

# Mathematics

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# 1 Calculus

## 1.1 Differentiation

**2 Series**

**3 Multivariable Calculus**

## 4 Vector Calculus

### 4.1 Operators

#### 4.1.1 Grad

#### 4.1.2 Div

#### 4.1.3 Curl

#### 4.1.4 Laplacian

### 4.2 Integral Theorems

#### 4.2.1 Divergence Theorem

#### 4.2.2 Stokes's Theorem

## 5 Fluid Mechanics

### 5.1 Kinematics

#### 5.1.1 Coordinates

**Lagrangian**  $\underline{x}(\underline{a}, t)$ : The *motion of individual particles* is studied; the position  $\underline{x}$  of a particle at time  $t$  is related to its position at a reference point in time  $\underline{a}$  (typically at  $t = 0$ ).

**Eulerian**  $(\underline{x}, t)$ : The *'flow field'* is considered as a whole and the state of a fluid is described in terms of the values at a fixed location  $\underline{x}$  and at a fixed time  $t$ .

#### 5.1.2 Velocity

In Cartesian coordinates the velocity of a fluid particle at position  $\underline{x}(x, y, z)$  is given by:

$$\underline{u}(x, y, z) = u(x, y, z)\hat{i} + v(x, y, z)\hat{j} + w(x, y, z)\hat{k}$$

#### 5.1.3 Stagnation Points

Stagnation points occur when the velocity vector  $\underline{u}$  is equal to  $\underline{0}$

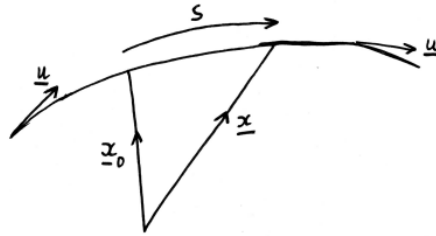
$$u = 0$$

$$v = 0$$

$$w = 0$$

#### 5.1.4 Streamlines

A streamline is a curve  $C$  drawn at one point in time such that the fluid velocity vector  $\underline{u}$  is tangent to  $C$  at every point along  $C$ .



$$\frac{d\underline{x}}{ds} = \underline{u}$$

$$\frac{dx}{ds} = u, \frac{dy}{ds} = v, \frac{dz}{ds} = w$$

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} (= ds)}$$

### 5.1.5 Particle Paths

Particle path is obtained by solving the initial value problem:

$$\frac{d\underline{x}}{dt} = \underline{u}(\underline{x}, t), \underline{x} = x_0 \text{ at } t = 0$$

$$\frac{dx}{dt} = u, x(0) = x_0$$

$$\frac{dy}{dt} = v, y(0) = y_0$$

$$\frac{dz}{dt} = w, z(0) = z_0$$

### 5.1.6 Steady Flow

**Steady Flow:** The flow velocity vector  $\underline{u}$  is independent of time  $t$

**Unsteady Flow:**  $\underline{u}$  depends on  $t$ ; the pattern of streamlines changes with  $t$

### 5.1.7 Convective Derivative

The convective derivative tells us how a property changes *as it moves with a flow*.

**General**

$$\boxed{\frac{D*}{Dt} = \frac{\partial*}{\partial t} + (\underline{u} \cdot \nabla)* = \frac{\partial*}{\partial t} + u \frac{\partial*}{\partial x} + v \frac{\partial*}{\partial y} + w \frac{\partial*}{\partial z}}$$

**Scalar**

$$\boxed{\frac{D\rho}{Dt} = \frac{\partial\rho}{\partial t} + (\underline{u} \cdot \nabla)\rho = \frac{\partial\rho}{\partial t} + u \frac{\partial\rho}{\partial x} + v \frac{\partial\rho}{\partial y} + w \frac{\partial\rho}{\partial z}}$$

**Vector**

$$\boxed{\frac{D\underline{u}}{Dt} = \frac{\partial\underline{u}}{\partial t} + (\underline{u} \cdot \nabla)\underline{u} = \frac{\partial\underline{u}}{\partial t} + u \frac{\partial\underline{u}}{\partial x} + v \frac{\partial\underline{u}}{\partial y} + w \frac{\partial\underline{u}}{\partial z}}$$

### 5.1.8 Vorticity

Vorticity  $\underline{\omega}$  is a measure of the local rotation of fluid particles in flow.

$$\underline{\omega} = \nabla \times \underline{u}$$

## 5.2 Pressure in a Fluid

## 5.3 Flow Dynamics

## 5.4 Two-dimensional Flow

## 5.5 Vorticity Dynamics

## 5.6 Free Surface Waves