BLG354E – Signals and Systems for CE Homework2

Şahin Olut - 150140124

1) Fourier series is important because in digital world we have limited space for storing informations. To store or process a file, we need the mathematical representation of file. Fourier transformation is doing it for us. Fourier transform allows us to simplify very difficult problems for analyzing.

Fourier Series Analysis

A signal x(t) can be represented as the sum of periodic signals.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \cdot e^{j(2\pi f_0)kt}$$

multiplying sides with $e^{-j(2\pi f_0)\ell t}$

$$x(t) \cdot e^{-j(2\pi f_0)\ell t} = \sum_{k=-\infty}^{\infty} a_k \, e^{j(2\pi f_0)kt} \cdot e^{-j(2\pi f_0)\ell t}$$

Integrating both sides in period T_0

$$\int_0^{T_0} x(t)\, e^{-j2\pi f_0 \ell t} dt = \int_0^{T_0} \sum_{k=-\infty}^{\infty} a_k \, e^{j2\pi f_0 kt} \cdot e^{-j2\pi f_0 \ell t} \, dt$$

this is interchangeble

$$= \sum_{k=-\infty}^{\infty} a_k \underbrace{\int_0^{T_0} e^{j2\pi f_0 kt} \cdot e^{-j2\pi f_0 \ell t} \, dt}_{\text{orthogonality property used}} \quad \begin{cases} \text{if } k=\ell & T_0 \\ \text{if } k\neq \ell & 0 \end{cases}$$

$$\int_0^{T_0} x(t) e^{-j2\pi f_0 \ell t} = \sum_{k=-\infty}^{\infty} a_k T_0 \quad \text{assuming } k=\ell$$

$$a_\ell = \frac{\int_0^{T_0} x(t)\, e^{-j2\pi f_0 \ell t}}{T_0}$$

Fourier Series Synthesis

(1) $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi f_0 k t}$ Lets call $e^{j2\pi f_0 k t} = V_k(t)$ as basis function.

zero integral property $= \int_0^{T_0} V_k(t)\, dt = 0 = \left. \frac{e^{j2\pi f_0 t}}{j2\pi f_0} \right|_0^{T_0}$

$$\frac{\overbrace{e^{j2\pi f_0 T_0}}^{1} - e^0}{j2\pi f_0 k} = \frac{e^{j2\pi k} - 1}{j2\pi k f_0} = 0 \qquad \boxed{e^{j2\pi k} = 1}$$

Lets assume $V_k(t)$ and $V_l(t)$ are ortagonal.

$$\int_0^{T_0} V_k(t) \cdot V_l(t) = \begin{cases} 0, & k \neq l \\ T_0, & k = l \end{cases} \leftarrow \text{orthogonality property}$$

if (1) is valid then we can multiply (1)'s both side with its complex conjugate $V_l(t)$ and integrate over its period.

$$\int_0^{T_0} x(t) \cdot e^{-j2\pi f_0 l t}\, dt = \int_0^{T_0} \left(\sum_{-\infty}^{\infty} a_k e^{j2\pi f_0 k t} \right) e^{-j2\pi f_0 l t}\, dt$$

$$\sum_{k=-\infty}^{\infty} a_k \left(\int_0^{T_0} \underbrace{e^{j2\pi f_0 (k-l)}}_{T_0 \text{ if } k=l} dt \right)$$

$$= a_k T_0$$

2) Orthogonality property is both used for analyzing and synthesing the Fourier Series. Without it, we can not say the result of integral in proof is equal to To.

3) Under Dirichlet conditions, periodic signals can be expanded to infinite set of complex exponents where frequencies are equal to +-k.f . If set has infinite elements, it turns out to be Fourier Transformation. Negative frequencies in Fourier Synthesis are similar to complex(imaginary) parts of signals. (Continuous-Time Signals, Springer, 2006)

4,5a)

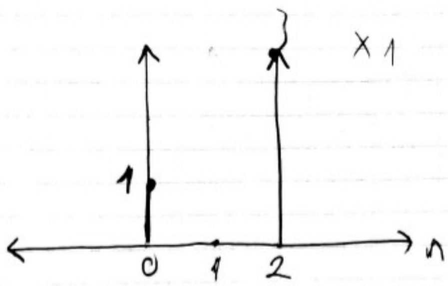
4) Defination of radian $= \frac{\text{radius}}{\text{circumreference}} = \frac{\text{length}}{\text{length}}$

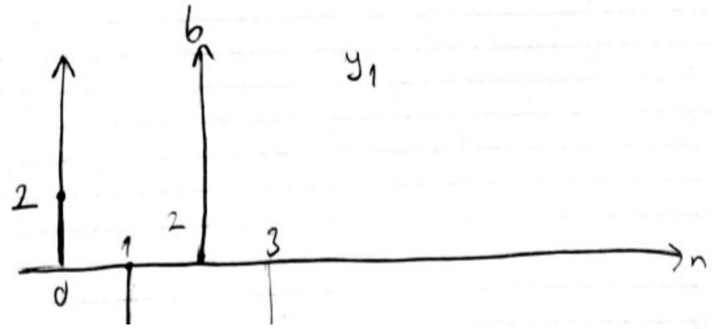
which Leads to a dimesionless measure

Degree is equal to 0,0174.. radians.

Which is dimensionless as well.

5)





5b)

5b) $x_1[N] = \delta[n] + 3\delta[n-2]$

$x_2[N] = 2\delta[n] - 3\delta[n-1] + 2\delta[n-2] - 9\delta[n-3] - 12\delta[n-4]$

$x_2[N] = a_1 x_1[n-n_1] + a_2 x_1[n-n_2] + a_3 x_1[n-n_3]$

$a_1 = -3 \qquad a_2 = 2 \qquad a_3 = -4$

$n_1 = n-1 \qquad n_2 = n \qquad n_3 = n-2$

5c)

5c)

$$x_2[N] = -3x_1[n-1] + 2x_1[n] - 4x_1[n-2]$$

$$x_1[N] \longrightarrow \boxed{\text{System}} \xrightarrow{y_1[N]}$$

$$y_2[N] = -3y_1[n-1] + 2y_1[n] - 4y_1[n-2]$$

$$-3y_1[n-1] = -6\delta[n-1] + 9\delta[n-2] - 18\delta[n-3] + 27\delta[n-4]$$

$$2y_1[n] = 4\delta[n] - 6\delta[n-1] + 12\delta[n-2] - 18\delta[n-3]$$

$$-4y_1[n-2] = -8\delta[n-2] + 12\delta[n-3] - 24\delta[n-4] + 36\delta[n-5]$$

5d,e)

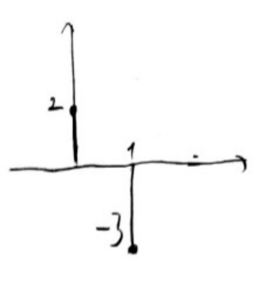
5d)

$$\overset{y_1[N]}{\overbrace{2\delta[n] - 3\delta[n-1] + 6\delta[n-2] - 9\delta[n-3]}} = \sum h[k]\cdot x[n-k]$$

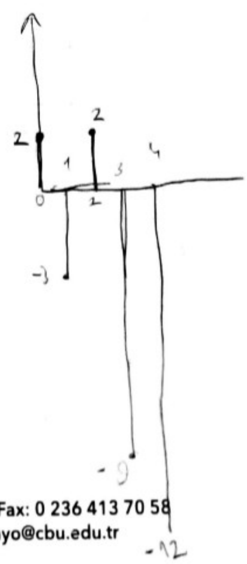
$$h[n] = 2\delta[n] - 3\delta[n-1]$$

5e) $h[n] * x[n] = y[n]$

$h[n]$



$x_2[N]$

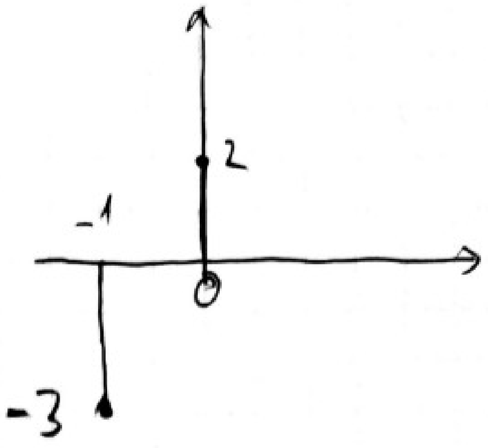


flip req. ②

$$y[0] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[-k]$$

h[-n]

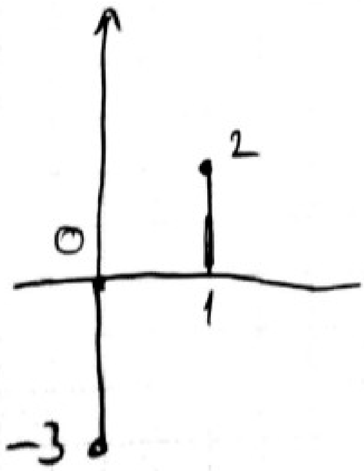
$y[0] = 4$



$$y[1] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[1-k]$$

h[1-n]

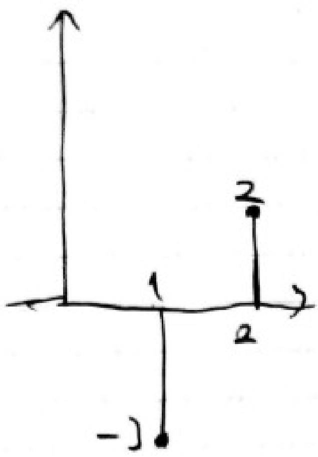
$y[1] = -\cancel{8} -6$



(3)

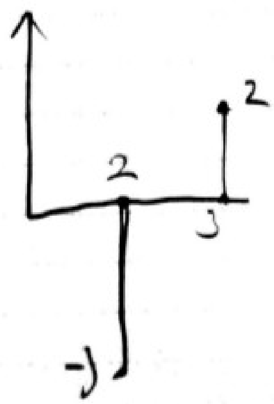
$$y[2] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[2-k]$$

$h[2-n]$



$y[2] = 9+4$

$$y[3] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[3-k]$$

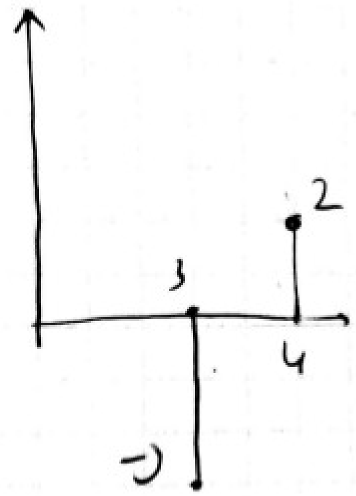


$y[3] = -6-18$

9

$$y[4] = \sum_{k=0}^{\infty} x[k] \cdot h[4-k]$$

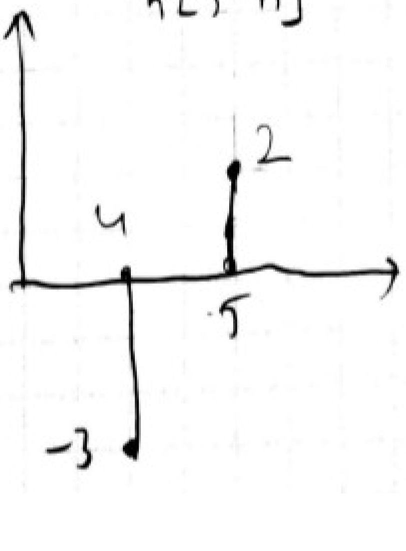
$h[4-n]$



$y[4] = 27 - 24$

$$y[5] = \sum_{k=0}^{\infty} x[k] \cdot h[5-k]$$

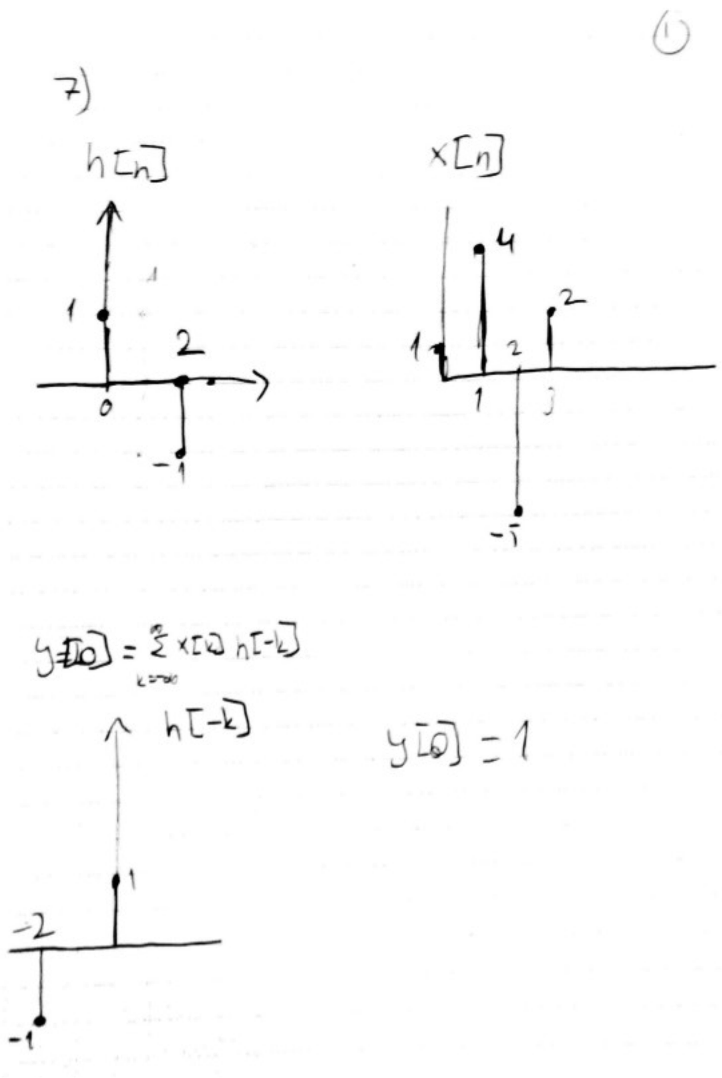
$h[5-n]$



$y[5] = 36$

Y2 is equal to 4δ[n]+12δ[n-1]+13δ[n-2]-24δ[n-3]+3δ[n-4]+36δ[n-5]

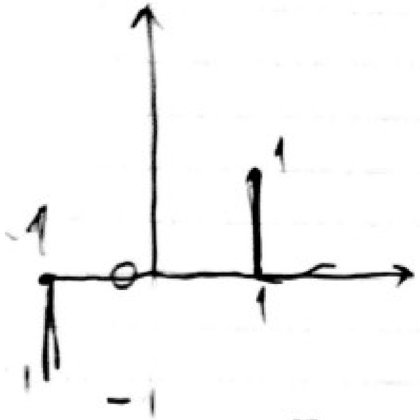
7)



②

$$y[1] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[1-k]$$

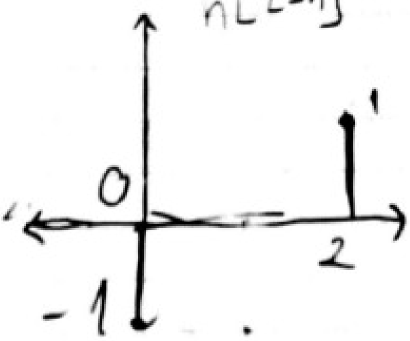
$h[1-n]$



$y[1] = -4$

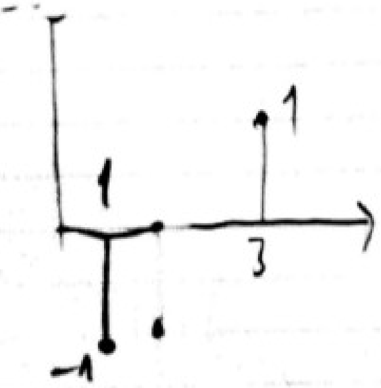
$$y[2] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[2-k]$$

$h[2-n]$



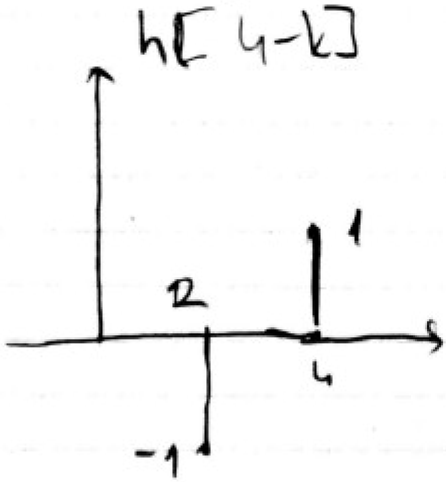
$y[2] = -6$

$$y[3] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[3-k]$$



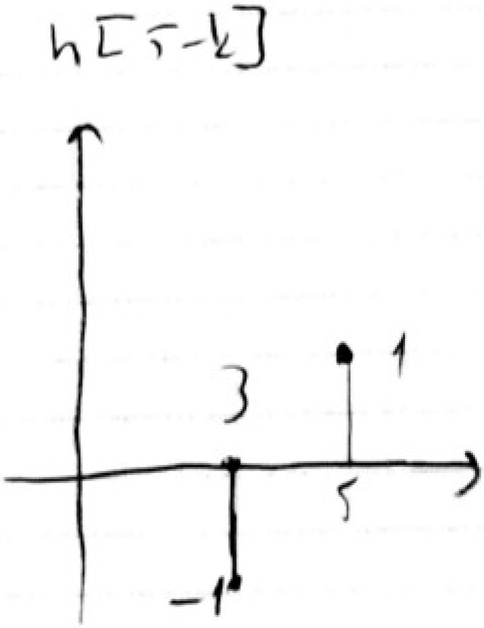
$y[3] = -2$

$$y[4] = \sum_{-\infty}^{\infty} x[k] \cdot h[4-k]$$



$y[4] = 5$

$$y[5] = \sum_{-\infty}^{\infty} x[k] \cdot h[5-k]$$



$y[5] = -2$

7b) $h[n] = x[n] - x[n-2]$

$y[n] = \delta[n] + 4\delta[n-1] - 6\delta[n-2] - 2\delta[n-3] + 5\delta[n-4] - 2\delta[n-5]$

7c) MATLAB code is provided in .zip file

REFERENCES USED IN HOMEWORK

1) Coursebook: Digital Signal Processing First

2) Continuous-Time Signals, Springer, 2006