CAPACITORS

Stores electrical potential energy and electric charge

→ made by insulating two conductors from e/o.

→ applications in flash photography, airbag sensors, and television recievers.

Capacitance

→ the ratio of charge on each conductor to the potential difference

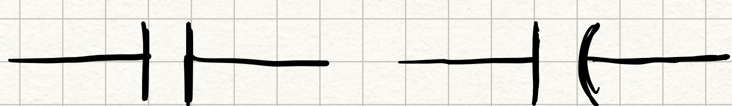
→ depends on
- → size & shape of conductors
- → material between the two conductors
  - → C ↑ w/ dielectric due to polarization

• energy stored in capacitor is "stored" in the electrical field itself.

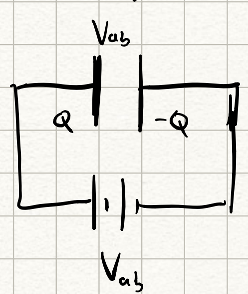
How to make a capacitor?

→ Separate any two conductors w/ insulator

→ each conductor initially has 0 charge and ends up with Q and −Q on each plate
- → charging the capacitor



→ we can charge a capacitor by connecting battery to two ends



$$Q \propto \vec{E} \propto V_{ab}$$

charge on plate

$$\boxed{C = \frac{Q}{V_{ab}}}$$

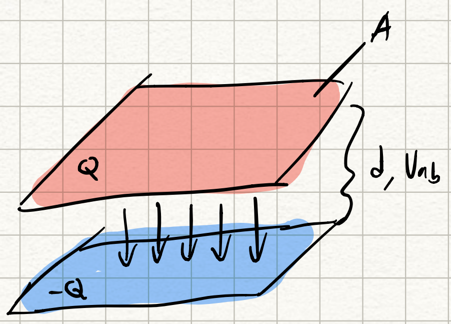
potential diff. btwn conductors

capacitance

→ the SI unit is a Farad $= 1\,\frac{C}{V} = 1$ coulomb / volt

For a given voltage, $C\uparrow \Rightarrow Q\uparrow \Rightarrow$ more NRG stored.

Capacitors in vacuum



parallel plate capacitor

From Gauss's Law,

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$V_{ab} = Ed = \frac{1}{\varepsilon_0}\frac{Qd}{A}$$

$$\boxed{C = \frac{Q}{V_{ab}} = \varepsilon_0 \frac{A}{d}}$$

$$1\,F = \left(\frac{C^2}{N\cdot m^2}\right)\frac{\cancel{m^2}}{m} = \frac{C^2}{N\cdot m} = \frac{C^2}{J} = \frac{C}{V}$$

$J = N\cdot m$ $\quad$ $1\,V = \frac{J}{C}$

$\Rightarrow \varepsilon_0$ has units F/m

ex. Find area of 1F capacitor if plates 1.0 mm apart.

$$A = \frac{Cd}{\varepsilon_0} = \frac{1(0.01)}{8.85E-12} = 1.1 \times 10^8\ m^2$$

ex. plates in // plate cap. are 5 mm apart and $2m^2$ in area. 10 KV applied across cap.

a) capacitance?

$$C = \varepsilon_0 \frac{A}{d} = (8.85E-12)\frac{(2)}{(0.005)} = 0.00354\ \mu F$$

b) Charge on each plate?

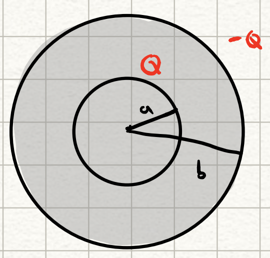
$$C = \frac{Q}{V} \Rightarrow Q = CV$$
$$= (0.00354)(10\ kV)$$
$$= 34.5\ \mu C$$

c) magnitude of $\vec{E}$?

$$\vec{E} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} = \frac{(34.5\ \mu C)}{(8.85E-12)(2)} = 2.00 \times 10^6\ \frac{N}{C}$$

$$\vec{E} = \frac{V_{ab}}{d} = \frac{10\ kV}{(0.005)} = 2.00 \times 10^6\ \frac{N}{C}$$

ex.



Scherical shell, a & b

Find capacitance:

$$C = \frac{Q}{V}$$

$$V = \int \vec{E} \cdot d\vec{r} \quad , \quad Q = \int dq$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$V(r) - V_\infty = \int_r^\infty \frac{Q}{4\pi\varepsilon_0 r^2}$$

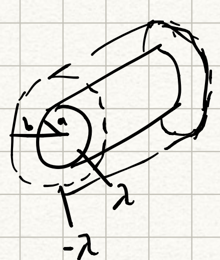
$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}\Big|_\infty^r$$

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V_{ab} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

$$\therefore C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) \approx \varepsilon_0\left(\frac{A_{gm}}{d}\right)$$ → geometric mean

ex.



coaxial cable

Find C per unit length:

$$C = \frac{Q}{V_{ab}}$$

$$\vec{E}(2\pi r l) = \frac{Q_{enc}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$V_{ab} = V_a - V_b = \int_a^b \vec{E} \cdot dl = \int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}\, dr = \frac{\lambda}{2\pi\varepsilon_0} \ln(r)\Big|_a^b = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{b}{a}\right)$$

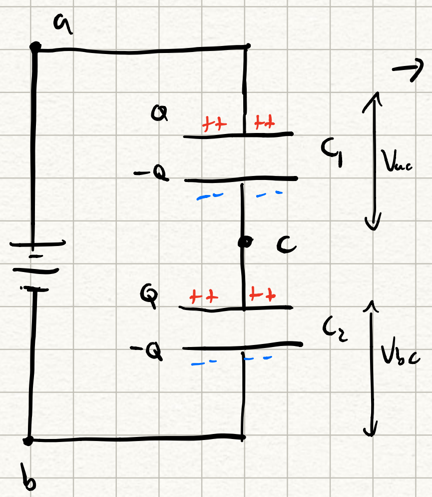
$$C = \frac{Q}{V_{ab}} = \frac{\cancel{\lambda} l\left(\frac{2\pi\varepsilon_0}{\cancel{\lambda}}\right)}{\ln\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$$

$$\frac{C}{l} = \frac{2\pi\varepsilon_0}{\ln(b/a)}$$

→ a typical cable for connecting television to cable TV has capacitance 69 pF/m

→ data is transferred via electromagnet field. In a coaxial cable, it can be stored only between the two conductors separated by an insulator.

Capacitors in Series



→ both capacitors share same charge magnitude!

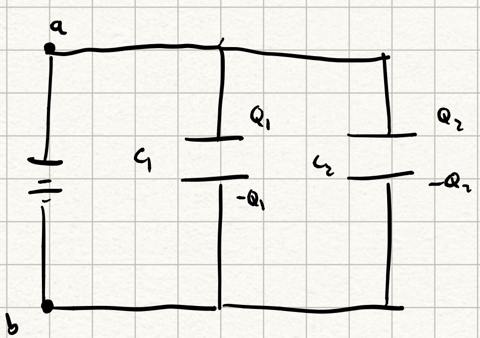
① top of C_1 acquires (+) charge
② this induces equal (−) charge on bottom of C_1
③ these charges come from top of C_2, inducing (+) charge
④ (−) charge is induced on bottom of C_2

$$V_{ac} = \frac{Q}{C_1} \quad \& \quad V_{cb} = \frac{Q}{C_2}$$

$$V_{ab} = V_{ac} + V_{cb} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\boxed{\frac{1}{C_{eq}} = \frac{V_{ab}}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \dots}$$

Capacitors in parallel



→ top plates are equipotential and so are bottom plates.

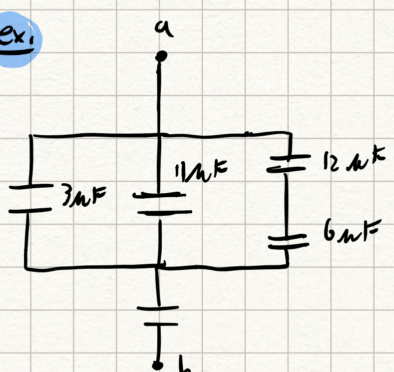
→ V_{ab} is same for top and bottom.

$$Q_1 = C_1 V, \quad Q_2 = C_2 V$$

$$Q = V(C_1 + C_2)$$

$$\boxed{C_{eq} = \frac{Q}{V_{ab}} = C_1 + C_2 + \dots}$$

ex:



$$C_{eq} = ((12 \leftrightarrow 6) + 11 + 3) \leftrightarrow 9$$

$$= \left(\frac{12(6)}{12+6} + 11 + 3\right) \leftrightarrow 9$$

$$= 18 \leftrightarrow 9$$

$$= \frac{18 \times 9}{18 + 9}$$

$$= 6\ \mu F$$

CAPACITORS w/ DIELECTRICS

→ most capacitors have dielectrics in btw.

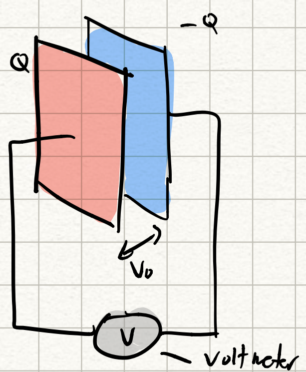
① solves problem of separating 2 conducting metal sheets

② increases maximum potential difference

→ any dielectric in presence of large enough $\vec{E}$ will be partially ionized and be somewhat conductive

→ known as dielectric breakdown

→ can store more charge / NRG



Original capacitance $C_0 = \frac{Q}{V_0}$

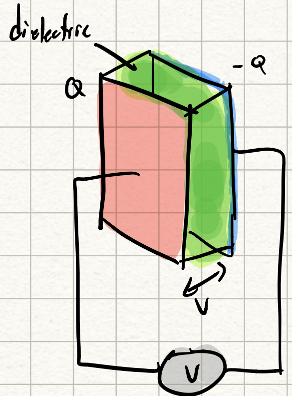
new capacitance $C = \frac{Q}{V}$

→ inserting dielectric decreases potential difference

$$\boxed{K = \frac{C}{C_0}}$$ (dielectric constant)

☆ $K = \varepsilon_r$

$$\frac{V}{V_0} = \frac{C_0}{C} \Rightarrow \boxed{V = \frac{V_0}{K}}$$



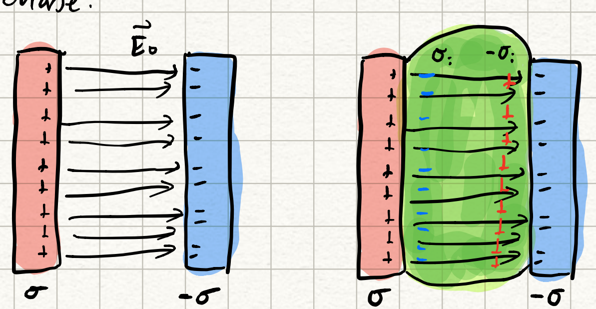
→ C is always greater than C_0

→ there will always be some current leakage, but we ignore it

Since $V \propto E$ for a capacitor,

$$\boxed{E = \frac{E_0}{K}}$$

→ adding dielectric doesn't change surface charge, but $\vec{E}$ decreases due to induced charge.



Since $E = \frac{\sigma_{net}}{\varepsilon_0}$ via Gauss's law.

Thus, $\boxed{E = \frac{\sigma - \sigma_i}{\varepsilon_0}}$ with dielectric and $\boxed{E_0 = \frac{\sigma}{\varepsilon_0}}$ without.

also, $\frac{E_0}{\kappa} = \frac{\sigma - \sigma_i}{\varepsilon_0}$

$$\Rightarrow \frac{\sigma}{\cancel{\varepsilon_0}\kappa} = \frac{\sigma}{\cancel{\varepsilon_0}} - \frac{\sigma_i}{\cancel{\varepsilon_0}}$$

$$\boxed{\sigma_i = \sigma\left(1 - \frac{1}{\kappa}\right)}$$ (induced charge density)

→ when κ very large, $\sigma_i \simeq \sigma$
→ almost no charge at plates

$\boxed{\varepsilon = \kappa \varepsilon_0}$ (def^n of permittivity)

$$C = \varepsilon_r C_0 = \varepsilon_r \varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d}$$