Tutorial - 1

Name.- Om Bhatt

Sec - CST SPL 2

Sem - IV

Class Roll no. — 36

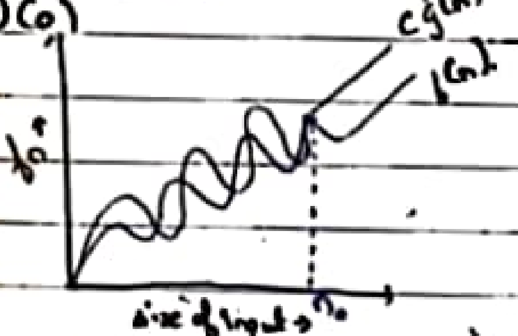
Uni. Roll no. - 2017519

Date - 10 March 2022

Om Bhatt

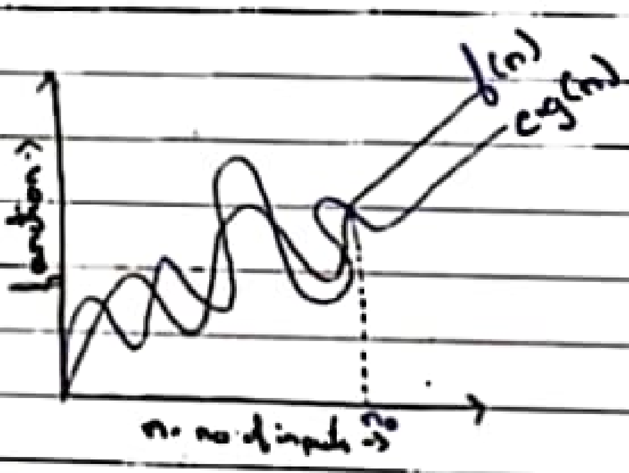
Que:1 Asymptotic Notations
↳ Tending to infinity
They help you find the complexity an algorith when input is very large.

i) Big O(o)

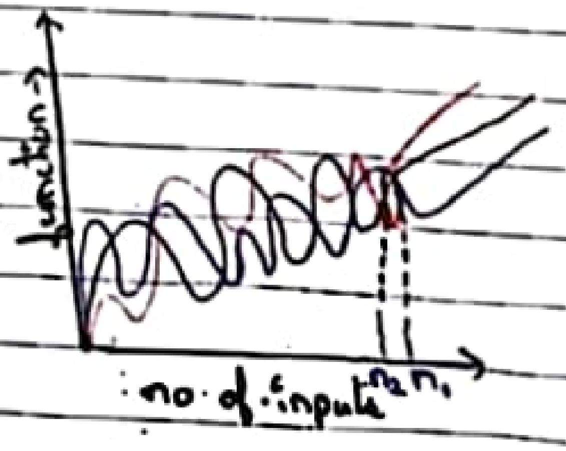


$f(n) = O(g(n))$
iff $f(n) \leq c \cdot g(n)$
$\forall n \geq n_0$
for some constant $c > 0$
⇒ g(n) is 'tight' upper bound of f(n)

ii) Big Omega (Ω)
$f(n) = \Omega(g(n))$
g(n) is 'tight' lower bound of f(n)
$f(n) = \Omega(g(n))$
iff $f(n) \geq c \cdot g(n)$
$\forall n \geq n_0$ for some constant $c > 0$



iii) Theta (Θ)
$f(n) = \Theta(g(n))$.
g(n) is both 'tight' upper & lower bound of fn f(n)
$f(n) = \Theta(g(n))$
iff
$c_1 g(n) \leq f(n) \leq c_2 \cdot g(n)$
$\forall n \geq \max(n_1, n_2)$
for some constant $c_1 > 0$ & $c_2 > 0$



4) Small o(O)

$f(n) = o(g(n))$

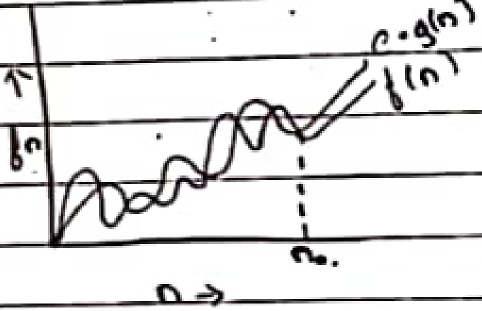
$g(n)$ is upper bound of fn $f(n)$

$f(n) = o(g(n))$

when $f(n) < c \cdot g(n)$

$\forall n > n_0$

& $\forall c > 0$



5) small omega (ω)

$f(n) = \omega(g(n))$

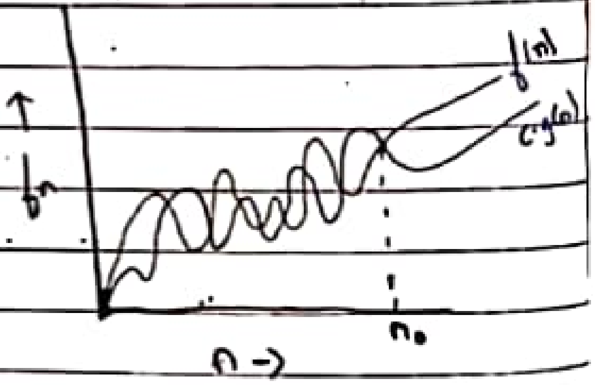
$g(n)$ is lower bound of fn $f(n)$

$f(n) = \omega(g(n))$

when $f(n) > c \cdot g(n)$

$\forall n > n_0$

& $\forall c > 0$



Ques-2 What should be time complexity of
for(i=1 to n) {i = i*2}

for(i=1 to n) // $i = 1, 2, 4, 8 \dots n$
{ i=i*2} // O(1)

$\Rightarrow \sum_{i=1} 1+2+4+8+\dots n$

GP kth value $\Rightarrow T_k = ar^{k-1}$

$\Rightarrow 1 \times 2^{k-1}$

$\Rightarrow n = \underline{2^{k-1}}$

$\Rightarrow 2n = 2^k$

$\Rightarrow \log 2n = k \log 2$

$\Rightarrow \log_2 + \log n = k \log 2$

$\Rightarrow \log n+1 = k$

$\Rightarrow O(k) = O(1+\log n)$

$= \underline{O(\log n)}$

Q-3 $T(n) = \{3T(n-1)$ if $n>0$, otherwise 1 }

$T(n) = 3T(n-1)$ — ①

put $n = n-1$

$T(n-1) = 3\{T(n-2)$ — ②

from 1 & 2

DOMS

putting $n = n-2$ in ①

$T(n) = 3(T(n-3))$ — ④

$\Rightarrow T(n) = 27(T(n-3))$

$\Rightarrow T(n) = 3^k(T(n-k))$

putting $n-k=0$

$\Rightarrow n = k$.

$\Rightarrow T(n) = 3^n[T(n-n)]$

$\Rightarrow T(n) = 3^n T(0)$

$\Rightarrow T(n) = 3^n \times 1$ $\quad$ [$T(0) =$

$\Rightarrow T(n) = O(3^n)$

4) $T(n) = \{ 2T(n-1)-1$ if $n>0$, otherwise

$T(n) = 2T(n-1)-1$ — ①

let $n = n-1$

$\Rightarrow T(n-1) = 2T(n-2) - 1$ — ②

$\Rightarrow$ from ① & ②

$\Rightarrow T(n) = 2[2T(n-2)-1] - 1$

$\Rightarrow T(n) = 4T(n-2) - 2 - 1$ — ③

let $n = n-2$

$\Rightarrow T(n-2) = 2T(n-3) - 1$ — ④.

from ③ & ④

$\Rightarrow T(n) = 4[2T(n-3)-1] - 2 - 1$

$\Rightarrow T(n) = 8T(n-3) - 4 - 2 - 1$

Scanned by CamScanner

DOMS

$\Rightarrow T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - \dots$

$\Rightarrow$ GP $= 2^{k-1} + 2^{k-2}, 2^{k-3}, \dots$

$a = 2^{k-1}$

$r = 1/2$

$\Rightarrow S_n = \dfrac{a(1-r^n)}{1-r}$

$= \dfrac{2^{k-1}(1-(1/2)^n)}{1-\frac{1}{2}}$

$= 2^k(1-(1/2)^k)$

$= 2^k - 1$ [illegible]

let $n-k = 0$

$\Rightarrow n = k$

$\Rightarrow T(n) = 2^n T(n-n) - (2^n - 1)$

$\Rightarrow T(n) = 2^n \cdot 1 - (2^n - 1)$

$\Rightarrow T(n) = 2^n - (2^n - 1)$

Qs.5 What should be time complexity of

```
int i=1, s=1;
while (s<=n)
{ i++; s=s+i;
    printf("#");
}
```

i=1 2 3 4 5 6 - - - - -

s = 1 3 6 10 15 21 - - - - - n - ①

~~Also see s~~

s becomes

Sum of s = 1+3+6+10+ - - - + T_n - ①

also s = 1+3+6+10 + - - T_{n-1} + T_n - ②

from ① - ②

0 = 1+2+3+4+ - - - - n - T_n

=> T_k = 1+2+3+4+ - - - k

=> T_k = $\frac{1}{2}$ k (k+1)

~~[illegible]~~

=> for k iterations

1+2+3+ - - - - + k <= n

=> $\frac{k(k+1)}{2}$ <= n

=> $\frac{k^2+k}{2}$ <= n

=> $O(k^2)$ <= n

=> k = $O(\sqrt{n})$

Ques-6 Time complexity of -

```
void fn(int n)
{ int i, count=0;
    for(i=1 ; i*i <=n ; ++i)
        count++              // O(1)
}
```

as $i^2 <= n$,

$\Rightarrow i <= \sqrt{n}$

$i = 1, 2, 3, 4 \ldots \sqrt{n}$

$\sum_{i=1} 1+2+3+4+ \ldots + \sqrt{n}$.

$\Rightarrow T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2}$

$\Rightarrow T(n) = \frac{n \times \sqrt{n}}{2}$

$\Rightarrow T(n) = \underline{O(n)}$

Ques-7 Time complexity of :-

```
void fn(int n)
{ int i, j, k, count=0;
    for(i=n/2 ; i<=n ; ++i)
        for(j=1 ; j<=n ; j=j*2)
            for(k=1 ; k<=n ; k=k*2)
                count++ ;
```

for $k = k * 2$

$k = 1, 2, 4, 8, \ldots n.$

$\Rightarrow$ GP $\rightarrow$ $a = 1, \quad r = 2.$

$S_n = \frac{a(r^k - 1)}{r - 1}$

$= \frac{1(2^k - 1)}{1}$

$n \Rightarrow 2^k$.

$\Rightarrow \therefore \underline{\underline{\log n = k}}$.

$\Rightarrow$

i	j	k
1	$\log n$	$\log n * \log n$
2	$\log n$	$\log n * \log n$
$\vdots$	$\vdots$	$\vdots$
n	$\log n$	$\log n * \log n$

$\Rightarrow O(n * \log n * \log n)$

Scanned by CamScanner

8) Time Complexity of

```
function (int n)
{ int (n==1)
    return;                      // O(1)
    for (i=1 to n)               // i = 1,2,3,4 --- n => O(n)
    {  for (j=1 to n)            // j = 1,2,3,4, --- n => O(n^2)
       { print ("*");
       }
    }
    function (n-3);              T(n/3).
}
```

$\Rightarrow$ $T(n) = T(n/3) + n^2$

$\Rightarrow$ $a = 1$, $b = 3$, $f(n) = n^2$.

$c = \log_3 1 = 0$

$\Rightarrow$ $n^0 = 1 > (f(n) = n^2)$.

$\Rightarrow$ $T(n) = \Theta(n^2)$

Ques 3 Time complexity of -

```
void function (int n)
{ for (i=1 to n)                      // O(n)
    { for (j=1 ; j<=n ; j=j+i)
          print ("*")                 // O(n)
    }
}
```

~~[illegible]~~

for i=1 :) j= 1, 2, 3, 4 . . . . n = n.
for i=2 :) j= 1, 3, 5, - - - - n = n/2
for i=3 :) j= 1, 4, 7, - - - - n = n/3
⋮
for i=n :) j= 1 ...

$\Rightarrow \sum_{j=n} n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \cdots + 1$

$\Rightarrow \sum_{j=n} n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \frac{1}{n}\right]$

$\sum_{j=n} n[\log n]$

$\Rightarrow T(n) = O[n \log n]$

Q-10. for functions, n^k & c^n, what is the asymptotic relation between these functions?
assume that $k >= 1$, & $c > 1$ are constant.
Find out the value of c & n_0 for which relation holds

as given n^k & c^n

relation b/w n^k & c^n is

$$n^k = O(c^n).$$

[illegible] as $n^k \leq ac^n$
$\forall\, n \geq n_0$ & some constant $a > 0$

for $n_0 = 1$
$c = 2$

$\Rightarrow \quad 1^k \leq a2^1$

$\Rightarrow \quad \underline{n_0 = 1}$ & $c = 2$