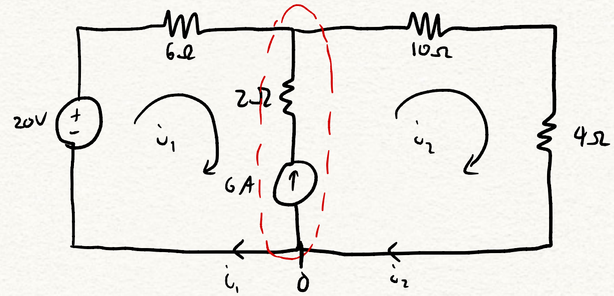
- if current source exists only in one mesh



$i_2 = -5$A

$$10 = 4i_1 + 6(i_1 - i_2)$$
$$10 = 10i_1 - 6i_2$$
$$i_1 = \frac{10 + 6(-5)}{10} = -2 \text{ A}$$

- if current source between two meshes, create supermesh
  - ↳ we ignore & run KVL on outer loop



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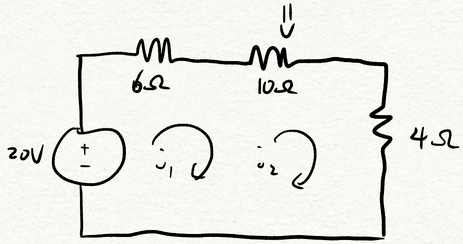


$20 = 6i_1 + 10i_2 + 4i_2$ (KVL)

$10 = 3i_1 + 7i_2$

Apply KCL to node in branch where two meshes intersect.

<u>at node 0:</u>

$i_2 = 6 + i_1$

$10 = 3i_1 + 7(6 + i_1)$

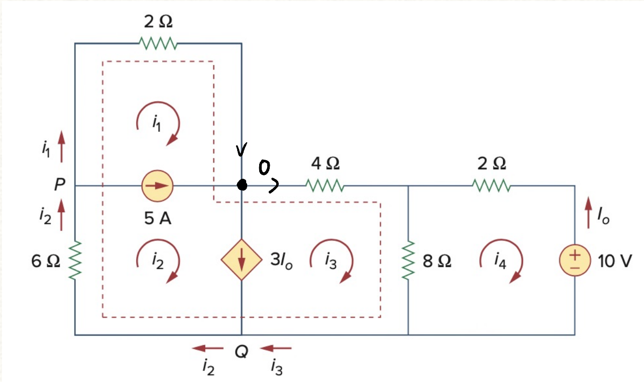
$10 = 10i_1 + 42$

$i_1 = \frac{-32}{10} = -\frac{16}{5} = -3.2A$

$i_2 = 2.8A$

Properties

- current source in supermesh gives eqn. to solve currents
- supermesh has no current of its own
- supermesh requires application of KCL & KVL.



<u>KVL on larger supermesh</u>

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0$$

$i_1 = -7.5A$
$i_2 = -2.5A$
$i_3 = 3.93A$
$i_4 = 2.14A$

<u>at point P:</u>

$$i_2 = i_1 + 5 \qquad -i_1 + i_2 = 5$$

<u>at point Q:</u>

$$3I_o + i_3 = i_2$$

$$-3i_4 + i_3 = i_2 \Rightarrow i_2 - i_3 + 3i_4 = 0$$

<u>at point O:</u>

$$5 + i_1 = 3I_o + i_3$$

$$I_o = -i_4$$

$$5 + i_1 = -3i_4 + i_3$$

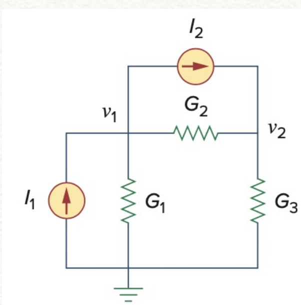
<u>in mesh 4:</u>

$$10 + 8(i_4 - i_3) + 2i_4 = 0$$

$$10 = 8i_3 - 10i_4$$

$$5 = 4i_3 - 5i_4$$

NODAL/MESH ANALYSES BY INSPECTION



- when all elements are indep current sources, we don't need to apply KCL to each node. By inspection,

ΣG to node 1

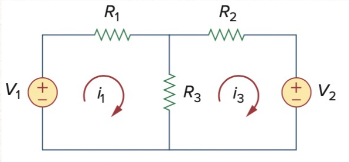
$$\begin{bmatrix} G_1+G_2 & -G_2 \\ -G_2 & G_2+G_3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

ΣI @ 1

ΣI @ 2

ΣG to node 2

negative of conductance btw. nodes



• if everything is voltage source,

ΣR in mesh 1

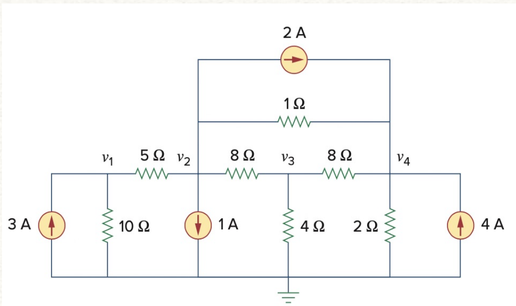
$$\begin{bmatrix} R_1+R_3 & -R_3 \\ -R_3 & R_2+R_3 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

algebraic sum taken clockwise

ΣR in mesh 2

negative of resistors in middle.

ex.



$$\begin{bmatrix} 0.3 & -\frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & 1.325 & -\frac{1}{8} & -1 \\ 0 & -\frac{1}{8} & 0.5 & -\frac{1}{8} \\ 0 & -1 & -\frac{1}{8} & 1.625 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

$V_1 = 13.90$ V

$V_2 = 5.845$ V

$V_3 = 3.348$ V

$V_4 = 7.547$ V

$$G_{11} = \frac{1}{10} + \frac{1}{5} = 0.3 \qquad G_{12} = G_{21} = -\frac{1}{5} = -0.2$$

$$G_{22} = \frac{1}{8} + \frac{1}{5} + \frac{1}{1} = 1.325$$

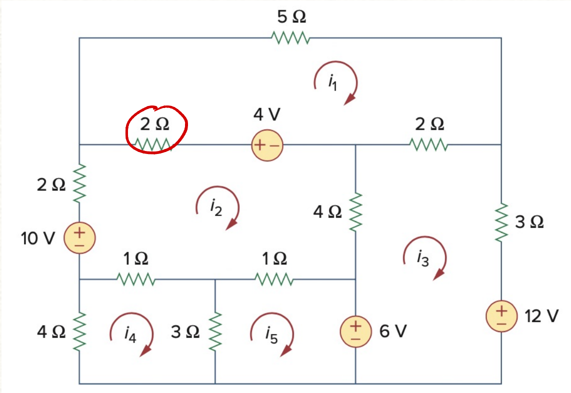
$$G_{33} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = 0.5$$

$$G_{44} = \frac{1}{8} + \frac{1}{2} + \frac{1}{1} = 1.625$$

conductance must be direct (no resistor in btw)

- G_{ii} is Σ conductance thru node i
- G_{ij} is Σ conductance btw node i and j

ex.



$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

- R_{ii} is Σ resistance in loop
- R_{ij} is $-\Sigma$ resistance in common w/ i and j.

NODAL VS. MESH ANALYSIS

Nodal Analysis	Mesh Analysis
↑ parallel elements	↑ series elements
↑ current sources	↑ voltage sources
↑ supernodes	↑ supermeshes
nodes < meshes	meshes < nodes
require node voltages	require currents