Fourier Series Analysis

A signal $x(t)$ can be represented as the sum of periodic signals.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \cdot e^{j(2\pi f_0)kt}$$

multiplying sides with $e^{-j(2\pi f_0)lt}$

$$x(t) \cdot e^{-j(2\pi f_0)lt} = \sum_{k=-\infty}^{\infty} a_k \, e^{j(2\pi f_0)kt} \cdot e^{-j(2\pi f_0)lt}$$

Integrating both sides in period T_0

$$\int_0^{T_0} x(t)\, e^{-j2\pi f_0 lt} dt = \int_0^{T_0} \sum_{k=-\infty}^{\infty} a_k \, e^{j2\pi f_0 kt} \cdot e^{-j2\pi f_0 lt} \, dt$$

this is interchangeble

$$= \sum_{k=-\infty}^{\infty} a_k \underbrace{\int_0^{T_0} e^{j2\pi f_0 kt} \cdot e^{-j2\pi f_0 lt} \, dt}_{\text{orthagonality property used}}$$

$$\begin{Bmatrix} \text{if } k = l & T_0 \\ \text{if } k \neq l & 0 \end{Bmatrix}$$

$$\int_0^{T_0} x(t) e^{-j2\pi f_0 lt} = \sum_{k=-\infty}^{\infty} a_k \, T_0$$

assuming $k = l$

$$a_l = \frac{\int_0^{T_0} x(t)\, e^{-j2\pi f_0 lt}}{T_0}$$

Fourier Series Synthesis

(1) $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi f_0 k t}$ Lets call $e^{j2\pi f_0 k t} = V_k(t)$ as basis function.

Zero integral property $= \int_0^{T_0} V_k(t)\,dt = 0 = \frac{e^{j2\pi f_0 t}}{j2\pi f_0} \Big|_0^{T_0}$

$$\frac{e^{j2\pi f_0 T_0 k} - e^0}{j2\pi f_0 k} = \frac{e^{j2\pi k} - 1}{j2\pi k f_0} = 0 \qquad \boxed{e^{j2\pi k} = 1}$$

Lets assume $V_k(t)$ and $V_l(t)$ are ortagonal.

$$\int_0^{T_0} V_k(t) \cdot V_l(t) = \begin{cases} 0, & k \neq l \\ T_0, & k = l \end{cases} \leftarrow \text{Orthogonality property}$$

if (1) is valid then we can multiply (1)'s both side with its complex conjugate $V_l(t)$ and integrate over its period.

$$\int_0^{T_0} x(t) \cdot e^{-j2\pi f_0 l t}\,dt = \int_0^{T_0} \left(\sum_{-\infty}^{\infty} a_k e^{j2\pi f_0 k t} \right) e^{-j2\pi f_0 l t}\,dt$$

$$\sum_{k=-\infty}^{\infty} a_k \left(\int_0^{T_0} \underbrace{e^{j2\pi f_0 (k-l)}}_{T_0 \text{ if } k=l}\,dt \right)$$

$$= a_k T_0$$