MAC LAURIN POLYNOMIALS

Suppose we want to find polynomial with sae derivatives at $f(x)$ at x_0.

→ start with n-th order polynomial in powers of $(x-x_0)$

$$p(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)^2 + \dots + a_n(x-x_0)^n$$

→ try to figure out coefficients to match $f(x)$.

Set $p(x_0) = f(x_0)$, $x = x_0$, $p(x_0) = a_0$

how to find a_1, differentiate first!

$$p'(x) = f'(x)$$

$$p'(x) = a_1 + 2a_2(x-x_0) + 3a_3(x-x_0)^2 + \dots + n a_n(x-x_0)^{n-1}$$

$\Rightarrow f'(x_0) = a_1$

how to find a_2?

$$p''(x) = f''(x)$$

$$p''(x_0) = 2a_2(x-x_0) + 6a_3(x-x_0)^2 + \dots + (n-1)a_n(x-x_0)^{n-2}$$

$$f''(x_0) = 2a_2$$

so $a_2 = \dfrac{f''(x_0)}{2}$

continuing...

$$p(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2!}f''(x_0)(x-x_0)^2 + \dots + \frac{1}{n!}f^{(n)}(x_0)(x-x_0)^n$$

⊘ the factorial comes from repeated use of the power rule!

Mac Laurin is Taylor Polynomial centered at $x_0 = 0$.

i.e. $$\sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!}x^k = f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 + \dots + \frac{1}{n!}f^{(n)}(0)x^n$$

Ex. consider $f(x)=\sqrt{x}$ near $x=4$.

$f(x)=\sqrt{x}$ $\quad f(4)=2.$

$f'(x)=\frac{1}{2\sqrt{x}}$ $\quad f'(4)=\frac{1}{4}$

$f''(x)=-\frac{1}{4x^{3/2}}$ $\quad f''(4)=-\frac{1}{32}$

$f'''(x)=\frac{3}{8x^{5/2}}$ $\quad f'''(4)=\frac{3}{256}$

$f^{(4)}(x)=-\frac{15}{16x^{7/2}}$ $\quad f^{(4)}(4)=-\frac{15}{2048}$

$P_{0,4}(x)=2$

$P_{1,4}(x)=2+\frac{1}{4}(x-4)$

$P_{2,4}(x)=2+\frac{1}{4}(x-4)-\frac{1}{64}(x-4)^2$

$P_{3,4}(x)=2+\frac{1}{4}(x-4)-\frac{1}{64}(x-4)^2+\frac{1}{512}(x-4)^3$

$P_{4,4}(x)=2+\frac{1}{4}(x-4)-\frac{1}{64}(x-4)^2+\frac{1}{512}(x-4)^3-\frac{5}{16384}(x-4)^4$

for $\sqrt{5}$,

$P_{0,4}=\sqrt{5}\approx 2$

$P_{1,4}=\sqrt{5}\approx 2.25$ (tangent line)

$P_{2,4}=\sqrt{5}\approx 2.234375$

$P_{3,4}=\sqrt{5}\approx 2.236328125$

$P_{4,4}=\sqrt{5}\approx 2.236022949$