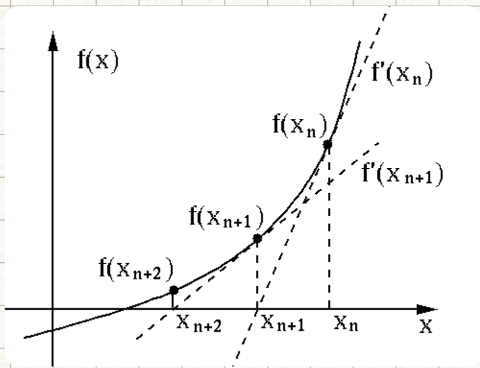
NEWTON'S METHOD

Newton's Suggestion: if we can't solve $f(x)=0$, replace $f(x)$ with linear approx. and find x-intercept of the line. Take that x-coord to be new center of linear approx & repeat.



ex. Find root of equation $x^3-2x-5=0$.

→ Start w/ bisection to find starting point

$f(0) = -5$
$f(1) = -6$
$f(2) = -1$
$f(3) = 16$

So root in $[2,3]$

Linearize at $x_0 = 2$:

$f(x) = x^3-2x-5$ | $f(2) = -1$
$f'(x) = 3x^2-2$ | $f(2) = 10$

$L_2(x) = -1 + 10(x-2)$
$= 10x - 21$

$L_2(x) = 0$ when $x = -2.1$

$f(2.1) = 0.061$
$f'(2.1) = 11.23$

$L_{2.1}(x) = 0.061 + 11.23(x-2.1)$
$= 11.23x - 23.522$

$L_{2.1}(x) = 0$ when $x = 2.09457$

... etc.

Iterative formula

- Pick x_0 using the Bisection Method / with sketch
- 1st approx. given by

$$L_{x_0}(x) = f(x_0) + f'(x_0)(x - x_0)$$

- Setting equal to zero, let $x = x_1$ since it's the next in sequence

$$f(x_0) + f'(x_0)(x - x_0)$$

$$x - x_0 = -\frac{f(x_0)}{f'(x_0)}$$

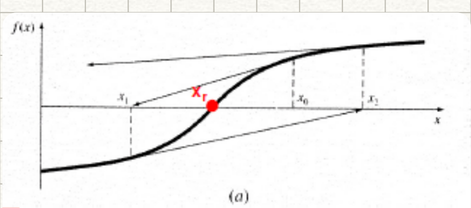
$$x_1 = x = x_0 - \frac{f(x_0)}{f'(x_0)}$$

repeating... $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

So general formula is

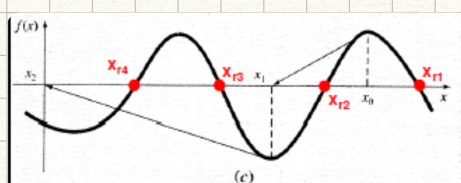
$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

When Newton's method fails



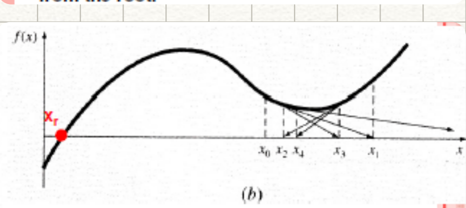
(a)

Case-a: Case where an inflection point ($f''(x)=0$) occurs in the vicinity of a root. Iterations begins at x_0 progressively diverge from the root.



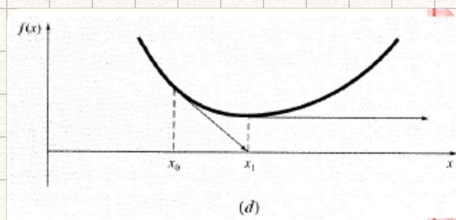
(c)

Case-c: Case shows how an initial guess that is close to one root can jump to a location several roots away. This is due to near zero slope.



(b)

Case-b: Case shows the tendency of the Newton's method to oscillate around a local maximum or minimum.



(d)

Case-d: Case shows an encounter of zero slope ($f'(x)=0$). In this case the solution shoots off horizontally and never hits the x-axis. (diverge- a disaster)