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|  | **BOSTON**  **UNIVERSITY** | **METROPOLITAN COLLEGE**  **DEPARTMENT OF ADMINISTRATIVE SCIENCES** |

**AD 616: Enterprise Risk Analytics**

**Assignment 2**

**What to submit?**

Please submit (i) a word file explaining in detail your answers to each question (you can use screenshots of the R Code to explain your answers) AND (ii) an R file for each question. For each question, make sure you develop the model and present the simulation results – the R code should be self-explanatory. **The assessment of your work will include both the accuracy and the clarity of your word file and the R Code.**

1. Answer the following two questions.
   1. According to the *central limit theorem*, the sum of *n independent identically distributed* random variables will start to resemble a normal distribution as *n* grows large. The mean of the resulting distribution will be *n* times the mean of the summands, and the variance *n* times the variance of the summands. Demonstrate this property using Monte Carlo simulation. Over 10,000 trials, take the sum of 100 uniform random variables (with min=0 and max=1). Note: the variance of the uniform distribution with min 0 and max 1 is 1/12. Include:
      1. A histogram of the results of the MC simulation

Chart, histogram

Description automatically generated

* + 1. A density plot of a normal distribution with the appropriate mean and standard deviation

Chart, histogram

Description automatically generated

* + 1. The mean and standard deviation of the MC simulation.

Mean = 50.03697 SD = 2.905557

* 1. According to probability theory, if are independent and *uniform distributed* random variables with min=0 and max=1, then the *kth* lowest will follow a *beta distribution* with parameters *shape1=k*, *shape2=n+1-k.* Demonstrate this property using Monte Carlo simulation. Simulate 10,000 trials. For each trial, generate 10 uniform random variables and select the 3rd lowest. Include:
     1. A histogram of the results of the MC simulation

Chart, histogram

Description automatically generated

* + 1. A density plot of the beta distribution with the appropriate parameters

Chart, histogram

Description automatically generated

* + 1. The mean and standard deviation of the MC simulation

Mean = 0.2752769, SD = 0.1290756

1. Assume you represent a worldwide distributor of classic cars. Create a Monte Carlo simulation with 10,000 trials to demonstrate the property that, if the amount of time it takes before your next customer makes a purchase can be modelled using an exponential distribution with a rate of 10 per day, then the number of times customers will make purchases in a day will follow a Poisson distribution with . Develop a histogram to reinforce your result.
2. A life insurance company is pricing a new policy to sell to a group of 45-year-old male non-smokers. They determine that the probability that a member of this group will die *X* years from the day they purchase the policy can be modeled with a Weibull distribution with shape parameter 4.5 and scale parameter 39, measured in years. The term of the policy is 20 years. At the end of every month, policy holders are expected to pay $115. If a policy holder in good standing dies during the term of the policy, his beneficiaries receive a lump sum of $1,000,000 at the end of the month. Every month there is a 0.3% chance that the policy holder will let the policy lapse (i.e. he will permanently stop paying premiums and forfeit his right to the benefit). The insurance company calculates cost of funds using a rate of 6.5%.

Create a Monte Carlo simulation with 10,000 trials of the above scenario to calculate the net present value of cash flows to the insurance company for one policyholder.

1. Create a histogram describing the NPV. How would you characterize the distribution?
2. What are the mean and standard deviation of the NPV? On balance, is the insurance company making a profit?
3. Provide a 95% confidence interval for the mean of the NPV. Interpret the result.
4. How many iterations would be necessary to provide a **99%** confidence interval with a half width of $200?
5. The company can be 90% sure their npv will be at least *x.* Solve for *x.*The company can be 99% sure their npv will be at least *y.* Solve for *y.*

Now assume the insurance company underwrites 1,000 policyholders. Create a Monte Carlo simulation with 1,000 trials to calculate the net present value of cashflows for the insurance company made to all the policy holders. *(Hint: Recycle your work above. Create a list of data frames, where each element of the list represents one trial.)* Answer questions (a)-(e) above under this assumption.