BLG 354E Homework - 2

Due 19.03.2017 22:00

Policy: Please do your homework on your own. The code and the report you submitted must be your own work. Cheating is highly discouraged for it could mean a zero or negative grade from the homework.

Only the problem parts denoted by [MATLAB] should be solved using Matlab. The rest is to be solved manually.

For your questions: albay@itu.edu.tr

1. (5 pt.) What is the theory behind Fourier Series? Why Fourier Series representation is important?

2. (25 pt.) Give derivation of Fourier Series Analysis and Synthesis equations step by step. Explain each step in detail. Why the orthogonality property is important in the derivation of analysis and synthesis equations? Emphasize how the orthogonality property is used during derivation of analysis and synthesis equations.

3. (5 pt.)Why does Fourier Synthesis equation

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T_0)kt}$$

   contain negative frequencies?

4. (5 pt.) Show that radian and degree are dimensionless measures.

5. (20 pt.) Suppose we have a linear time-invariant (LTI) FIR filter with an impulse response of $h[n]$. We do not know the impulse response, but we know that when the input signal is $x_1[n]$, output is $y_1[n]$, where $x_1[n]$ and $y_1[n]$ are as follows:

$$x_1[n] = \delta[n] + 3\delta[n-2]$$
$$y_1[n] = 2\delta[n] - 3\delta[n-1] + 6\delta[n-2] - 9\delta[n-3]$$

   (a) Plot the signals $x_1[n]$ and $y_1[n]$.

(b) Another signal, $x_2[n]$ is given as follows:

$$x_2[n] = 2\delta[n] - 3\delta[n-1] + 2\delta[n-2] - 9\delta[n-3] - 12\delta[n-4]$$

Plot $x_2[n]$. Write $x_2[n]$ as a linear combination of shifted $x_1[n]$ of three terms, i.e., find a_1, a_2, a_3, n_1, n_2 and n_3 values such that:

$$x_2[n] = a_1 x_1[n-n_1] + a_2 x_1[n-n_2] + a_3 x_1[n-n_3]$$

(c) Use linearity and time invariance of the system to determine the output $y_2[n]$ for the input signal $x_2[n]$.

(d) *System Identification:* Find the impulse response of the system, i.e., $h[n]$.

(e) With the impulse response, $h[n]$, you found; find $y_2[n]$ using convolution. Is the result same with the previous one?

6. (20 pt.) **[MATLAB]** Write a program that synthesize the periodic signals in Figure 1 using Fourier Series.

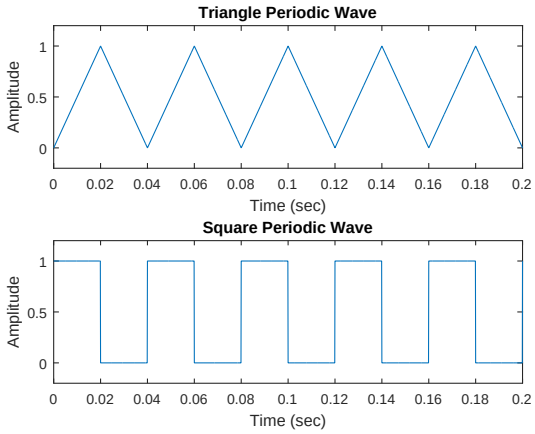

Figure 1

Your program add 1 component to the signal in each second, so you will create a kind of simulation of the signal synthesis, and at the end it will contain 60 harmonics. In your report, give the snapshots of the signal after 1 st, 2 nd, 3rd, 5 th, 10 th, 20 th, 40 th and 60 th harmonics.

7. (20 pt.) A discrete time signal $x[n]$ is given input to a linear time-invariant (LTI) FIR filter with an impulse response of $h[n]$ and the corresponding output signal is $y[n]$, where $x[n]$ and $h[n]$ are as follows:

$$x[n] = \delta[n] + 4\delta[n-1] - 5\delta[n-2] + 2\delta[n-3], \tag{1}$$

$$h[n] = \delta[n] - \delta[n-2].$$

(a) Find the output signal $y[n]$ using the graphical (sliding window) method of convolution. Write down $y[n]$ in terms of shifted $\delta[n]$ functions.

(b) Write down the difference equation for this signal, and find $y[n]$ by substituting (1) for input in the difference equation.

(c) **[MATLAB]** Find the output signal $y[n]$ using the *conv* function in Matlab. Plot $x[n]$, $h[n]$ and $y[n]$.