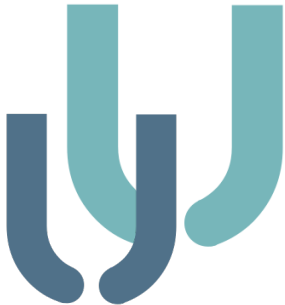
Introduction to Column-Generation
Theory and Application



Timo Gschwind

Thanks are due to Stefan Irnich(!), Christian Tilk, and many other colleagues

OeGOR Summer-School 2024, Krems
July 22-26, 2024

RPTU Rheinland-Pfälzische Technische Universität Kaiserslautern Landau
Logistik

In a Nutshell

'Column generation is the simplex algorithm for huge [=many(!) variables aka columns] *LPs.'*

Jacques Desrosiers

'Column generation is really simple if you ignore all the theory!'

unknown PhD colleague from Mainz

In a Nutshell

'Column generation is the simplex algorithm for huge [=many(!) variables aka columns] *LPs.'*

Jacques Desrosiers

'Column generation is really simple if you ignore all the theory!'

unknown PhD colleague from Mainz

Column Generation

// Input: LP with variables $\lambda_j, j \in J$

Select subset $J' \subset J$

repeat

  Solve LP restricted to $\lambda_j, j \in J'$

  if variable $\lambda_{j^\star}$ missing **then**

    $J' \leftarrow J' \cup \{j^\star\}$

until no variable was missing

// Output: Optimal solution $\bar{\lambda}_j, j \in J$ of LP

In a Nutshell

'Column generation is the simplex algorithm for huge [=many(!) variables aka columns] *LPs.'*

Jacques Desrosiers

'Column generation is really simple if you ignore all the theory!'

unknown PhD colleague from Mainz

Column Generation

// Input: LP with variables $\lambda_j, j \in J$

Select subset $J' \subset J$

repeat

  Solve LP restricted to $\lambda_j, j \in J'$

  if variable $\lambda_{j^\star}$ missing **then**

    $J' \leftarrow J' \cup \{j^\star\}$

until no variable was missing

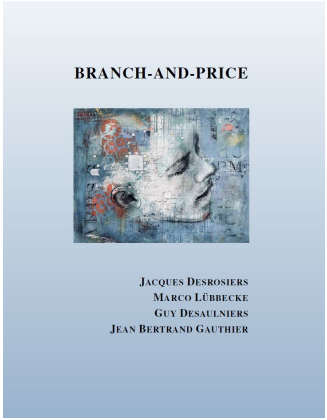
// Output: Optimal solution $\bar{\lambda}_j, j \in J$ of LP

Remark: Don't bother for LPs. Real benefits of CG are for integer programming

Where to Start – Collection of Main Reads

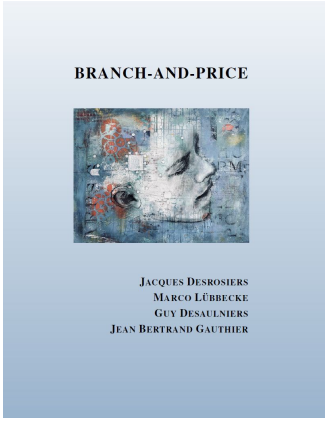
Where to Start – Collection of Main Reads

It's here! [Desrosiers et al., 2024]



Where to Start – Collection of Main Reads

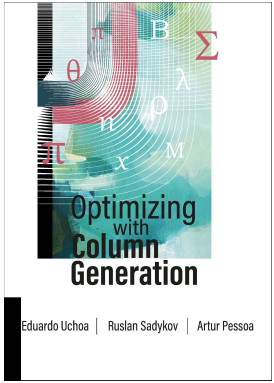
It's here! [Desrosiers et al., 2024]



And it's open access! `https://www.gerad.ca/en/papers/G-2024-36`

Where to Start – Collection of Main Reads

On the horizon:



- 'Scheduled to be finished by the end of the year [2023]'
- 'Beginner-friendly, starts from the basics'

Eduardo's presentation at Column Generation 2023 Workshop

Where to Start – Collection of Main Reads

Additionally:

- Desaulniers et al. [2005]
  - → CG book (not a classical/introductory textbook!)
- Desrosiers and Lübbecke [2005]
  - → the 'Primer on Column Generation'
- Lübbecke and Desrosiers [2005]
  - → survey: 'Selected Topics in Column Generation'
- Lübbecke [2010]
  - → CG in Wiley Encyclopedia of ORMS
- Lübbecke and Desrosiers [2010]
  - → Branch-Price-and-Cut in Wiley Encyclopedia of ORMS

Idea of the Course

- Transport a 'feeling' for CG-based approaches
  - → is a problem/formulation suitable for CG?
  - → how to design and implement a working B&P algorithm?
  - → what needs to be done to make it a good B&P algorithm?
    - > spoiler: many things to do/try...
- Examples, examples, some theory, more examples
- Hands-on approach
  - → implement your own algorithm
- Redundancy!! → I hope it helps ...

Agenda

1. Introduction to B&P: A full-fledged example for the VRPTW
2. Identifying suitable CG formulations of some fundamental optimization problems
3. Coding part 1
4. Basic theory (of CG and B&P)
   - → column generation
   - → Dantzig-Wolfe reformulation
   - → branching and cutting
5. Overcoming vanilla/textbook CG
   - → advice, suggestions, tips and tricks, things to try, ...
   - → pointers to more advanced topics
   - → include more coding parts

Guy Desaulniers, Jacques Desrosiers, and Marius M. Solomon, editors. Column Generation. Springer US, 2005.

Jacques Desrosiers and Marco E. Lübbecke. A primer in column generation. In Guy Desaulniers, Jacques Desrosiers, and Marius M. Solomon, editors, Column Generation, pages 1–32. Springer US, 2005.

Jacques Desrosiers, Marco Lübbecke, Guy Desaulniers, and Jean-Bertrand Gauthier. Branch-and-price. Les Cahiers du GERAD G-2024-36, Groupe d'études et de recherche en analyse des décisions, GERAD, Montréal QC H3T 2A7, Canada, 2024.

Marco E. Lübbecke. Column generation. Wiley Encyclopedia of Operations Research and Management Science, 2010.

Marco E. Lübbecke and Jacques Desrosiers. Selected topics in column generation. Operations Research, 53(6):1007–1023, 2005.

Marco E. Lübbecke and Jacques Desrosiers. Branch-price-and-cut algorithms. Wiley Encyclopedia of Operations Research and Management Science, 2010.