AMPERE'S LAW

For highly symmetric current distributions, we can use Ampere's Law to find the magnetic field.

→ similar yet different from Gauss's Law

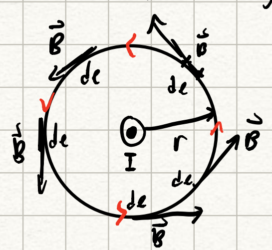
- Ampere's Law formulated as line integral of $\vec{B}$ around closed path.

$$\oint \vec{B} \cdot d\vec{l}$$

- recall that, for infinite straight conductor,

$\vec{B} = \dfrac{\mu_0 I}{2\pi r}$, where the field is circles around the line.

Let's take line integral of $\vec{B}$ around circle w/ radius r:

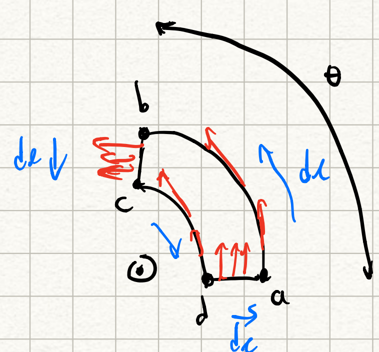


at all points $\vec{B} \parallel d\vec{l}$.

Thus, $\oint \vec{B} \cdot d\vec{l} = B \oint dl = 2\pi B r = \dfrac{\mu_0 I}{\cancel{2\pi r}} \cancel{(2\pi r)} = \mu_0 I$

↳ the line integral is independent of radius

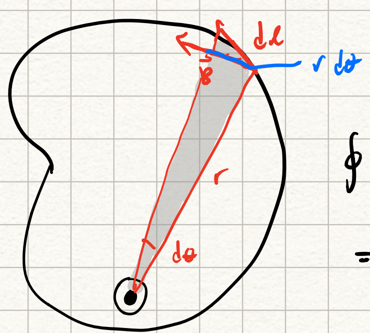
using right hand rule, currents that go along integration path are positive; against are negative.



$$\oint \vec{B} \cdot d\vec{l} = \oint \vec{B}_{\parallel} \, d\vec{l} = \int_a^b \vec{B}_{ab} \, d\vec{l} - \int_c^d \vec{B}_{cd} \, d\vec{l}$$

$$= \frac{\mu_0 I}{2\pi \cancel{r_1}} (\theta \cancel{r_1}) - \frac{\mu_0 I}{2\pi \cancel{r_2}} (\theta \cancel{r_2})$$

$$= 0$$



$d\ell \approx r\,d\theta$

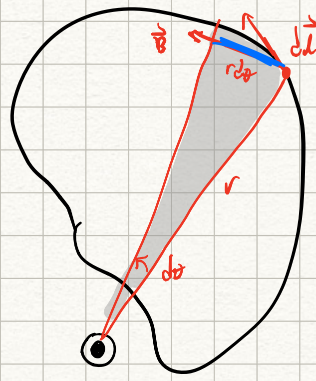
$$\oint \vec{B} \cdot d\vec{l} = B\,d\ell \cos\theta$$

$$= \oint \frac{\mu_0 I}{2\pi r} (r\,d\theta)$$

$$= \frac{\mu_0 I}{2\pi} \oint d\theta$$

$$= \mu_0 I$$

thus, as long as the wire is enclosed, $\oint \vec{B} \cdot d\vec{l}$ is always $\mu_0 I$.

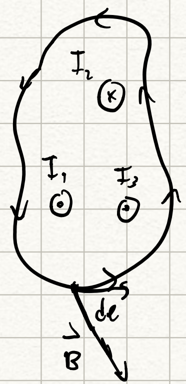


$$\oint \vec{B} \cdot d\vec{l} = B\,dl\cos\theta$$

$$= \frac{\mu_0 I}{2\pi} \oint d\theta$$

$$= 0$$

We can generalize this to multiple-wire enclosures!



$$I_{enc} = I_1 - I_2 + I_3$$

★ if $\oint \vec{B} \cdot d\vec{l} = 0$, doesn't mean no wires enclosed.

↳ only no net current

! $\oint \vec{E} \cdot d\vec{l} = 0$ for any closed path implies $\vec{F}_e = q\vec{E}$ is conservative.

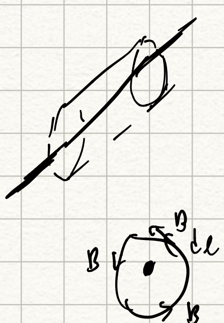
↳ no work done.

$\oint \vec{B} \cdot d\vec{l} = 0 \not\Rightarrow \vec{F}_{mag}$ is conservative.

$\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} \perp \vec{B}$, so $\vec{B} \cdot d\vec{l}$ not related.

↳ actually, non-conservative since depend on velocity

<u>ex:</u> use the fact that $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ to find $\vec{B}$ due to wire.

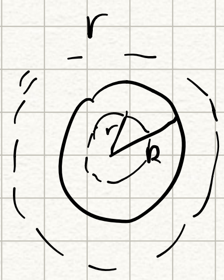


$$\vec{B} \oint d\vec{l} = \mu_0 I$$

$$\vec{B}(2\pi r) = \mu_0 I$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}$$

<u>ex.</u> Find $\vec{B}$ as func. of r for long cyl. w/ radius R.



$$\vec{B}\cdot\oint d\vec{\ell} = \mu_0 I_{enc}$$

<u>$r<R$:</u>

assume current density is constant.

$$I = JA$$

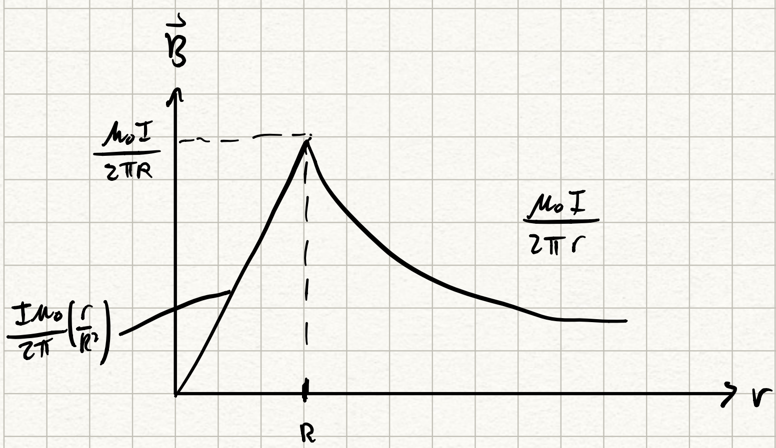
$$\frac{I_{enc}}{I_{tot}} = \frac{\cancel{J}A_{enc}}{\cancel{J}A_{tot}} \quad , A \propto r^2$$

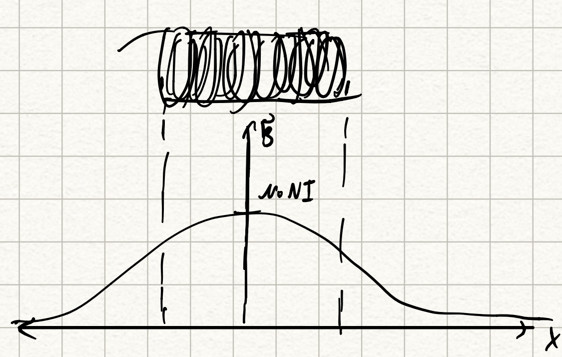
$$I_{enc} = \frac{r^2}{R^2} I$$

$$\vec{B} = \frac{r^{\cancel{2}}}{R^2}\left(\frac{I\mu_0}{2\pi \cancel{r}}\right) = \frac{I\mu_0}{2\pi}\left(\frac{r}{R^2}\right)$$

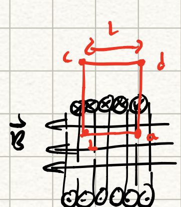
<u>$r>R$:</u>

$$\vec{B} = \frac{\mu_0 I}{2\pi r}$$





<u>ex.</u> Field of a solenoid



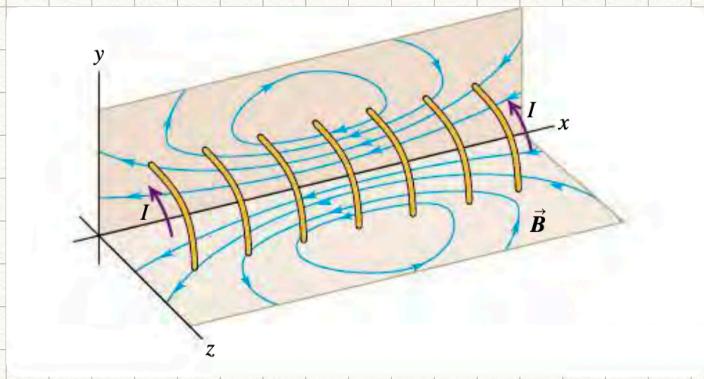
$$\oint \vec{B}\cdot d\vec{\ell} = \int_a^b \vec{B}\cdot d\vec{\ell}$$

$$= B\oint d\vec{\ell}$$

$$= BL$$

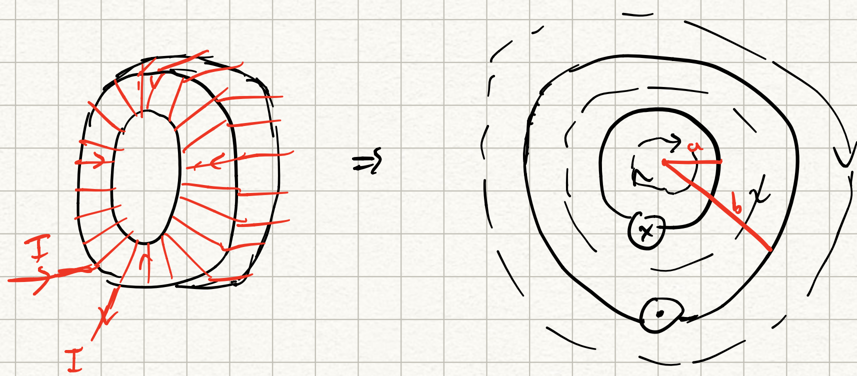
$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$\vec{B} = \frac{\mu_0 I_{enc}}{L} = \frac{\mu_0 \overset{\#\text{ turns}}{N} I}{L} = \mu_0 n I, \text{ where } n = \frac{N}{L} \text{ (density)}$$



magnetic field from solenoid

ex: field of toroidal solenoid w/ N turns, current I.



r<a:

$$\oint \vec{B} \cdot d\vec{l} = 0 \quad \text{(no current enclosed)}$$

a<r<b:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\vec{B} \cdot \oint dl = \mu_0 N I$$

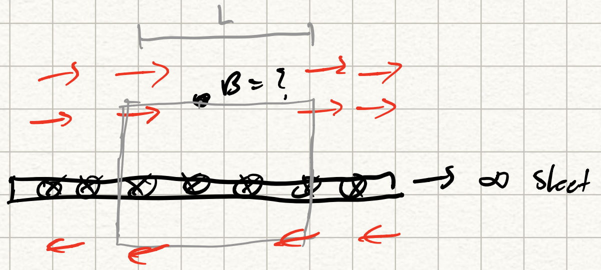
$$\vec{B} \cdot (2\pi r) = \mu_0 N I$$

$$\vec{B} = \frac{\mu_0 N I}{2\pi r} = \mu_0 n I \quad \text{(like normal solenoid)}$$

r>b:

$$\oint \vec{B} \cdot d\vec{l} = 0$$

ex.



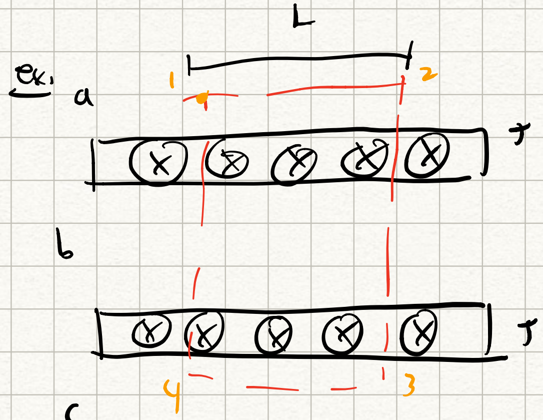
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$BL + BL = 2BL = \mu_0 I_{enc}$$

$$B = \frac{\mu_0 I_{enc}}{2L} = \frac{\mu_0 JL}{2L} = \frac{\mu_0 J}{2}$$

→ there is no dependence on distance ⇒ field is same everywhere.

ex.



find $\vec{B}$ everywhere.

1. using superposition,

$$\vec{B}_a = \vec{B}_1 + \vec{B}_2 = \mu_0 J \quad \text{(right)}$$

$$\vec{B}_b = \vec{B}_1 + \vec{B}_2 = 0$$

$$\vec{B}_c = \vec{B}_1 + \vec{B}_2 = \mu_0 J \quad \text{(left)}$$

2. Using Ampere's Law!

a) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

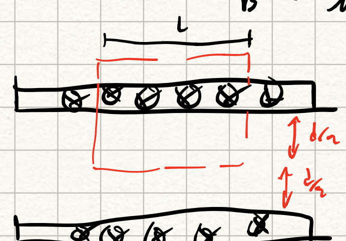
$$\int_1^2 \vec{B}\cdot d\vec{l} + \int_3^4 \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$2BL = \mu_0 I_{enc}$$

$$2BL = \mu_0 (2J\cdot L)$$

$$\vec{B} = \mu_0 J$$

b)



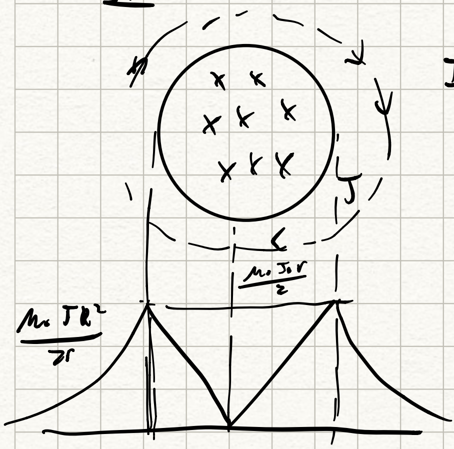
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$BL = \mu_0 (JL)$$

$$B = \mu_0 J$$

we know field is zero in sheet middle.

ex 1



r > R

$$I = \oint \vec{J} \cdot d\vec{s} = J_0 \pi R^2$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$\oint |\vec{B}||d\vec{l}| \cancel{\cos\theta}$$

$$\oint \vec{B}\, d\vec{l} = B \cdot 2\pi r$$

$$B \cdot 2\cancel{\pi} r = \mu_0 J_0 \cancel{\pi} R^2$$

$$\vec{B} = \frac{\mu_0 J_0 R^2}{2r}$$

r < R

$$\frac{I_{enc}}{I_{tot}} = \frac{\cancel{J} A_{enc}}{\cancel{J} A_{tot}}$$

$$I_{enc} = \frac{r^2}{R^2} I_{tot}$$

$$\vec{B} = \frac{\mu_0}{2\pi \cancel{r}} \left(\frac{r^{\cancel{2}}}{R^2} \right) I$$

$$= \frac{\mu_0}{2\pi} \left(\frac{r}{R^2} J_0 \pi R^2 \right)$$

$$= \frac{\mu_0 J_0 r}{2}$$