Outline

- Examples of Applications of Sparsity
- l_2, l_0, and l_1 Norms
- Solution Approaches
  - Matching Pursuit
  - Smooth Reformulations
- Sparse Solutions to Some Applications

What is Sparsity?

A vector is said to be sparse if it only has "*a few*" non-zero components

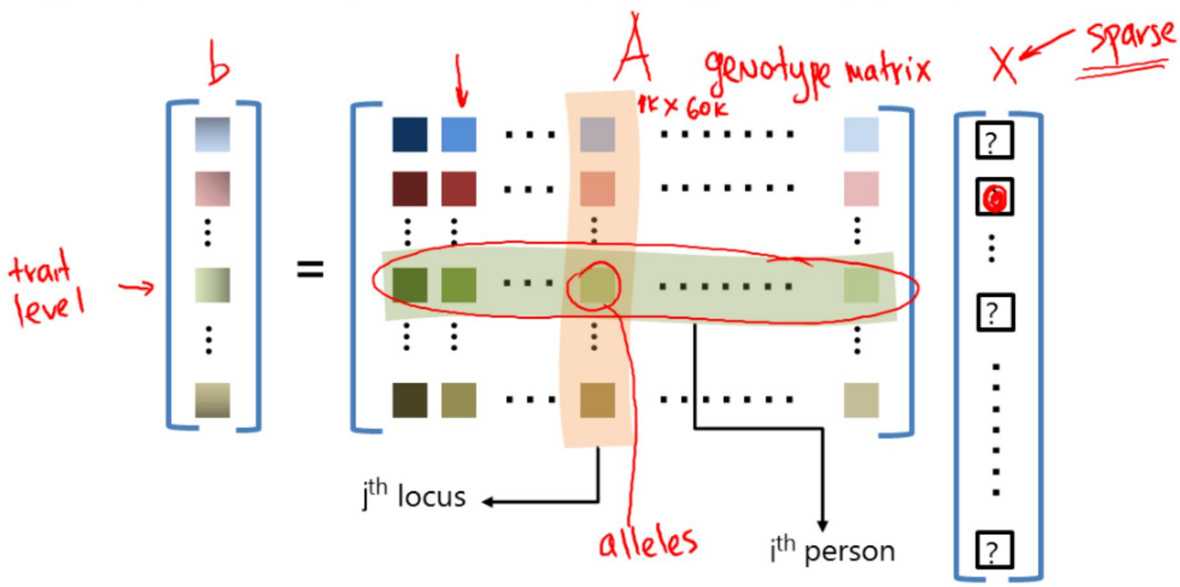
The vector can represent a signal (image), which may be sparse in its native domain (e.g., image of sky at night) or can be made sparse in another domain (e.g., natural images in the DFT domain)

A sparse vector may originate in numerous applications

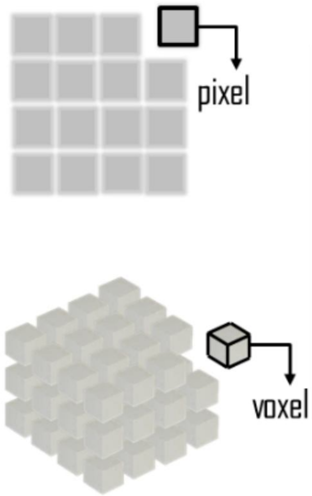
Applications

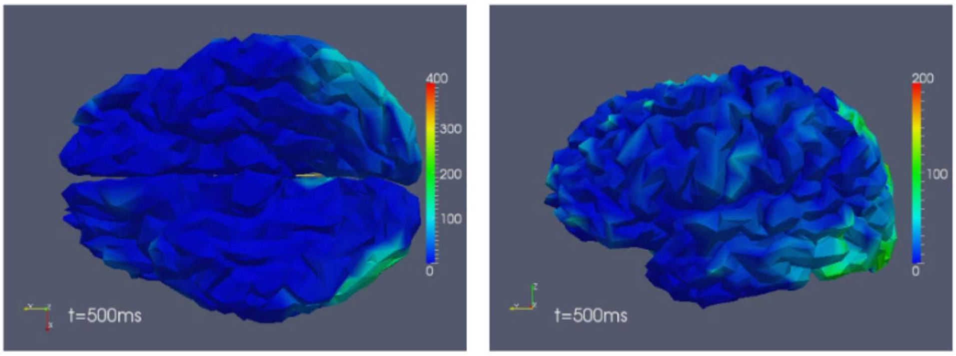
- Image and Video Processing
- Machine Learning
- Statistics
- Genetics
- Econometrics
- Neuroscience
- …

Genetics

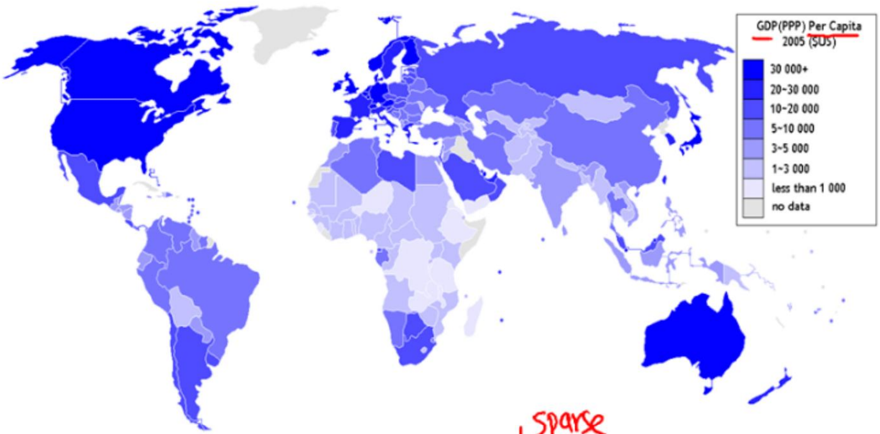


Neuroscience





Econometrics

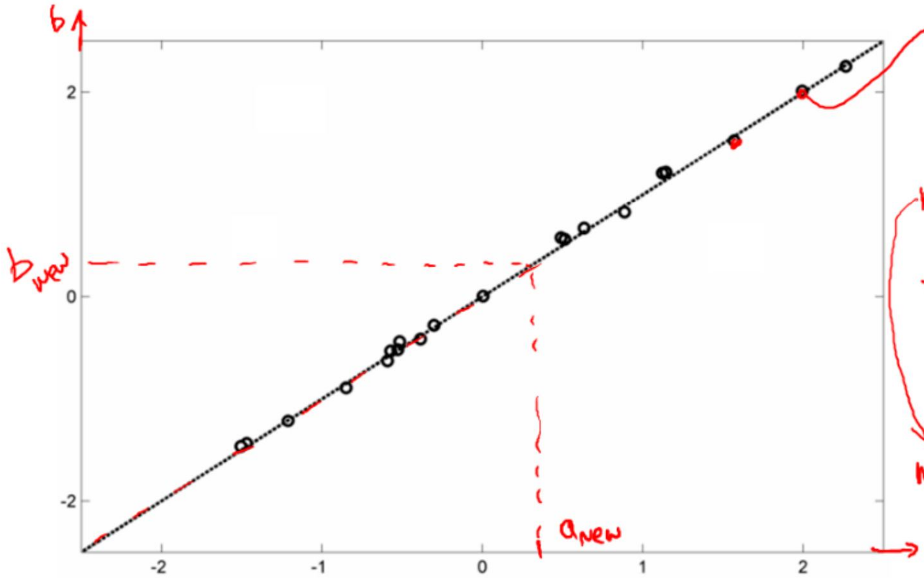


- Population density
- $\boxed{\text{Fraction of tropical area}}$
- Size of economy
- Defense spending
- $\boxed{\text{Life expectancy}}$
- Public investment
- ⋮
- Land area

$$A \overset{\text{sparse}}{x} = b$$

SOURCE: mapsof.net

Robust Regression



(a_i, b_i)

$a_i x \approx b_i$

$$\min_x \sum_i (a_i x - b_i)^2$$

$\to x^*$

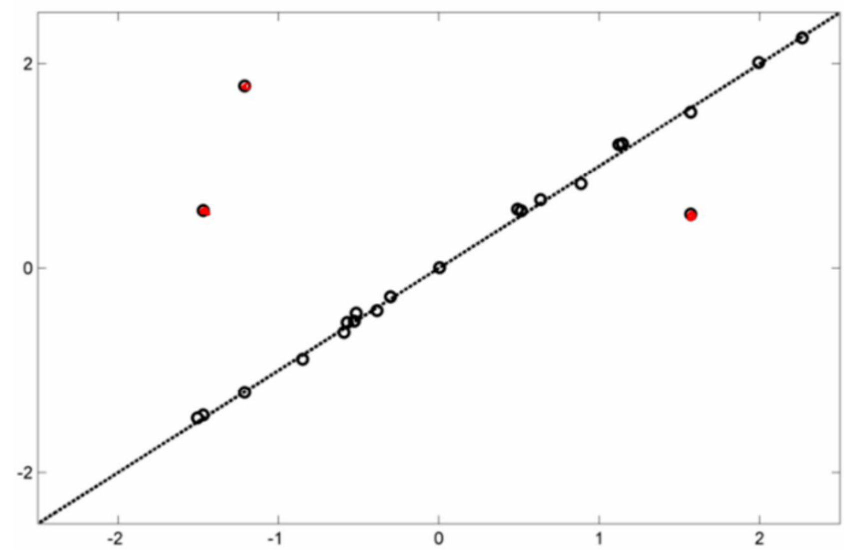
$a_{new} x^* = b_{new}$

$Ax = b$

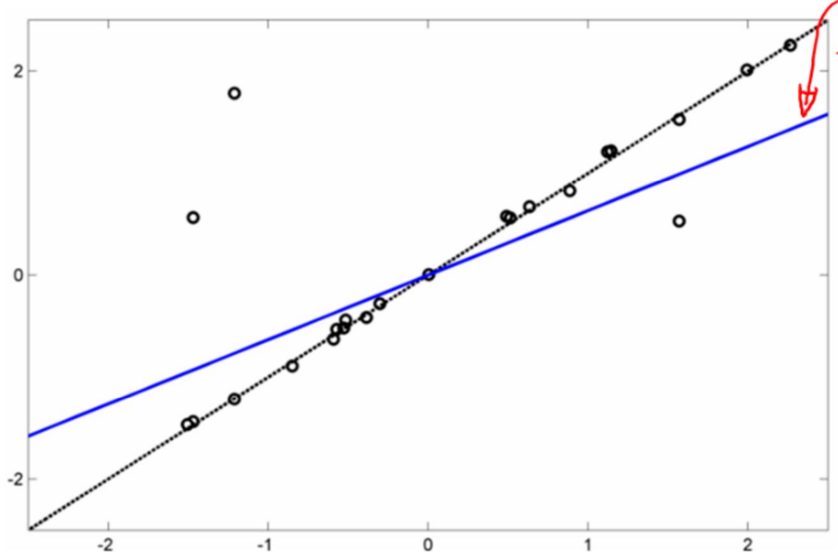
$$\min_x \|Ax - b\|_2^2$$

ℓ_2-norm

Robust Regression



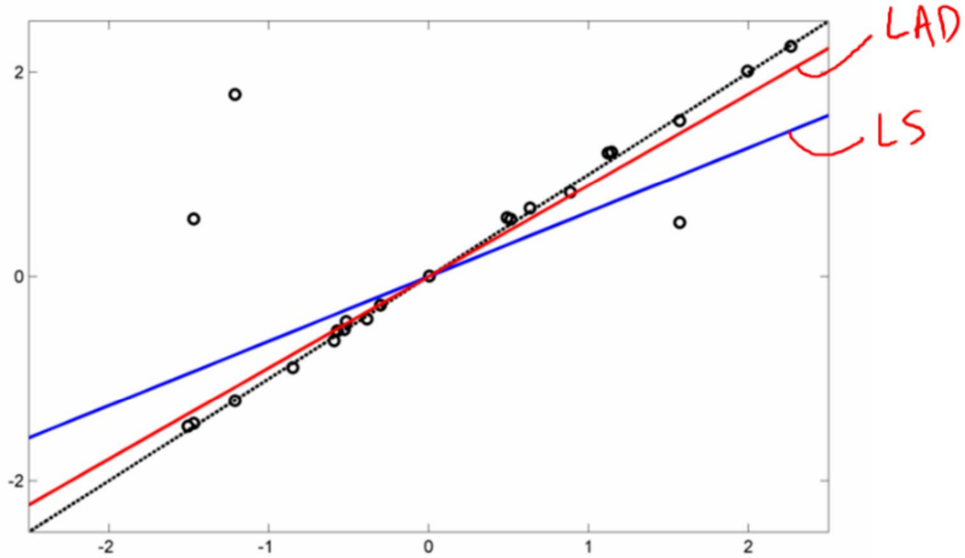
Robust Regression



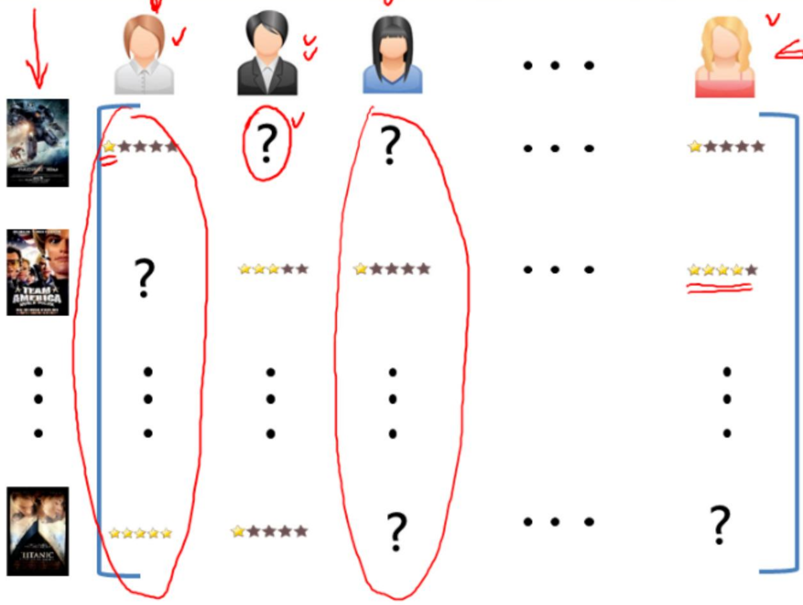
$\overset{x^*}{A}x + \underset{\text{sparse}}{e} \approx b$

Least Absolute Deviations

Robust Regression



Recommender Systems



Matrix Completion Problem

Rank Minimization Problem

Image Denoising

y_i

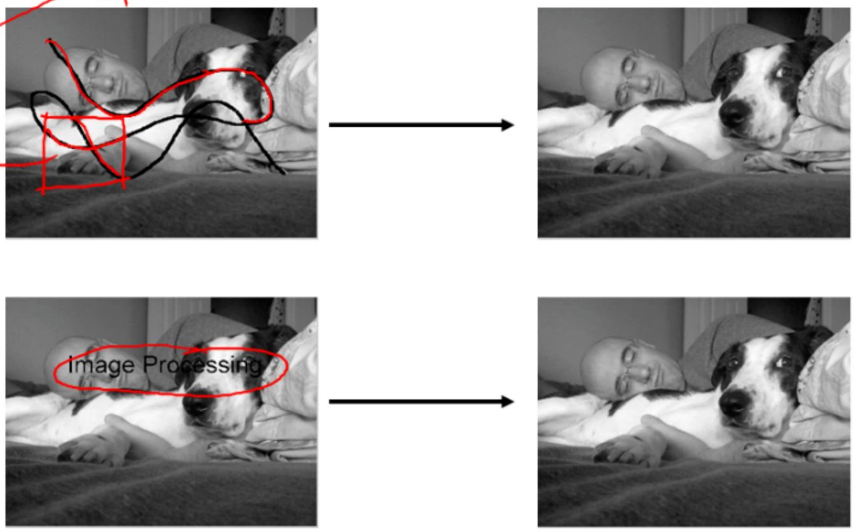
$y_i \cong A x_i$ ← sparse

$\min_{x_i} \|y_i - A x_i\|_2^2 + \lambda \|x_i\|_1$ ← ℓ_1-Norm

Image Inpainting

mask, dictionary, sparse

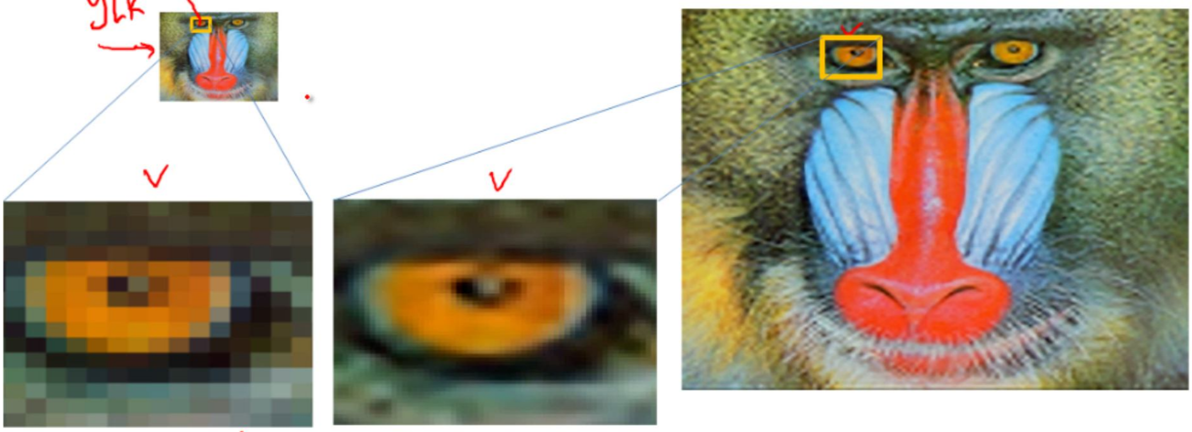
$$y_i \approx RAx_i$$



$$x_i^* = \arg\min_{x_i} \|y_i - RAx_i\|_2^2 + \lambda \|x_i\|_1$$

$$Ax_i^*$$

Image Super-Resolution



x^*_{SPARSE}

$$y_{LR} = A_{LR} x^*_{SPARSE}$$

$$\rightarrow y_{HR} = A_{HR} x^*_{SPARSE}$$

Video Surveillance

Background
Low-rank matrix

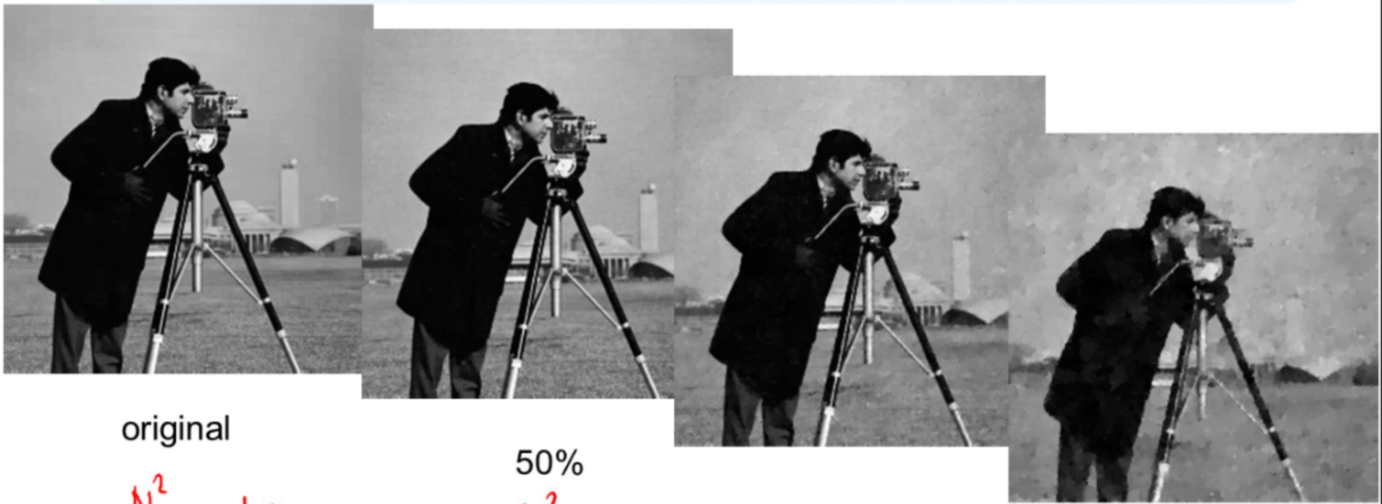
Foreground
Sparse matrix

Robust Face Recognition

$$\underline{\underline{b}} \approx A\,\underline{x} + \underline{\underline{e}}$$

sparse sparse

Compressive Sensing



original

N^2 samples

50%

$N^2/2$

25%

$N^2/4$

$N^2/10$

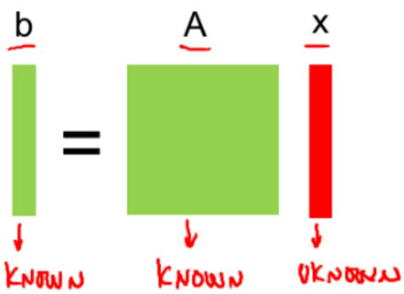
10%

Linear Inverse Problems

- Fully-determined system of equations
  \# equations = # unknowns

- Unique solution (if A is full rank):

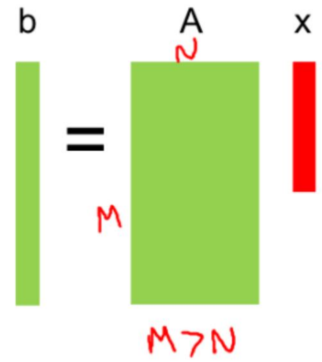
$$\mathbf{x}^* = A^{-1}\mathbf{b}$$



Linear Inverse Problems

- Over-determined system of equations
  # equations > # unknowns
- The Least Squares solution is given by

$$\mathbf{x}^* = \underbrace{(A^T A)}_{N \times N}^{-1} A^T \mathbf{b}$$

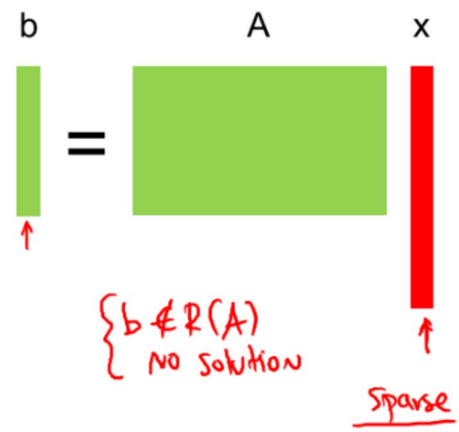


$$\min_x \|Ax-b\|_2^2 \;;\quad \nabla_x \|Ax-b\|_2^2 = 0 \rightarrow \nabla_x \{(Ax-b)^T(Ax-b)\} = 0$$

$$\rightarrow \nabla_x \{x^T A^T A x - 2x^T A^T b + b^T b\} = 0 \rightarrow 2A^T A x - 2A^T b = 0$$

Linear Inverse Problems

- under-determined system of equations
  # equations < # unknowns
- Infinitely many solutions (usually!)
- How to pick x? it depends on the application.
- Regularization



$$\min_{\mathbf{x}} J(\mathbf{x}) \;\text{ subject to }\; \mathbf{b} = \mathbf{A}\mathbf{x}$$

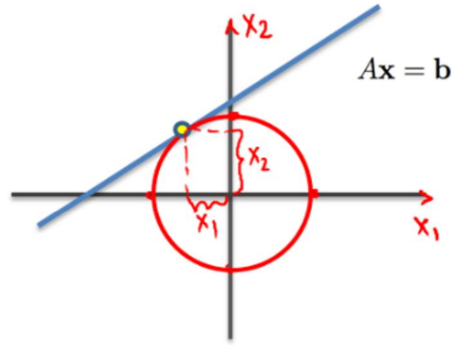
Minimum l_2 Norm Solution

- We want x to be 'small' (in the l_2 sense)

$$\|\mathbf{x}\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}$$

- The problem to solve is

$$\min_{\mathbf{x}} \|\mathbf{x}\|_2$$
$$subject\,to\; A\mathbf{x} = \mathbf{b}$$



- The closed form solution is given by

$$\mathbf{x}^* = A^T \left(AA^T\right)^{-1} \mathbf{b}$$

$x = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$ $\|x\|_2 = \sqrt{9+16}$ $= \sqrt{25} = 5$

$\|x\|_2^2 = 1 \Rightarrow x_1^2 + x_2^2 = 1$

Minimum l_2 Norm Solution

Derivation of closed form solution

$$\boxed{\begin{array}{l} \min_x \|x\|_2 \\ \text{s.t. } Ax-b \end{array}} \quad \min_x \Big(\underbrace{\|x\|_2 + \lambda^T (Ax-b)}_{L(x)} \Big) = \min_x L(x)$$

$\lambda^T A x = x^T A^T \lambda$

KKT

$\nabla_x L(x) = 0 \rightarrow x + A^T\lambda = 0 \Rightarrow x = -A^T\lambda \Rightarrow \boxed{x^* = A^T(AA^T)^{-1}b}$

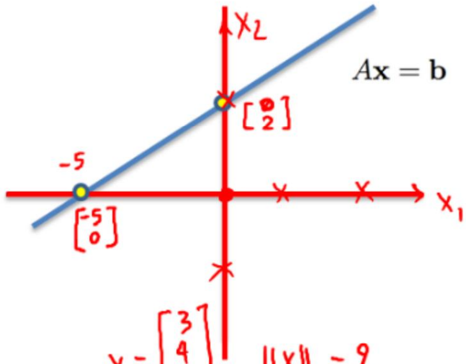
$\nabla_\lambda L(x) = 0 \rightarrow Ax - b = 0 \rightarrow -AA^T\lambda = b \Rightarrow \lambda = -(AA^T)^{-1}b$

Minimum l_0 Norm Solution

- We want x to be 'sparse' $\longrightarrow$ it should have few non-zero entries.
- Sparsity can be modeled via the l_0 norm*

$\|\alpha x\|_0 \neq \alpha \|x\|_0$

$$\|\mathbf{x}\|_0 = \# \, non-zero \; entries \; in \, \mathbf{x}$$



$x = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}, \; \|x\|_0 = 2$

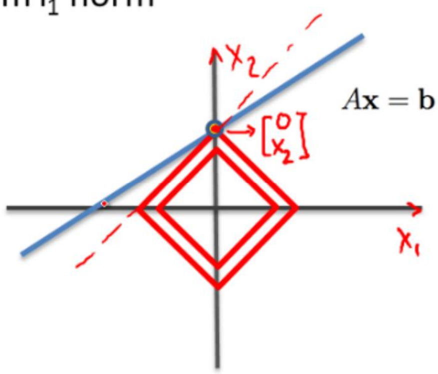
- The problem to solve is now

$$\boxed{\begin{array}{c} \min_{\mathbf{x}} \|\mathbf{x}\|_0 \\ subject\,to\; A\mathbf{x} = \mathbf{b} \end{array}}$$

Minimum l_1 Norm Solution

- Another special solution is the one with minimum l_1 norm

$$\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i|$$



- The problem to solve is now

$x = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$ $\|x\|_1 = 3+4=7$

$\|x\|_2 = 5$

$\|x\|_0 = 2$

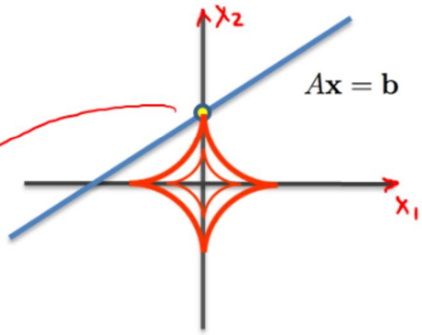
$$\boxed{\begin{array}{c} \min_{\mathbf{x}} \|\mathbf{x}\|_1 \\ subject\,to\; A\mathbf{x} = \mathbf{b} \end{array}}$$

Basis pursuit

Minimum l_p Norm Solution

- Another class of special solutions minimizes the l_p norm $(0<p<1)$

$$\rightarrow \quad \|\mathbf{x}\|_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}$$

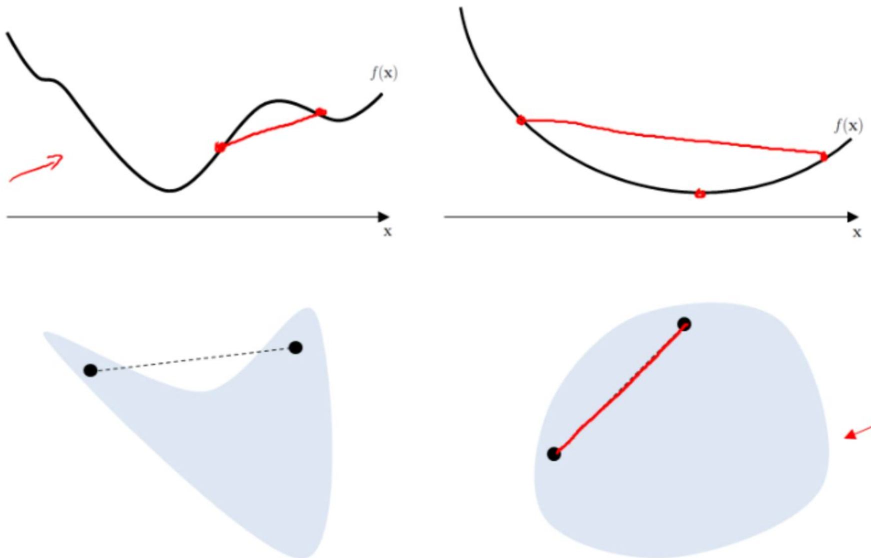


- The problem to solve becomes

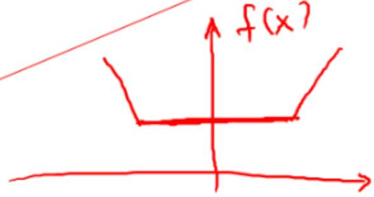
$$\boxed{\begin{array}{c} \min_{\mathbf{x}} \|\mathbf{x}\|_p \\ \text{subject to } A\mathbf{x} = \mathbf{b} \end{array}}$$

$$X = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} \quad \|X\|_{0.5} = \left(\sqrt{3} + \sqrt{4}\right)^2$$

On Convexity

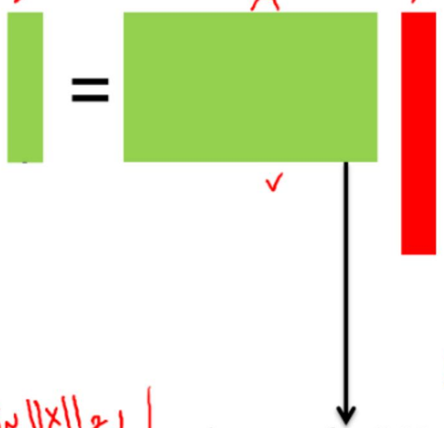


$$\min_{\mathbf{x}} f(\mathbf{x})$$
$$\text{subject to } \mathbf{x} \in \mathcal{S}$$

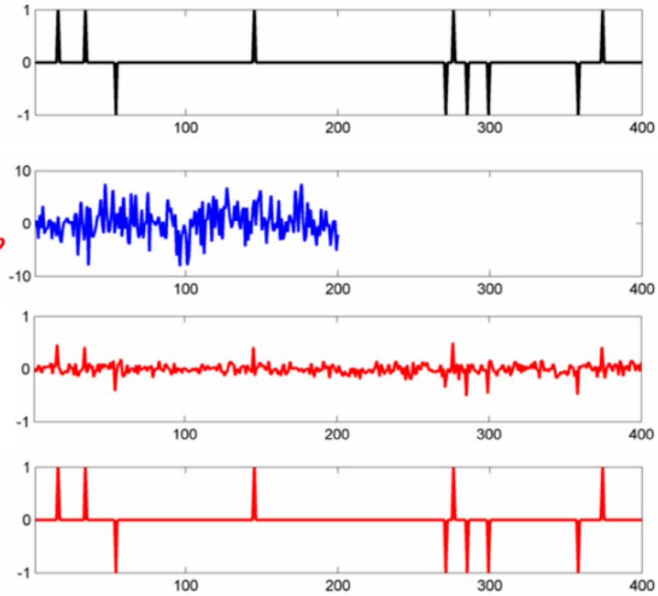


l_2 norm vs. l_1 norm

b A x



x



$\ell_2: x^*$

$\ell_1: x^*$

$\min \|x\|_2$
s.t. $Ax=b$

$A = \text{randn}\,(200, 400)$

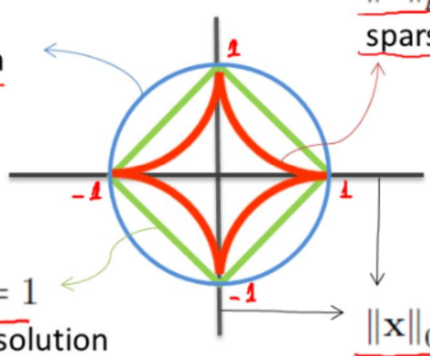
All Norm Balls in One Picture

convex
closed form
Solution

$\|\mathbf{x}\|_2 = 1$
small solution

$\|\mathbf{x}\|_p = 1$ $0<p<1$
sparse solution

NON-convex



convex

V

$\|\mathbf{x}\|_1 = 1$
sparse solution

$\|\mathbf{x}\|_0 = 1$
sparse solution

NP-hard

l_0 norm vs. l_1 norm

$$\boxed{\begin{aligned} &\min_{\mathbf{x}} ||\mathbf{x}||_0 \\ &subject\,to\; A\mathbf{x} = \mathbf{b} \end{aligned}}$$

- Models sparsity directly
- Non-convex
- NP-hard
- Greedy approaches (Matching Pursuit) approximate the solution

$$\boxed{\begin{aligned} &\min_{\mathbf{x}} ||\mathbf{x}||_1 \\ &subject\,to\; A\mathbf{x} = \mathbf{b} \end{aligned}}$$

- Models sparsity indirectly
- Convex
- Non-smooth
- Can be solved via convex optimization algorithms

Reformulation

$$\min_{\mathbf{x}} ||\mathbf{x}||_0$$
$$subject\,to\; A\mathbf{x} = \mathbf{b}$$

Noise in observation

$$\min_{\mathbf{x}} ||\mathbf{x}||_0 \quad ||Cx||_2$$
$$subject\,to\; ||A\mathbf{x} - \mathbf{b}||_2 \leq \epsilon$$

$x(\epsilon)$

Swapping constraint & objective

$$\min_{\mathbf{x}} ||A\mathbf{x} - \mathbf{b}||_2$$
$$subject\,to\; ||\mathbf{x}||_0 \leq S$$

$x(S)$

Reformulation

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to } A\mathbf{x} = \mathbf{b}$$

Noise in observation

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to } \|A\mathbf{x} - \mathbf{b}\|_2 \le \epsilon$$

$$\min_{x} \{ \|Ax-b\|_2^2 + \lambda \|Cx\|_2^2 \}$$

$$x^* = (A^TA + \lambda C^TC)^{-1} A^Tb$$

Swapping constraint & objective

$$\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|_2 \quad \text{subject to } \|\mathbf{x}\|_1 \le S$$

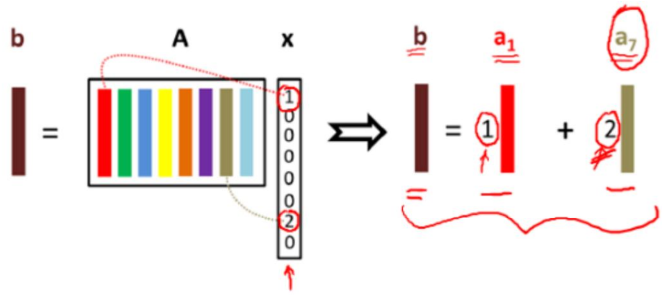
Bringing constraint to the objective

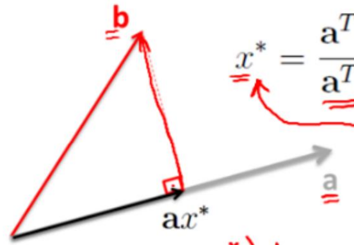
$$\underbrace{\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|_2 + \lambda \|\mathbf{x}\|_1}_{\|x(\lambda)\|_1 \le S}$$

LASSO

Matching Pursuit

$$\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|_2 \quad \text{subject to } \|\mathbf{x}\|_0 \le S$$





$$x^* = \frac{\mathbf{a}^T\mathbf{b}}{\mathbf{a}^T\mathbf{a}} = \mathbf{a}^T\mathbf{b}$$

$\|a\| = 1$

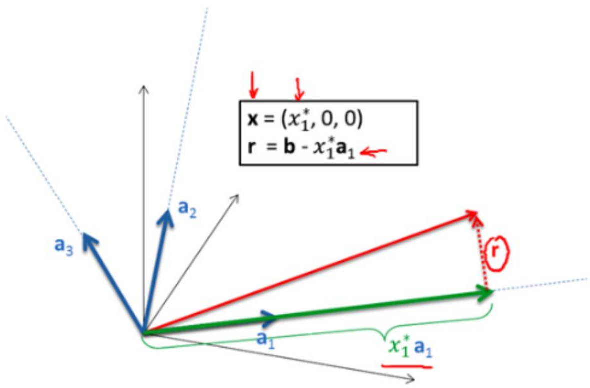
$$\|\mathbf{a}x^*\|_2 = |x^*| \underbrace{\|\mathbf{a}\|_2}_{=1} = |x^*|$$

$$i = \underset{k}{argmax} |x_k^*|$$

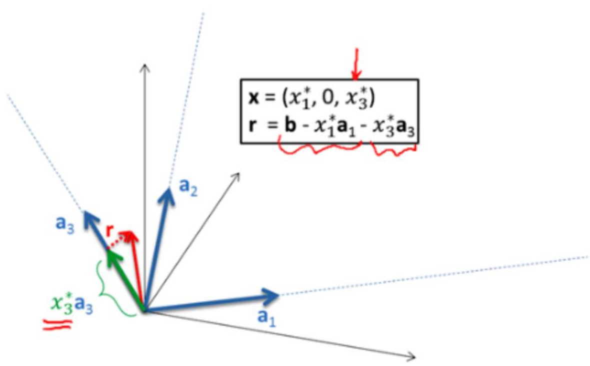
best column

$(b - ax^*) \perp a \Rightarrow (b - ax^*)^T a = 0$

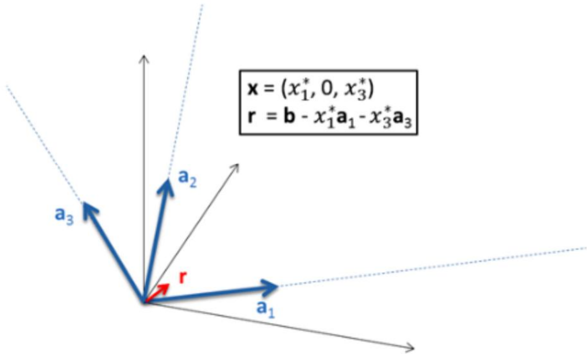
Matching Pursuit



Matching Pursuit



Matching Pursuit



Orthogonal Matching Pursuit

$$\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|_2$$
$$subject\,to\;\; \|\mathbf{x}\|_0 \leq S$$

Input: A (with unit norm columns), $\mathbf{b}$, and S.
Initialize $\mathbf{r} = \mathbf{b}$ and $\Omega = \emptyset$.
While $\|\mathbf{x}\|_0 < S$
compute $x_j = \mathbf{a}_j^T\mathbf{r}$ for all $j \notin \Omega$
$i = \underset{j \notin \Omega}{argmax}\, |x_j|$
$\Omega \longleftarrow \Omega \cup \{i\}$
$\boxed{\mathbf{x}_\Omega^* = \underset{\mathbf{x}}{argmin} \|A_\Omega \mathbf{x} - \mathbf{b}\|_2^2}$
$\mathbf{r} \longleftarrow \mathbf{b} - A_\Omega \mathbf{x}_\Omega^*$

Orthogonal Matching Pursuit

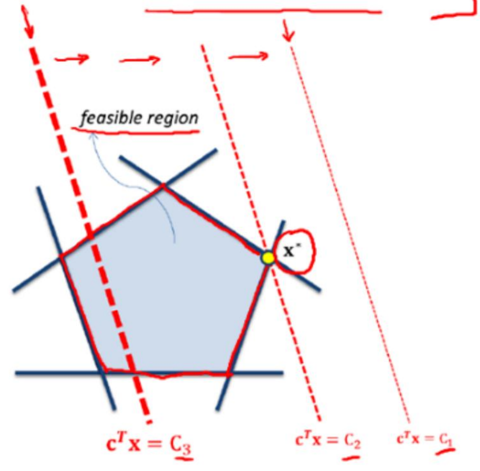
$$\boxed{\begin{array}{c} \min_{\mathbf{x}} \|\mathbf{x}\|_0 \\ subject\,to\;\|A\mathbf{x}-\mathbf{b}\|_2 \le \epsilon \end{array}}$$

Input: A (with unit norm columns), $\mathbf{b}$, and ϵ.
Initialize $\mathbf{r} = \mathbf{b}$ and $\Omega = \emptyset$.
While $\|\mathbf{r}\|_2^2 > \epsilon$
compute $x_j = \mathbf{a}_j^T\mathbf{r}$ for all $j \notin \Omega$
$i = \underset{j \notin \Omega}{argmax}\, |x_j|$
$\Omega \longleftarrow \Omega \cup \{i\}$
$\boxed{\mathbf{x}_\Omega^* = \underset{\mathbf{x}}{argmin} \|A_\Omega \mathbf{x} - \mathbf{b}\|_2^2}$
$\mathbf{r} \longleftarrow \mathbf{b} - A_\Omega \mathbf{x}_\Omega^*$

Linear Programs

$$\begin{array}{c} \min_{\mathbf{x}} \mathbf{c}^T\mathbf{x} \\ subject\ to\ F_i\mathbf{x} + \mathbf{g}_i \le \mathbf{0}\ \forall i \end{array}$$

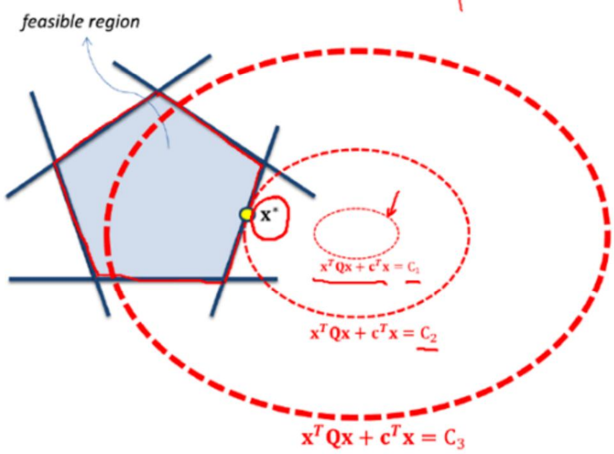
LINprog



Quadratic Programs

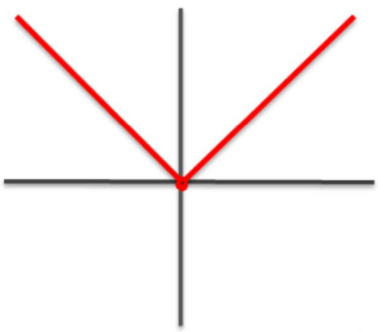
$$\begin{cases} \min_{\mathbf{x}} \ \mathbf{x}^T Q \mathbf{x} + \mathbf{c}^T \mathbf{x} \\ subject\ to\ F_i \mathbf{x} + \mathbf{g}_i \leq \mathbf{0}\ \forall i \end{cases}$$

quadprog



Smooth Reformulation Tricks

The L_1 norm is non-differentiable at the origin



We introduce two reformulation tricks that transform sparse optimization problems into well-studied Linear and Quadratic programs

Positive-Negative Split Trick

$$p_i = \begin{cases} x_i & if\ x_i > 0 \\ 0 & else \end{cases} \qquad n_i = \begin{cases} -x_i & if\ x_i < 0 \\ 0 & else \end{cases}$$

$$x_i = p_i - n_i$$

$x = \begin{bmatrix} 2 \\ -3 \\ 0 \\ 5 \end{bmatrix}$ $\quad p_1 = 2$, $n_2 = 3$, $p_4 = 5$

$x = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \\ 0 \\ 0 \end{bmatrix}$

$x = p - n$

$$\|\mathbf{x}\|_1 = \underline{\mathbf{1}}^T(\mathbf{p} + \mathbf{n}) = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 2 \\ 0 \\ 0 \\ 5 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} \right)$$

$p^T n = 0$

Positive-Negative Split Trick

$$\left.\begin{array}{c} \min\limits_{\mathbf{x}} \|\mathbf{x}\|_1 \\ subject\ to\ A\mathbf{x} = \mathbf{b} \end{array}\right] \rightarrow LP$$

$x = p - n$

$\min\limits_{p,n} 1^T(p+n)$

s.t. $A(p-n) = b$

$p, n \geq 0$

$\boxed{\cancel{p^T n = 0}}$

$z = \begin{bmatrix} p \\ n \end{bmatrix}$

$$\boxed{\begin{array}{l} \min\limits_{z} 1^T z \leftarrow \\ \text{s.t. } Cz = b \\ \quad z \geq 0 \end{array}}$$

$C = AF$

$F = \begin{bmatrix} I_{N \times N} & \vdots & -I_{NN} \end{bmatrix}$

$N \times 2N$

Positive-Negative Split Trick

$\underline{x} = p - n$

$$\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|_2^2 + \lambda\|\mathbf{x}\|_1$$

LASSO → QP

$$\min_{p,n} \|A(p-n) - b\|_2^2 + \lambda 1^T p + \lambda 1^T n$$

$p, n \geq 0$

$\boxed{\cancel{p^T n = 0}}$

$z = \begin{bmatrix} p \\ n \end{bmatrix}$

$$\Rightarrow \boxed{\begin{array}{c} \min_{z} z^T B z + c^T z \\ z \geq 0 \end{array}}$$

$B = \begin{bmatrix} A^T A & -A^T A \\ -A^T A & A^T A \end{bmatrix}$

$c = \lambda 1 + 2\begin{bmatrix} -A^T b \\ A^T b \end{bmatrix}$

Suppression Trick

$s, \; s_k$

$|x_k| \leq s_k$

$$\begin{cases} \min_{\mathbf{x}} \|\mathbf{x}\|_1 \\ subject\, to\; A\mathbf{x} = \mathbf{b} \end{cases}$$

$$\left\{\begin{array}{l} \min_{x,s} 1^T s \\ \text{s.t. } Ax = b \\ \quad |x_k| \leq s_k, \; \forall k \\ \quad s \geq 0 \end{array}\right\} \rightarrow \left\{\begin{array}{l} \min_{x,s} \underline{1^T s} \\ \text{s.t. } Ax = b \\ \quad x_k \leq s_k, \; \forall k \\ \quad x_k \geq -s_k, \; \forall k \\ \quad s \geq 0 \end{array}\right\}$$

$\underline{\underline{LP}}$

Advanced Methods

- Stagewise OMP (StOMP), compressive sampling matching pursuit (CoSaMP).
- FISTA (Fast Iterative Shrinkage Algorithm)
- ADMM (Alternating Direction Method of Multipliers)

$$\min_{x,y} f(x) + g(y)$$
$$\text{subject to } Ax + By = c \ .$$

Dictionary Learning

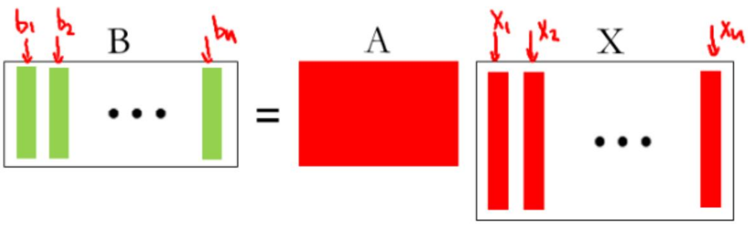
$$\min_{\mathbf{x}} \|A\mathbf{x}_1 - \mathbf{b}_1\|_2^2 \quad subject\,to\ \|\mathbf{x}_1\|_0 \le s$$

$$\vdots$$

$$\min_{\mathbf{x}} \|A\mathbf{x}_n - \mathbf{b}_n\|_2^2 \quad subject\,to\ \|\mathbf{x}_n\|_0 \le s$$

$$\min_{X} \|AX - B\|_F^2 \quad subject\,to\ \|\mathbf{x}_i\|_0 \le s\ 1 \le i \le n$$

$$\|A\|_F = \sqrt{\sum_i \sum_j |a_{ij}|^2}$$



What if we can choose A too?

$$\min_{A,X} \|AX - B\|_F^2 \quad subject\,to\ \|\mathbf{x}_i\|_0 \le s\ 1 \le i \le n$$

Method of Optimal Directions

(MOD)

$$\begin{cases} \min\limits_{A,X} \|AX - B\|_F^2 \\ subject\,to\ \|\mathbf{x}_i\|_0 \le S\ \forall i \end{cases}$$

Alternating minimization

Keep A fixed; solve for X

$$\begin{cases} \min\limits_{\mathbf{x}_i} \|A\mathbf{x}_i - \mathbf{b}_i\|_2^2 \\ subject\,to\ \|\mathbf{x}_i\|_0 \le S \end{cases}$$

Keep X fixed; solve for A

$$\min_A \|AX - B\|_F^2$$

$$\boxed{A = BX^T(XX^T)^{-1}}$$

Bi-convex Dictionary Learning

$$\min_{A,X} \|AX - B\|_F^2 + \lambda \|X\|_1$$

Alternating minimization

Keep A fixed; solve for X

$$\min_X \|AX - B\|_F^2 + \lambda \|X\|_1$$

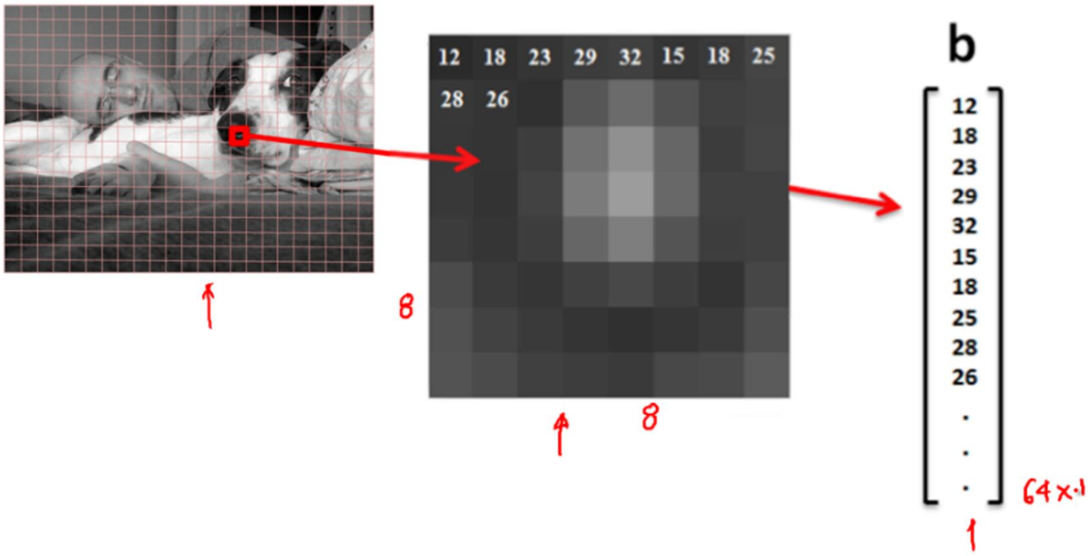
A series of LASSO problems

Keep A fixed; solve for X

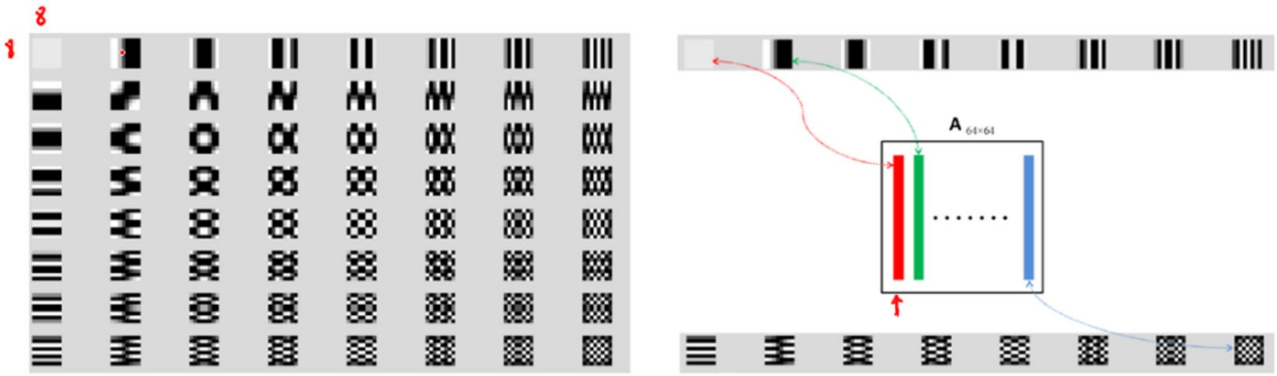
$$\min_A \|AX - B\|_F^2$$

Least Squares: $A = BX^T(XX^T)^{-1}$

Forming b



Forming A: DCT Dictionary



Experiment

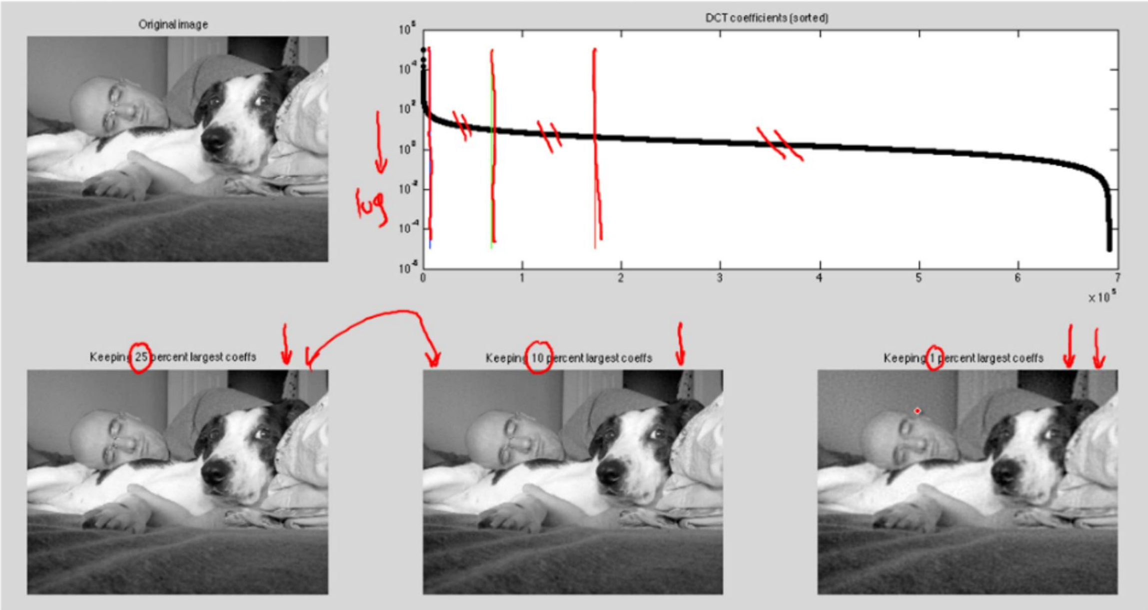


Image Denoising

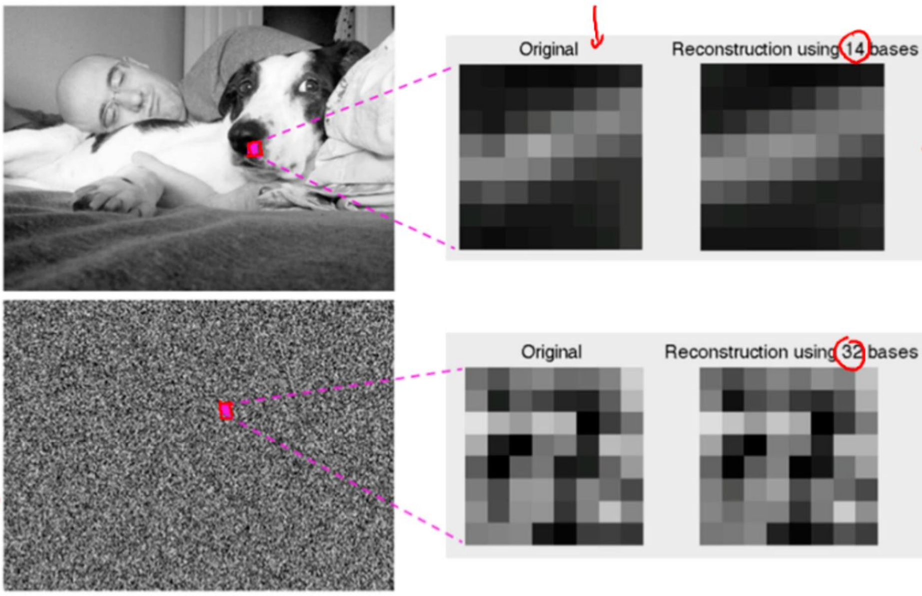


Image Denoising

$$\min_{A,X} \|AX - B\|_F^2 + \lambda \|X\|_1$$

Dictionary

Input noisy image

$AX^* \rightarrow$ Recovered image

Image Denoising

PSNR = 22.1 dB

PSNR = 33.4 dB

Image Inpainting

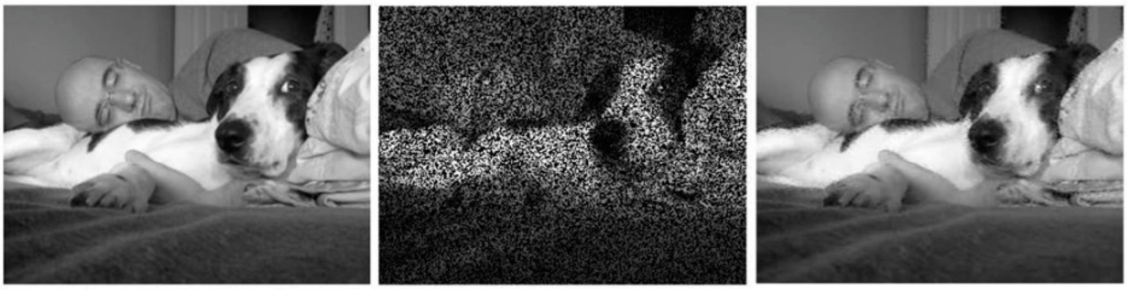
$$\min_X \|RAX - B\|_F^2 + \lambda \|X\|_1$$

Degradation matrix

Input image w\ missing pixels

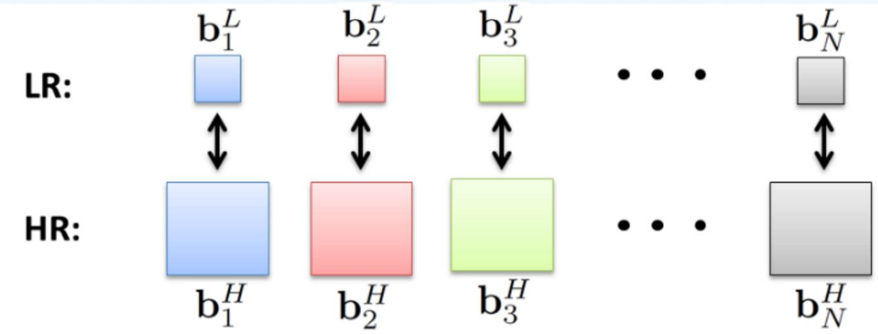
$AX^* \longrightarrow$ Recovered image

Image Inpainting



50%

Image Super-Resolution



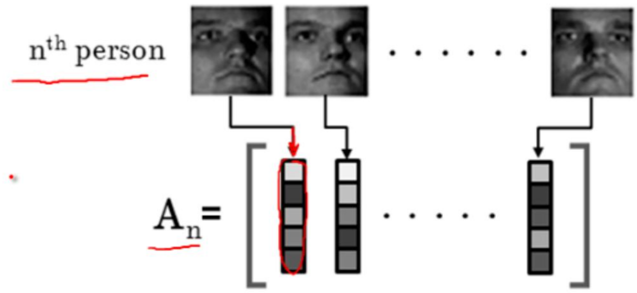
Training phase: $\min_{A^L, A^H, X} \|A^L X - B^L\|_F^2 + \mu \|A^H X - B^H\|_F^2 + \lambda \|X\|_1$

Reconstruction phase: $X^* = \underset{X}{argmin} \|A^L X - B^{new}\|_F^2 + \lambda \|X\|_1 \longrightarrow A^H X^*$

Super-resolved image

Image Super-Resolution

Robust Face Recognition



$$A = \begin{bmatrix} A_1 & A_2 & \cdots & A_n & \cdots & A_N \end{bmatrix}$$

Robust Face Recognition

$$\underline{\mathbf{b}} = A\mathbf{x} + \mathbf{e}$$

sparse sparse

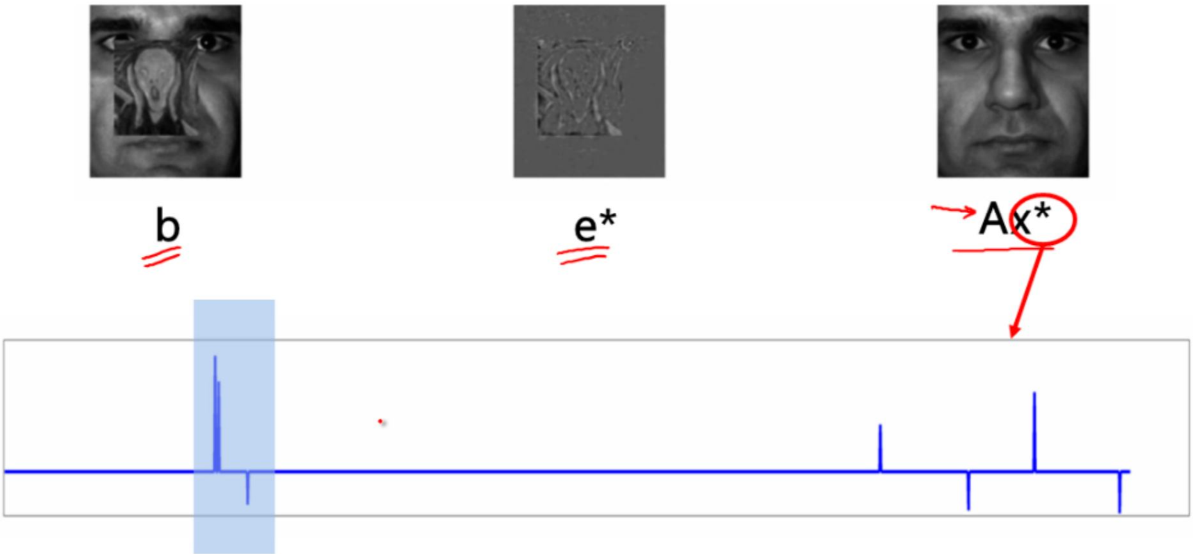
$$\begin{array}{l} \min\limits_{\mathbf{x},\mathbf{e}} \; \|\mathbf{x}\|_1 + \lambda\|\mathbf{e}\|_1 \\ \text{subject to } A\mathbf{x} + \mathbf{e} = \mathbf{b} \end{array}$$

$$\mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \lambda\mathbf{e} \end{bmatrix} \qquad F = [A \;\; \tfrac{1}{\lambda}I]$$

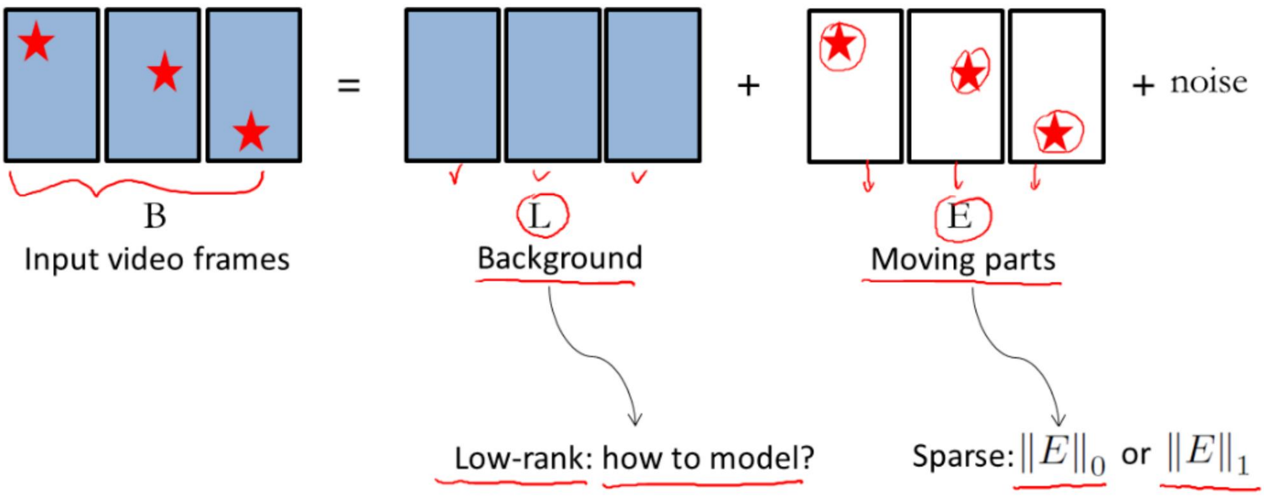
$$\begin{array}{l} \min\limits_{\mathbf{z}} \; \|\mathbf{z}\|_1 \\ \text{subject to } F\mathbf{z} = \mathbf{b} \end{array}$$

Basis pursuit

Robust Face Recognition



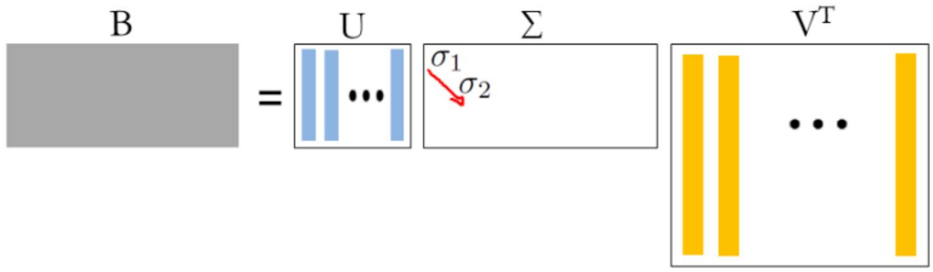
Video Surveillance



Singular Value Decomposition

$$\min_{L} \underbrace{\|B - L\|_F}$$
$$subject\ to\ rank\,(L) \le k$$

$$B = \sum_{i=1}^{r} \mathbf{u}_i \sigma_i \mathbf{v}_i^T \longrightarrow L = \sum_{i=1}^{k} \mathbf{u}_i \sigma_i \mathbf{v}_i^T$$



Video Surveillance

$$\min_{L,E} \|\underbrace{B - L - E}\|_F^2 + \lambda \|E\|_1$$
$$subject\,to\,rank(L) \le k$$

$$L = U\Sigma V^T$$

NXM Nxk kxM

$$L = AX$$

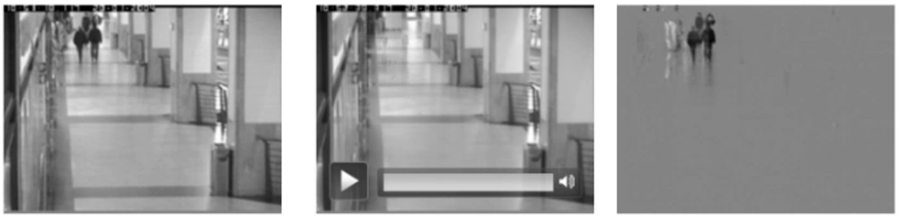
$$\min_{L,E} \|B - L - E\|_F^2 + \lambda \|E\|_1$$
$$subject\,to\ \|\Sigma\|_0 \le k$$

$$\min_{A,X,E} \|\underbrace{B - AX - E}\|_F^2 + \lambda \|E\|_1$$

$$\min_{L,E} \|B - L - E\|_F^2 + \lambda \|E\|_1 + \mu \|L\|_*$$

Nuclear Norm

Video Surveillance



B L E

Sensing by Sampling

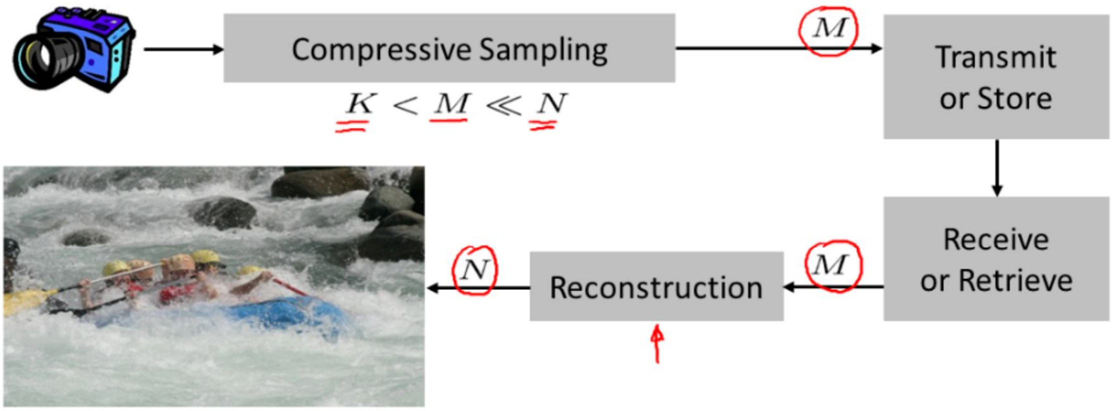
Sampling → N → Compression → K → Transmit or Store

$$\underbrace{\text{Sampling} \rightarrow N \rightarrow \text{Compression}}_{N \gg K}$$

Receive or Retrieve → K → Decompression → N

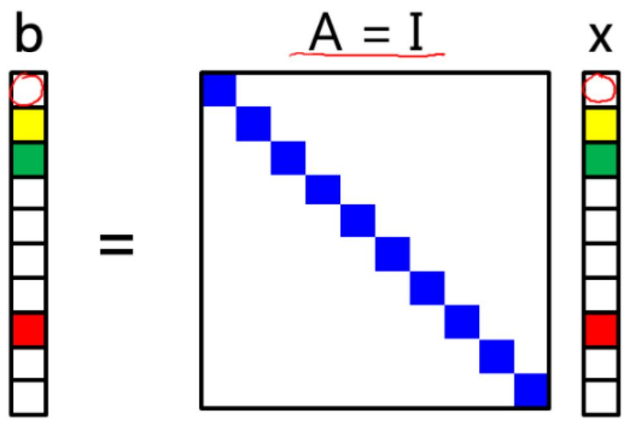
Compressive Sensing

- Directly acquire *"compressed" data*
- Replace samples by more general "measurements"



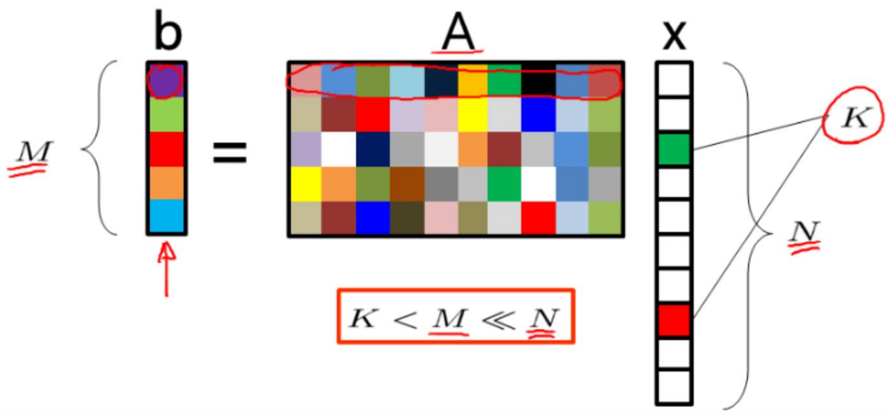
Sampling

- Signal x is K-*sparse* in basis/dictionary A
  - WLOG assume sparse in space domain

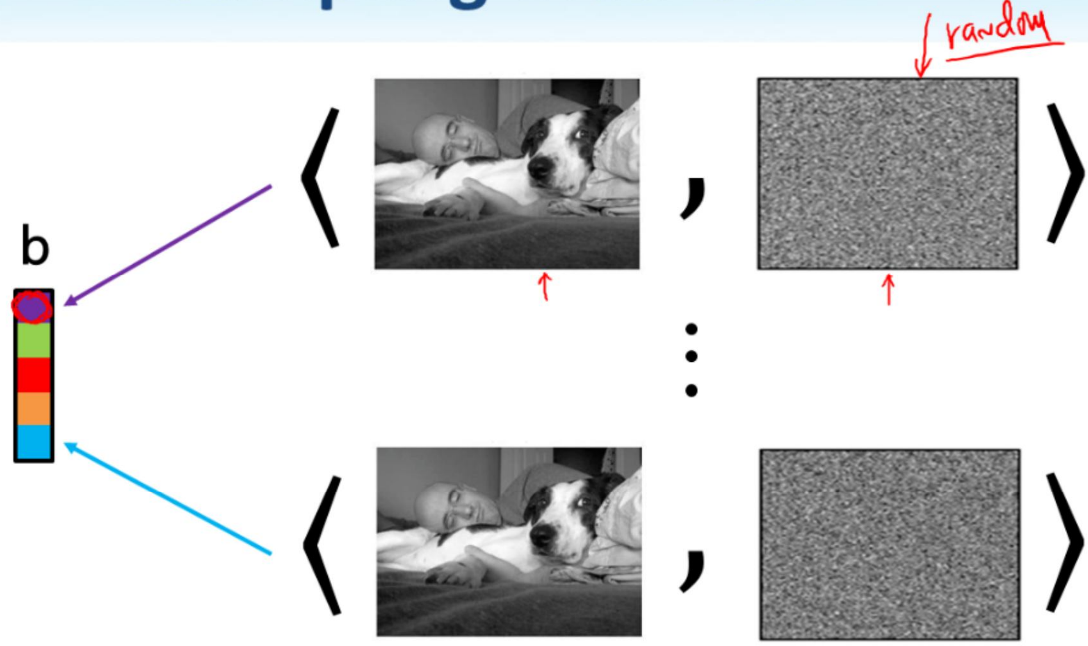


Compressive Data Acquisition

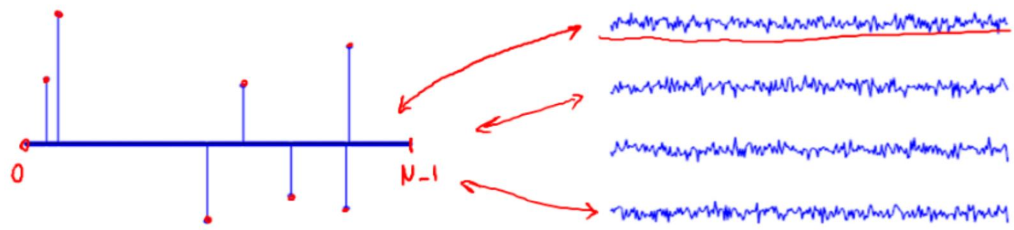
- When data is sparse/compressible, can directly acquire a condensed representation with no/little information loss through dimensionality reduction



Sampling Matrices



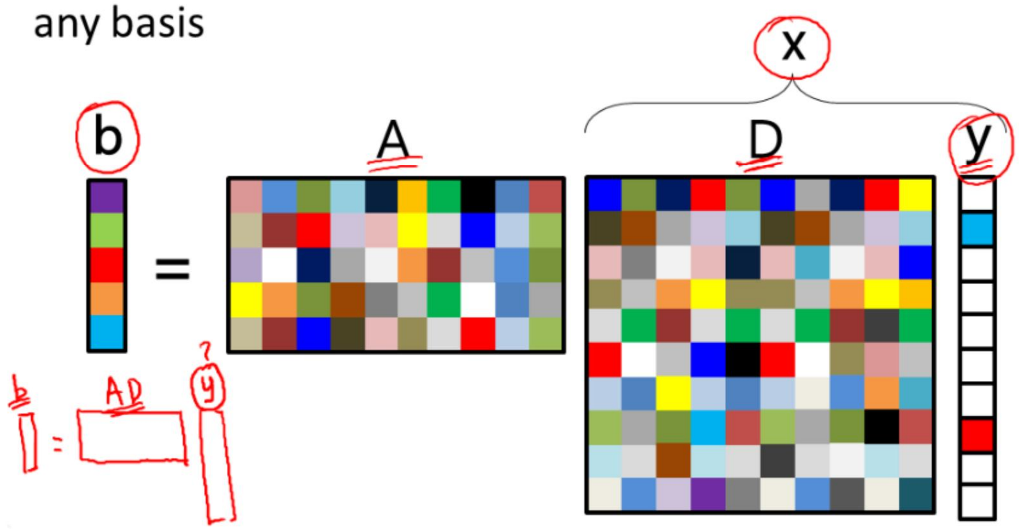
Intuition



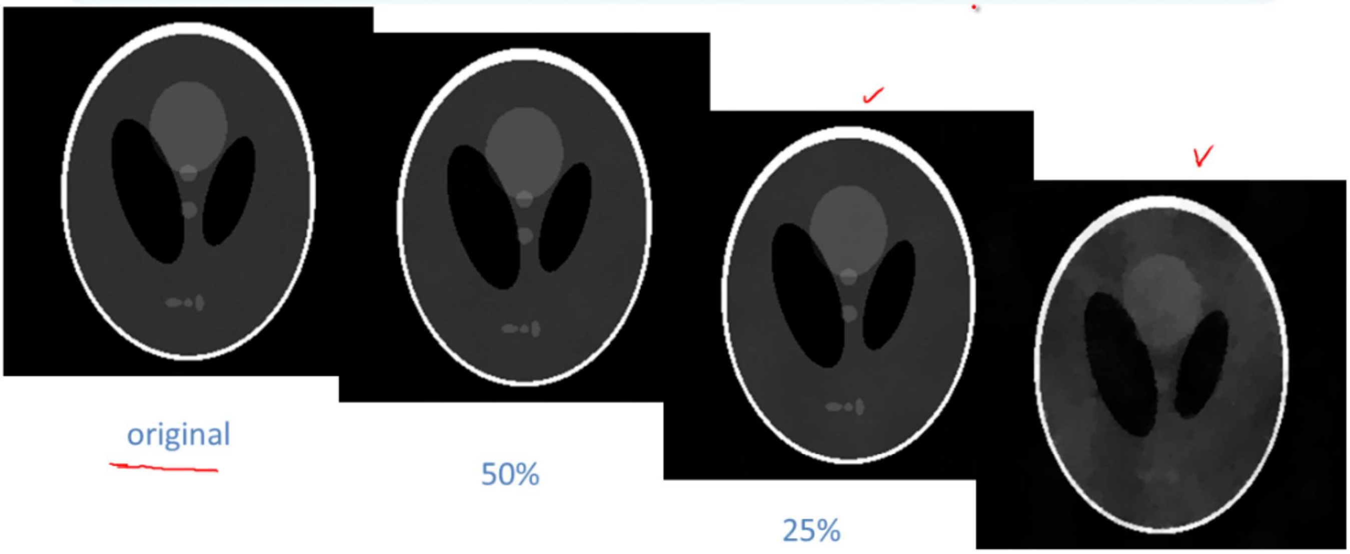
- Signal is local, measurements are global
- Each measurement picks up a little information about each component

Universality

- Random measurements can be used for signals sparse in any basis



Results Compressive Sensing



original

50%

25%

5%

Results