Module 5

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Module 5 Study Guide and Deliverables

Topics:	**Lecture 9:** Decision Analysis in the Enterprise – I **Lecture 10:** Decision Analysis in the Enterprise -II
Readings:	Lectures 9 and 10 online content
Discussions:	**Module 5 Discussion**
Assignments:	**Tutorial:** Understanding the AD616 Package for Decision Trees **Assignment 5**: Individual Assignment covering Lecture 9 and Lecture 10.
Assessments:	**Quiz 5**

Lecture 10

Learning Objectives

After you complete this lecture, you will be familiar with the following concepts:

Loading [Contrib]/a11y/accessibility-menu.js simulation in a decision tree

- Utility functions and exponential utility function
- Developing a decision tree with utility functions
- Certainty equivalents
- Risk premium

Decision Trees and Monte Carlo Simulation

In Lecture 9, in building our decision trees we assumed that all the inputs to the model are certain quantities. In realization of the fact that some of them may indeed be uncertain, we ran a sensitivity analysis to see how the different values of unit margins change the final decision. We can actually do better and develop a simulation model using a decision tree. In this section, we will learn how to do this using the Acme company decision tree that we built in Lecture 9.

Let us assume that the following are uncertain inputs to our model:

1. The possible quantities sold in national market
2. The unit margin

1) Let us first represent the uncertainty around the possible quantities sold in national market. To this end, let us assume that the possible quantities sold (in 1000s of units) in national market is lognormally distributed with a mean 600 and standard deviation 120 (great), with a mean 300 and standard deviation 60 (fair) and with a mean 90 and standard deviation 18 (awful). It is straightforward to incorporate these uncertainties to the decision tree that we developed in Lecture 9. More specifically, we update our decision tree as follows:

- We change our three values of great, fair and awful with the following command:

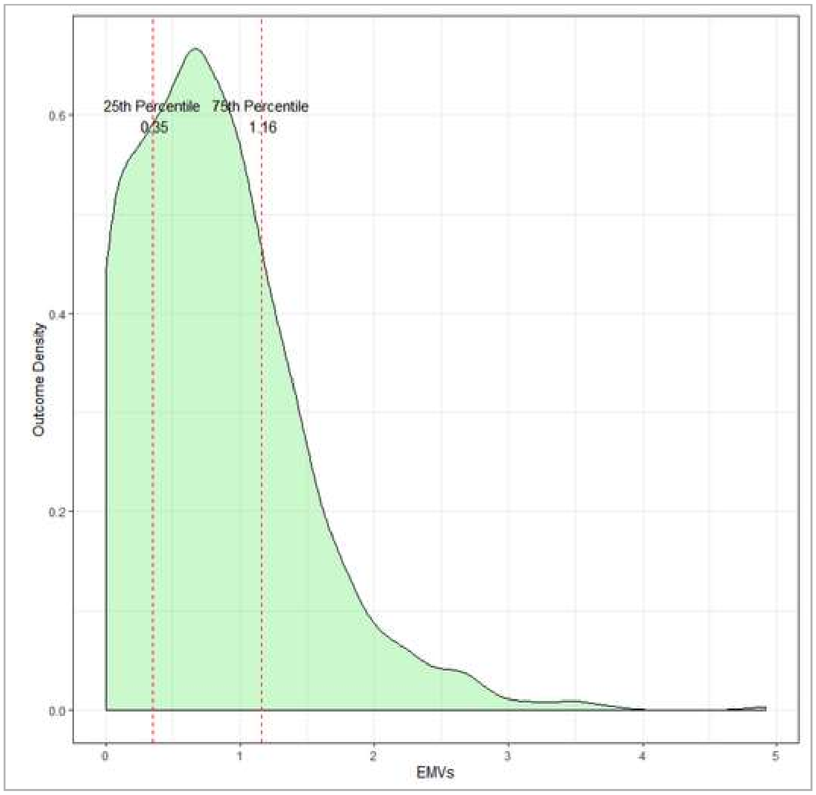
```
genGreat <- function(){profit_margin/1000*rlnorm2(600,120)}
genFair <- function(){profit_margin/1000*rlnorm2(300,60)}
genAwful <- function(){profit_margin/1000*rlnorm2(90,18)}
Great <- genGreat()
Fair <- genFair()
Awful <- genAwful()
```

Like in Module 4, we now can simulate the output from

- We find that 53% of the time the EMV will be smaller than $781K (the EMV value obtained when there is no uncertainty in the decision tree.

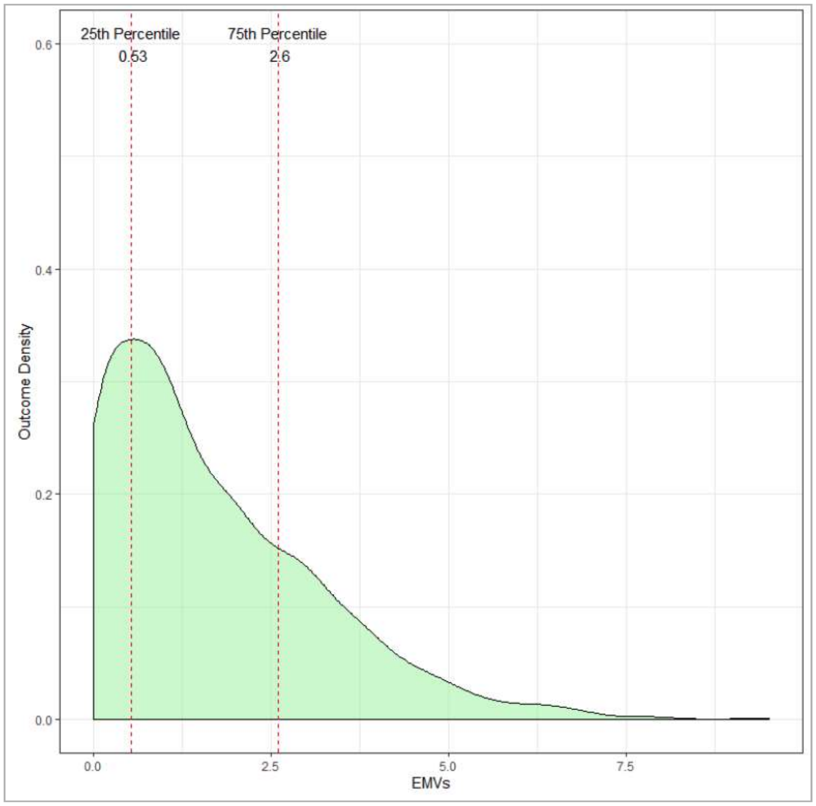
We further observe that almost 90% of the time, the EMV will lie between 0 and 2000 and the mean EMV is approximately 1037.

Figure 10.1: Simulation results of Acme company decision tree uncertain demand



2) Let us now represent the uncertainty around the unit margin. To this end, let us assume that the unit margin is uniformly distributed with parameters 14 and 28. In order to incorporate this into our decision tree, we perform the following:

Figure 10.2: Simulation results of Acme company with Demand and Profit uncertainty



- We find that that 25th Percentile revenue is at 530K and 75th percentile at 2.6M. The range is quite wide and mostly driven by uncertainty in demand and profit margin. This is a useful way to model an enterprise cash flow and ensure that the worst case eventualities are addressed.

Utility Theory

Although the EMV decision rule is widely used in practice, some decision makers may not necessarily choose the decision alternative with the largest EMV. This is especially true when a large amount of money is at stake. In this case, the decision makers may want to choose the less risky decision as opposed to the decision with the largest EMV.

For instance, consider the following situation where we could buy two companies (listed in the following payoff table) for the same price:[1]

State of Nature			
Company	**1**	**2**	**EMV**
A	150,000	–30,000	60,000

B	70,000	40,000	55,000
Chance	*50%*	*50%*	

The payoffs in this table represent the annual profits expected from this business. Thus, there is a 50% chance that company A will generate a profit of $150,000 and a 50% chance it will incur a loss of $30,000. Similarly, there is a 50% chance that company B will generate a profit of $70,000 and another 50% chance that it will generate a profit of $40,000. According to the EMV decision rule that we discussed in the previous module, we buy Company A as it has the largest EMV. However, it is important to note that "Company A represents a far more risky investment than Company B"[2]. With Company A, we can make a profit as big as $150,000 but we also have the chance of losing $30,000. With Company B, on the other hand, we do not have any chance of losing money and we will make at least $40,000 profit each year. Therefore, for a decision maker who does not want to take risk, a natural choice would be Company B as opposed to EMV-maximizer Company A.

This example shows that many rational decision makers do not behave as EMV maximizers. They incorporate their attitude toward risk to the decision-making process. "Indeed, most rational decision makers act as **expected utility** maximizers, i.e., they choose the decision alternative with the largest expected utility value."[3] In this module, we will discuss the utility theory and how it is used in the decision-making process.

Utility Functions

Utility theory assumes that every decision maker has a **utility function**, "which is a mathematical function that transforms monetary values such as costs and payoffs into nonmonetary utility values."[4] Different decision makers have different attitudes and preferences toward risk and return.

- "Decision makers who are risk neutral tend to make decisions using the maximum EMV rule."[5]
- "Decision makers who are risk averse are risk avoiders and are willing to sacrifice some EMV to avoid risky gambles."[6]
- Decision makers who are risk seekers are comfortable with making risky decisions.

Assuming that decision makers use utility functions to make decisions, how do we identify the utility function of a particular decision maker (e.g., individual or company)?

Identifying a utility function for both individuals and companies is a tedious task. Because different managers have different risk attitudes, it is difficult to identify a common utility function, especially for companies. Therefore, researchers have developed classes of utility functions that are approximations of the decision

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Exponential Utility Function

An exponential utility function has the following form:[7]

$$U(x)= 1-e^{-x/R} $$

In this representation, e is the natural logarithm (e = 2.718281…), *x* is a monetary value (positive if it is a payoff and negative if it is a cost) and R > 0 is a risk tolerance. R specifies an individual's aversion to risk. "More specifically, the larger the value of R, the less risk averse the decision maker is."[8] As R gets larger, the decision maker becomes an EMV maximizer.

The only value that we need to specify to use the exponential utility function is the value of R. One way of identifying R for large companies is proposed by Ronald Howard, who is a famous decision analyst.[9] According to Howard, the value of R is related to financial variables such as net sales, net income, and equity. More specifically, R is approximately:

- 6.4% of net sales,
- 124% of net income, and
- 15.7% of equity.

So, for a company with net sales of $50 million, the value of R would be ($50 million × 6.4%) = $3.2 million.

In the next section, we illustrate the use of the exponential utility function with an example.

An Example: Deciding Whether to Enter Risky Venture

In this section, we will work with the following problem:

Venture Limited is a company with net sales of $30 million. The company currently must decide whether to enter one of two risky ventures or invest in a sure thing. The gain from the latter is a sure $125,000. The possible outcomes for the less risky venture are a $0.5 million loss with probability 0.25, a $0.1 million gain with probability 0.5, and a $1 million gain with probability 0.25. The possible outcomes of the more risky venture are a $1 million loss with probability 0.35, a $1 million gain with probability 0.6, and a $3 million gain with probability 0.05. If Venture Limited must decide on exactly

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vhat should it do?

Source: Albright, S. C. and Winston, W. L. Business Analytics: Data Analysis and Decision Making, 5e, 2015, Cengage Learning, p. 275.

We assume that Venture Limited has an exponential utility function with an R value of $30 million × 6.4% = $1.92 million (based on Howard's guidelines). Now, let us calculate the utility value of each monetary outcome:

- • The gain from the riskless alternative (in $1000s) is 125 and its utility is:
  $$U(125)= 1-e^{-125/1920}= 1-0.9370=0.063$$
- The utility of a $0.5 million loss is:
  $$U(-500)= 1-e^{-(-500)/1920}= 1-1.298=-0.298$$
- The utility of a $0.1 million gain is:
  $$U(100)= 1-e^{-(100)/1920}= 1-0.949=-0.051$$
- The utility of a $1 million gain is:
  $$U(1000)= 1-e^{-(1000)/1920}= 1-0.594=-0.406$$
- The utility of a $1 million loss is:
  $$U(-1000)= 1-e^{-(-1000)/1920}= 1-1.683=-0.683$$
- The utility of a $3 million gain is:
  $$U(3000)= 1-e^{-(3000)/1920}= 1-0.210=-0.790$$

These probabilities will be used in the decision tree.

As you will see while going through the steps of the decision-tree development, we build the decision tree with an exponential utility function criterion, in exactly the same way we build a decision tree with the EMV criterion. In other words, we link the probabilities and the monetary values to the branches of the tree similarly. "However, the expected values shown in the tree will be expected utilities (so the expected utilities will be calculated by ASP automatically), and the optimal decision will be the one with the largest expected utility."[10]

Figure 10.3 Decision tree for the Risky Ventures

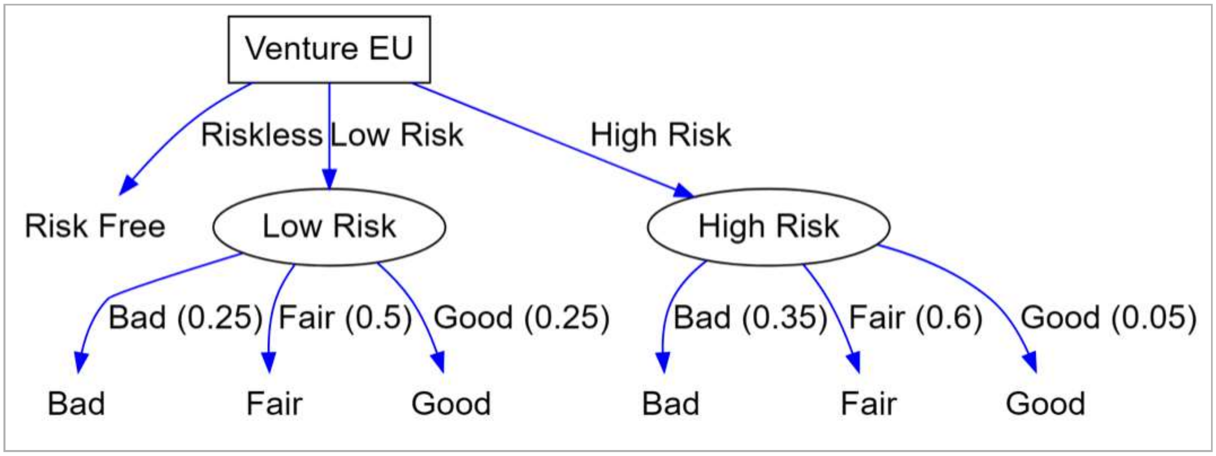

Figure 10.4: EMVs for Risky Ventures

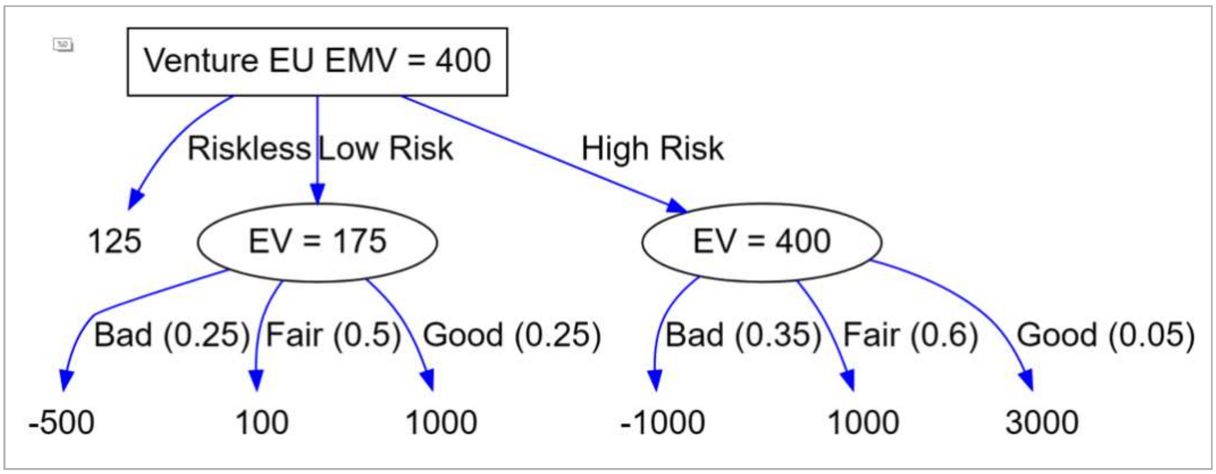

Figure 10.3 and 10.4 showcase the decision tree for the risky ventures. 10.5 shows the EMV calculations. Just going by EMV calculation the right decision to make would be the high risk option since the EMV is the highest. The ordering of the decisions according to the EMV criterion is more risky > less risky > riskless.

Fig 10.5 Utility for Risky Venture Problem

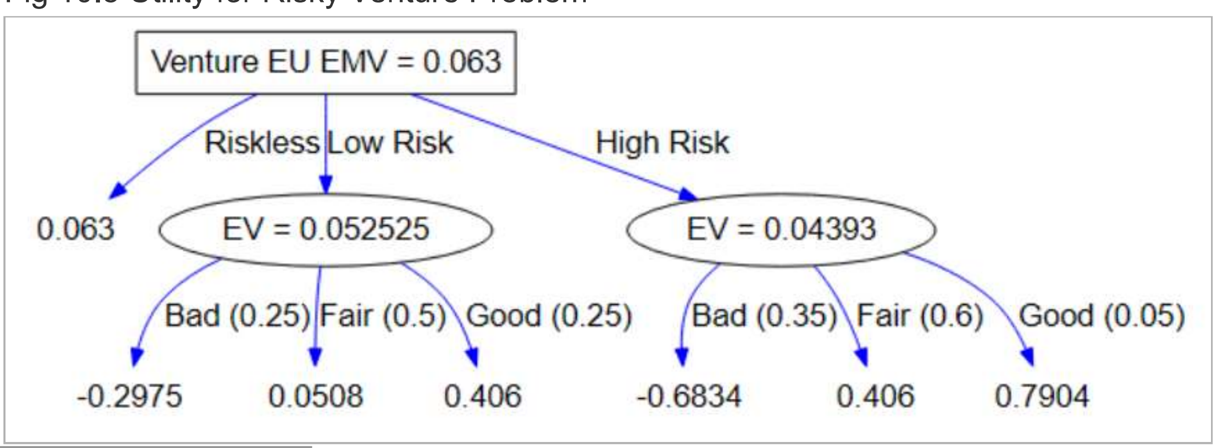

Figure 10.5 shows that the expected utility for the riskless option is 0.063. The utility is calculated by calling the **update_utility** function in R as explained in the tutorial.

Therefore, the best strategy in terms of the expected utility criterion is to go with the riskless option (as highlighted in the decision tree). The ordering of the decisions according to the expected utility criterion is riskless > less risky > more risky. Why? "Because the company is sufficiently risk averse, the company is willing to sacrifice $400,000 (the best option with the EMV criterion) – $125,000 (the EMV value of going with the riskless option, which is the optimal option according to the expected utility criterion) = $275,000 of EMV to avoid risk."[11]

Sensitivity Analysis

It is always a good idea to perform sensitivity analysis on the value of the key parameter, the risk tolerance. We see that when R is approximately about $2,210 million, the more risky venture becomes the best choice. When R is smaller than $1,920 million, the riskless decision is the best choice. So, this is more evidence that the value of R has a great impact on the final decision and thus the optimal decision heavily depends on top management's attitudes toward risk.

Certainty Equivalents

"Now let us change the Venture Limited problem slightly and let us assume that the company has two options: It can either enter the less risky venture, or receive a *certain* dollar amount xx and avoid the gamble altogether. Our goal is to find the value of xx so that the company is indifferent between these two options."[12]

- The utility of entering the less risky venture is 0.0525 as calculated in the decision tree.
- If the company receives a certain *x* dollars, then its utility will be:
  $$U(x)= 1-e^{-x/1920}$$
- We would like to find the value of *x* that the company is indifferent between these two options.
  Therefore we set $U(x) = 1-e^{-x/1920}$ equal to 0.0525 and solve for x.
  $$1-e^{-x/1920 }= 0.0525$$
  $$\Rightarrow e^{-x/1920 }= 0.9475$$

Taking the natural logarithm of both sides we have (LN is the Excel function for the natural logarithm):

$$LN(e^{-x/1920}) = LN(0.9475)$$

$$-x/1920 = -0.0539$$

$$x = 1920 \times 0.0539 = 104$$

Because of the unit measure (in $1000s), x is indeed $104,000. "This value is called the **certainty equivalent** of the risky venture. The company is indifferent between entering the less risky venture and receiving a sure $104,000 to avoid the risk. Note that the EMV of the less risky venture is $175,000 but the company acts as if it is $104,000. Thus, the company is willing to give up $175,000 – $104,000 = $71,000 to avoid the gamble, i.e., to avoid the risk."[13] The certainty equivalent is already tabulated in the decision tree of risky ventures — see cell E14 in Figure 10.6 (Note that we rounded the values in our calculations).

"The difference between the EMV of an uncertain situation and the certainty equivalent of the same situation is called the **risk premium**."[14] So, in this case the risk premium is $71,000.

With a similar approach, we can calculate the certainty equivalent and the risk premium for the more risky venture.

- The utility of entering the more risky venture is 0.0439 as calculated in the decision tree.
- If the company receives a certain *x* dollars, then its utility will be:
  $$U(x) = 1-e^{-x/1920}$$
- We would like to find the value of *x* that the company is indifferent between these two options. Therefore we set $U(x)= 1-e^{-x/1920}$ equal to 0.0439 and solve for x.
  $$1-e^{-x/1920} = 0.0439$$
  $$\Rightarrow e^{-x/1920} = 0.9561$$

Taking the natural logarithm of both sides we have (LN is the Excel function for the natural logarithm):

$$LN(e^{-x/1920})= LN(0.09561)$$

$$-x/1920 = -0.0449$$

Therefore,

$$x = -1920 \times 0.0449 = 86$$

Thus, the certainty equivalent of the more risky venture is $86,000. You can also see this certainty equivalent in cell E29 of Figure 10.6. The company is indifferent between entering the more risky venture and receiving a sure $86,000 for sure to avoid the risk. Note that the EMV of the more risky venture is $400,000 but the company acts as if it is $86,000. Thus, the company is willing to give up $400,000 – $86,000 = $314,000 to avoid the risk.

An Example: Deciding Whether to Enter Risky Venture (Continued)

Note: The problem studied here is taken from Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, pp. 278-279.

In Lecture 9, we assumed that Acme bases its decision on the EMV rule. Now, suppose Acme decides to use expected utility with an exponential utility function. How does this change its final decision about where to market its product?

We already have a decision tree for this problem which is based on the EMV rule. We modify this decision tree to accommodate the expected utility function as follows:

Class Exercise

Now use the same approach as above to calculate the expected Utility for the Acme problem with an exponential utility function with an R value of 1000. What do you observe?

Question: Why is it better to run the test market if the risk tolerance is sufficiently large?

Answer: When the risk tolerance is low, the company is risk averse and it will not market the product nationally. Therefore, running the test market is worthless when the final decision is not to market the product nationally. When the risk tolerance gets bigger, the company is willing to take risk, and in some scenarios its best decision is to market nationally. In these scenarios, the information gained by running the test market may be worth its price and the best strategy is to run the test market.

Source: Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 279.

Lecture 10 Footnotes

[1] This example is taken from Ragsdale, C. T., (2015). *Spreadsheet modeling and decision analysis: A practical introduction to business analytics* (7th edition). Cengage Learning, p. 776.

[2] Ragsdale, C. T., (2015). *Spreadsheet modeling and decision analysis: A practical introduction to business analytics* (7th edition). Cengage Learning, p. 776.

[3] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 272.

[4] Ragsdale, C. T., (2015). *Spreadsheet modeling and decision analysis: A practical introduction to business analytics* (7th edition). Cengage Learning, p. 776.

[5] Loading [Contrib]/a11y/accessibility-menu.js *adsheet modeling and decision analysis: A practical introduction to business*

analytics (7th edition). Cengage Learning, p. 776.

[6] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 273.

[7] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 274.

[8] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 274.

[9] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 274.

[10] Adapted from Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 275.

[11] Adapted from Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 277.

[12] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 277.

[13] Adapted from Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 277.

[14] Adapted from Ragsdale, C. T., (2015). *Spreadsheet modeling and decision analysis: A practical introduction to business analytics* (7th edition). Cengage Learning, p. 778.

Lecture 10 Resources

The decision tree for Acme problem simulation (uncertain quantity sold)

The decision tree for Acme problem simulation (uncertain unit margin)

The decision tree for Risky Ventures

The decision tree for Acme with expected utility functions

Lecture 10 Summary Questions

1. What is the purpose of incorporating Monte Carlo simulation into a decision tree?
2. Please comment on the following statement: *"It is always a good idea to choose the decision alternative with the largest EMV."*

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