BLG 354E Homework - 1

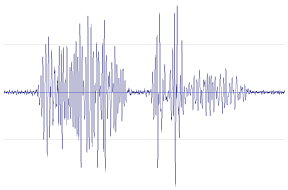
Due 26.02.2017 23:00

Policy: Please do your homework on your own. The code and the report you submitted must be your own work. Cheating is highly discouraged for it could mean a zero or negative grade from the homework. In Problems 4, 5, and 6, you will write code in Matlab. You should install Matlab R2015b to your computers immediately following the instructions in Ninova. Upload your solutions through Ninova. (Note: If you answer all of the questions correctly, you will get 110 points.)

For your questions: albay@itu.edu.tr

1. (5 pts.) From signals and systems perspective, draw a simple block diagram for a system composed of a digital photograph camera and a projector. The system is capable of taking picture and projecting the image on a curtain. (Note: You do not have to draw all the details, just give a rough diagram.)

2. (5 pts.) Which of the following images represent signal? Give reasoning for your answer, that is, why you think the image represents a signal or not.



3. (40 pts.) Give answer the followings

(a) What is an odd function? Define an odd function and give an example.

(b) What is an even function? Define an even function and give an example.

(c) Match the followings.

a.	$\sin\theta$	_____	$\cos\theta$
b.	$\cos(\theta + 2\pi k)$	_____	$-\sin\theta$
c.	$\cos(-\theta)$	_____	1, when k is integer
d.	$\sin(-\theta)$	_____	0, when k is integer
e.	$\sin(\pi k)$	_____	$\cos\theta$, when k is integer
f.	$\cos(2\pi k)$	_____	-1, when k is integer
g.	$\cos[2\pi(k + 1/2)]$	_____	$\cos(\theta - \pi/2)$

(d) Give derivation of the following identities.

i. $\sin^2\theta + \cos^2\theta = 1$
ii. $cos(2\theta) = \cos^2\theta - \sin^2\theta$
iii. $sin(2\theta) = 2\sin\theta\cos\theta$
iv. $sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
v. $cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

(e) What is the orthogonality of functions? Show that the following functions are orthogonal.

i. $\sin(2\pi nft)$ and $\sin(2\pi mft)$ on $-L \leq t \leq L, n \neq m$ and m, n integer
ii. $\cos(2\pi nft)$ and $\cos(2\pi mft)$ on $-L \leq t \leq L, n \neq m$ and m, n integer
iii. $\sin(2\pi nft)$ and $\cos(2\pi mft)$ on $-L \leq t \leq L, m$ and n integer

4. (20 pts.) Let

$$z_1(t) = \frac{e^{j(\omega t - \frac{1}{3}\pi)} + e^{-j(\omega t - \frac{1}{3}\pi)}}{2}$$
$$z_2(t) = 3\sin(\omega t - \frac{5}{4}\pi)$$
$$z_3(t) = \mathrm{Re}\left\{2e^{j(\omega(t - \frac{4.7124}{\omega}))}\right\}$$

$x(t)$ is defined as follows:

$$x(t) = z_1(t) + z_2(t) + z_3(t)$$

(a) Express $x(t)$ in the form $x(t) = A\cos(\omega t + \phi)$ by finding the numerical values of A and ϕ. Use complex phasor manipulations to obtain the answer.

(b) Plot all the phasors used to solve the problem in (a) in the complex plane.

(c) Write a script that will plot the signal $x(t)$ using MATLAB. Please select suitable sampling space that makes the curve a faithful representation of the cosine function. (Select suitable ω)

5. (20 pts.) A signal is given by the equation

$$x(t) = 2 + 4\cos(500\pi t + \frac{5}{4}\pi) - 3\sin(60\pi t) + 3\cos(250\pi(t - 10^{-3}))$$

(a) Sketch the spectrum of this signal, indicating the complex size of each frequency component. Make separate plots for real/imaginary or magnitude/phase of the complex amplitudes at each frequency. (Make plots on MATLAB, but not necessary.)

(b) Is $x(t)$ periodic? If so, what is the period?

(c) What is the fundamental frequency of this signal? Which harmonics does $x(t)$ contain?

6. (20 pts.) Let

$$\begin{aligned} y_1(t) &= 2\cos(10\pi t) \\ y_2(t) &= 7\sin(10000\pi t - \frac{1}{3}\pi) \\ x(t) &= y_1(t) * y_2(t) \end{aligned}$$

   (a) Find spectrum of $x(t)$ and express $x(t)$ as a sum of complex exponential signals.

   (b) Plot the spectrum of this signal.

   (c) Express $x(t)$ as a sum of two sinusoids, i.e., find the numbers A_1, A_2, ω_1, ω_2, ϕ_1 and ϕ_2 such that:

$$x(t) = A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2)$$

   (d) There is an interesting audio effect that can be created by multiplying two sinusoids having different frequencies called a **beat note**. $x(t)$ is an example of a beat note. Please write a MATLAB script to listen $x(t)$ (Hint: To listen the sound use *soundsc* MATLAB command. You must be careful about default sample rate, which is 8192, of *soundsc*.). Also plot, in turn, $y_1(t)$, $y_2(t)$, and $x(t)$ on the same figure using MATLAB for $0 \leq t \leq 0.4$. Find fundamental period of $x(t)$ and show the fundamental frequency on the plot (Hint: It is not necessary but to ease the localization of the fundamental period on the plot, you can change tick value locations of the plot axis using *ax = gca; ax.XTick = 0:0.01:0.4;* and show the grid of the plot using *grid on* command after *plot*.).

7. (Will not be graded.) Please solve questions that is shown below from your textbook for your own good.

   (a) From ch. 2: P-2.1, P-2.4, P-2.5, P-2.7, P-2.12, P-2.15, P-2.18

   (b) From ch. 3: P-3.1, P-3.4, P-3.11, P-3.14, P-3.15, P-3.17