POTENTIAL GRADIENT

recall that

$$V_{ab} = V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{l}$$

→ if we know $\vec{E}$, then we can calculate ΔV.
→ if we know V, we can calculate $\vec{E}$.

$$V_a - V_b = \int_b^a dV = -\int_a^b dV = \int_a^b \vec{E}\cdot d\vec{l}$$

$$-dV = \vec{E}\cdot d\vec{l}$$

$$\vec{E} = -\frac{dV}{dl} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

$$\vec{E} = -\nabla V$$

if E is radially symmetric,

$$\vec{E}_r = -\frac{\partial V}{\partial r}$$

Can compute $\vec{E}$ two ways

1. add $\vec{E}$ from every point charge
2. calculate potential then take gradient
   → easier

__ex.__ find electric field of point charge, q.

$$V_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}\right)$$

$$\vec{E} = -\frac{\partial V}{\partial r} = \frac{q}{4\pi\varepsilon_0 r^2}$$

__verify!__ $V_r - V_\infty = -\int_\infty^r \vec{E}\, d\vec{r} = -\frac{q}{4\pi\varepsilon_0}\int_\infty^r \frac{1}{r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{q}{4\pi\varepsilon_0 r}$

__alternatively:__ $r = \sqrt{x^2+y^2+z^2}$

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left(\frac{q}{4\pi\varepsilon_0\sqrt{x^2+y^2+z^2}}\right) = \frac{qx}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$$

$$\frac{\partial V}{\partial y} = \frac{qy}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$$

$$\frac{\partial V}{\partial z} = \frac{qz}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$$

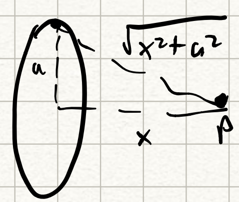
$$\vec{E} = -\left(\frac{qx\,\hat{i}}{4\pi\varepsilon_0(x^2+y^2+z^2)^{3/2}} + \frac{qy\,\hat{j}}{4\pi\varepsilon_0(x^2+y^2+z^2)^{3/2}} + \frac{qz\,\hat{k}}{4\pi\varepsilon_0(x^2+y^2+z^2)^{3/2}}\right)$$

$$= -\left(\hat{i}\frac{qx}{4\pi\varepsilon_0 r^3} + \hat{j}\frac{qy}{4\pi\varepsilon_0 r^3} + \hat{k}\frac{qz}{4\pi\varepsilon_0 r^3}\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\left(\frac{x\hat{i}+y\hat{j}+z\hat{k}}{r}\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

ex: find electric field for ring of charge



$$dV = \frac{K\,dQ}{r} = K\frac{dQ}{\sqrt{x^2+a^2}}$$

$$V = \frac{K}{\sqrt{x^2+a^2}}\int dQ$$

$$= \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+a^2}}$$

$$\vec{E}_x = -\frac{\partial V}{\partial x} = \frac{Qx}{4\pi\varepsilon_0(x^2+a^2)^{3/2}}$$

$$\vec{E}_y = -\frac{\partial V}{\partial y} = 0$$, since V is only fn of x.

ex: In certain region, $V = A + Bx + Cy^3 + Dxy$, $A, B, C, D > 0$
what is true

i) $A\uparrow \Rightarrow E\uparrow$ for all points
ii) $A\uparrow \Rightarrow E\downarrow$ for all points
iii) $\vec{E}$ has no z-component
iv) $\vec{E} = 0$ at origin ✗

$$E_x = -\frac{\partial V}{\partial x} = -(B + Dy)$$

$$E_x = -B \text{ at origin}$$