Signals & Systems

Week 8 –

03.04.2017

Lecture + Recitation

Chapter 9 (Continued) Continuous Time Systems

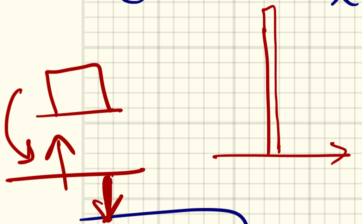
DT $\quad x[n] \longrightarrow \boxed{\text{DT LTI system}} \longrightarrow y[n]$

$\delta[n] \longrightarrow \boxed{\text{DT LTI system}} \longrightarrow h[n]$

$h[n]$

CT $\quad x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

$\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow h(t)$



ex: Delay System: LTI $\quad x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t-t_0)$

t_0

$y(t) = x(t-t_0)$

$h(t) = \delta(t-t_0)$

LTI CT System: Same as what we did in DT Systems

★ Know how to test for linearity & time-invariance.

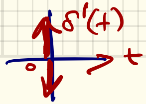
ex: Integrator System: $\quad x(t) \longrightarrow \boxed{\text{LTI Integrator}} \longrightarrow y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

$\delta(t) \longrightarrow$

$h(t) = ?$

$h(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$

ex: Differentiator System; $y(t) = \frac{d}{dt}x(t)$; $h(t) = \frac{d}{dt}\delta(t) = \delta^{(1)}(t)$



$\delta'(t)$?

CT Convolution:

For an LTI system

$$x(t) \longrightarrow \boxed{\text{LTI System}}_{h(t)} \longrightarrow y(t) = h(t) * x(t)$$

$$\left(y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]\right)$$

CT Convolution:

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Properties of Convolution:

CT

Same as DT Convolution

i. Commutative

ii. Associative

iii. Distributive over addition

iv. Has an identity element: $\delta(t)$.

Ex:

$$x(t) \longrightarrow \overset{\text{LTI}}{\boxed{u(t)}} \longrightarrow y(t)$$

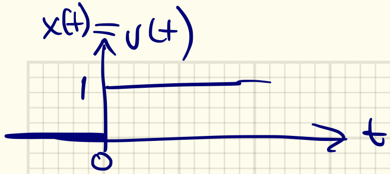
Let $x(t) = u(t) \longrightarrow \boxed{u(t)}$ $y(t) = ?$

$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, u(t-\tau)\, d\tau$$

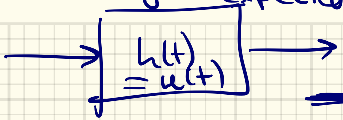
$$y(t) = \int_{0}^{\infty} u(t-\tau)\, d\tau$$

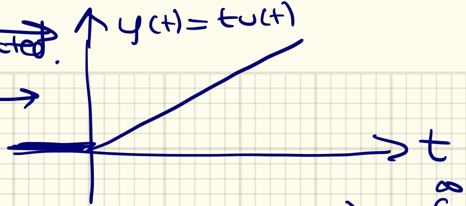
$$u(t-\tau) = \begin{cases} 1, & t-\tau \geq 0 = \tau \leq t \\ 0, & \tau > t \end{cases} \Rightarrow y(t) = \int_{0}^{t} 1 \cdot d\tau = t \cdot u(t)$$

$0 \leq \tau \leq t \Rightarrow t \geq 0$



Intuitively, this is expected.



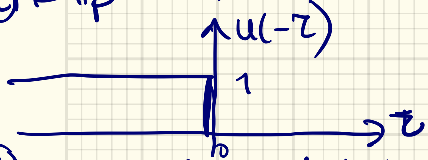


Using Graphical Method to perform Convolution:

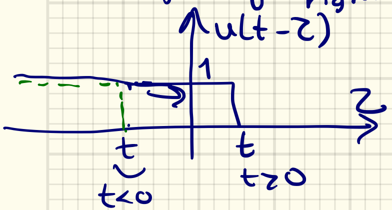
$$u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau)u(t-\tau)\,d\tau$$

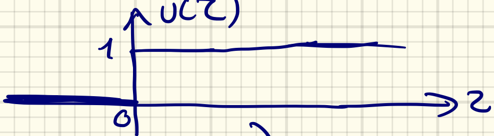
how to obtain $u(t-\tau)$

(i) Flip $u(\tau)$



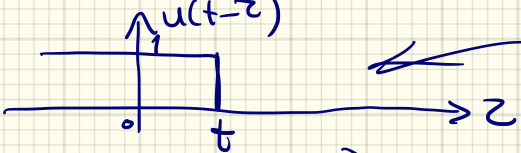
(ii) Now shift by t to the right



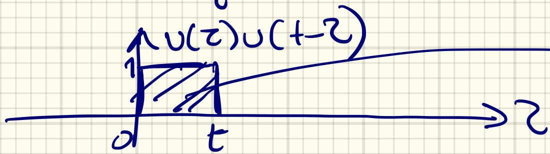


for $t < 0$

$u(\tau)u(t-\tau) = 0$



for $t \geq 0$



$$\int_{0}^{t} 1 \cdot d\tau = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$y(t) = t \cdot u(t)$$

Ex: Convolution w/ Impulse Fn:

$$y(t) = u(t) * \overbrace{\delta(t-t_0)}^{=x(t)} = ?$$

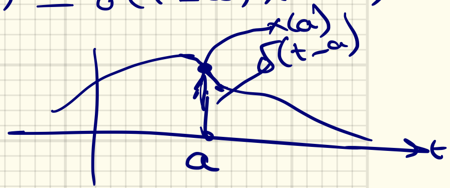
$$y(t) = \int_{-\infty}^{\infty} \underbrace{\overbrace{\delta(\tau-t_0)}^{x(t)}}_{\text{non-zero for } \tau=t_0} \cdot \overbrace{u(t-\tau)}^{u(t-\tau)}\,d\tau = u(t-t_0)$$

: from sampling property of $\delta(t)$.

* For any $x(t)$: $y(t) = x(t) * \delta(t-t_0) = x(t-t_0)$.

From :

→ Sampling property of $\delta(t)$: (i) $\delta(t-a)\,x(t) = \delta(t-a)\,x(t=a)$



(ii) $\displaystyle\int_{-\infty}^{\infty} \delta(t-a)\,x(t)\,dt = x(a)$

Q: $u(t-2) * \delta(t-3) = ?$

CASCADE SYSTEMS

LTI systems connected in series (same as in DT systems)

$\rightarrow \boxed{h_1(t)} \rightarrow \boxed{h_2(t)} \rightarrow \cdots$

$h_0(t) = ?$

Ex : $x(t) \rightarrow \underset{\text{LTI 1}}{\boxed{h(t)}} \xrightarrow{y(t)} \underset{\text{LTI 2}}{\boxed{\text{Differentiator}}} \rightarrow y'(t)$

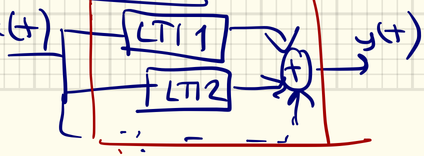
should be the same,

$x(t) \rightarrow \underset{\text{LTI 2}}{\boxed{\text{Differentiator}}} \xrightarrow{x'(t)} \underset{\text{LTI 1}}{\boxed{h(t)}} \rightarrow y'(t)$.

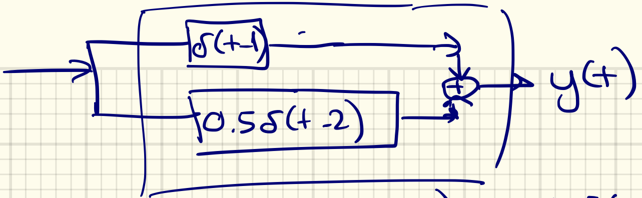
PARALLEL SYSTEMS

LTI systems connected

$h_0(t) = ?$



ex:

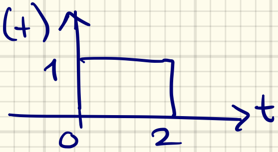


$$h_0(t) = \delta(t-1) + 0.5\,\delta(t-2)$$

$$y(t) = x(t) * h_0(t) = x(t) * \big(\delta(t-1) + 0.5\,\delta(t-2)\big)$$

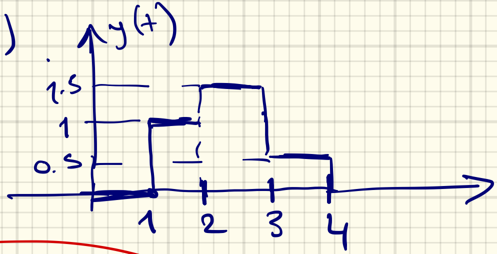
$$y(t) = x(t-1) + 0.5\,x(t-2).$$

Given: $x(t)$



$\longrightarrow$ Find $y(t)$

exercise
slow



System Properties: (Also LTI properties)

System	Stability	Causality
General	BIBO	Only use current & past values of the input.
LTI	$\int_{-\infty}^{\infty} \lvert h(t) \rvert \, dt < \infty$	$h(t) = 0$ for $t<0$.

$h(t)$

exercise: derive this condn.

exercise: derive this condn.

BIBO: If $|x(t)| \le \underbrace{M}_{<\infty}$

then $|y(t)| \le N < \infty$

ex: Integrator System : (check for LTI) $h(t) = u(t)$

Causality : ? Yes, b/c $h(t) = 0$ for $t < 0$. Yes.

Stability ? $\int_{-\infty}^{\infty} |u(t)|\,dt = \int_{0}^{\infty} 1 \cdot dt \not< \infty$ No

ex: $y(t) = x(-t)$: 1st check whether the system is LTI. $\longrightarrow$ Not an LTI system.

Causality : No, eg. $\underset{t=-2}{y(-2)} = x(2)$ $\hookrightarrow$ future input.

Stability : Let $|x(-t)| \leq M \rightarrow |y(t)| = |x(-t)| \leq M$.

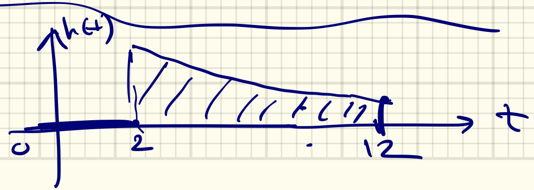
Yes (BIBO)

ex: Given a system by : $h(t) = \begin{cases} e^{-0.1(t-2)}, & 2 \leq t \leq 12 \\ 0, & \text{elsewhere.} \end{cases}$

$\Rightarrow$ LTI : Check

Causality ?

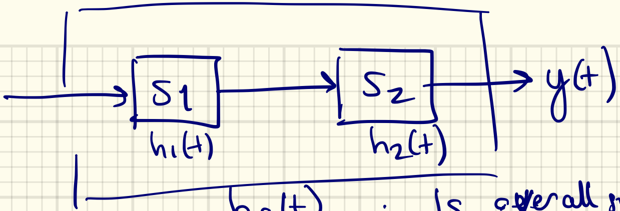
Stability : ?



Exercises:

(1)

$x(t)$



$h_0(t)$: Is overall system $h_0(t)$ causal & stable?

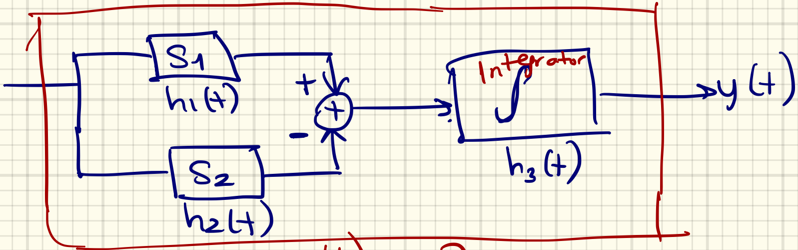
Given

$$h_1(t) = \begin{cases} e^{-2t}, & 0 \leq t \leq 1 \\ 0, & o/w \end{cases}$$

$h_2(t) = \delta^{(1)}(t) \rightarrow$ differentiator system: $y(t) = \frac{d}{dt}x(t)$

(2)

$x(t)$



Given

$h_1(t) = \delta(t+1)$

$h_2(t) = \delta(t-2)$

$h_3(t) = u(t)$

Compute $h_0(t) = ?$

Is overall system $h_0(t)$ causal and stable?