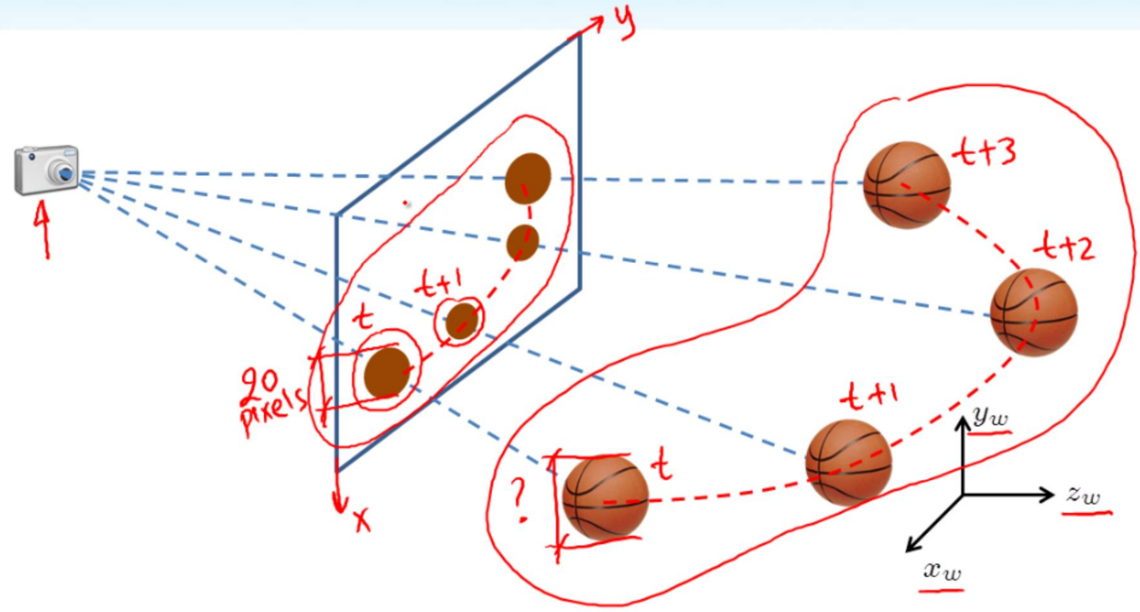
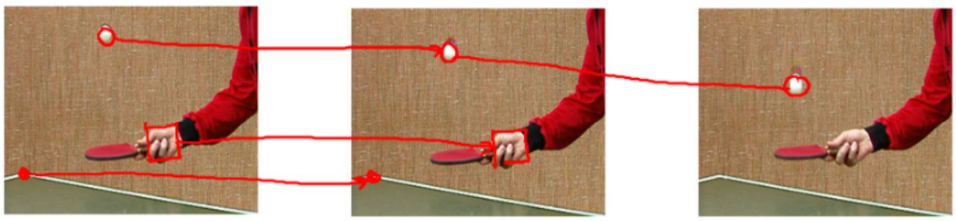
2D vs. 3D Motion



Basic Idea



$x_{k-1}(n_1, n_2)$ $x_k(n_1, n_2)$ $x_{k+1}(n_1, n_2)$

I(x, y, t-1)

"True" Motion vs. Optical Flow

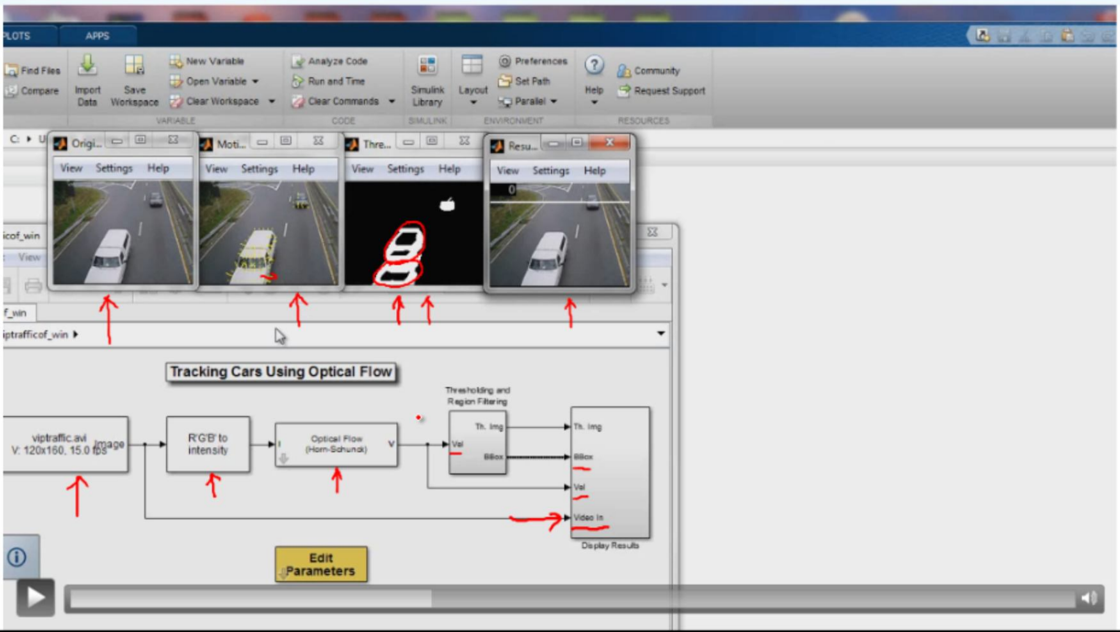
Non-zero true motion
Zero optical flow

Zero true motion
Non-zero optical flow

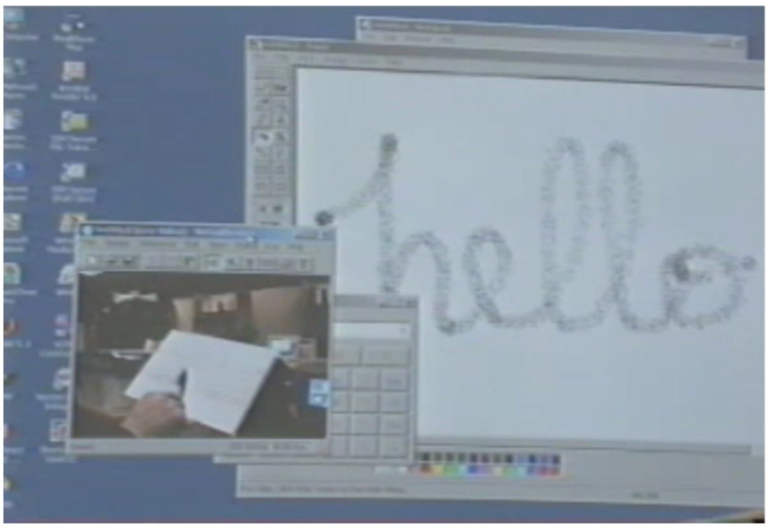
Motion Estimation Applications

- Object Tracking
- Human Computer Interaction (HCI)
- Temporal Interpolation
- Spatio-Temporal Filtering
- Compression

Object Tracking

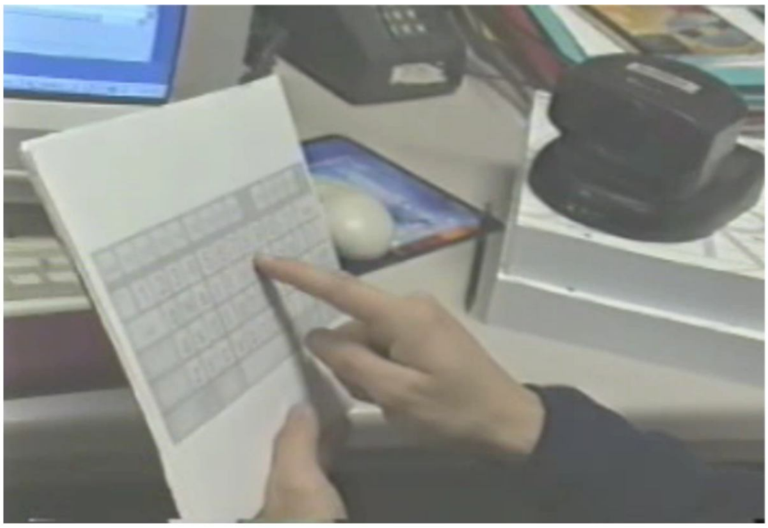


Human Computer Interaction



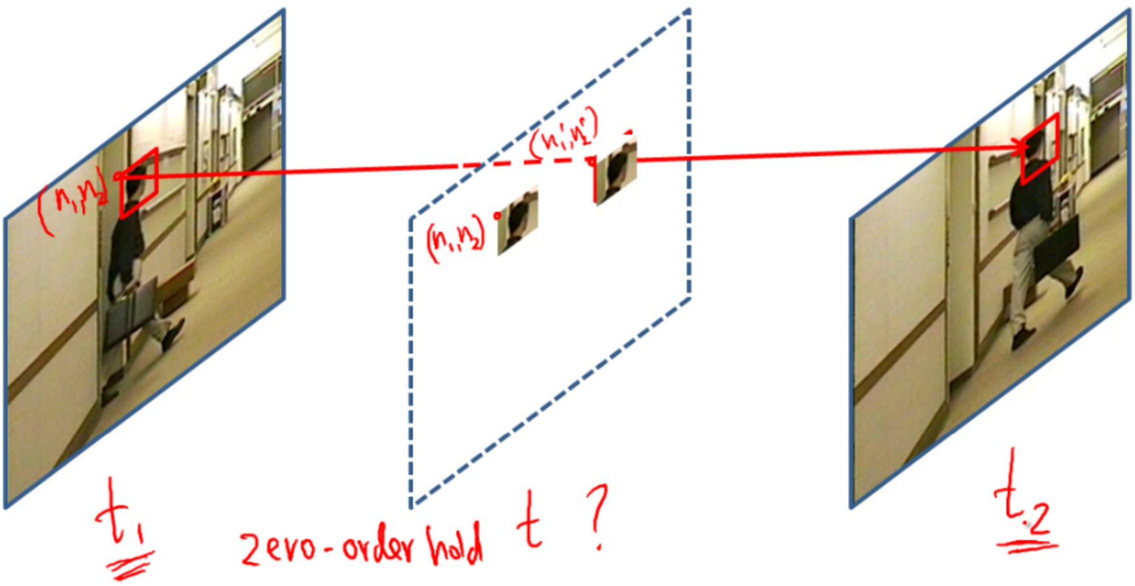
Courtesy of Prof. Ying Wu, Dept of EECS, Northwestern University

Human Computer Interaction

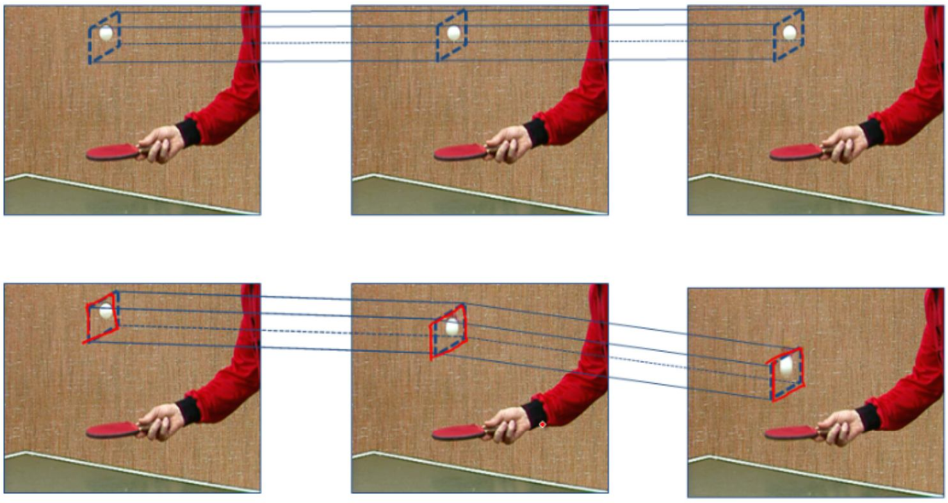


Courtesy of Prof. Ying Wu, Dept of EECS, Northwestern University

MC Temporal Interpolation



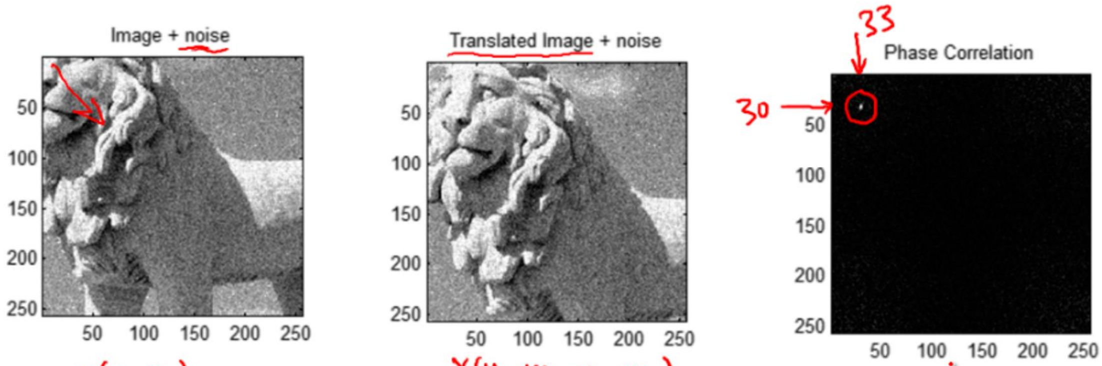
Motion Compensated Temporal Filtering



Classification of ME Methods

- Direct Methods
  - Phase Correlation
  - Block Matching
  - Spatio-temporal gradient
    - Optical Flow
    - Pel-Recursive
- Indirect Methods
  - Feature Matching

Phase Correlation Example



$x(n_1, n_2)$

$X(\omega_1, \omega_2)$

$x(n_1 - m_1, n_2 - m_2)$

$\boxed{e^{-j\omega_1 m_1} e^{-j\omega_2 m_2}} X(\omega_1, \omega_2)$

Phase Correlation

An image registration method

$$x_{t-1}(n_1, n_2) \leftrightarrow X_{t-1}(k_1, k_2) \qquad x_t(n_1, n_2) \leftrightarrow X_t(k_1, k_2)$$

Assume $$x_t(n_1, n_2) = x_{t-1}((n_1 - m_1)_{N_1}, (n_2 - m_2)_{N_2})$$ $N_1 \times N_2$ $\quad 0 - (N_1 - 1)$, $0 - (N_2 - 1)$

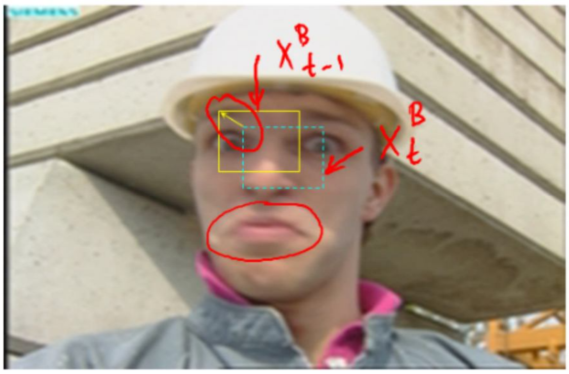
Then $$X_t(k_1, k_2) = X_{t-1}(k_1, k_2) e^{-j\frac{2\pi}{N_1} m_1 k_1} e^{-j\frac{2\pi}{N_2} m_2 k_2}$$

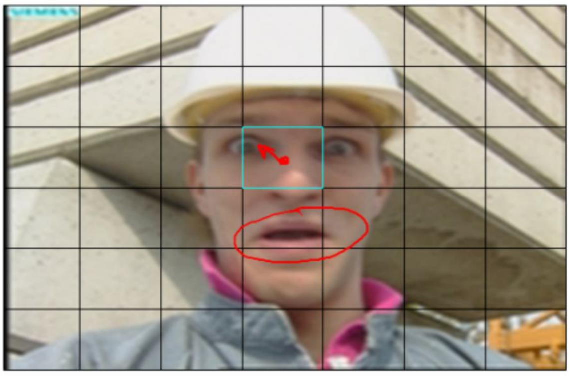
complex conjugate

Form: $$C(k_1, k_2) = \frac{X_t(k_1, k_2) \cdot X_{t-1}^*(k_1, k_2)}{|X_t(k_1, k_2) X_{t-1}^*(k_1, k_2)|} = \frac{|X_{t-1}(k_1, k_2)|^2 \, e^{-j\frac{2\pi}{N_1} m_1 k_1} e^{-j\frac{2\pi}{N_2} m_2 k_2}}{|X_{t-1}(k_1, k_2)|^2}$$

$$c(n_1, n_2) = \delta(n_1 - m_1, n_2 - m_2)$$

Block Matching





X_t

Basic underlying assumptions:

1. no change in the ambient lighting.
2. objects are rigid
3. objects are translated in the 3D world on a plane parallel to the image place
4. no objects appeared or left the scene

Matching Criteria

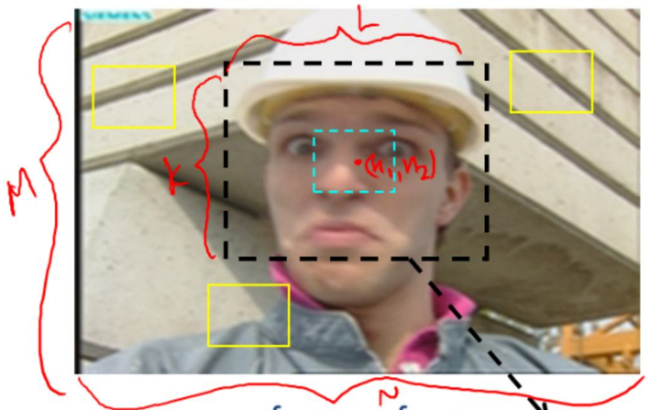
- A similarity or dissimilarity measure between regions (blocks)

$$\epsilon(d_1, d_2) = \sum_{(m_1,m_2)\in\mathcal{N}} \Phi\Big(x_t(n_1+m_1, n_2+m_2), x_{t-1}(n_1+m_2+d_1, n_2+m_2+d_2)\Big)$$

- Examples of $\Phi()$
  - Correlation Function
  - Mean Squared Error (MSE)
  - Mean Absolute Error (MAE) or Mean Absolute Difference (MAD)

$$\epsilon(d_1, d_2) = \sum_{(m_1,m_2)\in\mathcal{N}} |x_t(n_1+m_1, n_2+m_2) - x_{t-1}(n_1+m_2+d_1, n_2+m_2+d_2)|$$

Search Region



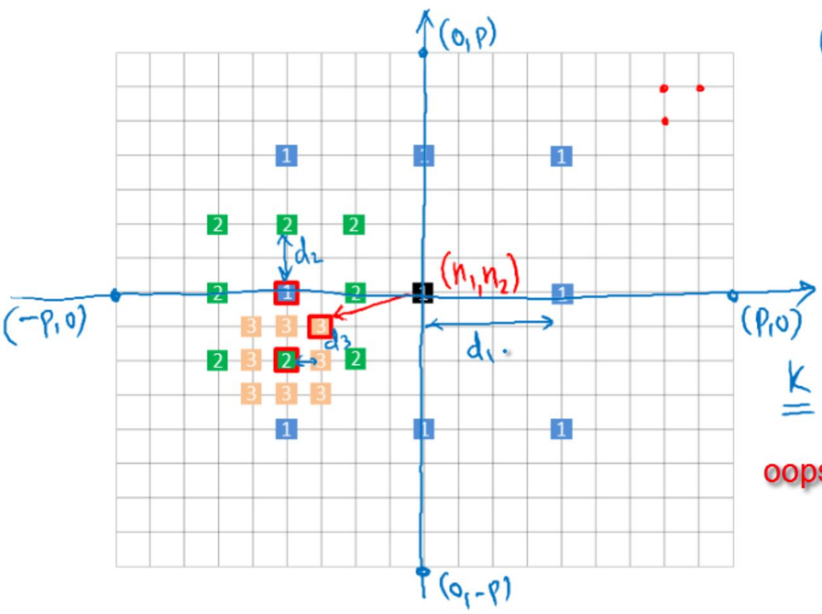
reference frame

search region

current frame

$M \times N$ matching comparisons
$K \times L$ " "

2D Logarithmic Search



$(2p+1) \times (2p+1)$

$K = \lceil \log_2 P \rceil$

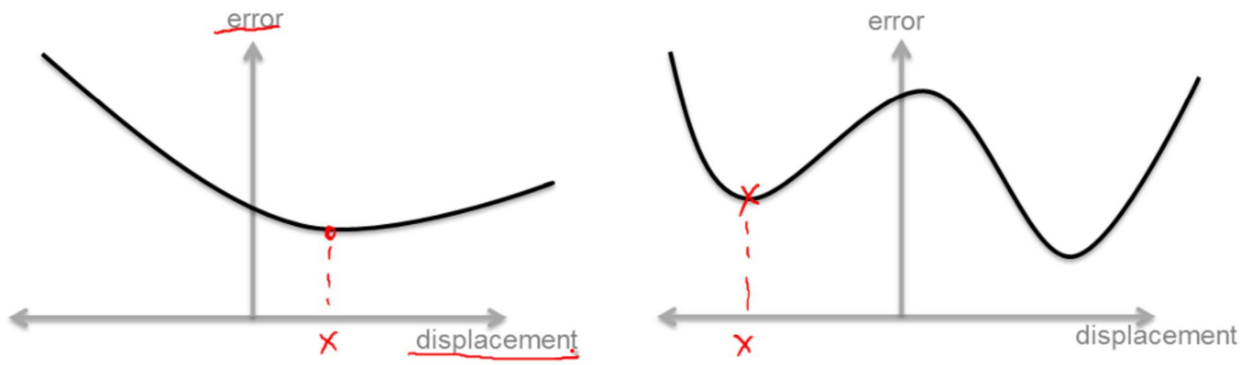
$P=7$, 15×15

$K=3$

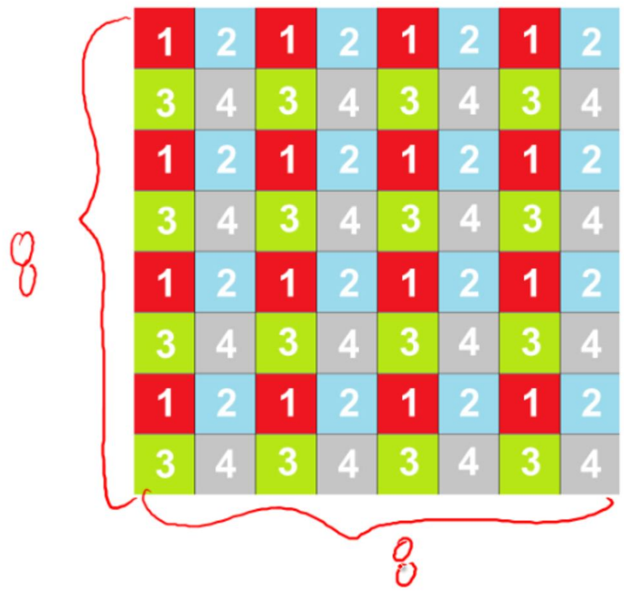
$$K \begin{cases} d_1 = 2^{k-1} \\ d_2 = \frac{d_1}{2} = 2^{k-2} \\ d_3 = \frac{d_2}{2} = 2^{k-1} \end{cases}$$

oops! >>> $d_3 = 2^{k-3}$

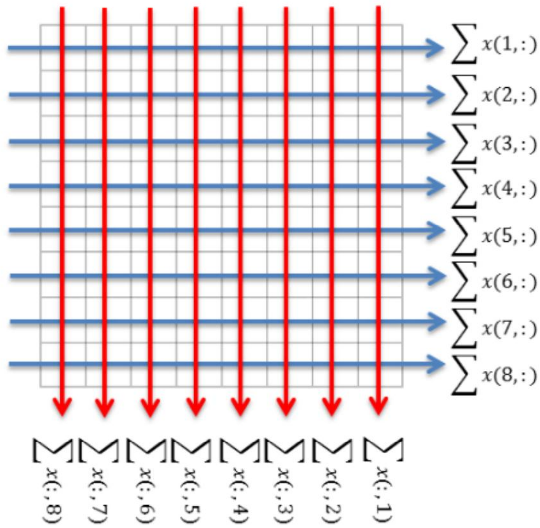
Global vs. Local Search



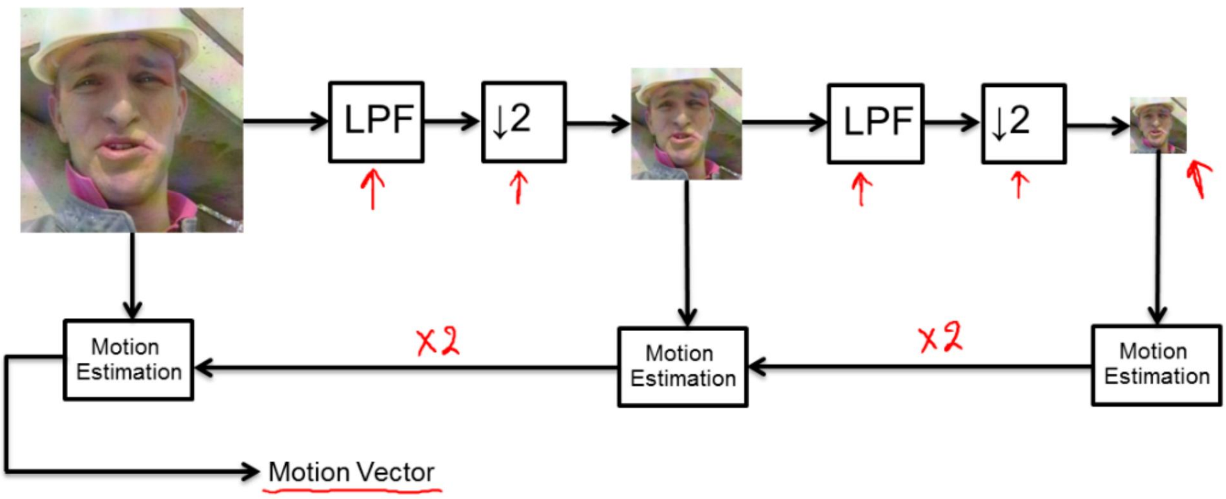
Pixel Sub-Sampling



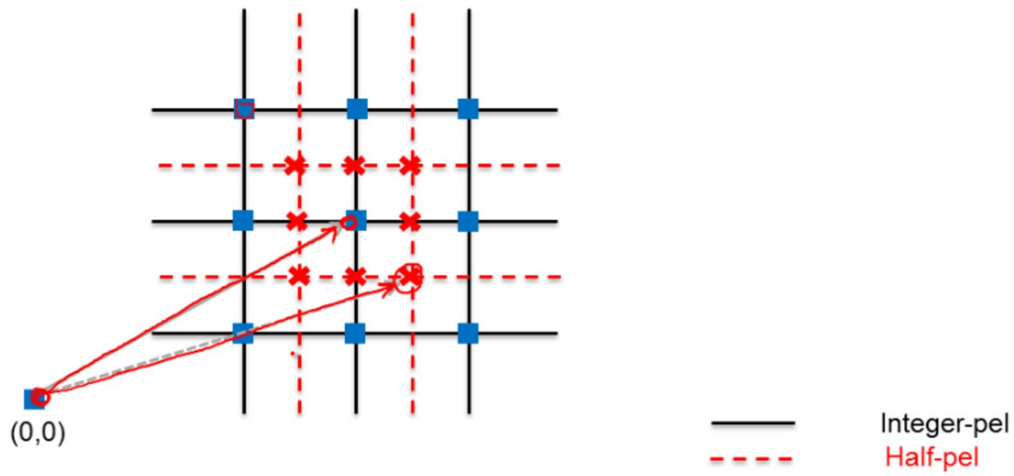
Pixel Projection



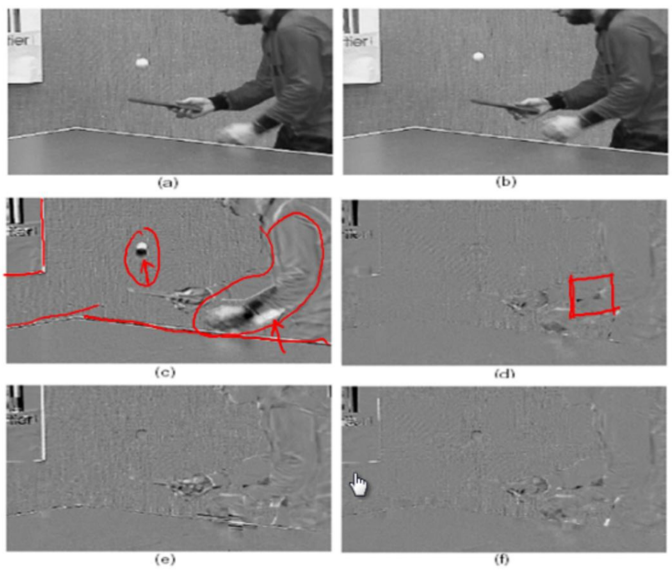
Hierarchical Motion Estimation



Sub-pixel Motion Estimation



Experimental Comparison



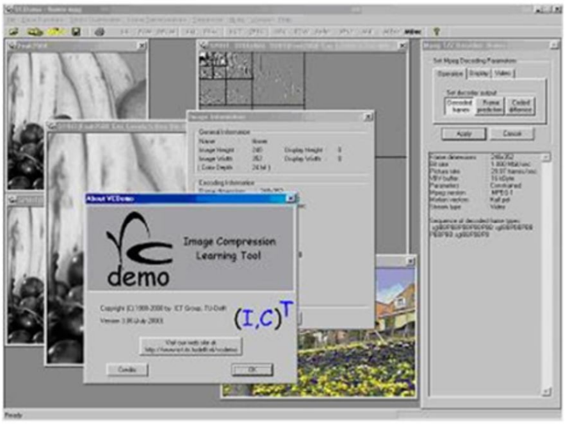
(a) Reference picture; (b) current picture;

(c) Frame difference; (d) full-search;

(e) Logarithmic search; (f) three level hierarchical search

V. Bhaskaran and K. Konstantinides, *Image and Video Compression Standards: Algorithms and Architectures*, Springer, 1997.

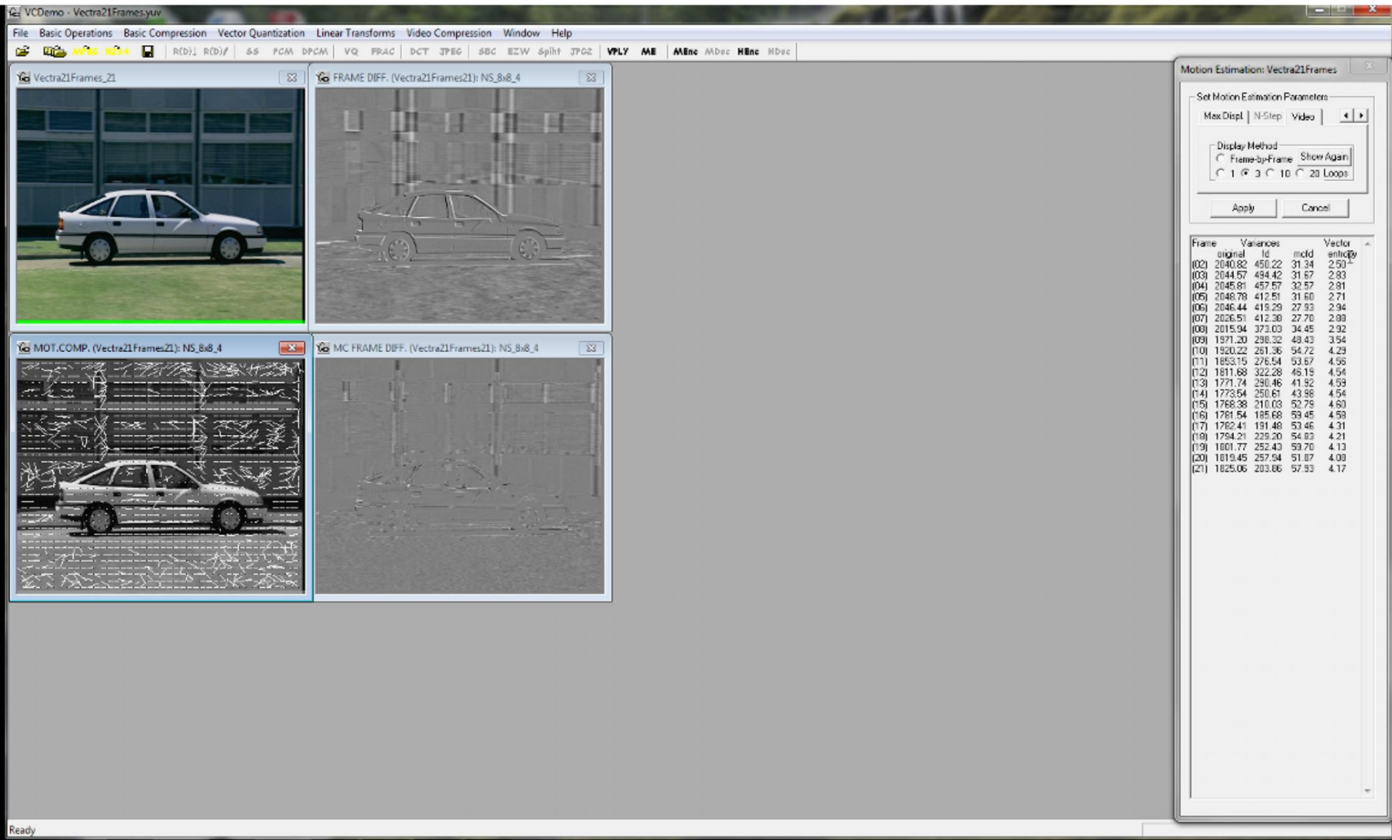
VcDemo



Tool to explore the possibilities of compression theory for images and videos

http://siplab.tudelft.nl/content/image-and-video-compression-learning-tool-vcdemo

Information and Communication Theory Group (ICT), Delft University of Technology, The Netherlands



Optical Flow Approach

Constant brightness constraint

$$I(x,y,0) = I(x+u, y+v, \tau)$$

Taylor series expansion

$$I(x+u,\cancel{y}+v,\tau) = I(x,\cancel{y},0) + \underbrace{\frac{\partial I(x,y,0)}{\partial x}}u + \underbrace{\frac{\partial I(x,y,0)}{\partial y}}v + \underbrace{\frac{\partial I(x,y,0)}{\partial t}}\tau + \text{H.O.T.}$$

$$I_x u + I_y v + I_t \tau = 0$$

Or

$$\boxed{I_x V_x + I_y V_y + I_t = 0}$$

Optical Flow Approach

Consider a neighborhood of the pixel:

$$\begin{aligned} I_x(q_1)V_x + I_y(q_1)V_y &= I_t(q_1) \\ I_x(q_2)V_x + I_y(q_2)V_y &= I_t(q_2) \\ \vdots \qquad\qquad \vdots \qquad &\qquad \vdots \\ I_x(q_n)V_x + I_y(q_n)V_y &= I_t(q_n) \end{aligned}$$

$$\underbrace{\begin{bmatrix} I_x(q_1) & I_y(q_1) \\ I_x(q_2) & I_y(q_2) \\ \vdots & \vdots \\ I_x(q_n) & I_y(q_n) \end{bmatrix}}_{A}{}_{n\times 2} \underbrace{\begin{bmatrix} V_x \\ V_y \end{bmatrix}_{2\times 1}}_{x} = \underbrace{\begin{bmatrix} I_t(q_1) \\ I_t(q_2) \\ \vdots \\ I_t(q_n) \end{bmatrix}_{n\times 1}}_{b}$$

We obtain $Ax = b$ $\quad x \rightarrow \boxed{A} \rightarrow b$

Min-norm Least-Squares Solution $A^T A x = A^T b \rightarrow x = (A^T A)^{-1} A^T b$

Regularized Solution $(A^T A + \lambda C^T C)x = A^T b \rightarrow x = (A^T A + \lambda C^T C)^{-1} A^T b$

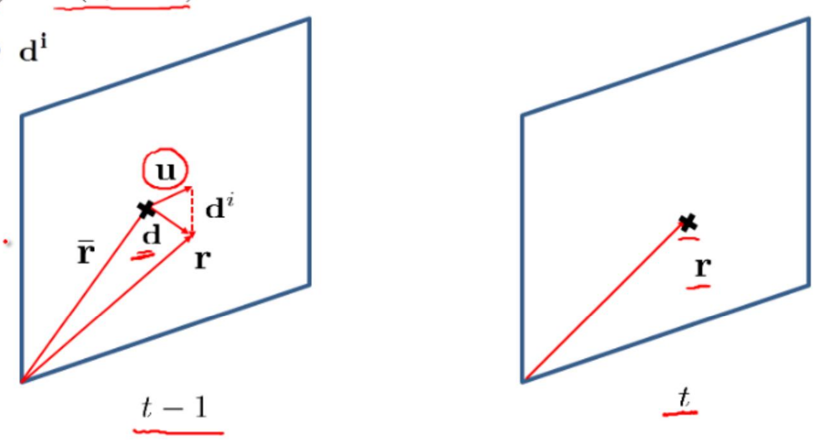
Pel-Recursive Algorithms

Constant brightness constraint

$$I(\mathbf{r},t) = I(\mathbf{r}-\mathbf{d},t-1) = I(\bar{\mathbf{r}},t-1)$$

Assume an initial estimate $\mathbf{d}^{\mathbf{i}}$

and set $\mathbf{u} = \mathbf{d} - \mathbf{d}^{\mathbf{i}}$



Pel-Recursive Algorithms

Displaced Frame Difference

$$\Delta(\mathbf{r},\mathbf{u}) = I(\mathbf{r},t) - I(\mathbf{r}-\mathbf{d}^{\mathbf{i}},t-1) = I(\mathbf{r},t) - I(\bar{\mathbf{r}}+\mathbf{u},t-1)$$

I(r̄+u,t-1)

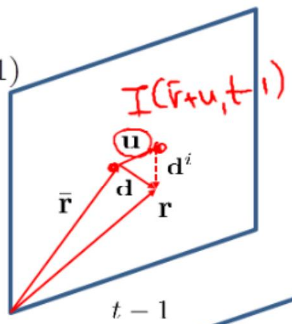
Taylor series expansion

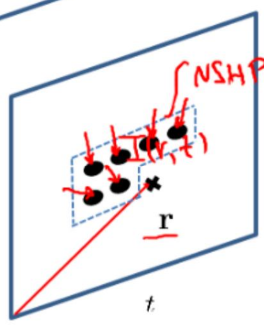
$$I(\bar{\mathbf{r}}+\mathbf{u},t-1) = I(\bar{\mathbf{r}},t-1) + \nabla^T I(\bar{\mathbf{r}}+\mathbf{u},t-1)\mathbf{u} + \epsilon(\mathbf{r},\mathbf{u})$$

I(r,t)

Finally $\boxed{\Delta(\mathbf{r},\mathbf{u}) = -\nabla^T I(\mathbf{r}-\mathbf{d}^{\mathbf{i}},t-1)\mathbf{u} - \epsilon(\mathbf{r},\mathbf{u})}$

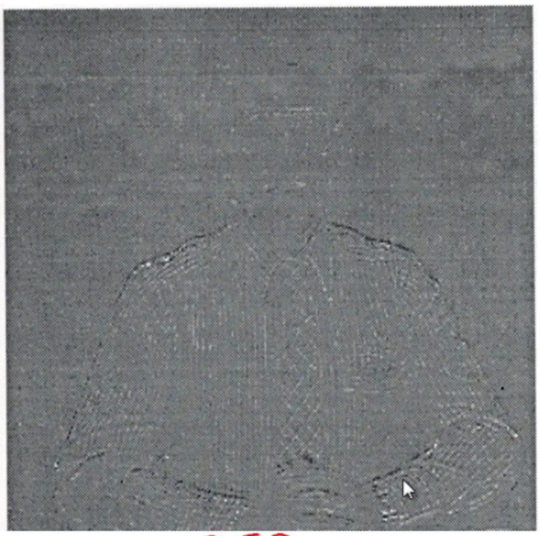
1. recursive computability
2. $d^i \longrightarrow u$ $\qquad u = d - d^i$





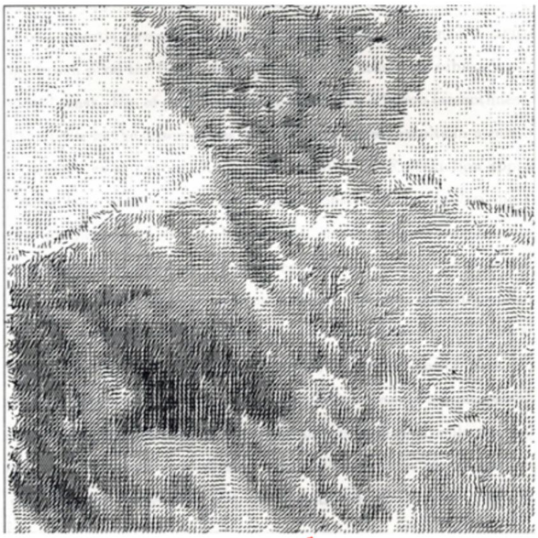
Frame Difference-- DFD

FD

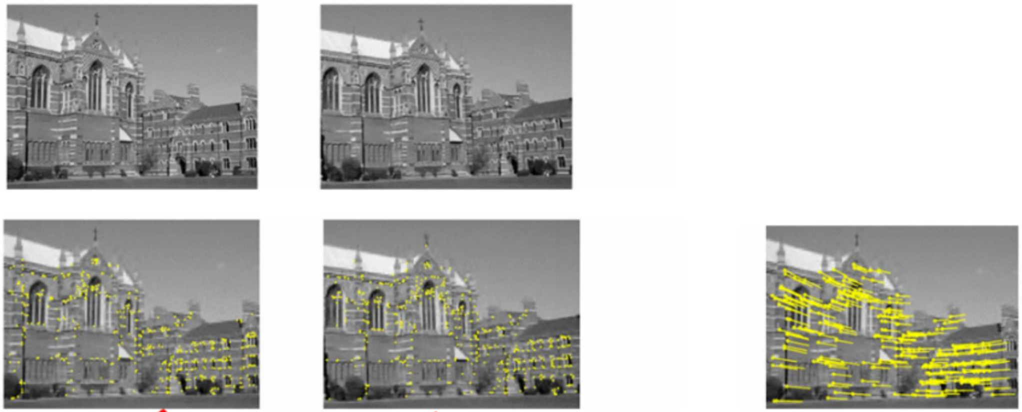


DFD

Estimated Motion Vectors



Feature-Based Methods



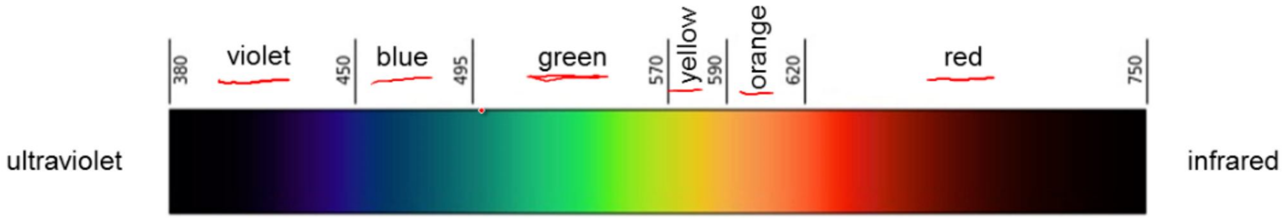
B. Triggs, A. Zisserman , **Feature Based Methods for Structure and Motion Estimation,** B. Triggs, A. Zisserman, R. Szeliski (Eds.): Vision Algorithms'99, LNCS 1883, pp. 278–294, 2000, Springer-Verlag Berlin Heidelberg 2000.

Harris corners, SIFT, SURF

Importance of Color

- Color is a powerful descriptor that can be used for various tasks, e.g., segmentation, object detection, tracking, and identification
- Humans can distinguish thousands of color shades and intensities, as compared to about only two dozen shades of gray

Color Spectrum



Light Characterization

- **Achromatic light**
  - Intensity
- **Chromatic light**
  - Radiance: total amount of energy that flows from the light source (Watts)
  - Luminance: perceived amount of energy (lumens)
  - Brightness: hard to measure; embodies the achromatic notion of intensity; used to describe color sensation

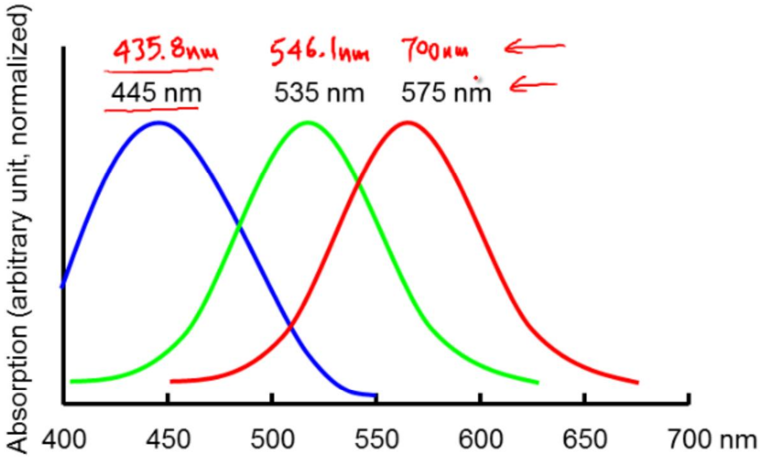
Light Absorption by the Human Eye

CIE (Commision International de l'Eclairage)

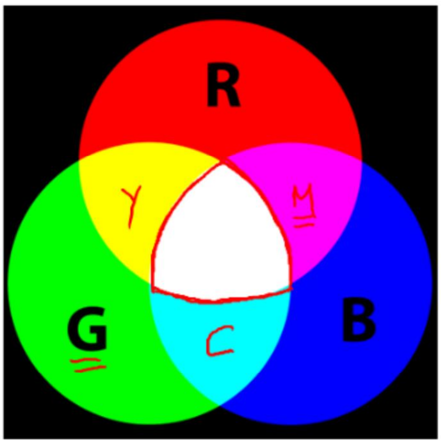
Cones
6-7 millions

65% → R
33% → G
2% → B

primary Colors



Primary Colors of Light

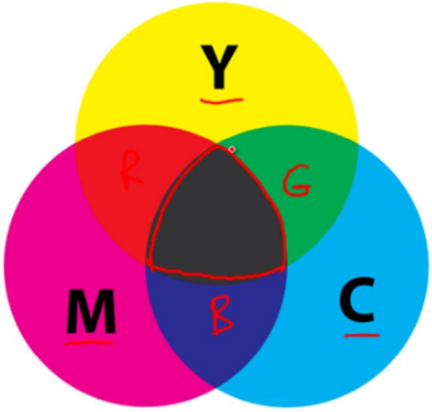


additive

Primary Colors of Pigments

$C = 1 - R$

$[0,1]$



subtractive

Color Distinguishing Characteristics

- **Brightness**
  - hard to measure; embodies the achromatic notion of intensity; used to describe color sensation
- **Hue**
  - indicates the dominant wavelength in a mixture of light waves
- **Saturation**
  - refers to the relative purity or the amount of white light mixed with its hue

Trichromatic Coefficients

$X,\ Y,\ Z$ tristimulus values

$x,\ y,\ z$ trichromatic coefficients

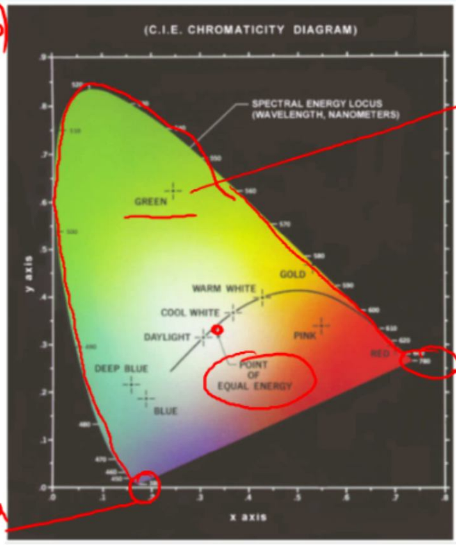
$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$z = \frac{Z}{X+Y+Z}$$

$$x + y + z = 1$$

y(green)



62% G
25% R
13% B

Z = 1-(x+y)
(blue)

780nm

x(red)

380nm

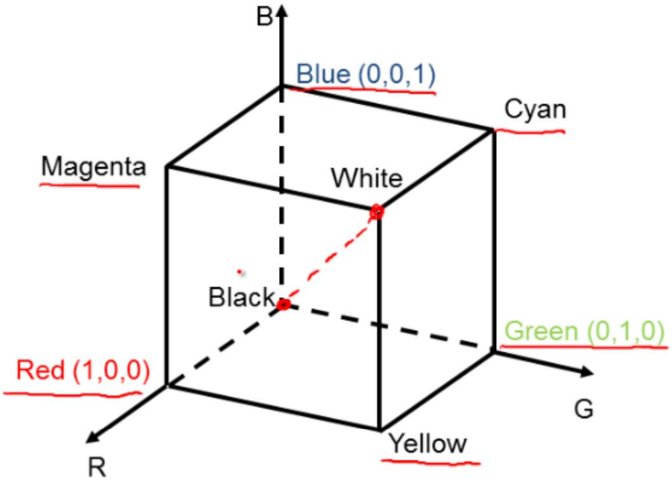
Rafael C. Gonzalez and Richard E. Woods, "Digital Image Processing," 3rd. ed., Pearson, Prentice Hall, 2008.

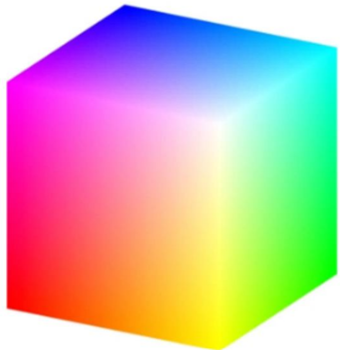
Color Models

They specify a coordinate system and a subspace within that system where each color is represented by a single point

- RGB
- CMY and CMYK
- HSI

RGB Model





RGB 24-bit color cube

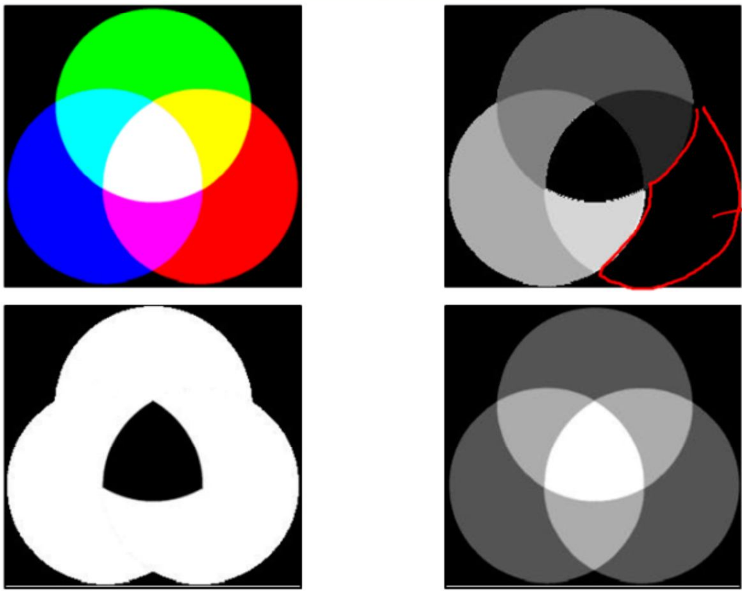
CMY Model

[0,1]

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Equal amounts of C, M, Y should produce black, but they produce instead in practice a muddy-looking black.

HSI Model



H

angle = 0°

S

[0, 255]

$I = \frac{1}{3}(R+G+B)$

YUV/YCbCr Model

YUV is used by the PAL, NTSC, and SECAM composite color video standards

YIQ is derived from the YUV color space

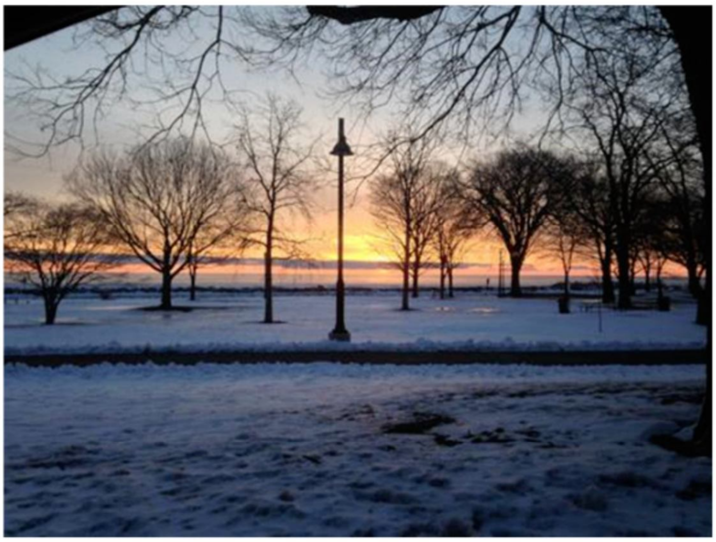
YCbCr (ITU-R BT.601) is a scaled and offset version of YUV

$$\begin{cases} Y = 0.299R + 0.587G + 0.114B \\ Cb = 128 - 0.168736R - 0.331264G + 0.5B \\ Cr = 128 + 0.5R - 0.418688G - 0.081312B \end{cases}$$

JPEG conversion formulae (there are other conversion formulae, for analog color, digital color, etc)

This takes input RGB values from 0 to 255, and output Y, Cb, Cr in the range 0 to 255.

Examples of Color Spaces



Original

Examples of Color Spaces

R G B

Examples of Color Spaces

C M Y

$C = 1 - R$

Examples of Color Spaces

H S I

Examples of Color Spaces

Y

~I , ~G

Cb

Cr

Color Image and Video Processing

- Choice of appropriate color space
- Independent channel processing
- Multi-channel processing

RGB vs HSI Filtering

RGB filtered separately

I filtered, H and S added

Difference

In both cases a 5X5 flat filter was used