Assignment ELC3050

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Resonance frequency calculation:

Adding walls to the guide forces boundary conditions in z direction:

$\beta = \frac{l\pi}{d}, l = 0, 1, 2, \ldots$ $\qquad \because \beta^2 = k^2 - k_c^2$ $\qquad \therefore (\frac{l\pi}{d})^2 = (\frac{w}{C})^2 - \left(\frac{m\pi}{a}\right)^2 - (\frac{n\pi}{b})^2$

$$\therefore f_{res} = \frac{C}{2\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + (\frac{n}{b})^2 + \left(\frac{l}{d}\right)^2}$$

Hence, there's a unique resonant frequency for each value of m, n, l. and for each resonant mode $TE_{m,n,l}$. The dominant TE resonant mode is always $\boldsymbol{TE_{101}}$. there are four possibilities for the next resonance mode: $\boldsymbol{TE_{102}}$, $\boldsymbol{TE_{011}}$, $\boldsymbol{TE_{201}}$, $\boldsymbol{TE_{110}}$.

Which mode is the next mode?:

$TE_{102} \rightarrow \omega_{m,n,l}\ \alpha \left(\frac{1}{a}\right)^2 + \left(\frac{2}{d}\right)^2$ $\qquad TM_{110} \rightarrow \omega_{m,n,l}\ \alpha \left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2$

$TE_{011} \rightarrow \omega_{m,n,l}\ \alpha \left(\frac{1}{b}\right)^2 + \left(\frac{1}{d}\right)^2$ $\qquad TE_{201} \rightarrow \omega_{m,n,l}\ \alpha \left(\frac{2}{a}\right)^2 + \left(\frac{1}{d}\right)^2$

We know that the given guide dimensions will be $d > a > b$:

$\because d > a$ $\qquad \therefore \boldsymbol{TE_{011}} \rightarrow TM_{110}$ $\qquad$ if $a \geqslant 2b$ & $\because d > a$ $\quad \therefore \boldsymbol{TE_{102}} \rightarrow TE_{201} \rightarrow TE_{011}$

if $a < 2b$, $\boldsymbol{TE_{011}} \rightarrow TE_{201}$*, now the next mode is either* $\boldsymbol{TE_{102}}$ *or* $\boldsymbol{TE_{011}}$*:*

if $\left(\frac{1}{a}\right)^2 + \left(\frac{2}{d}\right)^2 < \left(\frac{1}{b}\right)^2 + \left(\frac{1}{d}\right)^2$ *then the next mode is* $\boldsymbol{TE_{102}}$ *(i.e,* $\boldsymbol{TE_{102}} \rightarrow TE_{011}$)

$\frac{3}{d^2} > \frac{a^2 - b^2}{a^2 b^2}$ $\qquad \therefore d > \sqrt{\frac{3a^2b^2}{a^2 - b^2}}$ $\qquad put\ r = \sqrt{\frac{3a^2b^2}{a^2 - b^2}}$

Generally, if $d > r$, the next mode is $\boldsymbol{TE_{102}}$, otherwise the next mode is $\boldsymbol{TE_{011}}$.

Quality factor calculations:

$Q_{die} = \frac{1}{\tan \partial}$ $\qquad Q_{cond} = \frac{w_{res} E_{stored}}{P_{loss}}$ $\qquad \because E_{stored} = W_e + W_m$,

at resonace : $W_e = W_m$ $\qquad \therefore E_{stored} = 2W_e = 2W_m$ $\qquad \therefore Q_{cond} = \frac{2\, w_{res} W_e}{P_{loss}} = \frac{2\, w_{res} W_m}{P_{loss}}$

to calculate the quality factor, we need to calculate the power loss (P_{loss}), the stored energy (E_{stored}) and the resonance frequency (calculated via the above-stated relation).

Given that:

$\mathrm{H_z} = \mathrm{A} \cos(k_{xm}x) \cos(k_{yn}y) e^{-j\beta z}$ $\qquad \mathrm{E_y} = \frac{-j\omega\mu k_{xm}}{k_c^2} \mathrm{A} \sin(k_{xm}x) \cos(k_{yn}y) e^{-j\beta z} = -Z_{TE}\mathrm{H_x}$

$\mathrm{E_x} = \frac{j\omega\mu k_{yn}}{k_c^2} \mathrm{A} \cos(k_{xm}x) \sin(k_{yn}y) e^{-j\beta z} = Z_{TE}\mathrm{H_y}$

For $\boldsymbol{TE_{10}}$ ***mode :***

$\mathrm{H_z} = \mathrm{A} \cos\left(\frac{\pi}{a}x\right) e^{-j\beta z}$ $\qquad \mathrm{E_y} = \frac{-j\omega\mu}{k_c} \mathrm{A} \sin\left(\frac{\pi}{a}x\right) e^{-j\beta z} = -Z_{TE}\mathrm{H_x}$ $\qquad \mathrm{E_x} = \mathrm{H_y} = 0$

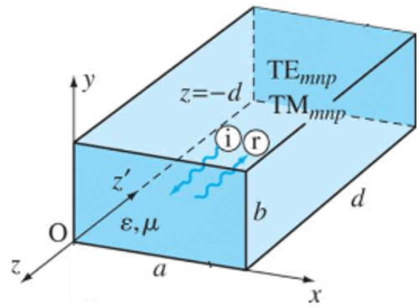
$k_c = \frac{\pi}{a}$ generally: $\beta = \frac{\omega\mu}{Z_{TE}}$

Due to the two walls closing the resonator, the wave is reflected, it must satisfy the boundary conditions:

$\mathrm{E_t} = 0$ $\mathrm{H_n} = 0$

$\therefore H_z{}^+ = -H_z{}^-$ $H_x{}^+ = H_x{}^-$ $E_y{}^+ = -E_y{}^-$

For TE_{10l} modes:



$$H_z = H_z{}^+ + H_z{}^- = \mathrm{A}\,(-2\mathrm{j})\cos\left(\frac{\pi}{a}x\right)\sin(\beta z)$$

$$E_y = E_y{}^+ + E_y{}^- = \frac{-2\omega\mu}{k_c}A\sin\left(\frac{\pi}{a}x\right)\sin(\beta z)$$

$$H_x = H_x{}^+ + H_x{}^- = \frac{(2j)\omega\mu}{k_c\,Z_{TE}}\,A\sin\left(\frac{\pi}{a}x\right)\cos(\beta z) = \frac{(2j)\beta}{k_c}A\,\sin\left(\frac{\pi}{a}x\right)\cos(\beta z)$$

Where $k_c = \frac{\pi}{a}$, $\beta = \frac{\mathrm{l}\pi}{\mathrm{d}}$.

Energy stored calculation:

$$W_e = \frac{\epsilon}{4}\int |E|^2 dv = \frac{\epsilon}{4}\int_{z=0}^{d}\int_{y=0}^{b}\int_{x=0}^{a}\left(\frac{2\omega\mu a}{\pi}\mathrm{A}\right)^2 \sin^2\left(\frac{\pi}{a}x\right)\sin^2\left(\frac{\mathrm{l}\pi}{\mathrm{d}}z\right)dxdydz = \frac{\epsilon\mathrm{abd}}{4}\left(\frac{\omega\mu}{k_c}\mathrm{A}\right)^2$$

$$\therefore E_{stored} = 2W_e = \frac{\epsilon\mathrm{abd}}{2}\left(\frac{\omega\mu}{k_c}\mathrm{A}\right)^2$$

Power loss calculations:

$P_{loss} = \frac{R_S}{2}\oint |H|^2 dS = \frac{R_S}{2}(2I_1 + 2I_2 + 2I_3)$, where I_1 , I_2 , I_3 are for the lateral, top and front faces, respectively.

$$I_1 = \int_{y=0}^{b}\int_{x=0}^{a}|H|^2{}_{z=0,d}\,dxdy = \int_{y=0}^{b}\int_{x=0}^{a}\left(\frac{2\beta}{k_c}A\right)^2\sin^2\left(\frac{\pi}{a}x\right)dxdy = \left(\frac{2al}{\mathrm{d}}A\right)^2\frac{\mathrm{ab}}{2}$$

$$I_2 = \int_{z=0}^{d}\int_{x=0}^{a}|H|^2{}_{y=0,b}\,dxdz = \int_{z=0}^{d}\int_{x=0}^{a}\left(\frac{2al}{\mathrm{d}}A\right)^2\sin^2\left(\frac{\pi}{a}x\right)\cos^2(\beta z) + (2\mathrm{A})^2\cos^2\left(\frac{\pi}{a}x\right)\sin^2(\beta z)\,dxdz$$

$$= (2\mathrm{A})^2\,\frac{\mathrm{ad}}{4}\left(1 + \left(\frac{al}{\mathrm{d}}\right)^2\right)$$

$$I_3 = \int_{z=0}^{d}\int_{y=0}^{b}|H|^2{}_{x=0,a}\,dydz = \int_{z=0}^{d}\int_{y=0}^{b}(2\mathrm{A})^2\,\sin^2(\beta z)\,dydz = (2\mathrm{A})^2\frac{\mathrm{db}}{2}$$

$$\therefore \boldsymbol{P_{loss} = R_S(\mathrm{A})^2\left[2\left(\frac{al}{\mathrm{d}}\right)^2\mathrm{ab} + 2\mathrm{db} + \mathrm{ad} + \left(\frac{al}{\mathrm{d}}\right)^2\mathrm{ad}\right]}$$

$$\therefore \boldsymbol{Q_{cond,10l} = \omega_{res}\frac{\frac{\epsilon}{2}\left(\frac{\omega\mu a}{\pi}\right)^2 abd}{R_s\left[2\left(\frac{al}{d}\right)^2 ab + 2db + ad + \left(\frac{al}{d}\right)^2 ad\right]}}$$

Thus far we have derived a formula for calculating $\boldsymbol{Q_{cond}}$ for $\boldsymbol{TE_{10l}}$ modes which includes the dominant mode $\boldsymbol{TE_{101}}$ and the next mode if it was $\boldsymbol{TE_{102}}$ (depending on the dimensions). Now we derive a general formula for $\boldsymbol{TE_{01l}}$ mode to calculate the quality factor if the next mode was $\boldsymbol{TE_{011}}$ (the other possibility). We'll basically go through the same steps.

For $\boldsymbol{TE_{10}}$ ***mode :***

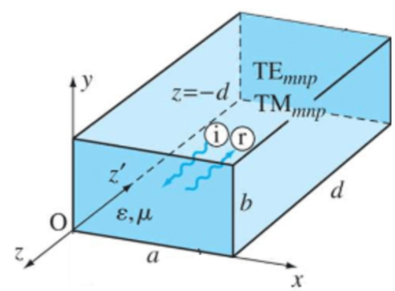
$H_z = A\cos\left(\frac{\pi}{b}y\right)e^{-j\beta z}$ $\qquad$ $E_x = \frac{j\omega\mu}{\pi/b}A\sin\left(\frac{\pi}{b}y\right)e^{-j\beta z} = z_{TE}H_y$ $\qquad$ $E_y = H_x = 0$

$k_c = \frac{\pi}{b}$ $\qquad$ generally: $\beta = \frac{\omega\mu}{Z_{TE}}$

boundary conditions: $\qquad$ $\mathrm{E_t} = 0$ $\qquad$ $\mathrm{H_n} = 0$

$\therefore H_z^+ = -H_z^-$ $\qquad$ $E_x^+ = -E_x^-$ $\qquad$ $H_y^+ = H_y^-$

For $\boldsymbol{TE_{01l}}$ ***modes:***



$$H_z = H_z{}^+ + H_z{}^- = \mathrm{A}\,(-2\mathrm{j})\cos\left(\frac{\pi}{b}y\right)\sin(\beta z)$$

$$E_x = E_x{}^+ + E_x{}^- = \frac{2\omega\mu}{k_c}A\sin\left(\frac{\pi}{b}y\right)\sin\left(\frac{l\pi}{d}z\right)$$

$$H_y = H_y{}^+ + H_y{}^- = \frac{(2j)\omega\mu}{k_c\,Z_{TE}}A\sin\left(\frac{\pi}{b}y\right)\cos(\beta z) = \frac{(2j)\beta}{k_c}A\sin\left(\frac{\pi}{b}y\right)\cos(\beta z)$$

Where $k_c = \frac{\pi}{b}$, $\beta = \frac{\mathrm{l}\pi}{\mathrm{d}}$.

<u>Energy stored calculation:</u>

$$W_e = \frac{\epsilon}{4}\int |E|^2 dv = \frac{\epsilon}{4}\int_{z=0}^{d}\int_{y=0}^{b}\int_{x=0}^{a}\left(\frac{2\omega\mu b}{\pi}\mathrm{A}\right)^2 \sin^2\left(\frac{\pi}{b}y\right)\sin^2\left(\frac{\mathrm{l}\pi}{\mathrm{d}}z\right)dxdydz = \frac{\epsilon\mathrm{abd}}{4}\left(\frac{\omega\mu}{k_c}\mathrm{A}\right)^2$$

$$\therefore E_{stored} = 2W_e = \frac{\epsilon\mathrm{abd}}{2}\left(\frac{\omega\mu}{k_c}\mathrm{A}\right)^2$$

<u>Power loss calculations:</u>

$P_{loss} = \frac{R_S}{2}\oint |H|^2 dS = \frac{R_S}{2}(2I_1 + 2I_2 + 2I_3)$, where I_1 , I_2 , I_3 are for the lateral, top and front faces, respectively.

$$I_1 = \int_{y=0}^{b}\int_{x=0}^{a}|H|^2{}_{z=0,d}\,dxdy = \int_{y=0}^{b}\int_{x=0}^{a}\left(\frac{2\beta}{k_c}A\right)^2\sin^2\left(\frac{\pi}{b}y\right)dxdy = \left(\frac{2bl}{\mathrm{d}}A\right)^2\frac{\mathrm{ab}}{2}$$

$$I_2 = \int_{z=0}^{d}\int_{x=0}^{a}|H|^2{}_{y=0,b}\,dxdz = \int_{z=0}^{d}\int_{x=0}^{a}(2\mathrm{A})^2\ \sin^2(\beta z)\ dxdz = (2\mathrm{A})^2\ \frac{\mathrm{ad}}{2}$$

$$I_3 = \int_{z=0}^{d}\int_{y=0}^{b}|H|^2{}_{x=0,a}\,dydz = \int_{z=0}^{d}\int_{y=0}^{b}\left(\frac{2\beta}{k_c}A\right)^2\sin^2\left(\frac{\pi}{b}y\right)\cos^2(\beta z) + (2\mathrm{A})^2\ \cos^2\left(\frac{\pi}{b}y\right)\sin^2(\beta z)\,dydz$$

$$= (2\mathrm{A})^2\ \frac{\mathrm{bd}}{4}\ \left(1 + \left(\frac{bl}{\mathrm{d}}\right)^2\right)$$

$$\therefore \boldsymbol{P_{loss} = R_S(\mathrm{A})^2\left[2\left(\frac{bl}{\mathrm{d}}\right)^2\mathrm{ab} + \mathrm{bd} + 2\mathrm{ad} + \left(\frac{bl}{\mathrm{d}}\right)^2\mathrm{bd}\right]}$$

$$\therefore Q_{cond,01l} = \omega_{res} \frac{\frac{\epsilon}{2}\left(\frac{\omega\mu b}{\pi}\mathrm{A}\right)^2 \mathrm{abd}}{R_s\left[2\left(\frac{bl}{\mathrm{d}}\right)^2 \mathrm{ab} + \mathrm{bd} + 2\mathrm{ad} + \left(\frac{bl}{\mathrm{d}}\right)^2 \mathrm{bd}\right]}$$

$$\therefore \frac{1}{Q_{\text{total}}} = \frac{1}{Q_{\text{die}}} + \frac{1}{Q_{\text{cond}}}$$

The fractional bandwidth is given by:

$$BW_{fractional} = \frac{\omega_2 - \omega_1}{\omega_o} = \frac{1}{Q_{\text{total}}}$$

Program code: (I used MATLAB R2021)

Run the “main” function via command window.

```
function main
clc;
clear;
close all;
format compact;

%cavity resonator dimensions in cm
global a;
global b;
global d;

%other cavity parameteers
global epsilon_r;            %relative permitivity
global sigma;                %metal conductivity
global Q_die;                %loss tangent at specific freq.

% epsilon_r = 2.5;
% sigma = 5.8*10^7;
% loss_tanget = 0.00004;
% Q_die = 1/loss_tanget;
% a = 0.05;
% b = 0.04;
% d = 0.07;

%get inputs from user
a = input("a (in cm)= ")*10^-2;
b = input("b (in cm)= ")*10^-2;
d = input("d (in cm)= ")*10^-2;
epsilon_r = input("relative permeability ε_r = ");
sigma = input("metal conductivity σ = ");
Q_die = 1/input("loss_tanget tan(δ) = ");

%constants
global c_0;
global epsilon_0;
global u_0;

c_0 = 3*10^8;
epsilon_0 = 8.8542* 10^-12;
u_0 = 4*pi*10^-7;

f_res_dominant =  resonance_freq(1,0,1); %#ok<*NOPRT,*NASGU>
Q_dominant = Q_total_10l(1,2*pi*f_res_dominant);

r = sqrt( (3*(a^2)*(b^2))/(a^2-b^2));

if d > r % next mode is TE102
   f_res_next = resonance_freq(1,0,2);
   Q_next = Q_total_10l(2,2*pi*f_res_next);
else
   f_res_next = resonance_freq(0,1,1);
   Q_next = Q_total_01l(1,2*pi*f_res_next);
end

print_output(f_res_dominant,Q_dominant,f_res_next,Q_next,r);
end

function [f_resonance] = resonance_freq(m,n,l)

global a;
global b;
global d;
global epsilon_r;

c_0= 3*10^8;
c = c_0/sqrt(epsilon_r);
kx = m*pi/(a);
ky = n*pi/(b);
beta = l*pi/(d);
f_resonance = (c/(2*pi)) * sqrt(kx^2 + ky^2 +beta^2);
end
```

```
%TE101 , TE102
function [Q_10l] = Q_total_10l(l,w_res)
global a;
global b;
global d;
global epsilon_r;
global epsilon_0;
global u_0;
global sigma;
global Q_die;

k_c= pi/a;
R_s = sqrt( (w_res * u_0)/(2*sigma));
Q_num = w_res *(((epsilon_r*epsilon_0*a*b*d)/2) * ((w_res*u_0)/k_c)^2);
Q_den = R_s * (2*(((a*l)/d)^2)*a*b + 2*d*b + a*d + (((a*l)/d)^2)*a*d);

Q_cond_10l = Q_num/Q_den;
dummy = 1/Q_cond_10l + 1/Q_die;
Q_10l = 1/dummy;
end

%TE011
function [Q_01l] = Q_total_01l(l,w_res)

global a;
global b;
global d;
global epsilon_r;
global epsilon_0;
global u_0;
global sigma;
global Q_die;

R_s=sqrt( (w_res * u_0)/(2*sigma));
k_c=pi/b;

Q_num = w_res *(((epsilon_r*epsilon_0*a*b*d)/2) * ((w_res*u_0)/k_c)^2);
Q_den = R_s * (2*(((b*l)/d)^2)*a*b + 2*a*d + b*d + (((b*l)/d)^2)*b*d);
Q_cond_01l = Q_num/Q_den;

dummy = 1/Q_cond_01l + 1/Q_die;
Q_01l = 1/dummy;
end

function print_output(f_res_dominant,Q_dominant,f_res_next,Q_next,r)
global d;
if d >r
    next_mode = "TE102";
else
    next_mode = "TE011";
end

BW_dominant = 1/Q_dominant;
BW_next = 1/Q_next;

fprintf("\n=================================\n");
fprintf("The dominant mode: TE101\n");
fprintf("the 1st resonance frequency = %f GHz\n",f_res_dominant*10^-9);
fprintf("the quality factor Q (TE101) = %f \n",Q_dominant);
fprintf("the fractional bandwidth (TE101) = %e \n",BW_dominant);
fprintf("=================================\n");
fprintf("The next mode: %s\n",next_mode);
fprintf("the 2nd resonance frequency = %f GHz\n",f_res_next*10^-9);
fprintf("the quality factor Q(%s) = %f \n",next_mode,Q_next);
fprintf("the fractional bandwidth (%s) = %e \n",next_mode,BW_next);
fprintf("=================================\n");
end
```