Problem 3

(a) $a_1 = 10$

$a_2 = 12 \times 10 = 120$

$a_3 = 12 \times 10 \times 10 + 10 \times 2 \times 10$
$= 1200 + 200$
$= 1400$

(b) For all $n \geq 3$, $a_n = 10a_{n-1} + 20a_{n-2}$

explanation:

In order to construct a_n from a_{n-1}, we can add one ~~to~~ character to rightmost position. Therefore we got $10 \cdot a_{n-1}$ combinations, but since in this situation, we ignored the possibility that the rightmost position ~~ab~~ within a_{n-1} could be '-' or '+', thus, it need to be added back, which indicate ~~the~~ ~~term~~ term ~~$2 \cdot a_{n-2}$~~ ~~$20 a_{n-2}$~~ $2 \cdot 10 \cdot a_{n-2} = 20a_{n-2}$. Finally, add them together $a_n = 10a_{n-1} + 20a_{n-2}$

[redacted]

c,

base case: when $n=1$, $a_1 = 10 < 11.8^1 = 11.8$

when $n=2$, $a_2 = 120 < 11.8^2 = 139.24$

induction step:

let $A(n)$ represent that ~~the~~ $a_n \le 11.8^n$

we can assume there exist n and $(n-1)$ that makes ~~the~~ $A(n)$ true, since we have proved in base case with $n=1$ and 2, the assumption is valid.

$$a_{n+1} = 10a_n + 20a_{n-1}$$

since $A(n)$ and $A(n-1)$ is true according to assumption

$a_{n-1} \le 11.8^{n-1}$

$a_n \le 11.8^n$

$\therefore$ ~~a_{n+1}~~ $a_{n+1} \le 10 \cdot 11.8^n + 20 \cdot 11.8^{n-1}$

$10 \cdot 11.8^n + 20 \cdot 11.8^{n-1}$

$= 10 \cdot 11.8^n + \frac{20}{11.8} \cdot 11.8^n$

$= 11.8^n \left(10 + \frac{20}{11.8}\right)$

$10 + \frac{20}{11.8} = 11.6949 < 11.8$

$\therefore$ $10 \cdot 11.8^n + 20 \cdot 11.8^{n-1} \le 11.8^{n+1}$ I replaced $<$ with $\le$, which is valid

$\therefore$ $a_{n+1} \le 11.8^{n+1}$

~~induction step can~~

therefore, according to induction, ~~the~~ $a_n \le 11.8^n$ is true ~~the A(n) is tru~~ for all $n \ge 1$ ~~and~~ n is integer

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bonus mark for Problem 3.

let $a_n \leq x^n$

$a_{n+1} = 10a_n + 20a_{n-1} \leq x^{n+1}$

~~ie~~ since $a_n \leq x^n$ and $a_{n-1} \leq x^{n-1}$

$\cancel{a_{n+1} \leq}$ $10 \cdot x^n + 20 \cdot x^{n-1} \leq x^{n+1}$

$x^n(10 + \frac{20}{x}) \leq x^{n+1}$

$10 + \frac{20}{x} \leq x$

$x^2 - 10x - 20 \geq 0$

$\therefore x \geq \frac{10+\sqrt{100+80}}{2} = \frac{10+\sqrt{180}}{2} \approx 11.708203932\,5$

$\therefore$ the lowest number is 11.7082039325