Ch 12 Filtering, Modulation & Sampling

- Sampling & Reconstruction

$$x_s(t) = x(t)\,p(t)$$

$$= x(t)\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} x(t)\,\delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s)$$

Fourier transform of periodic impulse train is

$$a_k = \frac{1}{T_s}$$

$$x_s(t) = x(t)\sum_{k=-\infty}^{\infty} \frac{1}{T_s} e^{jk\omega_s t} = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} x(t)\,e^{jk\omega_s t}$$

$\omega_s = 2\pi/T_s$

$$\hat{X}_s(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} \hat{X}(j(\omega - k\omega_s))$$

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DTFT

$$x_s(t) = x_c(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$= x_c(t) \sum_{k=-\infty}^{\infty} \frac{1}{T_s} e^{jk\omega_s t}$$

$$= \sum_{n=-\infty}^{\infty} x_c(nT_s)\, \delta(t - nT_s)$$

FT of $x_s(t)$

$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_c(j(\omega - k\omega_s))$$

$$X_s(j\omega) = \sum_{n=-\infty}^{\infty} x_c(nT_s)\, e^{-j\omega n T_s} = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n T_s}$$

$$\sum_{n=-\infty}^{n=\infty} x[n]\, e^{-j\omega n T_s} = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_c(j(\omega - 2\pi k/T_s))$$

$\omega T_s = \hat{\omega}$

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\hat{\omega} n}$$

Exercise 12.15:

Show that $X(e^{j\hat{\omega}})$ is periodic in $\hat{\omega}$ with period 2π

Show that $X(e^{j\omega T_s})$ is periodic in ω with period $2\pi/T_s$

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jn\hat{\omega}}$$

$$X(e^{j(\hat{\omega}+2\pi)}) = \sum_{n=-\infty}^{\infty} x(n)\, \underbrace{e^{-jn(\hat{\omega}+2\pi)}}_{e^{-jn\hat{\omega}}\, \underbrace{e^{-j2\pi n}}_{1}\, = e^{-jn\hat{\omega}}}$$

$$= X(e^{j\hat{\omega}})$$

$$X\left(e^{j(\omega+\frac{2\pi}{T_s})T_s}\right) = X\left(e^{j(\omega T_s+2\pi)}\right)$$

$$= X(e^{j\omega T_s})$$

Inverse DTFT

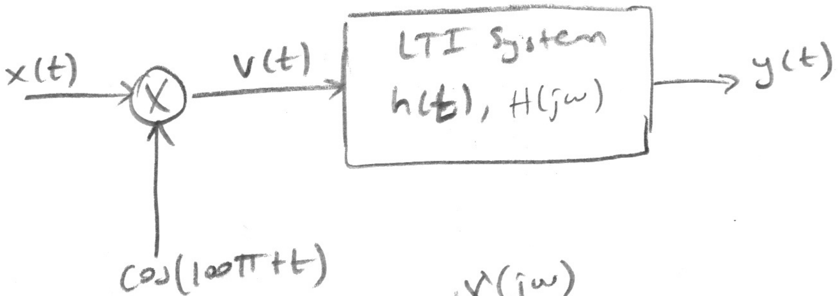
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\hat{\omega}})\, e^{j\hat{\omega} n}\, d\hat{\omega}$$

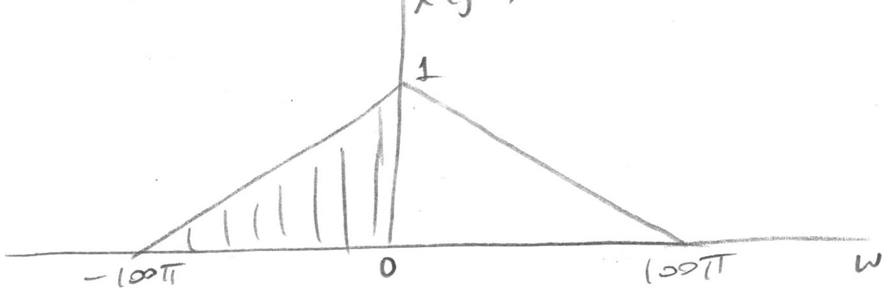
— Relation between the DTFT and the CTFT in sampling

$$X(e^{j\omega T_s}) = X(e^{j\hat{\omega}}) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X_c\left(j(\omega - 2\pi k/T_s)\right)$$

Problems

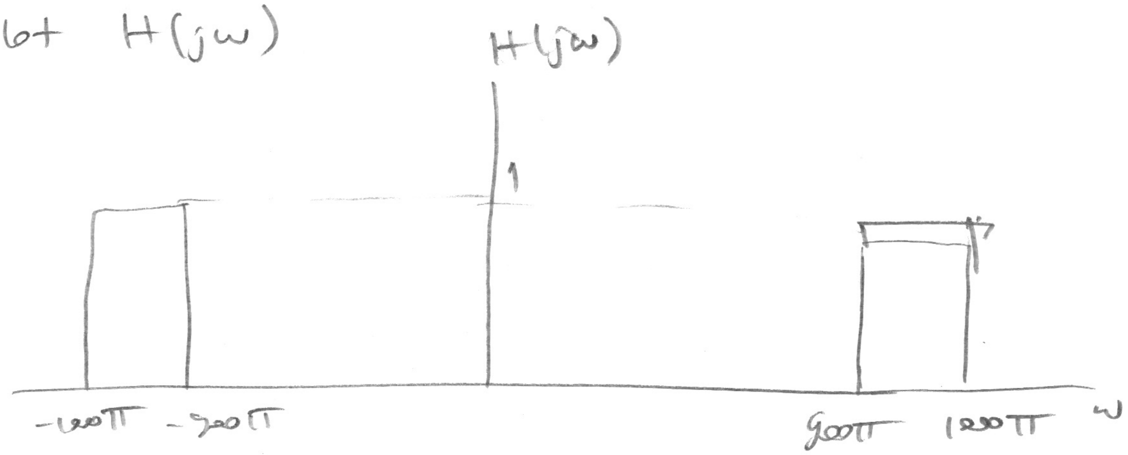
12.4 : Amplitude modulation system





$$H(j\omega) = \begin{cases} 1, & 900\pi < |\omega| < 1000\pi \\ 0, & \text{else} \end{cases}$$

a) Plot $H(j\omega)$

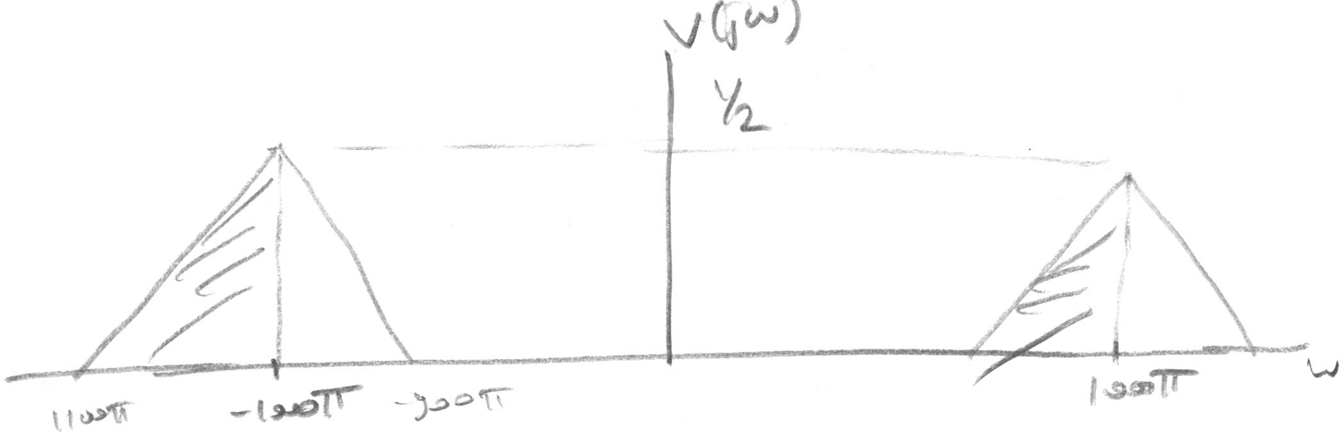


b) Plot $V(j\omega)$

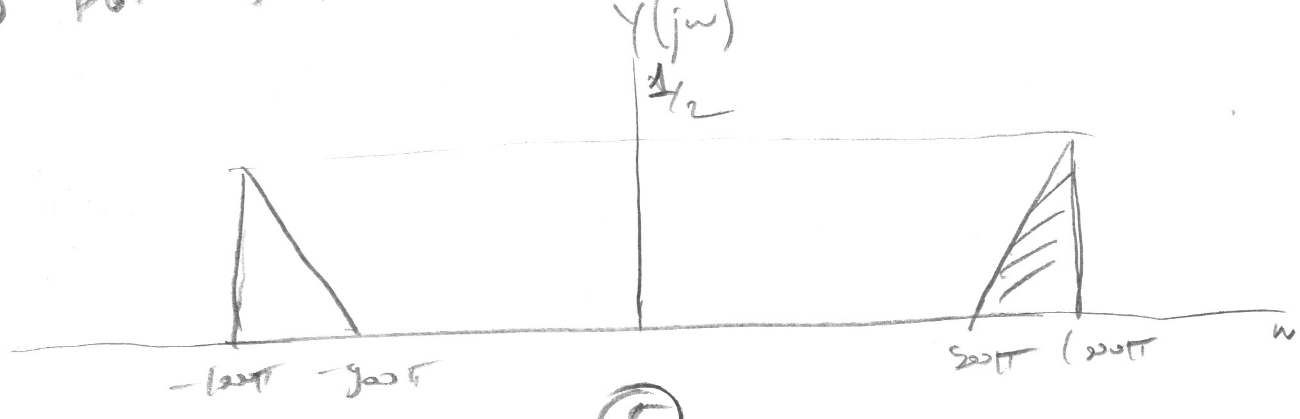
$$v(t) = x(t)\cos(1000\pi t)$$

$$V(j\omega) = X(j\omega) * \left[\frac{1}{2}\delta(\omega - 1000\pi) + \frac{1}{2}\delta(\omega + 1000\pi)\right]$$

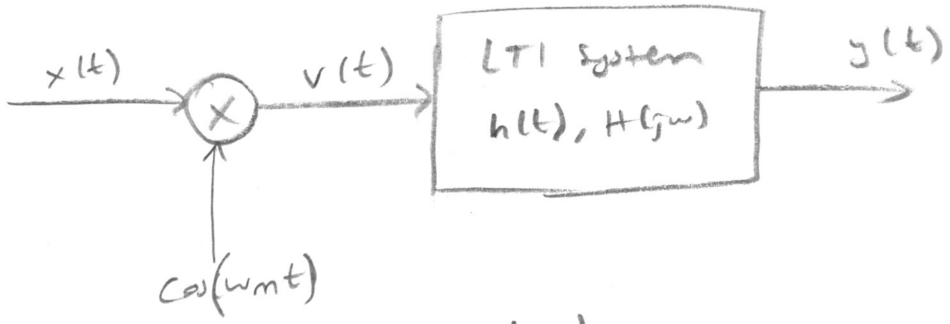
$$= \frac{1}{2}X(j(\omega - 1000\pi)) + \frac{1}{2}X(j(\omega + 1000\pi))$$

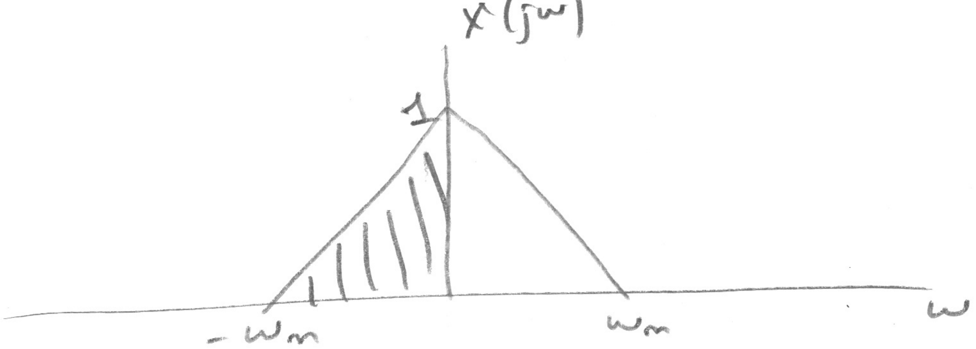


c) Plot $Y(j\omega)$



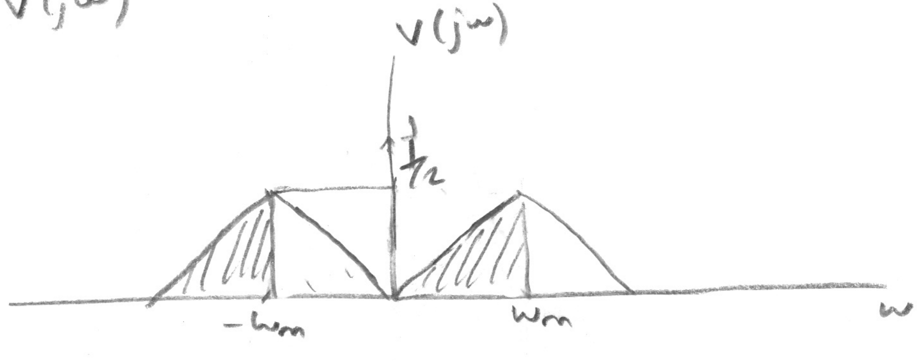
P-12b :





$$H(j\omega) = \begin{cases} 2, & |\omega| \le \omega_0 \\ 0, & \text{else} \end{cases}$$

a) Plot $V(j\omega)$



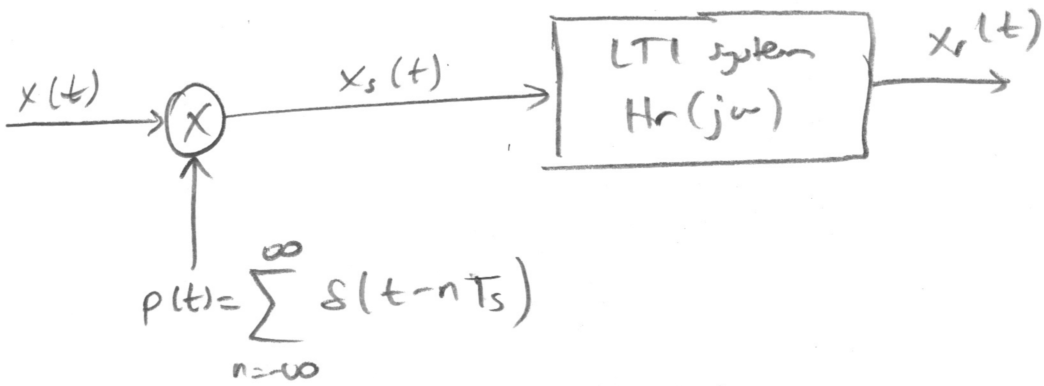
$Y(j\omega) = H(j\omega)\ Y(j\omega)$

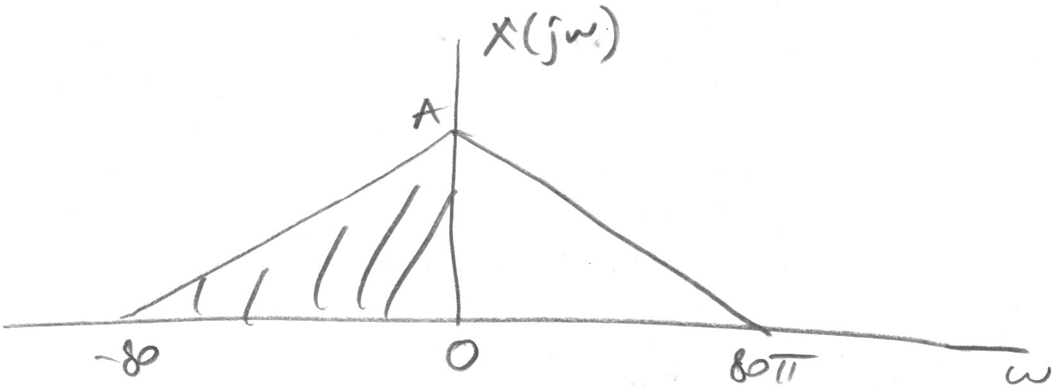
b)

c)

Solve on your own problem 6 in ch 12 in your book

P. 12-12 :





The derivation of sampling theorem involves the operations of impulse train sampling and reconstruction as shown in the Fig.

a) Choose sampling rate $w_s = 2\pi/T_s$ so that $x_r(t) = x(t)$ when

$$H_r(jw) = \begin{cases} T_s & |w| \leq \pi/T_s \\ 0 & |w| > \pi/T_s \end{cases}$$

Plot $X_s(jw)$ for the value of $w_s = 2\pi/T_s$ that is equal to the Nyquist rate

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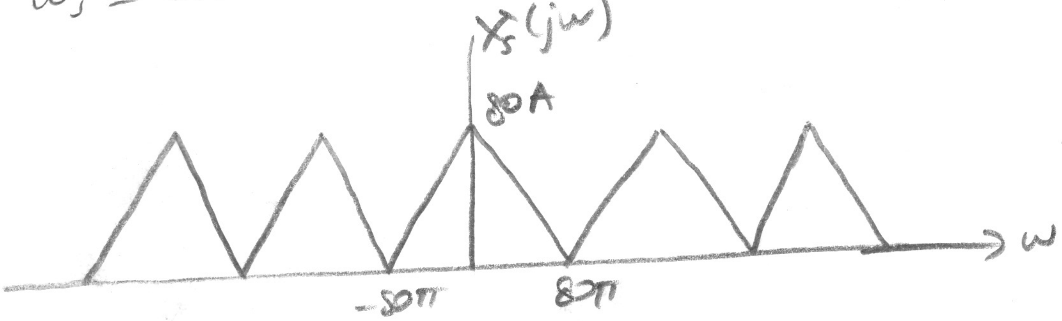
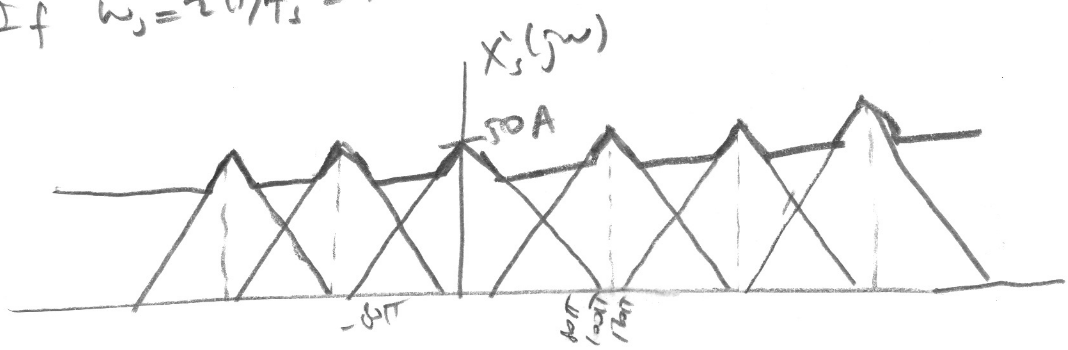
$w_s = 2w_b$ at least because of Nyquist

$w_s = 2 \cdot 80\pi = 160\pi$



b) If $w_s = 2\pi/T_s = 100\pi$, show the aliasing



c)