Module 5

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Module 5 Study Guide and Deliverables

Topics:	**Lecture 9:** Decision Analysis in the Enterprise – I **Lecture 10:** Decision Analysis in the Enterprise -II
Readings:	Lectures 9 and 10 online content
Discussions:	**Module 5 Discussion**
Assignments:	**Tutorial:** Understanding the AD616 Package for Decision Trees **Assignment 5**: Individual Assignment covering Lecture 9 and Lecture 10.
Assessments:	**Quiz 5**

Lecture 9

Learning Objectives

After you complete this lecture, you will be familiar with the following:

- The elements of a typical decision analysis problem
- Payoff tables and decision trees
- Optimistic, Conservative, and Minimax Regret approach for making decisions
- Expected monetary value approach for making decisions
- Sensitivity analysis for decision analysis problems
- Building a risk profile for decision analysis problems
- Multistage decision problems
- The application of Bayes' Rule in decision analysis
- Expected value of sample information
- Expected value of perfect information

Introduction to Decision Analysis

Throughout this class, we have discussed how to analyze data and build analytic models to deal with uncertainty (and manage risk). "Analytic models provide managers a wealth of information; however, managers make the final decision. Successful managers do not only implement the results of the analytic models but they also assess the intangible factors and incorporate the company's risk attitude to the decision-making process. *Decision analysis* is the study of how people make decisions, particularly when faced with imperfect or uncertain information."[1] "The main distinguishing feature of decision analysis from other techniques that we covered in this course is that decision analysis explicitly considers the individual's preferences and attitudes toward risk."[2]

The topic of decision analysis is discussed in the course AD 715: Quantitative and Qualitative Decision Making. In AD 616, we will go deeper and discuss risk profiles, sensitivity analysis, and how attitudes toward risk can affect final decisions. For those of you who have taken AD 715, most of this module will be a quick review of the decision analysis module in AD 715. Different from AD 715, we will discuss using Bayes' rule when developing a decision tree, and using ASP for creating a decision tree and for performing sensitivity analysis.

Although the decision-making process will be different from one problem to another, it has certain characteristics that are common for all problems:[3]

- The set of decisions or strategies available to the decision maker (often called **decision alternatives**).
- Uncertain future events, referred to as **chance events.**
- The set of possible outcomes (and the probabilities of these outcomes) associated with each combination of a decision alternative and chance event outcome. The possible outcomes for a

chance event are referred to as **states of nature**. States of nature are events that cannot be controlled by the decision maker.

- A value model that prescribes monetary values for the various decision-outcome combinations.

We will start discussing the elements of decision analysis with a very simple problem.

Assume that a decision maker must choose among three decisions, D1, D2, and D3. Each of these decisions has three possible outcomes, O1, O2, and O3 (in this simple example, we omit the concept of state of nature and directly state the outcomes associated with each decision) as listed in the following table.

			Outcome	
		O1	**O2**	**O3**
Decision	D1	10	10	10
	D2	–10	20	30
	D3	–30	30	80

Source: Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 225.

Payoff Tables

The table shown above is called a payoff table. **A payoff table** shows all possible combinations of decision alternatives and the outcomes. The payoffs may be positive or negative. Positive values generally correspond to rewards or gains whereas negative values correspond to costs or losses.

For example, in our simple problem if the decision maker chooses decision D3 and outcome O1 occurs, then a payoff of –$30 (cost) is received whereas if the decision maker chooses decision D3 and outcome O3 occurs, then a payoff of $80 (reward) is received.

By looking at this table, we observe that if the decision maker chooses decision D1, s/he will get a profit of $10 for sure. With decision D2, rewards of $20 and $30 are possible but there is also a possibility of losing $10. Decision D3 is riskier with a possibility of losing $30, but it also offers a bigger reward of $80. So, which decision should the decision maker choose?

Decision makers generally use decision rules that help them choose the best decision alternative. None of these rules work best in all situations; however, they help us build an intuition about the decision problem and make more informed decisions. In the next two sections, we will present several decision rules distinguished by the availability of information about the likelihoods of outcomes.

Decision Analysis without Probabilities

In this section, we assume that we do not have any information about how *likely* each outcome is. This is the case in our simple problem discussed above. We will review the following decision rules that help the decision maker in making decisions:

- Optimistic Approach
- Conservative Approach
- Minimax Regret Approach

Optimistic Approach

"The optimistic approach evaluates each alternative based on the *best* payoff that can occur."[4]

- For a maximization problem (where the output measure is profit), the optimistic approach is often referred to as the *maximax* approach, and it leads the decision maker to choose the alternative corresponding to the largest payoff.
- For a minimization problem (where the output measure is cost as opposed to profit), the optimistic approach often is referred to as the *minimin* approach, and it leads the decision maker to choose the alternative corresponding to the smallest payoff.

Adapted from Camm, J. D., Cochran, J. J., Fry, M. J., Ohlmann, J. W., Anderson, D. R., Sweeney, D. J., & Williams, T. A. (2015). *Essentials of business analytics* (1st edition). Cengage Learning, p. 554

Let us apply this approach to our simple decision problem (Table 9.1), where we assume that the output measures are profits.

Table 9.1: Maximum payoff for each decision alternative

Decision Alternative	Maximum Payoff
D1	10
D2	30
D3	80

"We first find the maximum payoff for each decision alternative, then we select the decision alternative that provides the overall maximum payoff."[5] In this case, the maximum payoff for D1 is 10; the maximum payoff for

D2 is 30; and the maximum payoff for D3 is 80. The maximum of the maximum payoffs is 80. Therefore, the optimistic approach suggests choosing the decision alternative D3.

The major drawback of the optimistic approach is that it ignores large losses. This may easily bankrupt a company; therefore, the optimistic approach is seldom used in practice.

Conservative Approach: "The conservative approach evaluates each alternative based on the *worst* payoff that can occur."[6]

- For a maximization problem, the conservative approach is often referred to as the *maximin* approach, and it leads the decision maker to choose the alternative that maximizes the minimum payoff.
- For a minimization problem, the conservative approach is often referred to as the *minimax* approach, and it leads the decision maker to choose the alternative that minimizes the maximum payoff.

Source: Adapted from Camm, J. D., Cochran, J. J., Fry, M. J., Ohlmann, J. W., Anderson, D. R., Sweeney, D. J., & Williams, T. A. (2015). *Essentials of business analytics* (1st edition). Cengage Learning, p. 555.

Let us apply this approach to our simple decision problem (Table 9.2):

Table 9.2: Minimum payoff for each decision alternative

Decision Alternative	Maximum Payoff
D1	10
D2	–10
D3	–30

"We first find the minimum payoff for each decision alternative then we select the decision alternative that maximizes the minimum payoff."[7] In this case, the minimum payoff for D1 is 10; the minimum payoff for D2 is -10; and the minimum payoff for D3 is -30. The maximum of the minimum payoffs is 10. Therefore, the conservative approach suggests choosing the decision alternative D1.

"The advantage of the conservative approach is that it avoids large losses but on the down side it fails to consider large rewards. Therefore, similar to the optimistic approach it is seldom used in practice."[8]

Minimax Regret Approach

"In decision analysis, **regret** is the difference between the payoff associated with a particular decision alternative and the payoff associated with the decision that would yield the most desirable payoff for a given state of nature."[9] "Under the minimax regret approach, the decision maker chooses the decision alternative that minimizes the maximum regret that could occur over all possible states of nature."[10]

The advantage of this approach is that it is neither conservative nor optimist. It is a balanced approach.

We use the following formula to calculate regret:

$$R_{ij} = |V_j^* - V_{ij}|$$

where

R_{ij} = the regret associated with a decision alternative d_i and state of nature s_j.

V_j^* = the payoff value corresponding to the best decision for the state of nature s_j.

V_{ij} = the payoff corresponding to the decision alternative di and state of nature s_j.

For maximization problems, the best payoff V_j^* is the largest entry in column j whereas it is the smallest entry in column j for minimization problems.

Source: Camm, J. D., Cochran, J. J., Fry, M. J., Ohlmann, J. W., Anderson, D. R., Sweeney, D. J., & Williams, T. A. (2015). *Essentials of business analytics* (1st edition). Cengage Learning, p. 556.

Let us apply this approach to our simple decision problem. We first identify that

V_1^* = 10 (the payoff value corresponding to the best decision for the state of nature O1)

V_2^* = 30 (the payoff value corresponding to the best decision for the state of nature O2)

V_3^* = 80 (the payoff value corresponding to the best decision for the state of nature O3).

Then, we calculate

R_{11} = 10 – 10 = 0

R_{21} = 10 – (–10) = 20

R_{31} = 10 – (–30) = 40

R_{12} = 30 – 10 = 20

R_{22} = 30 – 20 = 10

R_{32} = 30 – 30 = 0

R_{13} = 80 – 10 = 70

R_{23} = 80 – 30 = 50

R_{33} = 80 – 80 = 0

The regret table is given in Table 9.3.

Table 9.3: Regret table for the simple decision problem

Decision Alternative	**O1**	**O2**	**O3**
D1	0	20	70
D2	20	10	50
D3	40	0	0

We then identify the maximum regret associated with each decision alternative (Table 9.4) and select the decision alternative with the minimum regret value. It corresponds to the decision alternative D3.

Table 9.4: Maximum regret for the simple decision problem

Decision Alternative	**Maximum Regret**
D1	70
D2	50
D3	40

Decision Analysis with Probabilities

In many decision-making situations, we can obtain information about how likely each state of nature (outcome) is. In the presence of the probabilities for each outcome, we can use the expected monetary value (EMV) approach to identify the best decision alternative. Under this approach, we choose the decision alternative with the largest EMV.

The expected monetary value of a decision alternative d_i is calculated by

$$EMV(d_i) = \sum_{j=1}^{N} P(s_j)V_{ij}$$

where

$P(s_j)$ = the probability of state of nature s_j

N = the number of states of nature

Source: Camm, J. D., Cochran, J. J., Fry, M. J., Ohlmann, J. W., Anderson, D. R., Sweeney, D. J., & Williams, T. A. (2015). *Essentials of business analytics* (1st edition). Cengage Learning, p. 557.

Now, let us assume that we are given the following probabilities for the outcomes of our simple decision problem (Table 9.5):

Table 9.5: Probabilities for the outcomes of the simple decision problem

Decision Alternative	O1	O2	O3
D2	0.3	0.5	0.2
D3	0.5	0.2	0.3

EMV(d_1) = 10 (sure thing)

EMV(d_2) = 0.3 × (–10) + 0.5 × (20) + 0.2 × (30) = 13

EMV(d_3) = 0.5 × (–30) + 0.2 × (30) + 0.3 × (80) = 15

We conclude that under the EMV approach, the optimal decision is to select decision alternative D3 as it is the decision with the largest EMV.

It is important to note that EMV of 15 for D3 does NOT mean that if we choose D3, then our monetary gain will be 15. The EMV values of 10, 13, and 15 should only be used for comparison purposes, i.e., the decision with the largest EMV is the best decision while the decision with the smallest EMV is the worst decision.

Source: Adapted from Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 228.

Sensitivity Analysis

"The probabilities used in a decision analysis problem are intelligent guesses at best."[11] Therefore, it is important to check how the recommended decision alternative changes with the probabilities for the states of nature or changes in payoffs. "Sensitivity analysis helps the decision maker understand which of the inputs are critical and may change the optimal decision alternative. If a small change in the value of the input (for instance, the probability for a state of nature) leads to a change in the recommended decision alternative, we say that the solution to the decision analysis problem is very sensitive to this particular input and extra effort should be taken to use the most accurate value of this input. On the other hand, if the recommended decision alternative does not change with a modest-to-large change in the value of one input, then we conclude that the solution to the decision analysis problem is not sensitive to that particular input. No extra effort is needed for finding the most accurate value of this input."[12]

Let us apply sensitivity analysis to our simple decision analysis problem. For this easy problem, this is easy to do in a spreadsheet (Figure 9.1).

Figure 9.1: Spreadsheet model for the simple decision problem

	A	B	C	D	E	F
1	**Simple Decision Problem Under Uncertainty**					
2			Outcome			
3			**O1**	**O2**	**O3**	**EMV**
4	**Decision**	**D1**	10	10	10	10
5		**D2**	-10	20	30	13
6		**D3**	-30	30	80	15
7						
8	**Probabilities**					
9		**D2**	0.3	0.5	0.2	
10		**D3**	0.5	0.2	0.3	
11						

Using the spreadsheet, we can change any of the inputs and see whether the optimal decision continues to be D3.

- Change the probabilities of D3 slightly from 0.5, 0.2, and 0.3 to 0.6, 0.2, and 0.2. You will see that EMV for D3 will change to 4 indicating D3 will be the worst decision and D2 will be the best decision. So, we conclude that the recommended decision alternative is very sensitive to the assessed probabilities.
- Let the probabilities remain the same but the last payoff for D2 changes from 30 to 45. In this case, the EMV for D2 changes to 16 and D2 becomes the best decision.

Source: Adapted from Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 229.

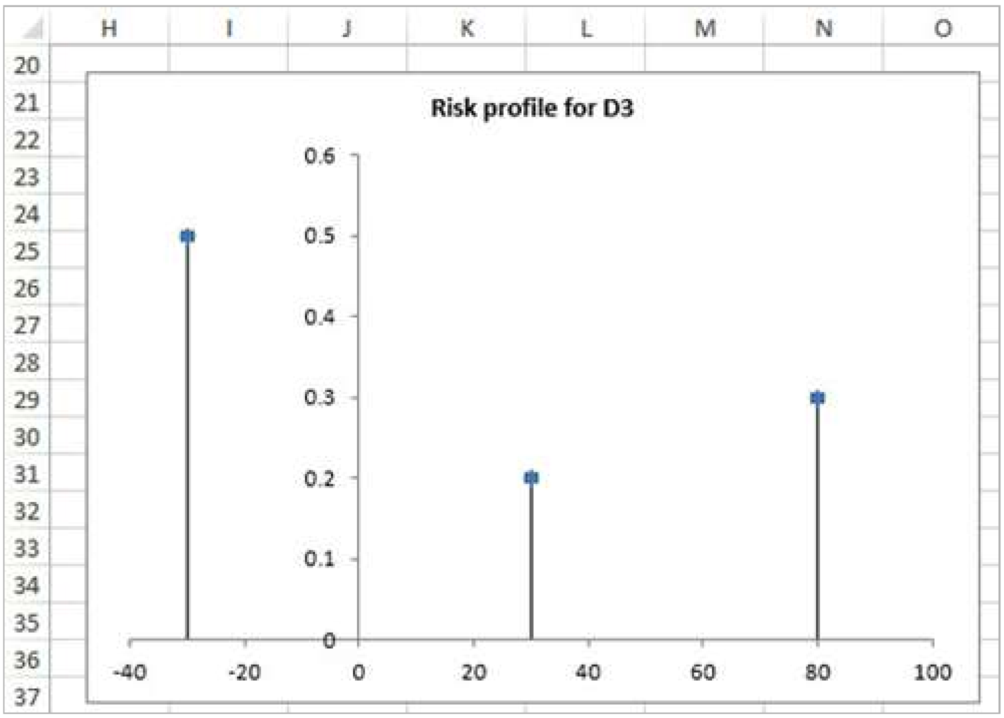
For a simple problem like this, it is easy to perform sensitivity analysis by making a number of ad hoc changes to the inputs. However, for larger problems it is important to perform more systematic sensitivity analysis. We will learn more about sensitivity analysis for decision problems in Tutorial 4.

Risk Profiles

"Risk analysis helps the decision maker recognize the difference between the expected value of a decision alternative and the payoff that may actually occur."[13] The **risk profile** for a decision alternative represents the monetary outcomes for this decision along with the associated probabilities. "Risk profile for a single decision allows one to see the risks and rewards associated with this decision. Furthermore, by comparing the risk profiles for different decisions, one can gain more insight into their relative weaknesses and strengths."[14]

The risk profile for D3 is given in Figure 9.2.

Figure 9.2: Risk profile for D3



Source: Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 232.

This risk profile tells us that a loss of $30 has probability 0.5, a gain of $30 has probability 0.2, and a gain of $80 has probability 0.3. The risk profile for D2 is very similar except that its spikes are at –10, 20, and 30, and the risk profile for D1 is single spike at 10.

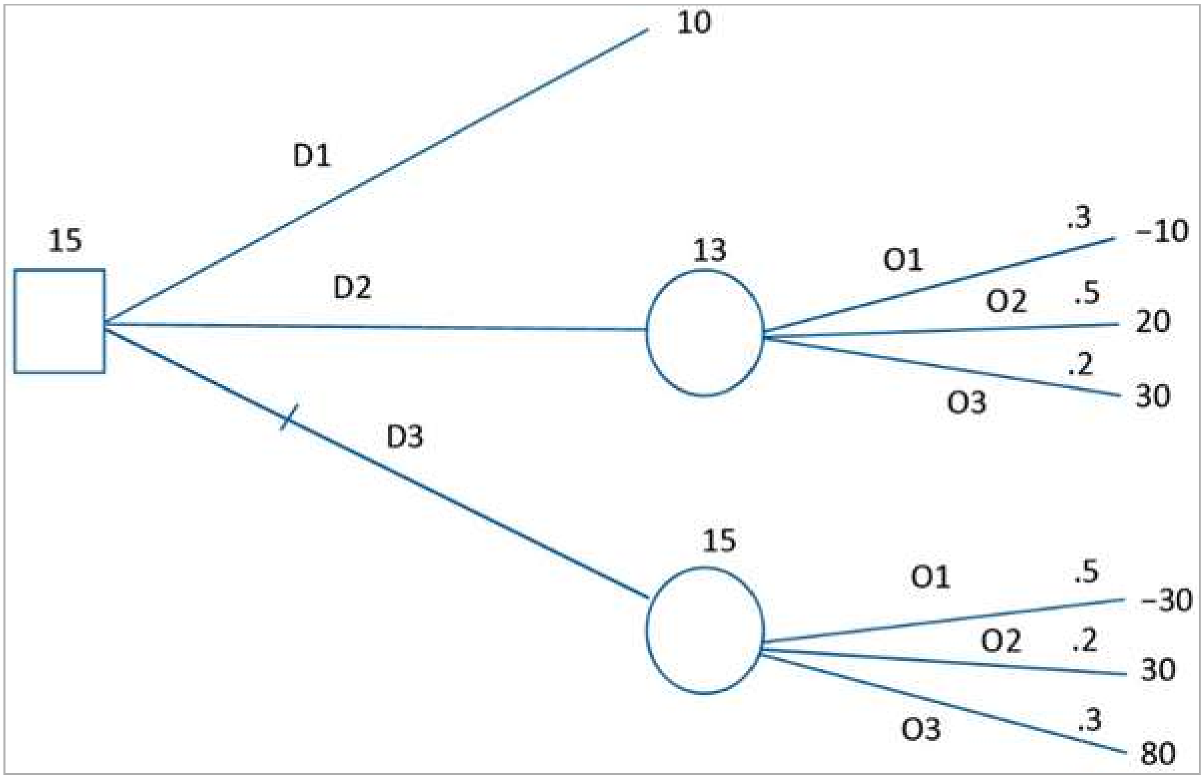
The review of a risk profile for the recommended decision alternative may cause the decision maker to switch to another decision alternative with less EMV value. "This is because of the fact that the EMV for any decision is actually a summary measure of the risk profile, i.e., it is the mean of the probability distribution."[15] When using EMV, the decision maker is not using the whole information in the risk profiles. This is why the extra information in the risk profile may lead the decision maker to change his/her decision. In the end, the decision maker may see too much risk in the EMV-maximizing decision and instead choose a less risky decision.

Decision Trees

"A decision tree provides a graphical representation of the decision-making process."[16]

The decision tree for our simple problem is given in Figure 9.3.

Figure 9.3: The decision tree for our simple problem



Source: Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 230.

- Decision trees are composed of *nodes* (circles, squares, and triangles) and *branches* (lines).
- The nodes represent points in time. A *decision node* depicted by a square represents a time when the decision maker makes a decision. A *chance node* depicted by a circle represents a time when the result of an uncertain outcome becomes known. An *end node* depicted by a triangle indicates that the problem is completed — all decisions have been made, all uncertainty has been resolved, and all payoffs and costs have been incurred. When drawing decision trees by hand, people usually omit the triangles (as is the case in Figure 9.3).
- Time proceeds from *left* to *right.* This means that any branches leading into a node (from the left) have already occurred. Any branches leading out of a node (to the right) have not yet occurred.
- Branches leading out of a decision node represent the possible decisions; the decision maker can choose the preferred branch. Branches leading out of chance nodes represent the

possible outcomes of uncertain events; the decision maker has no control over which of these will occur.

- Probabilities are listed on chance branches. These probabilities are conditional on the events that have already been observed (those to the left). Also the probabilities on branches leading out of any chance node must sum to 1.
- Monetary values are shown to the right of the end nodes.
- EMVs are calculated through a "folding-back" process, which will be discussed next. They are shown above the various nodes. It is then customary to mark the optimal decision branch(es) in some way. The optimal decision of the decision tree in Figure 9.3 is marked with a small notch.

Source: Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 230.

One of the advantages of working with a decision tree is that it allows you to make the EMV calculations easily with the following folding-back procedure:

The folding-back procedure for the calculation of EMVs in a decision tree works as follows. We start from the right of the decision tree and work back to the left performing the following actions:

- At each chance node, calculate an EMV – a sum of the products of monetary values and probabilities.
- At each decision node, take the maximum of EMVs to identify the optimal decision.

Source: Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 231.

For instance, the EMV for the first chance node (the one on the above) in Figure 9.3 is calculated as (–10) × 0.3 + 20 × 0.5 + 30 × 0.2 = 13. Similarly, the second chance node (the one on the bottom) is calculated as (–30) × 0.5 + 30 × 0.2 + 80 × 0.3 = 15. For D1, we do not need to calculate the EMV — it is 10. At the decision node we choose max{10, 13, 15}, which leads to the third decision alternative.

For such simple problems, it is easy to calculate the EMV in a decision tree by hand. However, for more complex problems it may not be possible to perform the calculations by hand. Fortunately, Frontline's ASP allows us to create a decision tree in Excel and performs the folding-back procedure automatically. We discuss how to create a decision tree using ASP in Tutorial 4. In the tutorial, we first build a decision tree for our simple decision problem using ASP. We then move to a more complex decision problem and build its decision tree and then we perform sensitivity analysis for this complex decision problem.

Multistage Decision Problems and the Value of Information

In our simple decision analysis problem, we made a single decision. In many realistic business problems, a decision maker must make at least two decisions that are separated in time. This type of decision problem is called a *multistage* decision problem. "In such problems, sometimes the first-stage decision is whether to purchase information that will make a better second-stage decision. If the information is obtained in the first-stage, this usually changes the probabilities of later outcomes."[17] In order to revise the probabilities, one often applies *Bayes' rule*, which will be discussed in this section. "Furthermore, decision usually comes with a price and thus the decision maker wants to learn whether the information is worth its price."[18] This leads to a concept called the value of information, which will also be discussed in this section.

Bayes' theorem is discussed in Unit of SwAM. We will briefly summarize the rule here.

Bayes' Rule

"In a multistage decision tree, all chance branches toward the *right* of the decision tree are conditional on outcomes that have occurred earlier, to their left. Therefore, the probabilities on these branches are of the form P(A|B) read as "probability of A given B," where A is an event corresponding to a current chance branch and B is an event that occurs *before* event A in time. However, it is usually more natural to obtain information of the form P(B|A) when gathering data for the problem."[19] Therefore, we will need a mechanism that will convert the probabilities of the form P(B|A) to the probabilities of the form P(A|B) — this mechanism is called Bayes' rule. This rule is also very useful for revising probabilities as new information becomes available.

- Let A_1 through A_n represent outcomes. The probabilities associated with these outcomes are $P(A_1)$ through $P(A_n)$ — these are called *prior* probabilities as these are the probabilities obtained before collecting more information (i.e., data).
- "Assume that there are several information outcomes you may observe and let one of these be labeled as B. Let us further assume that probabilities of B given that any of the As will occur are known."[20] These probabilities $P(B|A_1)$, … , $P(B|A_n)$ are called *likelihoods*.
- As the new information B may affect our thinking about the probabilities of the As, we will find the conditional probability for each outcome $P(A_i|B)$ called as the posterior probability of Ai via Bayes' rule as follows:

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B|A_i)P(A_i) + \ldots + P(B|A_n)P(A_n)}$$

The denominator in this equation is especially useful in multistage decision trees. It is basically the probability of the information outcome B and it is sometimes called the *law of total probability*.

The Value of Information

Given that we can gather some information that will impact our final decision, some of the questions that may come into our mind are

- How much is the information worth, and if it costs a given amount, should we purchase it?
- Even if the information is worth something, is it worth as much as its actual price?

Source: Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 260.

The good news is that we can obtain answers to these questions using the decision tree. We will find the values of two types of information: sample information and perfect information.

Sample Information

"Sample information (also called the imperfect information) is the information from the experiment itself whereas the **perfect information** is the information from a perfect test (i.e., a test that indicates with certainty which ultimate outcome will occur). It is important to note that perfect information is not available at any price, but it provides an upper bound on the value of *any* information."[21]

We will focus on the expected value of sample information (EVSI) and the expected value of perfect information (EVPI).

- The **EVSI** is the most you would be willing to pay for *sample* information.

  EVSI = EMV with (free) sample information – EMV without information

- The **EVPI** is the most you would be willing to pay for *perfect* information.

  EVPI = EMV with (free) perfect information – EMV without information

Fundamental Insights

- The amount you should be willing to spend for information is the expected increase in EMV you can obtain from having the information. If the actual price of the information is less than or equal to this amount, you should purchase it; otherwise, the information is not worth its price.

- The information that never affects your decision is worthless, and it should not be purchased at any price.
- The value of any information can never be greater than the value of perfect information that would eliminate all uncertainty.

Source: Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 261.

In the next section, we will apply the decision-analysis tools that we have learned so far including Bayes' rule and value of information to a realistic multistage decision problem.

Note

It is a good time now to review Tutorial 4 before you begin the next section.

An Example - Marketing a New Product at Acme

Acme Company is trying to decide whether to market a new product. As in many new-product situations, there is considerable uncertainty about whether the new product will eventually succeed. Acme believes that it might be wise to introduce the product in a regional test market before introducing it nationally. Therefore, the company's first decision is whether to conduct the test market.

Acme estimates that the net cost of the test market is $100,000. We assume this is mostly fixed costs, so that the same cost is incurred regardless of the test-market results. If Acme decides to conduct the test market, it must then wait for the results. Based on the results of the test market, it can then decide whether to market the product nationally, in which case it will incur a fixed cost of $7 million. On the other hand, if the original decision is not to run a test market, then the final decision — whether to market the product nationally — can be made without further delay. Acme's unit margin, the difference between its selling price and its unit variable cost, is $18. We assume this is relevant only for the national market.

Acme classifies the results in either the test market or the national market as great, fair, or awful. Each of these results in the national market is accompanied by a forecast of total units sold. The sales volumes (in 1000s of units) are 600 (great), 300 (fair), and 90 (awful). In the absence of any test market information, Acme estimates that probabilities of the three national market outcomes are 0.45, 0.35, and 0.20, respectively.

In addition, Acme has the following historical data from products that were introduced into both test markets and national markets.

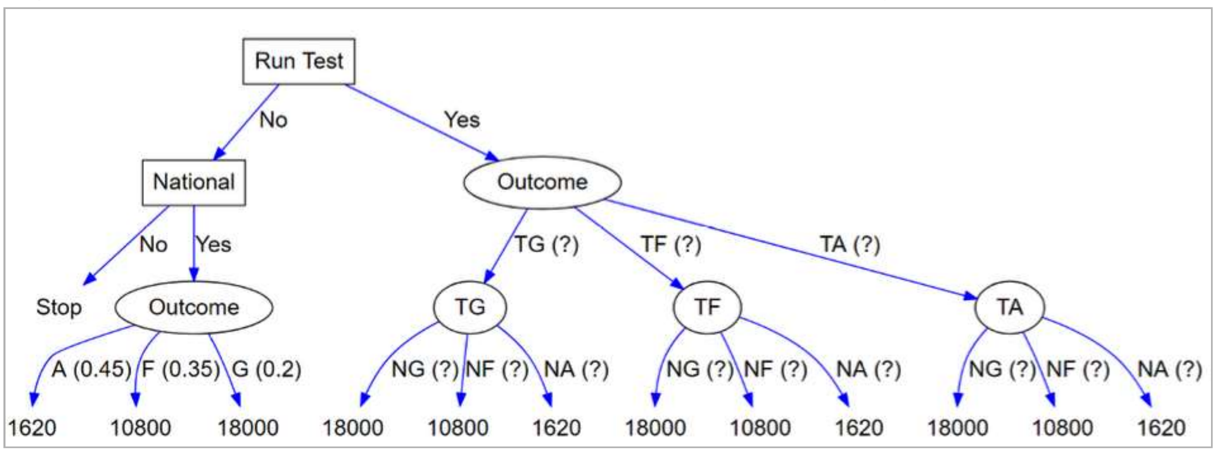
- Of the products that eventually did great in the national market, 64% did great in the test market, 26% did fair in the test market, and 10% did awful in the test market.
- Of the products that eventually did fair in the national market, 18% did great in the test market, 57% did fair in the test market, and 25% did awful in the test market.
- Of the products that eventually did awful in the national market, 9% did great in the test market, 48% did fair in the test market, and 43% did awful in the test market.

The company wants to use a decision tree approach to find the best strategy. It also wants to find the expected value of the information provided by the test market.

Source: Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, pp. 261-262.

Before drawing the actual decision-tree using ASP, let us draw a sketch of it to see what information we would need for this decision tree. Figure 9.4 shows the sketch of the decision tree (you can easily draw this by hand).

Figure 9.4: A sketch of the decision tree for the Acme company problem

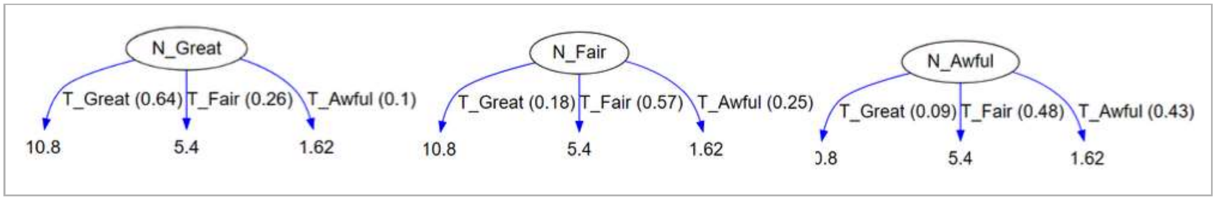


The question marks represent the probabilities needed to complete this tree. By looking at the description of the problem, we realize that we have information about the historical probabilities of the type P(Great test market | Great national market) = 0.64. More specifically, letting NG, NF, and NA represent great, fair, and awful national-market results, respectively, and TG, TF, and TA represent similar events for the test market, the historical probabilities that is given in the problem description are

$P(TG\|NG) = 0.64$	$P(TF\|NG) = 0.26$	$P(TA\|NG) = 0.10$
$P(TG\|NF) = 0.18$	$P(TF\|NF) = 0.57$	$P(TA\|NF) = 0.25$
$P(TG\|NA) = 0.09$	$P(TF\|NA) = 0.48$	$P(TA\|NA) = 0.43$

The list above can be represented graphically in Figure 9.5.

Figure 9.5: Inputs and probability calculations using Bayes' rule for Acme company problem



However, we realize that we cannot use these probabilities in the decision tree because in the decision tree we *first* realize the test-market results *then* the national- market results. We need conditional probabilities of the form P(National market | Test market). We calculate such probabilities via Bayes' rule as follows:

$$P(NG|TG) = \frac{P(TG|NG)P(NG)}{P(TG|NG)P(NG) + P(TG|NF)P(NF) + P(TG|NA)P(NA)}$$
$$= \frac{0.64 * 0.45}{(0.64 * 0.45) + (0.18 * 0.35) + (0.09 * 0.20)} = 0.7805$$

$$P(NF|TG) = \frac{P(TG|NF)P(NF)}{P(TG|NF)P(NF) + P(TG|NG)P(NG) + P(TG|NA)P(NA)}$$
$$= \frac{0.18 * 0.35}{(0.18 * 0.35) + (0.64 * 0.45) + (0.09 * 0.20)} = 0.1707$$

$$P(NA|TG) = \frac{P(TG|NA)P(NA)}{P(TG|NA)P(NA) + P(TG|NG)P(NG) + P(TG|NF)P(NF)}$$
$$= \frac{0.09 * 0.20}{(0.09 * 0.20) + (0.64 * 0.45) + (0.18 * 0.35)} = 0.0488$$

$$P(NG|TF) = \frac{P(TF|NG)P(NG)}{P(TF|NG)P(NG) + P(TF|NF)P(NF) + P(TG|NA)P(NA)}$$
$$= \frac{0.26 * 0.45}{(0.26 * 0.45) + (0.57 * 0.35) + (0.48 * 0.24)} = 0.2836$$

$$P(NF|TF) = \frac{P(TF|NF)P(NF)}{P(TF|NF)P(NF) + P(TF|NG)P(NG) + P(TF|NA)P(NA)}$$

$$= \frac{0.57 * 0.35}{(0.57 * 0.35) + (0.26 * 0.45) + (0.48 * 0.20)} = 0.4836$$

$$P(NA|TF) = \frac{P(TF|NA)P(NA)}{P(TF|NA)P(NA) + P(TF|NG)P(NG) + P(TF|NF)P(NF)}$$

$$= \frac{0.48 * 0.20}{(0.48 * 0.20) + (0.26 * 0.45) + (0.57 * 0.35)} = 0.2327$$

$$P(NG|TA) = \frac{P(TA|NG)P(NG)}{P(TA|NG)P(NG) + P(TA|NF)P(NF) + P(TA|NA)P(NA)}$$

$$= \frac{0.10 * 0.45}{(0.10 * 0.45) + (0.25 * 0.35) + (0.43 * 0.20)} = 0.2059$$

$$P(NF|TA) = \frac{P(TA|NF)P(NF)}{P(TA|NF)P(NF) + P(TA|NG)P(NG) + P(TA|NA)P(NA)}$$

$$= \frac{0.25 * 0.35}{(0.25 * 0.35) + (0.10 * 0.45) + (0.43 * 0.20)} = 0.4005$$

$$P(NA|TA) = \frac{P(TA|NA)P(NA)}{P(TA|NF)P(NF) + P(TA|NG)P(NG) + P(TA|NA)P(NA)}$$

$$= \frac{0.43 * 0.20}{(0.25 * 0.35) + (0.10 * 0.45) + (0.43 * 0.20)} = 0.3946$$

We can further obtain the probabilities related to the test-market result being great, fair, and awful. More specifically,

$P(TG) = P(TG|NG)P(NG) + P(TG|NF)P(NF) + P(TG|NA)P(NA) = 0.369$ which is the denominator of $P(NG|TG)$, $P(NF|TG)$, and $P(NA|TG)$.

$P(TF) = P(TF|NG)P(NG) + P(TF|NF)P(NF) + P(TF|NA)P(NA) = 0.4125$ which is the denominator of $P(NG|TF)$, $P(NF|TF)$, and $P(NA|TF)$.

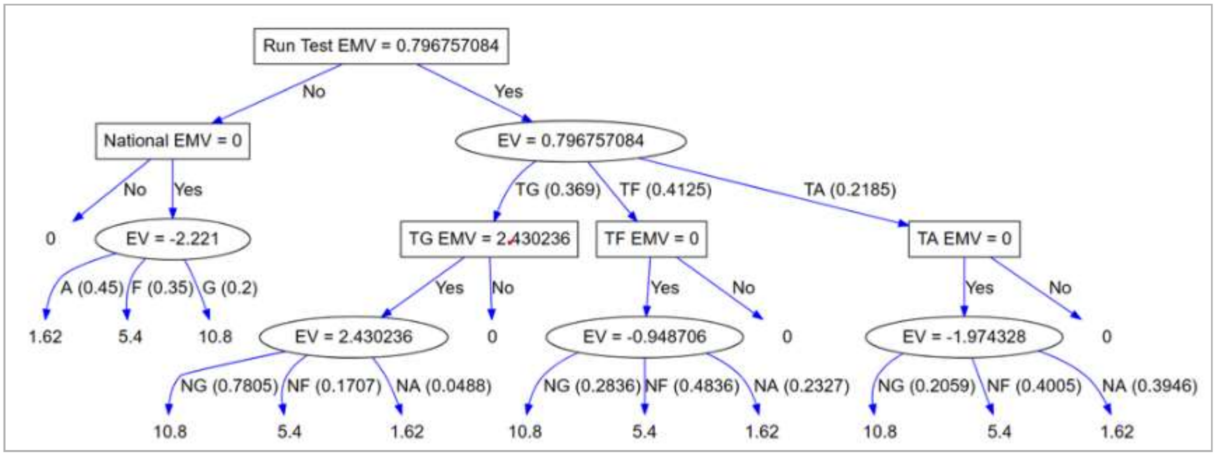
$P(TA) = P(TA|NG)P(NG) + P(TA|NF)P(NF) + P(TA|NA)P(NA) = 0.2185$ which is the denominator of $P(NG|TA)$, $P(NF|TA)$, and $P(NA|TA)$.

Decision Tree for Acme Problem

We build our decision tree using the step-by-step approach described in Tutorial 4. We will use Data Tree package in R to build all our trees. The following code demonstrates how to build the tree. After building the tree, we will call the **update_payoff** function with the root node to update the payoff calculations across all the tree nodes.

```
Great <- 600*18/1000
Fair <- 300*18/1000
Awful <- 90*18/1000

TM <- Node$new("Run Test",type="decision")
NM <- TM$AddChild("National",type="decision",route='No')
NM$AddChild("Stop",type="terminal",route='No',payoff=0)

NM_ <- NM$AddChild("Outcome",type="chance",route="Yes",cost=-7.0)
NM_$AddChild("A",type="terminal",p=0.45,payoff=Awful)
NM_$AddChild("F",type="terminal",p=0.35,payoff=Fair)
NM_$AddChild("G",type="terminal",p=0.2,payoff=Great)

TM_ <- TM$AddChild("Outcome",type="chance",route="Yes",cost=-0.1)

TM_G <- TM_$AddChild("TG",type='decision',p=0.369)
TM_TG <- TM_G$AddChild('Outcome',type='chance',route='Yes',cost=-7.0)
TM_G$AddChild('Stop',type='terminal',route='No',payoff=0)

TM_TG_NG <- TM_TG$AddChild("NG",type='terminal',p = 0.7805,payoff = Great)
TM_TG_NF <- TM_TG$AddChild("NF",type='terminal',p = 0.1707,payoff = Fair)
TM_TG_NA <- TM_TG$AddChild("NA",type='terminal',p = 0.0488,payoff = Awful)

TM_F <- TM_$AddChild("TF",type='decision',p=0.4125)
TM_TF <- TM_F$AddChild('Outcome',type='chance',route='Yes',cost=-7.0)
TM_F$AddChild('Stop',type='terminal',route='No',payoff=0)

TM_TF_NG <- TM_TF$AddChild("NG",type='terminal',p = 0.2836,payoff = Great)
TM_TF_NF <- TM_TF$AddChild("NF",type='terminal',p = 0.4836, payoff = Fair)
TM_TF_NA <- TM_TF$AddChild("NA",type='terminal',p = 0.2327,payoff = Awful)
TM_A <- TM_$AddChild("TA",type='decision',p=0.2185)
TM_TA <- TM_A$AddChild('Outcome',type='chance',route='Yes',cost=-7.0)
TM_A$AddChild('Stop',type='terminal',route='No',payoff=0)

TM_TA_NG <- TM_TA$AddChild("NG",type='terminal',p = 0.2059,payoff = Great)
TM_TA_NG <- TM_TA$AddChild("NF",type='terminal',p = 0.4005,payoff = Fair)
TM_TA_NG <- TM_TA$AddChild("NA",type='terminal',p = 0.3946,payoff = Awful)
SetEdgeStyle(TM, fontname = 'helvetica',fontsize=18, label = GetEdgeLabel,color='blue')
SetNodeStyle(TM, fontname = 'helvetica', fontsize=18,shape = GetNodeShape, label = GetNodeLabel)

plot(TM)
```

Figure 9.6 Updated Payoff Diagram



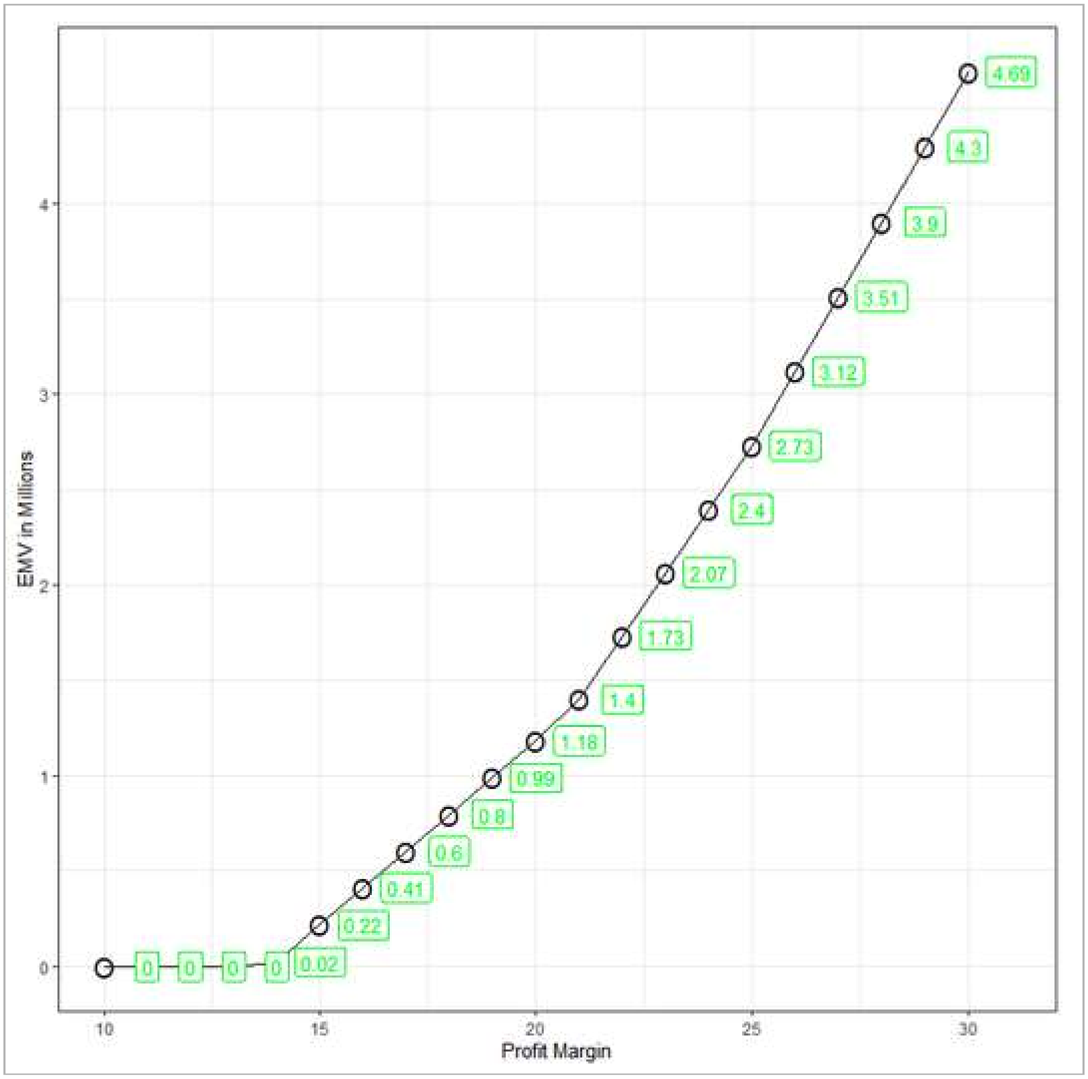
From the updated payoff diagram we notice that the best path to follow is to only proceed if the outcome of the market testing is great and our Expected MV is 0.79M.

Sensitivity Analysis

Once we have built the model, one of the analyses that might be useful to perform is sensitivity analysis. In this case we have made several assumptions from transition probabilities, to profit margins. It would be useful to know how the decision might change with the change in assumptions.

We will just do a quick analysis by changing the profitability margin from $8 to $30 and noting if our decision changes. We will execute the following code to get to the sensitivity analysis.

Fig 9.7 Sensitivity analysis Chart



The sensitivity dchart shows that the EMV value goes up exponentially with the profit margins. Further, looking at EMV for alternative options, we notice that conducting the test market seems to be the right thing to do to have a positive EMV, until the profit margins cross $27. Even then, the EMV is still high by running the test market. It is important to realize that this is still EMV (Expected Monetary Value) hence the right decision to make is to still run the test market and not take the other branch even if it is positive.

```
emv_sim <- function(profit_margin){
Great <- 600*profit_margin/1000
Fair <- 300*profit_margin/1000
Awful <- 90*profit_margin/1000
TM <- Node$new("Run Test",type="decision")
NM <- TM$AddChild("National",type="decision",route='No')
NM$AddChild("Stop",type="terminal",route='No',payoff=0)
NM_ <- NM$AddChild("Outcome",type="chance",route="Yes",cost=-7.0)
NM_$AddChild("A",type="terminal",p=0.45,payoff=Awful)
NM_$AddChild("F",type="terminal",p=0.35,payoff=Fair)
NM_$AddChild("G",type="terminal",p=0.2,payoff=Great)

TM_ <- TM$AddChild("Outcome",type="chance",route="Yes",cost=-0.1)

TM_G <- TM_$AddChild("TG",type='decision',p=0.369)
TM_TG <- TM_G$AddChild('Outcome',type='chance',route='Yes',cost=-
7.0)
TM_G$AddChild('Stop',type='terminal',route='No',payoff=0)
TM_TG_NG <- TM_TG$AddChild("NG",type='terminal',p = 0.7805,payoff =
Great)
TM_TG_NF <- TM_TG$AddChild("NF",type='terminal',p = 0.1707,payoff =
Fair)
TM_TG_NA <- TM_TG$AddChild("NA",type='terminal',p = 0.0488,payoff =
Awful)
TM_F <- TM_$AddChild("TF",type='decision',p=0.4125)
TM_TF <- TM_F$AddChild('Outcome',type='chance',route='Yes',cost=-
7.0)
TM_F$AddChild('Stop',type='terminal',route='No',payoff=0)
TM_TF_NG <- TM_TF$AddChild("NG",type='terminal',p = 0.2836,payoff =
Great)
TM_TF_NF <- TM_TF$AddChild("NF",type='terminal',p = 0.4836, payoff
= Fair)
TM_TF_NA <- TM_TF$AddChild("NA",type='terminal',p = 0.2327,payoff =
Awful)
TM_A <- TM_$AddChild("TA",type='decision',p=0.2185)
TM_TA <- TM_A$AddChild('Outcome',type='chance',route='Yes',cost=-
7.0)
TM_A$AddChild('Stop',type='terminal',route='No',payoff=0)
TM_TA_NG <- TM_TA$AddChild("NG",type='terminal',p = 0.2059,payoff =
Great)
TM_TA_NG <- TM_TA$AddChild("NF",type='terminal',p = 0.4005,payoff =
Fair)
TM_TA_NG <- TM_TA$AddChild("NA",type='terminal',p = 0.3946,payoff =
Awful)
update_payoff(TM)
return(c(TM$payoff,TM_G$payoff,TM_F$payoff, TM_A$payoff,
NM$payoff))
}
prof_margin <- seq(10,30,1)
emvs <- sapply(prof_margin,emv_sim)[1,]
ggplot() + geom_line(aes(x=prof_margin,y=emvs))+
geom_point(aes(x=prof_margin,y=emvs),shape="O",size=5) +
geom_label(aes(x=prof_margin+1,y=emvs),
label=round(emvs,2),color='green')+theme_bw()+
      xlab("Profit Margin") + ylab("EMV in Millions")
```

	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Total EMV	0	0	0	0	0.000000	0.00000	0.000000	0.000000	0.000000	0.000000	0.00000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.168500	0.434000
EMV_TG	0	0	0	0	0.334628	0.85853	1.382432	1.906334	2.430236	2.954138	3.47804	4.001942	4.525844	5.049746	5.573648	6.097550	6.621452	7.145354	7.669256
EMV_TF	0	0	0	0	0.000000	0.00000	0.000000	0.000000	0.000000	0.000000	0.00000	0.059843	0.396026	0.732209	1.068392	1.404575	1.740758	2.076941	2.413124
EMV_TA	0	0	0	0	0.000000	0.00000	0.000000	0.000000	0.000000	0.000000	0.00000	0.000000	0.000000	0.000000	0.000000	0.000000	0.259304	0.538508	0.817712
EMV_NO_TEST	0	0	0	0	0.000000	0.00000	0.000000	0.000000	0.000000	0.000000	0.00000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.168500	0.434000

This table shows that when the unit margin is between at the top row and each of the rows represents the EMVs for various branches. As we notice, the EMV for the no test scenario does not turn positive until it is $27 of profit margin.

Expected Value of Sample Information

The test market is used to provide information (more specifically, more accurate probabilities) of national-market results. The information usually costs money and as it is given in problem description the fixed cost of the test market is $100,000. A natural question to ask is how much this test market is really worth, i.e., the EVSI.

EVSI is easy to obtain from the decision tree.

- **EMV with (free) sample information:** From the completed decision tree we see that the EMV from test marketing is $796,760 and the cost of the test market is $100,000. "Therefore, if the test market were free, the expected profit would be $896,760."[22]
- **EMV without information:** The EMV from not running the test market is $74,000 (as seen from the completed decision tree).
- Thus, EVSI = $896,760 - $74,000 = $822,760.

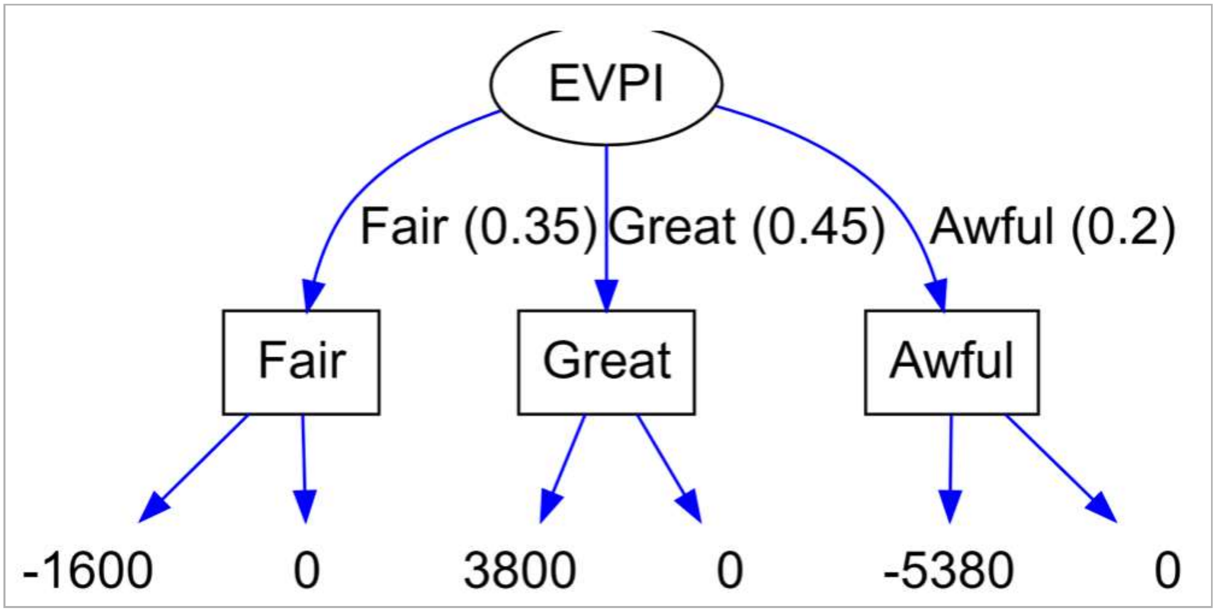
This means that for any value smaller than $822,760, the decision to test market will continue to be the best.

A close look at the decision tree reveals that running the test market is worth something as it changes the optimal decision. If we do not run the test market, the best decision is to market nationally. On the other hand, if we run the test market, the decision depends on the test-market results. So, running the test market provides valuable information because its outcome affects the optimal decision.

Expected Value of Perfect Information

"Imagine that Acme has access to an envelope that has the true national-market result – great, fair, or awful. By opening the envelope, Acme will remove all uncertainty and will make an easy decision. The good thing about the envelope is that Acme can open it *before* making the decision about the national market. EVPI is what this envelope is worth."[23] In order to calculate EVPI, we need to build the small decision tree in Figure 9.8. What is different in this decision tree is that the order of events is reversed in time: Acme first opens the envelope to see what is inside (thus, we have a chance node in the beginning). Then, it makes the final decision.

Figure 9.8



- **EMV with (free) perfect information:** If the envelope (perfect information) is free, then Figure 9.8 reveals that EMV is $1,710,000.
- **EMV without information:** If there is no information, the EMV is $74,000 (Figure 9.6).
- Therefore, EVPI = $1,710,000 - $74,000 = $1,636,000.

What does this information tell us? "It tells us no other information including test market could possibly be worth more than EVPI value. If Acme has access to market information which costs, say $1.7 million, then Acme knows that this information cannot be worth its cost."[24]

Lecture 9 Footnotes

[1] Evans, J. R. (2016). *Business analytics: Methods, models, and decisions* (2nd edition). Pearson Education, Inc., p. 554.

[2] Evans, J. R. (2016). *Business analytics: Methods, models, and decisions* (2nd edition). Pearson Education, Inc., p. 554.

[3] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 225.

[4] Camm, J. D., Cochran, J. J., Fry, M. J., Ohlmann, J. W., Anderson, D. R., Sweeney, D. J., & Williams, T. A. (2015). *Essentials of business analytics* (1st edition). Cengage Learning, p. 554.

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[6] Camm, J. D., Cochran, J. J., Fry, M. J., Ohlmann, J. W., Anderson, D. R., Sweeney, D. J., & Williams, T. A. (2015). *Essentials of business analytics* (1st edition). Cengage Learning, p. 555.

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[11] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 228.

[12] Adapted from Camm, J. D., Cochran, J. J., Fry, M. J., Ohlmann, J. W., Anderson, D. R., Sweeney, D. J., & Williams, T. A. (2015). *Essentials of business analytics* (1st edition). Cengage Learning, p. 560.

[13] Camm, J. D., Cochran, J. J., Fry, M. J., Ohlmann, J. W., Anderson, D. R., Sweeney, D. J., & Williams, T. A. (2015). *Essentials of business analytics* (1st edition). Cengage Learning, p. 559.

[14] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 231.

[15] Adapted from Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 232.

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[17] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 256.

[18] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 256.

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[20] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 252.

[21] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 260.

[22] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 269.

[23] Albright, S. C., & Winston, W. L. (2015). *Business analytics: Data analysis and decision making* (5th Edition). Cengage Learning, p. 270.

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Lecture 9 References

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Lecture 9 Resources

Spreadsheet model for the simple decision problem

The decision tree for Acme problem

The decision tree for calculating ACME EVPI

Lecture 9 Summary Questions

1. What is decision analysis? What are the elements of a decision analysis problem?
2. How do we define “regret” in the context of decision-analysis problems?
3. What is a decision tree? What are the elements of a decision tree?
4. Please comment on the following statement: “*The value of perfect information is the maximum you would like to pay for any information you may gain.”*

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