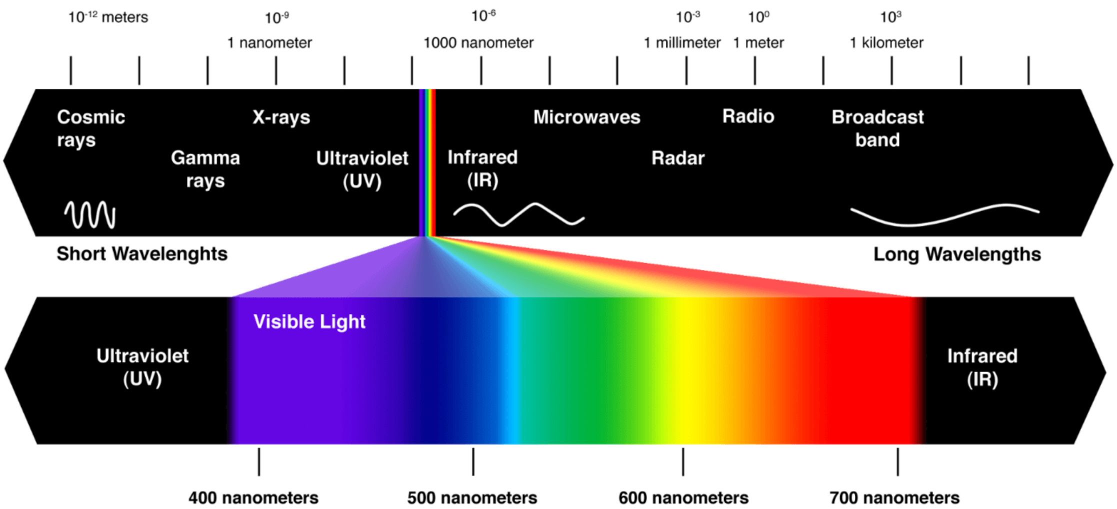
Aristotle University of Thessaloniki
Department of Electrical and Computer Engineering
Telecommunications Division

Communication Systems II

Modulations for visible light communication systems with shot noise



Picture taken from https://www.once.lighting/visible-light-spectrum/?fbclid=IwAR2rRgl1B7rZi1OYsJ5NmeicRKJrTNIyjFZEB6FR534fD-25mZ2Ve2VWbRg

Contents

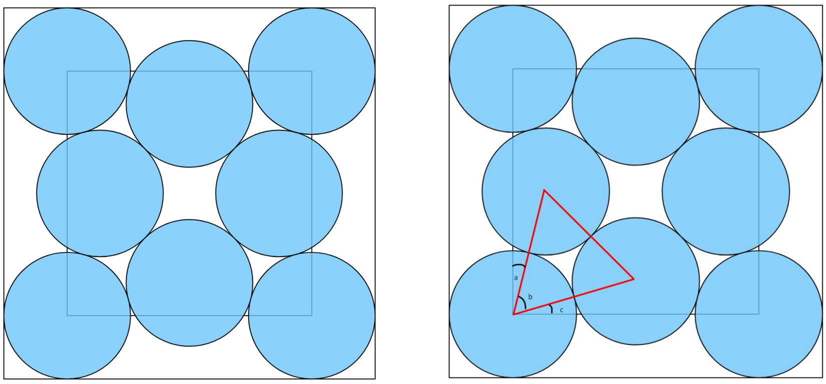


Figure 1: Optimized packing of 8 circles in a square.
Taken from Parcly Taxel - Own work, FAL, https://commons.wikimedia.org/w/index.php?curid=67466413

1 Optimal constellation design

Considering the given space S_{ABCD} and in order to maximize the minimum Euclidean distance, we intuitively put the 4 first constellation points on the 4 edges of S_{ABCD}, to leave more space to position the remaining 4. Therefore, we opted to have the same Euclidean distance between neighboring points. To succeed that, having 8 points, we positioned them considering the Circle packing in a square problem, as Figure 26 shows. We will try to find the radius **r** of the circles. Because the red triangle is **equilateral** and $\hat{a} + \hat{b} + \hat{c} = 90°$, therefore $\hat{b} = 60°$ and $\hat{a} = \hat{c} = 15°$. Given (by [1]) that $\mathbf{T} = \mathbf{1}$ and $\mathbf{P} = \mathbf{1}$, every side of the square is $\sqrt{0.5}/2$, hence

$$\cos 15° = \frac{\frac{\sqrt{0.5}}{2}}{2r} \Rightarrow r = 0.1830 \tag{1}$$

In order to find the coordinates of the 4 remaining points, we solve the following systems. Note that all of them have 2 solutions, so, we only keep the one that is inside S_{ABCD}.

$$\begin{cases} (2r)^2 = (x_1 - 0)^2 + (y_1 - 0)^2 \\ (2r)^2 = (x_1 - 0.5)^2 + (y_1 - 0.5)^2 \end{cases}, \text{ where } (x_1, y_1) = (0.1830, 0.3169)$$

$$\begin{cases} (2r)^2 = (x_2 - 0)^2 + (y_2 - 0)^2 \\ (2r)^2 = (x_2 + 0.5)^2 + (y_2 - 0.5)^2 \end{cases}, \text{ where } (x_2, y_2) = (-0.1830, 0, 3169)$$

$$\begin{cases} (2r)^2 = (x_3 + 0.5)^2 + (y_3 - 0.5)^2 \\ (2r)^2 = (x_3 + 0)^2 + (y_3 - 1)^2 \end{cases}, \text{ where } (x_3, y_3) = (-0.1830, 0.6830)$$

$$\begin{cases} (2r)^2 = (x_4 - 0)^2 + (y_4 - 1)^2 \\ (2r)^2 = (x_4 - 0.5)^2 + (y_4 - 0.5)^2 \end{cases}, \text{ where } (x_4, y_4) = (0.1830, 0.6830)$$

Finally, the optimal designed constellation is shown in Figure 27. Note that, it considerably looks like circular 8-QAM, shifted on the y-axis, although not intended.

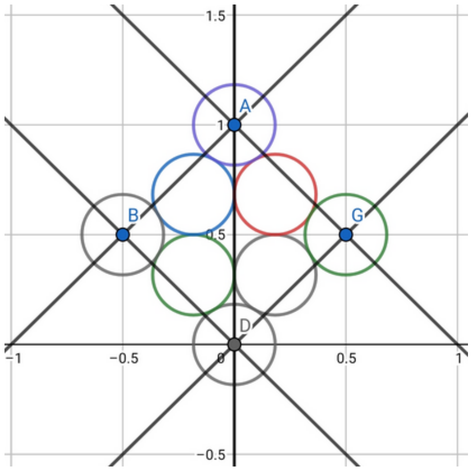


Figure 2: Optimal constellation design

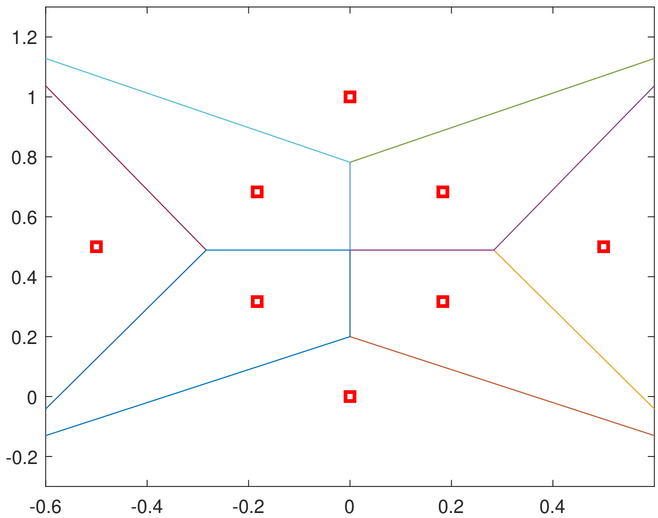


Figure 3: Constellation's decision areas

2 Simulation for Symbol Error Rate

The simulation consists of the following 5 files:

1. **simSER.m**
2. **create_lines.m**
3. **find_section_points.m**
4. **add_noise.m**
5. **demodulate.m**

2.1 Comparison and Discussion

Given the fact that the proposed constellation of [1] (the given paper) for $M = 8$ was the output of a constellation optimization problem, we expected that the constellation that we designed would have worse SER (higher SER) under the same signal-to-noise ratio. Surprisingly enough,

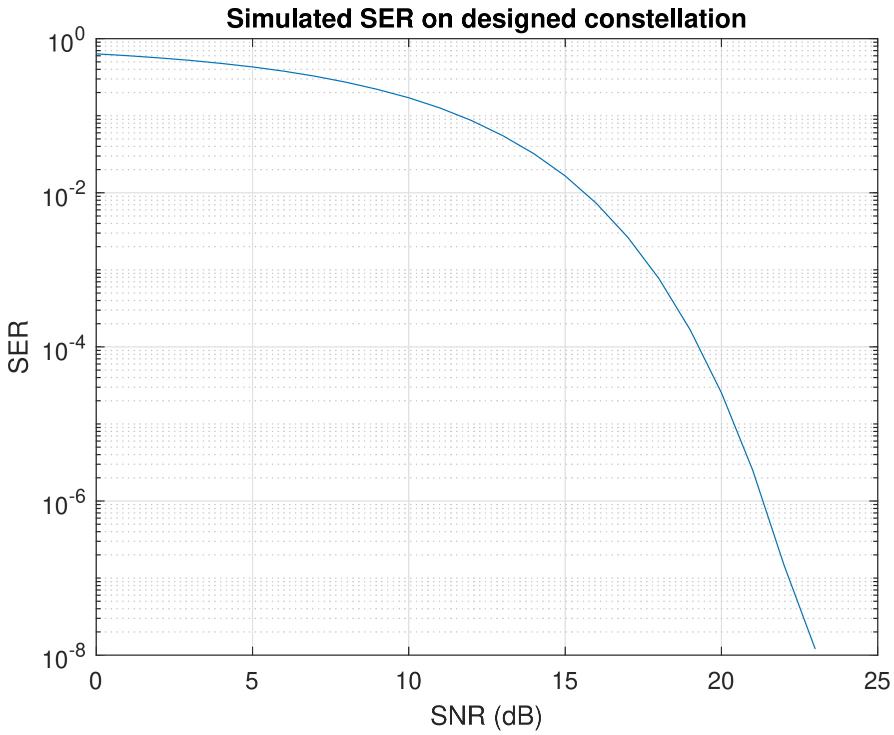


Figure 4: Symbol Error Rate versus SNR

the results we got tell the opposite. It is clear that SER drops to 10^{-8} for an SNR lower than **25** dB. In fig. 4 of [1], at **25** dB, SER is higher that 10^{-4}, indicating that we have achieved a much better performance.
Supposed that the above result is not due to simulation/coding errors or false understanding of the problem, we can explain it as follows. As stated in section B of the paper, between (21) and (22), $\sigma_{x_i}^2 = \sigma_0^2$ and $\sigma_{y,i}^2 = y_i \eta^2 \sigma_0^2$ represent the variance of signal independent noise and SDSN respectively. "Only y_i is related to SDSN, as the average power of $\phi_2(t)$ is zero in a symbol period, and x_i has no effect on SDSN", as stated in the paper. Therefore, given that the scaling factor $\eta^2 = 1$ and $0 \leq \ y_i \leq 1$, the noise will affect less y_i than x_i. This, of course, makes the variance $\sigma_{y,i}^2$ of (0,0) equal to zero, which is easily noticeable if you scatter-plot the demodulated signals.
Finally, considering the restrictions provided by the paper, the results are over all expected.
In an attempt to explain the gap between the paper's results and ours, we can say that our designed constellation has **a better minimum Euclidean distance** between the symbols. Also, given that the scaling factor $\eta^2 = 1$, among other restrictions, the SDSN was, in fact, **more favorable** than AWGN. The above, can justify the constellation's SER vs SNR performance, shown in Figure 29.

2.2 Comments on MATLAB code

You should run **simSER.m**, the other 4 files are supplementary functions.
Note that the SDSN was modeled by modifying a function to add AWGN. The only change needed was the following line:
```
n = sqrt(N0/2)*(randn(size(s))+ 1i*(sqrt(imag(s)).*randn(size(s))));
```
Extra comments on how the simulation works can be found in the code.

References

[1] Hanjie Chen and Zhengyuan Xu. A two-dimensional constellation design method for visible light communications with signal-dependent shot noise. *IEEE Communications Letters*, 22(9):1786–1789, sep 2018.