A pair of black-rimmed glasses is placed on an open book. The book's pages are filled with text, though it is out of focus. The scene is lit with a warm, golden light, creating a soft and contemplative atmosphere.

자기 아들을 아끼지 아니하시고 우리 모든 사람을 위하여 내주신 이가 어찌 그 아들과 함께 모든 것을 우리에게 주시지 아니하겠느냐 (로마서 8:32)

우리가 알거니와 하나님을 사랑하는 자 곧 그의 뜻대로 부르심을 입은 자들에게는 모든 것이 합력하여 선을 이루느니라 (로마서 8:28)

Overview

- Hash Table Data Structure: Purpose
- Implementations So Far

Array of size n	unsorted list	sorted array	Trees BST – average AVL – worst	Heap, Priority Queue	Hash Table
insert	find+ $O(1)$	$O(n)$	$O(\log n)$	$O(\log n)$	
find	$O(n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	
remove	find+ $O(1)$	$O(n)$	$O(\log n)$	$O(\log n)$	

Overview

- Hash Table Data Structure: Purpose
support insertion, deletion and search in average case constant time $O(1)$
- Implementations So Far

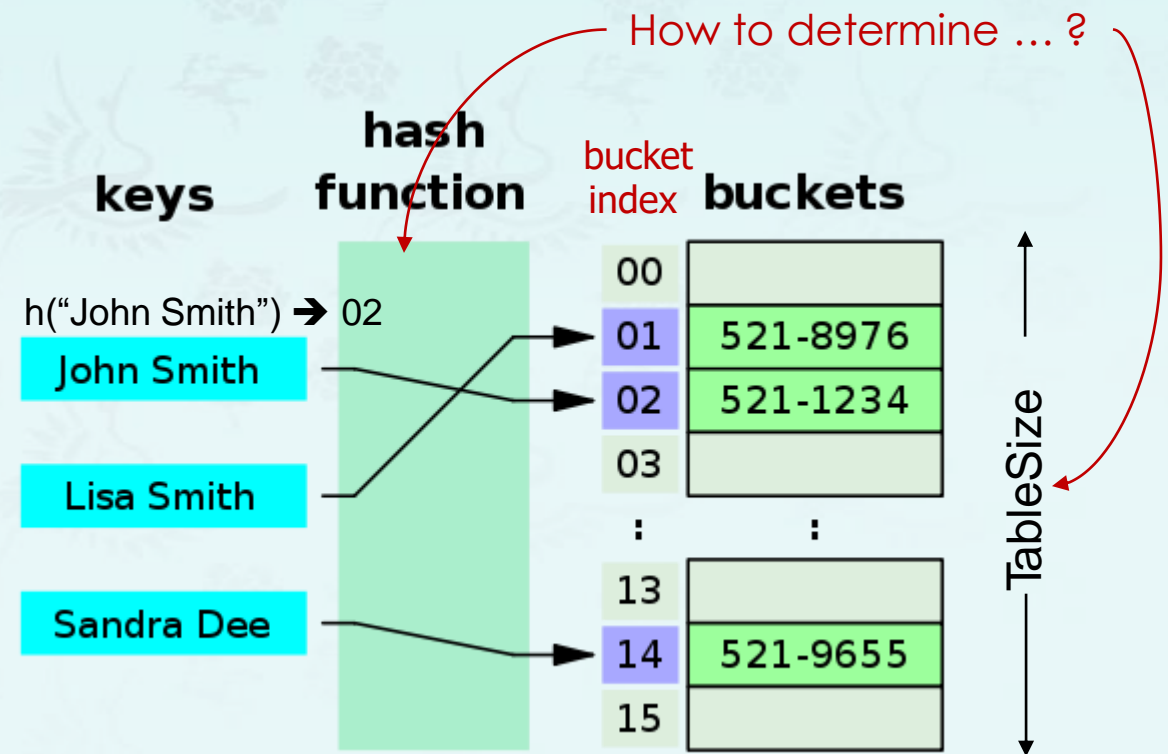
Array of size n	unsorted list	sorted array	Trees BST – average AVL – worst	Heap, Priority Queue	Hash Table
insert	find+ $O(1)$	$O(n)$	$O(\log n)$	$O(\log n)$	$O(1)$
find	$O(n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(1)$
remove	find+ $O(1)$	$O(n)$	$O(\log n)$	$O(\log n)$	$O(1)$

Overview

- Hash Table Data Structure: Purpose
support insertion, deletion and search in average case constant time $O(1)$
- Hash function
 - Hash[int key] \rightarrow integer value
 - Hash["string key"] \rightarrow integer value
- Hash table ADT
 - Implementations, Analysis, Applications

Hash Table Main components

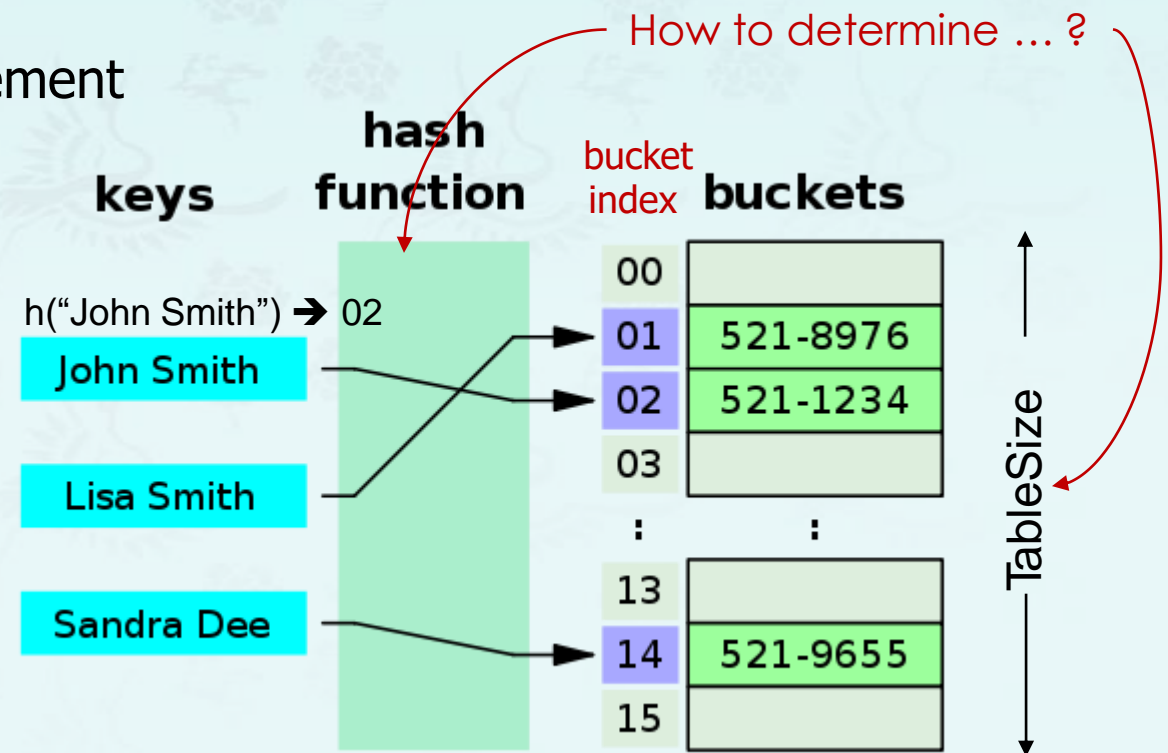
- Hash table is an array of fixed size elements
- Array elements indexed by a key mapped to an **bucket index** (0 ... TableSize-1)
- Mapping (hash function) h from key to index
 - e.g., $h(\text{"John Smith"}) \rightarrow 02$



Hash Table Operations

- insert
 - $\text{HashTable}[\text{h}(\text{"John Smith"})] = \langle \text{"John Smith"}, 521-1234 \rangle$
- remove
 - $\text{HashTable}[\text{h}(\text{"John Smith"})] = \text{NULL}$
- find
 - $\text{HashTable}[\text{h}(\text{"John Smith"})]$ returns the element hashed for "John Smith"

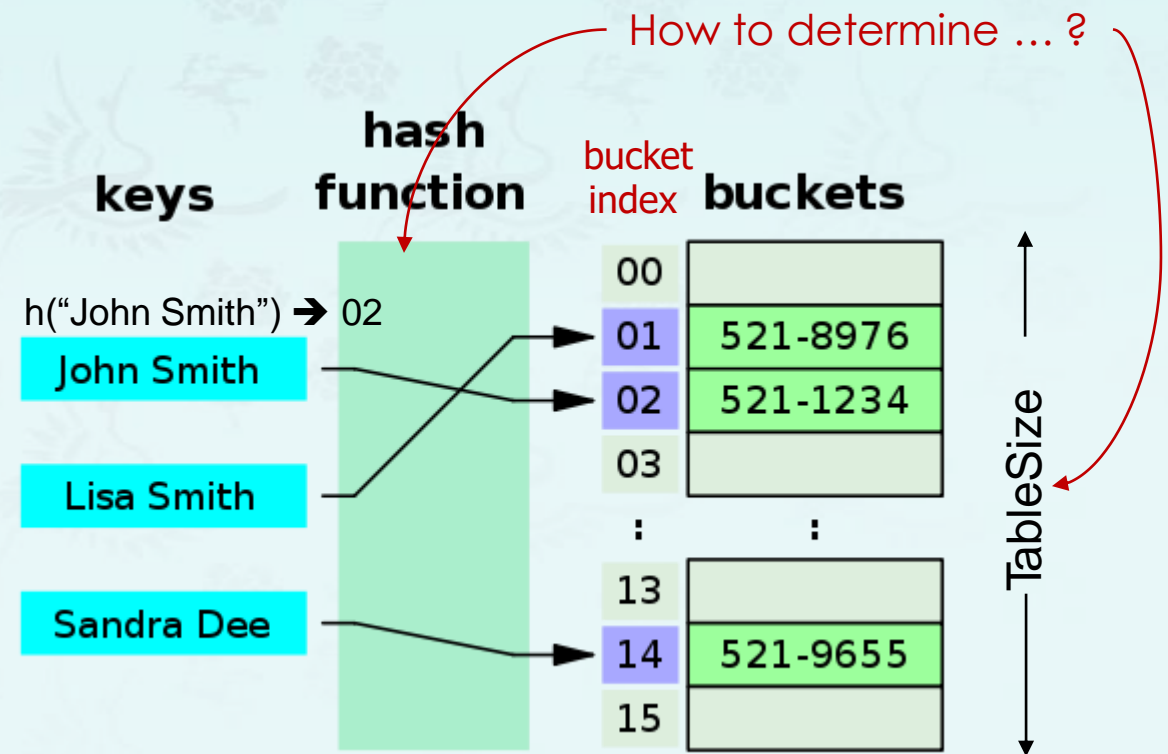
- What happens if $\text{h}(\text{"John Smith"}) == \text{h}(\text{"Joe Blow"})$
 - "Collision"



Hash Table Design

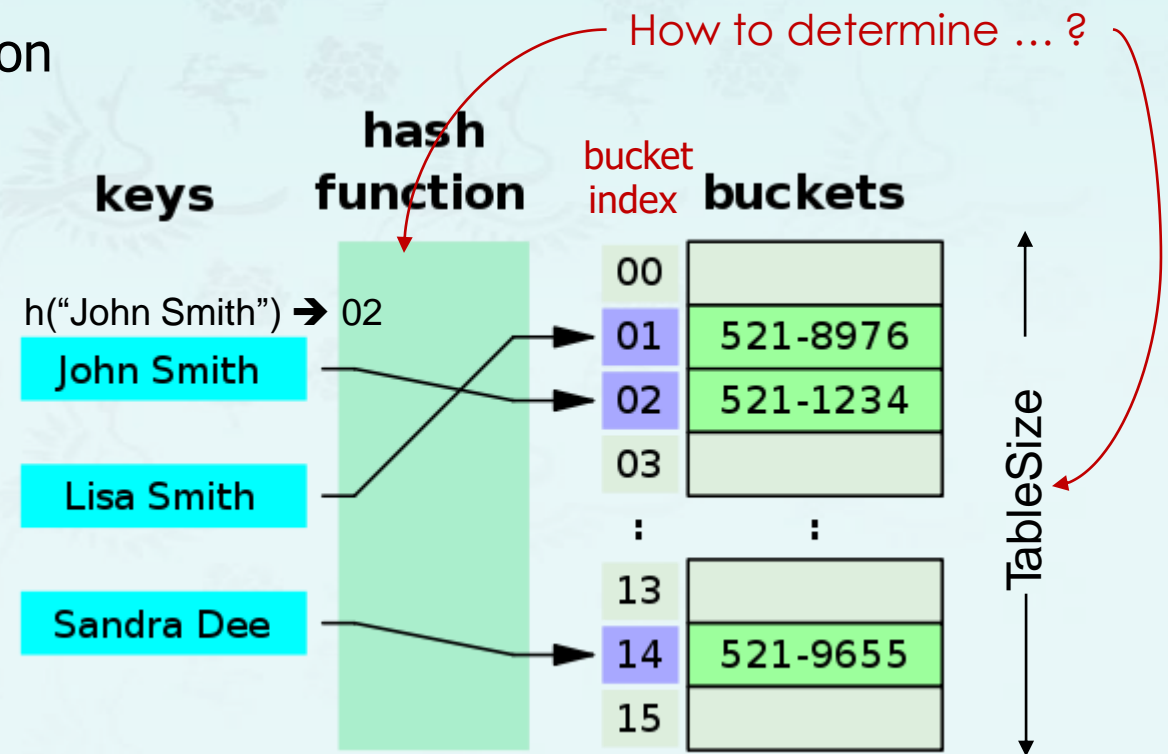
Factors affecting Hash Table Design

- Hash function
- Table size or
 - Usually fixed at the start
- Collision handling schemes



Hash Function

- It maps an element's key into a valid hash table index
 - $h(\text{key}) \rightarrow \text{hash table index}$
- Note that this is (slightly) different from saying:
 - $h(\text{string}) \rightarrow \text{int}$
 - Because the key can be of any type
 - e.g., " $h(\text{int}) \rightarrow \text{int}$ " is also a hash function



Hash Function Properties

- It maps an element's key into a valid hash table index
 - $h(\text{key}) \rightarrow$ hash table index
- It maps key to integer
 - Constraint: Integer should be between **[0, TableSize-1]**
- A hash function can result in a many-to-one mapping (causing collision)
 - Collision occurs when hash function maps two or more keys to same array index
- Collisions cannot be avoided but its chances can be reduced using a "good" hash function

Hash Function - Effective use of table size

- Simple hash function (assume integer keys)
 - $h(\text{Key}) = \text{Key} \% \text{TableSize}$
- For random keys, $h()$ distributes keys evenly over table
 - What if $\text{TableSize} = 100$ and keys are ALL multiples of 10?
 - Better if TableSize is a prime number

Different Ways to Design a Hash Function for String Keys

- A very simple function to map strings to integers:
 - Add up character ASCII values (0-255) to produce integer keys
 - e.g., "abcd" = 97 + 98 + 99 + 100 = 394
 - $\rightarrow h(\text{"abcd"}) = 394 \% \text{TableSize}$
- Potential problems:
 - Anagrams will map to the same index
 - $h(\text{"abcd"}) = h(\text{"dbac"})$
 - Small strings may not use all of table
 - $\text{strlen}(s) * 255 < \text{TableSize}$
 - Time proportional to length of the string

Different Ways to Design a Hash Function for String Keys

- Another approach:

- Treat first 3 characters of string as base-27 integer (26 letters plus space)
 - e.g., $\text{Key} = s[0] + (27^1 * s[1]) + (27^2 * s[2])$
 - Better than previous approach because ...

- Potential problems:

- Assumes first 3 characters randomly distributed
 - This is not true in English

Apple
Apply
Appointment
Apricot



collision

Different Ways to Design a Hash Function for String Keys

■ Last approach:

- *Use all N characters of string as an N -digit base- K number*
- Choose K to be prime number larger than number of different digits (characters)
 - i.e., $K = 29, 31, 37$
 - If $L = \text{Length of string } S$, then

$$h(S) = \sum_{i=0}^{L-1} S[L - i - 1] * 37^i \% \text{TableSize}$$

- Use Horner's rule to compute $h(S)$.
 - Limit L for long strings
-
- Potential problems
 - Overflow
 - Larger runtime

```
// a hash function for strings
int hash(const string& key, int tablesize) {
    int value = 0;
    for (auto x : key)
        value = value * 37 + x;
    value %= tablesize;
    if (value < 0) value += tablesize;
    return value;
}
```


Techniques to Deal with Collisions

- Chaining
- Open addressing
 - Linear Probing
 - quadratic Probing
 - Double hashing.

Resolving Collisions

- What happens when $h(k_1) = h(k_2)$? → Collision!
- Collision resolution strategies
 - **Chaining**
 - Store colliding keys in a linked list at the same hash table index
 - **Open addressing**
 - Store colliding keys elsewhere in the table

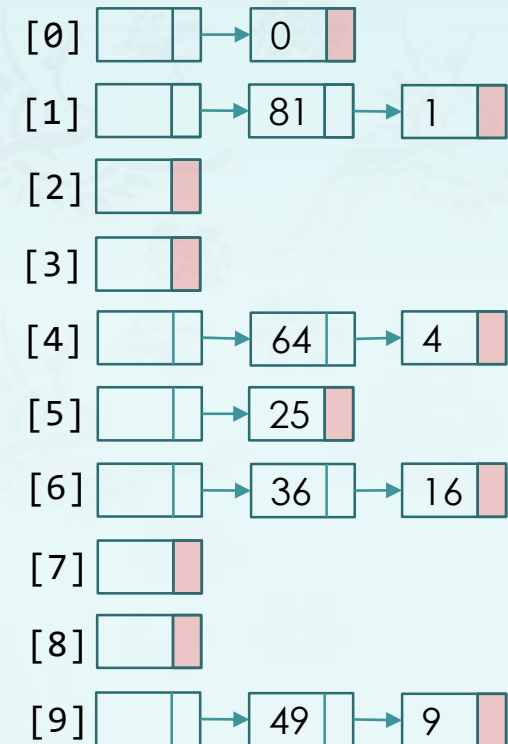
Chaining: Collision resolution technique 1

Collision Resolution by Chaining: Analysis

- Chaining strategy: maintains a linked list at **every hash index for collided elements**
 - Hash table T is a vector of linked lists
 - Insert element at the head (as shown here) or at the tail
 - Key k is stored in list a $\text{HashTable}[h(k)]$
 - e.g., $\text{TableSize} = 10$
 - $h(k) = k \% 10$
 - insert first 10 perfect squares

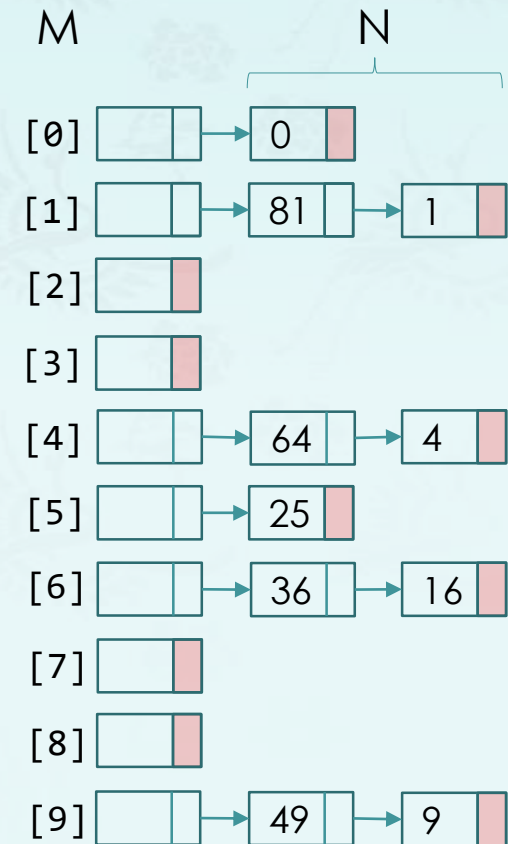


Insertion sequence:
{ 0 1 4 9 16 25 36 49 64 81 }



Collision Resolution by Chaining: Analysis

- Load factor λ of a hash table T is defined as follows:
 - Element size: N = number of elements in T
 - Table size: M = size of T
 - **Load factor:** $\lambda = N/M$ (적재율)
 - i.e., λ is the average length of a chain
- Unsuccessful search time: $O(\lambda)$
 - Same for insert time
- Successful search time average: $O(\lambda/2)$
- Ideally, want $\lambda \leq 1$ (then, not a function of N)



Potential disadvantages of Chaining

- Linked lists could get long
 - Especially when N approaches M
 - Longer linked lists could negatively impact performance
- More memory because of pointers
- Absolute worst-case (even if $N \ll M$):
 - All N elements in one linked list!
Typically the result of a bad hash function

Open Addressing: Collision resolution technique 2

- Linear Probing 선형조사법
- Quadratic Probing 이차조사법
- Double Hashing 이중해싱법

Collision Resolution by Open Addressing

- When a collision occurs, **look elsewhere in the table** for an empty slot
- Advantages over chaining
 - No need for list structures
 - No need to allocate/deallocate memory during insertion/deletion (slow)
- Disadvantages
 - Slower insertion – May need several attempts to find an empty slot
 - Table needs to be bigger (than chaining-based table) to achieve average-case constant-time performance
 - Load factor $\lambda \approx 0.5$

Collision Resolution by Open Addressing

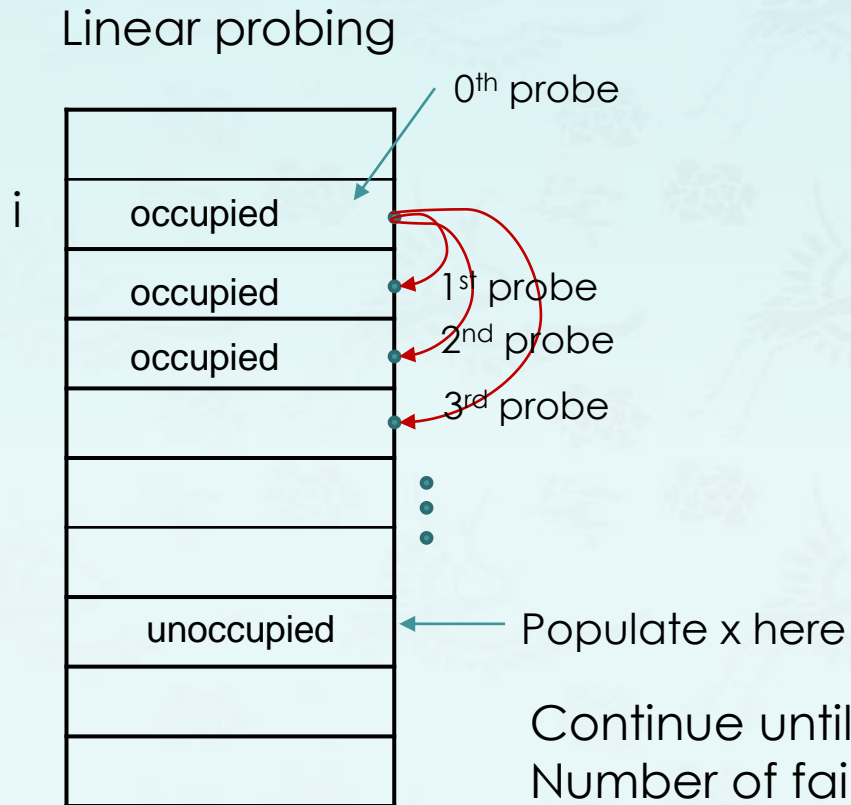
- A "Probe sequence" is a sequence of slots in hash table while searching for an element x
 - $h_0(x), h_1(x), h_2(x), \dots$
 - Needs to visit each slot exactly once
 - Needs to be repeatable (so we can find/delete what we've inserted)
- Hash function
 - $h_i(x) = (h(x) + f(i)) \% \text{TableSize}$
 - $f(0) = 0 \rightarrow$ position for the 0th probe
 - $f(i)$ is "the distance to be traveled relative to the 0th probe position, during the i^{th} probe". It can be linear, quadratic etc.

- $f(i)$ = is a linear function of i , e.g., $f(i) = i$

$$h_i(x) = (h(x) + i) \% \text{TableSize}$$

i^{th} probe index 0^{th} probe index

Probe sequence: $+0, +1, +2, +3, +4, \dots$



Continue until an empty slot is found
Number of failed probes is a measure of performance

Linear Probing Example

- $f(i)$ = is a linear function of i , e.g., $f(i) = i$

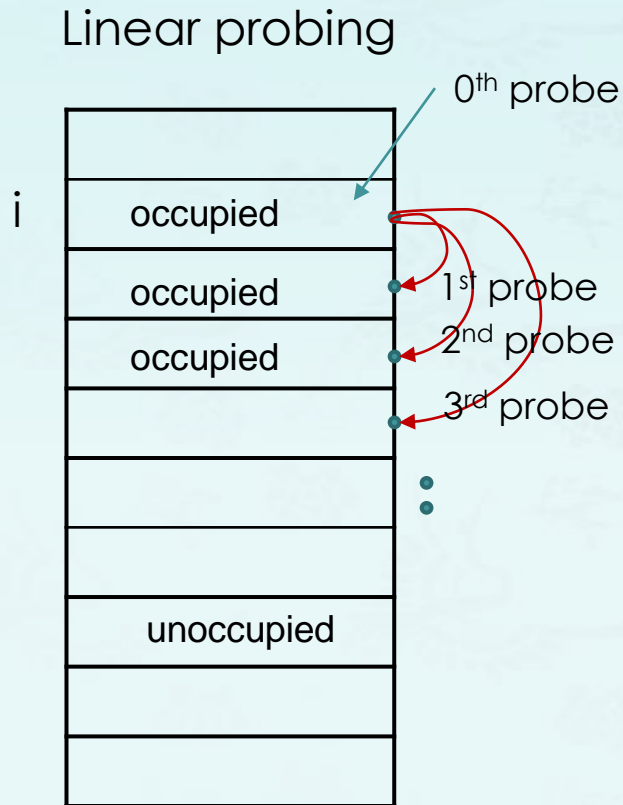
$$h_i(x) = (h(x) + i) \% \text{ TableSize}$$

i^{th} probe index 0^{th} probe index

Probe sequence: $+0, +1, +2, +3, +4, \dots$

- Example: $h(x) = x \% \text{ TableSize}$

- $h_0(89) = (h(89) + 0) \% 10 = 9$
- $h_0(18) = (h(18) + 0) \% 10 = 8$
- $h_0(49) = (h(49) + 0) \% 10 = 9$ (collision)
 $h_1(49) = (h(49) + 1) \% 10 = (h(49) + 1) \% 10 = 0$



Linear Probing Example

Insert sequence: **89, 18, 49, 58, 69**

$$h(x) = k \% 10$$

Complete the table and hash function

Empty Table	After 89	After 18	After 49	After 58	After 69
0			49	49	49
1				58	58
2					
3					
4					
5					
6					
7					
8		18	18	18	18
9	89	89	89	89	89

For example, linear probing for 58

$$h_0(58) = (h(58) + f(0)) \% 10$$

$$= (8 + 0) \% 10 = 8 \text{ (collision)}$$

$$h_1(58) = (h(58) + 1) \% 10 = 0 \text{ (collision)}$$

$$h_2(58) = (h(58) + 2) \% 10 = 2$$

0

0

1

3

3

7

Unsuccessful no. of probes

Total

Linear Probing Example

Insert sequence: **89, 18, 49, 58, 69**

$$h(x) = k \% 10$$

Complete the table and hash function

Empty Table	After 89	After 18	After 49	After 58	After 69
0			49	49	49
1				58	58
2					
3					
4					
5					
6					
7					
8		18	18	18	18
9	89	89	89	89	89

For example, linear probing for 58
 $h_0(58) = (h(58) + f(0)) \% 10$
 $= (8 + 0) \% 10 = 8$ (collision)
 $h_1(58) = (h(58) + 1) \% 10 = 0$ (collision)
 $h_2(58) = (h(58) + 2) \% 10 = 2$

For example, linear probing for 69
 $h_0(69) = (h(69) + f(0)) \% 10$
 $=$

0

0

1

3

3

7

Unsuccessful no. of probes

Total

Linear Probing Issues

- Probe sequences can get longer with time
- Primary clustering
 - Keys tend to cluster in one part of table
 - Keys that hash into cluster will be added to the end of the cluster (making it even bigger)
 - Side effect:
Other keys could also get affected if mapping to a crowded neighborhood

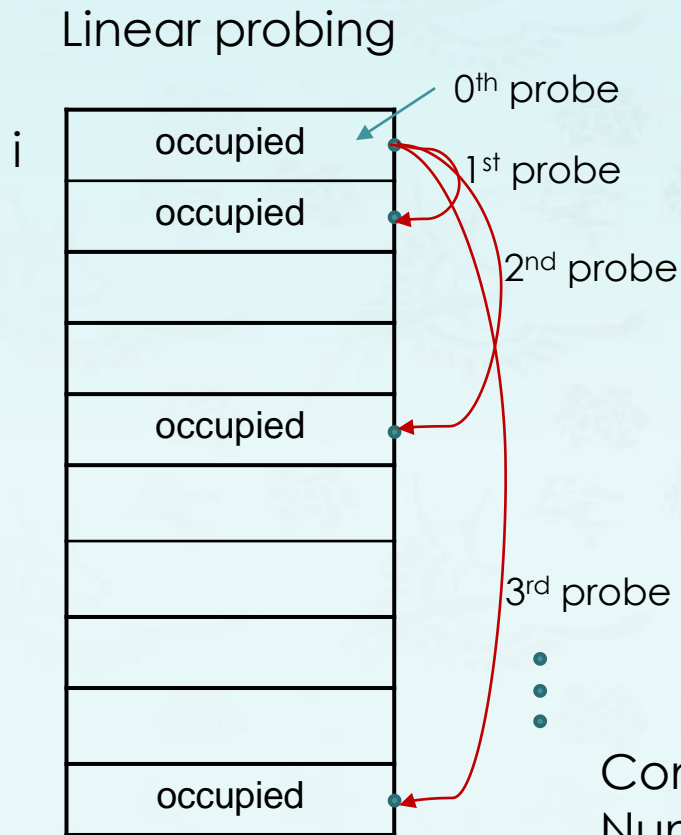
Quadratic Probing이차조사법

- Avoids primary clustering
- $f(i)$ is quadratic in i , e.g., $f(i) = i^2$

$$h_i(x) = (h(x) + i^2) \% \text{TableSize}$$

i^{th} probe index 0^{th} probe index

Probe sequence: $+0, +1, +4, +9, +16, \dots$

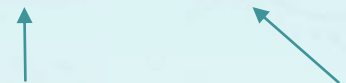


Continue until an empty slot is found
Number of failed probes is a measure of performance

Quadratic Probing^{이차조사법}

- Avoids primary clustering
- $f(i)$ is quadratic in i , e.g., $f(i) = i^2$

$$h_i(x) = (h(x) + i^2) \% \text{TableSize}$$


 i^{th} probe index 0^{th} probe index

Probe sequence: $+0, +1, +4, +9, +16, \dots$

- Example:

$$h_0(58) = (h(58) + 0^2) \% 10 = 8 \text{ (collision)}$$

$$h_1(58) = (h(58) + 1^2) \% 10 = 9 \text{ (collision)}$$

$$h_2(58) = (h(58) + 2^2) \% 10 = 2$$

Quadratic Probing Example

Insert sequence: **89, 18, 49, 58, 69**

$$h(x) = k \% 10$$

Complete the table and hash function

	Empty Table	After 89	After 18	After 49	After 58	After 69
0				49	49	49
1						
2					58	58
3						
4						
5						
6						
7						
8			18	18	18	18
9		89	89	89	89	89
		0	0	1	2	2

For example, linear probing for 58

$$h_0(58) = (h(58)+f(0)) \% 10$$
$$= (8 + 0) \% 10 = 8 \text{ (collision)}$$
$$h_1(58) = (h(58)+1) \% 10 = 9 \text{ (collision)}$$
$$h_2(58) = (h(58)+4) \% 10 = 2$$

Unsuccessful no. of probes

Total

Quadratic Probing Example

Insert sequence: 89, 18, 49, 58, 69					
Empty Table	After 89	After 18	After 49	After 58	After 69
0			49	49	49
1					
2				58	58
3					
4					
5					
6					
7					
8		18	18	18	18
9	89	89	89	89	89
	0	0	1	2	2

Unsuccessful no. of probes

5

Total

$h(x) = k \% 10$

Complete the table and hash function

For example, linear probing for 58

$h_0(58) = (h(58)+f(0)) \% 10$
 $= (8 + 0) \% 10 = 8$ (collision)

$h_1(58) = (h(58)+1) \% 10 = 9$ (collision)

$h_2(58) = (h(58)+4) \% 10 = 2$

For example, linear probing for 69

$h_0(69) =$

Quadratic Probing Analysis

- Difficult to analyze
- Theorem:
 - New element can always be inserted into a table that is at least half empty and TableSize is prime
 - Otherwise, may never find an empty slot, even if one exists
- Ensure table never gets half full
 - If close, then expand it
- May cause "secondary clustering"

Double Hashing 이중해싱법

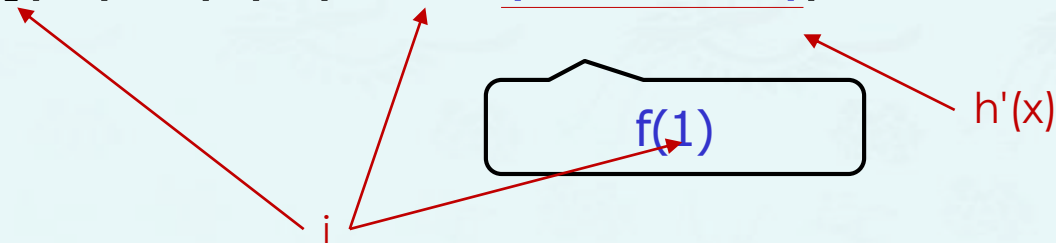
- Keep two hash functions h_1 and h_2
- Use **a second hash function** for all tries i other than 0
 $f(i) = i * h'(x)$

- Good choices for $h(x)$?
 - Should never evaluate to 0
 - $h'(x) = R - (x \% R)$
 - R is prime number less than TableSize

- Previous example with $R = 7$

$$h_0(49) = (h(49) + f(0)) \% 10 = 9 \text{ (collision)}$$

$$h_1(49) = (h(49) + 1 * (7 - 49 \% 7)) \% 10 = 6$$



Double Hashing Example

Insert sequence: **89, 18, 49, 58, 69, 23**

$h(x) = x \% 10$

	Empty Table	After 89	After 18	After 49	After 58	After 69	After 23
0							
1							
2							
3							
4							
5							
6				49	49	49	49
7							
8			18	18	18	18	18
9		89	89	89	89	89	89
		0	0	1	2	2	

Complete the table and hash function
No coloring is required in this page.

$h_0(49) = (h(49) + f(0)) \% 10 = 9$ (collision)
 $h_1(49) = (h(49) + 1 * (7 - 49 \% 7)) \% 10 = 6$

$h_0(58) =$
 $h_1(58) =$

$h_0(69) =$
 $h_1(69) =$

$h_0(23) =$
 $h_1(23) =$
:

Unsuccessful no. of probes

Double Hashing Analysis

- Imperative that **TableSize is prime**
 - e.g., insert 23 into previous table
- Empirical tests show **double hashing** close to random hashing
- Extra hash function takes extra time to compute

Rehashing

- Rehashing is the reconstruction of the hash table:
 - All the elements in the container **are rearranged** according to their hash value into the new set of buckets. This may alter the order of iteration of elements within the container.
- Increases the size of the hash table when load factor becomes "too high" (defined by a cutoff)
 - Anticipating that collisions would become higher
- Typically expand the table to **twice** its size (**but still prime**)
 - $\text{TableSize}_{\text{new}} = \text{nextprime}(2 * \text{TableSize}_{\text{old}})$
 - e.g., $2 \rightarrow 5$, $5 \rightarrow 11$, $11 \rightarrow 23$
- Need to **reinsert all existing elements** into new hash table

Rehashing Example

$$h(x) = x \% 7$$
$$\lambda = 0.57$$

0	6
1	15
2	
3	24
4	
5	
6	13

Insert 23



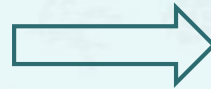
$$\lambda = 0.71$$

0	6
1	15
2	
3	24
4	
5	
6	13

TableSize = 7

$$\lambda_{max} = 0.6$$

Rehashing
since $\lambda > \lambda_{max}$



0	
1	
2	
3	
4	
5	
6	6
7	23
8	24
9	
10	
11	
12	
13	13
14	
15	15
16	

$$h(x) = x \% 17$$
$$\lambda = 0.29$$

TableSize = 17
 $\text{nextprime}(\text{TableSize} * 2)$

Rehashing Analysis

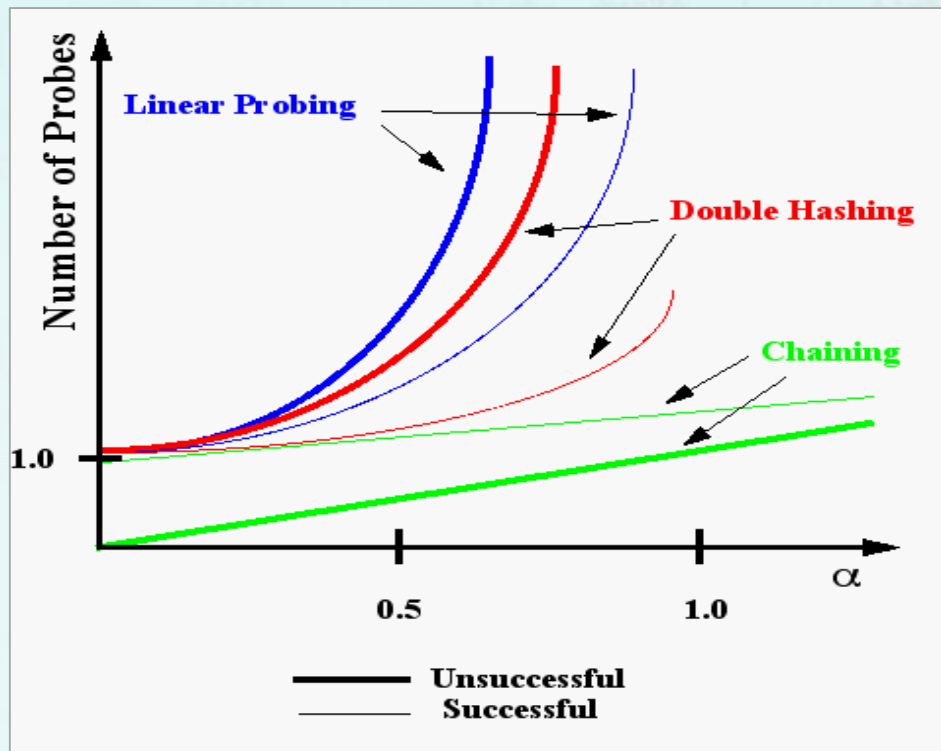
- Rehashing takes time to do N insertions
- Therefore should do it infrequently
- Specifically
 - Must have been $N/2$ insertions since last rehash
 - Amortizing the $O(N)$ cost over the $N/2$ prior insertions yields only constant additional time per insertion

Rehashing Implementation

- When to rehash
 - When load factor reaches some **threshold** (e.g., $\lambda \geq 1.0$), OR
 - When an insertion fails (open addressing cases)
- Applies across collision handling schemes

Summary(1/2)

- Hash tables support fast insert and search
 - $O(1)$ average case performance
 - Deletion possible, but degrades performance (but, not in chaining)
- Not suited if ordering of elements is important
- Expected number of probe vs. Load factor



Summary(2/2)

- Many applications
 - Symbol table in compilers
 - Accessing tree or graph nodes by name
 - e.g., city names in Google maps
 - Maintaining a transposition table in games
 - Remember previous game situations and the move taken (avoid re-computation)
 - Dictionary lookups
 - Spelling checkers
 - Natural language understanding (word sense)
 - Heavily used in text processing languages
 - e.g., Perl, Python, etc.

Points to remember - Hash tables

- Table size prime
- Table size larger than number of inputs (to maintain $\lambda \ll 1.0$)
- Tradeoffs between chaining vs. probing
- Collision chances decrease in this order:
linear probing^{선형조사법} → quadratic probing^{이차조사법} → double hashing^{이중해싱법}
- Rehashing recommended to resize hash table at a time when λ exceeds 0.5
- Good for searching.
Not good if there is some order implied by data

Hash Table(1) Using **list** in STL

```
struct Hash {
    int          tablesize;           // hash table size
    list<string>* hashtable;          // pointer to an array of buckets M
    int          nelements;           // number of elements in table N
    double        threshold;          // max_loadfactor N/M

    Hash(int size = 2, double lf = 1.0) { // a magic number, use a small prime
        tablesize = size;                // using list<string>* for a pedagogical purpose
        hashtable = new list<string>[size]; // but vector<list<string>> may be used
        nelements = 0;
        threshold = lf;                  // rehashes if loadfactor >= threshold
    }
    ~Hash() {
        delete[] hashtable;
    }
};
```

Hash Table(1) Using **list** in STL

```
// Notice that ht is passed by reference when its pointer may be changed inside
// except erase() and show(). Passing by reference may help to run the code faster.

int hashfunction(Hash* ht, int key);           // hash function for int key
int hashfunction(Hash* ht, string key);        // hash for string key

void rehash(Hash*& ht);                        // rehashes - doubles its tablesize
bool insert(Hash*& ht, string key);            // inserts key
                                              // rehashes if loadfactor >= threshold
bool erase(Hash*& ht, string key);             // erases key, returns true if successful
list<string> find(Hash* ht, string key);        // returns its list if found

void clear(Hash*& ht);                         // clear the table
void show(Hash*& ht, bool show_empty);         // show the table
int nextprime(int x);                          // returns the next prime
int tablesize(Hash* ht);                      // returns hash table size
int nelements(Hash* ht);                     // returns number of elements in table
double loadfactor(Hash* ht);                 // returns nelements/tablesize
double threshold(Hash* ht);                  // returns threshold(or max_loadfactor)
void threshold(Hash* &ht, double th);         // sets threshold and rehashes if needed
```

Hash Table(2) Using **list** in STL

```
// std::pair is a STL container or class which a pair of public members called
// 'first' and 'second'. The types of two members may be the same or different.

typedef std::pair<string, int> wordcount;

struct Hash {
    int            tablesize;           // hash table size or bucket_count()
    list<wordcount>* hashtable;         // pointer to an array of buckets
    int            nelements;          // number of elements in table or size()
    double         threshold;          // max_loadfactor

    Hash(int size = 2, double lf = 1.0) { // a magic number, use a small prime
        tablesize = size;                // using list<wordcount> for pedagogical purpose
        hashtable = new list<wordcount>[size]; // but vector<list<wordcount>> may be used
        nelements = 0;
        threshold = lf;                  // rehashes if loadfactor >= threshold
    }
    ~Hash() {
        delete[] hashtable;
    }
};
```

Hash Table(2) Using **list** in STL

```
// Notice that ht is passed by reference when its pointer may be changed inside
// except erase() and show(). Passing by reference may help to run the code faster.

int hashfunction(Hash* ht, int key);           // hash function for int key
int hashfunction(Hash* ht, string key);        // hash function for string key

void rehash(Hash*& ht);                        // rehashes - doubles its tablesize
void rehash(Hash*& ht, int usersize);           // rehashes using user-specified tablesize
bool insert(Hash*& ht, string key);            // inserts key & rehashes if loadfactor>=threshold
bool erase(Hash* ht, string key);             // erases key and returns true if successful
list<wordcount> find(Hash* ht, string key);    // returns its bucket list if found
void clear(Hash*& ht);                         // clear the table

void show(Hash*& ht, bool show_empty=false, int show_n=0); // show the table

int nextprime(int x);                          // returns the next prime
int tablesize(Hash* ht);                      // returns the table size
int nelements(Hash* ht);                     // returns number of elements in the table
double loadfactor(Hash* ht);                 // returns nelements/tablesize
double threshold(Hash* ht);                  // returns threshold(or max_loadfactor)
void threshold(Hash*& ht, double th);         // sets threshold and rehashes if needed
```


Hash Table(3) Using **unordered_map** in STL

- Unordered maps are associative containers that store elements formed by the combination of **a key value** and **a mapped value**, and which allows for fast retrieval of individual elements based on their keys.
- The key value is generally used to **uniquely identify the element**, while the mapped value is an object with the content associated to this key.
- Internally, the elements are **not** sorted in any particular order with respect to either their key or mapped values, **but organized into buckets depending on their hash values to allow for fast access to individual elements directly by their key values** (with a **constant average time complexity** on average).
- It is faster than **map** containers to access individual elements by their key, although they are generally less efficient for range iteration through a subset of their elements.
- It implements **the direct access operator** (**operator[]**) which allows for direct access of the mapped value using its key value as argument.
- Iterators in the container are at least forward iterators.

unordered_map::begin/end example

```
#include <iostream>
#include <unordered_map>
using namespace std;

int main () {
    unordered_map<string, int> mymap;
    mymap = {{"all", 3}, {"the", 2}, {"time", 5}};

    cout << "mymap contains:";
    for (auto it = mymap.begin(); it != mymap.end(); ++it)
        cout << " " << it->first << ":" << it->second;
    cout << endl;

    cout << "mymap's buckets contain:\n";
    for (unsigned i = 0; i < mymap.bucket_count(); ++i) {
        cout << "bucket #" << i << " contains:";
        for (auto ir = mymap.begin(i); ir != mymap.end(i); ++ir)
            cout << " " << ir->first << ":" << ir->second;
        cout << endl;
    }
    return 0;
}
```

```
mymap contains: time:5 the:2 all:3
mymap's buckets contain:
bucket #0 contains:
bucket #1 contains:
bucket #2 contains:
bucket #3 contains:
bucket #4 contains: the:2
bucket #5 contains:
bucket #6 contains:
bucket #7 contains: time:5
bucket #8 contains:
bucket #9 contains:
bucket #10 contains: all:3
bucket #11 contains:
bucket #12 contains:
```

unordered_map::begin/end example

```
#include <iostream>
#include <unordered_map>
using namespace std;

int main () {
    unordered_map<string, int> mymap;
    mymap = {{"all", 3}, {"the", 2}, {"time", 5}};

    cout << "mymap contains:";
    for (auto it = mymap.begin(); it != mymap.end(); ++it)
        cout << " " << it->first << ":" << it->second;
    cout << endl;

    cout << "mymap's buckets contain:\n";
    for (unsigned i = 0; i < mymap.bucket_count(); ++i) {
        cout << "bucket #" << i << " contains:";
        for (auto ir = mymap.begin(i); ir != mymap.end(i); ++ir)
            cout << " " << ir->first << ":" << ir->second;
        cout << endl;
    }
    return 0;
}
```

```
for (auto x : mymap)
    cout << " " << x.first << ":" << x.second;
```

A pair of glasses with a dark frame and light-colored lenses is resting on a piece of white paper. The background is a soft, out-of-focus yellow and orange gradient.

Data Structures: Hashing & Hash Tables

1. Hashing & Hash Table
2. Collision
3. Rehashing
4. Coding
 - Using list in STL
 - Using unordered_map in STL