Part 1

Import Libraries

In [2]:
```
import numpy as np
import modern_robotics as mr
import matplotlib.pyplot as plt
import random

# Set print options for numpy arrays
np.set_printoptions(precision=3, suppress=True)
```

Util Functions

In [11]:
```
def pi2pi(angle):
    while angle > np.pi:
        angle -= 2*np.pi
    while angle < -np.pi:
        angle += 2*np.pi
    return angle

def print_iteration(M, B, V_b, index, joint_vector):
    joint_vector = np.array(joint_vector)
    joint_vector = joint_vector.tolist()

    print(f"Iteration {index}:\n")

    print("joint vector:")
    joint_vector_to_print = joint_vector
    for i in range(len(joint_vector_to_print)):
        joint_vector_to_print[i] = round(joint_vector_to_print[i], 3)
    print(f"{str(joint_vector_to_print)[1:-1]}\n")

    print("SE(3) end-effector config:")
    Tsb = mr.FKinBody(M, B, joint_vector)
    decimal_places = 3
    width = 6
    print('\n'.join(' '.join(f"{cell:{width}.{decimal_places}f}" for cell ir
    print()

    decimal_places = 3
    Vb_to_print = V_b.tolist()
    for i in range(len(Vb_to_print)):
        Vb_to_print[i] = round(Vb_to_print[i], decimal_places)
    Vb_to_print = tuple(Vb_to_print)
    print(f"          error twist V_b: {Vb_to_print}")
    print(f"angular error ||omega_b||: {np.linalg.norm([V_b[0], V_b[1], V_b[
    print(f"     linear error ||v_b||: {np.linalg.norm([V_b[3], V_b[4], V_b[
    print("=============================================================

def save_csv(filename, data):
```

```
    np.savetxt(filename, data, delimiter=',')

def Vb(M, B, T_sd, joint_vector):
    Vb = mr.se3ToVec(mr.MatrixLog6(np.dot(mr.TransInv(mr.FKinBody(M, B, joir
    return Vb

def check_convergence(V_b, eps_w, eps_v):
    return (np.linalg.norm([V_b[0], V_b[1], V_b[2]]) > eps_w \
         or np.linalg.norm([V_b[3], V_b[4], V_b[5]]) > eps_v)

def IKinBodyIterates(M, B, T_sd, theta_0, traj_filename, max_iterations=100,
    index = 1
    joint_vector = np.array(theta_0)
    joint_vector = joint_vector.tolist()
    V_b = Vb(M, B, T_sd, theta_0)

    # For saving later
    traj = np.array(joint_vector)
    angular_error = np.array(np.linalg.norm([V_b[0], V_b[1], V_b[2]]))
    linear_error = np.array(np.linalg.norm([V_b[3], V_b[4], V_b[5]]))

    iterating = check_convergence(V_b, eps_w, eps_v)

    while iterating and index < max_iterations:
        joint_vector = joint_vector + np.dot(np.linalg.pinv(mr.JacobianBody(
        joint_vector = [np.arctan2(np.sin(theta), np.cos(theta)) for theta i
        V_b = Vb(M, B, T_sd, joint_vector)
        if(printing):
            print_iteration(M, B, V_b, index, joint_vector)
        traj = np.vstack([traj, joint_vector])
        angular_error = np.vstack([angular_error, np.array(np.linalg.norm([V
        linear_error = np.vstack([linear_error, np.array(np.linalg.norm([V_b
        iterating = check_convergence(V_b, eps_w, eps_v)

        if(iterating == False):
            if(printing):
                print(f"Converged. It took {index} iterations.")
                print("Trajectory:")
                print(traj)
                save_csv(traj_filename+'.csv', traj)
                save_csv(traj_filename+'_angular_error.csv', angular_error)
                save_csv(traj_filename+'_linear_error.csv', linear_error)
            break

        index += 1

    return index

def joint_traj2position_traj(joint_traj):
    joint_vector = joint_traj[0]
    pose_matrix = mr.FKinBody(M, B, joint_vector)
    position = pose_matrix[:3, 3]
    position_traj = np.array(position)
    for idx in range(1, joint_traj.shape[0]):
        joint_vector = joint_traj[idx]
        pose_matrix = mr.FKinBody(M, B, joint_vector)
```

```
        position = pose_matrix[:3, 3]
        position_traj = np.vstack([position_traj, position])
    return position_traj

def plot_position_traj(long_filename, short_filename):
    long_angle_traj = np.genfromtxt(long_filename+'.csv', delimiter=',')
    short_angle_traj = np.genfromtxt(short_filename+'.csv', delimiter=',')
    long_position_traj = joint_traj2position_traj(long_angle_traj)
    short_position_traj = joint_traj2position_traj(short_angle_traj)

    # Plot two trajectories in 3D space
    fig = plt.figure()
    ax = fig.add_subplot(111, projection='3d')
    ax.plot(long_position_traj[0,0], long_position_traj[0,1], long_position_
    ax.plot(long_position_traj[:, 0], long_position_traj[:, 1], long_positic
    ax.plot(long_position_traj[-1,0], long_position_traj[-1,1], long_positic

    ax.plot(short_position_traj[0,0], short_position_traj[0,1], short_positi
    ax.plot(short_position_traj[:, 0], short_position_traj[:, 1], short_posi
    ax.plot(short_position_traj[-1,0], short_position_traj[-1,1], short_posi

    ax.set_xlabel('X')
    ax.set_ylabel('Y')
    ax.set_zlabel('Z')
    ax.legend()
    plt.show()

def plot_angular_error(long_filename, short_filename):
    long_angular_error = np.genfromtxt(long_filename+'_angular_error.csv', d
    short_angular_error = np.genfromtxt(short_filename+'_angular_error.csv',
    plt.plot(long_angular_error, label='long_traj')
    plt.plot(short_angular_error, label='short_traj')
    plt.xlabel('Iterations')
    plt.ylabel('Angular Error')
    plt.legend()
    plt.show()

def plot_linear_error(long_filename, short_filename):
    long_linear_error = np.genfromtxt(long_filename+'_linear_error.csv', del
    short_linear_error = np.genfromtxt(short_filename+'_linear_error.csv', d
    plt.plot(long_linear_error, label='long_traj')
    plt.plot(short_linear_error, label='short_traj')
    plt.xlabel('Iterations')
    plt.ylabel('Linear Error')
    plt.legend()
    plt.show()

def filter_initial_guess(M, B, T_sd, lb, ub):
    # give a random initial guess
    ites = 0
    while (ites>=ub or ites<=lb):
        random_initial_guess = [random.uniform(-np.pi, np.pi) for i in range
        # round to 3 decimal places
        random_initial_guess = [round(i, 3) for i in random_initial_guess]
        random_traj_name = "random_traj"
        ites = IKinBodyIterates(M, B, T_sd, random_initial_guess, random_tra
```

```
    print(f"Found initial guess: {random_initial_guess} with {ites} iteratic
    return random_initial_guess
```

Constants

In [4]:
```
L1 = 0.425
L2 = 0.392
H1 = 0.089
H2 = 0.095
W1 = 0.109
W2 = 0.082

# Home Configuration:
M = np.array([[-1, 0, 0, L1+L2],
              [0, 0, 1, W1+W2],
              [0, 1, 0, H1-H2],
              [0, 0, 0, 1]])

# B:
B_list = []
B_list.append(np.array([0, 1, 0, W1+W2, 0, L1+L2]))
B_list.append(np.array([0, 0, 1, H2, -L1-L2, 0]))
B_list.append(np.array([0, 0, 1, H2, -L2, 0]))
B_list.append(np.array([0, 0, 1, H2, 0, 0]))
B_list.append(np.array([0, -1, 0, -W2, 0, 0]))
B_list.append(np.array([0, 0, 1, 0, 0, 0]))
B = np.column_stack(B_list)

# T_sd:
T_sd = np.array([[1, 0, 0, 0.3],
                 [0, 1, 0, 0.3],
                 [0, 0, 1, 0.4],
                 [0, 0, 0, 1]])
```

Find Appropriate Initial Theta Lists

In [5]:
```
# converges after 2–4 Newton-Raphson steps (short iterates), not more or les
short_traj_initial_guess = filter_initial_guess(M, B, T_sd, 2, 4)

# converges after 10 Newton-Raphson steps, or never converges (long iterates
long_traj_initial_guess = filter_initial_guess(M, B, T_sd, 11, np.inf)
```

```
Found initial guess: [-1.554, 2.462, 1.464, 0.288, 1.801, 0.389] with 3 iter
ations to converge.
Found initial guess: [0.834, -0.491, 3.134, -1.729, -0.532, 2.921] with 22 i
terations to converge.
```

Solve IK With the Initial Guess Resulting in **Long** Trajectory

In [16]:
```
long_traj_name = "long iterates"
IKinBodyIterates(M, B, T_sd, long_traj_initial_guess, long_traj_name)
```

```
Iteration 1:

joint vector:
-2.47, -2.957, -3.093, 2.928, -0.025, 1.527

SE(3) end-effector config:
-0.020  0.798  0.602  0.144
-0.014  0.602 -0.798 -0.129
-1.000 -0.024 -0.000  0.171
 0.000  0.000  0.000  1.000

          error twist V_b: (-0.705, -1.459, 0.739, -0.178, 0.52, 0.09)
angular error ||omega_b||: 1.781107191540879
     linear error ||v_b||: 0.5573832230715222

================================================================================
Iteration 2:

joint vector:
0.863, 1.843, 2.831, 1.255, -2.59, -0.519

SE(3) end-effector config:
 0.909  0.259  0.328 -0.119
 0.362 -0.097 -0.927 -0.078
-0.209  0.961 -0.182 -0.033
 0.000  0.000  0.000  1.000

          error twist V_b: (-1.688, -0.479, -0.092, 0.357, 0.661, 0.104)
angular error ||omega_b||: 1.7572528705011987
     linear error ||v_b||: 0.757821864184124

================================================================================
Iteration 3:

joint vector:
-1.232, -0.326, 0.048, -0.741, 0.916, -1.345

SE(3) end-effector config:
 0.420  0.562  0.713  0.447
-0.656  0.730 -0.190 -0.791
-0.627 -0.388  0.675  0.338
 0.000  0.000  0.000  1.000

          error twist V_b: (0.125, -0.842, 0.766, -0.585, 0.972, 0.003)
angular error ||omega_b||: 1.1454603223849076
     linear error ||v_b||: 1.1343751666568116

================================================================================
Iteration 4:

joint vector:
-0.453, -3.005, 2.133, -0.638, 1.634, -0.806

SE(3) end-effector config:
 0.952 -0.304  0.027 -0.017
 0.305  0.949 -0.083  0.124
```

```
-0.000  0.088  0.996  0.523
 0.000  0.000  0.000  1.000

          error twist V_b: (-0.087, -0.014, -0.309, 0.342, 0.121, -0.127)
angular error ||omega_b||: 0.3216527672596053
     linear error ||v_b||: 0.3842815880566039

========================================================================
Iteration 5:

joint vector:
1.408, 2.501, 1.999, 0.483, 1.31, -2.981

SE(3) end-effector config:
 0.977 -0.000 -0.213 -0.179
 0.065  0.953  0.297 -0.285
 0.203 -0.304  0.931  0.269
 0.000  0.000  0.000  1.000

          error twist V_b: (0.307, 0.212, -0.034, 0.504, 0.555, 0.168)
angular error ||omega_b||: 0.3748876422480335
     linear error ||v_b||: 0.7682236713147313

========================================================================
Iteration 6:

joint vector:
2.405, 2.254, -2.1, -1.477, 1.835, 2.347

SE(3) end-effector config:
 1.000 -0.007  0.000 -0.230
 0.006  0.936  0.352  0.090
-0.002 -0.352  0.936 -0.247
 0.000  0.000  0.000  1.000

          error twist V_b: (0.36, -0.002, -0.006, 0.53, 0.089, 0.679)
angular error ||omega_b||: 0.36050466495917344
     linear error ||v_b||: 0.8653642517998467

========================================================================
Iteration 7:

joint vector:
-0.519, -2.52, 2.071, -0.367, 0.86, -1.074

SE(3) end-effector config:
 0.550 -0.312  0.774  0.184
 0.102  0.946  0.309  0.082
-0.829 -0.091  0.552  0.487
 0.000  0.000  0.000  1.000

          error twist V_b: (0.239, -0.959, -0.248, 0.171, 0.208, 0.006)
angular error ||omega_b||: 1.0194319799551061
     linear error ||v_b||: 0.269193625412744

========================================================================
```

```
Iteration 8:

joint vector:
2.684, -1.831, 2.487, -0.428, -2.56, 2.513

SE(3) end-effector config:
 0.275  0.451  0.849 -0.140
-0.631 -0.582  0.513  0.024
 0.726 -0.677  0.124  0.178
 0.000  0.000  0.000  1.000

          error twist V_b: (1.626, -0.169, 1.478, 0.17, 0.284, 0.519)
angular error ||omega_b||: 2.203660220481679
     linear error ||v_b||: 0.615730495534247

================================================================================
Iteration 9:

joint vector:
1.472, 3.132, -2.488, 2.215, 1.449, 0.056

SE(3) end-effector config:
-0.973  0.082 -0.215 -0.140
 0.229  0.265 -0.937 -0.203
-0.020 -0.961 -0.277 -0.082
 0.000  0.000  0.000  1.000

          error twist V_b: (0.297, 2.399, -1.809, 1.088, -0.311, -0.492)
angular error ||omega_b||: 3.0188432600470803
     linear error ||v_b||: 1.2335653472027712

================================================================================
Iteration 10:

joint vector:
1.674, -1.674, 3.094, 1.809, -1.406, 2.8

SE(3) end-effector config:
-0.906 -0.332 -0.264 -0.133
-0.278 -0.007  0.961  0.091
-0.320  0.943 -0.086  0.212
 0.000  0.000  0.000  1.000

          error twist V_b: (0.671, -2.185, -2.096, -0.011, -0.386, 0.667)
angular error ||omega_b||: 3.1016566843180366
     linear error ||v_b||: 0.7710985577089325

================================================================================
Iteration 11:

joint vector:
2.127, -1.86, 2.396, 2.494, 2.166, 1.314

SE(3) end-effector config:
-0.161  0.381  0.911 -0.126
-0.140  0.905 -0.403  0.084
```

```
-0.977 -0.192 -0.092  0.383
 0.000  0.000  0.000  1.000

          error twist V_b: (-0.187, -1.674, 0.461, 0.255, 0.322, 0.331)
angular error ||omega_b||: 1.7460721283841358
     linear error ||v_b||: 0.527797904970423

================================================================================
Iteration 12:

joint vector:
0.426, -2.375, -2.1, 2.99, -1.821, 2.609

SE(3) end-effector config:
-0.822  0.569  0.027 -0.319
 0.543  0.798 -0.260 -0.048
-0.169 -0.199 -0.965 -0.084
 0.000  0.000  0.000  1.000

          error twist V_b: (-0.894, -2.88, 0.369, -0.59, 0.754, 0.724)
angular error ||omega_b||: 3.0383396277112946
     linear error ||v_b||: 1.2004161739935544

================================================================================
Iteration 13:

joint vector:
0.888, 1.779, 2.736, -1.903, 2.474, 2.883

SE(3) end-effector config:
 0.959 -0.076  0.272 -0.196
 0.231 -0.344 -0.910 -0.171
 0.163  0.936 -0.313  0.114
 0.000  0.000  0.000  1.000

          error twist V_b: (-1.898, -0.113, -0.315, 0.574, 0.519, -0.197)
angular error ||omega_b||: 1.9269984582887372
     linear error ||v_b||: 0.7983816415495258

================================================================================
Iteration 14:

joint vector:
2.074, -2.967, -2.724, -0.496, 1.826, -0.387

SE(3) end-effector config:
-0.880 -0.409 -0.243 -0.066
-0.260 -0.015  0.965 -0.063
-0.398  0.913 -0.093 -0.158
 0.000  0.000  0.000  1.000

          error twist V_b: (0.726, -2.124, -2.037, -0.311, -0.197, 0.901)
angular error ||omega_b||: 3.0311864415604486
     linear error ||v_b||: 0.973389846034667

================================================================================
```

```
Iteration 15:

joint vector:
-0.306, -3.139, 2.181, 1.018, 1.668, -2.07

SE(3) end-effector config:
-0.238  0.317  0.918 -0.088
-0.425  0.816 -0.392  0.134
-0.874 -0.483 -0.060  0.311
 0.000  0.000  0.000  1.000

          error twist V_b: (0.085, -1.675, 0.693, 0.141, 0.276, 0.384)
angular error ||omega_b||: 1.8144148324657048
     linear error ||v_b||: 0.49302623573459875

================================================================================
Iteration 16:

joint vector:
-0.681, 2.522, -2.904, -1.079, 0.567, -1.548

SE(3) end-effector config:
 0.778  0.249  0.577  0.203
-0.615  0.490  0.618  0.065
-0.129 -0.836  0.534  0.022
 0.000  0.000  0.000  1.000

          error twist V_b: (0.919, -0.447, 0.545, -0.048, 0.054, 0.474)
angular error ||omega_b||: 1.1578766648413341
     linear error ||v_b||: 0.4792205903660268

================================================================================
Iteration 17:

joint vector:
-2.785, 1.457, 1.848, 0.963, 1.958, -1.783

SE(3) end-effector config:
-0.927  0.287  0.241  0.295
-0.137 -0.859  0.493  0.026
 0.348  0.424  0.836 -0.160
 0.000  0.000  0.000  1.000

          error twist V_b: (0.454, 0.707, 2.795, -0.105, 0.047, 0.635)
angular error ||omega_b||: 2.9185858191554925
     linear error ||v_b||: 0.6454991586603375

================================================================================
Iteration 18:

joint vector:
-2.265, 2.548, 2.313, -0.141, 2.327, 0.748

SE(3) end-effector config:
 0.843  0.091 -0.531  0.168
 0.179  0.883  0.435  0.119
```

```
 0.508 -0.461  0.727  0.298
 0.000  0.000  0.000  1.000

          error twist V_b: (0.494, 0.573, -0.048, 0.166, 0.152, 0.103)
angular error ||omega_b||: 0.7580659643891672
     linear error ||v_b||: 0.24780753933581995

================================================================================
Iteration 19:

joint vector:
-1.751, 2.461, 1.918, -0.183, 1.575, -0.033

SE(3) end-effector config:
 0.978  0.188  0.084  0.182
-0.205  0.849  0.486  0.390
 0.020 -0.493  0.870  0.310
 0.000  0.000  0.000  1.000

          error twist V_b: (0.515, -0.034, 0.207, 0.127, -0.099, 0.068)
angular error ||omega_b||: 0.5564965404269925
     linear error ||v_b||: 0.17435573296645318

================================================================================
Iteration 20:

joint vector:
-2.09, 2.807, 1.709, 0.196, 1.793, 0.471

SE(3) end-effector config:
 0.980  0.058 -0.191  0.269
-0.037  0.993  0.110  0.287
 0.196 -0.100  0.975  0.414
 0.000  0.000  0.000  1.000

          error twist V_b: (0.106, 0.195, 0.047, 0.029, 0.014, -0.016)
angular error ||omega_b||: 0.22714814415109005
     linear error ||v_b||: 0.03646046948008749

================================================================================
Iteration 21:

joint vector:
-2.103, 2.786, 1.65, 0.285, 1.571, 0.534

SE(3) end-effector config:
 1.000 -0.002 -0.005  0.302
 0.002  1.000 -0.007  0.298
 0.005  0.007  1.000  0.399
 0.000  0.000  0.000  1.000

          error twist V_b: (-0.007, 0.004, -0.001, -0.002, 0.002, 0.001)
angular error ||omega_b||: 0.008523650641431324
     linear error ||v_b||: 0.003055527820815518

================================================================================
```

```
Iteration 22:

joint vector:
-2.096, 2.785, 1.651, 0.276, 1.571, 0.526

SE(3) end-effector config:
 1.000 -0.001  0.000  0.300
 0.001  1.000  0.000  0.300
-0.000 -0.000  1.000  0.400
 0.000  0.000  0.000  1.000

          error twist V_b: (0.0, 0.0, -0.0, 0.0, 0.0, -0.0)
angular error ||omega_b||: 6.221323477282383e-05
     linear error ||v_b||: 1.2429527304356725e-05

================================================================================
Converged. It took 22 iterations.
Trajectory:
[[ 0.834 -0.491  3.134 -1.729 -0.532  2.921]
 [-2.47  -2.957 -3.093  2.928 -0.025  1.527]
 [ 0.863  1.843  2.831  1.255 -2.59  -0.519]
 [-1.232 -0.326  0.048 -0.741  0.916 -1.345]
 [-0.453 -3.005  2.133 -0.638  1.634 -0.806]
 [ 1.408  2.501  1.999  0.483  1.31  -2.981]
 [ 2.405  2.254 -2.1   -1.477  1.835  2.347]
 [-0.519 -2.52   2.071 -0.367  0.86  -1.074]
 [ 2.684 -1.831  2.487 -0.428 -2.56   2.513]
 [ 1.472  3.132 -2.488  2.215  1.449  0.056]
 [ 1.674 -1.674  3.094  1.809 -1.406  2.8  ]
 [ 2.127 -1.86   2.396  2.494  2.166  1.314]
 [ 0.426 -2.375 -2.1    2.99  -1.821  2.609]
 [ 0.888  1.779  2.736 -1.903  2.474  2.883]
 [ 2.074 -2.967 -2.724 -0.496  1.826 -0.387]
 [-0.306 -3.139  2.181  1.018  1.668 -2.07 ]
 [-0.681  2.522 -2.904 -1.079  0.567 -1.548]
 [-2.785  1.457  1.848  0.963  1.958 -1.783]
 [-2.265  2.548  2.313 -0.141  2.327  0.748]
 [-1.751  2.461  1.918 -0.183  1.575 -0.033]
 [-2.09   2.807  1.709  0.196  1.793  0.471]
 [-2.103  2.786  1.65   0.285  1.571  0.534]
 [-2.096  2.785  1.651  0.276  1.571  0.526]]
```

Out[16]: 22

Solve IK With the Initial Guess Resulting in **Short** Trajectory

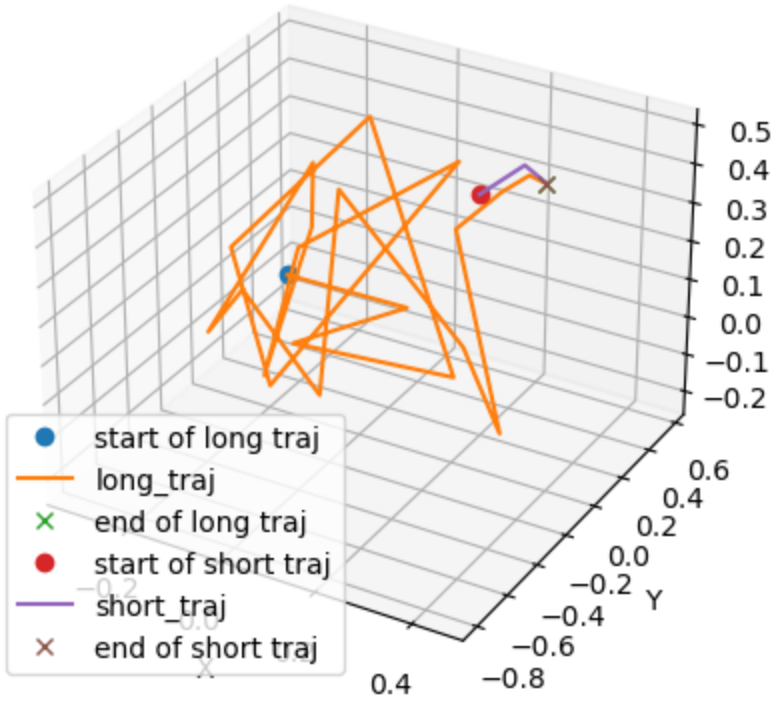
In [17]:
```
short_traj_name = "short iterates"
IKinBodyIterates(M, B, T_sd, short_traj_initial_guess, short_traj_name)
```

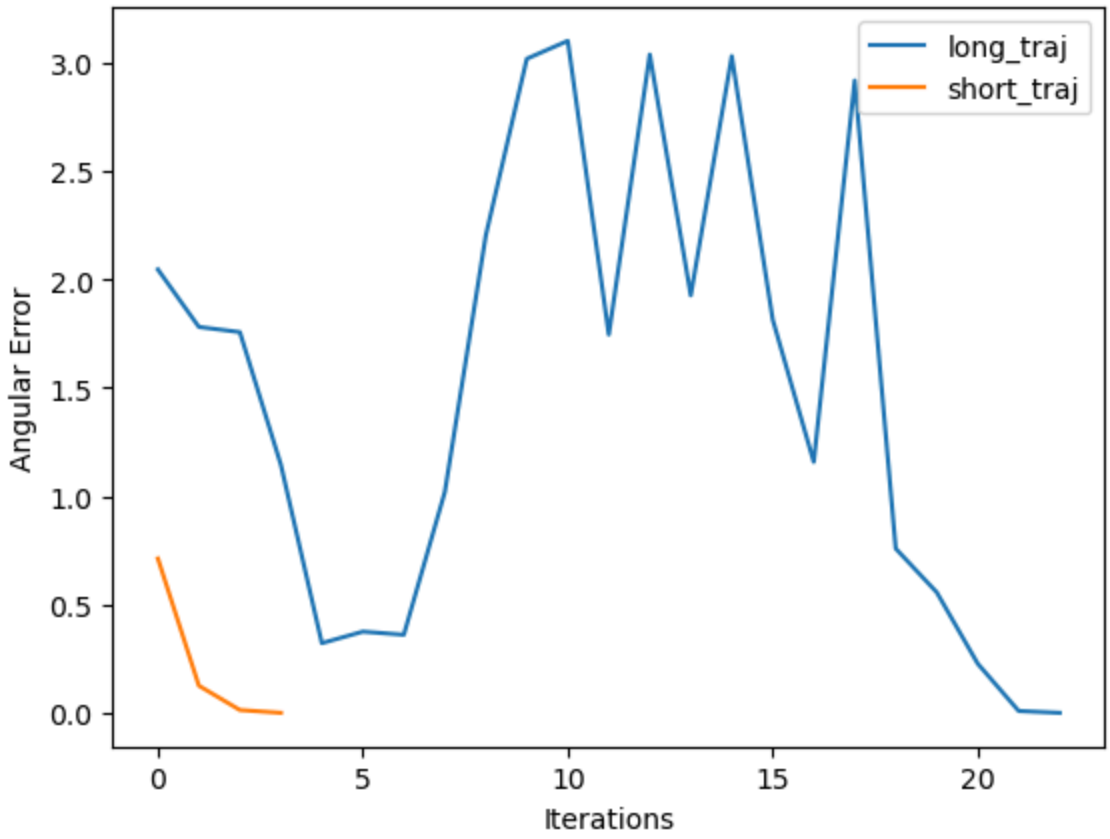
```
Iteration 1:

joint vector:
-1.991, 2.845, 1.655, 0.261, 1.686, 0.434

SE(3) end-effector config:
 0.992 -0.011 -0.125  0.250
 0.011  1.000  0.003  0.316
 0.125 -0.004  0.992  0.425
 0.000  0.000  0.000  1.000

          error twist V_b: (0.004, 0.125, -0.011, 0.048, -0.016, -0.028)
angular error ||omega_b||: 0.12534507476104986
     linear error ||v_b||: 0.057951035716249905

================================================================================
Iteration 2:

joint vector:
-2.102, 2.792, 1.651, 0.281, 1.566, 0.529

SE(3) end-effector config:
 1.000  0.002 -0.002  0.301
-0.002  1.000 -0.012  0.296
 0.002  0.012  1.000  0.402
 0.000  0.000  0.000  1.000

          error twist V_b: (-0.013, 0.002, 0.002, -0.001, 0.004, -0.002)
angular error ||omega_b||: 0.013039491663839251
     linear error ||v_b||: 0.004693607444187299

================================================================================
Iteration 3:

joint vector:
-2.096, 2.785, 1.651, 0.276, 1.571, 0.526

SE(3) end-effector config:
 1.000 -0.001  0.000  0.300
 0.001  1.000  0.000  0.300
-0.000 -0.000  1.000  0.400
 0.000  0.000  0.000  1.000

          error twist V_b: (0.0, 0.0, -0.0, 0.0, -0.0, -0.0)
angular error ||omega_b||: 6.943420225727724e-05
     linear error ||v_b||: 2.316914360262802e-05

================================================================================
Converged. It took 3 iterations.
Trajectory:
[[-1.554  2.462  1.464  0.288  1.801  0.389]
 [-1.991  2.845  1.655  0.261  1.686  0.434]
 [-2.102  2.792  1.651  0.281  1.566  0.529]
 [-2.096  2.785  1.651  0.276  1.571  0.526]]
```

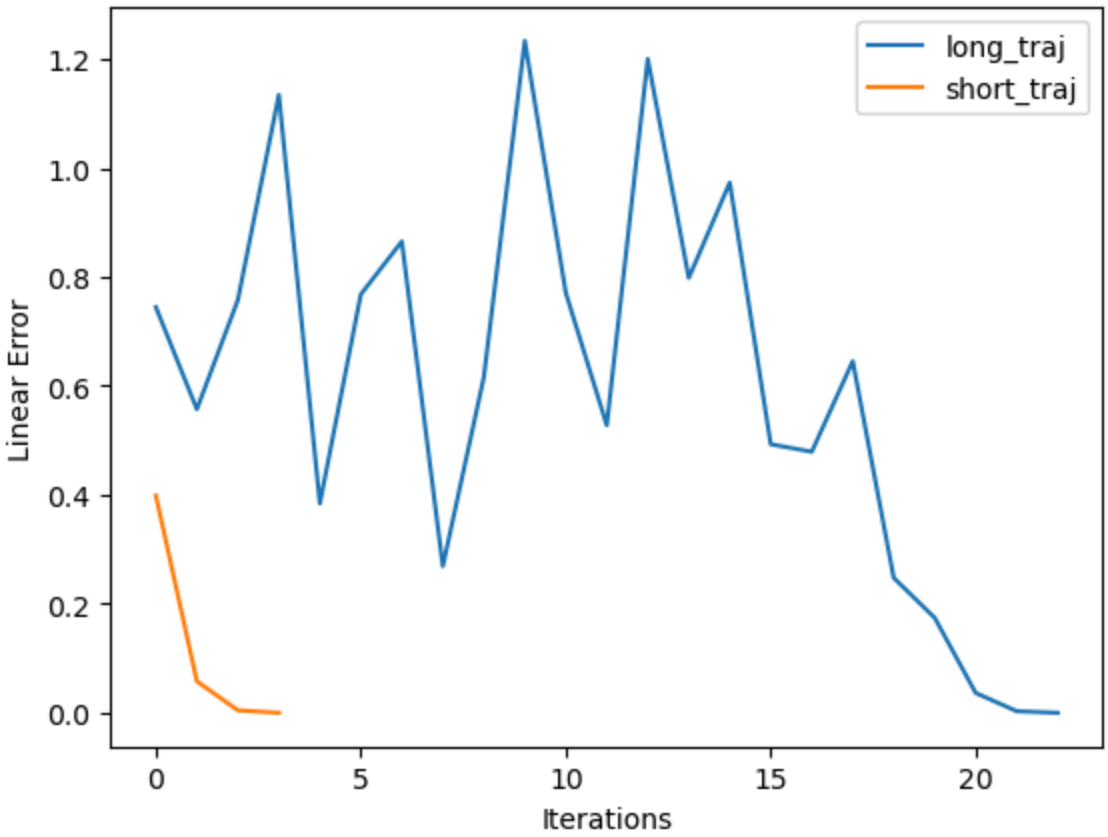
Out[17]: 3

Plot the results

In [18]:
```
plot_position_traj(long_traj_name, short_traj_name)
plot_angular_error(long_traj_name, short_traj_name)
plot_linear_error(long_traj_name, short_traj_name)
```

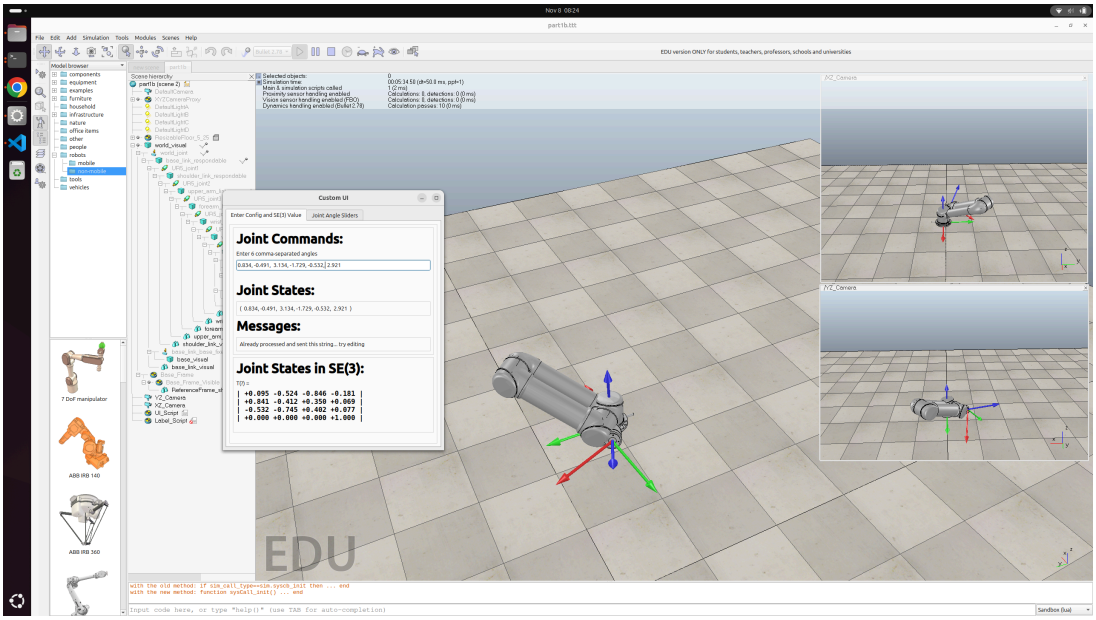




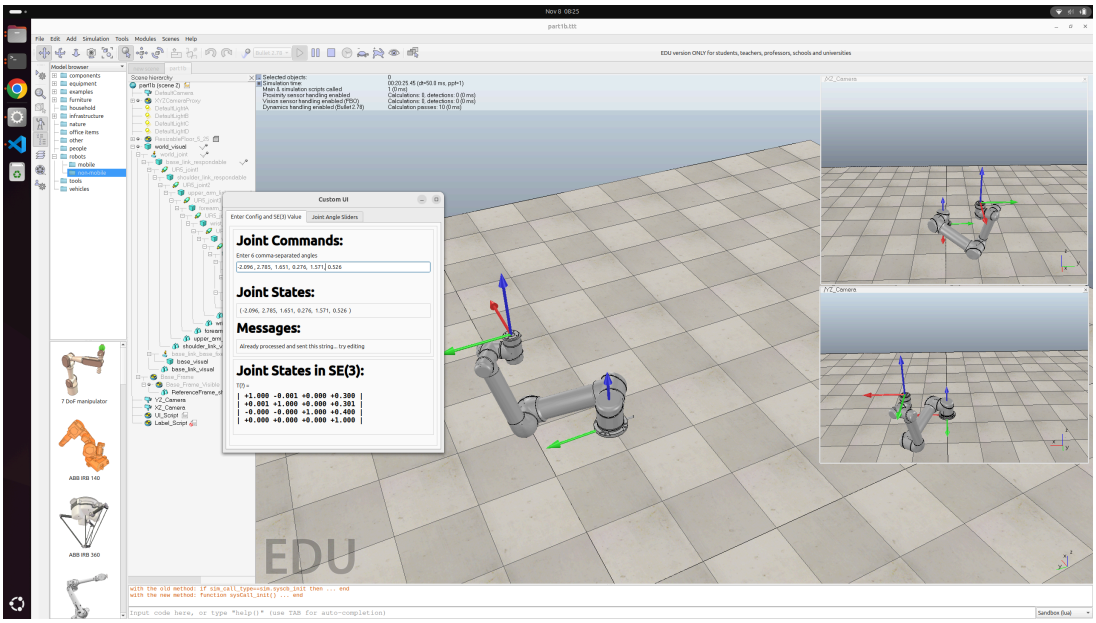


Long Trajectory IK in Simulation Visualization

- Start

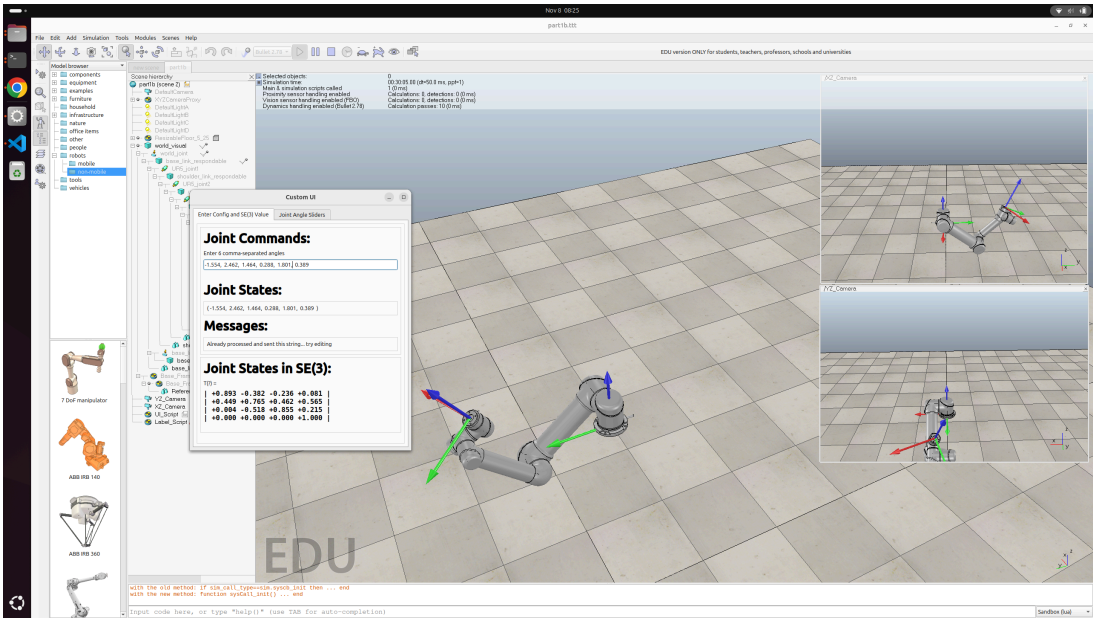


- End

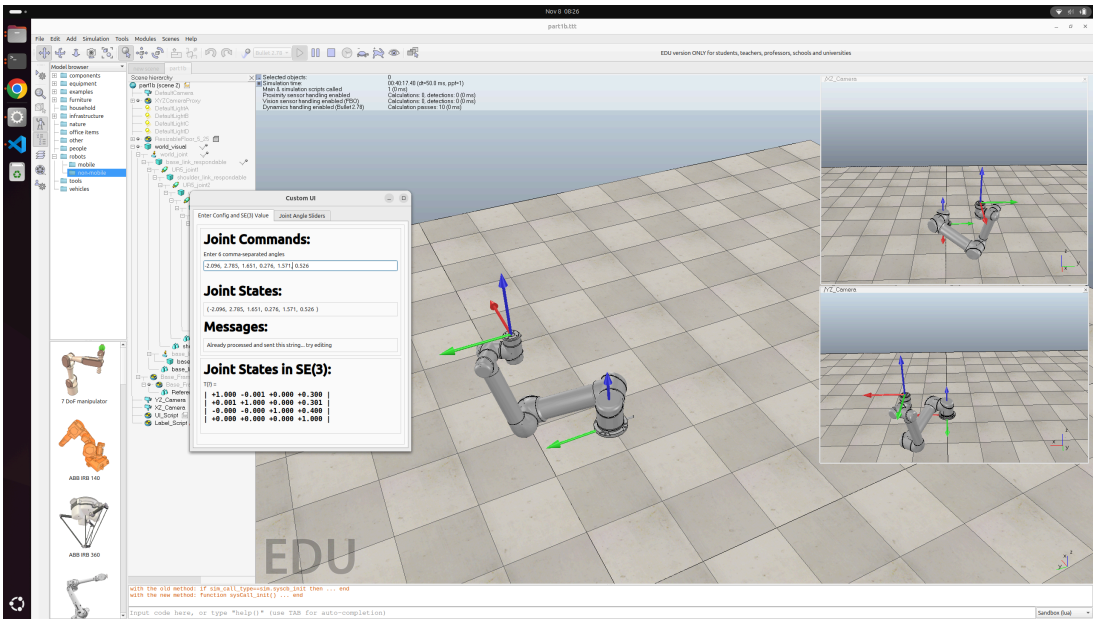


Short Trajectory IK in Simulation Visualization

- Start



- End



Part 2

Imagine the robot is powered up at a random configuration θ^0 such that $T_{sb}(\theta^0) = T^0$, and its first task is to move to a θ^* satisfying $T_{sb}(\theta^*) = T_{sd}$. Part 1 of this assignment deals with calculating an appropriate θ^*. In this part, you will consider how to actually control the robot to move from rest at θ^0 to rest at θ^* in t_f seconds.

(a) If you decide to move at constant joint speeds, what joint speed vector $\dot{\theta}$ do you command to the joints? Your answer should be symbolic in terms of relevant variables

Answer:

$$\Delta\theta = \theta^* - \theta^0 = \dot{\theta} \cdot t_f$$

$$Thus,\ \dot{\theta} = \frac{\theta^* - \theta^0}{t_f}$$

(b) If you decide to make the last link of the robot follow a single constant twist from T^0 to R_{sd}, what joint speed vector $\dot{\theta}(0)$ do you command to the joints at time $t = 0$? What is the commanded joint speed vector $\dot{\theta}(t_f/2)$, halfway through the motion? Your answers should be symbolic in terms of relevant variables. Use the function ***vec([$\mathcal{V}$])*** to convert $[\mathcal{V}] \in se(3)$ to the twist $\mathcal{V} \in \mathrm{R}^6$, and assume you have access to the body Jacobian $J_b(\theta)$.

Answer:

- Determine the Twist $\mathcal{V}$ to move from T^0 to T_{sb}

$$\mathcal{V} = \frac{vec(log((T^0)^{-1}T_{sd}))}{t_f}$$

- Determine $\dot{\theta}(0)$

$$\mathcal{V} = J_b(\theta^0)\dot{\theta}(0)$$
$$\dot{\theta}(0) = J_b(\theta^0)^{-1}\mathcal{V}$$
$$Thus,\ \dot{\theta}(0) = \frac{J_b(\theta^0)^{-1}vec(log((T^0)^{-1}T_{sd}))}{t_f}$$

- Determine $\dot{\theta}(t_f/2)$

$$\dot{\theta}(t_f/2) = J_b(\theta^{t_f/2})^{-1}\mathcal{V}$$
$$Thus,\ \dot{\theta}(t_f/2) = \frac{J_b(\theta^{t_f/2})^{-1}vec(log((T^0)^{-1}T_{sd}))}{t_f}$$

(c) Do you see any advantages to using the approach in (a) relative to (b), or vice-versa?

Answer:

- (a) is more simple in control strategy and has lower computational cost.
- (a) lacks direct cartesian path control for end effector, which can be dangerous in real world. (b) directly uses the information in cartesian space.