Sources of MAGNETIC FIELD

- charge creates electric field and electric field exerts force on charge
- magnetic field exerts force only on moving charge
- only moving charges (current) create magnetic fields
- Starting with the magnetic field generated by a single point charge, we can integrate the magnetic field for any shape of conductor.

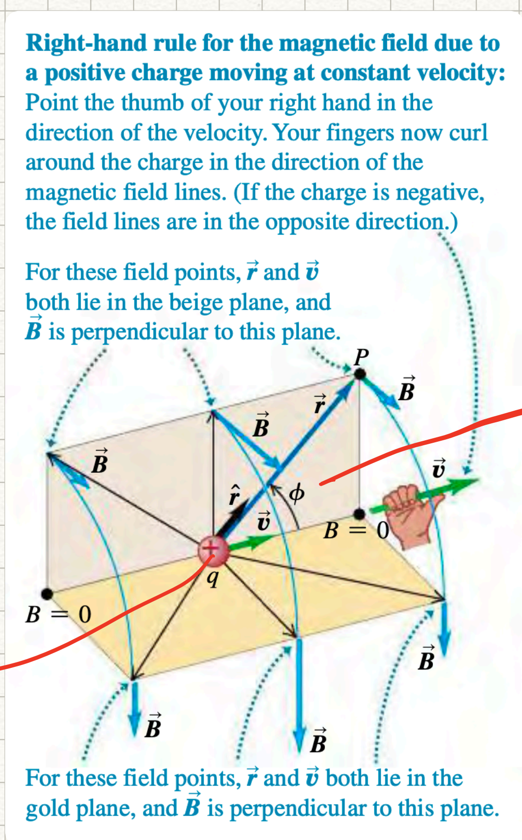
magnetic field of point charge

- recall that $E \propto \frac{1}{r^2}$ and $E \propto |q|$, pointing as line from source to field point.
- Similarly, $B \propto \frac{1}{r^2}$ and $B \propto |q|$, but is instead $\perp$ to plane containing line from source → field point and $\vec{v}$.

↳ also, $B \propto v$ and $B \propto \sin\phi$.

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{|q| v \sin\phi}{r^2}}$$, where μ_0 is magnetic constant $\left(1.2566\times10^{-6}\ \frac{H}{m} \text{ or } \frac{N}{A^2}\right)$ $\left(4\pi\times10^{-7}\ \frac{Ns^2}{C^2} = \frac{Wb}{A\cdot m} = \frac{T\cdot m}{A}\right)$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}}$$



smallest @ $\phi = 0$

largest @ $\phi = \frac{\pi}{2}$

if q negative, use left hand instead

- if charge accelerates, things get complicated

  → ex. when a wire bends

- μ_0 is exactly $4\pi \times 10^{-7}$ $\frac{N \cdot s^2}{C^2}$, $\frac{Wb}{A \cdot m}$, $\frac{T \cdot m}{A}$

  → why?

  speed of light

  recall $k = \frac{1}{4\pi\varepsilon_0} = 10^{-7} \frac{N \cdot s^2}{C^2} \cdot c^2$

$$\Rightarrow \varepsilon_0 = \frac{10^7}{4\pi c^2}$$

$$c^2 = \frac{1}{\varepsilon_0 \mu_0}$$

$$\cancel{c^2} = \frac{4\pi \cancel{c^2}}{10^7 \mu_0}$$

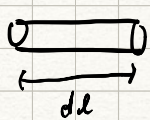
$$\mu_0 = 4\pi \times 10^{-7}$$

magnetic field for current element

- total magnetic field is vector sum of fields of individual charges.

for a current-carrying conductor, consider segment $d\vec{l}$

$$V = A\,dl$$



if $\exists$ n charged particles / unit volume, each w/ charge q,

$$dQ = nqA\,dl$$

dQ is equivalent to a single charge Q moving with velocity $\vec{v}_d$

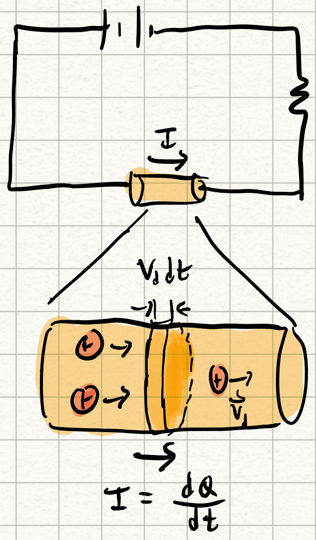
thus,

$$dB = \frac{\mu_0}{4\pi} \frac{|dQ|\, v_d \sin\phi}{r^2} = \frac{\mu_0}{4\pi} \frac{nqA\,dl\, v_d \sin\phi}{r^2} = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\phi}{r^2}$$

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{I\,dl \times \hat{r}}{r^2}}$$

(Law of Biot-Savart)

Current density



let n be concentration of particles (m^{-3}).

let V_d be drift velocity; all particles move @ this speed.

$$V_{cyl} = A\vec{v}_d\, dt,$$

$$\#\text{ particles} = nA\vec{v}_d\, dt$$

$$dQ = q(nA\vec{v}_d\, dt)$$

$$\boxed{I = \frac{dQ}{dt} = qnA\vec{v}_d}$$

$$\boxed{J = \frac{I}{A} = nq\vec{v}_d}$$

Current density = Current / cross-sectional area = $\frac{A}{m^2}$

ex: wire w/ diam. 1.02 mm has current of 1.67 A to 200-W lamp.

free-electron density is $8.5\times10^{28}/m^3$

a) current density?

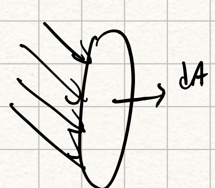
$$J = \frac{I}{A} = \frac{1.67}{\pi\left(\frac{1.02\times10^{-3}}{2}\right)^2} = 2.04\times10^{6}\ \frac{A}{m^2}$$

b) drift velocity?

$$J = nq\vec{v}_d$$

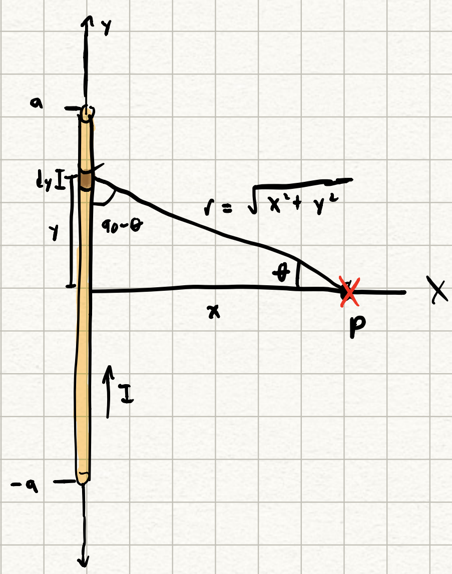
$$\vec{v}_d = \frac{J}{n|q|} = \frac{2.04\times10^{6}}{(8.5\times10^{28})(1.60\times10^{-19})} = 1.5\times10^{-4}\ m/s$$

What if non-uniform?



$$I = \oint \vec{J}\cdot d\vec{s}$$

MAGNETIC FIELD CONFIGURATIONS



$$dB = \frac{\mu_0}{4\pi} \frac{I\, d\vec{y} \times \vec{r}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\left(\frac{dy}{x^2+y^2}\right) \sin(90-\theta)$$

$$= \frac{\mu_0 I}{4\pi}\left(\frac{1}{x^2+y^2}\right)\left(\frac{x}{\sqrt{x^2+y^2}}\right) dy$$

$$B = \int dB = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{dy}{(x^2+y^2)^{3/2}}$$

$$= \frac{\mu_0 I x}{4\pi}$$

$y = x\tan\theta$

$dy = x\sec^2\theta\, d\theta$

$$\frac{1}{x^3} \cdot \int \frac{x\sec^2\theta}{\sec^3\theta}\, d\theta$$

$$= \frac{1}{x^2}\int \cos\theta\, d\theta$$

$$= \frac{1}{x^2}\sin\theta \qquad \sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

thus, $$\int_{-a}^{a} \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\left(\frac{a}{\sqrt{x^2+a^2}} + \frac{a}{\sqrt{x^2+a^2}}\right)$$

$$= \frac{2a}{x^2\sqrt{x^2+a^2}}$$

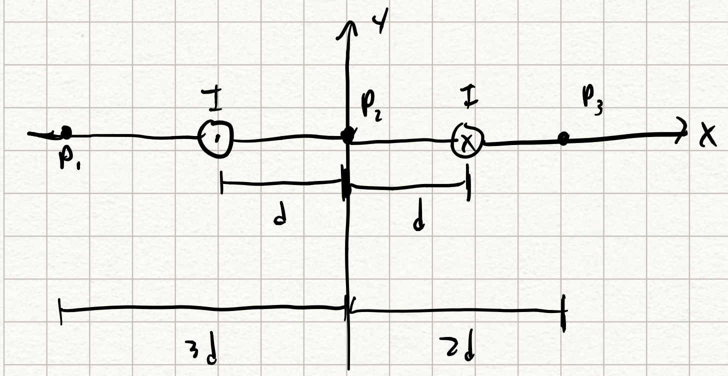
$$= \frac{\mu_0 I \cancel{x}}{2\cancel{4}\pi}\left(\frac{\cancel{2}a}{x^{\cancel{2}}\sqrt{x^2+a^2}}\right)$$

$$= \frac{\mu_0 I a}{2\pi x\sqrt{x^2+a^2}}$$

When $a >> x$ $\left(\frac{x}{a} \to 0\right)$

$$= \frac{\mu_0 I}{2\pi x}$$

ex. two wires, find $\vec{B}$ at P_1, P_2, P_3.



$\boxed{P_1}$

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi(2d)} = -\frac{\mu_0 I}{4\pi d}\hat{y}$$

$$\vec{B}_2 = \frac{\mu_0 I}{2\pi(4d)} = \frac{\mu_0 I}{8\pi d}\hat{y}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = -\frac{\mu_0 I}{8\pi d}\hat{y}$$

$\boxed{P_2}$

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi(d)}\hat{y}, \quad \vec{B}_2 = \frac{\mu_0 I}{2\pi(d)}\hat{y}$$

$$\Sigma\vec{B} = \frac{\mu_0 I}{\pi d}\hat{y}$$

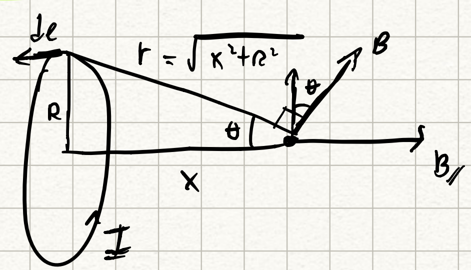
$\boxed{P_3}$

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi(3d)}\hat{y}, \quad \vec{B}_2 = -\frac{\mu_0 I}{2\pi d}\hat{y}$$

$$= -\frac{2\mu_0 I}{6\pi d}\hat{y}$$

$$= -\frac{\mu_0 I}{3\pi d}\hat{y}$$

circular current loop



$$dB = \frac{\mu_0}{4\pi}\left(\frac{I\,d\vec{l} \times \hat{r}}{r^2}\right) = \frac{\mu_0}{4\pi}\left(\frac{I\,dl}{r^2}\right)$$

$$dB_\perp = \frac{\mu_0}{4\pi}\left(\frac{I\,dl}{x^2+R^2}\right)\cos\theta$$

$$= \frac{\mu_0}{4\pi}\left(\frac{I\,dl}{x^2+R^2}\right)\left(\frac{x}{\sqrt{x^2+R^2}}\right)$$

$$dB_\parallel = \frac{\mu_0}{4\pi}\left(\frac{I\,dl}{x^2+R^2}\right)\left(\frac{R}{\sqrt{x^2+R^2}}\right)$$

$$\cancel{B_\perp = \frac{\mu_0}{4\pi}\frac{I}{(x^2+R^2)^{3/2}}\int dl}$$

$$\cancel{= \frac{\mu_0}{4\pi}\frac{I}{(x^2+R^2)^{3/2}}(2\pi R)}$$

$$\cancel{= \frac{\mu_0 I R}{2(x^2+R^2)^{3/2}}}$$

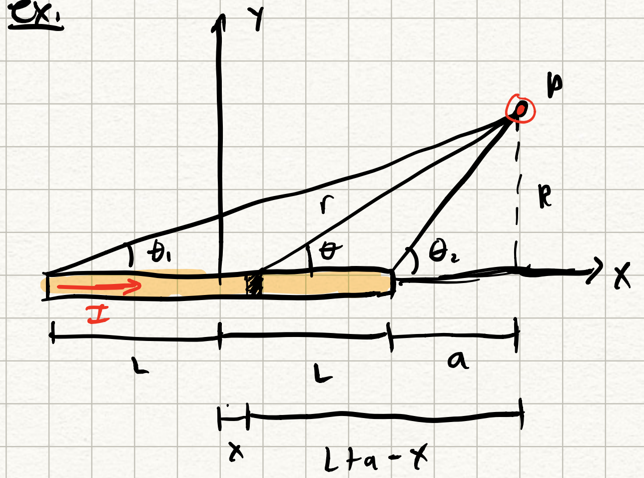
Cancels out w/ $\forall\, dl$, $\exists\, dl'$ on other side w/ opposite $\vec{B}$.

$$B_\parallel = \frac{\mu_0 I R}{4\pi(x^2+R^2)^{3/2}}\int dl$$

$$= \frac{\mu_0 I R}{4\pi(x^2+R^2)^{3/2}}(2\pi R)$$

$$= \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

ex.



find $\vec{B}$ at P.

$$dB = \frac{\mu_0}{4\pi}\left(\frac{I\, d\vec{l} \times \hat{r}}{r^2}\right)$$

$b = L+a-x$

$h = R$

$$r = \sqrt{(L+a-x)^2 + R^2}$$

$$\tan\theta = \frac{R}{L+a-x}$$

$$L+a-x = R\cot\theta$$

$$\sec^2\theta = \cot^2\theta + 1$$

$$= \frac{\cos^2\theta}{\sin^2\theta} + 1$$

$$\frac{1}{\sin^2\theta} = \cos^2\theta + \sin^2\theta$$

$\csc^2\theta$

$$r = R\sqrt{\cot^2\theta + 1}$$

$$r = R\csc\theta$$

$$x = L+a-\frac{R}{\tan\theta}$$

$$dx = \frac{R\sec^2\theta}{\tan^2\theta}\,d\theta$$

$$= R\left(\frac{1}{\cancel{\cos^2\theta}}\right)\left(\frac{\cancel{\cos^2\theta}}{\sin^2\theta}\right)d\theta$$

$$= R\csc^2\theta\, d\theta$$

$$d\vec{l} = dx\,\hat{i}$$

$$\vec{\hat{r}} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$d\vec{l}\times\hat{r} = \sin\theta\, dx\,\hat{n}$$

$$= R\csc^2\theta\sin\theta\,\hat{n}$$

$$dB = \frac{\mu_0}{4\pi}\left(\frac{IR\cancel{\csc^2\theta}\sin\theta\, d\theta}{R^2\cancel{\csc^2\theta}}\right)\hat{n}$$

$$dB = \frac{\mu_0 I}{4\pi R}\sin\theta\, d\theta\,\hat{n}$$

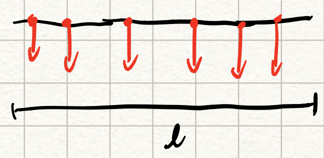
$$B = \int dB = \frac{\mu_0 I}{4\pi R}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta\,\hat{n}$$

$$B = \frac{\mu_0 I}{4\pi R}\left(\cos\theta_2 - \cos\theta_1\right)\hat{n}$$

what is force on parallel conductors?

$F = qv \times B$ for charge

$= |q||v|B\sin\theta$



$F = Q\vec{v}_d \times B = (q n A \cdot l)\vec{v}_d\,(B)(\cancel{\sin\theta})^{1}$

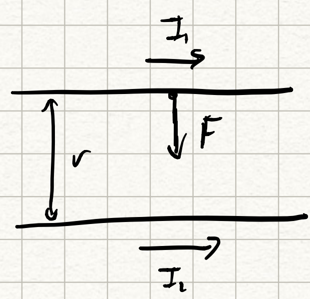
$= (qnA\vec{v}_d)\, l B$

$= IlB$

$V = A \cdot l$

$B = \frac{\mu_0 I_2}{2\pi r}$

So $F_{12} = I_1 l \left(\frac{\mu_0 I_2}{2\pi r}\right)$ for this config:



$$\boxed{\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

→ Current in same direction! attractive force

→ Current in opposite direction! repulsive force

defining the ampere

1 amp is the current that would cause a force of $2\times10^{-7}\,\frac{N}{l}$ for two parallel conductors of infinite length.