Since GATE Overflow (GO) started in August 2014, a lot of people have dedicated their time and effort in bringing this book now. Initiated by Omesh Pandita and Arjun Suresh as a Q/A platform for CSE students, Kathleen Bankson was instrumental in getting all previous year GATE questions here. Then experts like Praveen Saini, Happy Mittal, Sankaranarayanan P.N., Suraj Kumar etc. have contributed a lot to the answers in GO. Pragy Agarwal even after topping GATE has continuously contributed in GO with his knowledge as well as in making the contents beautiful with fine latex skills. We also have to thank the work done by our editors Jotheeswari Arasakumar, Misbah Ghaya, Ishrat Jahan, Nataliyah Ahmad, Kanza Bayad, Andrijana Milicevik and Pavan Singh who are continuously adding and keeping the contents in GO neat and clean. There are also many toppers of GATE 2015-18 who are contributing a lot in GO. The list of all the contributors can be found on GO site but even that does not include the contributions of some like Arif Ali Anapparakkal in helping design this book, Arvind Devaraj and others who have provided guidance and help, Aravind S.R. who helped fixing the page numbers of this book, Resmi A.C. who lend her Macbook for this book making, Marlyn Joseph in providing support for book printing and Akshat Joshi for book design support. Our final proof reading and editing were done with the help of M. Chandrakiruthiga, Pooja Khatri, Shikha Mallick, Manoja Rajalakshmi Aravindakshan, Subarna Das and Sukanya Das.

From the GO family, we thank the contributions of Silpa V.S., Digvijay Pandey, Rahul Kumar Yadav and others for getting the GATECSE Last-Rank page maintained. Bikram Ballav is behind most of the exams on GO (http://mockgate.com) and our exam interface is contributed by Arindam Sarkar. Pragy Agarwal is also behind the rank and score predictor tool, (http://mymarks.gatecse.in) used by GO which has 99-100\% accuracy over the last 3 years. SSpecial thanks to Sachin Mittal for making the How to Use GO videos, Silpa V.S. for classifying the questions topic-wise for the book, Pooja Palod for making the GATE 2018 schedule and Debashish Deka, Priti Sharma for GO classroom contributions. We also thank all the GATE toppers who took time to write a review for GO. Finally, on behalf of GO family and all GATE aspirants, we thank the wonderful faculties of IISc., IITs and other premier institutes who are behind the creation of such wonderful questions.

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Vol.1

Arjun Suresh

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This book was created programmatically by GATE Overflow for GATE CSE 2019. This book is having more than 3000 questions which were all from previous year standard exams like GATE, TIFR, CMI and ISI. Our site is having more than 10 times questions but due to difficulty in organizing them and filtering bad ones as of now we do not have them available in any book form but you can see all of them by visiting the site. If you feel any doubt regarding the answer, click on the question link and give a comment on the site. Studying all these questions might get you 60 marks in GATE but that might not be enough for an IIT. So, read standard books whenever you have a doubt, solve exercise questions and use these questions for cementing the concepts and aim 85+. At least if you are not getting the solution to a given problem first refer standard book and if not standard videos. For more preparation doubts you may see the FAQ section. If any error is found on any question it shall be updated at https://gateoverflow.in/corrections. If you want a hardcopy of this book you can get it from this link.

PDFs for the remaining subjects will be made available at http://classroom.gateoverflow.in and you can enroll in this course to get notification for the same. Enrollment is free and account details are of GATE Overflow with a new password which have been sent to all registered emails on GATE Overflow. New users will receive this email within a day of confirming their email address.

You can now join our Facebook group for GATE CSE discussions.

You can visit http://playlists.gatecse.in for high quality videos for GATE CSE and how to use GO site/ebook.

This book consists of only previous year GATE, TIFR, ISI and CMI questions (CS from 1987 and all 5 years of IT) all of which are relevant for GATE. Out of syllabus subjects as of GATE 2019 are removed from this book except in rare cases.

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FAQ

Best Books for GATE CSE

Standard books are recommended for a good understanding of Computer Science subjects which is very essential for GATE. For each subject you may choose ANY ONE of the books and not all. You do not need to read every chapter or even every line of a chapter as GATE is not an essay writing examination. What to cover and how to cover for each subject is given in GO Classroom. For full list of recommended standard books please visit Best books by GATE CSE.

Best Videos for GATE CSE

Videos are for those who are not able to follow standard books. Like standard books, to top GATE one must follow standard videos. You should not go behind popular GATE videos which are publicized by marketing guys because nothing can beat quality videos from great professors like Shai Simonson. Also some people try to instigate fear in students that they cannot study from standard videos - again this is just to take your money and fetch you a low rank in GATE. So, if you want to score well in GATE and struggling with some topics, please watch the videos listed in GO Youtube Channel.

Is Coaching necessary for GATE?

I have seen a lot of questions regarding COACHING. It is something which is currently being injected to the blood of students that without coaching success is not possible. When questions are being asked from a predefined text those who practice them a lot can succeed. But GATE is different. Only with proper knowledge and problem solving skills one can crack GATE. In doing this coaching might help a few people. Again coaching alone does not do everything for you to top GATE. You may read here to know more about how coaching is useful for GATE.

What is GO Classroom for GATE 2019?

Since a lot of aspirants were looking for schedule and materials for GATE 2019, we are adding all the stuffs to GO Classroom (All GO users get GO classroom access once they confirm their email, if you do not have a GO account you can create here). For GATE 2019, the classroom materials are updated daily based on the live discussions happening at the offline location in Thiruvananthapuram. Materials include, reading part of standard text books, other reference slides/web pages, problems and programming assignments to make the understanding of topics crystal clear. While GO creates a lot of toppers aim of GO Classroom is not to create any loser - all deserving people should get to IITs.

Is there any necessary condition to start GATE preparation?

Only required condition is the dedication and attitude to top GATE. You can read more here.

How to crack GATE?

First of all, do not say that you want to crack GATE. Cracking GATE is very difficult- there are more than 100,000 people trying to do it and only about 2000 seats including NITs. But if you study your Bachelor subjects in the right way, GATE will automatically come to you. No extra preparation is needed. Take a look at GATE syllabus, note down the important ones, and note down the important topics in them. For those subjects at least, use Standard text books, understand the topics well by doing practice problems at the end of the chapters (like doing exercises in Cormen). For practice you should use previous year GATE papers as that tells you how GATE questions are framed. Strictly avoid badly framed test series questions while preparing. For more tips you can see here

How to use GATE Overflow to top GATE?

Since GATE Overflow was started in 2014, we have added a lot of features and some of these are unknown to many. You may watch these videos to know about these and how to use GATE Overflow site and PDF in the best way.

What rank I need to get to old IITs?

There are a lot of mosconceptions regarding this. As of now one can get direct admits to old IITs upto around 330 GATE rank in general category and around 1000 rank for OBC NCL. But general category people with even above 1500 rank can get to old IITs for Research admits via interviews. This is one more reason why one should prepare using standard resources and do good problems and assignments. You can see the respective tabs here to know the exact criteria and cutoffs of each IIT and other institutes.

Does GATE Overflow provide test series for GATE CSE?

You can find all the tests on GO here. Under Tests by Mentors, you can find all the tests made by Bikram Ballav which includes 4 full length tests and 2 subject wise tests for each subject. Under GATE Overflow tests we will be adding subjectwise tests as well as full length tests for ensuring every important point is covered. For more details on tests you may refer here.

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1 Discrete Mathematics: Combinatory (62) top

1.1 Generating Functions(6) top

1.1.1 Generating Functions: GATE1987-10b top https://gateoverflow.in/82451

What is the generating function $G(z)$ for the sequence of Fibonacci numbers?

gate1987 permutations-and-combinations generating-functions descriptive

Answer

1.1.2 Generating Functions: GATE2005-50 top https://gateoverflow.in/1175

Let $G(x) = \frac{1}{(1-x)^2} = \sum_{i=0}^{\infty} g(i)x^i$, where $|x| < 1$. What is $g(i)$?

A. i
B. $i+1$
C. $2i$
D. 2^i

gate2005 normal generating-functions

Answer

1.1.3 Generating Functions: GATE2016-1-26 top https://gateoverflow.in/39693

The coefficient of x^{12} in $\left(x^3 + x^4 + x^5 + x^6 + \ldots\right)^3$ is _____.

gate2016-1 permutations-and-combinations generating-functions normal numerical-answers

Answer

1.1.4 Generating Functions: GATE2017-2-47 top https://gateoverflow.in/118392

If the ordinary generating function of a sequence $\{a_n\}_{n=0}^{\infty}$ is $\frac{1+z}{(1-z)^3}$, then $a_3 - a_0$ is equal to _____ .

gate2017-2 permutations-and-combinations generating-functions numerical-answers normal

Answer

1.1.5 Generating Functions: GATE2018-1 top https://gateoverflow.in/204075

Which one of the following is a closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0, 1, 2, \ldots$?

A. $\frac{3}{(1-x)^2}$
B. $\frac{3x}{(1-x)^2}$
C. $\frac{2-x}{(1-x)^2}$
D. $\frac{3-x}{(1-x)^2}$

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gate2018 generating-functions normal

Answer

1.1.6 Generating Functions: TIFR2010-A-12 top https://gateoverflow.in/18391

The coefficient of x^3 in the expansion of $(1+x)^3(2+x^2)^{10}$ is.

A. 2^{14}
B. 31
C. $\binom{3}{3}+\binom{10}{1}$
D. $\binom{3}{3}+2\binom{10}{1}$
E. $\binom{3}{3}\binom{10}{1}2^9$

tifr2010 generating-functions

Answer

Answers: Generating Functions

1.1.1 Generating Functions: GATE1987-10b top https://gateoverflow.in/82451

Selected Answer

The general form is $G(x)=\frac{x}{1-x-x^2}$

and after solving this using partial fraction, we will get

$f_n=\frac{1}{\sqrt{5}}((\frac{1+\sqrt{5}}{2})^n-(\frac{1-\sqrt{5}}{2})^n)$

11 votes -- Manu Madhavan (1.3k points)

1.1.2 Generating Functions: GATE2005-50 top https://gateoverflow.in/1175

Selected Answer

$\frac{1}{1-x}=1+x+x^2+x^3+x^4+x^5+\cdots+\infty$

Differentiating it w.r.to x

$\frac{1}{(1-x)^2}=1+2x+3x^2+4x^3+5x^4+\cdots+\infty$

$\sum_{i=0}^{\infty}g(i)x^i=g(0)+g(1)x+g(2)x^2+g(3)x^3+\cdots+\infty$

Comparing above two, we get $g(1)=2, g(2)=3 \Rightarrow g(i)=i+1$

17 votes -- Manish Joshi (27.9k points)

1.1.3 Generating Functions: GATE2016-1-26 top https://gateoverflow.in/39693

Selected Answer

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we will get x^{12} as

1. $(x^4)^3$ having coefficient ${}^3C_0 = 1$
2. $(x^3)^2(x^6)$ having coefficient ${}^3C_1 = 3$
3. $(x^3)(x^4)(x^5)$ having coefficient ${}^3C_2 \times {}^2C_1 = 6$

So it is 10

Second Method:

$$\left[\mathbf{x^{12}}\right]\left(x^3+x^4+x^5+x^6+\ldots\right)^3$$
$$\left[\mathbf{x^{12}}\right]\left[x^3\left(1+x^1+x^2+x^3+\ldots\right)\right]^3$$
$$\left[\mathbf{x^{12}}\right]\left[x^9\left(1+x^1+x^2+x^3+\ldots\right)^3\right]$$
$$\left[\mathbf{x^{3}}\right]\left[\left(1+x^1+x^2+x^3+\ldots\right)^3\right]$$
$$\left[\mathbf{x^{3}}\right]\left[\left(\frac{1}{1-x}\right)^3\right]$$
$$\left[\mathbf{x^{3}}\right]\left[\sum_{k=0}^{\infty}\binom{3+k-1}{k}x^k\right]$$
$$\text{Now , put k} = 3$$
$$\text{Coefficient of } \left[\mathbf{x^{3}}\right] = \binom{2+3}{3} = 5C3 = 5C2 = 10$$

$$\left[\mathbf{x^{12}}\right]\left(x^3+x^4+x^5+x^6+\ldots\right)^3 \Rightarrow \mathbf{10}$$

40 votes -- Praveen Saini (54.5k points)

1.1.4 Generating Functions: GATE2017-2-47 top https://gateoverflow.in/118392

Selected Answer

$\frac{1+z}{(1-z)^3} = (1+z)(1-z)^{-3}$

$(1-z)^{-3} = 1 + \binom{3}{1}z + \binom{4}{2}z^2 + \binom{5}{3}z^3 + \ldots\infty$

$(1+z)(1-z)^{-3} = (1+z) * (1 + \binom{3}{1}z + \binom{4}{2}z^2 + \binom{5}{3}z^3 + \ldots\infty)$

a_0 is the first term in the expansion of above series and a_3 is the fourth term (or) coefficient of z^3

a_0 = coefficient of $z^0 = 1$
a_3 = coefficient of $z^3 = \binom{5}{3} + \binom{4}{2} = 10 + 6$

$\Rightarrow a_3 - a_0 = 16 - 1 = 15$

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52 votes -- Manish Joshi (27.9k points)

1.1.5 Generating Functions: GATE2018-1 top

https://gateoverflow.in/204075

Selected Answer

Given that $a_n = 2n + 3$

Let $G(x)$ be the generating function for the sequence $\{a_n\}$.

So, $G(x) = \sum_{n=0}^{\infty} a_n x^n$
$= \sum_{n=0}^{\infty} (2n+3)x^n$
$= \sum_{n=0}^{\infty} (2n)x^n + \sum_{n=0}^{\infty} (3)x^n$
$= 2\sum_{n=0}^{\infty} nx^n + 3\sum_{n=0}^{\infty} x^n$
$= 2A + 3B$

Now, $A = \sum_{n=0}^{\infty} nx^n$. By expanding, it will look like: $0 + 1x + 2x^2 + 3x^3 + \ldots$ which is an AGP series with first term, $(a) = 0$, common difference, $(d) = 1$, ratio, $(r) = x$.

Sum of infinite AGP series $= \frac{a}{1-r} + \frac{dr}{(1-r)^2}$.

So, $A = \frac{0}{1-x} + \frac{x}{(1-x)^2} = \frac{x}{(1-x)^2}$

and $B = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}$

Therefore, $2A + 3B = \frac{2x}{(1-x)^2} + \frac{3}{1-x}$

$= \frac{2x+3-3x}{(1-x)^2} = \frac{3-x}{(1-x)^2}$

Option (D) is correct.

13 votes -- Pinaki Dash (1.6k points)

1.1.6 Generating Functions: TIFR2010-A-12 top

https://gateoverflow.in/18391

Selected Answer

$(1+x)^3 = (1 + x^3 + 3x + 3x^2)$

and

$(2+x^2)^{10} = {}^{10}C_0 * 2^0 * (x^2)^{10} + {}^{10}C_1 * 2^1 * (x^2)^9 + \ldots + {}^{10}C_9 * 2^9 * (x^2)^1 + {}^{10}C_{10} * 2^{10} * (x^2$

So,coefficient of $x^3 = {}^{10}C_{10} * 2^{10} + 3 * {}^{10}C_9 * 2^9$

$= 2^9(32) = 2^{14}$.

As here we need to multiply last term of second expansion with first term of first coefficient x^3 and $3x$ with x^2 in the second expansion.

20 votes -- worst_engineer (4.1k points)

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1.2 Modular Arithmetic(1) top

1.2.1 Modular Arithmetic: GATE2016-2-29 top

https://gateoverflow.in/39588

The value of the expression $13^{99} \pmod{17}$ in the range 0 to 16, is _____.

gate2016-2 modular-arithmetic normal numerical-answers

Answer

Answers: Modular Arithmetic

1.2.1 Modular Arithmetic: GATE2016-2-29 top

https://gateoverflow.in/39588

Selected Answer

By Fermat's Little Theorem, if p is prime, then

$a^{p-1} \equiv 1 \bmod p$.

So, $13^{16} \equiv 1 \bmod 17$.

And, $13^{96} = 13^{16\times 6} \equiv 1 \bmod 17$.

We are left with $13^{99} = 13^{96} \times 13^3 \equiv 13^3 \bmod 17 \equiv 2197 \bmod 17$ which is 4.

43 votes -- Gaurav Sharma (2.4k points)

1.3 Permutations And Combinations(42) top

1.3.1 Permutations And Combinations: CMI2010-A-02 top

https://gateoverflow.in/46132

We need to choose a team of 11 from a pool of 15 players and also select a captain. The number of different ways this can be done is

A. $\binom{15}{11}$

B. $11 . \binom{15}{11}$

C. $15.14.13.12.11.10.9.8.7.6.5$

D. $(15.14.13.12.11.10.9.8.7.6.5).11$

cmi2010 permutations-and-combinations normal discrete-mathematics

Answer

1.3.2 Permutations And Combinations: CMI2014-A-01 top

https://gateoverflow.in/46864

For the inter-hostel six-a-side football tournament, a team of 6 players is to be chosen from 11 players consisting of 5 forwards, 4 defenders and 2 goalkeepers. The team must include at least 2 forwards, at least 2 defenders and at least 1 goalkeeper. Find the number of different ways in which the team can be chosen.

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A. 260
B. 340
C. 720
D. 440

cmi2014 permutations-and-combinations discrete-mathematics normal

Answer

1.3.3 Permutations And Combinations: GATE1989-4-i top

https://gateoverflow.in/87874

Provide short answers to the following questions:

How many substrings (of all lengths inclusive) can be formed from a character string of length n? Assume all characters to be distinct, prove your answer.

gate1989 descriptive permutations-and-combinations discrete-mathematics normal

Answer

1.3.4 Permutations And Combinations: GATE1990-3-iii top

https://gateoverflow.in/84060

Choose the correct alternatives (More than one may be correct).

The number of rooted binary trees with n nodes is,

A. Equal to the number of ways of multiplying $(n+1)$ matrices.
B. Equal to the number of ways of arranging n out of $2n$ distinct elements.
C. Equal to $\frac{1}{(n+1)}\binom{2n}{n}$.
D. Equal to $n!$.

gate1990 normal permutations-and-combinations

Answer

1.3.5 Permutations And Combinations: GATE1990-3-ix top

https://gateoverflow.in/84841

Choose the correct alternatives (More than one may be correct).

The number of ways in which 5 A's, 5 B's and 5 C's can be arranged in a row is:

A. $15!/(5!)^3$
B. $15!$
C. $\binom{15}{5}$
D. $15!(5!3!)$.

gate1990 normal permutations-and-combinations

Answer

1.3.6 Permutations And Combinations: GATE1991-02,iv top

https://gateoverflow.in/514

Match the pairs in the following questions by writing the corresponding letters only.

A. The number distinct binary trees with n nodes.	P. $\frac{n!}{2}$
B. The number of binary strings of length of $2n$ with an equal number of $0's$ and $1's$.	Q. $\binom{3n}{n}$

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C. The number of even permutation of n objects.	R. $\binom{2n}{n}$
D. The number of binary strings of length $6n$ which are palindromes with $2n$ $0's$.	S. $\frac{1}{1}\binom{2n}{n}$ $+n$

gate1991 permutations-and-combinations normal match-the-following

Answer

1.3.7 Permutations And Combinations: GATE1991-16,a top

Find the number of binary strings w of length $2n$ with an equal number of $1's$ and $0's$ and the property that every prefix of w has at least as many $0's$ as $1's$.

gate1991 permutations-and-combinations normal descriptive

Answer

1.3.8 Permutations And Combinations: GATE1994-1.15 top

The number of substrings (of all lengths inclusive) that can be formed from a character string of length n is

A. n
B. n^2
C. $\frac{n(n-1)}{2}$
D. $\frac{n(n+1)}{2}$

gate1994 permutations-and-combinations normal

Answer

1.3.9 Permutations And Combinations: GATE1998-1.23 top

How many sub strings of different lengths (non-zero) can be found formed from a character string of length n?

A. n
B. n^2
C. 2^n
D. $\frac{n(n+1)}{2}$

gate1998 permutations-and-combinations normal

Answer

1.3.10 Permutations And Combinations: GATE1999-1.3 top

The number of binary strings of n zeros and k ones in which no two ones are adjacent is

A. ${}^{n-1}C_k$

B. ${}^{n}C_k$

C. ${}^{n}C_{k+1}$

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D. None of the above

gate1999 permutations-and-combinations normal

Answer

1.3.11 Permutations And Combinations: GATE1999-2.2 top

https://gateoverflow.in/1480

Two girls have picked 10 roses, 15 sunflowers and 15 daffodils. What is the number of ways they can divide the flowers among themselves?

A. 1638
B. 2100
C. 2640
D. None of the above

gate1999 permutations-and-combinations normal

Answer

1.3.12 Permutations And Combinations: GATE2000-5 top

https://gateoverflow.in/676

A multiset is an unordered collection of elements where elements may repeat any number of times. The size of a multiset is the number of elements in it, counting repetitions.

a. What is the number of multisets of size 4 that can be constructed from n distinct elements so that at least one element occurs exactly twice?
b. How many multisets can be constructed from n distinct elements?

gate2000 permutations-and-combinations normal descriptive

Answer

1.3.13 Permutations And Combinations: GATE2001-2.1 top

https://gateoverflow.in/719

How many 4-digit even numbers have all 4 digits distinct

A. 2240
B. 2296
C. 2620
D. 4536

gate2001 permutations-and-combinations normal

Answer

1.3.14 Permutations And Combinations: GATE2002-13 top

https://gateoverflow.in/866

a. In how many ways can a given positive integer $n \geq 2$ be expressed as the sum of 2 positive integers (which are not necessarily distinct). For example, for $n = 3$ the number of ways is 2, i.e., $1 + 2, 2 + 1$. Give only the answer without any explanation.
b. In how many ways can a given positive integer $n \geq 3$ be expressed as the sum of 3 positive integers (which are not necessarily distinct). For example, for $n = 4$, the number of ways is 3, i.e., $1 + 2 + 1, 2 + 1 + 1$. Give only the answer without explanation.
c. In how many ways can a given positive integer $n \geq k$ be expressed as the sum of k positive integers (which are not necessarily distinct). Give only the answer without explanation.

gate2002 permutations-and-combinations normal descriptive

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Answer

1.3.15 Permutations And Combinations: GATE2003-34 top

https://gateoverflow.in/924

m identical balls are to be placed in n distinct bags. You are given that $m \geq kn$, where k is a natural number ≥ 1. In how many ways can the balls be placed in the bags if each bag must contain at least k balls?

A. $\binom{m-k}{n-1}$
B. $\binom{m-kn+n-1}{n-1}$
C. $\binom{m-1}{n-k}$
D. $\binom{m-kn+n+k-2}{n-k}$

gate2003 permutations-and-combinations normal

Answer

1.3.16 Permutations And Combinations: GATE2003-4 top

https://gateoverflow.in/895

Let A be a sequence of 8 distinct integers sorted in ascending order. How many distinct pairs of sequences, B and C are there such that

i. each is sorted in ascending order,
ii. B has 5 and C has 3 elements, and
iii. the result of merging B and C gives A

A. 2
B. 30
C. 56
D. 256

gate2003 permutations-and-combinations normal

Answer

1.3.17 Permutations And Combinations: GATE2003-5 top

https://gateoverflow.in/896

n couples are invited to a party with the condition that every husband should be accompanied by his wife. However, a wife need not be accompanied by her husband. The number of different gatherings possible at the party is

A. ${}^{2n}\mathrm{C}_n \times 2^n$
B. 3^n
C. $\frac{(2n)!}{2^n}$
D. ${}^{2n}\mathrm{C}_n$

gate2003 permutations-and-combinations normal

Answer

1.3.18 Permutations And Combinations: GATE2004-75 top

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Mala has the colouring book in which each English letter is drawn two times. She wants to paint each of these 52 prints with one of k colours, such that the colour pairs used to colour any two letters are different. Both prints of a letter can also be coloured with the same colour. What is the minimum value of k that satisfies this requirement?

A. 9
B. 8
C. 7
D. 6

gate2004 permutations-and-combinations

Answer

1.3.19 Permutations And Combinations: GATE2004-IT-35 top

https://gateoverflow.in/3678

In how many ways can we distribute 5 distinct balls, B_1, B_2, ..., B_5 in 5 distinct cells, C_1, C_2, ..., C_5 such that Ball B_i is not in cell C_i, $\forall i = 1, 2, \ldots 5$ and each cell contains exactly one ball?

A. 44
B. 96
C. 120
D. 3125

gate2004-it permutations-and-combinations normal

Answer

1.3.20 Permutations And Combinations: GATE2005-IT-46 top

https://gateoverflow.in/3807

A line L in a circuit is said to have a $stuck-at-0$ fault if the line permanently has a logic value 0. Similarly a line L in a circuit is said to have a $stuck-at-1$ fault if the line permanently has a logic value 1. A circuit is said to have a multiple $stuck-at$ fault if one or more lines have stuck at faults. The total number of distinct multiple $stuck-at$ faults possible in a circuit with N lines is

A. 3^N
B. $3^N - 1$
C. $2^N - 1$
D. 2

gate2005-it permutations-and-combinations normal

Answer

1.3.21 Permutations And Combinations: GATE2007-84 top

https://gateoverflow.in/1275

Suppose that a robot is placed on the Cartesian plane. At each step it is allowed to move either one unit up or one unit right, i.e., if it is at (i, j) then it can move to either $(i+1, j)$ or $(i, j+1)$.

How many distinct paths are there for the robot to reach the point $(10, 10)$ starting from the initial position $(0, 0)$?

A. ${}^{20}\mathrm{C}_{10}$
B. 2^{20}
C. 2^{10}
D. None of the above

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gate2007 permutations-and-combinations

Answer

1.3.22 Permutations And Combinations: GATE2007-85 top

https://gateoverflow.in/43509

Suppose that a robot is placed on the Cartesian plane. At each step it is allowed to move either one unit up or one unit right, i.e., if it is at (i, j) then it can move to either $(i+1, j)$ or $(i, j+1)$.

Suppose that the robot is not allowed to traverse the line segment from $(4,4)$ to $(5,4)$. With this constraint, how many distinct paths are there for the robot to reach $(10,10)$ starting from $(0,0)$?

A. 2^9
B. 2^{19}
C. ${}^{8}\mathrm{C}_4 \times {}^{11}\mathrm{C}_5$
D. ${}^{20}\mathrm{C}_{10} - {}^{8}\mathrm{C}_4 \times {}^{11}\mathrm{C}_5$

gate2007 permutations-and-combinations normal discrete-mathematics

Answer

1.3.23 Permutations And Combinations: GATE2008-IT-25 top

https://gateoverflow.in/3286

In how many ways can b blue balls and r red balls be distributed in n distinct boxes?

A. $\frac{(n+b-1)!\,(n+r-1)!}{(n-1)!\,b!\,(n-1)!\,r!}$
B. $\frac{(n+(b+r)-1)!}{(n-1)!\,(n-1)!\,(b+r)!}$
C. $\frac{n!}{b!\,r!}$
D. $\frac{(n+(b+r)-1)!}{n!\,(b+r-1)}$

gate2008-it permutations-and-combinations normal

Answer

1.3.24 Permutations And Combinations: GATE2014-1-49 top

https://gateoverflow.in/1929

A pennant is a sequence of numbers, each number being 1 or 2. An $n-$pennant is a sequence of numbers with sum equal to n. For example, $(1,1,2)$ is a $4-$pennant. The set of all possible $1-$pennants is (1), the set of all possible $2-$pennants is $(2),(1,1)$ and the set of all $3-$pennants is $(2,1),(1,1,1),(1,2)$. Note that the pennant $(1,2)$ is not the same as the pennant $(2,1)$. The number of $10-$pennants is_____

gate2014-1 permutations-and-combinations numerical-answers normal

Answer

1.3.25 Permutations And Combinations: GATE2015-3-5 top

https://gateoverflow.in/8399

The number of 4 digit numbers having their digits in non-decreasing order (from left to right) constructed by using the digits belonging to the set $\{1,2,3\}$ is _____.

gate2015-3 permutations-and-combinations normal numerical-answers

Answer

1.3.26 Permutations And Combinations: GATE2018-46 top

https://gateoverflow.in/204121

The number of possible min-heaps containing each value from $\{1,2,3,4,5,6,7\}$

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exactly once is _____

gate2018 permutations-and-combinations numerical-answers

Answer

1.3.27 Permutations And Combinations: ISI 2004 MIII top

https://gateoverflow.in/123714

The number of permutation of $\{1, 2, 3, 4, 5\}$ that keep at least one integer fixed is.

A. 81
B. 76
C. 120
D. 60

permutations-and-combinations isi2004 discrete-mathematics normal

Answer

1.3.28 Permutations And Combinations: ISI 2004 MIII top

https://gateoverflow.in/123727

In how many ways can three person, each throwing a single die once, make a score of 11

A. 22
B. 27
C. 24
D. 38

permutations-and-combinations isi2004

Answer

1.3.29 Permutations And Combinations: ISI 2004 MIII top

https://gateoverflow.in/123721

A subset S of set of numbers $\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is said to be good if has exactly 4 elements and their $gcd = 1$, Then number of good subset is

A. 126
B. 125
C. 123
D. 121

permutations-and-combinations isi2004 discrete-mathematics normal

Answer

1.3.30 Permutations And Combinations: ISI 2004 MIII top

https://gateoverflow.in/123718

A club with x members is organized into four committees such that

i. each member is in exactly two committees,
ii. any two committees have exactly one member in common .

Then x has

A. exactly two values both between 4 and 8.
B. exactly one value and this lies between 4 and 8.
C. exactly two values both between 8 and 16.
D. exactly one value and this lies between 8 and 16.

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permutations-and-combinations isi2004

Answer

1.3.31 Permutations And Combinations: ISI 2017 top https://gateoverflow.in/124372

For each positive integer n consider the set S_n defined as follows: $S_1 = \{1\}$, $S_2 = \{2, 3\}$, $S_3 = \{4, 5, 6\}$, $\ldots$ and in general, S_{n+1} consists of $n+1$ consecutive integers the smallest of which is one more than the largest integer in S_n. Then the sum of all the integers in S_{21} equals to

A. 1113
B. 53361
C. 5082
D. 4641

isi2017 permutations-and-combinations discrete-mathematics normal

Answer

1.3.32 Permutations And Combinations: ISI2016 top https://gateoverflow.in/45161

A palindrome is a sequence of digits which reads the same backward or forward. For example, 7447, 1001 are palindromes, but 7455, 1201 are not palindromes. How many 8 digit prime palindromes are there?

isi2016 permutations-and-combinations discrete-mathematics normal descriptive

Answer

1.3.33 Permutations And Combinations: ISI2017-MMA-22 top

https://gateoverflow.in/214179

The five vowels—A, E, I, O, U—along with 15 $X's$ are to be arranged in a row such that no X is at an extreme position. Also, between any two vowels, there must be at least 3 $X's$. The number of ways in which this can be done is

A. 1200
B. 1800
C. 2400
D. 3000

isi2017 engineering-mathematics permutations-and-combinations

Answer

1.3.34 Permutations And Combinations: TIFR2011-A-2 top

https://gateoverflow.in/19628

In how many ways can the letters of the word ABACUS be rearranged such that the vowels always appear together?

A. $\dfrac{(6+3)!}{2!}$

B. $\dfrac{6!}{2!}$

C. $\dfrac{3!3!}{2!}$

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D. $\frac{4!3!}{2!}$

E. None of the above.

tifr2011 permutations-and-combinations

Answer

1.3.35 Permutations And Combinations: TIFR2012-A-10 top

https://gateoverflow.in/25014

In how many different ways can r elements be picked from a set of n elements if

i. Repetition is not allowed and the order of picking matters?
ii. Repetition is allowed and the order of picking does not matter?

A. $\frac{n!}{(n-r)!}$ and $\frac{(n+r-1)!}{r!(n-1)!}$, respectively.
B. $\frac{n!}{(n-r)!}$ and $\frac{n!}{r!(n-1)!}$, respectively.
C. $\frac{n!}{r!(n-r)!}$ and $\frac{(n-r+1)!}{r!(n-1)!}$, respectively.
D. $\frac{n!}{r!(n-r)!}$ and $\frac{n!}{(n-r)!}$, respectively.
E. $\frac{n!}{r!}$ and $\frac{r!}{n!}$, respectively.

tifr2012 permutations-and-combinations discrete-mathematics normal

Answer

1.3.36 Permutations And Combinations: TIFR2012-A-7 top

https://gateoverflow.in/21004

It is required to divide the $2n$ members of a club into n disjoint teams of 2 members each. The teams are not labelled. The number of ways in which this can be done is:

A. $\frac{(2n)!}{2^n}$
B. $\frac{(2n)!}{n!}$
C. $\frac{(2n)!}{2^n.n!}$
D. $\frac{n!}{2}$
E. None of the above.

tifr2012 permutations-and-combinations

Answer

1.3.37 Permutations And Combinations: TIFR2013-A-9 top

https://gateoverflow.in/25431

There are n kingdoms and $2n$ champions. Each kingdom gets 2 champions. The number of ways in which this can be done is:

A. $\frac{(2n)!}{2^n}$
B. $\frac{(2n)!}{n!}$
C. $\frac{(2n)!}{2^n.n!}$
D. $\frac{n!}{2}$
E. None of the above.

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tifr2013 permutations-and-combinations discrete-mathematics normal

Answer

1.3.38 Permutations And Combinations: TIFR2015-A-7 top

https://gateoverflow.in/29568

A 1×1 chessboard has one square, a 2×2 chessboard has five squares. Continuing along this fashion, what is the number of squares on the regular 8×8 chessboard?

A. 64
B. 65
C. 204
D. 144
E. 256

tifr2015 permutations-and-combinations

Answer

1.3.39 Permutations And Combinations: TIFR2015-A-8 top

https://gateoverflow.in/29571

There is a set of $2n$ people: n male and n female. A good party is one with equal number of males and females (including the one where none are invited). The total number of good parties is.

A. 2^n
B. n^2
C. $\left(\frac{n}{n/2}\right)^2$
D. $\left(\frac{2n}{n}\right)$
E. None of the above.

tifr2015 permutations-and-combinations discrete-mathematics normal

Answer

1.3.40 Permutations And Combinations: TIFR2016-A-15 top

https://gateoverflow.in/97624

In a tournament with 7 teams, each team plays one match with every other team. For each match, the team earns two points if it wins, one point if it ties, and no points if it loses. At the end of all matches, the teams are ordered in the descending order of their total points (the order among the teams with the same total are determined by a whimsical tournament referee). The first three teams in this ordering are then chosen to play in the next round. What is the minimum total number of points a team must earn in order to be guaranteed a place in the next round?

A. 13
B. 12
C. 11
D. 10
E. 9

tifr2016 permutations-and-combinations discrete-mathematics normal

Answer

1.3.41 Permutations And Combinations: TIFR2017-A-5 top

https://gateoverflow.in/94853

How many distinct ways are there to split 50 identical coins among three people so that each person

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gets at least 5 coins?

A. 3^{35}
B. $3^{50} - 2^{50}$
C. $\binom{35}{2}$
D. $\binom{50}{15}.3^{35}$
E. $\binom{37}{2}$

tifr2017 permutations-and-combinations discrete-mathematics normal

Answer

1.3.42 Permutations And Combinations: TIFR2017-A-6 top

https://gateoverflow.in/95033

How many distinct words can be formed by permuting the letters of the word $ABRACADABRA$?

A. $\frac{11!}{5!\ 2!\ 2!}$
B. $\frac{11!}{5!\ 4!}$
C. $11!\ 5!\ 2!\ 2!$
D. $11!\ 5!\ 4!$
E. $11!$

tifr2017 permutations-and-combinations discrete-mathematics easy

Answer

Answers: Permutations And Combinations

1.3.1 Permutations And Combinations: CMI2010-A-02 top

https://gateoverflow.in/46132

Selected Answer

Number of ways selecting a captain from 15 players $= \binom{15}{1}$

Number of ways selecting remaining team members from remaining 14 players $= \binom{14}{10}$

The number of different ways to choose a team of 11 from a pool of 15 players and also select a captain,

$$= \binom{15}{1} * \binom{14}{10}$$

$$= 15 * 13 * 11 * 7$$

$$= 11 * \binom{15}{11}$$

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Hence,Option **(B)** is the correct choice.

11 votes -- Leen Sharma (40k points)

1.3.2 Permutations And Combinations: CMI2014-A-01 top

https://gateoverflow.in/46964

Selected Answer

There are three ways to choose 6 Players.

1. ${}^5C_3 *^4 C_2 *^2 C_1 = 120.$
2. ${}^5C_2 *^4 C_2 *^2 C_2 = 60.$
3. ${}^5C_2 *^4 C_3 *^2 C_1 = 80.$

So total No. of ways is $260.$

10 votes -- Manoj Kumar (38.7k points)

1.3.3 Permutations And Combinations: GATE1989-4-i top

https://gateoverflow.in/87874

Selected Answer

Lets take an example . lets consider the given string is $\{GATE\}.$

So, set of string of length $1 = \{G, A, T, E\}$; cardinality of set $= 4.$
Set of strings of length $2 = \{GA, AT, TE\}.$
Set of strings of length $3 = \{GAT, ATE\}.$
Set of strings of length $4 = \{GATE\}.$
Set of strings of length $0 = \{\}.$

We cannot have any substring of length 5 as given string has only 4 length .

So total no of substrings possible,

$$= 0 \text{ length substring} + 1 \text{ length substrings} + 2 \text{ length substrings} + 3 \text{ length substrings} + 4 \text{ length substrings}.$$

$$= (1 + 4 + 3 + 2 + 1).$$

This means for 1 length substring to n length substrings, countt will sum of the n natural numbers from 1 to $n.$

$$= 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}.$$

So total no. of substrings possible $= 0 \text{ length strings} + \frac{n(n+1)}{2} = 1 + \left[\frac{n(n+1)}{2}\right].$

17 votes -- Amit Pal (4k points)

1.3.4 Permutations And Combinations: GATE1990-3-iii top

https://gateoverflow.in/84060

Selected Answer

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Number of rooted binary trees (unlabeled) with n nodes is given by n^{th} Catalan number which equals $\frac{^{2n}C_n}{n+1}$.

Here, both options A and C are true as option A corresponds to n multiply operations of $n+1$ matrices, the number of ways for this is again given by the nth Catalan number.

Ref: https://math.stackexchange.com/questions/1630457/how-many-ways-to-multiply-n-matrices

1 votes -- Arjun Suresh (348k points)

1.3.5 Permutations And Combinations: GATE1990-3-ix top

Selected Answer

https://gateoverflow.in/84841

(**A**) Use permutation with repetitions formula as we have to arrange 15 elements where 5 each are identical.

1 votes -- NRN (333 points)

1.3.6 Permutations And Combinations: GATE1991-02,iv top

Selected Answer

https://gateoverflow.in/514

A. - S Catalan number http://http://gatecse.in/wiki/Number_of_Binary_trees_possible_with_n_nodes
B. - R Choosing n locations out of $2n$ to place 0. Remaining automatically become 1.
C. - P An even permutation is a permutation obtainable from an even number of two-element swaps, For a set of n elements and $n > 2$, there are $n!/2$ even permutations.
Ref - http://mathworld.wolfram.com/EvenPermutation.html
D. - Q

Length $= 6n$, as it is palindrome, we need to only consider half part.

Total Length to consider $3n$ (Remaining $3n$ will be revese of this $3n$)

Now, Choosing n $0's$ out of $3n$. So **Q** is correct for **D**.

20 votes -- Akash Kanase (42.5k points)

1.3.7 Permutations And Combinations: GATE1991-16,a top

Selected Answer

https://gateoverflow.in/543

Answer to a is $\frac{^{2n}C_n}{(n+1)}$ which is the Catalan number.

This is also equal to the number of possible combinations of balanced parenthesizes.

See the 5th proof here http://en.wikipedia.org/wiki/Catalan_number

19 votes -- Arjun Suresh (348k points)

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1.3.8 Permutations And Combinations: GATE1994-1.15 top

https://gateoverflow.in/2458

Selected Answer

no. of substrings of length n is 1.

no. of substrings of length $(n-1)$ is 2.

no. of substrings of length $(n-2)$ is 3.

so $\frac{n(n+1)}{2}$.

16 votes -- Bhagirathi Nayak (14.1k points)

1.3.9 Permutations And Combinations: GATE1998-1.23 top

https://gateoverflow.in/1660

Selected Answer

Assuming in the string of length n provided, all alphabets are distinct.

No. of strings of length $1 = n$
No. of strings of length $2 = (n-1)$
No. of strings of length $3 = (n-2)$
$\vdots$
No. of strings of length $n = 1$

$$\text{Hence, total no. of strings} = n + (n-1) + (n-2) + (n-2) + \cdots + 1 = \frac{n(n+1)}{2}$$

20 votes -- Digvijay (54.8k points)

1.3.10 Permutations And Combinations: GATE1999-1.3 top

https://gateoverflow.in/1457

Selected Answer

Fnswer - (**D**)

First place n zeroes side by side _ 0 _ 0 _ 0 ... 0 _

k $1's$ can be placed in any of the $(n+1)$ available gaps. Hence, number of ways $={}^{n+1}C_k$

22 votes -- Ankit Rokde (9k points)

1.3.11 Permutations And Combinations: GATE1999-2.2 top

https://gateoverflow.in/1480

Selected Answer

For each flower type, say there are n number of flowers. We apply star and bars method for each flower type. n flowers of a type will generate $(n+1)$ spaces we just need to place one bar which will separate them into 2 for the two girls. To do that we need to select a position:

For roses: $\binom{10+1}{1}$

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For sunflowers: $\binom{15+1}{1}$
For daffodils: $\binom{15+1}{1}$

Total number of ways distribution can take place $= 11 \times 16 \times 16 = 2816.$

18 votes -- Amar Vashishth (30.5k points)

Answer - D
Number of ways roses can be distributed $= \{(0,10),(1,9),(2,8),\ldots,(10,0)\}$- 11 ways
Similarly, sunflowers and daffodils can be distributed in 16 ways each
So, total number of ways $= 11 \times 16 \times 16 = 2816.$

22 votes -- Ankit Rokde (9k points)

1.3.12 Permutations And Combinations: GATE2000-5 top

Selected Answer

https://gateoverflow.in/676

A. There are four places to be filled in the multiset using the n distinct elements. At least one element has to occur exactly twice. That would leave 2 more places in the multiset. This means, at most two elements can occur exactly twice. We can thus divide this into 2 mutually exclusive cases as follows:

1. Exactly one element occurs exactly twice:
2. Select this element in n ways.

Fill up the remaining two spots using 2 distinct elements from the remaining $n-1$ elements in ${}^{(n-1)}C_2$ ways.

Exactly two elements that occur twice each: These two will fill up the multiset.

So, we only have to select two elements out of n in ${}^{n}C_2$ ways.
Since, these are mutually exclusive, the total number of ways to form the multiset is:
${}^{n}C_2 + n.{}^{(n-1)}C_2.$

B. There are infinite number of sets as n is unbounded.

ref: http://http://cs.stackexchange.com/questions/7578/multisets-of-a-given-set

12 votes -- Pranay Datta (10.2k points)

1.3.13 Permutations And Combinations: GATE2001-2.1 top

Selected Answer

https://gateoverflow.in/719

- If the number ends with a 0 then there are 9 choices for the first digit, 8 for the second and 7 for the third, which makes $1 \times 9 \times 8 \times 7 = 504$ possibilities.
- If the number is even ending with something else than 0 then there are 4 choices for the last digit, 8 choices for the first digit (no 0 nor the last digit), 8 for the second digit and 7 for the third digit, which makes $4 \times 8 \times 8 \times 7 = 1792$

Together, this gives 2296 numbers with 4 distinct digits that are even. Note that this does not allow leading 0, as you see to want it based from the question.

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35 votes -- yallasrikanthreddy (337 points)

1.3.14 Permutations And Combinations: GATE2002-13 top

https://gateoverflow.in/866

Selected Answer

a.
$n = 2\left(1+1\right),\ n = 3\left(1+2, 2+1\right), n = 4\left(1+3, 3+1, 2+2\right),\ n = 5\left(1+4, 4+1, 2+3,\right.$
so $x_1 + x_2 = n$ and $x_1, x_2 > 0$ (no.of integral sol)
This is same as number of ways of putting $(n-2)$ (as we can't have 0 for either x_1 or x_2) identical balls into two distinct bins, which is obtained by putting a divider across $(n-2)$ balls and taking all possible permutations with $(n-2)$ being identical. i.e., $\frac{(n-2+1)!}{(n-2)!} = (n-1)$.
We can also use the following formula ,
${}^{(n-2+2-1)}C_{(2-1)} = {}^{n-1}C_1$.

b.
$n = 3\left(1+1+1\right),\ n = 4\left(1+1+2, 1+2+1, 2+1+1\right), n = 5\left(1+1+3, 1+3+1, 3+\right.$
so $x_1 + x_2 + x_3 = n$ and $x_1, x_2, x_3 > 0$ (no.of integral sol)
Here, we can permute $(n-3)$ items with 2 dividers which will give $\frac{(n-3+2)!}{(n-3)!2!}$

$$= \frac{(n-1)!}{(n-1-2)!2!}$$

$$= {}^{n-1}C_2$$

c. ${}^{(n-k+k-1)}C_{k-1} = {}^{n-1}C_{k-1}$.

20 votes -- Supromit Roy (795 points)

1.3.15 Permutations And Combinations: GATE2003-34 top

https://gateoverflow.in/924

Selected Answer

As there have to be atleast k balls in each bag, so firstly put k balls in each bag i.e $(k*n)$ balls.

Then now we have total $(m-k*n)$ balls remaining.

We can use balls $\&$ sticks method now !

n bags$=$ n variables, they need to equal to $(m-k*n)$, no restrictions on how many balls in each bag !

$x_1 + x_2 + \cdots + x_n = (m-k*n), x_1, x_2 \cdots x_n >= 0.$

So when we solve it

We get

$C(m-k*n+n-1, n-1) = C(m-k*n+n-1, m-k*n)$

18 votes -- Akash Kanase (42.5k points)

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1.3.16 Permutations And Combinations: GATE2003-4 top

https://gateoverflow.in/895

Selected Answer

If you pick any 3 numbers in the given order from the array(sorted) remaining 5 elements are already sorted. You can not change the relative position of those 5 elements because they are distinct and already sorted. So no of ways $= {}^8C_3$.

Same argument holds for picking up 5 elements initially. No of ways $= {}^8C_5$.

22 votes -- Debashish Deka (56.6k points)

1.3.17 Permutations And Combinations: GATE2003-5 top

https://gateoverflow.in/896

Selected Answer

Possible outcome for a couple:

1. only wife comes
2. both come
3. none come

Thus 3 possibilities for each couple, so $3 * 3 * 3 * \cdots * n$ times = 3^n

42 votes -- Palash Nandi (1.5k points)

1.3.18 Permutations And Combinations: GATE2004-75 top

https://gateoverflow.in/1069

Selected Answer

This question is slightly ambigous. So first let us understand what question is asking. So in a book, we have letters A to Z and each letter is printed twice, so there are 52 letters. Now we have to color each letter, so we need a pair of colors for that, because each letter is printed twice. Also in a pair, both colors can be same. Now condition is that a pair of colors can't be used more than once.

So suppose Mala has 3 colors : $\mathrm{Red}, \mathrm{Blue}, \mathrm{Green}$. She can color as follows :

$1 : (Red, Red), 2 : (Blue, Blue), 3 : (Green, Green),$

$4 : (Red, Blue), 5 : (Red, Green), 6 : (Blue, Green).$

Now we don't have more pairs of colors left, we have used all pairs, but could color only 6 letters out of 26. So question is to find minimum no. of colors, so that we could color all 26 letters.

So if Mala has k colors, she can have k pairs of same colors, thus coloring k letters, then ${}^kC_2$ other pairs of colors, thus coloring ${}^kC_2$ more letters.
So total no. of letters colored = $k + \binom{k}{2} = k + k\left(\frac{k-1}{2}\right) = k\left(\frac{k+1}{2}\right)$.
So we want $k\left(\frac{k+1}{2}\right) \geq 26$ i.e. $k(k+1) \geq 52 \implies k \geq 7$, so option **(C) is correct.**

Ref: http://www.cse.iitd.ac.in/~mittal/gate/gate_math_2004.html

32 votes -- Anu (5.9k points)

1.3.19 Permutations And Combinations: GATE2004-IT-35 top

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Selected Answer

https://gateoverflow.in/3678

Adding another answer to explain how the formula came in a detailed way..

Derangement : arrangement where no element is in its designated position

Number of derangements can be calculated easily using principle of mutual exclusion and inclusion (http://math.mit.edu/~fox/MAT307-lecture04.pdf)

In this question, we take the designated position of ball B_i as C_i.

We need to find all possible arrangements where **no** B_i is in its designated cell C_i.

we take 5 arrangements for positions $1, 2, 3, 4$ and 5 where, in each of them, one element is in its designated position and denote them as S_i.

1	x	x	x	x

$\rightarrow S_1$

$|S_1| = 4!$ as the other 4 balls can be arranged in $4!$ ways inside S_1

Similarly, we have S_2 which is arrangement of all balls except B_2 which is in its designated position

x	2	x	x	x

$\rightarrow S_2$

We have $|S_2| = 4!$ similar to S_1.

Similarly, $|S_i| = 4!$.

We have total number of non-derangements $= |S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5|$

according to the principle of mutual exclusion and inclusion,

$|S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5|$ = $\Sigma|S_i| - \Sigma|S_i \cap S_j| + \Sigma|S_i \cap S_j \cap S_k| + \ldots \longrightarrow$ ©

$|S_1 \cap S_2|$ = number of arrangements where B_1 and B_2 are in their designated position

1	2	x	x	x

this can be done in $3!$ ways.

similarly all the other two intersections,$|S_i \cap S_j| = 3!$

and three intersections,$|S_i \cap S_j \cap S_k| = 2!$ and similarly all.

Substituting in equation ©,

$|S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5| = 5 * 4! - \binom{5}{2} * 3! + \binom{5}{3} * 2! - \binom{5}{4} * 1! + \binom{5}{5}) * 0! = 76$ (= number of non derangements)

```
Derangements = all arrangements - non derangements
```

$= 5! - 76 = 44$

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13 votes -- junk_mayavi (3.9k points)

1.3.20 Permutations And Combinations: GATE2005-IT-46 top

https://gateoverflow.in/3807

Selected Answer

Answer should be $3^N - 1$.

This is because the total possible combinations (i.e a line may either be at fault (in 2 ways i.e stuck at fault 0 or 1) or it may not be , so there are only 3 possibilities for a line) is 3^N. In only one combination the circuit will have all lines to be correct (i.e not at fault.) Hence $3^N - 1$. (as it has been said that circuit is said to have multiple stuck up fault if one or more line is at fault)

47 votes -- Afaque Ahmad (893 points)

1.3.21 Permutations And Combinations: GATE2007-84 top

https://gateoverflow.in/1275

Selected Answer

In ques given, $r = \text{Move Right}$ and $u = \text{Move Up}$
so using 10 combination of r and 10 combinations of u moves we get a solution.

Convert the graphical moves to text and one such solution we get,

$= \{u, u, u, u, u, u, u, u, u, u, r, r, r, r, r, r, r, r, r, r\}$

now all possible arrangements of them is given by $= \frac{20!}{10! \times 10!} = \binom{20}{10}$.

Hence, **option A** is true.

40 votes -- Amar Vashishth (30.5k points)

1.3.22 Permutations And Combinations: GATE2007-85 top

https://gateoverflow.in/43509

Selected Answer

Say, $r = \text{Move Right}$ and $u = \text{Move Up}$
so using 10 combination of r and 10 combinations of u moves we get a solution.

Convert the graphical moves to text and one such solution we get,

$= \{u, u, u, u, u, u, u, u, u, u, r, r, r, r, r, r, r, r, r, r\}$

now all possible arrangements of them is given by = $\frac{20!}{10! \times 10!} = \binom{20}{10}$

now we need to discard the segment move from $(4, 4)$ to $(5, 4)$:

to do that we first calculate how many solutions to our problem to reach $(10, 10)$ involves that segment.

We'll then subtract those solutions from the total number of solutions.
Number of solutions to reach from$(0, 0)$ to $(4, 4)$,

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$=$all possible arrangements of $\{r,r,r,r,u,u,u,u\}$

$= \frac{(4+4)!}{4!\times 4!} = \binom{8}{4}$

definitely we take the segment $(4,4)$ to $(5,4) = 1.$

now, Number of solutions to reach from $(5,4)$ to $(10,10)$,

$=$ all possible arrangements of $\{r,r,r,r,r,r,u,u,u,u,u\}$

$= \frac{(6+5)!}{6!\times 5!} = \binom{11}{5}$

so required number of solutions for Q.85 is given by **option D**
i.e. $= \binom{20}{10} - \binom{8}{4} \times 1 \times \binom{11}{5}$

33 votes -- Amar Vashishth (30.5k points)

1.3.23 Permutations And Combinations: GATE2008-IT-25 top

Selected Answer

https://gateoverflow.in/3286

r red balls can be distributed into n-distinct boxes in $C(n+r-1,r) = \frac{(n+r-1)!}{(n-1)!r!}$

b blue balls can be distributed in $C(n+b-1,b) = \frac{(n+b-1)!}{(n-1)!b!}$

By product rule total ways are $\frac{(n+b-1)!(n+r-1)!}{(n-1)!b!(n-1)!r!}$

SO THE ANSWER IS (**A**).

35 votes -- Madhu Veluguri (261 points)

1.3.24 Permutations And Combinations: GATE2014-1-49 top

Selected Answer

https://gateoverflow.in/1929

Let us denote number of $n-$pennants by $f(n)$, so $f(10)$ is number of 10-pennants.

A $10-$pennant means sum of numbers in sequence is 10. If we look at any $9-$pennant, we can make it a $10-$pennant by adding 1 into that sequence. Similarly, we can make any $8-$pennant a $10-$pennant by adding 2 into that sequence.

So all $10-$pennants can be formed by $8-$pennants and $9-$pennants, and no other pennant (since we can add only 1 or 2 into a sequence)

So $f(10) = f(9) + f(8)$

This is in fact a Fibonacci sequence, in which $f(1) = 1, f(2) = 2$, so this sequence becomes

$1, 2, 3, 5, 8, 13, 21, 34, 55, 89 \ldots$

So $f(10) = 89.$

55 votes -- Happy Mittal (11.2k points)

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Numbers could be any one of

$\{(1,1,1,1,1,1,1,1,1,1),(1,1,1,1,1,1,1,1,1,2),(1,1,1,1,1,1,2,2),$
$(1,1,1,1,2,2,2),(1,1,2,2,2,2),(2,2,2,2,2)\}$

So, the number of 10 pennants $= 1 + \frac{9!}{8!} + \frac{8!}{6!2!} + \frac{7!}{4!3!} + \frac{6!}{2!4!} + 1 = 89.$

44 votes -- srestha (86.6k points)

1.3.25 Permutations And Combinations: GATE2015-3-5 top

Selected Answer

https://gateoverflow.in/8399

We can arrive at a solution by constructing a graph for each starting digit. For example root 3 means - starting with 3 it can have 3 children $1, 2, 3$ and the construction goes.

3 can have three children $1, 2, 3$

2 can have two children $1, 2$

1 can have only 1 as child.

Graph need to be done till four levels as we need 4 digits and we have 3 such graphs starting with 3, 2 and 1.
And finally count the total number of leaves of all the graphs gives our answer as 15.

36 votes -- Sankaranarayanan P.N (11.5k points)

Dynamic programming Approach

	1 digit	**2 digit**	**3 digit**	**4 digits**
Starting 3	1	1	1	1
Starting 2	1	2	3	4
Starting 1	1	3	6	10

Here Starting 1 means numbers starting with 1. And cell (i, j) is for number of numbers starting with i and having j digits. We can have the relation

$$c(i,j) = \Sigma_{k=1}^{i} c(k, j-1)$$

as per the non-decreasing condition given in the question. So, our answer will be
$c(1,4) + c(2,4) + c(3,4) = 1 + 4 + 10 = 15$

Brute force

3 3 3 3
2 2 2 2
2 2 2 3
2 2 3 3

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```
2 3 3 3
1 1 1 1
1 1 1 2
1 1 1 3
1 1 2 2
1 1 2 3
1 1 3 3
1 2 2 2
1 2 2 3
1 2 3 3
1 3 3 3
```

38 votes -- Arjun Suresh (348k points)

1.3.26 Permutations And Combinations: GATE2018-46 top

Selected Answer

https://gateoverflow.in/204121

Lets answer this question in an easier way :

Now do $\frac{7!}{7\times3\times3} = 80$

Here $7!$ because 7 items to be filled, Now 7 because root has only 7 nodes as its decedent including itself and only one can be the root. In same way we get 3 and 3 for the second level nodes and 1 and 1 for the third level.

15 votes -- Prashant Singh (59.8k points)

Ans: 80

Explanation:

Number of min-heaps possible with keys $\{1, 2, 3, 4, 5, 6, 7\}$.

A min-heap is a binary tree such that.

- the data contained in each node is less than (or equal to) the data in that node's children.

- the binary tree is complete.

Since a binary heap is always a complete binary tree, so with 7 nodes, we can have 2 levels (root at level 0). It's structure will be like this:

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Now because of min-heap property, every node's value must be smaller than all of it's children. So, this ensure that the minimum will always be at the root. $\therefore 1$ will be at the root.

The second minimum element(i.e. 2) can be present at the first level only(root is at the zeroth level). So we have two options. Let's, for now, fix 2 at the left side.

We are now left with 5 more elements $\{3, 4, 5, 6, 7\}$. For the left subtree's two leaves, we have $5 * 4$ ways. i.e. first choosing one of the 5 nodes, then choosing one of the remaining 4 nodes.

Now 3 nodes left. Out of these 3, one will be the least among them, and that will definitely become the parent of the two remaining leaves(in the right subtree). Now with 2 nodes left, we can arrange them in 2 ways.

This gives $(5 * 4) * 2 = 40$ ways.

We can have the same number of ways, if we fixed 2 at the right subtree instead of left. So total ways:

$= 40 * 2$

$= \mathbf{80}$

34 votes -- Rishabh Gupta (13.9k points)

1.3.27 Permutations And Combinations: ISI 2004 MIII top

https://gateoverflow.in/123714

Selected Answer

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It is based on inclusion exclusion principle:

Total arrangements - derangements

$$\begin{aligned} &= 5! - [5! -^5 C_1(4!) +^5 C_2(3!) -^5 C_3(2!) +^5 C_4(1!) -^5 C_5(0!)] \\ &=^5 C_1(4!) -^5 C_2(3!) +^5 C_3(2!) -^5 C_4(1!) +^5 C_5(0!) \\ &= 120 - 60 + 20 - 5 + 1 \\ &= 76. \end{aligned}$$

7 votes -- Heisenberg (2.3k points)

1.3.28 Permutations And Combinations: ISI 2004 MIII top

https://gateoverflow.in/123727

Selected Answer

We can solve it by star and bar method.

Suppose 1st person's score is x_1.

Suppose 2nd person's score is x_2.

Suppose 3rd person's score is x_3.

We want $x_1 + x_2 + x_3 = 11$ with constraints $1 \leq x_1 \leq 6$, $1 \leq x_2 \leq 6$, $1 \leq x_2 \leq 6$

So x_1, x_2, x_3 have minimum values 1 and remaining 8 can be scored as $x_1 + x_2 + x_3 = 8$.

Total no. of ways to score $8 = \binom{3+8-1}{2} = \binom{10}{2}$.

Now we have to subtract the ways in which either x_1 or x_2 or x_3 have values ≥ 7.

Putting $x_1 = 7$. We get $x_2 + x_3 = 2$ (we subtract 6 from RHS because x_1 is already assumed to be ≥ 1 and $6 + 1 = 7$).

No. of ways $= \binom{3+2-1}{2} = \binom{4}{2}$

Doing same for x_2 and x_3.

Note: Only one of the 3 can have score ≥ 7 as if 2 suppose x_1 and x_2 both is 7 then total will exceed 8.

So, total no. of ways = $\binom{10}{2} - \left[\binom{4}{2} + \binom{4}{2} + \binom{4}{2}\right] = 27.$

(B) is the correct answer.

14 votes -- Soumya Jain (9.7k points)

1.3.29 Permutations And Combinations: ISI 2004 MIII top

https://gateoverflow.in/123721

Selected Answer

$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}, |S| = 9$

Total no of subsets with cardinality $4 = C(9, 4) = 126$

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The subsets with cardinality 4 which having $gcd \neq 1$ is possible only when subset contains only even numbers.

Here even no $E = \{2, 4, 6, 8, 10\}$

No. of subsets with cardinality 4 from set $E = C(5, 4) = 5$.

1. $\{2, 4, 6, 8\}$
2. $\{2, 4, 6, 10\}$
3. $\{2, 4, 8, 10\}$
4. $\{2, 6, 8, 10\}$
5. $\{4, 6, 8, 10\}$

So,there are 5 subsets which having $gcd = 2$ i.e $gcd \neq 1$

So, Total no subsets with cardinality 4 which having $gcd = 1$ is $126 - 5 = 121$

The correct answer is (D)121

12 votes -- Warrior (6.3k points)

1.3.30 Permutations And Combinations: ISI 2004 MIII top

https://gateoverflow.in/123718

Selected Answer

(**B**) is ans.

12 votes -- 2018 (6.8k points)

1.3.31 Permutations And Combinations: ISI 2017 top https://gateoverflow.in/124372

Selected Answer

Every new set S_{n+1} starts after n elements from the starting element of S_n . This means that we can find the starting number of S_{21} using Arithmetic progression formula.

Let $\text{Sum}\,(n)$denote sum of natural numbers until n :

S_1 starts with $= 1$

S_2 starts with $= \text{Sum}\,(1) + 1 = 2$

S_3 starts with $= \text{Sum}\,(2) + 1 = (1 + 2) + 1 = 4$

$\text{Similarly, } S_{21}$ starts with $S\,(20) + 1 = \frac{20(20+1)}{2} + 1 = 211$

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Now we need to find sum of 21 consecutive natural numbers starting from 211

Using A.P. sum formula $S_n = \frac{n}{2}[2a + (n-1) * d]$ where, $a =$ starting term , $d =$ difference

Sum of elements in $S_{21} = \frac{21}{2}[2(211) + 20] = \frac{21*442}{2} = 4641.$ Option (**D**) is correct

10 votes -- Heisenberg (2.3k points)

1.3.32 Permutations And Combinations: ISI2016 top https://gateoverflow.in/45161

Selected Answer

I think the answer should be 0, as any even digit palindrome(other than 11) cannot be prime.
Even digit palindromes will always be divisible by 11 (you can check the divisibility test by 11).

11 votes -- sourb (171 points)

1.3.33 Permutations And Combinations: ISI2017-MMA-22 top

https://gateoverflow.in/214179

Selected Answer

First, we'll place the vowels

& we can place the 5 vowels in $5!$ ways

Now, X can't be seated or placed in extreme position.

So,

Now, between any 2 vowels, there can be at least 3 X's

So, we're done with $3 \times 4 = 12$ X's

& 12 X's can be placed in 4 ways (as all X's are identicals)

Now, we're left with $(15 - 12) = 3$ X's to place

Now, these 3 X's can be placed in many ways-

- When all the 3 X's are placed between the same 2 vowels

$\therefore$ Total ways = 4

- Place 2 X's between the same pair of vowels, & place the remaining X in other pair of vowels

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$\therefore$ Total ways = 4×3 $\quad [\because \text{Once we fill 2 X's, then we'll have only 3 ways to fill 1 X}]$

- Place X's in different pair of vowels

$\therefore$ Total ways = 4

Or, we can compute it in this way, ------

We have 4 possibilities left & we have to place 3 X's.

So, ${}^4C_3 = \dfrac{4!}{3! \times (4-1)!} = \dfrac{4!}{3!} = 4$

$$\text{The total no. of ways} = 5! \times (4 + 4 \times 3 + 4)$$
$$= 120 \times (4 + 12 + 4)$$
$$= 120 \times 20$$
$$= 2400\, ways$$

2 votes -- Subarna Das (10.5k points)

1.3.34 Permutations And Combinations: TIFR2011-A-2 top

https://gateoverflow.in/19829

Selected Answer

Take AAU together and treat it like 1 entity. Now arrange $\boxed{\text{AAU}}\text{BCS}$ in $4!$ ways.

Then, the AAU can be arranged in $\dfrac{3!}{2!}$ ways because A has been repeated twice.

So, total arrangements $= \dfrac{4!3!}{2!}$

Option (**D**) is the correct answer.

19 votes -- yes (1.7k points)

1.3.35 Permutations And Combinations: TIFR2012-A-10 top

https://gateoverflow.in/25014

Selected Answer

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i. Repetition is not allowed and the order of picking matters $=$r arrangement with no repetition $={}^{n}\mathrm{P}_{r} = \frac{n!}{(n-r)!}$

ii. Repetition is allowed and the order of picking does not matter $=$ combination with unlimited repetition $={}^{(n-1+r)}\mathrm{C}_{r} = \frac{n-1+r!}{(n-1)!r!}$

Hence, ans is Option (**A**).

14 votes -- Umang Raman (15.7k points)

1.3.36 Permutations And Combinations: TIFR2012-A-7 top

Selected Answer

https://gateoverflow.in/21004

$2n$ member to be n teams with 2 member each and teams are unordered so we can exchange n team member among them.

$$= \frac{(2n)!}{\underbrace{2!.2!.2!...2!}_{n \text{ times}} \times n!}$$
$$= \frac{(2n)!}{2^n \times n!}$$

Option c.

15 votes -- Umang Raman (15.7k points)

1.3.37 Permutations And Combinations: TIFR2013-A-9 top

Selected Answer

https://gateoverflow.in/25431

We have n Kingdoms as $k_1, k_2, \ldots, k_n$.

Firstly we can select 2 champions from $2n$ champions and assign to $k_1 = \binom{2n}{2}$ ways (Say w_1)

Then we can select next 2 champions and assign to $k_2 = \binom{2n-2}{2}$ ways (Say w_2)

and so on..

For last kingdom , we have 2 champions left$= \binom{2}{2}$ ways (Say w_n)

$$\text{Total ways for assigning } 2n \text{ champions to } n \text{ kingdoms} = w_1 * w_2 * \ldots * w_n$$
$$= \binom{2n}{2} * \binom{2n-2}{2} * \ldots * \binom{2}{2}$$
$$= \frac{(2n)!}{2^n}$$

So, Option **A (Ans) .**

17 votes -- Himanshu Agarwal (15.3k points)

1.3.38 Permutations And Combinations: TIFR2015-A-7 top

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Selected Answer

https://gateoverflow.in/29568

No. of squares on chessboard of $n \times n$ is equal to sum of squares of n terms for 8×8 chessboard,

$$= \frac{n(n+1)(2n+1)}{6}$$
$$= \frac{8 \times 9 \times 17}{6}$$
$$= 204.$$

11 votes -- Pooja Palod (31.3k points)

1.3.39 Permutations And Combinations: TIFR2015-A-8 top

Selected Answer

https://gateoverflow.in/29571

There are n men and n women

Now we can select 1 woman from n in ${}^nC_1$

Same way 1 man can be selected ${}^nC_1$ ways

So, for 1 woman and 1 man we can get ${}^nC_1 \times^n C_1$ ways $\ldots\ldots(1)$

Similarly, we can select 2 woman from n women in ${}^nC_2$

2 man can be selected in ${}^nC_2$ ways

So, for 2 woman and 2 man we can get ${}^nC_2 \times^n C_2$ ways $\ldots\ldots(2)$

$\vdots$

For n woman and n man we can get ${}^nC_n \times^n C_n$ ways. $\ldots\ldots(n)$

Now, by adding these equations $(1),(2),\ldots(n)$ we get ,

$${}^nC_0 \times^n C_0 +^n C_1 \times^n C_1 +^n C_2 \times^n C_2 +^n C_3 \times^n C_3 + \cdots +^n C_n \times^n C_n = \left({}^{2n}C_n\right)$$

Hence, Ans will be (**E**).

14 votes -- srestha (86.6k points)

1.3.40 Permutations And Combinations: TIFR2016-A-15 top

Selected Answer

https://gateoverflow.in/97624

Let the 7 Teams be A, B, C, D, E, F, G and so each team plays total 6 matches.

Suppose, Team A wins over E, F, G and draws with B, C, D hence total points scored by Team A $= 9$ points

Now, Team B wins over E, F, G and draws with A, C, D hence total points scored by Team B $= 9$ points

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Similarly, happens for next two teams C and D .

Hence, Finalized scores are $\Rightarrow$

```
A = 9
B = 9
C = 9
D = 9
E = ? (Less than or equal to 4)
F = ? ("...")
G = ? ("...")
```

Given that the order among the teams with the same total are determined by a whimsical tournament referee.

So, He/She can order the top 3 teams like $ABC, ABD, BCD, ACD, \ldots$

But, Question says " team must earn in order to be guaranteed a place in the next round "

Hence, Not to depend on that whimsical referee, the minimum total number of points a team must earn in order to be guaranteed a place in the next round = $9 + 1 = 10$ points

14 votes -- Kapil Phulwani (50k points)

1.3.41 Permutations And Combinations: TIFR2017-A-5 top

https://gateoverflow.in/94953

Selected Answer

Distinct ways are there to split 50 identical coins among three people so that each person gets at least 5 coins

$x_1 + 5 + x_2 + 5 + x_3 + 5 = 50$

$x_1 + x_2 + x_3 = 35$

Solving Non-integral solution $n = 35, r = 3$

${}^{(n+r-1)}C_{r-1} = {}^{(35+3-1)}C_{3-1} = {}^{37}C_2.$

Hence E is Answer

17 votes -- Prajwal Bhat (11.3k points)

1.3.42 Permutations And Combinations: TIFR2017-A-6 top

https://gateoverflow.in/95033

Selected Answer

ABRACADABRA

$A \rightarrow 5B \rightarrow 2R \rightarrow 2$

Total Permutation of words $= 11!$

Now,we have to remove word from total permutation of words which have repetition of letter,

$= \dfrac{11!}{5!2!2!}$

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Hence option (**A**) is correct.

10 votes -- kunal chalotra (20.4k points)

1.4 Pigeonhole Principle(4) top

1.4.1 Pigeonhole Principle: GATE2000-1.1 top

https://gateoverflow.in/624

The minimum number of cards to be dealt from an arbitrarily shuffled deck of 52 cards to guarantee that three cards are from same suit is

A. 3
B. 8
C. 9
D. 12

gate2000 easy pigeonhole-principle permutations-and-combinations

Answer

1.4.2 Pigeonhole Principle: GATE2005-44 top

https://gateoverflow.in/1170

What is the minimum number of ordered pairs of non-negative numbers that should be chosen to ensure that there are two pairs (a, b) and (c, d) in the chosen set such that, $a \equiv c \mod 3$ and $b \equiv d \mod 5$

A. 4
B. 6
C. 16
D. 24

gate2005 set-theory&algebra normal pigeonhole-principle

Answer

1.4.3 Pigeonhole Principle: TIFR2014-A-5 top

https://gateoverflow.in/25990

The rules for the University of Bombay five-a-side cricket competition specify that the members of each team must have birthdays in the same month. What is the minimum number of mathematics students needed to be enrolled in the department to guarantee that they can raise a team of students?

A. 23
B. 91
C. 60
D. 49
E. None of the above.

tifr2014 permutations-and-combinations discrete-mathematics normal pigeonhole-principle

Answer

1.4.4 Pigeonhole Principle: TIFR2018-A-6 top

https://gateoverflow.in/179275

What is the minimum number of students needed in a class to guarantee that there are at least 6

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students whose birthdays fall in the same month ?

a. 6
b. 23
c. 61
d. 72
e. 91

tifr2018 pigeonhole-principle permutations-and-combinations

Answer

Answers: Pigeonhole Principle

1.4.1 Pigeonhole Principle: GATE2000-1.1 top

https://gateoverflow.in/624

Selected Answer

There are 4 sets of cards. So, up till 8 cards there is a chance that no more than 2 cards are from a given set. But, once we pick the 9th one, it should make 3 cards from any one of the sets. So, (C) is the answer.

23 votes -- gatecse (18k points)

1.4.2 Pigeonhole Principle: GATE2005-44 top

https://gateoverflow.in/1170

Selected Answer

Let us pick any tuple (p, q) from $\mathbb{N}^2$

What can happen?

Well, $p \mod 3$ can be $0, 1$ or 2. And $q \mod 5$ can be $0, 1, 2, 3$ or 4. So, there are 15 possibilities.

Now if we have 16 of these tuples, surely two of these will map to same combination. Hence answer is 16.

16 votes -- Dhruv Patel (2k points)

Order pairs for (a, b) are
$(0,0), (0,1), (0,2), (0,3), (0,4)(1,0), (1,1), (1,2), (1,3), (1,4)(2,0), (2,1), (2,2), (2,3), (2,4)$
Take any other combination for (c, d) that will surely match with one of the above 15 combinations

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(Pigeon Hole principle)
Total $15+1=16$ combinations

27 votes -- Digvijay (54.8k points)

1.4.3 Pigeonhole Principle: TIFR2014-A-5 top https://gateoverflow.in/25990

Selected Answer

There are 12 months and we have to get 5 people having birthdays in the same month in order to form a team . we can apply the pigeon hole principal :

$\lceil \frac{N}{12} \rceil = 5$

On solving we get $N = 49$.

Hence answer is (**D**).

16 votes -- Riya Roy(Arayana) (7.2k points)

1.4.4 Pigeonhole Principle: TIFR2018-A-6 top https://gateoverflow.in/179275

Selected Answer

Using pigeonhole principle: With n pigeon and m holes atleast p pigeo will be on m holes are:

$\frac{n-1}{m} + 1 = p$

$n - 1 + 12 = 6 \times 12$

$n - 1 = 72 - 12 = 60$

$n = 60 + 1 = 61$

4 votes -- Anu007 (16.8k points)

1.5 Recurrence(6) top

1.5.1 Recurrence: GATE2004-IT-34 top https://gateoverflow.in/3677

Let $H_1, H_2, H_3, \ldots$ be harmonic numbers. Then, for $n \in Z^+$, $\sum_{j=1}^{n} H_j$ can be expressed as

A. $nH_{n+1} - (n+1)$
B. $(n+1)H_n - n$
C. $nH_n - n$
D. $(n+1)H_{n+1} - (n+1)$

gate2004-it recurrence permutations-and-combinations normal

Answer

1.5.2 Recurrence: GATE2007-IT-76 top https://gateoverflow.in/3528

Consider the sequence $\langle x_n \rangle$, $n \geq 0$ defined by the recurrence relation $x_{n+1} = c.x_n^2 - 2$, where

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$c > 0$.

Suppose there exists a ***non-empty, open*** interval (a, b) such that for all x_0 satisfying $a < x_0 < b$, the sequence converges to a limit. The sequence converges to the value?

A. $\frac{1+\sqrt{1+8c}}{2c}$
B. $\frac{1-\sqrt{1+8c}}{2c}$
C. 2
D. $\frac{2}{2c-1}$

gate2007-it permutations-and-combinations normal recurrence

Answer

1.5.3 Recurrence: GATE2016-1-2 top

https://gateoverflow.in/39636

Let a_n be the number of n-bit strings that do **NOT** contain two consecutive $1's$. Which one of the following is the recurrence relation for a_n?

A. $a_n = a_{n-1} + 2a_{n-2}$
B. $a_n = a_{n-1} + a_{n-2}$
C. $a_n = 2a_{n-1} + a_{n-2}$
D. $a_n = 2a_{n-1} + 2a_{n-2}$

gate2016-1 permutations-and-combinations recurrence easy

Answer

1.5.4 Recurrence: GATE2016-1-27 top

https://gateoverflow.in/39714

Consider the recurrence relation $a_1 = 8, a_n = 6n^2 + 2n + a_{n-1}$. Let $a_{99} = K \times 10^4$. The value of K is _____.

gate2016-1 permutations-and-combinations recurrence normal numerical-answers

Answer

1.5.5 Recurrence: TIFR2014-A-3 top

https://gateoverflow.in/25988

The Fibonacci sequence is defined as follows: $F_0 = 0, F_1 = 1,$ and for all integers $n \geq 2, F_n = F_{n-1} + F_{n-2}$. Then which of the following statements is FALSE?

a. $F_{n+2} = 1 + \sum_{i=0}^{n} F_i$ for any integer $n \geq 0$
b. $F_{n+2} \geq \emptyset^n$ for any integer $n \geq 0$, where $\emptyset = \left(\sqrt{5}+1\right)/2$ is the positive root of $x^2 - x - 1 = 0$.
c. F_{3n} is even, for every integer $n \geq 0$.
d. F_{4n} is a multiple of 3, for every integer $n \geq 0$.
e. F_{5n} is a multiple of 4, for every integer $n \geq 0$.

tifr2014 recurrence easy

Answer

1.5.6 Recurrence: TIFR2017-A-7 top

https://gateoverflow.in/95037

Consider the sequence $S_0, S_1, S_2, \ldots$ defined as follows: $S_0 = 0,\ S_1 = 1$ and $S_n = 2S_{n-1} + S_{n-2}$ for $n \geq 2$. Which of the following statements is FALSE?

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A. for every $n \geq 1$, S_{2n} is even
B. for every $n \geq 1$, S_{2n+1} is odd
C. for every $n \geq 1$, S_{3n} is multiple of 3
D. for every $n \geq 1$, S_{4n} is multiple of 6
E. none of the above

tifr2017 recurrence

Answer

Answers: Recurrence

1.5.1 Recurrence: GATE2004-IT-34 top

https://gateoverflow.in/3677

Selected Answer

The nth ***Harmonic Number*** is defined as the summation of the reciprocals of all numbers from 1 to n.

$$H_n = \sum_{i=1}^{n} \frac{1}{i} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}$$

Lets call the value of $\sum_{j=1}^{n} H_j$ as S_n

Then,

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$$S_n = H_1 + H_2 + H_3 + \cdots + H_n$$

$$= \overbrace{\left(\frac{1}{1}\right)}^{H_1} + \underbrace{\left(\frac{1}{1} + \frac{1}{2}\right)}_{H_2} + \overbrace{\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3}\right)}^{H_3} + \cdots + \underbrace{\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)}_{H_n}$$

$$= n \times \frac{1}{1} + (n-1) \times \frac{1}{2} + (n-2) \times \frac{1}{3} + \cdots + 1 \times \frac{1}{n}$$

$$= \sum_{i=1}^{n} (n - i + 1) \times \frac{1}{i}$$

$$= \sum_{i=1}^{n} \left(\frac{n+1}{i} - 1\right)$$

$$= \left(\sum_{i=1}^{n} \frac{n+1}{i}\right) - \left(\sum_{i=1}^{n} 1\right)$$

$$= \left((n+1) \times \underbrace{\sum_{i=1}^{n} \frac{1}{i}}_{=H_n}\right) - n$$

$$S_n = (n+1) \cdot H_n - n$$

Hence, the answer is option (B).

44 votes -- Pragy Agarwal (20.6k points)

1.5.2 Recurrence: GATE2007-IT-76 top

https://gateoverflow.in/3528

Since it converges, we can write:

$x = cx^2 - 2$

or

$cx^2 - x - 2 = 0$

Solving for x:

$x = \frac{1 \pm \sqrt{1+8c}}{2c}$

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So both (**A**) and (**B**) can be the values.

12 votes -- Ashis Kumar Sahoo (865 points)

1.5.3 Recurrence: GATE2016-1-2 top

https://gateoverflow.in/39636

Selected Answer

n	n-bit strings that do NOT contain consecutive 11	a_n	those contain 11
1	$\{0,1\}$	$a_1 = 2$	-
2	$\{00, 01, 10\}$	$a_2 = 3$	$\{11\}$
3	$\{000, 100, 001, 101\}$ $010,$	$a_3 = 5$	$\{011, 110, 111\}$

$a_n = a_{n-1} + a_{n-2}$

Rest of the options are already out.

Alternatively, we can get a string in a_n by appending "0" to any string in a_{n-1} as well as by appending "01" to any string in a_{n-2} and the two cases are mutually exclusive (no common strings) as well as exhaustive (covers all cases).

46 votes -- Praveen Saini (54.5k points)

1.5.4 Recurrence: GATE2016-1-27 top

https://gateoverflow.in/39714

Selected Answer

$a_n = 6n^2 + 2n + a_{n-1}$

$= 6n^2 + 2n + 6(n-1)^2 + 2(n-1) + a_{n-2}$

$= 6n^2 + 2n + 6(n-1)^2 + 2(n-1) + 6(n-2)^2 + 2(n-2) + \ldots\ldots + a_1$

$= 6n^2 + 2n + 6(n-1)^2 + 2(n-1) + 6(n-2)^2 + 2(n-2) + \ldots\ldots + 6.1^2 + 2.1$

$= 6(n^2 + (n-1)^2 + \ldots + 2^2 + 1^2) + 2(n + (n-1) + \ldots + 2 + 1)$

$= 6 \times \frac{n(n+1)(2n+1)}{6} + 2 \times \frac{n(n+1)}{2}$

$= n(n+1)(2n+1+1)$

$a_n = 2n(n+1)^2$

for $n = 99$ $a_{99} = 2 \times 99 \times (99+1)^2 = 198 \times 10^4$

97 votes -- Praveen Saini (54.5k points)

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1.5.5 Recurrence: TIFR2014-A-3 top

https://gateoverflow.in/25988

Selected Answer

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7
0	1	1	2	3	5	8	13

Option (**e**) is FALSE. F_{5n} is a multiple of 4, for every integer $n \geq 0$.

7 votes -- Umang Raman (15.7k points)

1.5.6 Recurrence: TIFR2017-A-7 top

https://gateoverflow.in/95037

Selected Answer

$S_n = 2S_{n-1} + S_{n-2}$

Characterstic polynomial for this recurrence is $x^2 = 2x + 1$

$x^2 - 2x - 1 = 0 \Rightarrow x_1 = (1+\sqrt{2}), x_2 = (1-\sqrt{2})$

The solution to the recurrence relation is of the form : $S_n = C_1 \times x_1^n + C_2 \times x_2^n$

Putting $S(0) = 0$, $C_1 + C_2 = 0$

Putting $S(1) = 1$, $C_1 \times (1+\sqrt{2}) + C_2 \times (1-\sqrt{2}) = 1$

Solving these two, we get $C_1 = \frac{1}{2\sqrt{2}}$ and $C_2 = -\frac{1}{2\sqrt{2}}$

$$S_n = \frac{1}{2\sqrt{2}}\left((1+\sqrt{2})^n - (1-\sqrt{2})^n\right)$$

$S_0 = 0, S_1 = 1, S_2 = 2, S_3 = 5, S_4 = 12, S_5 = 29, S_6 = 70$

Clearly S_3 and S_6 are not a multiple of 3

Hence (**C**) is correct answer.

10 votes -- Manish Joshi (27.9k points)

1.6 Summation(3) top

1.6.1 Summation: GATE1994-15 top

https://gateoverflow.in/2511

Use the patterns given to prove that

a. $\sum_{i=0}^{n-1}(2i+1) = n^2$

(You are not permitted to employ induction)

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b. Use the result obtained in (a) to prove that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

gate1994 permutations-and-combinations proof summation descriptive

Answer

1.6.2 Summation: GATE2008-24 top

https://gateoverflow.in/422

Let $P = \sum_{\substack{1 \le i \le 2k \\ i\ odd}} i$ and $Q = \sum_{\substack{1 \le i \le 2k \\ i\ even}} i$, where k is a positive integer. Then

A. $P = Q - k$
B. $P = Q + k$
C. $P = Q$
D. $P = Q + 2k$

gate2008 permutations-and-combinations easy summation

Answer

1.6.3 Summation: GATE2015-1-26 top

https://gateoverflow.in/8248

$\sum_{x=1}^{99} \frac{1}{x(x+1)}$ = _____.

gate2015-1 permutations-and-combinations normal numerical-answers summation

Answer

Answers: Summation

1.6.1 Summation: GATE1994-15 top

https://gateoverflow.in/2511

Selected Answer

a. Using the pattern we can see that n^2 is obtained by summing all the odd numbers from 1 to $2n-1$. For i^{th} row of the pattern if we sum the dots to right end and then down we get $2i-1$. Then sum up the values of all rows and we get n^2.

b. $\sum_{i=0}^{n-1}(2i+1) = n^2$
$\implies 1 + \sum_{i=1}^{n}(2i+1) = n^2 + 2n + 1$
$\implies \sum_{i=1}^{n} 2i + \sum_{i=1}^{n} 1 = n^2 + 2n$
$\implies 2.\sum_{i=1}^{n} i + n = n^2 + 2n$
$\implies 2.\sum_{i=1}^{n} i = n^2 + n$
$\implies \sum_{i=1}^{n} i = \frac{n.(n+1)}{2}$

7 votes -- Arjun Suresh (348k points)

1.6.2 Summation: GATE2008-24 top

https://gateoverflow.in/422

Selected Answer

$\mathbf{P} = 1 + 3 + 5 + 7 + \ldots\ldots + (2k-1)$

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$$= (2-1)+(4-1)+(6-1)+(8-1)+.....+(2k-1)$$

$$= (2+4+6+8+.....2k)+(-1-1-1-1-1.....k \text{ times})$$

$$= \mathbf{Q}+(-k) = \mathbf{Q\text{-}k}$$

13 votes -- Pranabesh Ghosh (4k points)

1.6.3 Summation: GATE2015-1-26 top

https://gateoverflow.in/8248

Selected Answer

$$\frac{1}{1.2}+\frac{1}{2.3}+\cdots+\frac{1}{99.100} = 1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{99}-\frac{1}{100} = 1-\frac{1}{100} = 0.99$$

36 votes -- Arjun Suresh (348k points)

It can be answered by using the concept of partial fraction

$$\frac{1}{x(x+1)} = \frac{A}{x}+\frac{B}{(x+1)}$$

solving this we will get $A=1$ and $B=-1$

So, this will form a sequence in which 2 terms will remain $\left(1-\frac{1}{100}\right)$ and we will get $\frac{99}{100} = 0.99$ as answer.

34 votes -- Aditi Tiwari (1.2k points)

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2 Discrete Mathematics: Graph Theory (75) top

2.1 Adjacency Matrix(1) top

2.1.1 Adjacency Matrix: GATE1988-2xvi top

https://gateoverflow.in/94340

Write the adjacency matrix representation of the graph given in below figure.

gate1988 descriptive graph-theory adjacency-matrix

Answer

Answers: Adjacency Matrix

2.1.1 Adjacency Matrix: GATE1988-2xvi top

https://gateoverflow.in/94340

Selected Answer

	p1	p2	p3	p4	p5
p1	0	1	0	0	1
p2	0	0	0	1	0
p3	1	0	0	0	0
p4	0	0	1	0	1
p5	0	1	0	0	0

3 votes -- shruti pujar (303 points)

2.2 Counting(7) top

2.2.1 Counting: CMI2013-B-02 top

https://gateoverflow.in/46612

A complete graph on n vertices is an undirected graph in which every pair of distinct vertices is connected by an edge. A simple path in a graph is one in which no vertex is repeated. Let G be a complete graph on 10 vertices. Let u, v, w be three distinct vertices in G. How many simple paths are there from u to v going through w?

cmi2013 descriptive graph-theory counting

Answer

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2.2.2 Counting: GATE1994-1.6, ISRO2008-29 top

https://gateoverflow.in/2443

The number of distinct simple graphs with up to three nodes is

A. 15
B. 10
C. 7
D. 9

gate1994 graph-theory permutations-and-combinations normal isro2008 counting

Answer

2.2.3 Counting: GATE2001-2.15 top

https://gateoverflow.in/733

How many undirected graphs (not necessarily connected) can be constructed out of a given set $V = \{v_1, v_2, \ldots v_n\}$ of n vertices?

A. $\frac{n(n-1)}{2}$
B. 2^n
C. $n!$
D. $2^{\frac{n(n-1)}{2}}$

gate2001 graph-theory normal counting

Answer

2.2.4 Counting: GATE2004-79 top

https://gateoverflow.in/1073

How many graphs on n labeled vertices exist which have at least $\frac{(n^2-3n)}{2}$ edges ?

A. $\left(\frac{n^2-n}{2}\right) C_{\left(\frac{n^2-3n}{2}\right)}$
B. $\sum_{k=0}^{\left(\frac{n^2-3n}{2}\right)} .(n^2-n) C_k$
C. $\left(\frac{n^2-n}{2}\right) C_n$
D. $\sum_{k=0}^{n} .\left(\frac{n^2-n}{2}\right) C_k$

gate2004 graph-theory permutations-and-combinations normal counting

Answer

2.2.5 Counting: GATE2005-35 top

https://gateoverflow.in/1371

How many distinct binary search trees can be created out of 4 distinct keys?

A. 5
B. 14
C. 24
D. 42

gate2005 graph-theory counting normal

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Answer

2.2.6 Counting: GATE2012-38 top

https://gateoverflow.in/473

Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to

A. 15
B. 30
C. 90
D. 360

gate2012 graph-theory normal marks-to-all counting

Answer

2.2.7 Counting: TIFR2017-B-12 top

https://gateoverflow.in/95819

An undirected graph is complete if there is an edge between every pair of vertices. Given a complete undirected graph on n vertices, in how many ways can you choose a direction for the edges so that there are no directed cycles?

A. n
B. $\frac{n(n-1)}{2}$
C. $n!$
D. 2^n
E. $2^m, \text{ where } m = \frac{n(n-1)}{2}$

tifr2017 graph-theory counting

Answer

Answers: Counting

2.2.1 Counting: CMI2013-B-02 top

https://gateoverflow.in/46612

Let us take the three vertices as $(1, 2, 10)$.

Now the simplest case is when only 3 vertices are involved , i.e. $(1, 2, 10) = 1$ way
Now we make room for one more vertex $(1, _, 2, 10)$ or $(1, 2, _, 10)$ which can be filled in 7 ways giving a total of $7 * 2$ ways
For one more vertex we have $7 * 6 * 3$ ways and so on till we have accompanied all the 7 vertices and along with 2 a total of 8 vertices are there with total ordering of $7! * 8$

So, to sum it : $1 + 7 * 2 + 7 * 6 * 3 + 7 * 6 * 5 * 4 + \ldots + 7! * 8$

$$= 7P0 * 1 + 7P1 * 2 + 7P2 * 3 + \ldots + 7P7 * 8$$

2 votes -- Prasita Mukherjee (429 points)

2.2.2 Counting: GATE1994-1.6, ISRO2008-29 top

https://gateoverflow.in/2443

Selected Answer

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answer = **option (C)**

50 votes -- Amar Vashishth (30.5k points)

Answer: C

The number of max edges a simple graph can have is $\frac{n \times (n-1)}{2}$.

So, for a graph with 3 nodes the max number of edges is 3.
Now there can be 0 edges, 1 edge, 2 edges or 3 edges in a 3 node simple graph.
So the total number of unlabeled simple graphs on 3 nodes will be 4.
Similarly for two node graph we have option of 0 or 1 edge and for one node graph we have option of 0 edge.
So the total number of simple graphs upto three nodes $= 4 + 2 + 1 = 7$.

28 votes -- Rajarshi Sarkar (34.1k points)

2.2.3 Counting: GATE2001-2.15 top

https://gateoverflow.in/733

Selected Answer

With n vertices we have max possible ${}^{n}C_2$ edges in a simple graph. and each subset of these edges will form a graph, so total number of undirected graph possible = $2^{\frac{n(n-1)}{2}}$

34 votes -- Vikrant Singh (13.5k points)

2.2.4 Counting: GATE2004-79 top

https://gateoverflow.in/1073

Selected Answer

Let $a = \frac{n(n-1)}{2}, b = \frac{n^2-3n}{2}$

Minimum no of edges has to be $\frac{n^2-3n}{2} = b$.

Maximum no of edges in simple graph = $\frac{n(n-1)}{2} = a$.

So, no of graph with minimum b edges :

$$= C(a,b) + C(a,b+1) + C(a,b+2) + \cdots + C(a,a)$$

$$= C(a,a-b) + C(a,a-(b+1)) + C(a,a-(b+2)) + \cdots + C(a,0)$$

$$= C(a,n) + C(a,n-1) + C(a,n-2) + \cdots + C(a,0)) \ (\because a-b=n)$$

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$$= C\left(\frac{n(n-1)}{2}, n\right) + C\left(\frac{n(n-1)}{2}, n-1\right) + C\left(\frac{n(n-1)}{2}, n-2\right) + \cdots + C\left(\frac{n(n-1)}{2}, 0\right)$$

$$= \sum_{k=0}^{n} \binom{\frac{n^2-n}{2}} C_k$$

Option (**D**)..

53 votes -- Digvijay (54.8k points)

2.2.5 Counting: GATE2005-35 top

https://gateoverflow.in/1371

Selected Answer

answer - (B)

number of distinct BSTs = $\frac{{}^{2n}C_n}{n+1}$ (or) $= \frac{(2n)!}{(n+1)!n!}$

For a given Binary tree structure, there can be only 1 BST. Hence, no. of different BSTs with n nodes will be equal to the no. of different binary tree structures possible for n nodes.

For derivation:
http://gatecse.in/number-of-binary-trees-possible-with-n-nodes/

23 votes -- Ankit Rokde (9k points)

2.2.6 Counting: GATE2012-38 top

https://gateoverflow.in/473

Selected Answer

From 6 vertices we can select 4 distinct vertices in ${}^6C_4 = 15$ ways.
Now, with 4 vertices, we can form only 3 distinct cycles. [See below]
So, total no. of distinct cycles of length $4 = 15 \times 3 = 45$.

No. of cyclic permutations of n objects $= (n-1)!$ and for $n = 4$, we get $3! = 6$ ways. But number of distinct cycles in a graph is exactly half the number of cyclic permutations as there is no left/right ordering in a graph. For example $a - b - c - d$ and $a - d - c - b$ are different permutations but in a graph they form the same cycle.

Since, 45 was not in the choice, marks were given to all in GATE.

88 votes -- gatecse (18k points)

2.2.7 Counting: TIFR2017-B-12 top

https://gateoverflow.in/95819

Selected Answer

They are asking to convert Complete Undirected graph into Directed graph without cycle by choosing direction for the edges.

See this K_3 graph-

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(Image ref)

By this time you must have got Problem statement meaning. Your resultant graph should be acyclic.

Lets say you have a complete graph G which has n vertices, $v_1, v_2, \ldots v_n$. To convert it into the resultant graph we have to assign direction to each edge. Now see, our resultant graph is acyclic therefore it must have topological order.

(I have not drawn all edges except V_1 edges.)

here every rearrangement of vertices in topological sort leads to one particular combination to choose the direction of edges.

Hence - $n!$ is answer.

Just to illustrate more, see one of the permutation out of $n!$

These two permutation shows that undirected edge between V_1 and V_2, was first chosen as $V_1 \rightarrow V_2$ and then $V_2 \rightarrow V_1$

Don't think about the labeling of vertices, If I do unlabelling of all $n!$ permutations then all structures are same. But it doesn't matter If I am arriving at the same structure, What matters is, In how many ways I can reach to that.

See this-

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All these structures are Isomorphic...But still, there are $3!$ ways to reach such structure.

C.

24 votes -- Sachin Mittal (15.6k points)

2.3 Degree Of Graph(16) top

2.3.1 Degree Of Graph: CMI2013-A-06 top

https://gateoverflow.in/46596

A simple graph is one in which there are no self-loops and each pair of distinct vertices is connected by at most one edge.Let G be a simple graph on 8 vertices such that there is a vertex of degree 1, a vertex of degree 2, a vertex of degree 3, a vertex of degree 4, a vertex of degree 5, a vertex of degree 6 and a vertex of degree 7. Which of the following can be the degree of the last vertex?

A. 3
B. 0
C. 5
D. 4

cmi2013 graph-theory normal degree-of-graph

Answer

2.3.2 Degree Of Graph: CMI2015-A-05 top

https://gateoverflow.in/47039

An undirected graph has 10 vertices labelled $1, 2, \ldots, 10$ and 37 edges. Vertices $1, 3, 5, 7, 9$ have degree 8 and vertices $2, 4, 6, 8$ have degree 7. What is the degree of vertex 10 ?

A. 5
B. 6
C. 7
D. 8

cmi2015 graph-theory degree-of-graph easy

Answer

2.3.3 Degree Of Graph: GATE1987-9c top

https://gateoverflow.in/82438

Show that the number of odd-degree vertices in a finite graph is even.

gate1987 graph-theory degree-of-graph descriptive

Answer

2.3.4 Degree Of Graph: GATE1991-16,b top

https://gateoverflow.in/26647

Show that all vertices in an undirected finite graph cannot have distinct degrees, if the graph has at least two vertices.

gate1991 graph-theory degree-of-graph descriptive

Answer

2.3.5 Degree Of Graph: GATE1995-24 top

https://gateoverflow.in/2663

Prove that in finite graph, the number of vertices of odd degree is always even.

gate1995 graph-theory degree-of-graph descriptive

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Answer

2.3.6 Degree Of Graph: GATE2003-40 top

https://gateoverflow.in/931

A graph $G = (V, E)$ satisfies $|E| \leq 3|V| - 6$. The min-degree of G is defined as $\min_{v \in V} \{degree(v)\}$. Therefore, min-degree of G cannot be

A. 3
B. 4
C. 5
D. 6

gate2003 graph-theory normal degree-of-graph

Answer

2.3.7 Degree Of Graph: GATE2006-71 top

https://gateoverflow.in/1850

The 2^n vertices of a graph G corresponds to all subsets of a set of size n, for $n \geq 6$. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements.
The number of vertices of degree zero in G is:

A. 1
B. n
C. $n + 1$
D. 2^n

gate2006 graph-theory normal degree-of-graph

Answer

2.3.8 Degree Of Graph: GATE2006-72 top

https://gateoverflow.in/43566

The 2^n vertices of a graph G corresponds to all subsets of a set of size n, for $n \geq 6$. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements.

The maximum degree of a vertex in G is:

A. $\binom{\frac{n}{2}}{2}.2^{\frac{n}{2}}$
B. 2^{n-2}
C. $2^{n-3} \times 3$
D. 2^{n-1}

gate2006 graph-theory normal degree-of-graph

Answer

2.3.9 Degree Of Graph: GATE2009-3 top

https://gateoverflow.in/804

Which one of the following is **TRUE** for any simple connected undirected graph with more than 2 vertices?

A. No two vertices have the same degree.
B. At least two vertices have the same degree.
C. At least three vertices have the same degree.
D. All vertices have the same degree.

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gate2009 graph-theory normal degree-of-graph

Answer

2.3.10 Degree Of Graph: GATE2010-28 top

https://gateoverflow.in/1154

The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences can not be the degree sequence of any graph?

I. $7, 6, 5, 4, 4, 3, 2, 1$
II. $6, 6, 6, 6, 3, 3, 2, 2$
III. $7, 6, 6, 4, 4, 3, 2, 2$
IV. $8, 7, 7, 6, 4, 2, 1, 1$

A. I and II
B. III and IV
C. IV only
D. II and IV

gate2010 graph-theory degree-of-graph

Answer

2.3.11 Degree Of Graph: GATE2013-25 top

https://gateoverflow.in/1536

Which of the following statements is/are TRUE for undirected graphs?

P: Number of odd degree vertices is even.
Q: Sum of degrees of all vertices is even.

A. P only
B. Q only
C. Both P and Q
D. Neither P nor Q

gate2013 graph-theory easy degree-of-graph

Answer

2.3.12 Degree Of Graph: GATE2014-1-52 top

https://gateoverflow.in/1932

An ordered $n-$tuple $(d_1, d_2, \ldots, d_n)$ with $d_1 \geq d_2 \geq \ldots \geq d_n$ is called *graphic* if there exists a simple undirected graph with n vertices having degrees $d_1, d_2, \ldots, d_n$ respectively. Which one of the following 6-tuples is NOT graphic?

A. $(1, 1, 1, 1, 1, 1)$
B. $(2, 2, 2, 2, 2, 2)$
C. $(3, 3, 3, 1, 0, 0)$
D. $(3, 2, 1, 1, 1, 0)$

gate2014-1 graph-theory normal degree-of-graph

Answer

2.3.13 Degree Of Graph: GATE2017-2-23 top

https://gateoverflow.in/118594

G is an undirected graph with n vertices and 25 edges such that each vertex of G has degree at least 3. Then the maximum possible value of n is _____ .

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gate2017-2 graph-theory numerical-answers degree-of-graph

Answer

2.3.14 Degree Of Graph: TIFR2010-B-36 top

https://gateoverflow.in/19248

In a directed graph, every vertex has exactly seven edges coming in. What can one always say about the number of edges going out of its vertices?

A. Exactly seven edges leave every vertex.
B. Exactly seven edges leave some vertex.
C. Some vertex has at least seven edges leaving it.
D. The number of edges coming out of vertex is odd.
E. None of the above.

tifr2010 graph-theory degree-of-graph

Answer

2.3.15 Degree Of Graph: TIFR2012-B-2 top

https://gateoverflow.in/25047

In a graph, the degree of a vertex is the number of edges incident (connected) on it. Which of the following is true for every graph G?

a. There are even number of vertices of even degree.
b. There are odd number of vertices of even degree.
c. There are even number of vertices of odd degree.
d. There are odd number of vertices of odd degree.
e. All the vertices are of even degree.

tifr2012 graph-theory degree-of-graph

Answer

2.3.16 Degree Of Graph: TIFR2018-B-8 top

https://gateoverflow.in/179292

In an undirected graph G with n vertices, vertex 1 has degree 1, while each vertex $2, \ldots, n-1$ has degree 10 and the degree of vertex n is unknown, Which of the following statement must be TRUE on the graph G?

a. There is a path from vertex 1 to vertex n.
b. There is a path from vertex 1 to each vertex $2, \ldots, n-1$.
c. Vertex n has degree 1.
d. The diameter of the graph is at most $\frac{n}{10}$
e. All of the above choices must be TRUE

tifr2018 graph-theory degree-of-graph

Answer

Answers: Degree Of Graph

2.3.1 Degree Of Graph: CMI2013-A-06 top

https://gateoverflow.in/46596

Selected Answer

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We know, sum of degrees of all vertices $= 2 \times \text{no. of edges}$

Say no. of edges is E

Degree of last vertices is x

then,

$1+2+3+4+5+6+7+x = 2 \times E.$

$28 + x = 2 \times E.$

Now, putting options we get answer B) 0 or D) 4

But One vertex of degree 7 means it connected to all other vertex

So, degree must be (**D**) 4.

9 votes -- srestha (86.6k points)

2.3.2 Degree Of Graph: CMI2015-A-05 top

https://gateoverflow.in/47039

Selected Answer

Vertices $1, 3, 5, 7, 9$ have degree 8 and vertices $2, 4, 6, 8$ have degree 7.

We know tht in a graph **Sum of degrees**
$= 2 \times \text{No. of edges}$

Let $X =$ degree of vertex 10

$8+7+8+7+8+7+8+7+8+X = 2 \times 37$

$68 + X = 74$

$X = 6$

Hence,Degree of vertex 10 is 6.

Hence, Option (**B**) 6 is the correct choice.

9 votes -- Leen Sharma (40k points)

2.3.3 Degree Of Graph: GATE1987-9c top

https://gateoverflow.in/82438

Selected Answer

For proving this we should know that

$\sum_d(V) = 2e \quad \rightarrow (1)$

Because one edge consist of two vertices and hence contributes two degrees.

For any Graph G

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$\sum_d(V) = \sum_d(V_{odd}) + \Sigma_d(V_{even}) \quad \rightarrow (2)$

From (1) we can say that $\sum_d(V)$ should be **even.**

$\sum_d(V_{even})$ will be always **even** ($\because$ sum of even numbers is always even)

So, for (2) to be true $\sum_d(V_{odd})$ should be **even** ($\because$ even+odd=odd & even+even=even)

In $\sum_d(V_{odd})$, every **degree is odd.**

So, for $\sum_d(V_{odd})$ to be **Even,**

Number of odd-degree vertices should be even (
$\because$ only when we add an even number of odd numbers we get an even number).

3 votes -- saurabh dange (399 points)

2.3.4 Degree Of Graph: GATE1991-16,b top https://gateoverflow.in/26647

Selected Answer

Let $n > 2$ and all the vertices have distinct degrees. Now, let the degrees be $0, 1, 2, \ldots, (n-1)$ which are all distinct and possible as a vertex can be connected to $(n-1)$ other vertices. But, there is a problem here if a vertex is connected to $(n-1)$ other vertices, it means there cannot be a vertex with 0 degree anymore. Thus for n vertices we now have only$(n-1)$ possible degrees meaning at least one must repeat- pigeon comes here :)

31 votes -- Arjun Suresh (348k points)

2.3.5 Degree Of Graph: GATE1995-24 top https://gateoverflow.in/2663

Selected Answer

In any finite graph,

Sum of degree of all the vertices $= 2 \times \text{number of edges}$

Sum of degree of all the vertices with even degree $+$ sum of degree of all the vertices with odd degree $= 2 \times \text{number of edges}$

Even number $+$ sum of degree of all the vertices with odd degree $=$ a even number.

It is possible only if the number of odd degree vertices is even.

16 votes -- suraj (5.6k points)

2.3.6 Degree Of Graph: GATE2003-40 top https://gateoverflow.in/931

Selected Answer

Say every vertex has a minimum degree, therefore, least number of edges that will be in the graph is given by the handshaking lemma as $\frac{min \times |v|}{2}$

But the maximum number of edges for such a graph is defined in the question as $3 \times |v| - 6$

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Putting the minimum number of edges obtained by handshaking lemma in the given inequality, we get:

$$\frac{min \times |v|}{2} \leq 3 \times |v| - 6$$

$$\frac{6 \times |v|}{2} \leq 3 \times |v| - 6; \text{ putting min degree as } 6$$

$$3 \times |v| \leq 3 \times |v| - 6$$

$$0 \leq -6$$

which is definitely inconsistent.

Hence, answer = **option (D)**

34 votes -- Amar Vashishth (30.5k points)

Let the min-degree of G be x. Then G has at least $\left[|v| \times \frac{x}{2}\right]$ edges.

$$\left[|v| \times \frac{x}{2}\right] \leq [(3 \times |v|) - 6]$$

for $x = 6$, we get $0 \leq -6$, Therefore, min degree of G cannot be 6.

Correct answer is **(D)**.

An alternative approach,

Let the min_degree of a graph be 'x' , then

$x \leq \left(\frac{2e}{n}\right)$,
given , $e \leq (3n - 6)$ $\{\text{it will be planner graph}\}$
put the value of e ,then min_degree will be ,

$x \leq \frac{(2(3n-6))}{n}$
$x \leq \frac{(6n-12)}{n}$
$x \leq \left(\frac{6n}{n} - \frac{12}{n}\right)$
$x \leq \left(6 - \frac{12}{n}\right)$,
when number of vertices is more , then value of

$\left(\frac{12}{n}\right)$ will be less , $\left(\frac{12}{n} = 0.000001\text{assume}\right)$,

then min_degree will be ,

$x \leq (6 - 0.000001)$
$x \leq 5.999999$, max value
$x \leq \text{floor value}\,(5.9999999\ldots)$
$x = 5$, maximum value of min_degree of defined graph (i.e. planner graph)

22 votes -- suraj (5.6k points)

2.3.7 Degree Of Graph: GATE2006-71 top

https://gateoverflow.in/1850

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Selected Answer

Ans is **(C)**.

no. of vertices with degree zero $=$ no. of subsets with size $(\leq 1) = n+1$.

as edges are there for every vertex with two or more elements as we have a vertex for all subsets of n.

32 votes -- Vikrant Singh (13.5k points)

2.3.8 Degree Of Graph: GATE2006-72 top

https://gateoverflow.in/43566

Selected Answer

(C) $\max_k({}^kC_2.2^{n-k}) = {}^3C_2.\,2^{n-3} = 3.2^{n-3}$.

Let the vertex having the max degree contain k elements. Now, as per the given condition, it can have edges to all vertices having two common elements (exactly 2 common). So, we can choose the 2 common elements in ${}^kC_2$ ways. Now, for each of these 2 pair of elements, it can have an edge to a vertex containing $n-k$ elements $+$ the 2 common elements. This will be equal to 2^{n-k} possible subsets as the 2 common elements must always be present and other k elements must always be absent. So, we get the degree as

${}^kC_2.\,2^{n-k}$

Now, our answer will be the maximum value for this. We can differentiate this (w.r.t k) and equate to 0. But in other way we can try different values for k starting with 2. As we see if we increase k from 2 on wards, the 2^{n-k} term gets divided by 2. The other term is ${}^kC_2$, which goes like $1, 3, 6, 10\ldots$ for $k = 2, 3, 4, 5, \ldots$. So, we get the max. degree for $k = 3$ or 4 and this will be $3.\,2^{n-3}$.

26 votes -- Vikrant Singh (13.5k points)

2.3.9 Degree Of Graph: GATE2009-3 top

https://gateoverflow.in/804

Selected Answer

answer = **option (B)**

There are n vertices and at least $(n-1)$ edges. So, for each vertex, degree should range from 1 (since graph is connected) to $(n-1)$ (since graph is simple).

But we have n such vertices- filling n things with $(n-1)$ numbers.

$$\left\lceil \frac{n}{n-1} \right\rceil = \lceil 1. \sim \rceil = 2$$

So, at least 2 of them must be equal (pigeonhole principle).

38 votes -- gatecse (18k points)

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2.3.10 Degree Of Graph: GATE2010-28 top

https://gateoverflow.in/1154

Selected Answer

A degree sequence $d_1, d_2, d_3 \ldots d_n$ of non negative integer is graphical if it is a degree sequence of a graph. We now introduce a powerful tool to determine whether a particular sequence is graphical due to Havel and Hakimi

Havel–Hakimi Theorem :

→ According to this theorem, Let D be the sequence $d_1, d_2, d_3 \ldots dn$ with $d_1 \geq d_2 \geq d_3 \geq \ldots d_n$ for $n \geq 2$ and $d_i \geq 0$.
→ If each $d_i = 0$ then D is graphical
→ Then D_0 be the sequence obtained by:
→ Discarding d_1, and
→ Subtracting 1 from each of the next d_1 entries of D.
→ That is Degree sequence D_0 would be : $d_2 - 1, d_3 - 1, \ldots, d_{d_1+1} - 1, \ldots, d_n$
→ Then, D is graphical if and only if D_0 is graphical.

Now, we apply this theorem to given sequences:

I. $7,6,5,4,4,3,2,1 \rightarrow 5,4,3,3,2,1,0 \rightarrow 3,2,2,1,0,0 \rightarrow 1,1,0,0,0 \rightarrow 0,0,0,0$so it is graphical.
II. $6,6,6,6,3,3,2,2 \rightarrow 5,5,5,2,2,1,2$(arrange in ascending order) $\rightarrow 5,5,5,2,2,2,1 \rightarrow 4,4,1,1,1,1 \rightarrow 3,0,0,0,1$we cannot continue to get all $0's$, so it is not graphical.
III. $7,6,6,4,4,3,2,2 \rightarrow 5,5,3,3,2,1,1 \rightarrow 4,2,2,1,1,0 \rightarrow 1,1,0,0,0 \rightarrow 0,0,0,0$so it is graphical.
IV. $8,7,7,6,4,2,1,1$, here degree of a vertex is 8 and total number of vertices are 8, which is impossible, hence it is not graphical.

Hence, only option (I) and (III) are graphic sequence and answer is option-(**D**)

13 votes -- Madhab Paul Choudhury (4.1k points)

2.3.11 Degree Of Graph: GATE2013-25 top

https://gateoverflow.in/1536

Selected Answer

Both are correct

P: sum of odd degree $+$ sum of even degree$= 2 \times \text{no. of edges}$

sum of odd degree$= 2 \times \text{no. of edges}$ - $\text{sum of even degree}$

The right hand side must be even as the difference of 2 even numbers is always even.

Q: each edge is counted twice so sum of degree is always even

24 votes -- Bhagirathi Nayak (14.1k points)

2.3.12 Degree Of Graph: GATE2014-1-52 top

https://gateoverflow.in/1932

Selected Answer

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This can be solved using havel-hakimi theorem.

$\text{The idea is simple}:$ Remove a vertex, which results into decrease of degree by 1 of each vertex which was connected to it. Keep removing like this, and if we get any negative degree, the degree sequence was not possible.

We need not check (A) and (B) as they are clearly graphs : (A) is 3 disconnected edges and(B) is 2 disconnected triangles.

For (C), we remove first vertex of degree 3, and thus decrease degree by 1 of next 3 vertices, so we get $(2,2,0,0,0)$, then we remove vertex of degree 2, and decrease degree of next 2 vertices to get $(1,-1,0,0)$. Since we get negative degree, original degree sequence is impossible.

For (D) : $(3,2,1,1,1,0) \Rightarrow (1,0,0,1,0)$. Now since this list is not sorted (which is required to apply further steps of algorithm), we sort it to get $(1,1,0,0,0)$. Then we continue our algorithm on this list to get $(0,0,0,0)$, which is valid (4 isolated vertices).

So (C) is answer.

32 votes -- Happy Mittal (11.2k points)

2.3.13 Degree Of Graph: GATE2017-2-23 top

https://gateoverflow.in/118594

Selected Answer

Let m be minimum degree and M be maximum degree of a graph, then $m \leq \frac{2E}{V} \leq M$

$m = 3, E = 25, V = \ldots ?$

So, $3 \leq \frac{2*25}{V}$

$V \leq \frac{50}{3}$

$V \leq 16.667 \Rightarrow V = 16$

41 votes -- Manish Joshi (27.9k points)

2.3.14 Degree Of Graph: TIFR2010-B-36 top

https://gateoverflow.in/19248

Selected Answer

Since 7 edges come to every vertex, total no. of edges leaving n vertices must be $7n$. So, option (A) is a possibility but it needn't be always true. We can have 8 edges leave one vertex and 6 edges leave another (and similarly any other combination of outgoing edges ensuring total no. of outgoing edges remain constant). But option (C) must always be true as if none of the n vertices have at least 7 edges leaving, sum of outgoing edges can never be $7n$.

21 votes -- Arjun Suresh (348k points)

2.3.15 Degree Of Graph: TIFR2012-B-2 top

https://gateoverflow.in/25047

Selected Answer

As we know that sum of degree of vertex $= 2 \times edges$.

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let there is u vertex with odd degrees and v vertex with even degrees.

Then $\sum(u) + \sum(v) = 2e$.
now $2e = \text{even}$.
$\sum(v) =$ sum of even number will be even.
$\sum(u) =$ if you consider odd number of vertices of odd degree then sum will be odd and this will violate $2e$
so there will be always the even number of vertices with odd degree

Hence, Ans is (**c**)There are the even number of vertices of odd degree.

9 votes -- Umang Raman (15.7k points)

2.3.16 Degree Of Graph: TIFR2018-B-8 top

https://gateoverflow.in/179292

Only one statement is important to resolve this question

Number of vertices of odd degree in a graph must be even

so degree of vertex 1 is 1 and rest $2, \ldots, n-1$ are even

which concludes degree of vertex n must be odd

degree of vertex n can be 1,3,5,7...$

if degree of vertex n is 1 then statement b becomes false

if degree of vertex n is $3, 5, 7\ldots$ then statement c and d become false

only statement hold is (**a**).

3 votes -- Keval Malde (16.1k points)

2.4 Euler Graph(1) top

2.4.1 Euler Graph: GATE2007-23 top

https://gateoverflow.in/1221

Which of the following graphs has an Eulerian circuit?

A. Any k-regular graph where k is an even number.
B. A complete graph on 90 vertices.
C. The complement of a cycle on 25 vertices.
D. None of the above

gate2007 graph-theory normal graph-connectivity euler-graph

Answer

Answers: Euler Graph

2.4.1 Euler Graph: GATE2007-23 top

https://gateoverflow.in/1221

Selected Answer

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A connected Graph has Euler Circuit $\iff$ `all of its vertices` have even degree
A connected Graph has Euler Path $\iff$ `exactly 2 of its vertices` have odd degree

A. k-regular graph where k is even number.
a k-regular graph need not be connected always. Example : The given below graph is a 2 regular graph is not a Euler graph. This is so because there is no single walk which covers all edges.

Graph G with 2 Components

B. The complete graph of 90 vertices
In such a graph `every vertex will have an odd degree` = 89, Hence it cannot have a Euler path/Circuit.

C. To get degree of all vertices of the complement of cycle on 25 vertices we need to `subtract the degree of a complete graph of 25 vertices with degree of vertices in the original given graph` i.e. cycle on 25 vertices.

Degree of complement $= 24 - 2 = 22$. Since, every degree is Even, and it is connected also, therefore `Graph Might be a Euler Cycle` but we need to check graph is connected or not.

Now Question comes it is connected or not :

`It is connected` because, there is a theorem which says, "G be a graph with n vertices and if every vertex has a degree of at least $\frac{n-1}{2}$ then G is connected." [check this]

Here Degree of each vertex is 22, which is, of course, greater than $\frac{25-1}{2} = 12$.

Hence Graph must be Euler graph.

`Option` **C**

62 votes -- Mithlesh Upadhyay (5.8k points)

2.5 Graph Coloring(11) top

2.5.1 Graph Coloring: GATE2002-1.4 top

https://gateoverflow.in/808

The minimum number of colours required to colour the vertices of a cycle with n nodes in such a way that no two adjacent nodes have the same colour is

A. 2
B. 3
C. 4
D. $n - 2\left\lfloor \frac{n}{2} \right\rfloor + 2$

gate2002 graph-theory graph-coloring normal

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Answer

2.5.2 Graph Coloring: GATE2004-77 top

https://gateoverflow.in/1071

The minimum number of colours required to colour the following graph, such that no two adjacent vertices are assigned the same color, is

A. 2
B. 3
C. 4
D. 5

gate2004 graph-theory graph-coloring easy

Answer

2.5.3 Graph Coloring: GATE2006-IT-25 top

https://gateoverflow.in/3564

Consider the undirected graph G defined as follows. The vertices of G are bit strings of length n. We have an edge between vertex u and vertex v if and only if u and v differ in exactly one bit position (in other words, v can be obtained from u by flipping a single bit). The ratio of the chromatic number of G to the diameter of G is,

A. $\frac{1}{(2^{n-1})}$
B. $\left(\frac{1}{n}\right)$
C. $\left(\frac{2}{n}\right)$
D. $\left(\frac{3}{n}\right)$

gate2006-it graph-theory graph-coloring normal

Answer

2.5.4 Graph Coloring: GATE2008-IT-3 top

https://gateoverflow.in/3263

What is the chromatic number of the following graph?

A. 2
B. 3
C. 4
D. 5

gate2008-it graph-theory graph-coloring normal

Answer

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2.5.5 Graph Coloring: GATE2009-2 top

https://gateoverflow.in/796

What is the chromatic number of an n vertex simple connected graph which does not contain any odd length cycle? Assume $n > 2$.

A. 2
B. 3
C. $n-1$
D. n

gate2009 graph-theory graph-coloring normal

Answer

2.5.6 Graph Coloring: GATE2016-2-03 top

https://gateoverflow.in/39553

The minimum number of colours that is sufficient to vertex-colour any planar graph is _____.

gate2016-2 graph-theory graph-coloring normal numerical-answers

Answer

2.5.7 Graph Coloring: GATE2018-18 top

https://gateoverflow.in/204092

The chromatic number of the following graph is _____

graph-theory graph-coloring numerical-answers gate2018

Answer

2.5.8 Graph Coloring: TIFR2013-B-1 top

https://gateoverflow.in/25508

Let $G = (V, E)$ be a simple undirected graph on n vertices. A colouring of G is an assignment of colours to each vertex such that endpoints of every edge are given different colours. Let $\chi(G)$ denote the chromatic number of G, i.e. the minimum numbers of colours needed for a valid colouring of G. A set $B \subseteq V$ is an independent set if no pair of vertices in B is connected by an edge. Let $a(G)$ be the number of vertices in a largest possible independent set in G. In the absence of any further information about G we can conclude.

A. $\chi(G) \geq a(G)$
B. $\chi(G) \leq a(G)$
C. $a(G) \geq \frac{n}{\chi(G)}$
D. $a(G) \leq \frac{n}{\chi(G)}$
E. None of the above.

tifr2013 graph-theory graph-coloring

Answer

2.5.9 Graph Coloring: TIFR2017-B-1 top

https://gateoverflow.in/95669

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A vertex colouring with three colours of a graph $G = (V, E)$ is a mapping $c : V \to \{R, G, B\}$ so that adjacent vertices receive distinct colours. Consider the following undirected graph.

How many vertex colouring with three colours does this graph have?

A. 3^9
B. 6^3
C. 3×2^8
D. 27
E. 24

tifr2017 graph-theory graph-coloring

Answer

2.5.10 Graph Coloring: TIFR2017-B-10 top

https://gateoverflow.in/95817

A vertex colouring of a graph $G = (V, E)$ with k coulours is a mapping $c : V \to \{1, \ldots, k\}$ such that $c(u) \neq c(v)$ for every $(u, v) \in E$. Consider the following statements:

i. If every vertex in G has degree at most d then G admits a vertex coulouring using $d + 1$ colours.
ii. Every cycle admits a vertex colouring using 2 colours
iii. Every tree admits a vertex colouring using 2 colours

Which of the above statements is/are TRUE? Choose from the following options:

A. only i
B. only i and ii
C. only i and iii
D. only ii and iii
E. i, ii, and iii

tifr2017 graph-theory graph-coloring

Answer

2.5.11 Graph Coloring: TIFR2018-A-9 top

https://gateoverflow.in/179388

How many ways are there to assign colours from range $\{1, 2, \ldots, r\}$ to vertices of the following graph so that adjacent vertices receive distinct colours?

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A. r^4
B. $r^4 - 4r^3$
C. $r^4 - 5r^3 + 8r^2 - 4r$
D. $r^4 - 4r^3 + 9r^2 - 3r$
E. $r^4 - 5r^3 + 10r^2 - 15r$

tifr2018 graph-theory graph-coloring

Answer

Answers: Graph Coloring

2.5.1 Graph Coloring: GATE2002-1.4 top

https://gateoverflow.in/808

Selected Answer

Chromatic number will be 3 for when n is odd and will be 2 when n is even. Option (**D**) is a representation for this, hence the correct answer

24 votes -- Madhur Rawat (2.3k points)

2.5.2 Graph Coloring: GATE2004-77 top

https://gateoverflow.in/1071

Selected Answer

4 colors are required to color the graph in the prescribed way.

answer = **option C**

20 votes -- Amar Vashishth (30.5k points)

2.5.3 Graph Coloring: GATE2006-IT-25 top

https://gateoverflow.in/3564

Selected Answer

Answer is (C)

For the given condition we can simply design a K-MAP and mark an edge between every two

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adjacent cells in K-Map. (adjacency has to seen just as we do for minimization)

That will give us a Bipartite graph. chromatic number for this $= 2$.

Also from the same we can conclude that we need ,for a $'n'$ bit string, to traverse NO MORE than $(n-1)$ edges or $'n'$ vertices to get a path b/w two arbitrary points.

So ratio is $\left(\frac{2}{n}\right)$.

The given graph is actually hypercubegraph.

https://en.wikipedia.org/wiki/Hypercube_graph

See problem 4 here:
http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-042j-mathematics-for-computer-science-spring-2005/assignments/pset5_soln.pdf

24 votes -- Sandeep_Uniyal (7.5k points)

2.5.4 Graph Coloring: GATE2008-IT-3 top

https://gateoverflow.in/3263

Selected Answer

The chromatic number of a graph is the smallest number of colors needed to color the vertices so that no two adjacent vertices share the same color.

Hence minimum number of colors needed to color given graph is equal to 3 (**option B**)

For odd length cycles we need minimum 3 colors for vertex coloring and for even length cycles we need just 2.

24 votes -- vinodmits (393 points)

2.5.5 Graph Coloring: GATE2009-2 top

https://gateoverflow.in/796

Selected Answer

Lemma 1 : G is bipartite, if and only if it does not contain any cycle of odd length.

Proof. Suppose G has an odd cycle. Then obviously it cannot be bipartite, because no odd cycle is 2-colorable. Conversely, suppose G has no odd cycle. Then we can color the vertices greedily by 2 colors, always choosing a different color for a neighbor of some vertex which has been colored already. Any additional edges are consistent with our coloring, otherwise they would close a cycle of odd length with the edges we considered already. The easiest extremal question is about the maximum possible number of edges in a bipartite graph on n vertices. 1 ref@ http://math.mit.edu/~fox/MAT307-lecture07.pdf

Bipartite Graph: A graph which is 2-colorable is called bipartite. We have already seen several bipartite graphs, including paths, cycles with even length, and the graph of the cube (but not any other regular polyhedra)

ref@ http://ocw.mit.edu/high-school/mathematics/combinatorics-the-fine-art-of-counting/lecture-notes/MITHFH_lecturenotes_9.pdf
3. Bipartite graphs: By definition, every bipartite graph with at least one edge has chromatic number 2. (otherwise 1 if graph is null graph)

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ref@
http://math.ucsb.edu/~padraic/mathcamp_2011/introGT/MC2011_intro_to_GT_wk1_day4.pdf

28 votes -- Mithlesh Upadhyay (5.8k points)

2.5.6 Graph Coloring: GATE2016-2-03 top

https://gateoverflow.in/39553

Selected Answer

Four color theorem is a famous result and it says that any planar graphs can be colored with only 4 colors.

Ref: https://en.wikipedia.org/wiki/Four_color_theorem

Note for confused people $\Rightarrow$ 😬

Here ANY is used in sense of FOR ALL x. i.e., ANY means literally any one of graph can be selected !

Any man alive is gonna die $\Rightarrow$ All men are gonna die and not any specific one.
Hope this clears thing a bit !

34 votes -- Akash Kanase (42.5k points)

2.5.7 Graph Coloring: GATE2018-18 top

https://gateoverflow.in/204092

Selected Answer

3 is answer.

6 votes -- Prashant Singh (59.8k points)

2.5.8 Graph Coloring: TIFR2013-B-1 top

https://gateoverflow.in/25508

Selected Answer

Independence number : Size of largest maximum independent set.$a(G)$ (it covers all adjacent vertices)
Chromatic Number : Minimum No. of color required to properly color the graph .$\chi(G)$

The vertices of G can be partitioned into $\chi(G)$ monochromatic classes. Each class is an independent set, and hence cannot have size larger than $\alpha(G)$.

$\alpha(G)\chi(G) \geq n$. (its a theorem),
option C.

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15 votes -- Umang Raman (15.7k points)

2.5.9 Graph Coloring: TIFR2017-B-1 top

https://gateoverflow.in/95669

Selected Answer

Start with the Inner one which can be filled in $\Rightarrow 3 \times 2 \times 1 = 6$ ways.

Then, middle one can be filled in $\Rightarrow 2 \times 1 \times 1 = 2$ ways.

Then, similarly outermost can be filled in $\Rightarrow 2 \times 1 \times 1 = 2$ ways.

Hence, Total number of ways to fill this figure $\Rightarrow 6 \times 2 \times 2 = 24$ ways.

29 votes -- Kapil Phulwani (50k points)

2.5.10 Graph Coloring: TIFR2017-B-10 top

https://gateoverflow.in/95817

Selected Answer

i. is true, since in worst case the graph can be complete. So, $d+1$ colours are necessary for graph containing vertices with degree atmost $'d'$. Example : Consider a complete graph of 4 vertices ... K_4

ii. is false since cyles with odd no of vertices require 3 colours.

iii. is true, since each level of the tree must be coloured in an alternate fashion. We can do this with two colours. Its a theorem that a tree is 2 colourable ...

Therefore, option **C** is correct.

13 votes -- tarun_svbk (1.5k points)

2.5.11 Graph Coloring: TIFR2018-A-9 top

https://gateoverflow.in/179388

Selected Answer

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The graph can be colored using minimum 3 colors and maximum 4 colors.

No. of ways using 3 colors $= r(r-1)(r-2)$.

No. of ways using 4 colors $= r(r-1)(r-2)(r-3)$.

Total no. of ways $= r(r-1)(r-2) + r(r-1)(r-2)(r-3)$

$$= r^4 - 5r^3 + 8r^2 - 4r$$

OPTION (C)

13 votes -- ZAHID WAKEEL (1.2k points)

2.6 Graph Connectivity(22) top

2.6.1 Graph Connectivity: CMI2011-A-07 top https://gateoverflow.in/46194

Let $G = (V, E)$ be a graph. Define $\bar{G}$ to be $(V, \bar{E})$, where for all $u,\ v \in V$, $(u, v) \in \bar{E}$ if and only if $(u, v) \notin E$. Then which of the following is true?

A. $\bar{G}$ is always connected.
B. $\bar{G}$ is connected if G is not connected.
C. At least one of G and $\bar{G}$ connected.
D. G is not connected or $\bar{G}$ is not connected

cmi2011 graph-theory graph-connectivity

Answer

2.6.2 Graph Connectivity: CMI2012-B-01 top https://gateoverflow.in/46545

Let $G = (V, E)$ be a graph where $|V| = n$ and the degree of each vertex is strictly greater than $\frac{n}{2}$. Prove that G has a Hamiltonian path. (Hint: Consider a path of maximum length in G.)

cmi2012 descriptive graph-theory graph-connectivity

Answer

2.6.3 Graph Connectivity: GATE1990-1-viii top https://gateoverflow.in/83854

Fill in the blanks:

A graph which has the same number of edges as its complement must have number of vertices congruent to _____ or _____ modulo 4.

gate1990 graph-theory graph-connectivity

Answer

2.6.4 Graph Connectivity: GATE1991-01,xv top https://gateoverflow.in/510

The maximum number of possible edges in an undirected graph with n vertices and k components is _____.

gate1991 graph-theory graph-connectivity normal

Answer

2.6.5 Graph Connectivity: GATE1993-8.1 top https://gateoverflow.in/2299

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Consider a simple connected graph G with n vertices and n edges $(n > 2)$. Then, which of the following statements are true?

A. G has no cycles
B. The graph obtained by removing any edge from G is not connected
C. G has at least one cycle
D. The graph obtained by removing any two edges from G is not connected
E. None of the above

gate1993 graph-theory graph-connectivity easy

Answer

2.6.6 Graph Connectivity: GATE1995-1.25 top https://gateoverflow.in/2612

The minimum number of edges in a connected cyclic graph on n vertices is:

A. $n-1$
B. n
C. $n+1$
D. None of the above

gate1995 graph-theory graph-connectivity easy

Answer

2.6.7 Graph Connectivity: GATE1999-1.15 top https://gateoverflow.in/1468

The number of articulation points of the following graph is

A. 0
B. 1
C. 2
D. 3

gate1999 graph-theory graph-connectivity normal

Answer

2.6.8 Graph Connectivity: GATE1999-5 top https://gateoverflow.in/1504

Let G be a connected, undirected graph. A cut in G is a set of edges whose removal results in G being broken into two or more components, which are not connected with each other. The size of a cut is called its cardinality. A min-cut of G is a cut in G of minimum cardinality. Consider the following graph.

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a. Which of the following sets of edges is a cut

   i. $\{(A,B),(E,F),(B,D),(A,E),(A,D)\}$
   ii. $\{(B,D),(C,F),(A,B)\}$

b. What is cardinality of min-cut in this graph?
c. Prove that if a connected undirected graph G with n vertices has a min-cut of cardinality k, then G has at least $\left(\frac{n\times k}{2}\right)$ edges.

gate1999 graph-theory graph-connectivity normal

Answer

2.6.9 Graph Connectivity: GATE2002-1.25, ISRO2008-30, ISRO2016-6 top

https://gateoverflow.in/830

The maximum number of edges in a n-node undirected graph without self loops is

A. n^2
B. $\frac{n(n-1)}{2}$
C. $n-1$
D. $\frac{(n+1)(n)}{2}$

gate2002 graph-theory easy isro2008 isro2016 graph-connectivity

Answer

2.6.10 Graph Connectivity: GATE2003-8, ISRO2009-53 top

https://gateoverflow.in/899

Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie down between

A. k and n
B. $k-1$ and $k+1$
C. $k-1$ and $n-1$
D. $k+1$ and $n-k$

gate2003 graph-theory graph-connectivity normal isro2009

Answer

2.6.11 Graph Connectivity: GATE2004-IT-37 top

https://gateoverflow.in/3680

What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree $2, 3$ vertices of degree 4 and remaining of degree 3?

A. 10
B. 11
C. 18
D. 19

gate2004-it graph-theory graph-connectivity normal

Answer

2.6.12 Graph Connectivity: GATE2004-IT-5 top

https://gateoverflow.in/3646

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What is the maximum number of edges in an acyclic undirected graph with n vertices?

A. $n-1$
B. n
C. $n+1$
D. $2n-1$

gate2004-it graph-theory graph-connectivity normal

Answer

2.6.13 Graph Connectivity: GATE2005-IT-56 top https://gateoverflow.in/3817

Let G be a directed graph whose vertex set is the set of numbers from 1 to 100. There is an edge from a vertex i to a vertex j iff either $j=i+1$ or $j=3i$. The minimum number of edges in a path in G from vertex 1 to vertex 100 is

A. 4
B. 7
C. 23
D. 99

gate2005-it graph-theory graph-connectivity normal

Answer

2.6.14 Graph Connectivity: GATE2006-73 top https://gateoverflow.in/43567

The 2^n vertices of a graph G corresponds to all subsets of a set of size n, for $n \geq 6$. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements.

The number of connected components in G is:

A. n
B. $n+2$
C. $2^{\frac{n}{2}}$
D. $\frac{2^n}{n}$

gate2006 graph-theory normal graph-connectivity

Answer

2.6.15 Graph Connectivity: GATE2006-IT-11 top https://gateoverflow.in/3550

If all the edge weights of an undirected graph are positive, then any subset of edges that connects all the vertices and has minimum total weight is a

A. Hamiltonian cycle
B. grid
C. hypercube
D. tree

gate2006-it graph-theory graph-connectivity normal

Answer

2.6.16 Graph Connectivity: GATE2008-IT-27 top https://gateoverflow.in/3317

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G is a simple undirected graph. Some vertices of G are of odd degree. Add a node v to G and make it adjacent to each odd degree vertex of G. The resultant graph is sure to be

A. regular
B. complete
C. Hamiltonian
D. Euler

gate2008-it graph-theory graph-connectivity normal

Answer

2.6.17 Graph Connectivity: GATE2014-1-51 top https://gateoverflow.in/1931

Consider an undirected graph G where self-loops are not allowed. The vertex set of G is $\{(i,j) \mid 1 \leq i \leq 12, 1 \leq j \leq 12\}$. There is an edge between (a,b) and (c,d) if $|a-c| \leq 1$ and $|b-d| \leq 1$. The number of edges in this graph is_____.

gate2014-1 graph-theory numerical-answers normal graph-connectivity

Answer

2.6.18 Graph Connectivity: GATE2014-2-3 top https://gateoverflow.in/1955

The maximum number of edges in a bipartite graph on 12 vertices is____

gate2014-2 graph-theory graph-connectivity numerical-answers normal

Answer

2.6.19 Graph Connectivity: GATE2014-3-51 top https://gateoverflow.in/2085

If G is the forest with n vertices and k connected components, how many edges does G have?

A. $\left\lfloor \frac{n}{k} \right\rfloor$
B. $\left\lceil \frac{n}{k} \right\rceil$
C. $n-k$
D. $n-k+1$

gate2014-3 graph-theory graph-connectivity normal

Answer

2.6.20 Graph Connectivity: GATE2015-2-50 top https://gateoverflow.in/8252

In a connected graph, a bridge is an edge whose removal disconnects the graph. Which one of the following statements is true?

A. A tree has no bridges
B. A bridge cannot be part of a simple cycle
C. Every edge of a clique with size ≥ 3 is a bridge (A clique is any complete subgraph of a graph)
D. A graph with bridges cannot have cycle

gate2015-2 graph-theory graph-connectivity easy

Answer

2.6.21 Graph Connectivity: ISI2015-PCB-C3 top https://gateoverflow.in/120885

For a positive integer n, let $G = (V,E)$ be a graph, where $V = \{0,1\}^n$, i.e., V is the set of

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vertices has one to one correspondence with the set of all n-bit binary strings and $E = \{(u,v) \mid u,v \text{ belongs to } V, u \text{ and } v \text{ differ in exactly one bit position}\}$.

i. Determine size of E
ii. Show that G is connected

graph-theory discrete-mathematics isi2015 graph-connectivity

Answer

2.6.22 Graph Connectivity: TIFR2015-B-5 top

https://gateoverflow.in/29858

Suppose

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

is the adjacency matrix of an undirected graph with six vertices: that is, the rows and columns are indexed by vertices of the graph, and an entry is 1 if the corresponding vertices are connected by an edge and is 0 otherwise; the same order of vertices is used for the rows and columns. Which of the graphs below has the above adjacency matrix?

A. Only (i)
B. Only (ii)
C. Only (iii)
D. Only (iv)
E. (i) and (ii)

tifr2015 graph-connectivity graph-theory

Answer

Answers: Graph Connectivity

2.6.1 Graph Connectivity: CMI2011-A-07 top

https://gateoverflow.in/46194

Selected Answer

Correct answer would be **(C) At least one of G and G-bar is connected.**

Option (A): It is straight forward wrong.

Option (B): This is a **subset** of Option (C).

Option (D): This also implies that **G and G-bar are not connected at the same time, which is impossible**.

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Here are the total possibilities:

G	G-bar	Possible/Not-Possible
Connected	Connected	Possible
Connected	Disconnected	Possible
Disconnected	Connected	Possible
Disconnected	Disconnected	Not-Possible

18 votes -- Muktinath Vishwakarma (35.3k points)

2.6.2 Graph Connectivity: CMI2012-B-01 top https://gateoverflow.in/46545

The proof is similar to **Dirac theorem** In an n-vertex graph in which each vertex has degree at least $\frac{n}{2}$ must have a Hamiltonian cycle.

So, we can say that **If a graph contain Hamiltonian cycle, it will surely contain a Hamiltonian Path.**

But the converse of this is not true.

Here, consider a graph with 4 vertices and 6 edges which is $K4$ and the degree of each vertex is 3 $\left(\text{i.e} > \frac{n}{2}\right)$.

So, the graph contains $a\ b\ c\ d$ one path.

$b\ c\ d\ a$ another and even more.

And even it contains a Hamiltonian cycle.

3 votes -- Manoj Kumar (38.7k points)

2.6.3 Graph Connectivity: GATE1990-1-viii top https://gateoverflow.in/83854

It is the definition of self complementary graph..The definition of self complementary graph is :

It is a graph which is isomorphic to its complement.

By using invariant of isomorphism and property of edges of graph and its complement , we have **:**

a. No of edges of ismorphic graphs must be the same.
b. no of edge of a graph $+$ no of edges of complementary graph = No of edges in K_n (complete graph), where n is the no of vertices in each of the 2 graphs which will be the same

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So we know no of edges in $K_n = \frac{n(n-1)}{2}$.

So no of edges of each of the above 2 graph (a graph and its complement) $= \frac{n(n-1)}{4}$

So this means the number of vertices in each of the 2 graphs should be of the form " $4x$ " or " $4x+1$ " for integral value of no of edges which is necessary..

Hence the required answer is $4x$ or $4x+1$ . . .So that on doing modulo we get 0 which is the definition of congruence.

6 votes -- HABIB MOHAMMAD KHAN (98.7k points)

2.6.4 Graph Connectivity: GATE1991-01,xv top https://gateoverflow.in/510

Selected Answer

N vertices and K components.
(Component means they are non connected subgraphs)

We want maximum edges in total graph. How to get maximum edges ?

To get maximum, take one vertex each for each component, except last component.
Now $k-1$ components have 1 vertex each and so no edges.
The last component has $n-(k-1)$ vertices.
So make the last component complete.
i.e., It has ${}^{n-(k-1)}C_2 = \frac{(n-k)(n-k+1)}{2}$ edges.

Must do a similar model qsn on forest: https://gateoverflow.in/580/gate1992_03-iii

13 votes -- Ahwan Mishra (10.5k points)

Hopefully it should be clear that in any such graph all components will be complete, i.e., have all possible edges. Thus the only remaining question is how large each component should be?

If there are two components with a and b vertices, $a > 1, b > 1$, then together they can have at most

$\binom{a}{2} + \binom{b}{2} = \frac{1}{2}\left(a^2 - a + b^2 - b\right)$ edges.

However, if we place all but one of the vertices in a single component, we could have

$\binom{a+b-1}{2} + \binom{1}{2} = \frac{1}{2}(a+b-1)(a+b-2)$
$= \frac{1}{2}\left(a^2 + 2ab - 3a + b^2 - 3b + 2\right)$ edges.

Subtracting the first quantity from the second gives

$\frac{1}{2}((2ab - 3a - 3b + 2) - (-a - b)) = ab - a - b + a = (a-1)(b-1)$ which is > 0

Hence it is better not to have two components with multiple vertices.

This leaves us with the answer that all components should have one vertex except one, which will have $n-k+1$ vertices, for a total of $\binom{n-k+1}{2}$ edges.

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in simple connected graph , number of edges ,

$(n-1) \leq e \leq n.\frac{(n-1)}{2}$

in simple unconnected graph with k component , number of edges ,

$(n-k) \leq e \leq (n-k).\frac{(n-k+1)}{2}$

note :- put $k=1$ then it will be connected graph .

reference @ http://www.quora.com/What-is-the-maximum-number-of-edges-in-graph-with-n-vertices-and-k-components
another read @ http://stackoverflow.com/questions/24003861/maximum-number-of-edges-in-undirected-graph-with-n-vertices-with-k-connected-com

31 votes -- Mithlesh Upadhyay (5.8k points)

2.6.5 Graph Connectivity: GATE1993-8.1 top https://gateoverflow.in/2299

Selected Answer

This seems like multiple answer questions.

Here we have n vertices & n edges. So we must have cycle.

So (C) has at least one cycle is True & (A) is false.

(D) The graph obtained by removing any two edges from G is not connected $\rightarrow$ This is true, for graph of n vertices to be connected, we need at least $n-1$ edges. If we remove 2 out of n, we get $n-2$ edges, which can connect at max $n-1$ vertices. 1 Vertex at least will be disconnected. So D is true.

(B) is false as if graph is cyclic graph then removing any edge will not disconnect graph.

Answer $\rightarrow$ **(C)** & **(D)**.

16 votes -- Akash Kanase (42.5k points)

2.6.6 Graph Connectivity: GATE1995-1.25 top https://gateoverflow.in/2612

Selected Answer

(b). For making a cyclic graph, the minimum number of edges have to be equal to the number of vertices.

24 votes -- Gate Keeda (19.6k points)

2.6.7 Graph Connectivity: GATE1999-1.15 top https://gateoverflow.in/1468

Selected Answer

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ARTICULATION POINT: are those points whose removal from the graph makes the graph disconnected.

here if we remove the vertex no. 2 than we get disconnected graph.

similarly if we remove the vertex no. 3 than we get disconnected graph.

similarly if we remove the vertex no. 5 than we get disconnected graph.

So, **(D)** choice.

24 votes -- kunal chalotra (20.4k points)

2.6.8 Graph Connectivity: GATE1999-5 top

https://gateoverflow.in/1504

Selected Answer

a. Not a cut. We have spanning tree after removing this edges.
This is cut.we break graph into two pieces.

b. Min cut size is $2.\{BC, CF\}$. Removal these two edges disconnects C from the remaining graph.

c. Always cardinality of min-cut will be $\leq$ min-degree of the graph
So, $k \leq$ min-degree of the graph
min-degree of the graph $\geq k$
WKT sum of degrees of all vertices $= 2*$ number of edges.
minimum degree $\times n \leq 2 \times |E|$, where n is number of vertices, $|E|$ is number of edges.
$= k * n \leq 2|E|$

$\Rightarrow |E| \geq (n * k/2)$

16 votes -- Akash Kanase (42.5k points)

2.6.9 Graph Connectivity: GATE2002-1.25, ISRO2008-30, ISRO2016-6 top

https://gateoverflow.in/830

Selected Answer

In a graph of n vertices you can draw an edge from a vertex to $(n-1)$ vertex we will do it for n vertices so total number of edges is $n(n-1)$ now each edge is counted twice so the required maximum number of edges is $\frac{n(n-1)}{2}$.

31 votes -- Bhagirathi Nayak (14.1k points)

2.6.10 Graph Connectivity: GATE2003-8, ISRO2009-53 top

https://gateoverflow.in/899

Selected Answer

If a vertex is removed from the graph G,

Lower Bound: number of components decreased by one $= k-1$ (remove an isolated vertex which

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was a component)

Upper Bound: number of components $= n - 1$ (consider a vertex connected to all other vertices in a component as in a star and all other vertices outside this component being isolated. Now, removing the considered vertex makes all other $n - 1$ vertices isolated making $n - 1$ components)

Therefore (**C**).

48 votes -- Danish (3.8k points)

2.6.11 Graph Connectivity: GATE2004-IT-37 top https://gateoverflow.in/3680

Selected Answer

sum of degree of all the vertices $= 2*$ number of edges.

$2 \times 6 + 4 \times 3 + 3 \times x = 27 \times 2$

$x = 10.$

Number of vertices $= 6 + 3 + x = 19.$

The correct answer is (**D**).

22 votes -- suraj (5.6k points)

2.6.12 Graph Connectivity: GATE2004-IT-5 top https://gateoverflow.in/3646

Selected Answer

This is possible with spanning tree.

A spanning tree with n nodes has $n - 1$ edges.

Therefore, Answer is (**A**)

23 votes -- Dhananjay (1.1k points)

2.6.13 Graph Connectivity: GATE2005-IT-56 top https://gateoverflow.in/3817

Selected Answer

Edge set consists of edges from i to j using either

1. $j = i + 1$ (or)
2. $j = 3i.$

Second option will help us reach from 1 to 100 rapidly. The trick to solve this question is to **think in reverse way**. Instead of finding a path from 1 to 100, try to find a path from 100 to 1.

The edge sequence with minimum number of edges is
$1 \rightarrow 3 \rightarrow 9 \rightarrow 10 \rightarrow 11 \rightarrow 33 \rightarrow 99 - 100$ which consists of 7 edges.
The answer is option (2).

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44 votes -- Shridhar (393 points)

2.6.14 Graph Connectivity: GATE2006-73 top https://gateoverflow.in/43567

Selected Answer

(**B**) $n+1$ (subsets of size <2 are all disconnected) $+1$ (subsets of size ≥ 2 are all connected) $=n+2$.

32 votes -- Vikrant Singh (13.5k points)

2.6.15 Graph Connectivity: GATE2006-IT-11 top https://gateoverflow.in/3550

Selected Answer

A. **Hamiltonian cycle** $\Rightarrow$ This is cycle guys. Cycle will not only connect all vertices, it will have 1 extra edge than necessary. So I can just remove that edge & get better cost "subset of edges" which connect all vertices. So this is false.
B. **grid** $\Rightarrow$ This is unrelated concept. This is false.
ref $\Rightarrow$ https://en.wikipedia.org/wiki/Electrical_grid
C. **Hypercube** $\Rightarrow$ This is also unrelated concept. Also it have cycles too..This is false.
D. **Tree** $\Rightarrow$ This is answer. We need to have Minimum spanning Tree to be exact.
Ref $\Rightarrow$ https://en.wikipedia.org/wiki/Minimum_spanning_tree
"If all the edge weights of an undirected graph are positive, then any subset of edges that connects all the vertices and has minimum total weight is a Minimum Spanning Tree". !

(D) is true.

31 votes -- Akash Kanase (42.5k points)

2.6.16 Graph Connectivity: GATE2008-IT-27 top https://gateoverflow.in/3317

Selected Answer

In any simple undirected graph, total degree of all vertices is even (since each edge contributes 2 degrees). So number of vertices having odd degrees must be even, otherwise their sum would have been odd, making total degree also odd.

Now Single vertex v is connected to all these even number of vertices (which have odd degrees). So degree of v is also even. Moreover, now degree of all vertices which are connected to v is increased by 1, hence vertices which had odd degree earlier now have even degree.

So now, all vertices in graph have even degree, which is necessary and sufficient condition for euler graph. So (**D**) is correct.

40 votes -- Happy Mittal (11.2k points)

2.6.17 Graph Connectivity: GATE2014-1-51 top https://gateoverflow.in/1931

Selected Answer

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If you think of a 12×12 grid (like a chess board of size 12×12), then each each point (i, j), which is in i^{th} row and j^{th} column, is a vertex (i, j).

Now we are allowed to connect only those points which are atmost 1 distance apart (in both horizontal and vertical direction). So we will connect only horizontal neighbours, vertical neighbours, and diagonal neighbours.

So horizontal edges on each row are 11 i.e. $11 \times 12 = 132$ horizontal edges. Similarly we have 132 vertical edges.

To count diagonal edges, think of 1×1 square boxes in which diagonals meet each other. There are 11×11 such square boxes, and each box contains 2 diagonals, so total diagonals $= 242$.

So total edges $= 132 + 132 + 242 = 506$.

96 votes -- Happy Mittal (11.2k points)

Total number of vertices $= 12 \times 12 = 144$.

The graph formed by the description contains 4 (corner) vertices of degree 3 and 40 (external) vertices of degree 5
and 100 (remaining) vertices of degree 8.

According to (handshake theorem's)

$2|E| =$ sum of the degrees

$2|E| = 4 \times 3 + 40 \times 5 + 100 \times 8 = 1012.$

$|E| = \frac{1012}{2} = 506$ edges.

61 votes -- Mithlesh Upadhyay (5.8k points)

2.6.18 Graph Connectivity: GATE2014-2-3 top https://gateoverflow.in/1955

Selected Answer

Maximum no. of edges occur in a complete bipartite graph i.e. when every vertex has an edge to every opposite vertex.

Number of edges in a complete bipartite graph is mn, where m and n are no. of vertices on each side. This quantity is maximum when $m = n$ i.e. when there are 6 vertices on each side, so answer is 36.

31 votes -- Happy Mittal (11.2k points)

2.6.19 Graph Connectivity: GATE2014-3-51 top https://gateoverflow.in/2085

Selected Answer

A forest is a collection of trees. here we are given a forest with n vertices and k components. a component is itself a tree.

Since there are k components means that every component has a root (every tree has one), therefore we have k roots.

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Introduction of each new vertex to the forest introduces a single edge to a forest. so for remaining $n-k$ vertices when introduced, to make up to n vertices, contributes to $n-k$ edges.

Hence, ans $=$ **option (C)** $= (n-k)$

33 votes -- Amar Vashishth (30.5k points)

2.6.20 Graph Connectivity: GATE2015-2-50 top https://gateoverflow.in/8252

Selected Answer

Bridge / cut edge : A single edge whose removal will disconnect the graph is known as Bridge or cut edge.

A. Every edge of a tree is a bridge.
B. A bridge cannot be a part of a single cycle because in a cycle every vertex will be connected with every other vertex in 2 ways. Even if you remove 1 way by deleting an edge still the other way will make sure that the graph is connected.
C. A Clique will never have a bridge because though we remove 1 edge between any 2 vertices those 2 vertices will still be connected with the remaining $(n-2)$ vertices using an edge each.
D. Black edge in the graph is clearly a Bridge as removing that will produce an isolated edge.

14 votes -- Vicky rix (6.9k points)

2.6.21 Graph Connectivity: ISI2015-PCB-C3 top https://gateoverflow.in/120885

Selected Answer

These types of graphs are also known as hypercube graphs.

i. Consider any vertex v. Exactly n vertices have a hamming distance of 1 from v. (Reason: Consider the vertex which is exactly the same bit pattern as v except the first bit as v_1. v_2 differs from v in only the 2nd bit and so on till v_n which is different from v in last bit).
Now there are total of 2^n vertices (Since each vertex corresponds to a bit string and for n bits there are 2^n bit strings.).
Consider the basic theorem for undirected graphs which says that sum of degrees of vertices is equal to twice the no. of edges.
Using that here we get:
$2^n * n = 2 * e$ where e is the no. of edges.
Thus no. of edges = $2^{n-1}n$

ii. A graph is called connected if there's a path between any two vertices.
Consider any vertices v_x and v_y. Suppose hamming distance between them is k. Consider the first different bit k_1. You simply reach v_{k1} from v_x which differs from v_x only in that bit. Now v_{k1} and v_y differ from each other by $k-1$ bits. You continue to do this until you reach v_y. Thus any two vertices are connected by a path of k where k is the hamming distance between them.

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10 votes -- Akshay Arora (2.5k points)

2.6.22 Graph Connectivity: TIFR2015-B-5 top

https://gateoverflow.in/29858

Selected Answer

Yes, Option (**E**) must be the right answer.

Number of edges in the graph:

Since the graphs are undirected, it can be observed that there will be two 1's in the adjacency matrix corresponding to each edge in the graph.

For example, suppose two there is an edge between nodes A & B, then there will be 1 in position $[A, B]$ & there will be a 1 in position $[B, A]$ of the adjacency matrix.

That's why the given adjacency matrix is symmetric.

So the number of edges in the graph must be equal to half the number of 1's in the adjacency matrix.

Hence number of edges will be 7 in the graph.

All the other graphs except (iii), have 7 edges.So it is clear that the adjacency matrix does not represents graph (iii).

Isomorphism:

From the definition of Isomorphic graphs, it can be inferred that,

Isomorphic graphs must have same (adjacency matrix) representation.

Thus after eliminating graph (iii) we have to check for isomorphism among graphs (i), (ii) & (iv).

It can clearly be observed that graphs (ii) & (iv) are not isomorphic to each other.

It can also be observed that graph (i) & (ii) are isomorphic(Rotate graph (i) by 90 degree left/right.

Graph (ii) is looking like a closed envelope in the figure, try to view it like an open envelope, like a trapezium over a rectangle.)

So now it can be inferred that either the adjacency matrix is representing both graphs (i) & (ii) or it is only representing (iv).

Cycles of length 6 :

Now from the adjacency matrix it can be observed that there should be a cycle of length 6 in the graph, since $[1, 2], [2, 3], [3, 4], [4, 5], [5, 6], [6, 1]$are all 1's in the matrix.(as 1 at any position $[x, y]$ represents an edge between x & y in the graph).

& both graphs (i) & (ii) have cycles of length 6, but graph (iv) does not has any cycle of length 6, it has cycles of length 4 & 5 only.

Thus graph (iv) can not have the above adjacency matrix.

Hence the adjacency matrix represents graphs (i) & (ii).

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20 votes -- Anurag Pandey (13.6k points)

2.7 Graph Isomorphism(2) top

2.7.1 Graph Isomorphism: GATE2012-26 top

https://gateoverflow.in/1611

Which of the following graphs is isomorphic to

A.

B.

C.

D.

gate2012 graph-theory graph-isomorphism normal

Answer

2.7.2 Graph Isomorphism: GATE2014-2-51 top

https://gateoverflow.in/2018

A cycle on n vertices is isomorphic to its complement. The value of n is _____.

gate2014-2 graph-theory numerical-answers normal graph-isomorphism

Answer

Answers: Graph Isomorphism

2.7.1 Graph Isomorphism: GATE2012-26 top

https://gateoverflow.in/1611

Selected Answer

For these type of questions find which are not isomorphic.

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The graph in option (A) has a 3 length cycle whereas the original graph does not have a 3 length cycle
The graph in option (C) has vertex with degree 3 whereas the original graph does not have a vertex with degree 3
The graph in option (D) has a 4 length cycle whereas the original graph does not have a 4 length cycle

so option B must be correct.

28 votes -- Bhagirathi Nayak (14.1k points)

2.7.2 Graph Isomorphism: GATE2014-2-51 top

https://gateoverflow.in/2018

Selected Answer

A cycle with n vertices has n edges.
Number of edges in cycle $= n$
Number of edges in its complement $= \frac{n(n-1)}{2} - n.$

Fo isomorphism, both graphs should have equal number of edges.
This gives:

$$\frac{n(n-1)}{2} - n = n.$$
$$\Rightarrow n = 5.$$

58 votes -- Sachin Mittal (15.6k points)

its $n = 5$ only.

only C_5 is isomorphic to its complement.

25 votes -- jayendra (8.3k points)

2.8 Graph Matching(1) top

2.8.1 Graph Matching: GATE2003-36 top

https://gateoverflow.in/926

How many perfect matching are there in a complete graph of 6 vertices?

A. 15
B. 24
C. 30

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D. 60

gate2003 graph-theory graph-matching normal

Answer

Answers: Graph Matching

2.8.1 Graph Matching: GATE2003-36 top

https://gateoverflow.in/926

Selected Answer

Perfect matching is a set of edges such that each vertex appears only once and all vertices appear at least once (EXACTLY one appearance). So for n vertices perfect matching will have $n/2$ edges and there won't be any perfect matching if n is odd.

For $n = 6$, we can choose the first edge in ${}^6C_2 = 15$ ways, second in ${}^4C_2 = 6$ ways and third in ${}^2C_2 = 1$ way. So, total number of ways $= 15 \times 6 = 90$. But perfect matching being a set, order of elements is not important. i.e., the $3!$ permutations of the 3 edges are same only. So, total number of perfect matching $= \frac{90}{3!} = \frac{90}{6} = 15$.

Alternatively we can also say there are 3 identical buckets to be filled from 6 vertices such that 2 should go to each of them. Now the first vertex can combine with any of the other 5 vertices and go to bucket $1 - 5$ ways. Now only 4 vertices remain and 2 buckets. We can take one vertex and it can choose a companion in 3 ways and go to second bucket- 3 ways. Now only a single bucket and 2 vertices remain. so just 1 way to fill the last one. So total ways$= 5 \times 3 = 15$.

46 votes -- Arjun Suresh (348k points)

Note: To understand the solution please go through the definitions of perfect matching

The complete graph k_n have a perfect matching only when n is even. So let $n = 2m$.

Let the vertices be $V_1, V_2, \ldots, V_{2m}$.

v_1 can be joined to any other $(2m - 1)$ vertices.

v_2 can be joined to any other $(2m - 3)$ vertices.

Similarly, go till V_{2m} which will have only one vertex to be joined with.

No. of Perfect matches$= (2m - 1)(2m - 3)(2m - 5) \ldots (3)(1)$

In the above question $2m = 6$.

So, No. of perfect matches = $5 \times 3 \times 1 = 15$.

28 votes -- Hunaif (541 points)

2.9 Graph Planarity(4) top

2.9.1 Graph Planarity: GATE1990-3-vi top

https://gateoverflow.in/87129

Answer the following:

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Which of the following graphs is/are planner? (see Fig. 2)

gate1989 normal graph-theory graph-planarity descriptive

Answer

2.9.2 Graph Planarity: GATE1990-3-xi top

https://gateoverflow.in/85384

Choose the correct alternatives (More than one may be correct).

A graph is planar if and only if,

A. It does not contain subgraphs homeomorphic to k_5 and $k_{3,3}$.
B. It does not contain subgraphs isomorphic to k_5 or $k_{3,3}$.
C. It does not contain a subgraph isomorphic to k_5 or $k_{3,3}$
D. It does not contain a subgraph homeomorphic to k_5 or $k_{3,3}$.

gate1990 normal graph-theory graph-planarity

Answer

2.9.3 Graph Planarity: GATE2005-10 top

https://gateoverflow.in/1159

Let G be a simple connected planar graph with 13 vertices and 19 edges. Then, the number of faces in the planar embedding of the graph is:

A. 6
B. 8
C. 9
D. 13

gate2005 graph-theory graph-planarity

Answer

2.9.4 Graph Planarity: GATE2008-23 top

https://gateoverflow.in/421

Which of the following statements is true for every planar graph on n vertices?

A. The graph is connected
B. The graph is Eulerian
C. The graph has a vertex-cover of size at most $\frac{3n}{4}$
D. The graph has an independent set of size at least $\frac{n}{3}$

gate2008 graph-theory normal graph-planarity

Answer

Answers: Graph Planarity

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2.9.1 Graph Planarity: GATE1990-3-vi top

https://gateoverflow.in/87129

Selected Answer

$G1$ is $K_{3,3}$ which is a non-planar graph with the minimum number of edges.

Proof: Let $K_{3,3}$ is a planar graph.
Therefore it must satisfy this useful corollary. As there is no triangle in $K_{3,3}$.

- Let G be a connected planar simple graph with n vertices and m edges, and no triangles. Then $m \leq 2n-4$

$m=9, n=6.$
$9 \leq 12-4.$
$\Rightarrow 9 \leq 8,$ which is false. So our assumption that $K_{3,3}$ is planar is false.

$G2$ can be redrawn like this.

Therefore $G2$ is a planar graph.

For $G3$, we assume that it is a planar graph. Then it must satisfy the above corollary as it does not have a triangle.

$m=9, n=6.$
$9 \leq 12-4.$

$\Rightarrow 9 \leq 8,$ which is false. So our assumption is wrong. $G3$ is not a planar graph.

Note: G1 and G3 are isomorphic graphs.

Ans:
$G2$ **only.**

9 votes -- Hemant Parihar (14.8k points)

2.9.2 Graph Planarity: GATE1990-3-xi top

https://gateoverflow.in/85384

Selected Answer

A graph is non planar if and only if it contains a sub graph which is homomorphic to K_5 or $K_{3,3}$.

This is KURATOWSKI theorem.

When a function and its inverse function are both homomorphic, two graphs are isomorphic

Hence **(D)** is the answer.

https://math.stackexchange.com/questions/183133/difference-between-graph-homomorphism-and-graph-isomorphism

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8 votes -- vaishalitanna (209 points)

2.9.3 Graph Planarity: GATE2005-10 top

https://gateoverflow.in/1159

Selected Answer

$f = e - n + 2$ where f denotes number of faces E the number of edges n the number of vertices
So $f = 19 - 13 + 2 = 8$ faces

11 votes -- Bhagirathi Nayak (14.1k points)

2.9.4 Graph Planarity: GATE2008-23 top

https://gateoverflow.in/421

Selected Answer

Independent Set $\geq$ ceil(n/k) where,
n is the number of vertices and k is the chromatic number
For any planar graph $k \geq 3$ and $k \geq 4$, therefore, the Independent set is at least ceil$(n/4)$.
Hence (**D**) is false.

Now we know that ***Vertex cover + Independent Set Number***
$= n$.
If Independent set $\geq$ ceil$(n/4)$ then,
Vertex cover $\leq n - (n/4)$
Vertex Cover $\leq 3n/4$

Vertex Cover is at most 3n/4. So **(C)** is the correct answer.

12 votes -- nvedansh (303 points)

2.10 Graph Search(1) top

2.10.1 Graph Search: GATE2018-30 top

https://gateoverflow.in/204104

Let G be a simple undirected graph. Let T_D be a depth first search tree of G. Let T_B be a breadth first search tree of G. Consider the following statements.

I. No edge of G is a cross edge with respect to T_D. (A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D).
II. For every edge (u, v) of G, if u is at depth i and v is at depth j in T_B, then $|i - j| = 1$.

Which of the statements above must necessarily be true?

A. I only
B. II only
C. Both I and II
D. Neither I nor II

gate2018 graph-theory graph-search normal

Answer

Answers: Graph Search

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2.10.1 Graph Search: GATE2018-30 top

https://gateoverflow.in/204104

Selected Answer

I. **Undirected graph** cant have cross edges in DFS forest. (Only directed graphs can have) (Hence **I is True**)
(If you do not agree just try it with taking examples. You cant draw)

II. Just draw a triangle SAB. Source is S. Vertex A and B are at same level hence distance 1. There is an edge between A and B too. So here $|i-j|=0$. (Hence **II is false**)

Hence answer is **(A).**

Anyway, **II** is simple to understand from the above explanation.
Those who did not get **I**, you may see Theorem 22.10 in Cormen)

7 votes -- Ahwan Mishra (10.5k points)

2.11 Line Graph(2) top

2.11.1 Line Graph: GATE2013-26 top

https://gateoverflow.in/1537

The line graph $L(G)$ of a simple graph G is defined as follows:

There is exactly one vertex $v(e)$ in $L(G)$ for each edge e in G.

For any two edges e and e' in G, $L(G)$ has an edge between $v(e)$ and $v(e')$, if and only if e and e' are incident with the same vertex in G.

Which of the following statements is/are TRUE?

- (P) The line graph of a cycle is a cycle.
- (Q) The line graph of a clique is a clique.
- (R) The line graph of a planar graph is planar.
- (S) The line graph of a tree is a tree.

A. P only
B. P and R only
C. R only
D. P, Q and S only

gate2013 graph-theory normal line-graph

Answer

2.11.2 Line Graph: TIFR2017-B-13 top

https://gateoverflow.in/95821

For an undirected graph $G=(V,E)$, the line graph $G'=(V',E')$ is obtained by replacing each edge in E by a vertex, and adding an edge between two vertices in V' if the corresponding edges in G are incident on the same vertex. Which of the following is TRUE of line graphs?

A. the line graph for a complete graph is complete
B. the line graph for a connected graph is connected

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C. the line graph for a bipartite graph is bipartite
D. the maximum degree of any vertex in the line graph is at most the maximum degree in the original graph
E. each vertex in the line graph has degree one or two

tifr2017 graph-theory line-graph

Answer

Answers: Line Graph

2.11.1 Line Graph: GATE2013-26 top

https://gateoverflow.in/1537

Selected Answer

P) **True**. Because every edge in cycle graph will become a vertex in new graph $L(G)$ and every vertex of cycle graph will become an edge in new graph.

R) False. We can give counter example. Let G has 5 vertices and 9 edges which is a planar graph. Assume degree of one vertex is 2 and of all others are 4. Now, $L(G)$ has 9 vertices (because G has 9 edges) and 25 edges. (See below). But for a graph to be planar $|E| <= 3|V| - 6$.

For 9 vertices $|E| \leq 3 * 9 - 6$

$\Rightarrow |E| \leq 27 - 6$

$\Rightarrow |E| \leq 21$. But $L(G)$ has 25 edges and so is not planar.

As R) is False option (B), (C) are eliminated.
http://www.personal.kent.edu/~rmuhamma/GraphTheory/MyGraphTheory/planarity.htm

S) False. By counter example. Try drawing a simple tree which has a Root node ,Root node has one child A, node A has two child B and C. Draw its Line graph acc. to given rules in question you will get a cycle graph of 3 vertices.

So (D) also not correct.

$\therefore$ option (**A**) is correct.

For a graph *G* with *n* vertices and *m* edges, the number of vertices of the line graph *L*(*G*) is *m*, and the number of edges of *L*(*G*) is half the sum of the squares of the degrees of the vertices in *G*, minus *m*.

34 votes -- prashant singh (445 points)

2.11.2 Line Graph: TIFR2017-B-13 top

https://gateoverflow.in/95821

option **B** is the right answer

We can solve this question by eliminating options

Option (A) False

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Let us take a complete graph of 4 vertices:

In a line graph, no. of edges is

$\sum_{i=0}^{n} {}^{d_i}C_2,$ d_i is degree of each vertex

$= \frac{3\times2}{2} + \frac{3\times2}{2} + \frac{3\times2}{2} + \frac{3\times2}{2} = 3 \times 4 = 12$

No. of vertices in line graph = No. of edges in original graph
No. of vertices in line graph $= 6$
So, no. of edges to make complete graph with 6 vertices $= \frac{6\times5}{2} = 3 \times 5 = 15$
But for given line graph from complete graph of 4 vertices we have only 12 edges.
Contradiction.

Option (B) True

1. Smallest line graph for original graph one edge

which is also connected graph

If a graph is connected with more then one edge, it will never be disconnected

Option (C) False

This cannot be 2-colorable and hence is not bipartite.

Option (D) False

Because line graph degree of vertex depends on the attribute

e.g., $[A, B]$ as point in line graph, then degree of this vertex depends on degree of A and degree of B in the original graph.

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I'm drawing degree for a point $[AB]$ in line graph.

So, this is wrong.

Option (E) False (wrong as proved in above option (D))

6 votes -- SAKET NANDAN (4.8k points)

2.12 Regular Graph(1) top

2.12.1 Regular Graph: GATE1994-2.5 top https://gateoverflow.in/2472

The number of edges in a regular graph of degree d and n vertices is _____

gate1994 graph-theory easy regular-graph descriptive

Answer

Answers: Regular Graph

2.12.1 Regular Graph: GATE1994-2.5 top https://gateoverflow.in/2472

Selected Answer

Consider the example of complete graph, In a complete graph which is $(n-1)$ **regular** (where n is the no of vertices) has edges $\frac{n(n-1)}{2}$
n vertices are adjacent to $n-1$ vertices and an edge contributes two degree so dividing by 2.

$d*n=2*|E|$
Therefore, in d regular graph No of edges will be $\frac{n*d}{2}$

13 votes -- Manu Thakur (39.6k points)

2.13 Spanning Tree(2) top

2.13.1 Spanning Tree: GATE2007-IT-25 top https://gateoverflow.in/3458

What is the largest integer m such that every simple connected graph with n vertices and n edges contains at least m different spanning trees ?

A. 1
B. 2
C. 3

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D. n

gate2007-it graph-theory spanning-tree normal

Answer

2.13.2 Spanning Tree: TIFR2015-B-11 top

https://gateoverflow.in/30043

Let K_n be the complete graph on n vertices labeled $\{1, 2, \ldots, n\}$ with $m = \frac{n(n-1)}{2}$ edges. What is the number of spanning trees of K_n?

A. $\frac{m}{n-1}$
B. m^{n-1}
C. n^{n-2}
D. n^{n-1}
E. None of the above.

tifr2015 graph-theory spanning-tree

Answer

Answers: Spanning Tree

2.13.1 Spanning Tree: GATE2007-IT-25 top

https://gateoverflow.in/3458

Selected Answer

OPTION (C) is Correct, reason is as follows:

A graph is **connected** and has 'n' vertices and edges means, **exactly one cycle** is there.

Now we can make a different spanning tree by removing one edge from the cycle, one at a time.

Minimum cycle length can be 3, So, there must be at least 3 spanning trees in any such graph.

Consider the above graph. Here $n = 4$ and three spanning trees possible at max (removing edges of cycle one at a time, alternatively).

So, any such Graph with minimum cycle length '3' will have at least 3 spanning trees.

42 votes -- Himanshu Agarwal (15.3k points)

2.13.2 Spanning Tree: TIFR2015-B-11 top

https://gateoverflow.in/30043

Selected Answer

Answer will be (C)

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e.g. for K_4 no. of spanning tree will be 16.

Number of Spanning trees of a Complete Graph K_n is given by Cayley's theorem = n^{n-2}.

8 votes -- srestha (86.6k points)

2.14 Trees(3) top

2.14.1 Trees: GATE2010-1 top

https://gateoverflow.in/1147

Let $G = (V, E)$ be a graph. Define $\xi(G) = \sum_d i_d * d$, where i_d is the number of vertices of degree d in G. If S and T are two different trees with $\xi(S) = \xi(T)$, then

A. $|S| = 2|T|$
B. $|S| = |T| - 1$
C. $|S| = |T|$
D. $|S| = |T| + 1$

gate2010 graph-theory normal trees

Answer

2.14.2 Trees: ISI Entrance Exam MTech (CS) top

https://gateoverflow.in/124367

Consider all possible trees with n nodes. Let k be the number of nodes with degree greater than 1 in a given tree. What is the maximum possible value of k?

isi2016 graph-theory trees descriptive

Answer

2.14.3 Trees: TIFR2011-B-33 top

https://gateoverflow.in/20624

Which of the following is NOT a sufficient and necessary condition for an undirected graph G to be a tree?

A. G is connected and has $n-1$ edges.
B. G is acyclic and connected.
C. G is acyclic and has $n-1$ edges.
D. G is acyclic, connected and has $n-1$ edges.
E. G has $n-1$ edges.

tifr2011 graph-theory trees

Answer

Answers: Trees

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2.14.1 Trees: GATE2010-1 top

https://gateoverflow.in/1147

Selected Answer

Sum of degrees in a graph $= 2|E|$, as each edge contributes two to the sum of degrees. So, when sum of degrees are same, number of edges must also be same.

Trees with equal no of edges has to have equal no of vertices as No of Edges $=$ No of vertices -1, in a tree.

So, should be $|S| = |T|$

51 votes -- Digvijay (54.8k points)

2.14.2 Trees: ISI Entrance Exam MTech (CS) top

https://gateoverflow.in/124367

Selected Answer

Maximum value of k is $n-2$ which is example of line graph because every tree should contain at least 2 pendent vertices (i.e vertex with degree 1). Therefore value of k cannot exceed $n-2$.

11 votes -- Heisenberg (2.3k points)

2.14.3 Trees: TIFR2011-B-33 top

https://gateoverflow.in/20624

Selected Answer

Option a
$\iff$ **Option b**
$\iff$ **Option c**
$\iff$ **Option d.**

- You need atleast $n-1$ edges to have a connected graph. This leaves no edges to make any cycles. Thus, Option a $\implies G$ is acyclic.
- A connected graph with $n-1$ edges is acyclic, as shown above. Now, if we add any more edges to this graph, we will be connecting two vertices that are already connected. Thus, adding any more than edges to a connected graph will cause cycles. So, if a graph is acyclic and connected, it has exactly $(n-1)$ edges.
- You can't fit $(n-1)$ edges between $(n-1)$ vertices without causing cycles. Thus, if graph with $(n-1)$ edges is acyclic, it must connect n vertices. Hence, an acyclic graph with $(n-1)$ edges is connected.

Thus, all options, a to d are equivalent.

Option b
$\iff$ **G is a tree.**

- Any acyclic connected graph is a tree by definition.
- A graph G is a tree if it is both acyclic and connected by definition.

Thus, all option a to d are both necessary and sufficient for an undirected graph G to be a tree.

Option e
$\not\Rightarrow$ **G is a tree.**

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- Since G is not constrained to be acyclic, we can create a cyclic graph with $(n-1)$ edges. This graph will be cyclic, and it won't be connected. And thus, it won't be a tree.

Hence, option E is the correct answer.

17 votes -- Pragy Agarwal (20.6k points)

2.15 Vertex Cover(1) top

2.15.1 Vertex Cover: GATE2005-11 top https://gateoverflow.in/1161

Let G be a simple graph with 20 vertices and 100 edges. The size of the minimum vertex cover of G is 8. Then, the size of the maximum independent set of G is:

A. 12
B. 8
C. less than 8
D. more than 12

gate2005 graph-theory normal vertex-cover

Answer

Answers: Vertex Cover

2.15.1 Vertex Cover: GATE2005-11 top https://gateoverflow.in/1161

Selected Answer

Vertex cover: A set of vertices such that each edge of the graph is incident to at least one vertex of the set.

Therefore, removing all the vertices of the vertex cover from the graph results in an isolated graph and the same set of nodes would be the independent set in the original graph.

Size of minimum vertex cover $= 8$
Size of maximum independent set $= 20 - 8 = 12$

Therefore, correct answer would be (**A**).

Reference :- http://mathworld.wolfram.com/MaximumIndependentVertexSet.html

35 votes -- suraj (5.6k points)

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3 Discrete Mathematics: Mathematical Logic (82) top

3.1 First Order Logic(34) top

3.1.1 First Order Logic: GATE1989-14a top

https://gateoverflow.in/93179

Symbolize the expression "Every mother loves her children" in predicate logic.

gate1989 descriptive first-order-logic mathematical-logic

Answer

3.1.2 First Order Logic: GATE1991-15,b top

https://gateoverflow.in/26748

(b) Consider the following first order formula:

$$\left(\begin{array}{c} \forall x \exists y : R(x,y) \\ \wedge \\ \forall x \forall y : (R(x,y) \implies \neg R(y,x)) \\ \wedge \\ \forall x \forall y \forall z : (R(x,y) \wedge R(y,z) \implies R(x,z)) \\ \wedge \\ \forall x : \neg R(x,x) \end{array}\right)$$

Does it have finite models?

Is it satisfiable? If so, give a countable model for it.

gate1991 first-order-logic descriptive

Answer

3.1.3 First Order Logic: GATE1992-92,xv top

https://gateoverflow.in/256

Which of the following predicate calculus statements is/are valid?

A. $(\forall(x))P(x) \vee (\forall(x))Q(x) \implies (\forall(x))(P(x) \vee Q(x))$
B. $(\exists(x))P(x) \wedge (\exists(x))Q(x) \implies (\exists(x))(P(x) \wedge Q(x))$
C. $(\forall(x))(P(x) \vee Q(x)) \implies (\forall(x))P(x) \vee (\forall(x))Q(x)$
D. $(\exists(x))(P(x) \vee Q(x)) \implies \sim (\forall(x))P(x) \vee (\exists(x))Q(x)$

gate1992 mathematical-logic normal first-order-logic

Answer

3.1.4 First Order Logic: GATE2003-32 top

https://gateoverflow.in/922

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Which of the following is a valid first order formula? (Here α and β are first order formulae with x as their only free variable)

A. $((\forall x)[\alpha] \Rightarrow (\forall x)[\beta]) \Rightarrow (\forall x)[\alpha \Rightarrow \beta]$
B. $(\forall x)[\alpha] \Rightarrow (\exists x)[\alpha \wedge \beta]$
C. $((\forall x)[\alpha \vee \beta] \Rightarrow (\exists x)[\alpha]) \Rightarrow (\forall x)[\alpha]$
D. $(\forall x)[\alpha \Rightarrow \beta] \Rightarrow (((\forall x)[\alpha]) \Rightarrow (\forall x)[\beta])$

gate2003 mathematical-logic first-order-logic normal

Answer

3.1.5 First Order Logic: GATE2003-33 top

https://gateoverflow.in/923

Consider the following formula and its two interpretations I_1 and I_2.

$\alpha : (\forall x)\,[P_x \Leftrightarrow (\forall y)\,[Q_{xy} \Leftrightarrow \neg Q_{yy}]] \Rightarrow (\forall x)\,[\neg P_x]$

I_1 : Domain: the set of natural numbers

P_x = 'x is a prime number'

Q_{xy} = 'y divides x'

I_2 : same as I_1 except that P_x = 'x is a composite number'.

Which of the following statements is true?

A. I_1 satisfies α, I_2 does not
B. I_2 satisfies α, I_1 does not
C. Neither I_1 nor I_2 satisfies α
D. Both I_1 and I_2 satisfies α

gate2003 mathematical-logic difficult first-order-logic

Answer

3.1.6 First Order Logic: GATE2004-23, ISRO2007-32 top

https://gateoverflow.in/1020

Identify the correct translation into logical notation of the following assertion.

Some boys in the class are taller than all the girls

Note: $\text{taller}(x, y)$ is true if x is taller than y.

A. $(\exists x)(\text{boy}(x) \rightarrow (\forall y)(\text{girl}(y) \wedge \text{taller}(x, y)))$
B. $(\exists x)(\text{boy}(x) \wedge (\forall y)(\text{girl}(y) \wedge \text{taller}(x, y)))$
C. $(\exists x)(\text{boy}(x) \rightarrow (\forall y)(\text{girl}(y) \rightarrow \text{taller}(x, y)))$
D. $(\exists x)(\text{boy}(x) \wedge (\forall y)(\text{girl}(y) \rightarrow \text{taller}(x, y)))$

gate2004 mathematical-logic easy isro2007 first-order-logic

Answer

3.1.7 First Order Logic: GATE2004-IT-3 top

https://gateoverflow.in/3644

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Let $a(x,y), b(x,y,)$ and $c(x,y)$ be three statements with variables x and y chosen from some universe. Consider the following statement:

$$(\exists x)(\forall y)[(a(x,y) \wedge b(x,y)) \wedge \neg c(x,y)]$$

Which one of the following is its equivalent?

A. $(\forall x)(\exists y)[(a(x,y) \vee b(x,y)) \rightarrow c(x,y)]$
B. $(\exists x)(\forall y)[(a(x,y) \vee b(x,y)) \wedge \neg c(x,y)]$
C. $\neg(\forall x)(\exists y)[(a(x,y) \wedge b(x,y)) \rightarrow c(x,y)]$
D. $\neg(\forall x)(\exists y)[(a(x,y) \vee b(x,y)) \rightarrow c(x,y)]$

gate2004-it mathematical-logic normal first-order-logic

Answer

3.1.8 First Order Logic: GATE2005-41 top

https://gateoverflow.in/1166

What is the first order predicate calculus statement equivalent to the following?

"Every teacher is liked by some student"

A. $\forall(x)\,[\text{teacher}\,(x) \rightarrow \exists(y)\,[\text{student}\,(y) \rightarrow \text{likes}\,(y,x)]]$
B. $\forall(x)\,[\text{teacher}\,(x) \rightarrow \exists(y)\,[\text{student}\,(y) \wedge \text{likes}\,(y,x)]]$
C. $\exists(y)\forall(x)\,[\text{teacher}\,(x) \rightarrow [\text{student}\,(y) \wedge \text{likes}\,(y,x)]]$
D. $\forall(x)\,[\text{teacher}\,(x) \wedge \exists(y)\,[\text{student}\,(y) \rightarrow \text{likes}\,(y,x)]]$

gate2005 mathematical-logic easy first-order-logic

Answer

3.1.9 First Order Logic: GATE2005-IT-36 top

https://gateoverflow.in/3783

Let $P(x)$ and $Q(x)$ be arbitrary predicates. Which of the following statements is always **TRUE**?

A. $((\forall x\,(P\,(x) \vee Q\,(x)))) \implies ((\forall x P\,(x)) \vee (\forall x Q\,(x)))$
B. $(\forall x\,(P\,(x) \implies Q\,(x))) \implies ((\forall x P\,(x)) \implies (\forall x Q\,(x)))$
C. $(\forall x\,(P\,(x)) \implies \forall x\,(Q\,(x))) \implies (\forall x\,(P\,(x) \implies Q\,(x)))$
D. $(\forall x\,(P\,(x)) \Leftrightarrow (\forall x\,(Q\,(x)))) \implies (\forall x\,(P\,(x) \Leftrightarrow Q\,(x)))$

gate2005-it mathematical-logic first-order-logic normal

Answer

3.1.10 First Order Logic: GATE2006-26 top

https://gateoverflow.in/989

Which one of the first order predicate calculus statements given below correctly expresses the following English statement?

Tigers and lions attack if they are hungry or threatened.

A. $\forall x[(\text{tiger}(x) \wedge \text{lion}(x)) \rightarrow (\text{hungry}(x) \vee \text{threatened}(x)) \rightarrow \text{attacks}(x)]$
B. $\forall x[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow (\text{hungry}(x) \vee \text{threatened}(x)) \wedge \text{attacks}(x)]$
C. $\forall x[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow \text{attacks}(x) \rightarrow (\text{hungry}(x) \vee \text{threatened}(x))]$
D. $\forall x[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow (\text{hungry}(x) \vee \text{threatened}(x)) \rightarrow \text{attacks}(x)]$

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gate2006 mathematical-logic normal first-order-logic

Answer

3.1.11 First Order Logic: GATE2006-IT-21 top

https://gateoverflow.in/3560

Consider the following first order logic formula in which R is a binary relation symbol.

$$\forall x \forall y (R(x,y) \implies R(y,x))$$

The formula is

A. satisfiable and valid
B. satisfiable and so is its negation
C. unsatisfiable but its negation is valid
D. satisfiable but its negation is unsatisfiable

gate2006-it mathematical-logic normal first-order-logic

Answer

3.1.12 First Order Logic: GATE2007-22 top

https://gateoverflow.in/1220

Let $\text{Graph}(x)$ be a predicate which denotes that x is a graph. Let $\text{Connected}(x)$ be a predicate which denotes that x is connected. Which of the following first order logic sentences **DOES NOT** represent the statement:

$$\text{"Not every graph is connected"}$$

A. $\neg \forall x \left(\text{Graph}(x) \implies \text{Connected}(x) \right)$
B. $\exists x \left(\text{Graph}(x) \wedge \neg \text{Connected}(x) \right)$
C. $\neg \forall x \left(\neg \text{Graph}(x) \vee \text{Connected}(x) \right)$
D. $\forall x \left(\text{Graph}(x) \implies \neg \text{Connected}(x) \right)$

gate2007 mathematical-logic easy first-order-logic

Answer

3.1.13 First Order Logic: GATE2007-IT-21 top

https://gateoverflow.in/3454

Which one of these first-order logic formulae is valid?

A. $\forall x (P(x) \implies Q(x)) \implies (\forall x P(x) \implies \forall x Q(x))$
B. $\exists x (P(x) \vee Q(x)) \implies (\exists x P(x) \implies \exists x Q(x))$
C. $\exists x (P(x) \wedge Q(x)) \iff (\exists x P(x) \wedge \exists x Q(x))$
D. $\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$

gate2007-it mathematical-logic normal first-order-logic

Answer

3.1.14 First Order Logic: GATE2008-30 top

https://gateoverflow.in/441

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Let fsa and pda be two predicates such that $\text{fsa}(x)$ means x is a finite state automaton and $\text{pda}(y)$ means that y is a pushdown automaton. Let equivalent be another predicate such that $\text{equivalent}(a, b)$ means a and b are equivalent. Which of the following first order logic statements represent the following?

Each finite state automaton has an equivalent pushdown automaton

A. $(\forall x\ \text{fsa}\,(x)) \implies (\exists y\ \text{pda}\,(y) \land \text{equivalent}\,(x, y))$
B. $\neg\forall y\,(\exists x\ \text{fsa}\,(x) \implies \text{pda}\,(y) \land \text{equivalent}\,(x, y))$
C. $\forall x \exists y\,(\text{fsa}\,(x) \land \text{pda}\,(y) \land \text{equivalent}\,(x, y))$
D. $\forall x \exists y\,(\text{fsa}\,(y) \land \text{pda}\,(x) \land \text{equivalent}\,(x, y))$

gate2008 easy mathematical-logic first-order-logic

Answer

3.1.15 First Order Logic: GATE2008-IT-21 top https://gateoverflow.in/3282

Which of the following first order formulae is logically valid? Here $\alpha(x)$ is a first order formula with x as a free variable, and β is a first order formula with no free variable.

A. $[\beta \rightarrow (\exists x, \alpha(x))] \rightarrow [\forall x, \beta \rightarrow \alpha(x)]$
B. $[\exists x, \beta \rightarrow \alpha(x)] \rightarrow [\beta \rightarrow (\forall x, \alpha(x))]$
C. $[(\exists x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$
D. $[(\forall x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

gate2008-it first-order-logic normal

Answer

3.1.16 First Order Logic: GATE2008-IT-22 top https://gateoverflow.in/3283

Which of the following is the negation of $[\forall x, \alpha \rightarrow (\exists y, \beta \rightarrow (\forall u, \exists v, y))]$

A. $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, y))]$
B. $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, \neg y))]$
C. $[\forall x, \neg\alpha \rightarrow (\exists y, \neg\beta \rightarrow (\forall u, \exists v, \neg y))]$
D. $[\exists x, \alpha \land (\forall y, \beta \land (\exists u, \forall v, \neg y))]$

gate2008-it mathematical-logic normal first-order-logic

Answer

3.1.17 First Order Logic: GATE2009-23 top https://gateoverflow.in/800

Which one of the following is the most appropriate logical formula to represent the statement?

"Gold and silver ornaments are precious".
The following notations are used:

- $G(x)$: x is a gold ornament
- $S(x)$: x is a silver ornament
- $P(x)$: x is precious

A. $\forall x(P(x) \implies (G(x) \land S(x)))$
B. $\forall x((G(x) \land S(x)) \implies P(x))$
C. $\exists x((G(x) \land S(x)) \implies P(x))$

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D. $\forall x((G(x) \vee S(x)) \implies P(x))$

gate2009 mathematical-logic easy first-order-logic

Answer

3.1.18 First Order Logic: GATE2009-26 top

https://gateoverflow.in/803

Consider the following well-formed formulae:

I. $\neg \forall x(P(x))$
II. $\neg \exists x(P(x))$
III. $\neg \exists x(\neg P(x))$
IV. $\exists x(\neg P(x))$

Which of the above are equivalent?

A. I and III
B. I and IV
C. II and III
D. II and IV

gate2009 mathematical-logic normal first-order-logic

Answer

3.1.19 First Order Logic: GATE2010-30 top

https://gateoverflow.in/1156

Suppose the predicate $F(x, y, t)$ is used to represent the statement that person x can fool person y at time t.

Which one of the statements below expresses best the meaning of the formula,

$$\forall x \exists y \exists t(\neg F(x, y, t))$$

A. Everyone can fool some person at some time
B. No one can fool everyone all the time
C. Everyone cannot fool some person all the time
D. No one can fool some person at some time

gate2010 mathematical-logic easy first-order-logic

Answer

3.1.20 First Order Logic: GATE2011-30 top

https://gateoverflow.in/2132

Which one of the following options is CORRECT given three positive integers x, y and z, and a predicate

$$P(x) = \neg(x = 1) \wedge \forall y(\exists z(x = y * z) \Rightarrow (y = x) \vee (y = 1))$$

A. $P(x)$ being true means that x is a prime number
B. $P(x)$ being true means that x is a number other than 1
C. $P(x)$ is always true irrespective of the value of x
D. $P(x)$ being true means that x has exactly two factors other than 1 and x

gate2011 mathematical-logic normal first-order-logic

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Answer

3.1.21 First Order Logic: GATE2012-13 top https://gateoverflow.in/45

What is the correct translation of the following statement into mathematical logic?

"Some real numbers are rational"

A. $\exists x(real(x) \vee rational(x))$
B. $\forall x(real(x) \rightarrow rational(x))$
C. $\exists x(real(x) \wedge rational(x))$
D. $\exists x(rational(x) \rightarrow real(x))$

gate2012 mathematical-logic easy first-order-logic

Answer

3.1.22 First Order Logic: GATE2013-27 top https://gateoverflow.in/1538

What is the logical translation of the following statement?

"None of my friends are perfect."

A. $\exists x(F(x) \wedge \neg P(x))$
B. $\exists x(\neg F(x) \wedge P(x))$
C. $\exists x(\neg F(x) \wedge \neg P(x))$
D. $\neg \exists x(F(x) \wedge P(x))$

gate2013 mathematical-logic easy first-order-logic

Answer

3.1.23 First Order Logic: GATE2013-47 top https://gateoverflow.in/80

Which one of the following is **NOT** logically equivalent to $\neg \exists x(\forall y(\alpha) \wedge \forall z(\beta))$?

A. $\forall x(\exists z(\neg \beta) \rightarrow \forall y(\alpha))$
B. $\forall x(\forall z(\beta) \rightarrow \exists y(\neg \alpha))$
C. $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg \beta))$
D. $\forall x(\exists y(\neg \alpha) \rightarrow \exists z(\neg \beta))$

mathematical-logic normal marks-to-all gate2013 first-order-logic

Answer

3.1.24 First Order Logic: GATE2014-1-1 top https://gateoverflow.in/769

Consider the statement

"Not all that glitters is gold"

Predicate glitters(x) is true if x glitters and predicate gold(x) is true if x is gold. Which one of the following logical formulae represents the above statement?

A. $\forall x : glitters(x) \Rightarrow \neg gold(x)$
B. $\forall x : gold(x) \Rightarrow glitters(x)$

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C. $\exists x : gold(x) \land \neg glitters(x)$
D. $\exists x : glitters(x) \land \neg gold(x)$

gate2014-1 mathematical-logic first-order-logic

Answer

3.1.25 First Order Logic: GATE2014-3-53 top https://gateoverflow.in/2087

The CORRECT formula for the sentence, "not all Rainy days are Cold" is

A. $\forall d(\text{Rainy}(d) \land \text{~Cold}(d))$
B. $\forall d(\text{~Rainy}(d) \rightarrow \text{Cold}(d))$
C. $\exists d(\text{~Rainy}(d) \rightarrow \text{Cold}(d))$
D. $\exists d(\text{Rainy}(d) \land \text{~Cold}(d))$

gate2014-3 mathematical-logic easy first-order-logic

Answer

3.1.26 First Order Logic: GATE2015-2-55 top https://gateoverflow.in/8259

Which one of the following well-formed formulae is a tautology?

A. $\forall x\, \exists y\, R(x,y) \leftrightarrow \exists y \forall x\, R(x,y)$
B. $(\forall x\, [\exists y\, R(x,y) \rightarrow S(x,y)]) \rightarrow \forall x\, \exists y\, S(x,y)$
C. $[\forall x\, \exists y\, (P(x,y) \rightarrow R(x,y))] \leftrightarrow [\forall x\, \exists y\, (\neg P(x,y) \lor R(x,y))]$
D. $\forall x\, \forall y\, P(x,y) \rightarrow \forall x\, \forall y\, P(y,x)$

gate2015-2 mathematical-logic normal first-order-logic

Answer

3.1.27 First Order Logic: GATE2016-2-27 top https://gateoverflow.in/39618

Which one of the following well-formed formulae in predicate calculus is **NOT** valid ?

A. $(\forall_x p(x) \implies \forall_x q(x)) \implies (\exists_x \neg p(x) \lor \forall_x q(x))$
B. $(\exists x p(x) \lor \exists x q(x)) \implies \exists x (p(x) \lor q(x))$
C. $\exists x (p(x) \land q(x)) \implies (\exists x p(x) \land \exists x q(x))$
D. $\forall x (p(x) \lor q(x)) \implies (\forall x p(x) \lor \forall x q(x))$

gate2016-2 mathematical-logic first-order-logic normal

Answer

3.1.28 First Order Logic: GATE2017-1-02 top https://gateoverflow.in/118701

Consider the first-order logic sentence $F : \forall x(\exists y R(x,y))$. Assuming non-empty logical domains, which of the sentences below are *implied* by F?

I. $\exists y(\exists x R(x,y))$
II. $\exists y(\forall x R(x,y))$
III. $\forall y(\exists x R(x,y))$
IV. $\neg \exists x(\forall y \neg R(x,y))$

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A. IV only
B. I and IV only
C. II only
D. II and III only

gate2017-1 mathematical-logic first-order-logic

Answer

3.1.29 First Order Logic: GATE2018-28 top https://gateoverflow.in/204102

Consider the first-order logic sentence

$$\varphi \equiv \exists s \exists t \exists u \forall v \forall w \forall x \forall y \psi(s,t,u,v,w,x,y)$$

where $\psi(s,t,u,v,w,x,y,)$ is a quantifier-free first-order logic formula using only predicate symbols, and possibly equality, but no function symbols. Suppose φ has a model with a universe containing 7 elements.

Which one of the following statements is necessarily true?

A. There exists at least one model of φ with universe of size less than or equal to 3
B. There exists no model of φ with universe of size less than or equal to 3
C. There exists no model of φ with universe size of greater than 7
D. Every model of φ has a universe of size equal to 7

gate2018 mathematical-logic normal first-order-logic

Answer

3.1.30 First Order Logic: TIFR2010-A-8 top https://gateoverflow.in/18239

Which of the following is NOT necessarily true? { Notation: The symbol "$\neg$"notes negation; $P(x,y)$ means that for given x and y, the property $P(x,y)$ is true }.

a. $(\forall x \forall y P(x,y)) \Rightarrow (\forall y \forall x P(x,y))$
b. $(\forall x \exists y \neg P(x,y)) \Rightarrow \neg(\exists x \forall y P(x,y))$
c. $(\exists x \exists y P(x,y)) \Rightarrow (\exists y \exists x P(x,y))$
d. $(\exists x \forall y P(x,y)) \Rightarrow (\forall y \exists x P(x,y))$
e. $(\forall x \exists y P(x,y)) \Rightarrow (\exists y \forall x P(x,y))$

tifr2010 mathematical-logic first-order-logic

Answer

3.1.31 First Order Logic: TIFR2012-A-2 top https://gateoverflow.in/20939

If $Mr.M$ is guilty, then no witness is lying unless he is afraid. There is a witness who is afraid. Which of the following statements is true?

(Hint: Formulate the problem using the following predicates
$G - Mr.M$ is guilty
$W(x) - x$ is a witness
$L(x) - x$ is lying
$A(x) - x$ is afraid)

A. $Mr.M$ is guilty.

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B. $Mr. M$ is not guilty.
C. From these facts one cannot conclude that $Mr. M$ is guilty.
D. There is a witness who is lying.
E. No witness is lying.

tifr2012 mathematical-logic first-order-logic

Answer

3.1.32 First Order Logic: TIFR2012-B-3 top

https://gateoverflow.in/25048

For a person p, let $w(p)$, $A(p,y)$, $L(p)$ and $J(p)$ denote that p is a woman, p admires y, p is a lawyer and p is a judge respectively. Which of the following is the correct translation in first order logic of the sentence: "All woman who are lawyers admire some judge"?

a. $\forall x : [(w(x) \Lambda L(x)) \Rightarrow (\exists y : (J(y) \Lambda w(y) \Lambda A(x,y)))]$
b. $\forall x : [(w(x) \Rightarrow L(x)) \Rightarrow (\exists y : (J(y) \Lambda A(x,y)))]$
c. $\forall x \forall y : [(w(x) \Lambda L(x)) \Rightarrow (J(y) \Lambda A(x,y))]$
d. $\exists y \forall x : [(w(x) \Lambda L(x)) \Rightarrow (J(y) \Lambda A(x,y))]$
e. $\forall x : [(w(x) \Lambda L(x)) \Rightarrow (\exists y : (J(y) \Lambda A(x,y)))]$

tifr2012 mathematical-logic first-order-logic

Answer

3.1.33 First Order Logic: TIFR2016-B-4 top

https://gateoverflow.in/97634

In the following, A stands for a set of apples, and $S(x,y)$ stands for "x is sweeter than y. Let

$$\Psi \equiv \exists x : x \in A$$

$$\Phi \equiv \forall x \in A : \exists y \in A : S(x,y).$$

Which of the following statements implies that there are infinitely many apples (i.e.,, A is an inifinite set)?

A. $\Psi \wedge \Phi \wedge [\forall x \in A : \neg S(x,x)]$
B. $\Psi \wedge \Phi \wedge [\forall x \in A : S(x,x)]$
C. $\Psi \wedge \Phi \wedge [\forall x, y \in A : S(x,x) \wedge S(x,y) \rightarrow S(y,y)]$
D. $\Psi \wedge \Phi \wedge [\forall x \in A : \neg S(x,x)] \wedge [\forall x, y, z \in A : S(x,y) \wedge S(y,z) \rightarrow s(y,x)]$
E. $\Psi \wedge \Phi \wedge [\forall x \in A : \neg S(x,x)] \wedge [\forall x, y, z \in A : S(x,y) \wedge S(y,z) \rightarrow s(x,z)]$

tifr2016 mathematical-logic first-order-logic

Answer

3.1.34 First Order Logic: TIFR2017-B-11 top

https://gateoverflow.in/95818

Given that

- $B(x)$ means "x is a bat",
- $F(x)$ means "x is a fly", and
- $E(x,y)$ means "x eats y",

what is the best English translation of

$$\forall x(F(x) \rightarrow \forall y(E(y,x) \rightarrow B(y)))?$$

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A. all flies eat bats
B. every fly is eaten by some bat
C. bats eat only flies
D. every bat eats flies
E. only bats eat flies

tifr2017 first-order-logic

Answer

Answers: First Order Logic

3.1.1 First Order Logic: GATE1989-14a top

https://gateoverflow.in/93179

Selected Answer

$M(x) \rightarrow x$ is mother
$Ch(y,x) \rightarrow y$ is child of x
$Lo(x,y) \rightarrow x$ loves y

$$\forall x \forall y \left[(M(x) \wedge C(y,x)) \rightarrow Lo(x,y) \right]$$

4 votes -- bhuv (3.6k points)

3.1.2 First Order Logic: GATE1991-15,b top

https://gateoverflow.in/26748

Selected Answer

Let's break it down. Consider an ordered structure (directed graph).

A. $\forall x \exists y : R(x,y) \equiv$ Every vertex has atleast 1 outgoing edge.

B. $\forall x \forall y : \Big(R(x,y) \implies \neg R(y,x) \Big) \equiv$ If there is a directed edge from vertex u to vertex v, there should not be an edge back from v to u. That is, our relation $R(x,y)$ is antisymmetric.

C. $\forall x \forall y \forall z : \Big(R(x,y) \wedge R(y,z) \implies R(x,z) \Big) \equiv$ If $u \rightarrow v \rightarrow z$, then $u \rightarrow z$ is also true. That is, our relation $R(x,y)$ is transitive.

D. $\forall x : \neg R(x,x) \quad \equiv$ We cannot have a self-loop in the graph. That is, $R(x,y)$ is irreflexive.

Now, such a non-trivial $(size > 0)$ finite structure cannot exist.

Proof by contradiction:

Assume, for the sake of contradiction, that such a finite structure $S = (V,E)$ exists. Since it is finite, let the number of vertices in this structure be $|V| = n, n \in \mathbb{N}, n > 0$.

Edit: A summarized version of the following proof is in the comments. You can directly skip to that.

Lemma

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1:
v_n has an incoming edge from every vertex $v_i, i < n$

Proof by Induction:

Induction Hypothesis: $P(n) =$For every $1 \leq i < j \leq n$, there is an out edge from vertex v_i to vertex v_j, that is $v_i \longrightarrow v_j$.

Base Cases:

- **Let $n = 2$.**
  $v_1 \longrightarrow v_2$ must be true since there has to be an out edge from v_1 (Property **A**) and the only available vertex is v_2 (no self loops allowed - Property **D**).
  Hence, our hypothesis $P(2)$ is satisfied.
- **Let $n = 3$.**
  There must be an out edge from v_1 to some vertex. Let's call that vertex v_2, that is $v_1 \longrightarrow v_2$. Similarly, there must be an out edge from v_2. But due to property **B**, we can't have an out edge from v_2 back to v_1. Hence, the out edge from v_2 must lead us to a new vertex. Lets call that v_3.
  Since $v_1 \longrightarrow v_2 \longrightarrow v_3$, due to Property **C**, we must have $v_1 \longrightarrow v_3$.
  Hence, our hypothesis $P(3)$ is satisfied.

Inductive Step:

For $P(n+1)$: The nth vertex v_n must have an out edge. Since $P(n)$ is true, the nth vertex has incoming edges from all vertices $v_i, i < n$. Hence, the out edge from v_n cannot be to any of those vertices. Self loops aren't allowed either.

Hence, the out edge from vertex v_n must be to the new vertex v_{n+1}. That is, $\begin{array}{c} \nearrow \\ \longrightarrow \\ \searrow \end{array} v_n \longrightarrow v_{n+1}$

Since every vertex $v_i, i < n$ has an out edge to v_n, and v_n has an out edge to v_{n+1}, due to Property **C**, we have that v_i has an out edge to v_{n+1}. That is, $v_i \longrightarrow v_{n+1}, \forall i \leq n$.

This is exactly what $P(n+1)$ states.

Hence, $P(n) \implies P(n+1)$.

Q.E.D

Since $P(n)$ is true as proven above, every vertex v_i must have an out edge to the vertex v_n.

Since the nth vertex has incoming edges from all other vertices (Lemma 1), it cannot have an out edge to any vertex. It can't have self loop either. Thus, it fails to satisfy Property A.

Hence, our assumption that S exists leads to a contradiction.

Q.E.D

The given logic formula can be satisfied by an infinite model.

For example, $R(x, y) \iff x < y, \quad x, y \in S$, where S is any infinite ordered set, satisfies the given formula.

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17 votes -- Pragy Agarwal (20.6k points)

3.1.3 First Order Logic: GATE1992-92,xv top

https://gateoverflow.in/256

Selected Answer

A. The corresponding English meaning: If $P(x)$ is true for all x, or if $Q(x)$ is true for all x, then for all x, either $P(x)$ is true or $Q(x)$ is true. This is always true and hence valid. To understand deeply, consider $X = \{3, 6, 9, 12\}$. For LHS of implication to be true, either $P(x)$ must be true for all elements in X or $Q(x)$ must be true for all elements in X. In either case, if we take each element x in X, either one of $P(x)$ or $Q(x)$ will be true. Hence, this implication is always valid.
If still in doubt, let $P(x)$ mean x is a multiple of 3 and $Q(x)$ means x is a multiple of 2)
B. The corresponding English meaning: If $P(x)$ is true for at least one x, and if $Q(x)$ is true for at least one x, then there is at least one x for which both $P(x)$ and $Q(x)$ are true. This is not always true as $P(x)$ can be true for one x and $Q(x)$ can be true for some other x . To understand deeply, consider $X = \{3, 6, 9, 12\}$. Let $P(x)$ be x is a multiple of 9 and $Q(x)$ be x is a multiple of 6. Now, LHS of implication is true, since $P(x)$ is true for $x = 9$, and $Q(x)$ is true for $x = 6$. But RHS of implication is not true as there is no x for which both $P(x)$ and $Q(x)$ holds. Hence, this implication is not valid.
C. If for each x, either $P(x)$ is true or $Q(x)$ is true then $P(x)$ is true for all x or $Q(x)$ is true for all x. Just one read is enough to see this is an invalid implication. Consider set {2,4,5}. Here every element is either a multiple or 2 or 5. But all elements are neither multiple of 2 nor 5.
D. If there is at least one x for which either $P(x)$ or $Q(x)$ is true then either it is not the case that $P(x)$ is true for all x or $Q(x)$ is true for at least one x. This is clearly invalid as LHS of implication becomes true if $P(x)$ is true for some x and $Q(x)$ is not true for any x, but RHS will be false (if $P(x)$ is true for all x).
A little modification to the statement is enough to make it valid:

$$\exists(x)(P(x) \vee Q(x)) \implies \sim (\forall(x) \sim P(x)) \vee \exists(x)Q(x)$$

which means if there is at least one x for which either $P(x)$ or $Q(x)$ is true then either it is not the case that $\sim P(x)$ is true for all x (which means P(x) is true for some x) or $Q(x)$ is true for some x.

Note
De Morgan's law is applicable in first order logic and is quite useful:

$$\forall(x)(P(x)) \equiv \neg\exists(x)(\neg P(x))$$

This is a logical reasoning statement which means if $P(x)$ is true for all x, then there can never exist an x for which $P(x)$ is not true. This formula is quite useful in proving validity of many statements as is its converse given below:

$$\exists(x)(P(x)) \equiv \neg\forall(x)(\neg P(x))$$

34 votes -- gatecse (18k points)

3.1.4 First Order Logic: GATE2003-32 top

https://gateoverflow.in/922

Selected Answer

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(**D**) is the answer.

A. Let $X = \{3, 6, 9, 8\}$. Let α denote multiple of 3 and β denote multiple of 4.$(\forall x)[\alpha]$ becomes false as 8 is not a multiple of 3, and so $(\forall x)[\alpha] \Rightarrow (\forall x)[\beta]$ becomes TRUE. Now, this won't imply $(\forall x)[\alpha \Rightarrow \beta]$ as multiple of 3 doesn't imply multiple of 4 for $3, 6$ or 9.

B. Let $X = \{3, 6, 9\}$. Let α denote multiple of 3 and β denote multiple of 4. Now LHS is TRUE but RHS is false as none of the x in X, is a multiple of 4.

C. Let $X = \{3, 6, 9, 7\}$. Let α denote multiple of 3 and β denote multiple of 4. Now $(\forall x)[\alpha \vee \beta]$ becomes false and hence LHS $= ((\forall x)[\alpha \vee \beta] \Rightarrow (\exists x)[\alpha])$ becomes true. But RHS is false as 7 is not a multiple of 3.

D. This is valid. LHS is saying that if α is holding for any x, then β also holds for that x. RHS is saying if α is holding for all x, then β also holds for all x. Clearly LHS $\implies$ RHS (but RHS does not imply LHS).
For example, let $X = \{4, 8, 12\}, \alpha$ denote multiple of 2 and β denote multiple of 4. LHS $= (\forall x)[\alpha \Rightarrow \beta]$, is TRUE. RHS is also true. If we add '3' to X, then LHS is true, first part of RHS becomes false and thus RHS also becomes TRUE. There is no way we can make LHS true and RHS false here. But if we add 2 and 3 to X, RHS will be true and LHS will be false. So, we can't say RHS implies LHS.

28 votes -- Arjun Suresh (348k points)

3.1.5 First Order Logic: GATE2003-33 top

https://gateoverflow.in/923

Selected Answer

Given: $(\alpha : (\forall x)\,[P_x \Leftrightarrow (\forall y)\,[Q_{xy} \Leftrightarrow \neg Q_{yy}]] \Rightarrow (\forall x)\,[\neg P_x])$
Q_{yy} is always True, this makes $\neg Q_{yy}$ False.

Writing $(\forall y)[Q_{xy} \Leftrightarrow \neg Q_{yy}]$ is same as writing $(\forall y)[Q_{xy} \Leftrightarrow False]]$

This is equivalent to saying that, for all y Q_{xy} is false and finally we can rewrite $(\forall y)[Q_{xy} \Leftrightarrow \neg Q_{yy}]]$ as $(\forall y)[\neg Q_{xy}]$

$\alpha : (\forall \mathbf{x})[\mathbf{P_x} \Leftrightarrow (\forall \mathbf{y})[\neg \mathbf{Q_{xy}}]] \Rightarrow (\forall x)[\neg P_x]$

$LHS : (\forall x)[P_x \Leftrightarrow (\forall y)[\neg Q_{xy}]]$

consider only $(\forall \mathbf{y})[\neg \mathbf{Q_{xy}}]$ it says all values of y does not divide x, but there will be atleast one value of y (when $y = x$, or when $y = 1$) that divides x, i.e. $[\neg \mathbf{Q_{xy}}]$ is not true for all values of y. $(\forall \mathbf{y})[\neg \mathbf{Q_{xy}}]$ is false.

Now LHS becomes $(\forall x)[P_x \Leftrightarrow False]$, "$P_x \Leftrightarrow False$" this means P_x is False, which is same as writing "$\neg P_x$".

Finally, we reduced LHS to $(\forall x)[\neg P_x]$

$\alpha : (\forall \mathbf{x})[\neg \mathbf{P_x}] \Rightarrow (\forall x)[\neg P_x]$ (which is trivial, $P(x) \Rightarrow P(x)$ is trivially true)

Hence $(\forall \mathbf{x})[\neg \mathbf{P_x}] \Rightarrow (\forall x)[\neg P_x]$ is trivially true for any $P(x)$, doesn't matter if $P(x)$ is for prime number or for composite number $(I1$ or $I2)$.

$\Rightarrow I1$ and $I2$ both satisfies α.

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Option **D**.

64 votes -- Sachin Mittal (15.6k points)

$\alpha : (\forall x)\,[P_x \Leftrightarrow (\forall y)\,[Q_{xy} \Leftrightarrow \neg Q_{yy}]] \Rightarrow (\forall x)\,[\neg P_x]$

This is can be interpreted as:

- $\alpha : ((\forall x)\,[P_x \Leftrightarrow (\forall y)\,[Q_{xy} \Leftrightarrow \neg Q_{yy}]]) \Rightarrow ((\forall x)\,[\neg P_x])$

See the RHS. It says $P(x)$ is false for any natural number. But there are natural numbers which are prime and hence this RHS is FALSE. Now, to make α TRUE, LHS must be FALSE for any x. Here, LHS is bit complex, so lets consider it separately.

$(\forall x)\,[P_x \Leftrightarrow (\forall y)\,[Q_{xy} \Leftrightarrow \neg Q_{yy}]]$

LHS is TRUE only if the given implication is TRUE for all x. Here the rightmost double implication $(\forall y)\,[Q_{xy} \Leftrightarrow \neg Q_{yy}]$ is always FALSE, because x can be equal to y and hence **forall** can never be TRUE. So the LHS reduces to just $(\forall x)\neg P(x)$ and returns FALSE as we have prime as well as non-prime natural numbers. So, FALSE $\Rightarrow$ FALSE returns TRUE making both I_1 and I_2 satisfy α. D choice.

35 votes -- Arjun Suresh (348k points)

3.1.6 First Order Logic: GATE2004-23, ISRO2007-32 top

https://gateoverflow.in/1020

Selected Answer

Now many people get confused when to use $\wedge$ and when to use $\implies$. This question tests exactly that.
We use $\wedge$ when we want to say that the both predicates in this statement are always true, no matter what the value of x is. We use $\implies$ when we want to say that although there is no need for left predicate to be true always, but whenever it becomes true, right predicate must also be true.
Now we have been given the statement "$\text{Some boys in the class are taller than all the girls}$".
Now we know for sure that there is at least a boy in class. So we want to proceed with "$(\exists x)\,(boy\,(x) \wedge$" and not "$(\exists x)\,(boy\,(x) \implies$", because latter would have meant that we are putting no restriction on the existence of boy i.e. there may be a boy-less class, which is clearly we don't want, because in the statement itself, we are given that there are some boys in the class.
So options (A) and (C) are ruled out.
Now if we see option (B), it says, every y in class is a girl i.e. every person in class is a girl, which is clearly false. So we eliminate this option also, and we get correct option (D). Let us see option (D)explicitly also whether it is true or not. So it says that if person y is a girl, then x is taller than y, which is really we wanted to say.
So option (**D**) is correct.

http://www.cse.iitd.ac.in/~mittal/gate/gate_math_2004.html

84 votes -- Anu (5.9k points)

3.1.7 First Order Logic: GATE2004-IT-3 top

https://gateoverflow.in/3644

Selected Answer

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$(\exists x)(\forall y)[(a(x,y) \wedge b(x,y)) \wedge \neg c(x,y)]$

$\equiv \neg(\forall x)\neg(\forall y)[(a(x,y) \wedge b(x,y)) \wedge \neg c(x,y)]$
$(\because (\exists x)F(x) = \neg\forall x \neg F(x))$

$\equiv \neg(\forall x)(\exists y)\neg[(a(x,y) \wedge b(x,y)) \wedge \neg c(x,y)]$
$(\because (\forall x)F(x) = \neg\exists x \neg F(x)), \neg\neg F(x) = F(x))$

$\equiv \neg(\forall x)(\exists y)[\neg(a(x,y) \wedge b(x,y)) \vee c(x,y)]$

$\equiv \neg(\forall x)(\exists y)[(a(x,y) \wedge b(x,y)) \rightarrow c(x,y)]$

(**C**) choice.

25 votes -- Arjun Suresh (348k points)

3.1.8 First Order Logic: GATE2005-41 top

https://gateoverflow.in/1166

Selected Answer

Answer is (**B**). In simpler way we can say **If**
X **is a teacher then there exists some**
Y **who is a student and likes**
X**.**

(A) choice: If X is a teacher, then there exists a Y such that if Y is a student, then Y likes X.
(C) choice: There exist a student who likes all teachers.
(D) choice: Everyone is a teacher and there exists a Y such that if Y is student then y likes X.
Assuming one cannot be both student and teacher at same time, this just means, everyone is a teacher.

24 votes -- Manali (2.6k points)

3.1.9 First Order Logic: GATE2005-IT-36 top

https://gateoverflow.in/3783

Selected Answer

Procedure to be followed: For each option assume LHS to be TRUE and try to make RHS be False by selecting some values which makes LHS true in every condition.
(one can also start from RHS assuming it to be false and trying to make LHS to be true)

A. $[(\forall \mathbf{x}(\mathbf{P}(\mathbf{x}) \vee \mathbf{Q}(\mathbf{x})))] \Longrightarrow [(\forall \mathbf{x}\mathbf{P}(\mathbf{x})) \vee (\forall \mathbf{x}\mathbf{Q}(\mathbf{x}))]$

Let us assume the domain of x such that, for the first half of the values $P(x)$ is True & $Q(x)$ is False while for the other half, $Q(x)$ is True and $P(x)$ is False.

LHS: Since for LHS to be true either $P(x)$ or $Q(x)$ should be true. Hence our assumption of the domain will make LHS be TRUE.

RHS: for $(\forall \mathbf{x}\mathbf{P}(\mathbf{x}))$ or $(\forall \mathbf{x}\mathbf{Q}(\mathbf{x}))$ to be true, each must be true for all values which is not possible as per assumption we have made.Thus, RHS becomes FALSE.

Thus, $T \rightarrow F$ makes **statement A False.**

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C. $[\forall \mathbf{x}(\mathbf{P}(\mathbf{x})) \implies \forall \mathbf{x}(\mathbf{Q}(\mathbf{x}))] \implies [\forall \mathbf{x}(\mathbf{P}(\mathbf{x}) \implies \mathbf{Q}(\mathbf{x}))]$

Let us assume some values of $P(x)$ and $Q(x)$ as follows :

x	$P(x)$	$Q(x)$
x_1	F	T
x_2	T	F

LHS: for assumed domain $\forall \mathbf{x}(\mathbf{P}(\mathbf{x}))$ will becomes False and $\forall \mathbf{x}(\mathbf{Q}(\mathbf{x}))$ will also be false. Since $F \rightarrow F$ is True, LHS becomes TRUE.

RHS: for x_1 $[(\mathbf{P}(\mathbf{x}) \implies \mathbf{Q}(\mathbf{x}))$ is True and for x_2 $(\mathbf{P}(\mathbf{x}) \implies \mathbf{Q}(\mathbf{x}))$ is False. As a whole $\forall \mathbf{x}(\mathbf{P}(\mathbf{x}) \implies \mathbf{Q}(\mathbf{x}))$ becomes False, thus RHS becomes FALSE**.**

Thus, $T \rightarrow F$ makes **statement C as False**.

D. $[\forall \mathbf{x}(\mathbf{P}(\mathbf{x})) \Leftrightarrow (\forall \mathbf{x}(\mathbf{Q}(\mathbf{x})))] \implies [\forall \mathbf{x}(\mathbf{P}(\mathbf{x}) \Leftrightarrow \mathbf{Q}(\mathbf{x}))]$

if we assume same domain as in above option C,then observations are as following :

LHS: $\forall \mathbf{x}(\mathbf{P}(\mathbf{x}))$ becomes false as x_1 is false. Also $(\forall \mathbf{x}(\mathbf{Q}(\mathbf{x})))$ becomes false as x_2 is false.Thus $F \leftrightarrow F$ implies LHS is TRUE.

RHS: $**\mathbf{(P(x) \Leftrightarrow Q(x))}**$ will be false for both x_1 and x_2. Hence $\forall \mathbf{x}(\mathbf{P}(\mathbf{x}) \Leftrightarrow \mathbf{Q}(\mathbf{x}))$ becomes False which makes RHS to be FALSE.

Thus $T \rightarrow F$ makes **statement D as False.**

B. $[\forall \mathbf{x}(\mathbf{P}(\mathbf{x}) \implies \mathbf{Q}(\mathbf{x}))] \implies [(\forall \mathbf{x}\mathbf{P}(\mathbf{x})) \implies (\forall \mathbf{x}\mathbf{Q}(\mathbf{x}))]$

as we are assuming LHS to be TRUE then we'll not make any selection in which $P(x)$ is True and $Q(x)$ is false as it will make our assumption false.Thus values can be like this:

x	$P(x)$	$Q(x)$
x_1	T	T
x_2	F	T
x_3	F	F

LHS: $(\mathbf{P}(\mathbf{x}) \implies \mathbf{Q}(\mathbf{x}))$ becomes TRUE for each value and thus $\forall \mathbf{x}(\mathbf{P}(\mathbf{x}) \implies \mathbf{Q}(\mathbf{x}))$ become TRUE.

RHS: $(\forall \mathbf{x}\mathbf{P}(\mathbf{x}))$ **and** $(\forall \mathbf{x}\mathbf{Q}(\mathbf{x}))$ both becomes false for assumed values which implies $F \rightarrow F$ and thus makes RHS to be TRUE.

Hence $T \rightarrow T$ makes **statement B to be TRUE. Thus Answer is B.**

8 votes -- AAKASH SAINI (1.5k points)

Answer: B

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Let P: Student is a girl.

and Q: Student is smart.

Option B says: IF for all student x if x is a girl then the student is smart THEN if the whole class comprises of girls then the whole class comprises of smart students.

26 votes -- Rajarshi Sarkar (34.1k points)

3.1.10 First Order Logic: GATE2006-26 top https://gateoverflow.in/989

Selected Answer

The statement ``Tigers and lions attack if they are hungry or threatened"means that if an animal is either tiger or lion, then if it is hungry or threatened, it will attack. **So option (D) is correct.**

Do not get confused by "and" between tigers and lions in the statement. This ``and" does not mean that we will write "$\text{tiger}(x) \wedge \text{lion}(x)$", because that would have meant that an animal is both tiger and lion, which is not what we want.

http://www.cse.iitd.ac.in/~mittal/gate/gate_math_2006.html

23 votes -- Anu (5.9k points)

3.1.11 First Order Logic: GATE2006-IT-21 top https://gateoverflow.in/3560

Selected Answer

The given relation is nothing but symmetry. We have both symmetric relations possible as well as anti-symmetric but neither always holds for all sets. So they both are not valid but are satisfiable. (**B**) option.

46 votes -- Arjun Suresh (348k points)

3.1.12 First Order Logic: GATE2007-22 top https://gateoverflow.in/1220

Selected Answer

D says "all graphs are not connected" but the question says " not every graph is connected" .i.e " there exists at least one graph which is not connected". Hence the answer is (**D**)

25 votes -- Manali (2.6k points)

3.1.13 First Order Logic: GATE2007-IT-21 top https://gateoverflow.in/3454

Selected Answer

(**A**) is the answer

A. LHS: For every x, if P holds then Q holds

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RHS: If P(x) holds for all x, then Q(x) holds for all x.
LHS implies RHS but RHS does not imply LHS.

B. LHS: An x exist for which either P(x) is true or Q(x) is true.
RHS: If an x exist for which P(x) is true then another x exist for which Q(x) is true.
LHS does not imply RHS, but on RHS if we change ∃xP(x) to ~∃xP(x), implication becomes TRUE.

C. LHS: There exist an x for which both P(x) and Q(x) are true.
RHS: There exist an x for which P(x) is true and there exist an x for which Q(x) is true.
LHS implies RHS but RHS does not imply LHS as the 'x' for P and Q can be different on the RHS

D. LHS: For every x, there exist a y such that P(x, y) holds.
RHS: There exist a y such that for all x P(x, y) holds.
Here RHS implies LHS but LHS does not imply RHS as the y on LHS can be different for each x.

33 votes -- Arjun Suresh (348k points)

3.1.14 First Order Logic: GATE2008-30 top

https://gateoverflow.in/441

Selected Answer

None of these.

A. If everything is a FSA, then there exists an equivalent PDA for everything.
B. It is not the case that for all y if there exist a FSA then it has an equivalent PDA.
C. Everything is a FSA and has an equivalent PDA.
D. Everything is a PDA and has exist an equivalent FSA.

The correct answer would be

$$\forall x\,(\text{fsa}\,(x) \implies (\exists y\,\text{pda}\,(y) \wedge \text{equivalent}\,(x,y)))$$

39 votes -- Arjun Suresh (348k points)

3.1.15 First Order Logic: GATE2008-IT-21 top

https://gateoverflow.in/3282

Selected Answer

A. $[\beta \rightarrow (\exists x, \alpha(x))] \rightarrow [\forall x, \beta \rightarrow \alpha(x)]$
LHS: If β (some condition) is true, then there exists an x for which $\alpha(x)$ is true.
RHS: For all x, if β is true then $\alpha(x)$ is true. This is same as saying if β is true then for all x, $\alpha(x)$ is true. $(\beta \implies \forall x, \alpha(x))$.

So,

$$\text{RHS} \implies \text{LHS and LHS} \implies \text{RHS}.$$

B. $[\exists x, \beta \rightarrow \alpha(x)] \rightarrow [\beta \rightarrow (\forall x, \alpha(x))]$
LHS: There exists an x such that if β is true then $\alpha(x)$ is true.
RHS: If β is true then for all $x, \alpha(x)$ is true.
So,

$$\text{RHS} \implies \text{LHS and LHS} \implies \text{RHS}.$$

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C. $[(\exists x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$
LHS: If there is an x such that $\alpha(x)$ is true, then β is true.
RHS: For all x, if $\alpha(x)$ is true, then β is true.
Here, both LHS and RHS are in fact same as β is a formula which is independent of x. (if β is true for one x, it is true for every x and vice versa).
So,

$$\text{RHS} \implies \text{LHS and LHS} \implies \text{RHS}.$$

D. $[(\forall x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$
LHS: If $\alpha(x)$ is true for every x, then β is true.
RHS: For every x, if $\alpha(x)$ is true then β is true.
So,

$$\text{RHS} \implies \text{LHS and LHS} \implies \text{RHS}.$$

So, answer here is **option C**. Any of options A, B or D could be valid if their implication is reversed. For option C, LHS and RHS being equivalent, even if the implication is reversed (or changed to double implies) it remains valid.

32 votes -- Arjun Suresh (348k points)

3.1.16 First Order Logic: GATE2008-IT-22 top

https://gateoverflow.in/3283

Selected Answer

$[\forall x, \alpha \rightarrow (\exists y, \beta \rightarrow (\forall u, \exists v, y))] \equiv [\forall x, \neg\alpha \vee (\exists y, \neg\beta v(\forall u, \exists v, y))]$

Now, doing complement gives (complement of $\forall$ is $\exists$ and vice versa while propagating negation inwards as $\forall x(P) \equiv \neg\exists x(\neg P)$ and $\exists x(P) \equiv \neg\forall x(\neg P)$)

$[\exists x, \alpha \wedge (\forall y, \beta \wedge (\exists u, \forall v, \neg y))]$

(**D**) choice

29 votes -- Arjun Suresh (348k points)

3.1.17 First Order Logic: GATE2009-23 top

https://gateoverflow.in/800

Selected Answer

The statement could be translated as,If x is either Gold or Silver, then it would be precious. Rather than,

If x is both Gold and Silver, as an item cannot both Gold and silver at the same time.

Hence Ans is (D).

33 votes -- Sona Praneeth Akula (4.2k points)

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3.1.18 First Order Logic: GATE2009-26 top

https://gateoverflow.in/803

Selected Answer

Option (B) is correct. I and IV are equivalent.

$\neg\forall x(P(x)) \equiv \exists x(\neg P(x))$ **[De morgan's Law]**

Alternate approach:

Let's take an **example.**

Let $P(x) \implies$ Student x is pass

$I \implies$ Not all students are pass. (which means "Some students are fail")

$II \implies$ There does not exist a student who is pass. (which means "Every student is fail")

$III \implies$ There does not exist a student who is not pass (which means "Every student is pass")

$IV \implies$ Some students are not pass. (which means "Some students are fail")

I and IV are equivalent.

10 votes -- Soumya Jain (9.7k points)

3.1.19 First Order Logic: GATE2010-30 top

https://gateoverflow.in/1156

Selected Answer

$F(x,y,t) \implies$ person x can fool person y at time t.

For the sake of simplicity propagate negation sign outward by applying De Morgan's law.

$\forall x\exists y\exists t(\neg F(x,y,t)) \equiv \neg\exists x\forall y\forall t(F(x,y,t))$ [By applying De Morgan's law.]

Now converting $\neg\exists x\forall y\forall t(F(x,y,t))$ to English is simple.

$\neg\exists x$
$\forall y$
$\forall t(F(x,y,t)) \implies$ There does not exist a person who can fool everyone all the time.

Which means No one can fool everyone all the time.

So, **option (B) is correct.**

36 votes -- Soumya Jain (9.7k points)

B is the correct answer. The trick is to bring the negate sign to the extreme left. Form a sentence without using negate and just negate that.

$\forall x\exists y\exists t(\neg F(x,y,t))$
$\equiv \neg(\neg\forall x\neg\exists y\neg\exists t)(\neg F(x,y,t))$
$\equiv \neg(\neg\forall x\neg\exists y\neg\exists t(F(x,y,t)))$
$\equiv \neg(\exists x\forall y\forall t(F(x,y,t)))$.

41 votes -- Bhagirathi Nayak (14.1k points)

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3.1.20 First Order Logic: GATE2011-30 top

https://gateoverflow.in/2132

Selected Answer

Answer is (**A**).

$$P(x) = (\neg(x=1) \wedge \forall y(\exists z(x = y * z) \implies ((y=x) \vee (y=1)))$$

Statement:
x **is not equal to**
1 **and if there exists some**
z **for all**
y **such that product of**
y **and**
z **is**
x**, then**
y **is either the number itself or**
1**. This is the definition of prime numbers.**

Alternative approach:
The formula

$$\exists x \forall y \forall z[\times(y,z,x) \rightarrow ((y=1) \vee (z=1))]$$

expresses the statement "there exists a prime number" (the number 1 also satisfies this statement).

Note here that $\times(y,z,x)$ is equivalent to $(x = y \times z)$.
but $\neg(x=1)$ removes 1 as satisfying given number in question's formula, so the option (A) is True.
ref@ https://en.wikibooks.org/wiki/Logic_for_Computer_Science/First-Order_Logic#Semantics
ref@ http://math.stackexchange.com/questions/1037795/what-is-the-meaning-of-this-predicate-statement

28 votes -- Sona Praneeth Akula (4.2k points)

3.1.21 First Order Logic: GATE2012-13 top

https://gateoverflow.in/45

Selected Answer

Meaning of each choices:

A. There exists a number which is either real or rational
B. If a number is real it is rational
C. There exists a number which is real and rational
D. There exists a number such that if it is rational, it is real

So, (**C**) is the answer.

27 votes -- Arjun Suresh (348k points)

3.1.22 First Order Logic: GATE2013-27 top

https://gateoverflow.in/1538

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Selected Answer

A. some of my friends are not perfect
B. some of those who are not my friends are perfect
C. some of those who are not my friends are not perfect
D. NOT (some of my friends are perfect) / none of my friends are perfect

40 votes -- Bhagirathi Nayak (14.1k points)

$F(x)$: x is my friend.

$P(x)$: x is perfect.

``None of my friends are perfect'' can be written like

$$\forall x[F(x) \implies \neg P(x)]$$
$$\equiv \forall x[\neg F(x) \vee \neg P(x)]$$
$$\equiv \forall x \neg [F(x) \wedge p(x)]$$
$$\equiv \neg \exists x[F(x) \wedge P(x)]$$

So, the answer is D.

21 votes -- vnc (2.6k points)

3.1.23 First Order Logic: GATE2013-47 top

https://gateoverflow.in/80

Selected Answer

A useful rule:

$$\forall x(\alpha) = \neg \exists (x)(\neg \alpha)$$

i.e.; If some property α is true for all x, then it is equivalent ot say that no x exists such that property α does not hold for it.

Starting with choices:

A. $\forall x(\exists z(\neg \beta) \rightarrow \forall y(\alpha))$
$\implies \forall x(\neg \exists z(\neg \beta) \vee \forall y(\alpha))$
$\implies \forall x(\forall z(\beta) \vee \forall y(\alpha))$
$\implies \neg \exists x \neg (\forall z(\beta) \vee \forall y(\alpha))$
$\implies \neg \exists x(\neg \forall z(\beta) \wedge \neg \forall y(\alpha))$

So, A is not matching with the logical statement in question.

B. $\forall x(\forall z(\beta) \rightarrow \exists y(\neg \alpha))$
$\implies \forall x(\neg \forall z(\beta) \vee \exists y(\neg \alpha))$
$\implies \neg \exists x \neg (\neg \forall z(\beta) \vee \exists y(\neg \alpha))$
$\implies \neg \exists x(\forall z(\beta) \wedge \neg \exists y(\neg \alpha))$
$\implies \neg \exists x(\forall z(\beta) \wedge \forall y(\alpha))$

Hence matches with the given statement.

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C. $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg\beta))$
$\implies \forall x(\neg\forall y(\alpha) \vee \exists z(\neg\beta))$
$\implies \neg\exists x\neg(\neg\forall y(\alpha) \vee \exists z(\neg\beta))$
$\implies \neg\exists x(\forall y(\alpha) \wedge \neg\exists z(\neg\beta))$
$\implies \neg\exists x(\forall y(\alpha) \wedge \forall z(\beta))$

Hence matches with the given statement.

D. $\forall x(\exists y(\neg\alpha) \rightarrow \exists z(\beta))$
$\implies \forall x(\neg\exists y(\neg\alpha) \vee \exists z(\beta))$
$\implies \forall x(\forall y(\alpha) \vee \exists z(\beta))$
$\implies \neg\exists x\neg(\forall y(\alpha) \vee \exists z(\beta))$
$\implies \neg\exists x(\neg\forall y(\alpha) \wedge \neg\exists z(\beta))$
$\implies \neg\exists x(\neg\forall y(\alpha) \wedge \forall z(\neg\beta))$

So, D is not matching with the logical statement in question.

Thus both (**A**) and (**D**) are not logically equivalent to the given statement.
In GATE 2013 marks were given to all for this question

32 votes -- Arjun Suresh (348k points)

3.1.24 First Order Logic: GATE2014-1-1 top

https://gateoverflow.in/769

Selected Answer

"Not all that glitters is gold" can be expressed as :

$$\neg(\forall x(glitters(x) \implies gold(x)))$$

(as restriction of universal quantification is same as universal quantification of a conditional statement.)

"Not all that glitters is gold" means **"some glitters are not gold"** which can be expressed as

$$\exists x(glitters(x) \wedge \neg gold(x))$$

(as restriction of an existential quantification is same as existential quantification of a conjunction.)

So option (D) is correct.

8 votes -- Soumya Jain (9.7k points)

3.1.25 First Order Logic: GATE2014-3-53 top

https://gateoverflow.in/2087

Selected Answer

Not all rainy days are cold.

In other words it says ``Some rainy days are not cold"

Given statement is
$\neg\forall d[R(d) \rightarrow C(d)]$

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$\equiv \neg\forall d[\neg R(d) \vee C(d)]$
$\equiv \exists d[R(d) \wedge \neg C(d)]$
Hence option (D) is correct.

22 votes -- Srinath Jayachandran (3.7k points)

A) No rainy days are cold

B) All non-rainy days are cold

C)Some non-rainy days are cold.

D) Some rainy days are not cold.

option D

20 votes -- Manali (2.6k points)

3.1.26 First Order Logic: GATE2015-2-55 top

https://gateoverflow.in/8259

Selected Answer

Ans (**C**).

$(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$

(D) is wrong as shown below.

Let $S = \{2,3,4,5\}$ and $P(x,y)$ be $x < y$.

Now, $P(2,3)$ is true but $P(3,2), P(4,2)$ etc are false and hence the implication also.

This is because the given formula is evaluated as:

$\forall x \forall y\ (P(x,y)\ \rightarrow\ \forall x \forall y P(y,x))$

For every (x,y) if P(x,y) is true then for every (x,y) P(y,x) is true.

On the RHS, $P(y,x)$ can be replaced with $P(x,y)$ and then also the formula means the same. So, here precedence rule used is $\implies$ having more precedence than quantification which is against the convention used in Wikipedia. I guess all books only talk about conventions and there is no standard here. (C) option being so straight forward I guess, GATE didnot even consider this as an ambiguity. Also, it works only if x, y belongs to same domain.

The below one is a tautology provided x, y have the same domain.

$(\forall x \forall y P(x,y)\ ) \rightarrow\ (\forall x \forall y\, P(y,x))$

If P(x,y) is true for all (x,y), then P(y,x) is true for all (x,y).

31 votes -- Vikrant Singh (13.5k points)

3.1.27 First Order Logic: GATE2016-2-27 top

https://gateoverflow.in/39618

Selected Answer

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Here, (**D**) is not valid

Let me prove by an example

What (D) is saying here is:

For all x (x is even no or x is odd no) $\implies$ For all x(x is even no) or For all x (x is odd no)

OR

If every x is either even or odd, then every x must be even or every x must be odd.

If our domain is the set of natural numbers LHS is true but RHS is false as not all natural numbers are even or odd.

Answer is (D).

34 votes -- Akash Kanase (42.5k points)

3.1.28 First Order Logic: GATE2017-1-02 top

https://gateoverflow.in/118701

Selected Answer

Ans is B.

1st Method: $F : \forall x(\exists y R(x,y))$

Take option 4: $\neg\exists x(\forall y \neg R(x,y))$

$\equiv \forall x(\exists y R(x,y))$ (Since we know that $\neg\forall x \equiv \exists x$ And $\neg\exists x = \forall x$)

F: For all girls there exist a boyfriend. (Given)
(x for girl and y for boys)

I. There exists some boys who have girlfriends. (**True**)
II. There exists some boys for which all the girls are girlfriend. (False)
III. For all boys there exists a girlfriend. (False)
IV. For all girls, there exists a boyfriend (**True**) (Same as given statement F)

35 votes -- Ahwan Mishra (10.5k points)

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Solution.

I. $\forall x \exists y R(x,y) \rightarrow \exists y \exists x R(x,y)$ True
II. $\forall x \exists y R(x,y) \rightarrow \exists y \forall x R(x,y)$ False
III. $\forall x \exists y R(x,y) \rightarrow \exists x \exists y R(x,y)$ False
IV. $\forall x \exists y R(x,y) \rightarrow \neg \exists x (\forall y \neg R(x,y)) \forall x \exists y R(x,y)$ True

So, I and IV are only true.

26 votes -- Akash Dinkar (23.7k points)

3.1.29 First Order Logic: GATE2018-28 top

https://gateoverflow.in/204102

Selected Answer

Answer **- A.**

Quick logic review -

$\alpha : \forall x \exists y \;\; y < x$

Is α true for domain of **all integers** ?, Yes it is true. You pick any number x, I can always give you y that is less than your number x.

Is α true for domain of **Non Negative integers** $\{0,1,2,3,...\}$? No, it is not true. (You pick any number x) If you pick 0 then I can not give you y which is less than 0.

Definition of Model - **Domain** for which my sentence is true. for above sentence α, **all integers** is model and there can be many other models, like - **real numbers.**

(Definition of Co Model - **Domain** for which my sentence is False.)

Given that Predicate $\Phi \equiv \exists s \exists t \exists u \forall v \forall w \forall x \forall y \Psi(s,t,u,v,w,x,y)$ has a model with universe containing 7 elements. I.e. there is a domain with 7 elements which satisfies my Φ .

Now let $\Phi \equiv \exists s \exists t \exists u \forall v \forall w \forall x \forall y \;\; s+t+u+v+w+x+y > 200$. Can you suggest me a set (domain) that is model for Φ ?. (i.e. the domain for which you can satisfy Φ).

$\{10,20,30,40,50,60,100\}$, now if I choose $s=50$, $t=60$ and $u=100$ [1] and let anyone choose values of v,u,x and y then Φ is always satisfiable for any values (or write like this - for all values) of v,u,x and y . The key point is you have to choose values of s,t and u carefully and once u fixed these values then it should work for all remaining universal quantifiers values.

Actually I have problem with first element of set "10". Can I remove it and the resultant set will still work as model ? $\{20,30,40,50,60,100\}$, Ofcourse this is model for Φ (Why ? - take $s=50$, $t=60$ and $u=100$ and let any other variable value be anything).

Similarly, $\{50,60,100\}$ is also model for Φ.

Idea is once you have your s,t and u in set then that set is model (because remaining quantifiers can take any value).

Even the singleton set $\{100\}$ is also model for Φ.

Now there is always one model of universe size 3, and depending on your predicate you can have model of less than size 3 too (like above singleton set).

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[1] $s = t = u = 100$ also works.

Now, Lets generalize this for better understanding-

let Φ has following model of size 7- $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$, and let $s = e_2, t = e_5, u = e_1$ is the **only** setting which works for any (for all) values of v, w, x and y, then Can we reduce model size ?- Yes, we can have a model of size 3 $\{e_1, e_2, e_5\}$. Can we reduce size further ?- We can not (Because $s = e_2, t = e_5, u = e_1$ is the **only** setting which works for any value of v, w. .)

19 votes -- Sachin Mittal (15.6k points)

3.1.30 First Order Logic: TIFR2010-A-8 top

https://gateoverflow.in/18239

Selected Answer

a. is TRUE as both LHS and RHS are equivalent- English would be for every x, and for every $y, P(x, y)$ is TRUE. Changing y and x wouldn't change the meaning.

b. is TRUE as both LHS and RHS are equivalent- RHS is obtained by double negation of LHS.

c. Similar to (a), both are equivalent.

d. LHS: For some x, for all $y, P(x, y)$ is TRUE.
RHS: For all y and for some $x, P(x, y)$ is TRUE.
Both are not equivalent. LHS is stronger and implies RHS. For example, on the natural number set, we have $x = 1$ such that for every $y, P(x \leq y)$ is TRUE. Clearly, this implies for all y there exists some x (here x could be different for different y but on LHS, it must be the same).

e. LHS: For all x and for some $y, P(x, y)$ is TRUE.
RHS: For some y and for all $x, P(x, y)$ is TRUE.

As explained in d, these are not equivalent and here RHS is stronger than LHS, making the implication false. For example consider the "<=" relation on the integer set. LHS is true here as for every integer we have another integer which is greater . But RHS is false as there is no single integer (infinity is not an integer) which is greater than all other integers.

Hence, (**e**). is not necessarily TRUE.

16 votes -- Arjun Suresh (348k points)

3.1.31 First Order Logic: TIFR2012-A-2 top

https://gateoverflow.in/20939

Selected Answer

If $\text{Mr. } M$ is guilty, then if we pick a witness, we know that the witness won't lie unless he is afraid. If the witness is afraid, it may lie or it may lot lie (nothing is guaranteed).

However, unless we know what the victim said in the court (whether he said that $\text{Mr. } M$ was guilty or not guilty), we can't say anything about $\text{Mr. } M$.

All we know is that we've a witness who is afraid, so he may or may not lie in the court. We haven't been told anything about what actually happened in the court proceeding.

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So, we can't logically conclude anything about $\text{Mr. } M$ being guilty or not guilty.

Thus, options (a) and (b) are False.

Furthermore, that witness who was afraid, he may or may not lie. Since he is afraid, we know that he "can" lie, but we're not guaranteed that he will lie.

Thus, options (d) and (e) are False too.

This leaves option c, and as we have seen earlier, we cannot conclude anything about $\text{Mr. } M$ being guilty or not guilty.

Hence, option (c) is the correct answer.

Although not necessary, the logic equivalent of the given statement will be:

$$\left[G \Rightarrow \neg\exists x : (W(x) \wedge L(x) \wedge \neg A(x))\right] \equiv$$

$$\left[G \Rightarrow \forall x : (W(x) \Rightarrow (\neg A(x) \Rightarrow \neg L(x)))\right]$$

15 votes -- Pragy Agarwal (20.6k points)

3.1.32 First Order Logic: TIFR2012-B-3 top

https://gateoverflow.in/25048

Selected Answer

Just translating to English:

a. Every women who is a lawyer admires some women judge.
b. If a person being women implies she is a lawyer then she admires some judge. OR If a person is not women or is a lawyer he/she admires some judge.
c. Every women who is a lawyer admires every judge.
d. There is some judge who is admired by every women lawyer.
e. Every women lawyer admire some judge.

So, option (**e**) is the answer.

21 votes -- Arjun Suresh (348k points)

3.1.33 First Order Logic: TIFR2016-B-4 top

https://gateoverflow.in/97634

Selected Answer

(E) is the answer

Let A be $\{1, 2\}$ (say apple 1, apple 2)

There is at least one element in A : satisfied
For every element x in A there is a y in A such that $S(x, y)$: Since nothing is told about the symmetry of the relation, we can have $S(1, 2)$ and $S(2, 1)$, so, satisfied

So, as of now, $A = \{1, 2\}$ and $S = \{(1, 2), (2, 1)\}$

Now we can go through the options :

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A. $S(x,x)$ is not possible . . . I do not have that in S . . . so satisfied . . . and still finite A
B. $S(x,x)$ should be there . . . well, I will make $S = \{(1,2)(2,1),(1,1),(2,2)\}$. . . satisfied and still finite A
C. take the same case as above . . . satisfied and still finite A
D. Now I can not have $(1,1)$ and $(2,2)$
but even with $S = \{(1,2).(2,1)\}$ this condition is satisfied . . . still finite A
E. I can not do this with $(1,2)$ and $(2,1)$ because the transitivity makes it $(1,1)$ which should not be there and whatever elements I add the transitivity will lead me to (x,x) because any element added to A should occur at least once on the left side of an ordered pair . . . so only solution is infinite A.

7 votes -- Anand Vijayan (1k points)

3.1.34 First Order Logic: TIFR2017-B-11 top

https://gateoverflow.in/95818

Selected Answer

If x is a fly, then for all y which eats x, y is a bat. This means only bats eat flies. Option (E).

20 votes -- Arjun Suresh (348k points)

3.2 Logical Reasoning(10) top

3.2.1 Logical Reasoning: CMI2013-A-07 top

https://gateoverflow.in/46597

Consider the following two statements.

1. There are infinitely many interesting whole numbers.
2. There are finitely many uninteresting whole numbers.

Which of the following is true?

A. Statements 1 and 2 are equivalent.
B. Statement 1 implies statement 2.
C. Statement 2 implies statement 1.
D. None of the above.

cmi2013 mathematical-logic logical-reasoning

Answer

3.2.2 Logical Reasoning: GATE2012-1 top

https://gateoverflow.in/33

Consider the following logical inferences.

I_1: If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.

I_2: If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.

Which of the following is **TRUE**?

A. Both I_1 and I_2 are correct inferences

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B. I_1 is correct but I_2 is not a correct inference
C. I_1 is not correct but I_2 is a correct inference
D. Both I_1 and I_2 are not correct inferences

gate2012 mathematical-logic easy logical-reasoning

Answer

3.2.3 Logical Reasoning: GATE2015-2-3 top https://gateoverflow.in/8049

Consider the following two statements.

- S_1: If a candidate is known to be corrupt, then he will not be elected
- S_2: If a candidate is kind, he will be elected

Which one of the following statements follows from S_1 and S_2 as per sound inference rules of logic?

A. If a person is known to be corrupt, he is kind
B. If a person is not known to be corrupt, he is not kind
C. If a person is kind, he is not known to be corrupt
D. If a person is not kind, he is not known to be corrupt

gate2015-2 mathematical-logic normal logical-reasoning

Answer

3.2.4 Logical Reasoning: GATE2015-3-24 top https://gateoverflow.in/8427

In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking the person replies the following

"The result of the toss is head if and only if I am telling the truth"

Which of the following options is correct?

A. The result is head
B. The result is tail
C. If the person is of Type 2, then the result is tail
D. If the person is of Type 1, then the result is tail

gate2015-3 mathematical-logic difficult logical-reasoning

Answer

3.2.5 Logical Reasoning: TIFR2010-A-4 top https://gateoverflow.in/18212

If the bank receipt is forged, then Mr. M is liable. If Mr. M is liable, he will go bankrupt. If the bank will loan him money, he will not go bankrupt. The bank will loan him money.

Which of the following can be concluded from the above statements?

a. Mr. M is liable
b. The receipt is not forged
c. Mr. M will go bankrupt
d. The bank will go bankrupt

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e. None of the above

tifr2010 logical-reasoning mathematical-logic

Answer

3.2.6 Logical Reasoning: TIFR2011-A-1 top

https://gateoverflow.in/237

If either wages or prices are raised, there will be inflation. If there is inflation, then either the government must regulate it or the people will suffer. If the people suffer, the government will be unpopular. Government will not be unpopular. Which of the following can be validly concluded from the above statements.

A. People will not suffer
B. If the inflation is not regulated, then wages are not raised
C. Prices are not raised
D. If the inflation is not regulated, then the prices are not raised
E. Wages are not raised

tifr2011 mathematical-logic normal logical-reasoning

Answer

3.2.7 Logical Reasoning: TIFR2011-A-12 top

https://gateoverflow.in/20221

The action for this problem takes place in an island of Knights and Knaves, where Knights always make true statements and Knaves always make false statements and everybody is either a Knight or a Knave. Two friends A and B lives in a house. The census taker (an outsider) knocks on the door and it is opened by A. The census taker says ''I need information about you and your friend. Which if either is a Knight and which if either is a Knave?". "We are both Knaves" says A angrily and slams the door. What, if any thing can the census taker conclude?

A. A is a Knight and B is a Knave.
B. A is a Knave and B is a Knight.
C. Both are Knaves.
D. Both are Knights.
E. No conclusion can be drawn.

tifr2011 mathematical-logic logical-reasoning

Answer

3.2.8 Logical Reasoning: TIFR2012-A-3 top

https://gateoverflow.in/20981

Long ago,in a planet far far away, there lived three races of intelligent inhabitants: the blues (who always tell the truth), the whites (who always lie), and the pinks (who, when asked a series of questions, start with a lie and then tell the truth and lie alternately). To three creatures, chosen from the planet and seated facing each other at A, B, and C (see figure), the following three questions are put:

i. What race is your left-hand neighbour?
ii. What race is your right-hand neighbour?
iii. What race are you?

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Here are their answers:

A. (i) White (ii) Pink (iii) Blue
B. (i) Pink (ii) Pink (iii) Blue
C. (i) White (ii) Blue (iii) Blue

What is the actual race of each of the three creatures?

A. A is Pink, B is White, C is Blue.
B. A is Blue, B is Pink, C is White.
C. A is Pink, B is Blue, C is Pink.
D. A is White, B is Pink, C is Blue.
E. Cannot be determined from the above data.

tifr2012 mathematical-logic logical-reasoning

Answer

3.2.9 Logical Reasoning: TIFR2013-A-3 top

https://gateoverflow.in/25384

Three candidates, Amar, Birendra and Chanchal stand for the local election. Opinion polls are conducted and show that fraction a of the voters prefer Amar to Birendra, fraction b prefer Birendra to Chanchal and fraction c prefer Chanchal to Amar. Which of the following is impossible?

a. $(a, b, c) = (0.51, 0.51, 0.51)$;
b. $(a, b, c) = (0.61, 0.71, 0.67)$;
c. $(a, b, c) = (0.68, 0.68, 0.68)$;
d. $(a, b, c) = (0.49, 0.49, 0.49)$;
e. None of the above.

tifr2013 set-theory&algebra logical-reasoning

Answer

3.2.10 Logical Reasoning: TIFR2014-A-8 top

https://gateoverflow.in/25994

All that glitters is gold. No gold is silver.

Claims:

1. No silver glitters.
2. Some gold glitters.

Then, which of the following is TRUE?

a. Only claim 1 follows.
b. Only claim 2 follows.
c. Either claim 1 or claim 2 follows but not both.
d. Neither claim 1 nor claim 2 follows.
e. Both claim 1 and claim 2 follow.

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tifr2014 mathematical-logic logical-reasoning

Answer

Answers: Logical Reasoning

3.2.1 Logical Reasoning: CMI2013-A-07 top

https://gateoverflow.in/46597

Selected Answer

As we know that there are infinite whole numbers.

Now if we say that all even whole numbers are interesting whole number then we will have **infinite interesting** whole numbers and **infinite uninteresting** whole numbers (odd whole number).

Now if we say that whole numbers between **1 to 100** are interesting whole number then we will have **finite interesting** whole numbers and **infinite uninteresting** whole numbers (whole numbers excluding 1 to 100).

So from second example we can say that -

If there are finitely many uninteresting whole numbers, then there are infinitely many interesting whole numbers.
But we can't say anything about the converse i.e., if there are infinitely many interesting whole numbers then there are finitely many uninteresting whole numbers.

Ans- (**C**)

12 votes -- Dhananjay Kumar Sharma (25.7k points)

3.2.2 Logical Reasoning: GATE2012-1 top

https://gateoverflow.in/33

Selected Answer

I_1 is a correct inference. I_2 is not a correct inference as it was not mentioned what would have happened if it hadn't rained- They might have played or they might not have played.

32 votes -- Arjun Suresh (348k points)

3.2.3 Logical Reasoning: GATE2015-2-3 top

https://gateoverflow.in/8049

Selected Answer

$S_1 = C \rightarrow \neg E$
$S_2 = K \rightarrow E$

so, writing them using primary operators :
$S_1 = \neg C \vee \neg E$
$S_2 = \neg K \vee E$

on using resolution principle
$\neg E$ and E cancels each other out
and conclusion = $\neg C \vee \neg K$

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which can also be written as $K \rightarrow \neg C$ which is translated into English as = **option C**

12 votes -- Amar Vashishth (30.5k points)

Option c. If a person is kind, he is not known to be corrupt

Let

$C(x)$: x is known to be corrupt
$K(x)$: x is kind
$E(x)$: x will be elected

- $S1 : C(x) \rightarrow \neg E(x)$
- $S2 : K(x) \rightarrow E(x)$

S1 can be written as $E(x) \rightarrow \neg C(x)$ as $A \rightarrow B = \neg B \rightarrow \neg A$.
Thus, from S1 and S2,

$K(x) \rightarrow E(x) \rightarrow \neg C(x)$.

Thus we get C option.

38 votes -- Anoop Sonkar (5k points)

3.2.4 Logical Reasoning: GATE2015-3-24 top

https://gateoverflow.in/8427

Selected Answer

We do not know the type of the person from whom those words are coming from and so can have two cases :

1. Truth-teller : definitely implies that result of toss is Head.
2. Liar : the reality will be the negation of the statement.

The negation of $(x \iff y)$ is exactly one of x or y holds. So, we negate the statement : "The result of the toss is head if and only if I am telling the truth". This give rise to two possibilities

- it is head and lie spoken
- it is not head and truth spoken

Clearly the second one cannot be true because it cannot be a reality that the liar speaks the truth.

So, this imply that even if we negate the statement to see the reality or don't do that; The reality is that the toss yielded a Head.

Answer = **option (A).**

58 votes -- Amar Vashishth (30.5k points)

Person 1 (truth teller), result is head. No doubt here as he is a truth teller.
Person 2(lie teller). result is head if and only if he is telling truth. He is telling lies. So, the truth is the opposite of his statement. We can analyze his statement as two

1. If I'm telling the truth result is head
2. If result is head I'm telling the truth

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Both these are of the form $A \rightarrow B = \neg A \vee B$. Now, the truth will be the negation of these statements which will be $A \wedge \neg B$ which can be expressed as

1. I'm telling the truth and result is not head
2. Result is head and I'm telling false

Both of these means, the result is head as the person is lie teller. (Actually even if iff is replaced with if, answer would be A)

So, option A.

37 votes -- Arjun Suresh (348k points)

3.2.5 Logical Reasoning: TIFR2010-A-4 top

https://gateoverflow.in/18212

Selected Answer

Let us denote sentences with variables:

- F : Bank receipt is forged
- L : Mr. M is liable
- B : He will go bankrupt
- M : Bank loan him money

1. $F \rightarrow L$
2. $L \rightarrow B$
3. $M \rightarrow B'$
4. M

From 3 and 4 modus ponens we get
$B' \quad \rightarrow (5)$

From 2 and 5 modus tollens we get
$L' \quad \rightarrow (6)$

From 1 and 6 modus tollens we get
F'

Ans is (**b**). Bank receipt is not forged.

13 votes -- Pooja Palod (31.3k points)

3.2.6 Logical Reasoning: TIFR2011-A-1 top

https://gateoverflow.in/237

Selected Answer

It is told in the question "If the people suffer, the government will be unpopular". And "government will not be unpopular" means, people will not suffer.
It is like $A \rightarrow B$ is true and ~B is given. So, ~A must be true.

So, (A) is valid (always true).

Lets take the English meaning

Government will not be unpopular

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$\implies$ People will not suffer
$\implies$ Either no inflation or government regulates it
$\implies$ If no regulation then no inflation
$\implies$ if no regulation then no wage or price rise

So, (B) and (D) are valid (always true) and (C) and (E) are not valid.

12 votes -- Arjun Suresh (348k points)

3.2.7 Logical Reasoning: TIFR2011-A-12 top

https://gateoverflow.in/20221

Selected Answer

Option (**B**) should be the correct answer, that is A is a Knave & B is a Knight.

A must be either a Knight or a Knave.

Suppose A is a Knight, it means that the statement "We are both Knaves." must be true.

This is contradicting our assumption.

So the assumption that "A is a Knight" is not logically satisfiable simultaneously with the statement he made, which implies that A must be a Knave.

Now since A is a Knave, the statement made by him : "We are both Knaves." must be false.

The statement "We are both Knaves." will be false in any one of the following 3 conditions :

1. A is a Knight, B is a Knave.
2. A is a Knave, B is a Knight.
3. A is a Knight, B is a Knight.

Bus since we have already deduced that A is a Knave so in order to make the statement "We are both Knaves." false, we are only left with condition 2.

So B must be a Knight.

19 votes -- Anurag Pandey (13.6k points)

3.2.8 Logical Reasoning: TIFR2012-A-3 top

https://gateoverflow.in/20981

Selected Answer

If A is Blue (honest), then

- Whatever A says about B and C must be True.
- A says that B is White(liar) and C is Pink(alternating). So, if A is Blue, B must be White and C must be Pink.
- B says that C is Pink. But B is a liar, and B agrees with A on the race of C (they must not agree). Thus, we reached a contradiction.

So, A can't be Blue.

If B is Blue (honest), then

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- Whatever B says about A and C must be True.
- B says that A is Pink(alternating) and C is Pink(alternating). So, if B is Blue, A must be Pink and C must be Pink.
- Since A is pink, it must lie about B, say the truth about C and then lie about itself. Which it does.
- Since C is pink, it must lie about A, say the truth about B, and then lie about itself. Which it does.

So we see that Blue
B**, Pink**
A **and Pink**
C **is a possible solution!**

Thus, option (C) is correct.

However, there is another option (E), which says Cannot be determined from the above data.

So, what if there are multiple solutions that satisfy these constraints? If that is the case, option e will be correct. Sadly, there is no way of proving that no other solutions work except checking each one of them (using branch and bound to somewhat improve). Sadly, that will be lengthy.

Here is a Python3 program that finds all solutions to this problem: http://ideone.com/7EFXCn

14 votes -- Pragy Agarwal (20.6k points)

3.2.9 Logical Reasoning: TIFR2013-A-3 top

https://gateoverflow.in/25384

Selected Answer

6 preference order for voter are possibe:

$ABC. ACB, BCA, BAC, CAB, CBA$ also Given that

$a = ABC + ACB + CAB(A \text{ prefer over } B) - - - (1)$

$b = BCA + BAC + ABC(B \text{ prefer over } C) - - - (2)$

$c = CAB + CBA + BCA(C \text{ prefer over } A) - - - (3)$

Adding $1, 2$ and 3 we get

$a + b + c = 2(ABC + BCA + CAB) + ACB + BAC + CAB$

Now we know that $ABC + ACB + BAC + BCA + CAB + CBA = 1$ therefore

$[ABC + ACB + BAC + BCA + CAB + CBA] < [2(ABC + BCA + CAB) + ACB + BAC$

Hence we can say that value of $a + b + c$ must be between 1 and 2

option (c) value greater than 2 hence correct answer is (**c**)

15 votes -- Saurav Shrivastava (1.6k points)

3.2.10 Logical Reasoning: TIFR2014-A-8 top

https://gateoverflow.in/25994

Selected Answer

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The correct answer is option (a) Only claim 1 follows.

$\text{Glitters}(x) \implies \text{Gold}(x) \implies \neg\,\text{Silver}(x)$. Hence, Claim 1 follows. If something Glitters, it cannot be Silver.

For claim 2:

The set of things that Glitter could be empty.

We can still assert that All that Glitters is Gold, because nothing Glitters in the first place.

So, in the case when nothing Glitters, there is no Gold that Glitters. Glitters is still a subset of Gold, but there is no element in the subset Glitters.

30 votes -- Pragy Agarwal (20.6k points)

3.3 Propositional Logic(38) top

3.3.1 Propositional Logic: CMI2017-A-03 top

https://gateoverflow.in/203260

Four siblings go shopping with their father. If Abhay gets shoes, then Asha does not get a necklace. If Arun gets a T-shirt, then Aditi gets bangles. If Abhay does not get shoes or Aditi gets bangles, the mother will be happy. Which of the following is TRUE?

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A. If the mother is happy, then Aditi got bangles.
B. If Aditi got bangles, then Abhay got shoes.
C. If the mother is not happy, then Asha did not get a necklace and Arun did not get a T-shirt.
D. None of the above.

mathematical-logic cmi2017 propositional-logic

Answer

3.3.2 Propositional Logic: GATE1987-10e top https://gateoverflow.in/82457

Show that the conclusion $(r \rightarrow q)$ follows from the premises:

$p, (p \rightarrow q) \vee (p \wedge (r \rightarrow q))$

gate1987 mathematical-logic propositional-logic proof descriptive

Answer

3.3.3 Propositional Logic: GATE1988-2vii top https://gateoverflow.in/93947

Define the validity of a well-formed formula(wff)

gate1988 descriptive mathematical-logic propositional-logic

Answer

3.3.4 Propositional Logic: GATE1989-3-v top https://gateoverflow.in/87126

Answer the following:

Which of the following well-formed formulas are equivalent?

A. $P \rightarrow Q$
B. $\neg Q \rightarrow \neg P$
C. $\neg P \vee Q$
D. $\neg Q \rightarrow P$

gate1989 normal mathematical-logic propositional-logic

Answer

3.3.5 Propositional Logic: GATE1990-3-x top https://gateoverflow.in/84861

Choose the correct alternatives (More than one may be correct).

Indicate which of the following well-formed formulae are valid:

A. $(P \Rightarrow Q) \wedge (Q \Rightarrow R) \Rightarrow (P \Rightarrow R)$
B. $(P \Rightarrow Q) \Rightarrow (\neg P \Rightarrow \neg Q)$
C. $(P \wedge (\neg P \vee \neg Q)) \Rightarrow Q$
D. $(P \Rightarrow R) \vee (Q \Rightarrow R) \Rightarrow ((P \vee Q) \Rightarrow R)$

gate1990 normal mathematical-logic propositional-logic

Answer

3.3.6 Propositional Logic: GATE1991-03,xii top https://gateoverflow.in/526

Choose the correct alternatives (more than one may be correct) and write the corresponding letters only:

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If F_1, F_2 and F_3 are propositional formulae such that $F_1 \wedge F_2 \rightarrow F_3$ and $F_1 \wedge F_2 \rightarrow \sim F_3$ are both tautologies, then which of the following is true:

A. Both F_1 and F_2 are tautologies
B. The conjunction $F_1 \wedge F_2$ is not satisfiable
C. Neither is tautologous
D. Neither is satisfiable
E. None of the above.

gate1991 mathematical-logic normal propositional-logic

Answer

3.3.7 Propositional Logic: GATE1992-02,xvi top https://gateoverflow.in/574

Choose the correct alternatives (more than one may be correct) and write the corresponding letters only:

Which of the following is/are a tautology?

A. $a \vee b \rightarrow b \wedge c$
B. $a \wedge b \rightarrow b \vee c$
C. $a \vee b \rightarrow (b \rightarrow c)$
D. $a \rightarrow b \rightarrow (b \rightarrow c)$

gate1992 mathematical-logic easy propositional-logic

Answer

3.3.8 Propositional Logic: GATE1993-18 top https://gateoverflow.in/2315

Show that proposition C is a logical consequence of the formula

$$A \wedge (A \rightarrow (B \vee C)) \wedge (B \rightarrow \neg A)$$

using truth tables.

gate1993 mathematical-logic normal propositional-logic proof descriptive

Answer

3.3.9 Propositional Logic: GATE1993-8.2 top https://gateoverflow.in/2300

The proposition $p \wedge (\sim p \vee q)$ is:

A. a tautology

B. logically equivalent to $p \wedge q$

C. logically equivalent to $p \vee q$

D. a contradiction

E. none of the above

gate1993 mathematical-logic easy propositional-logic

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Answer

3.3.10 Propositional Logic: GATE1994-3.13 top https://gateoverflow.in/2499

Let p and q be propositions. Using only the Truth Table, decide whether

$p \iff q$ does not imply $p \rightarrow \neg q$

is **True** or **False**.

gate1994 mathematical-logic normal propositional-logic descriptive

Answer

3.3.11 Propositional Logic: GATE1995-13 top https://gateoverflow.in/2649

Obtain the principal (canonical) conjunctive normal form of the propositional formula

$$(p \wedge q) \vee (\neg q \wedge r)$$

where $\wedge$ is logical and, $\vee$ is inclusive or and $\neg$ is negation.

gate1995 mathematical-logic propositional-logic normal descriptive

Answer

3.3.12 Propositional Logic: GATE1995-2.19 top https://gateoverflow.in/2631

If the proposition $\neg p \rightarrow q$ is true, then the truth value of the proposition $\neg p \vee (p \rightarrow q)$, where $\neg$ is negation, $\vee$ is inclusive OR and $\rightarrow$ is implication, is

A. True
B. Multiple Values
C. False
D. Cannot be determined

gate1995 mathematical-logic normal propositional-logic

Answer

3.3.13 Propositional Logic: GATE1996-2.3 top https://gateoverflow.in/2732

Which of the following is false? Read $\wedge$ as AND, $\vee$ as OR, $\neg$ as NOT, $\rightarrow$ as one way implication and $\leftrightarrow$ as two way implication

A. $((x \rightarrow y) \wedge x) \rightarrow y$

B. $((\neg x \rightarrow y) \wedge (\neg x \rightarrow \neg y)) \rightarrow x$

C. $(x \rightarrow (x \vee y))$

D. $((x \vee y) \leftrightarrow (\neg x \rightarrow \neg y))$

gate1996 mathematical-logic normal propositional-logic

Answer

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3.3.14 Propositional Logic: GATE1997-3.2 top https://gateoverflow.in/2233

Which of the following propositions is a tautology?

A. $(p \vee q) \rightarrow p$
B. $p \vee (q \rightarrow p)$
C. $p \vee (p \rightarrow q)$
D. $p \rightarrow (p \rightarrow q)$

gate1997 mathematical-logic easy propositional-logic

Answer

3.3.15 Propositional Logic: GATE1998-1.5 top https://gateoverflow.in/1642

What is the converse of the following assertion?

- I stay only if you go

A. I stay if you go
B. If I stay then you go
C. If you do not go then I do not stay
D. If I do not stay then you go

gate1998 mathematical-logic easy propositional-logic

Answer

3.3.16 Propositional Logic: GATE1999-14 top https://gateoverflow.in/1513

Show that the formula $[(\sim p \vee q) \Rightarrow (q \Rightarrow p)]$ is not a tautology.

Let A be a tautology and B any other formula. Prove that $(A \vee B)$ is a tautology.

gate1999 mathematical-logic normal propositional-logic proof descriptive

Answer

3.3.17 Propositional Logic: GATE2000-2.7 top https://gateoverflow.in/654

Let a, b, c, d be propositions. Assume that the equivalence $a \Leftrightarrow (b \vee \neg b)$ and $b \Leftrightarrow c$ hold. Then the truth-value of the formula $(a \wedge b) \rightarrow (a \wedge c) \vee d$ is always

A. True
B. False
C. Same as the truth-value of b
D. Same as the truth-value of d

gate2000 mathematical-logic normal propositional-logic

Answer

3.3.18 Propositional Logic: GATE2001-1.3 top https://gateoverflow.in/696

Consider two well-formed formulas in propositional logic

$F_1 : P \Rightarrow \neg P \qquad F_2 : (P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

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Which one of the following statements is correct?

A. F_1 is satisfiable, F_2 is valid
B. F_1 unsatisfiable, F_2 is satisfiable
C. F_1 is unsatisfiable, F_2 is valid
D. F_1 and F_2 are both satisfiable

gate2001 mathematical-logic easy propositional-logic

Answer

3.3.19 Propositional Logic: GATE2002-1.8 top

https://gateoverflow.in/812

"If X then Y unless Z" is represented by which of the following formulas in prepositional logic? ("$\neg$" is negation, "$\wedge$" is conjunction, and "$\rightarrow$" is implication)

A. $(X \wedge \neg Z) \rightarrow Y$
B. $(X \wedge Y) \rightarrow \neg Z$
C. $X \rightarrow (Y \wedge \neg Z)$
D. $(X \rightarrow Y) \wedge \neg Z$

gate2002 mathematical-logic normal propositional-logic

Answer

3.3.20 Propositional Logic: GATE2002-5b top

https://gateoverflow.in/3915

Determine whether each of the following is a tautology, a contradiction, or neither ("$\vee$" is disjunction, "$\wedge$" is conjunction, "$\rightarrow$" is implication, "$\neg$" is negation, and "$\leftrightarrow$" is biconditional (if and only if).

$A \leftrightarrow (A \vee A)$

$(A \vee B) \rightarrow B$

$A \wedge (\neg(A \vee B))$

gate2002 mathematical-logic easy descriptive propositional-logic

Answer

3.3.21 Propositional Logic: GATE2003-72 top

https://gateoverflow.in/959

The following resolution rule is used in logic programming.
Derive clause $(P \vee Q)$ from clauses $(P \vee R), (Q \vee \neg R)$
Which of the following statements related to this rule is FALSE?

A. $((P \vee R) \wedge (Q \vee \neg R)) \Rightarrow (P \vee Q)$ is logically valid
B. $(P \vee Q) \Rightarrow ((P \vee R)) \wedge (Q \vee \neg R))$ is logically valid
C. $(P \vee Q)$ is satisfiable if and only if $(P \vee R) \wedge (Q \vee \neg R)$ is satisfiable
D. $(P \vee Q) \Rightarrow \text{FALSE}$ if and only if both P and Q are unsatisfiable

gate2003 mathematical-logic normal propositional-logic

Answer

3.3.22 Propositional Logic: GATE2004-70 top

https://gateoverflow.in/1064

The following propositional statement is $(P \implies (Q \vee R)) \implies ((P \wedge Q) \implies R)$

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A. satisfiable but not valid
B. valid
C. a contradiction
D. None of the above

gate2004 mathematical-logic normal propositional-logic

Answer

3.3.23 Propositional Logic: GATE2004-IT-31 top https://gateoverflow.in/3674

Let p, q, r and s be four primitive statements. Consider the following arguments:

$P: [(\neg pvq) \wedge (r \rightarrow s) \wedge (pvr)] \rightarrow (\neg s \rightarrow q)$
$Q: [(\neg p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \neg r$
$R: [[(q \wedge r) \rightarrow p] \wedge (\neg qvp)] \rightarrow r$
$S: [p \wedge (p \rightarrow r) \wedge (qv\neg r)] \rightarrow q$

Which of the above arguments are valid?

A. P and Q only
B. P and R only
C. P and S only
D. P, Q, R and S

gate2004-it mathematical-logic normal propositional-logic

Answer

3.3.24 Propositional Logic: GATE2005-40 top https://gateoverflow.in/1165

Let P, Q and R be three atomic propositional assertions. Let X denote $(P \vee Q) \rightarrow R$ and Y denote $(P \rightarrow R) \vee (Q \rightarrow R)$. Which one of the following is a tautology?

A. $X \equiv Y$
B. $X \rightarrow Y$
C. $Y \rightarrow X$
D. $\neg Y \rightarrow X$

gate2005 mathematical-logic propositional-logic normal

Answer

3.3.25 Propositional Logic: GATE2006-27 top https://gateoverflow.in/990

Consider the following propositional statements:

- $P_1: ((A \wedge B) \rightarrow C)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$
- $P_2: ((A \vee B) \rightarrow C)) \equiv ((A \rightarrow C) \vee (B \rightarrow C))$

Which one of the following is true?

A. P_1 is a tautology, but not P_2
B. P_2 is a tautology, but not $P_1
C. P_1 and P_2 are both tautologies
D. Both P_1 and P_2 are not tautologies

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gate2006 mathematical-logic normal propositional-logic

Answer

3.3.26 Propositional Logic: GATE2008-31 top https://gateoverflow.in/442

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \neg Q$
II. $\neg(\neg P \wedge Q)$
III. $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$
IV. $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

A. Only I and II
B. Only I, II and III
C. Only I, II and IV
D. All of I, II, III and IV

gate2008 normal mathematical-logic propositional-logic

Answer

3.3.27 Propositional Logic: GATE2009-24 top https://gateoverflow.in/801

The binary operation □ is defined as follows

P	Q	P □ Q
T	T	T
T	F	T
F	T	F
F	F	T

Which one of the following is equivalent to $P \vee Q$?

A. $\neg Q \square \neg P$
B. $P \square \neg Q$
C. $\neg P \square Q$
D. $\neg P \square \neg Q$

gate2009 mathematical-logic easy propositional-logic

Answer

3.3.28 Propositional Logic: GATE2014-1-53 top https://gateoverflow.in/1933

Which one of the following propositional logic formulas is TRUE when exactly two of p, q and r are TRUE?

A. $((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$
B. $(\sim (p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$
C. $((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$
D. $(\sim (p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

gate2014-1 mathematical-logic normal propositional-logic

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Answer

3.3.29 Propositional Logic: GATE2014-2-53 top https://gateoverflow.in/2020

Which one of the following Boolean expressions is NOT a tautology?

A. $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$
B. $(a \rightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$
C. $(a \wedge b \wedge c) \rightarrow (c \vee a)$
D. $a \rightarrow (b \rightarrow a)$

gate2014-2 mathematical-logic propositional-logic normal

Answer

3.3.30 Propositional Logic: GATE2014-3-1 top https://gateoverflow.in/2035

Consider the following statements:

- P: Good mobile phones are not cheap
- Q: Cheap mobile phones are not good

L: P implies Q
M: Q implies P
N: P is equivalent to Q

Which one of the following about L, M, and N is CORRECT?

A. Only L is TRUE.
B. Only M is TRUE.
C. Only N is TRUE.
D. L, M and N are TRUE.

gate2014-3 mathematical-logic easy propositional-logic

Answer

3.3.31 Propositional Logic: GATE2015-1-14 top https://gateoverflow.in/8209

Which one of the following is NOT equivalent to $p \leftrightarrow q$?

A. $(\neg p \vee q) \wedge (p \vee \neg q)$
B. $(\neg p \vee q) \wedge (q \rightarrow p)$
C. $(\neg p \wedge q) \vee (p \wedge \neg q)$
D. $(\neg p \wedge \neg q) \vee (p \wedge q)$

gate2015-1 mathematical-logic easy propositional-logic

Answer

3.3.32 Propositional Logic: GATE2016-1-1 top https://gateoverflow.in/39663

Let p, q, r, s represents the following propositions.

- $p : x \in \{8, 9, 10, 11, 12\}$
- $q : x$ is a composite number.

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- r : x is a perfect square.
- s : x is a prime number.

The integer $x \geq 2$ which satisfies $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is _____.

gate2016-1 mathematical-logic normal numerical-answers propositional-logic

Answer

3.3.33 Propositional Logic: GATE2016-2-01 top https://gateoverflow.in/39568

Consider the following expressions:

i. $false$
ii. Q
iii. $true$
iv. $P \vee Q$
v. $\neg Q \vee P$

The number of expressions given above that are logically implied by $P \wedge (P \Rightarrow Q)$ is _____.

gate2016-2 mathematical-logic normal numerical-answers propositional-logic

Answer

3.3.34 Propositional Logic: GATE2017-1-01 top https://gateoverflow.in/118698

The statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statements below?

I. $p \Rightarrow q$
II. $q \Rightarrow p$
III. $(\neg q) \vee p$
IV. $(\neg p) \vee q$

A. I only
B. I and IV only
C. II only
D. II and III only

gate2017-1 mathematical-logic propositional-logic easy

Answer

3.3.35 Propositional Logic: GATE2017-1-29 top https://gateoverflow.in/118310

Let p, q and r be propositions and the expression $(p \rightarrow q) \rightarrow r$ be a contradiction. Then, the expression $(r \rightarrow p) \rightarrow q$ is

A. a tautology
B. a contradiction
C. always TRUE when p is FALSE
D. always TRUE when q is TRUE

gate2017-1 mathematical-logic propositional-logic

Answer

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3.3.36 Propositional Logic: GATE2017-2-11 top https://gateoverflow.in/118151

Let p, q, r denote the statements *"It is raining"*, *"It is cold"*, and *"It is pleasant"*, respectively. Then the statement *"It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold"* is represented by

A. $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$
B. $(\neg p \wedge r) \wedge ((p \wedge q) \rightarrow \neg r)$
C. $(\neg p \wedge r) \vee ((p \wedge q) \rightarrow \neg r)$
D. $(\neg p \wedge r) \vee (r \rightarrow (p \wedge q))$

gate2017-2 mathematical-logic propositional-logic

Answer

3.3.37 Propositional Logic: TIFR2015-A-5 top https://gateoverflow.in/29454

What is logically equivalent to "If Kareena and Parineeti go to the shopping mall then it is raining":

a. If Kareena and Parineeti do not go to the shopping mall then it is not raining.
b. If Kareena and Parineeti do not go to the shopping mall then it is raining.
c. If it is raining then Kareena and Parineeti go to the shopping mall.
d. If it is not raining then Kareena and Parineeti do not go to the shopping mall.
e. None of the above.

tifr2015 mathematical-logic propositional-logic

Answer

3.3.38 Propositional Logic: TIFR2018-B-4 top https://gateoverflow.in/179288

The notation "$\Rightarrow$" denotes "implies" and "$\wedge$" denotes "and" in the following formulae.

Let X denote the formula: $(b \Rightarrow a) \Rightarrow (a \Rightarrow b)$
Let Y denote the formula: $(a \Rightarrow b) \wedge b$

Which of the following is TRUE ?

A. X is satisfiable and Y is not satisfiable.
B. X is satisfiable and Y is tautology.
C. X is not tautology and Y is not satisfiable.
D. X is not tautology and Y is satisfiable.
E. X is a tautology and Y is satisfiable,

tifr2018 mathematical-logic propositional-logic

Answer

Answers: Propositional Logic

3.3.1 Propositional Logic: CMI2017-A-03 top https://gateoverflow.in/203260

Selected Answer

Let's name the events:

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$\mathbf{S}$ = Abhay gets **s**hoes

$\mathbf{N}$ = Asha gets a **n**ecklace

$\mathbf{T}$ = Arun gets a **T**-shirt

$\mathbf{B}$ = Aditi gets **b**angles

$\mathbf{M}$ = **M**other is happy.

Now understanding the questions in a logical way:

NOTE: *"if p, then q"* is equivalent to writing: $p \rightarrow q$

If Abhay gets shoes, then Asha does not get a necklace.

This can be written as:

$\mathbf{S} \rightarrow \sim \mathbf{N}$

If Arun gets a T-shirt, then Aditi gets bangles.

$\mathbf{T} \rightarrow \mathbf{B}$

If Abhay does not get shoes or Aditi gets bangles, the mother will be happy.

$(\sim \mathbf{S} \vee \mathbf{B}) \rightarrow \mathbf{M}$

We have to see that what can be inferred from these three statements.

Now looking at the options:

A. ***If the mother is happy, then Aditi got bangles.***
$M \rightarrow B$
This can not be inferred because it might be the case that Aditi does not get bangles, but the mother is still happy.

B. ***If Aditi got bangles, then Abhay got shoes.***
$B \rightarrow S$
This also can not be inferred. It can be that Aditi got bangles, but Abhay does not get shoes.

C. ***If the mother is not happy, then Asha did not get a necklace and Arun did not get a T-shirt.***
$\sim M \rightarrow (\sim N \wedge \sim T)$
This can be inferred as follows:
We have: $(\sim S \vee B) \rightarrow M$
Contrapositive of this is: $\sim M \rightarrow \sim (\sim S \vee B)$
$= \sim M \rightarrow (S \wedge \sim B)$ ----- (1)
We also have: $S \rightarrow \sim N$ ----- (2),
And: $T \rightarrow B$
whose contrapositive is: $\sim B \rightarrow \sim T$ ----- (3)
From $(1), (2)$ and (3) we can conclude:
$\sim M \rightarrow (\sim N \wedge \sim T)$

Hence, the **correct option should be (C)**

3 votes -- Rishabh Gupta (13.9k points)

3.3.2 Propositional Logic: GATE1987-10e top

https://gateoverflow.in/82457

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Selected Answer

$P_1 : P$

$P_2 : (P \to Q) \vee (P \wedge (R \to Q))$

So, $P_1 \wedge P_2 \to (R \to Q)$

$$\equiv [P \wedge [(\neg P \vee Q) \vee (P \wedge \neg R \vee (P \wedge Q))] \to (R \to Q)$$
$$\equiv [P \wedge [\neg P \vee Q \vee (P \wedge \neg R) \vee (P \wedge Q)]] \to (R \to Q)$$
$$\equiv [(P \wedge \neg R) \vee (P \wedge Q)] \to (R \to Q)$$
$$\equiv [(\neg P \vee R) \wedge (\neg P \vee \neg Q) \vee \neg R \vee Q]$$
$$\equiv [\neg P \vee (R \wedge \neg Q) \vee \neg R \vee Q]$$
$$\equiv [\neg P \vee \neg Q \vee \neg R \vee Q]$$
$$\equiv 1$$

Tautology

Hence proved.

11 votes -- Prashant Singh (59.8k points)

3.3.3 Propositional Logic: GATE1988-2vii top https://gateoverflow.in/93947

Selected Answer

Valid : A wff which is always true (tautology) is valid. i.e., its value is true for any set of assignment to its variables.

Satisfiable : A wff which is not a contradiction (always false) is satisfiable. It may be a tautology.

6 votes -- Kantikumar (4.6k points)

3.3.4 Propositional Logic: GATE1989-3-v top https://gateoverflow.in/87126

Selected Answer

A. $P \to Q \equiv \neg P \vee Q$
B. $\neg Q \to \neg P \equiv Q \vee \neg P$
C. $\neg P \vee Q \equiv \neg P \vee Q$

So, A, B, C are equivalent .

10 votes -- kunal chalotra (20.4k points)

3.3.5 Propositional Logic: GATE1990-3-x top https://gateoverflow.in/84861

Selected Answer

Ans is (A)

$\Rightarrow$ To make a formula valid it must be true for all cases (Tautology)

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a. $\underbrace{(P \implies Q) \wedge (Q \implies R)}_{T} \implies \underbrace{(P \implies R)}_{F}$:To make it invalid

assume:

$\underbrace{P}_{T} \implies \underbrace{R}_{F} : F$

Now,

$\underbrace{\underbrace{(P \implies Q)}_{T} \wedge \underbrace{(Q \implies R)}_{T}}_{T}$

We know,

$P = T;\ R = F$

$\underbrace{(T \implies Q)}_{T} \wedge \underbrace{(Q \implies F)}_{T}$

$Q \neq F;$ it should be T. $Q \neq T;$ it should be F.

Since ,value of Q is ambiguous. it tells that our original assumption of $P = T; R = F$ and $P \implies R : F;$ is wrong.

$T \implies F$ is not possible. **Hence its valid.**

b. $\underbrace{(P \implies Q)}_{T} \implies \underbrace{(\neg P \implies \neg Q)}_{F}$; to make it invalid the case must hold.

So, $\underbrace{\neg P}_{T} \implies \underbrace{\neg Q}_{F}$; for this to be false.

$\therefore P = F; Q = T$ which makes $P \implies Q : T$

So, $T \implies F$ does hold .**Hence invalid.**

c. $\underbrace{(P \implies R) \vee (Q \implies R)}_{T} \implies \underbrace{((P \vee Q) \implies R)}_{F}$

$\underbrace{(P \implies R)}_{T} \vee \underbrace{(Q \implies R)}_{T} \qquad \underbrace{(P \vee Q)}_{T} \implies \underbrace{R}_{F}$

or,

$(P \implies F) \vee (Q \implies F) \quad [\because R = F]$

assume $P = T$ and $Q = F$ which makes the above T (LHS) and F (RHS)

So,again $T \implies F$ hold.**Hence invalid.**

14 votes -- Tuhin Dutta (7.9k points)

3.3.6 Propositional Logic: GATE1991-03,xii top

https://gateoverflow.in/526

Selected Answer

answer = option (**B**)

$$\text{False} \rightarrow \text{anything} = \text{True, always}$$

17 votes -- Amar Vashishth (30.5k points)

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3.3.7 Propositional Logic: GATE1992-02,xvi top

https://gateoverflow.in/574

Selected Answer

Answer: (**B**)

$$(a \wedge b) \rightarrow b \vee c$$
$$\implies \neg(a \wedge b) \vee b \vee c$$
$$\implies \neg a \vee \neg b \vee b \vee c$$
$$\implies T$$

Option (A) is not TRUE when C is FALSE
Option (C) is not TRUE when b is TRUE and C is FALSE
Option (D) is not TRUE when a and b are TRUE and C is FALSE.

12 votes -- Rajarshi Sarkar (34.1k points)

3.3.8 Propositional Logic: GATE1993-18 top

https://gateoverflow.in/2315

Selected Answer

$$A \wedge (A \rightarrow (B \vee C)) \wedge (B \rightarrow \neg A)$$
$$\equiv A \wedge (\neg A \vee B \vee C)(\neg A \vee \neg B)$$
$$\equiv (A \wedge \neg B) \wedge (\neg A \vee B \vee C)$$
$$\equiv (A \wedge \neg B \wedge C)$$

C is logical consequence of a formula X if, $X \rightarrow C$ is true.

$$\text{Here, } X \equiv A \wedge (A \rightarrow (B \vee C)) \wedge (B \rightarrow \neg A)$$
$$\equiv A \wedge \neg B \wedge C$$

Checking,
$$(A \wedge \neg B \wedge C) \rightarrow C$$
$$\equiv \neg(A \wedge \neg B \wedge C) \vee C$$
$$\equiv \neg A \vee B \vee \neg C \vee C$$
$$\equiv 1.$$

So, C is logical consequence of $A \wedge (A \rightarrow (B \vee C)) \wedge (B \rightarrow \neg A)$.

14 votes -- Digvijay (54.8k points)

3.3.9 Propositional Logic: GATE1993-8.2 top

https://gateoverflow.in/2300

Selected Answer

$$p \wedge (\sim p \vee q)$$

$$\equiv (p \wedge \sim p) \vee (p \wedge q)$$

$$\equiv F \vee (p \wedge q)$$

$$\equiv (p \wedge q)$$

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Hence, Option(**B**) logically equivalent to $(p \wedge q)$.

9 votes -- Leen Sharma (40k points)

3.3.10 Propositional Logic: GATE1994-3.13 top https://gateoverflow.in/2499

Selected Answer

p	q	$p \leftrightarrow q$	$p \rightarrow \neg q$	$(p \rightarrow (p \leftrightarrow q) \rightarrow \neg q)$
0	0	1	1	1
0	1	0	1	1
1	0	0	1	1
1	1	1	0	0

So, "imply" is FALSE making does not imply TRUE.

20 votes -- Arjun Suresh (348k points)

3.3.11 Propositional Logic: GATE1995-13 top https://gateoverflow.in/2649

Selected Answer

Note:-..canonical conjunctive normal form means in our Digital logic ..it is Canonical Product of Sum term Form
Canonical Disjunctive normal form means in our Digital logic ..it is Canonical Sum of Product term Form

$pq + q'r$

put in $k-$map.

you will get

$\sum$(1,5,6,7)= $\prod$(0,2,3,4)=(p∨q∨r) ∧ (p∨¬q∨r) ∧ (p∨¬q∨¬r) ∧ (¬p∨q∨r) is Ans.

11 votes -- Rajesh Pradhan (22.6k points)

3.3.12 Propositional Logic: GATE1995-2.19 top https://gateoverflow.in/2631

Selected Answer

Cannot be determined.

From the axiom $\neg p \rightarrow q$, we can conclude that $p \vee q$.

So, either p or q must be TRUE.

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$$\neg p \vee (p \rightarrow q)$$

$$\equiv \neg p \vee (\neg p \vee q)$$

$$\equiv \quad \neg p \vee q$$

Since nothing can be said about the Truth value of p, it implies that $\neg p \vee q$ can also be True or False.

Hence, the value cannot be determined.

34 votes -- Pragy Agarwal (20.6k points)

3.3.13 Propositional Logic: GATE1996-2.3 top

https://gateoverflow.in/2732

Selected Answer

$$\begin{aligned}
A.\ & ((x \rightarrow y) \wedge x) \rightarrow y \\
& \equiv \neg((\neg x \vee y) \wedge x) \vee y \\
& \equiv ((x \wedge \neg y) \vee \neg x) \vee y \\
& \equiv ((x \vee \neg x) \wedge (\neg x \vee \neg y)) \vee y \\
& \equiv (T \wedge (\neg x \vee \neg y)) \vee y \\
& \equiv (\neg x \vee \neg y) \vee y \\
& \equiv (y \vee \neg y) \vee \neg x \\
& \equiv T \vee \neg x \\
& \equiv T
\end{aligned}$$

$$\begin{aligned}
B.\ & ((\neg x \rightarrow y) \wedge (\neg x \rightarrow \neg y)) \rightarrow x \\
& \equiv \neg((x \vee y) \wedge (x \vee \neg y)) \vee x \\
& \equiv ((\neg x \wedge \neg y) \vee (\neg x \wedge y)) \vee x \\
& \equiv ((\neg x \wedge (\neg y \vee y)) \vee x \\
& \equiv (\neg x \wedge T) \vee x \\
& \equiv (\neg x \wedge x) \\
& \equiv T
\end{aligned}$$

$$\begin{aligned}
C.\ & (x \rightarrow (x \vee y)) \\
& \equiv (\neg x \vee (x \vee y)) \\
& \equiv (\neg x \vee x) \vee y \\
& \equiv T \vee y \\
& \equiv T
\end{aligned}$$

$$\begin{aligned}
D.\ & ((x \vee y) \Leftrightarrow (\neg x \rightarrow \neg y)) \\
& \equiv ((x \vee y) \Leftrightarrow (x \vee \neg y)) \\
& \equiv ((x \vee y) \rightarrow (x \vee \neg y)) \wedge ((x \vee \neg y) \rightarrow (x \vee y)) \\
& \equiv (\neg(x \vee y) \vee (x \vee \neg y)) \wedge (\neg(x \vee \neg y) \vee (x \vee y)) \\
& \equiv ((\neg x \wedge \neg y) \vee (x \vee \neg y)) \wedge (\neg x \wedge y) \vee ((x \vee y)) \\
& \equiv (((\neg x \wedge \neg y) \vee x) \neg y) \wedge (((\neg x \wedge y) \vee x) \vee y) \\
& \equiv (((\neg x \vee x) \wedge (x \vee \neg y)) \vee \neg y) \wedge (((\neg x \vee x) \wedge (x \vee y)) \vee y) \\
& \equiv ((T \wedge (x \vee \neg y)) \vee \neg y) \wedge ((T \wedge (x \vee y)) \vee y) \\
& \equiv ((x \vee \neg y) \wedge (x \vee y)) \\
& \equiv (x \wedge x) \vee (x \wedge y) \vee (x \wedge \neg y) \vee (\neg y \wedge y) \\
& \equiv x \vee (x \wedge y) \vee (x \wedge \neg y) \vee F \\
& \equiv x \vee (x \wedge y) \vee (x \wedge \neg y)
\end{aligned}$$

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$$\equiv \quad x$$

Hence, Option(**D**). $((x \vee y) \Leftrightarrow (\neg x \rightarrow \neg y))$.

8 votes -- Leen Sharma (40k points)

3.3.14 Propositional Logic: GATE1997-3.2 top

https://gateoverflow.in/2233

Selected Answer

$$\begin{aligned}
A. \quad & (p \vee q) \rightarrow p \\
& \equiv \quad \neg(p \vee q) \vee p \\
& \equiv \quad (\neg p \wedge \neg q) \vee p \\
& \equiv \quad (\neg p \vee p) \wedge (\neg q \vee p) \\
& \equiv (p \vee \neg q)
\end{aligned}$$

$$\begin{aligned}
B. \quad & p \vee (q \rightarrow p) \\
& \equiv \quad p \vee (\neg q \vee p) \\
& \equiv \quad (p \vee p) \vee \neg q \\
& \equiv p \vee \neg q
\end{aligned}$$

$$\begin{aligned}
C. \quad & p \vee (p \rightarrow q) \\
& \equiv \quad p \vee (\neg p \vee q) \\
& \equiv \quad (p \vee \neg p) \vee q \\
& \equiv \quad T \vee q \\
& \equiv \quad T
\end{aligned}$$

$$\begin{aligned}
D. \quad & p \rightarrow (p \rightarrow q) \\
& \equiv \quad p \rightarrow (\neg p \vee q) \\
& \equiv \quad \neg p \vee (\neg p \vee q) \\
& \equiv \quad (\neg p \vee \neg p) \vee q \\
& \equiv \quad \neg p \vee q
\end{aligned}$$

Hence, Option(**C**) $p \vee (p \rightarrow q)$.

5 votes -- Leen Sharma (40k points)

3.3.15 Propositional Logic: GATE1998-1.5 top

https://gateoverflow.in/1642

Selected Answer

"I stay only if you go" **is equivalent to** "If I stay then you go."

$\because A$ only if $B \equiv (A \rightarrow B)$

$A =$ "I stay" **and** $B =$ "You go"

Converse $(A \rightarrow B) = B \rightarrow A$

"If you go then I stay"

Answer is (**A**).

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17 votes -- Manali (2.6k points)

3.3.16 Propositional Logic: GATE1999-14 top

https://gateoverflow.in/1513

Selected Answer

a. $[(\sim p \vee q) \Rightarrow (q \Rightarrow p)] = \sim (\sim p \vee q)) \vee (q \Rightarrow p) = (p \wedge \sim q) \vee (\sim q \vee p) = p \vee \sim q$.

Hence not tautology.

b. $(A \vee B) = T \vee B = T$

10 votes -- Arjun Suresh (348k points)

3.3.17 Propositional Logic: GATE2000-2.7 top

https://gateoverflow.in/654

Selected Answer

Given that $a \Leftrightarrow (b \vee \sim b)$ and $b \Leftrightarrow c$
Now,
$(a \wedge b) \rightarrow (a \wedge c) \vee d$
$\equiv (a \wedge b) \rightarrow (a \wedge b) \vee d$
$(\because b \Leftrightarrow c)$
$\equiv \neg(a \wedge b) \vee (a \wedge b) \vee d$
$\equiv T \vee d$
$\equiv T$
Hence, Option(**A**) True.

13 votes -- Leen Sharma (40k points)

3.3.18 Propositional Logic: GATE2001-1.3 top

https://gateoverflow.in/696

Selected Answer

F1: $P \rightarrow \neg P$

$= \neg P \wedge \neg P$

$= \neg P.$ can be true when P is false (Atleast one T hence satisfiable)

F2: $(P \rightarrow \neg P) \wedge (\neg P \rightarrow P)$

$= \neg P \vee (P \vee P)$

$= \neg P \vee P$

$= T.$

VALID

Option (**A**)

22 votes -- Manali (2.6k points)

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3.3.19 Propositional Logic: GATE2002-1.8 top

https://gateoverflow.in/812

Selected Answer

```
while ( not z )
{
if (X) then
    Y
}
or
unless( z )
{
if (X) then
    Y
}
```

this is what it means in programming. if you want to execute statement Y then X must be True and $Z\text{False}$, which is equivalent to $(X \wedge \neg Z)Y$.

option (A).

49 votes -- Vikrant Singh (13.5k points)

Answer is a) $(X \wedge \neg Z) \rightarrow Y$

((refer page 6,7 Discrete Math,ed 7, Kenneth H Rosen))

Implication "P implies Q" i.e., $(p \rightarrow Q)$, where P is premise and Q is Conclusion, can be equivalently expressed in many ways. And the two equivalent expression relevant to the question are as follows

one is " if P then Q "

another is "Q unless $\neg P$"

Both of these are equivalent to the propositional formula $(P \rightarrow Q)$,

Now compare "If X then Y unless Z" with "Q unless $\neg P$" , here $(\neg P = Z)$ so $(P = \neg Z)$ and $(Q = Y)$

Compare with "if P then Q", here $(P = X), (Q = Y)$

So we get premise $P =' X'$ and $'\neg Z'$, conclusion $Q =' Y'$

Equivalent propositional formula $(X \wedge \neg Z) \rightarrow Y$

PS: Someone messaged me that i have taken "If X then (Y unless Z)" in above explanation and how to know if we take "(If X then Y) unless Z" or "If X then (Y unless Z)". So let me show that both way gives the same answer.

"(If X then Y) unless Z" $\Leftrightarrow (X \rightarrow Y)$ unless Z

$$\begin{aligned}
&\Leftrightarrow \neg Z \rightarrow (X \rightarrow Y) \\
&\Leftrightarrow \neg Z \rightarrow (\neg X \vee Y) \\
&\Leftrightarrow Z \vee \neg X \vee Y \\
&\Leftrightarrow \neg(X \wedge \neg Z) \vee Y
\end{aligned}$$

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$\Leftrightarrow (X \wedge \neg Z) \rightarrow Y$

29 votes -- Sourav Das (297 points)

3.3.20 Propositional Logic: GATE2002-5b top https://gateoverflow.in/3915

Selected Answer

This can be solved by Truth table. But there is something else which can be done quickly. See what each formula means:

1. A↔(A∨A) It says if A then (A or A) and if (A or A) then A. Always a tautology

2. (A∨B)→B If A or B then B. No guarantee that if only A is true, B need to be true. Hence neither tautology nor contradiction

3. A∧(¬(A∨B)) When A is true ¬(A∨B) will be false. So, this formula is a contradiction

23 votes -- Arjun Suresh (348k points)

3.3.21 Propositional Logic: GATE2003-72 top https://gateoverflow.in/959

Selected Answer

Taking option (A)
$((P \vee R) \wedge (Q \vee \neg R)) \Rightarrow (P \vee Q)$ is logically valid.

$((P \vee R) \wedge (Q \vee \neg R)) \Rightarrow (P \vee Q)$
$\equiv \neg((P \vee R) \wedge (Q \vee \neg R)) \vee (P \vee Q)$
$\equiv ((\neg P \wedge \neg R) \vee (\neg Q \wedge R)) \vee (P \vee Q)$
$\equiv P \vee \neg R \vee Q \vee R$
$\equiv 1 \vee P \vee Q$
$\equiv 1.$

So, Tautology (Logically VALID). True

Option B
$(P \vee Q) \Rightarrow ((P \vee R)) \wedge (Q \vee \neg R))$ is logically valid

$(P \vee Q) \Rightarrow ((P \vee R)) \wedge (Q \vee \neg R))$
$\equiv \neg(P \vee Q) \vee ((P \vee R)) \wedge (Q \vee \neg R))$
$\equiv (\neg P \wedge \neg Q) \vee (P \wedge Q) \vee (P \wedge \neg R) \vee (Q \wedge R)$

which is a contingency $($can be TRUE or FALSE depending on values of P, Q and $R)$ and hence not logically valid.

Answer B.

14 votes -- Prashant Singh (59.8k points)

3.3.22 Propositional Logic: GATE2004-70 top https://gateoverflow.in/1064

Selected Answer

Answer a

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It is false when P = T, Q = T, R = F

It is true (satisfiable) when P = T. Q = T, R = T

16 votes -- Anu (5.9k points)

3.3.23 Propositional Logic: GATE2004-IT-31 top

https://gateoverflow.in/3674

Selected Answer

An argument form is **valid** if no matter what propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are all true.
i.e. $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$ is a tautology.

P
$: \neg \cancel{p}$
$\vee q$
$\neg \cancel{r}$
$\vee s$

$\cancel{p}$
$\underline{\vee \cancel{r}}$
also, $\neg s$ ✓
$s \quad \rightarrow q$
$\vee q$

$Q: \quad \neg p \quad \cancel{q} \quad \underline{\neg \cancel{q} \vee (\neg p \vee r)}$
$\neg p \vee r$

$\neq \neg r$

R
$: \neg(q = \neg q$
$\wedge r) \quad \underline{\vee \neg r \vee p \neg q \vee p}$
$\vee p$
$\neg q$
$\vee p$

$\neq r$

$p \vee \neg q \vee \neg r$

$S: \cancel{p} \neg \cancel{p} \vee \cancel{r} \underline{q \vee \neg \cancel{r}}$
q ✓

21 votes -- Amar Vashishth (30.5k points)

3.3.24 Propositional Logic: GATE2005-40 top

https://gateoverflow.in/1165

Selected Answer

$$\begin{aligned} X &\equiv (P \vee Q) \rightarrow R \\ &\equiv \neg(P \vee Q) \vee R \\ &\equiv (\neg P \wedge \neg Q) \vee R \\ &\equiv (\neg P \vee R) \wedge (\neg Q \vee R) \\ &\equiv (P \rightarrow R) \wedge (Q \rightarrow R) \end{aligned}$$

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So, $X \rightarrow Y$ is true as $(A \wedge B) \rightarrow (A \vee B)$ is always TRUE but reverse implication is not always true.

Hence, B.

21 votes -- Arjun Suresh (348k points)

3.3.25 Propositional Logic: GATE2006-27 top https://gateoverflow.in/990

Selected Answer

(D) Both P_1 and P_2 are not tautologies.

P_1 : If A is true and B is false, LHS of P_1 is true but RHS becomes false. Hence not tautology.

P_2 : Forward side is true. But reverse side is not true. When A is false and B is true and C is false, RHS is true but LHS is false.

LHS of P_2 can be simplified as follows:

$$\begin{aligned}((A \vee B) \rightarrow C) &\equiv ((A \vee B) \vee C) \\ &\equiv (A \wedge B) \vee C) \\ &\equiv (A \vee C) \wedge (B \vee C) \\ &\equiv (A \rightarrow C) \wedge (B \rightarrow C)\end{aligned}$$

20 votes -- Arjun Suresh (348k points)

3.3.26 Propositional Logic: GATE2008-31 top https://gateoverflow.in/442

Selected Answer

I and **II** are present in all options so need to check.

For **III** and **IV**

$$\begin{aligned}(P \wedge Q) \vee (P \wedge \neg Q) &\equiv P \wedge (QV\neg Q) \quad \text{(By distributive law)} \\ &\equiv P \wedge T \\ &\equiv P\end{aligned}$$

For III.

$P \vee (\neg P \wedge \neg Q) \equiv P \vee \neg Q$ (By Absorption Law)

For IV.

$P \vee (\neg P \wedge Q) \equiv P \vee Q$ (By Absorption Law)

So Option B is correct.

I, II, III are logically equivalent.

12 votes -- Soumya Jain (9.7k points)

3.3.27 Propositional Logic: GATE2009-24 top https://gateoverflow.in/801

Selected Answer

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Answer is B because the truth values for option B is same as that of P "or" Q.

The given truth table is for $Q \implies P$ which is $\bar{Q} + P$.

Now, with A option we get $\bar{\bar{Q}} + P = P + Q$

21 votes -- chetna (383 points)

3.3.28 Propositional Logic: GATE2014-1-53 top https://gateoverflow.in/1933

Selected Answer

A. will be true if P, Q, R are true, $((p \leftrightarrow q) \wedge r)$ will return true. So "exactly two" is false
C. if only r is true and p and q are false, first part of implication itself will result in true
D. if r is true or false, this returns false due to r and $\neg r$ present in conjunction. So, this is a CONTRADICTION.

B is the answer. B is true if p is TRUE and q is FALSE or vice verse, and r is true or if p and q are TRUE and r is FALSE.

PS: Actually the question should have been "TRUE **ONLY** when exactly two of p, q and r are TRUE"

18 votes -- Manu Thakur (39.6k points)

3.3.29 Propositional Logic: GATE2014-2-53 top https://gateoverflow.in/2020

Selected Answer

Another way to solve it...

Implication $A \rightarrow B$ is not tautology if B is false and A is true.

For b option Let RHS ie. $b \rightarrow (a \wedge c)$ be false ie b is false and $(a \wedge c)$ is false.

Now, a AND c is false if both a and c are false or one of them is true and other is false.

Now, if a and c both are false then $a \rightarrow c$ is true. Now ,LHS is true and RHS is false.

So option b is not tautology..

23 votes -- Pooja Palod (31.3k points)

3.3.30 Propositional Logic: GATE2014-3-1 top https://gateoverflow.in/2035

Selected Answer

D)
Lets break the given compound statements into atomic statements.

A : Good mobile phones.
B : Cheap mobile phones.

P :$(A \rightarrow \neg B) \iff (\neg A \vee \neg B)$
Q :$(B \rightarrow \neg A) \iff ((\neg B \vee \neg A) \iff \neg A \vee \neg B))$ (Disjunction is commutative),

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Hence, $(P \iff Q)$ which means $(P \rightarrow Q)$ and $(Q \rightarrow P)$.

34 votes -- Srinath Jayachandran (3.7k points)

3.3.31 Propositional Logic: GATE2015-1-14 top https://gateoverflow.in/8209

Selected Answer

$(p \iff q)$

$= (p \rightarrow q) \wedge (q \rightarrow p)$

$= (\neg p \vee q) \wedge (q \rightarrow p)$ As$(p \rightarrow q = \neg p \vee q)$

$= (\neg p \vee q) \wedge (\neg q \vee p)$

$= (\neg p \wedge \neg q) \vee (p \wedge q)$

So, answer C

27 votes -- Priya_das (797 points)

3.3.32 Propositional Logic: GATE2016-1-1 top https://gateoverflow.in/39663

Selected Answer

$$\begin{aligned}&\neg((p \rightarrow q) \wedge (\neg r \vee \neg s))\\ &\quad \equiv (\neg(\neg p \vee q)) \vee (\neg(\neg r \vee \neg s))\\ &\quad \equiv (p \wedge \neg q) \vee (r \wedge s)\end{aligned}$$

which can be read as $(x \in \{8, 9, 10, 11, 12\}$ AND x is not a composite number$)$ OR $(x$ is a perfect square AND x is a prime number$)$
Now for

- x is a perfect square and x is a prime number, this can never be true as every square has at least 3 factors, $1, x$ and x^2.

So, second condition can never be true.
which implies the first condition must be true.
$x \in \{8, 9, 10, 11, 12\}$ AND x is not a composite number

But here only 11 is not a composite number. so only 11 satisfies the above statement.

ANSWER 11.

53 votes -- Abhilash Panicker (9.4k points)

3.3.33 Propositional Logic: GATE2016-2-01 top https://gateoverflow.in/39568

Selected Answer

4 should be the correct answer.

PQ	$P \implies Q$	$P \wedge (P \implies Q)$
F F	T	F

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F T	T	F
T F	F	F
T T	T	T

Suppose $(P \wedge (P \implies Q)) \iff A$ (For notational convenience)

Thus for options, $(i), (ii), (iii), (iv), (v)$
If $(A \implies \text{option x})$ is a **tautology.**
then $P \wedge (P \implies Q)$ logically implies option x
else $P \wedge (P \implies Q)$ does not logically implies option x.

P	Q	A	Option(i)		Option(ii)		Option(iii)		Option(iv)		Option(v)	
			False	$A \Rightarrow F$	Q	$A \Rightarrow Q$	True	$A \Rightarrow True$	$(P \vee Q)$	$A \Rightarrow (P \vee Q)$	$\neg Q \vee P$	$A \Rightarrow (\neg Q \vee P)$
F	F	F		T		T		T		T		T
F	T	F		T		T		T		T		T
T	F	F		T		T		T		T		T
T	T	T	F	F	T	T	T	T	T	T	T	T

Answer=4

P.S: Blank entries in the above truth table are like don't care conditions because in those rows the value of A is set to False. Hence $(A \implies \text{Anything})$ would be set to True.

91 votes -- Anurag Pandey (13.6k points)

3.3.34 Propositional Logic: GATE2017-1-01 top https://gateoverflow.in/118698

Selected Answer

$(\neg P \rightarrow \neg Q)$ can also be written as $(P \vee \neg Q)$, so statement 3 is correct.

Now taking the contrapositive of $(\neg P \rightarrow \neg Q)$, we get $(Q \rightarrow P)$ hence statement 2 is correct.

So, the answer is OPTION (D).

15 votes -- sriv_shubham (3.3k points)

3.3.35 Propositional Logic: GATE2017-1-29 top https://gateoverflow.in/118310

Selected Answer

Given $(P \rightarrow Q) \rightarrow R$ is false. It is possible only when R is FALSE and $(P \rightarrow Q)$ is TRUE.
Now even without checking any other option we can directly conclude option D is correct as $(R \rightarrow P) \rightarrow Q$ can be written as $\neg(R \rightarrow P) \vee Q$.

Since, R is False, $R \rightarrow P$ is true and $\neg(R \rightarrow P)$ is False. So, it becomes $(False \vee Q)$.
which is TRUE whenever Q is TRUE

Hence, option (D)

30 votes -- sriv_shubham (3.3k points)

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3.3.36 Propositional Logic: GATE2017-2-11 top

https://gateoverflow.in/118151

Selected Answer

1. "It is not raining and it is pleasant" can be written as $(\neg p \wedge r)$

2. Now, "it is not pleasant only if it is raining and it is cold" is represented by $\neg r \implies (p \wedge q)$ but $(p \wedge q) \not\implies \neg r$. Why? Because if it is not pleasant then we can conclude it must be raining and it is cold. However, it is raining and cold does not assure that it will be unpleasant. i.e., p only if q can be written as if p then q (not double implication).

So, ANDing clause $1.$ and $2.$ we get $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$

option A is correct.

18 votes -- Prateek Kumar (1.1k points)

3.3.37 Propositional Logic: TIFR2015-A-5 top

https://gateoverflow.in/29454

Selected Answer

Answer will be (D)

"If Kareena and Parineeti go to the shopping mall then it is raining"

Let "Kareena and Parineeti go to the shopping mall" be represented by p and "it is raining" by q

Now, the statement says that $p \rightarrow q$

a. a. If Kareena and Parineeti do not go to the shopping mall then it is not raining.
   i.e., $\neg p \rightarrow \neg q$
   Not matching with the given implication.
b. b.If Kareena and Parineeti do not go to the shopping mall then it is raining.
   i.e., $\neg p \rightarrow q$
   Not matching with the given implication.
c. If it is raining then Kareena and Parineeti go to the shopping mall.
   i.e., $q \rightarrow p$
   Not matching with the given implication.
d. If it is not raining then Kareena and Parineeti do not go to the shopping mall.
   i.e., $\neg q \rightarrow \neg p \equiv q \vee \neg p \equiv p \rightarrow q$
   Matces with the given implication.

So, correct option is (D).

21 votes -- srestha (86.6k points)

3.3.38 Propositional Logic: TIFR2018-B-4 top

https://gateoverflow.in/179288

Selected Answer

Since we only have to deal with 2 variables(a and b), solving this with truth table is feasible.

Truth table for X will be:

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a	b	$b \Rightarrow a$	$a \Rightarrow b$	$(b \Rightarrow a) \Rightarrow (a \Rightarrow b)$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	1	1	1

So, clearly X is satisfiable but not a tautology .

Truth table for Y will be:

a	b	$a \Rightarrow b$	$(a \Rightarrow b) \wedge b$
0	0	1	0
0	1	1	1
1	0	0	0
1	1	1	1

So, Y is also satisfiable but not a tautology .

This gives **OPTION (D)** as the correct answer.

1 votes -- Rishabh Gupta (13.9k points)

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4 Discrete Mathematics: Set Theory & Algebra (178) top

4.1 Binary Operation(8) top

4.1.1 Binary Operation: GATE1989-1-v top https://gateoverflow.in/87051

The number of possible commutative binary operations that can be defined on a set of n elements (for a given n) is _____.

gate1989 descriptive set-theory&algebra binary-operation

Answer

4.1.2 Binary Operation: GATE1994-2.2 top https://gateoverflow.in/2469

On the set N of non-negative integers, the binary operation _____ is associative and non-commutative.

gate1994 set-theory&algebra normal binary-operation descriptive

Answer

4.1.3 Binary Operation: GATE2003-38 top https://gateoverflow.in/929

Consider the set $\{a, b, c\}$ with binary operators $+$ and $*$ defined as follows.

+	a	b	c
a	b	a	c
b	a	b	c
c	a	c	b

*	a	b	c
a	a	b	c
b	b	c	a
c	c	c	b

For example, $a + c = c, c + a = a, c * b = c$ and $b * c = a$.

Given the following set of equations:

$$(a * x) + (a * y) = c(b * x) + (c * y) = c$$

The number of solution(s) (i.e., pair(s) (x, y) that satisfy the equations) is

A. 0
B. 1
C. 2
D. 3

gate2003 set-theory&algebra normal binary-operation

Answer

4.1.4 Binary Operation: GATE2006-28 top https://gateoverflow.in/991

A logical binary relation $\odot$, is defined as follows:

A	B	$A \odot B$

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True	True	True
True	False	True
False	True	False
False	False	True

Let $\sim$ be the unary negation (NOT) operator, with higher precedence then $\odot$.

Which one of the following is equivalent to $A \wedge B$?

A. $(\sim A \odot B)$
B. $\sim (A \odot \sim B)$
C. $\sim (\sim A \odot \sim B)$
D. $\sim (\sim A \odot B)$

gate2006 set-theory&algebra binary-operation

Answer

4.1.5 Binary Operation: GATE2006-IT-2 top

https://gateoverflow.in/3539

For the set N of natural numbers and a binary operation $f : N \times N \rightarrow N$, an element $z \in N$ is called an identity for f, if $f(a, z) = a = f(z, a)$, for all $a \in N$. Which of the following binary operations have an identity?

I. $f(x, y) = x + y - 3$
II. $f(x, y) = \max(x, y)$
III. $f(x, y) = x^y$

A. I and II only
B. II and III only
C. I and III only
D. None of these

gate2006-it set-theory&algebra easy binary-operation

Answer

4.1.6 Binary Operation: GATE2013-1 top

https://gateoverflow.in/59

A binary operation $\oplus$ on a set of integers is defined as $x \oplus y = x^2 + y^2$. Which one of the following statements is **TRUE** about $\oplus$?

A. Commutative but not associative
B. Both commutative and associative
C. Associative but not commutative
D. Neither commutative nor associative

gate2013 set-theory&algebra easy binary-operation

Answer

4.1.7 Binary Operation: GATE2015-1-28 top

https://gateoverflow.in/8226

The binary operator ≠ is defined by the following truth table.

p	q	p ≠ q

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0	0	0
0	1	1
1	0	1
1	1	0

Which one of the following is true about the binary operator ≠?

A. Both commutative and associative
B. Commutative but not associative
C. Not commutative but associative
D. Neither commutative nor associative

gate2015-1 set-theory&algebra easy binary-operation

Answer

4.1.8 Binary Operation: GATE2015-3-2 top

https://gateoverflow.in/8393

Let $\#$ be the binary operator defined as

$X\#Y = X' + Y'$ where X and Y are Boolean variables.

Consider the following two statements.

- (S_1) $(P\#Q)\#R = P\#(Q\#R)$
- (S_2) $Q\#R = (R\#Q)$

Which are the following is/are true for the Boolean variables P, Q and R?

A. Only S_1 is true
B. Only S_2 is true
C. Both S_1 and S_2 are true
D. Neither S_1 nor S_2 are true

gate2015-3 set-theory&algebra binary-operation normal

Answer

Answers: Binary Operation

4.1.1 Binary Operation: GATE1989-1-v top

https://gateoverflow.in/87051

Selected Answer

Given the cardinality of the set $= n$.

Therefore the no: of entries in operation table (Cayley table) $= n^2$.

And hence if we consider lower triangular or upper triangular half , we have : $\frac{(n^2+n)}{2}$.

And in an operation table , each entry can be filled in n ways by any one element out of given n elements of the set.

So no. of ways we can fill the upper or lower triangular half $= n^{\frac{(n^2+n)}{2}}$

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Each of these is nothing but an instance of operation table of commutative operation as say (i,j) entry is filled in the table so (j,i) entry will also be the same hence the choice for (j,i) entry is constrained to 1 as we are concerned about commutative operation table here.

$\therefore$ ***No of possible binary operations which are commutative*** $= n^{\frac{(n^2+n)}{2}}$

19 votes -- HABIB MOHAMMAD KHAN (98.7k points)

4.1.2 Binary Operation: GATE1994-2.2 top

https://gateoverflow.in/2469

Selected Answer

Define Binary operation $*$ on (a,b) as : $a*b=a$

1. It is associative : $(a*b)*c=a*c=a$, and $a*(b*c)=a*b=a$

2. It is not commutative : $a*b=a$, whereas $b*a=b$.

21 votes -- Happy Mittal (11.2k points)

4.1.3 Binary Operation: GATE2003-38 top

https://gateoverflow.in/929

Selected Answer

Consider each pair

1. $(a,a):(a*a)+(a*a)=a+a=b\neq c$. So, (a,a) is not possible

2. $(a,b):(a*a)+(a*b)=a+b=a\neq c$. So, (a,b) is not possible

3. $(a,c):(a*a)+(a*c)=a+c=c$

   $(b*a)+(c*c)=b+b=b\neq c$. So, (a,c) is not possible

4. $(b,a):(a*b)+(a*a)=b+a=a\neq c$. So, (b,a) is not possible

5. $(b,b):(a*b)+(a*b)=b+b=b\neq c$. So, (b,b) is not possible

6. $(b,c):(a*b)+(a*c)=b+c=c$

   $(b*b)+(c*c)=c+b=c$. So, (b,c) is a solution

7. $(c,a):(a*c)+(a*a)=c+a=a\neq c$. So, (c,a) is not possible

8. $(c,b):(a*c)+(a*b)=c+b=c$

   $(b*c)+(c*b)=a+c=c$. So, (c,b) is a solution

9. $(c,c):(a*c)+(a*c)=c+c=b\neq c$. So, (c,c) is not possible

So, no. of possible solutions is 2.

20 votes -- Praveen Saini (54.5k points)

4.1.4 Binary Operation: GATE2006-28 top

https://gateoverflow.in/991

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Selected Answer

This question is easier to answer With Boolean Algebra.

$A \odot B \equiv B \rightarrow A$, i.e. $(\neg B \vee A)$.

Now, lets look at Option D

$$\begin{aligned}\neg(\neg A \odot B)\\ &\equiv \neg(B \rightarrow \neg A)\\ &\equiv \neg(\neg B \vee \neg A)\\ &\equiv B \wedge A\end{aligned}$$

So Answer is D.

Other options:

A. $\neg B \vee \neg A$
B. $\neg B \wedge \neg A$
C. $\neg B \wedge A$

21 votes -- Akash Kanase (42.5k points)

4.1.5 Binary Operation: GATE2006-IT-2 top

https://gateoverflow.in/3539

Selected Answer

Answer: A

I. Identity element is 3.
II. Identity element is 1.
III. There is no identity element. $(x^y \neq y^x$, when $x \neq y)$

26 votes -- Rajarshi Sarkar (34.1k points)

4.1.6 Binary Operation: GATE2013-1 top

https://gateoverflow.in/59

Selected Answer

Answer is (A) Commutative but not associative.

$y \oplus x = y^2 + x^2 = x \oplus y$. Hence, commutative.

$$(x \oplus y) \oplus z = (x^2 + y^2) \oplus z = (x^2 + y^2)^2 + z^2$$
$$x \oplus (y \oplus z) = x \oplus (y^2 + z^2) = x^2 + (y^2 + z^2)^2$$

So, $((x \oplus y) \oplus z) \neq (x \oplus (y \oplus z))$, hence not associative.

30 votes -- Arjun Suresh (348k points)

4.1.7 Binary Operation: GATE2015-1-28 top

https://gateoverflow.in/8226

Selected Answer

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option A : as it is XOR operation

17 votes -- GATERush (1.1k points)

4.1.8 Binary Operation: GATE2015-3-2 top

https://gateoverflow.in/8393

Selected Answer

Answer: B

$$(P\#Q)\#R = (P' + Q')\#R$$
$$= P.Q + R'$$

whereas,

$$P\#(Q\#R) = P' + (Q\#R)'$$
$$= P' + (Q' + R')'$$
$$= P' + QR$$

20 votes -- overtomanu (1.2k points)

4.2 Countable Set(1) top

4.2.1 Countable Set: GATE2018-27 top

https://gateoverflow.in/204101

Let N be the set of natural numbers. Consider the following sets,

- P : Set of Rational numbers (positive and negative)
- Q : Set of functions from $\{0,1\}$ to N
- R : Set of functions from N to $\{0,1\}$
- S : Set of finite subsets of N

Which of the above sets are countable?

A. Q and S only
B. P and S only
C. P and R only
D. P, Q and S only

gate2018 set-theory&algebra countable-set normal

Answer

Answers: Countable Set

4.2.1 Countable Set: GATE2018-27 top

https://gateoverflow.in/204101

Set of Rational numbers is countable whether positive or negative.

Reference: https://www.homeschoolmath.net/teaching/rational-numbers-countable.php

Set of function from $\{0,1\}$ to N is countable.

Reference: https://mafiadoc.com/homework-10-solutions-which-of-the-following-are-countable-_5a0331e71723dd63441df951.html

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Set of finite subsets of N is countable.

Reference: https://math.stackexchange.com/questions/908222/the-set-of-all-finite-subsets-of-the-natural-numbers-is-countable

But set of functions from N to $\{0,1\}$ is not countable.

Ans: D

8 votes -- Prashant Kumar (1.2k points)

4.3 Fields(1) top

4.3.1 Fields: GATE2008-IT-26 top

https://gateoverflow.in/3316

Consider the field C of complex numbers with addition and multiplication. Which of the following form(s) a subfield of C with addition and multiplication?

- **S1:** the set of real numbers
- **S2:** $\{(a+ib) \mid a \text{ and } b \text{ are rational numbers}\}$
- **S3:** $\{a+ib \mid (a^2+b^2) \leq 1\}$
- **S4:** $\{ia \mid a \text{ is real}\}$

A. only S1
B. S1 and S3
C. S2 and S3
D. S1 and S2

gate2008-it set-theory&algebra normal fields

Answer

Answers: Fields

4.3.1 Fields: GATE2008-IT-26 top

https://gateoverflow.in/3316

Selected Answer

A field is an algebraic structure over two operations $+$ and $*$ if :

1. It is closed under both $+$ and $*$.

2. $+$ and $*$ are both commutative and associative. $(+$ and $*$ in this question are already commutative and associative, so no need to check$)$

3. Existence of additive identity(0) and multiplicative identity (1)

4. Existence of additive and multiplicative inverses for each non-zero element.

5. Distributive property of $*$ over $+$ (This is also satisfied in question)

So, for each option, we have to check only properties $1, 3$ and 4.

(S1) : set of all real numbers

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1. Closed : Yes, $\text{real} + \text{real} = \text{real}, \text{real} * \text{real} = \text{real}$

3. Additive and multiplicative identity : Yes, 0 and 1 are real numbers

4. Additive and multiplicative inverse for each non-zero element: Yes, for any real number a, additive inverse is $-a$, which is also a real number, and multiplicative inverse is $1/a$, which is also a real number.

(S2) : $\{(a+ib) \mid a$ and b are rational numbers$\}$

1. Closed : Yes, $\text{rational} + \text{rational} = \text{rational}, \text{rational} * \text{rational} = \text{rational}$

3. Additive and multiplicative identity : Yes, $0+0i$ (additive identity) and $1+0i$ (multiplicative identity) belong to given set.

4. Additive and multiplicative inverse for each non-zero element : Additive inverse is $-a-ib$, which belongs to given set. Multiplicative identity is $\frac{1}{a+ib} = \frac{a-ib}{a^2+b^2} = \frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}$, which also belongs to given set.

(S3) : $\{a+ib \mid (a^2+b^2) \leq 1\}$

1. Closed : No, for example : $(0.3+0.4i)+(0.7+0.6i) = 1+i$. Here, both complex numbers which were added were in the given set, but resultant complex number is not.

(S4) : $\{ia \mid a$ is real$\}$

Here, this set does not contain 1 (multiplicative identity)

So, (S1) and (S2) are subfields of C. Option D) is correct.

24 votes -- Happy Mittal (11.2k points)

4.4 Functions(34) top

4.4.1 Functions: GATE1987-9b top

https://gateoverflow.in/82437

How many one-to-one functions are there from a set A with n elements onto itself?

gate1987 set-theory&algebra functions descriptive

Answer

4.4.2 Functions: GATE1988-13ii top

https://gateoverflow.in/94634

If the set S has a finite number of elements, prove that if f maps S onto S, then f is one-to-one.

gate1988 descriptive set-theory&algebra functions

Answer

4.4.3 Functions: GATE1989-13c top

https://gateoverflow.in/93178

Find the number of single valued functions from set A to another set B, given that the cardinalities of the sets A and B are m and n respectively.

gate1989 descriptive functions

Answer

4.4.4 Functions: GATE1993-8.6 top

https://gateoverflow.in/2304

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Let A and B be sets with cardinalities m and n respectively. The number of one-one mappings from A to B, when $m < n$, is

A. m^n
B. ${}^nP_m$
C. ${}^mC_n$
D. ${}^nC_m$
E. ${}^mP_n$

gate1993 set-theory&algebra functions easy

Answer

4.4.5 Functions: GATE1996-1.3 top

https://gateoverflow.in/2707

Suppose X and Y are sets and $|X|$ and $|Y|$ are their respective cardinality. It is given that there are exactly 97 functions from X to Y. From this one can conclude that

A. $|X| = 1, |Y| = 97$
B. $|X| = 97, |Y| = 1$
C. $|X| = 97, |Y| = 97$
D. None of the above

gate1996 set-theory&algebra functions normal

Answer

4.4.6 Functions: GATE1996-2.1 top

https://gateoverflow.in/2730

Let R denote the set of real numbers. Let $f : R \times R \rightarrow R \times R$ be a bijective function defined by $f(x, y) = (x + y, x - y)$. The inverse function of f is given by

A. $f^{-1}(x, y) = \left(\frac{1}{x+y}, \frac{1}{x-y}\right)$

B. $f^{-1}(x, y) = (x - y, x + y)$

C. $f^{-1}(x, y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$

D. $f^{-1}(x, y) = [2(x - y), 2(x + y)]$

gate1996 set-theory&algebra functions normal

Answer

4.4.7 Functions: GATE1997-13 top

https://gateoverflow.in/2273

Let F be the set of one-to-one functions from the set $\{1, 2, \ldots, n\}$ to the set $\{1, 2, \ldots, m\}$ where $m \geq n \geq 1$.

a. How many functions are members of F?

b. How many functions f in F satisfy the property $f(i) = 1$ for some $i, 1 \leq i \leq n$?

c. How many functions f in F satisfy the property $f(i) < f(j)$ for all $1 \leq i \leq j \leq n$?

gate1997 set-theory&algebra functions normal descriptive

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Answer

4.4.8 Functions: GATE1998-1.8 top

https://gateoverflow.in/1645

The number of functions from an m element set to an n element set is

A. $m+n$
B. m^n
C. n^m
D. $m*n$

gate1998 set-theory&algebra permutations-and-combinations functions easy

Answer

4.4.9 Functions: GATE2001-2.3 top

https://gateoverflow.in/721

Let $f: A \rightarrow B$ a function, and let E and F be subsets of A. Consider the following statements about images.

- $S1: f(E \cup F) = f(E) \cup f(F)$
- $S2: f(E \cap F) = f(E) \cap f(F)$

Which of the following is true about S1 and S2?

A. Only S1 is correct
B. Only S2 is correct
C. Both S1 and S2 are correct
D. None of S1 and S2 is correct

gate2001 set-theory&algebra functions normal

Answer

4.4.10 Functions: GATE2003-37 top

https://gateoverflow.in/927

Let $f: A \rightarrow B$ be an injective (one-to-one) function. Define $g: 2^A \rightarrow 2^B$ as: $g(C) = \{f(x) \mid x \in C\}$, for all subsets C of A.
Define $h: 2^B \rightarrow 2^A$ as: $h(D) = \{x \mid x \in A, f(x) \in D\}$, for all subsets D of B. Which of the following statements is always true?

A. $g(h(D)) \subseteq D$
B. $g(h(D)) \supseteq D$
C. $g(h(D)) \cap D = \phi$
D. $g(h(D)) \cap (B - D) \neq \phi$

gate2003 set-theory&algebra functions normal

Answer

4.4.11 Functions: GATE2003-39 top

https://gateoverflow.in/930

Let $\Sigma = \{a, b, c, d, e\}$ be an alphabet. We define an encoding scheme as follows:

$g(a) = 3, g(b) = 5, g(c) = 7, g(d) = 9, g(e) = 11.$

Let p_i denote the i-th prime number $(p_1 = 2)$.

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For a non-empty string $s = a_1 \ldots a_n$, where each $a_i \in \Sigma$, define $f(s) = \Pi_{i=1}^{n} P_i^{g(a_i)}$.

For a non-empty sequence $\langle s_j, \ldots, s_n \rangle$ of stings from Σ^+, define $h\left(\langle s_i \ldots s_n \rangle\right) = \Pi_{i=1}^{n} P_i^{f(s_i)}$

Which of the following numbers is the encoding, h, of a non-empty sequence of strings?

A. $2^7 3^7 5^7$

B. $2^8 3^8 5^8$

C. $2^9 3^9 5^9$

D. $2^{10} 3^{10} 5^{10}$

gate2003 set-theory&algebra functions normal

Answer

4.4.12 Functions: GATE2005-43 top

https://gateoverflow.in/1168

Let $f : B \rightarrow C$ and $g : A \rightarrow B$ be two functions and let $h = fog$. Given that h is an onto function which one of the following is TRUE?

A. f and g should both be onto functions
B. f should be onto but g need not to be onto
C. g should be onto but f need not be onto
D. both f and g need not be onto

gate2005 set-theory&algebra functions normal

Answer

4.4.13 Functions: GATE2005-IT-31 top

https://gateoverflow.in/3777

Let f be a function from a set A to a set B, g a function from B to C, and h a function from A to C, such that $h(a) = g(f(a))$ for all $a \in A$. Which of the following statements is always true for all such functions f and g?

A. g is onto $\implies$ h is onto
B. h is onto $\implies$ f is onto
C. h is onto $\implies$ g is onto
D. h is onto $\implies$ f and g are onto

gate2005-it set-theory&algebra functions normal

Answer

4.4.14 Functions: GATE2006-2 top

https://gateoverflow.in/881

Let X, Y, Z be sets of sizes x, y and z respectively. Let $W = X \times Y$ and E be the set of all subsets of W. The number of functions from Z to E is

A. $z^{2^{xy}}$
B. $z \times 2^{xy}$
C. $z^{2^{x+y}}$
D. 2^{xyz}

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gate2006 set-theory&algebra normal functions

Answer

4.4.15 Functions: GATE2006-25 top

https://gateoverflow.in/988

Let $S = \{1, 2, 3, \ldots, m\}, m > 3$. Let $X_1, \ldots, X_n$ be subsets of S each of size 3. Define a function f from S to the set of natural numbers as, $f(i)$ is the number of sets X_j that contain the element i. That is $f(i) = |\{j \mid i \in X_j\}|$ then $\sum_{i=1}^{m} f(i)$ is:

A. $3m$
B. $3n$
C. $2m + 1$
D. $2n + 1$

gate2006 set-theory&algebra normal functions

Answer

4.4.16 Functions: GATE2006-IT-6 top

https://gateoverflow.in/3545

Given a boolean function $f(x_1, x_2, \ldots, x_n)$, which of the following equations is NOT true?

A. $f(x_1, x_2, \ldots, x_n) = x_1' f(x_1, x_2, \ldots, x_n) + x_1 f(x_1, x_2, \ldots, x_n)$
B. $f(x_1, x_2, \ldots, x_n) = x_2 f(x_1, x_2, \ldots, x_n) + x_2' f(x_1, x_2, \ldots, x_n)$
C. $f(x_1, x_2, \ldots, x_n) = x_n' f(x_1, x_2, \ldots, 0) + x_n f(x_1, x_2, \ldots, 1)$
D. $f(x_1, x_2, \ldots, x_n) = f(0, x_2, \ldots, x_n) + f(1, x_2, \ldots, x_n)$

gate2006-it set-theory&algebra functions normal

Answer

4.4.17 Functions: GATE2007-3 top

https://gateoverflow.in/1202

What is the maximum number of different Boolean functions involving n Boolean variables?

A. n^2
B. 2^n
C. 2^{2^n}
D. 2^{n^2}

gate2007 permutations-and-combinations functions normal

Answer

4.4.18 Functions: GATE2012-37 top

https://gateoverflow.in/1759

How many onto (or surjective) functions are there from an n-element $(n \geq 2)$ set to a 2-element set?

A. 2^n
B. $2^n - 1$
C. $2^n - 2$
D. $2(2^n - 2)$

gate2012 set-theory&algebra functions normal

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Answer

4.4.19 Functions: GATE2014-1-50 top

https://gateoverflow.in/1930

Let S denote the set of all functions $f : \{0,1\}^4 \to \{0,1\}$. Denote by N the number of functions from S to the set $\{0,1\}$. The value of $\log_2 \log_2 N$ is _____.

gate2014-1 set-theory&algebra functions permutations-and-combinations numerical-answers

Answer

4.4.20 Functions: GATE2014-3-2 top

https://gateoverflow.in/2036

Let X and Y be finite sets and $f : X \to Y$ be a function. Which one of the following statements is TRUE?

A. For any subsets A and B of $X, |fA \cup B| = |f(A)| + |f(B)|$
B. For any subsets A and B of $X, f(A \cap B) = f(A) \cap f(B)$
C. For any subsets A and B of $X, |f(A \cap B)| = \min\{|f(A)|, |f(B)|\}$
D. For any subsets S and T of $Y, f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

gate2014-3 set-theory&algebra functions normal

Answer

4.4.21 Functions: GATE2014-3-49 top

https://gateoverflow.in/2083

Consider the set of all functions $f : \{0, 1, \ldots, 2014\} \to \{0, 1, \ldots, 2014\}$ such that $f(f(i)) = i$, for all $0 \leq i \leq 2014$. Consider the following statements:

P. For each such function it must be the case that for every $i, f(i) = i$.

Q. For each such function it must be the case that for some $i, f(i) = i$.

R. Each function must be onto.

Which one of the following is CORRECT?

A. P, Q and R are true
B. Only Q and R are true
C. Only P and Q are true
D. Only R is true

gate2014-3 set-theory&algebra functions normal

Answer

4.4.22 Functions: GATE2015-1-39 top

https://gateoverflow.in/8294

Consider the operations

$f\ (X,\ Y,\ Z) = X'YZ + XY' + Y'Z'$and $g\ (X,\ Y,\ Z) = X'YZ + X'YZ' + XY$

Which one of the following is correct?

A. Both $\{f\}$ and $\{g\}$ are functionally complete
B. Only $\{f\}$ is functionally complete

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C. Only $\{g\}$ is functionally complete
D. Neither $\{f\}$ nor $\{g\}$ is functionally complete

gate2015-1 set-theory&algebra functions difficult

Answer

4.4.23 Functions: GATE2015-1-5 top

https://gateoverflow.in/8025

If $g(x) = 1 - x$ and $h(x) = \frac{x}{x-1}$, then $\frac{g(h(x))}{h(g(x))}$ is:

A. $\frac{h(x)}{g(x)}$
B. $\frac{-1}{x}$
C. $\frac{g(x)}{h(x)}$
D. $\frac{x}{(1-x)^2}$

gate2015-1 set-theory&algebra functions normal

Answer

4.4.24 Functions: GATE2015-2-40 top

https://gateoverflow.in/8212

The number of onto functions (surjective functions) from set $X = \{1, 2, 3, 4\}$ to set $Y = \{a, b, c\}$ is _____.

gate2015-2 set-theory&algebra functions normal numerical-answers

Answer

4.4.25 Functions: GATE2015-2-54 top

https://gateoverflow.in/8257

Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denote the set of all possible functions defined from X to Y. Let f be randomly chosen from F. The probability of f being one-to-one is _____.

gate2015-2 set-theory&algebra functions normal numerical-answers

Answer

4.4.26 Functions: GATE2015-2-GA-3 top

https://gateoverflow.in/8030

Consider a function $f(x) = 1 - |x| \text{ on } -1 \leq x \leq 1$. The value of x at which the function attains a maximum, and the maximum value of the function are:

A. $0, -1$
B. $-1, 0$
C. $0, 1$
D. $-1, 2$

gate2015-2 set-theory&algebra functions normal

Answer

4.4.27 Functions: GATE2015-2-GA-9 top

https://gateoverflow.in/8040

If p, q, r, s are distinct integers such that:

$$f(p, q, r, s) = \max(p, q, r, s)$$

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$g(p,q,r,s) = \min(p,q,r,s)$

$h(p,q,r,s) = \text{remainder of } \frac{(p\times q)}{(r\times s)} \text{ if } (p\times q) > (r\times s)$
$\text{or remainder of } \frac{(r\times s)}{(p\times q)} \text{ if } (r\times s) > (p\times q)$

Also a function $fgh(p,q,r,s) = f(p,q,r,s) \times g(p,q,r,s) \times h(p,q,r,s)$
Also the same operations are valid with two variable functions of the form $f(p,q)$
What is the value of $fg(h(2,5,7,3),4,6,8)$?

gate2015-2 set-theory&algebra functions normal numerical-answers

Answer

4.4.28 Functions: GATE2016-1-28 top

https://gateoverflow.in/39717

A function $f:\mathbb{N}^+ \to \mathbb{N}^+$, defined on the set of positive integers $\mathbb{N}^+$,satisfies the following properties:

$$f(n) = f(n/2) \quad \text{if } n \text{ is even}$$

$$f(n) = f(n+5) \quad \text{if } n \text{ is odd}$$

Let $R = \{i \mid \exists j : f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is _____.

gate2016-1 set-theory&algebra functions normal numerical-answers

Answer

4.4.29 Functions: ISI2017-MMA-24 top

https://gateoverflow.in/214015

The number of polynomial function f of degree ≥ 1 satisfying

$$f(x^2) = (f(x))^2 = f(f(x))$$

for all real x, is

A. 0
B. 1
C. 2
D. infinitely many

isi2017 functions

Answer

4.4.30 Functions: TIFR2012-B-1 top

https://gateoverflow.in/25046

For $x, y \in \{0,1\}^n$, let $x \oplus y$ be the element of $\{0,1\}^n$ obtained by the component-wise exclusive-or of x and y. A Boolean function $F:\{0,1\}^n \to \{0,1\}$ is said to be linear if $F(x\oplus y) = F(x) \oplus F(y)$, for all x and y. The number of linear functions from $\{0,1\}^n$ to $\{0,1\}$ is.

a. 2^{2n}
b. 2^{n+1}
c. $2^{n-1}+1$

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d. $n!$
e. 2^n

tifr2012 set-theory&algebra functions

Answer

4.4.31 Functions: TIFR2013-B-16 top

https://gateoverflow.in/25859

Consider a function $T_{k,n} : \{0,1\}^n \rightarrow \{0,1\}$ which returns 1 if at least k of its n inputs are 1. Formally, $T_{k,n}(x) = 1$ if $\sum_1^n x_i \geq k$. Let $y \in \{0,1\}^n$ be such that y has exactly k ones. Then, the function $T_{k,n-1}\left(y_1, y_2, \ldots . y_{i-1}, y_{i+1}, \ldots, y_n\right)$ (where y_i is omitted) is equivalent to

a. $T_{k-1}, n(y)$
b. $T_{k,n}(y)$
c. y_i
d. $\neg y_i$
e. None of the above.

tifr2013 set-theory&algebra functions

Answer

4.4.32 Functions: TIFR2014-B-17 top

https://gateoverflow.in/27344

Let $f : \{0,1\}^n \rightarrow \{0,1\}$ be a boolean function computed by a logical circuit comprising just binary AND and binary OR gates (assume that the circuit does not have any feedback). Let PARITY : $\{0,1\}^n \rightarrow \{0,1\}$ be the boolean function that outputs 1 if the total number of input bits set to 1 is odd. Similarly, let MAJORITY be the boolean function that outputs 1 if the number of input bits that are set to 1 is at least as large as the number of input bits that are set to 0. Then, which of the following is NOT possible?

a. $f(0,0,\ldots,0) = f(1,1,\ldots,1) = 0.$
b. $f(0,0,\ldots,0) = f(1,1.\ldots,1) = 1$
c. f is the MAJORITY function.
d. f is the PARITY function.
e. f outputs 1 at exactly one assignment of the input bits.

tifr2014 set-theory&algebra functions

Answer

4.4.33 Functions: TIFR2014-B-18 top

https://gateoverflow.in/27351

Let k be an integer at least 4 and let $[k] = \{1,2,\ldots,k\}$. Let $f : [k]^4 \rightarrow \{0,1\}$ be defined as follows: $f(y_1, y_2, y_3, y_4) = 1$ if an only if the $y_i's$ are all distinct. For each choice $z = (z_1, z_2, z_3) \in [k]^3$, let $g_z : [k] \rightarrow \{0,1\}$ be defined by $g_z(Y) = f(Y, z_1, z_2, z_3)$. Let N be the number of distinct functions g_z that are obtained as z varies in $\{1,2,\ldots,k\}^3$, that is, $N = \left| \left\{ g_z : z \in \{1,2,\ldots,k\}^3 \right\} \right|$. What is N?

a. $k^3 + 1$
b. $2^{\left(\frac{k}{3}\right)}$
c. $\left(\frac{k}{3}\right)$
d. $\left(\frac{k}{3}\right) + 1$

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e. $4\left(\frac{k}{3}\right)$

tifr2014 set-theory&algebra functions

Answer

4.4.34 Functions: TIFR2017-A-11 top

https://gateoverflow.in/95289

Let $f \circ g$ denote function composition such that $(f \circ g)(x) = f(g(x))$. Let $f : A \to B$ such that for all $g : B \to A$ and $h : B \to A$ we have $f \circ g = f \circ h \Rightarrow g = h$. Which of the following must be true?

A. f is onto (surjective)
B. f is one-to-one (injective)
C. f is both one-to-one and onto (bijective)
D. the range of f is finite
E. the domain of f is finite

tifr2017 set-theory&algebra functions

Answer

Answers: Functions

4.4.1 Functions: GATE1987-9b top

https://gateoverflow.in/82437

Selected Answer

There are $n!$ one to one function possible from a set of n elements to itself.

i.e., $P\binom{n}{n} = n!$

11 votes -- Prashant Singh (59.8k points)

4.4.2 Functions: GATE1988-13ii top

https://gateoverflow.in/94634

Selected Answer

Let set $S = \{1, 2, 3, 4\}$.
Now see the mapping from S to S.
For f to be onto every element of codomain must be mapped by every element in domain.
Since, cardinality is same for both domain and codomain, we can not have mapping like $f(1) = 1, f(2) = 1$ because if it happened then at least one element remain unmapped in codomain, which result in f being not onto but it is given that f is onto. So every element in codomain has exactly one element in domain. Thus f must be an one-to-one function.

NOTE: If S is infinite then this result may not be true.

13 votes -- junaid ahmad (8.8k points)

4.4.3 Functions: GATE1989-13c top

https://gateoverflow.in/93178

Selected Answer

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A single-valued function is an emphatic term for a mathematical function in the usual sense. That is, each element of the function's domain maps to a single, well-defined element of its range. By default, we always consider function as a single valued function except when clearly mentioned that function is a multi-valued function.

So, the number of single valued functions from A to $B =$ number of functions from A to B.

Lets take an example:

$A = \{1, 2\}, B = \{a, b\}$

1. $f(1) = f(2) = a$
2. $f(1) = f(2) = b$
3. $f(1) = a$ and $f(2) = b$
4. $f(1) = b$ and $f(2) = a$

The total number of single valued functions from set A to another set $B = |B|^{|A|} = n^m$.
This is because for every element in A we have $|B|$ possibilities in the function.

The correct answer is n^m.

7 votes -- Warrior (6.3k points)

4.4.4 Functions: GATE1993-8.6 top

https://gateoverflow.in/2304

Selected Answer

Answer: B
Ref: Page 33 of http://www.cs.toronto.edu/~stacho/macm101-2.pdf

16 votes -- Rajarshi Sarkar (34.1k points)

4.4.5 Functions: GATE1996-1.3 top

https://gateoverflow.in/2707

Selected Answer

We can say $|Y|^{|X|} = 97$. Only option A satisfies this. Still, this can be concluded only because 97 is a prime number and hence no other power gives 97.

20 votes -- Arjun Suresh (348k points)

4.4.6 Functions: GATE1996-2.1 top

https://gateoverflow.in/2730

Selected Answer

to find inverse of the function take

$z_1 = x + y \rightarrow (1)$

$z_2 = x - y \rightarrow (2)$

Adding (1) and (2) we get,

$x = \frac{z_1 + z_2}{2}$ and $y = \frac{z_1 - z_2}{2}$

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So, $f\left(\frac{z_1}{2}, \frac{z_2}{2}\right) = \left(\frac{z_1+z_2}{2}, \frac{z_1-z_2}{2}\right) = (x, y) \implies f^{-1}(x, y) = \left(\frac{z_1}{2}, \frac{z_2}{2}\right) = \left\{\frac{x+y}{2}, \frac{x-y}{2}\right\}$

24 votes -- neha pawar (4.2k points)

Taking an example:

$f(2, 3) = (2 + 3, 2 - 3) = (5, -1)$

$f^{-1}(5, -1)$ should be $(2, 3)$.

Substituting the values we get (C) as answer.

29 votes -- Kumar Shikhar Deep (1.3k points)

4.4.7 Functions: GATE1997-13 top

https://gateoverflow.in/2273

Selected Answer

(a) A function from A to B must map every element in A. Being one-one, each element must map to a unique element in B. So, for n elements in A, we have m choices in B and so we can have ${}^{m}\mathbb{P}_{n}$ functions.

(b) Continuing from (a) part. Here, we are forced to fix $f(i) = 1$. So, one element from A and B gone with n possibilities for the element in A and 1 possibility for that in B, and we get $n \times {}^{m-1}\mathbb{P}_{n-1}$ such functions.

(c) $f(i) < f(j)$ means only one order for the n selection permutations from B is valid. So, the answer from (a) becomes ${}^{m}\mathbb{C}_{n}$ here.

19 votes -- Arjun Suresh (348k points)

4.4.8 Functions: GATE1998-1.8 top

https://gateoverflow.in/1645

Selected Answer

No. of functions from an m element set to an n element set is n^m as for each of the m element, we have n choices to map to, giving $\underbrace{n \times n \times \ldots n}_{m \text{ times}} = n^m$.

PS: Each element of the domain set in a function must be mapped to some element of the co-domain set.

20 votes -- Digvijay (54.8k points)

4.4.9 Functions: GATE2001-2.3 top

https://gateoverflow.in/721

Selected Answer

Say $E = \{1, 2\}$ and $F = \{3, 4\}$.

- $f(1) = a$
- $f(2) = b$
- $f(3) = b$
- $f(4) = d$

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$f(E \cup F) = f(1,2,3,4) = \{a,b,d\}$
$f(E) \cup f(F) = f(1,2) \cup f(3,4) = \{a,b\} \cup \{b,d\} = \{a,b,d\}$

Now, $E \cap F = \emptyset$
$f(E \cap F) = f(\emptyset) = \emptyset$

But, $f(E) \cap f(F) = f(1,2) \cap f(3,4) = \{a,b\} \cap \{b,d\} = \{b\}$

So, S2 is not true. S1 is always true (no counter example exists)

27 votes -- srestha (86.6k points)

4.4.10 Functions: GATE2003-37 top

https://gateoverflow.in/927

Selected Answer

$f : A \to B$ is a one to one function. Every element in A will have a corresponding element in B. Therefore, the size of range for this is n(A) and n(B) $\geq$ n(A).

$g : 2^A \to 2^B, g(C) = \{f(x) \mid x \in C\}$, since f is one to one, for every subset of A there will be corresponding subset of B. Therefore this is also a one to one function and size of range for this is n(2^A).

$h : 2^B \to 2^A, h(D) = \{x \mid x \in A, f(x) \in D\}$ this function is not a one to one function. Every subset of B will be mapped to subset of A for which it has all the images of subset of A. size of range for this function will be n(2^A)

That said, now $g(h(D))$ will also have the range of size n(2^A). Since $n(A) \leq n(b)$, n(2^A) must be lest than or equal to n(2^B). The answer is $g(h(D)) \subseteq D$.

For example let $A = \{1,2\}$ and $B = \{a,b,c\}$. Let $f(1) = a, f(2) = b$. Now,

$g(\{\}) = \{\}$

$g(\{1\}) = \{a\}$

$g(\{2\}) = \{b\}$

$g(\{1,2\}) = \{a,b\}$

$h(\{\}) = \{\}$

$h(\{a\}) = \{1\}$

$h(\{b\}) = \{2\}$

$h(\{c\}) = \{\}$

$h(\{a,b\}) = \{1,2\}$

$h(\{a,c\}) = \{1\}$

$h(\{b,c\}) = \{2\}$

$h(\{a,b,c\}) = \{1,2\}$

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Now we can see that for any $D \subseteq B, g(h(D)) \subseteq D$. Had the function f been bijective (one-one and onto or one-one and co-domain = range) , then we would have got $g(h(D)) = D$.

27 votes -- Mari Ganesh Kumar (2k points)

4.4.11 Functions: GATE2003-39 top

https://gateoverflow.in/930

Selected Answer

It is clear from the choices that there are 3 strings in the sequence as we have the first 3 prime numbers in the product. Now, in $f(s)$ the first term is 2^x for some x, so, A and C choices can be eliminated straight away as neither 7 nor 9 is a multiple of 2.

The sequence of strings are "a", "a" and "a"

$f(a) = 2^3 = 8$. So, we get $2^8 3^8 5^8$ as per the definition of h.

17 votes -- Arjun Suresh (348k points)

4.4.12 Functions: GATE2005-43 top

https://gateoverflow.in/1168

Selected Answer

B. g need not be onto.

Let,

$A = \{0, 1, 2\}, B = \{0, 3, 4, 25\}, C = \{3, 4, 5\}$

$f = \{(0, 3), (3, 5), (4, 4), (25, 3)\}$

$g = \{(1, 3), (2, 4), (0, 0)\}$ (25 in B not mapped to by g, hence g is not ONTO)

$h = \{(0, 3), (1, 5), (2, 4)\}$

Now, h is an onto function but g is not.

f must be an onto function as otherwise we are sure to miss some elements in range of h making it not onto.

25 votes -- Arjun Suresh (348k points)

4.4.13 Functions: GATE2005-IT-31 top

https://gateoverflow.in/3777

Selected Answer

Let h be onto (onto means co-domain = range). So, h maps to every element in C from A. Since $h(a) = g(f(a)), g$ should also map to all elements in C. So, g is also onto $\implies$ option (C).

20 votes -- Arjun Suresh (348k points)

4.4.14 Functions: GATE2006-2 top

https://gateoverflow.in/881

Selected Answer

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D is Correct.

$|E| = 2^{xy}$ which is the number of subsets of W.

Now, the mapping for a function from A to B with N and M elements respectively can be done in M^N ways.

Here,

$|E|^z = \{2^{xy}\}^z = 2^{xyz}$

25 votes -- Snehil Joshi (279 points)

4.4.15 Functions: GATE2006-25 top

https://gateoverflow.in/988

Selected Answer

$|S| = m$

No.of three elements subset $= n = \binom{m}{3}$

Function $f(i)$ is defined as No. of three elements subset which contain element i.

Without loss of generality, lets assume $i = 1$

Now we need to count three elements subset which contain element 1.
All those subset will be of the form $\{1, a, b\}$, where a and b are distinct and not equal to 1.

We can choose all those a and b in $\binom{m-1}{2}$ ways

So, $f(1) = \binom{m-1}{2}$, which is true for any i
Therefore $f(i) = \binom{m-1}{2}$

Now,

$$\sum_{i=1}^{m} f(i) = \sum_{i=1}^{m} \binom{m-1}{2}$$

$$= m\binom{m-1}{2} = 3\binom{m}{3} = 3n$$

3 votes -- Lakpa Tashi Bhutia (685 points)

Alternative Approach:

Total elements in $S = m$

Total number of subsets of size 3 each can be ${}^{m}c_3$.

Now suppose take 1st element 1. Out of ${}^{m}c_3$ subsets 1 wont be there in ${}^{(m-1)}c_3$ subsets.

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So 1 will be there in ${}^{m}c_3 - {}^{(m-1)}c_3 = \frac{(m-1)(m-2)}{2}$ subsets.

$\sum f(i) = \sum \frac{(m-1)(m-2)}{2} = \frac{m(m-1)(m-2)}{2}$.

We know, ${}^{m}c_3 = n$ (No. of X subset) $\quad \therefore \frac{m(m-1)(m-2)}{2} = 3n$.

45 votes -- Madhur Rawat (2.3k points)

4.4.16 Functions: GATE2006-IT-6 top

https://gateoverflow.in/3545

Selected Answer

Answer: D

Proceed by taking $f(x_1) = x_1$

LHS: $f(x_1) = 0$ when $x_1 = 0$

LHS: $f(x_1) = 1$ when $x_1 = 1$

RHS: $f(0) + f(1) = 0 + 1 =$ always 1

20 votes -- Rajarshi Sarkar (34.1k points)

4.4.17 Functions: GATE2007-3 top

https://gateoverflow.in/1202

Selected Answer

answer - C

size of domain = number of different combinations of inputs $= 2^n$.

size of codomain $= 2(\{0,1\})$.

number of functions $= (\text{size of co-domain})^{(\text{size of domain})}$

28 votes -- Ankit Rokde (9k points)

4.4.18 Functions: GATE2012-37 top

https://gateoverflow.in/1759

Selected Answer

No. onto (or surjective) functions are there from an n-element $(n \geq 2)$ set to a 2-element set $=$ Total No of functions $-$ (No of functions with 1 element from RHS not mapped) $+$ (No of functions with 2 element from RHS not mapped)...(So on Using Inclusion Excusion principle $= 2^n$ (Total no of functions) $- 2 * 1^n$ (No of functions in which one element is excluded) $+0$ (No element in RHS is selected) $= 2^n - 2$.

Hence Ans is (C).

alternate

https://gateoverflow.in/8212/gate2015-2_40

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23 votes -- Akash Kanase (42.5k points)

4.4.19 Functions: GATE2014-1-50 top

https://gateoverflow.in/1930

Selected Answer

For a function from set A to set B, we need to have a mapping for all elements of A and mapping must be unique.
Let number of elements in A be m and that in B be n

So, if we consider an element from A, it can be mapped to any of the element from B. i.e., it has n possibilities when a function is formed. Similarly, for all other members also there are n possibilities as one element from A can be mapped to only a single element in B (though reverse need not true). So, for n elements in A, we totally have $\underbrace{n \times \cdots \times n}_{m \text{ times}} = n^m$ possible functions.

In the question Number of elements (functions) in f is 2^{2^4} as $\{0,1\}^4$ contains 2^4 elements. So, number of functions from S to $\{0,1\}$ will be $2^{2^{2^4}}$. So, $\log_2 \log_2 N = 2^4 = 16$.

51 votes -- Arjun Suresh (348k points)

4.4.20 Functions: GATE2014-3-2 top

https://gateoverflow.in/2036

Selected Answer

D is answer.

3 out of 4 options can be eliminated with the help of a counter example.

Let $X = \{a, b, c\}$ and $Y = \{1, 2\}$
A Function f maps each element of X to exactly one element in Y.
Let $f(a) = 1, f(b) = 1, f(c) = 1$ and $A = \{a\}, B = \{b, c\}$
A)
LHS: $|f(A \cup B)| = |f(\{a, b, c\})| = |\{1\}| = 1$

RHS: $|f(A)| + |f(B)| = 1 + 1 = 2$,

LHS != RHS.
B)
LHS: $f(A \cap B) = f(\{\}) = \{\}$.

RHS: $f(A) \cap f(B) = \{1\} \cap \{1\} = \{1\}$

LHS!=RHS.
C)
LHS: $|f(A \cap B)| = |f(\{\})| = |\{\}| = 0$
RHS: $\min\{|f(A)|, |f(B)|\} = \min(1, 1) = 1$

LHS!=RHS.
D) Its easy to see that this is true because in a function a value can be mapped only to one value. The option assumes inverse of function f exists.

40 votes -- Srinath Jayachandran (3.7k points)

4.4.21 Functions: GATE2014-3-49 top

https://gateoverflow.in/2083

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Selected Answer

Let $f(i) = j$. Now, we have $f(j) = i$, as per the given condition $f(f(i)) = i$.

For any $i \neq j$, we can have a mapping $f(i) = j, f(j) = i$ thus avoiding the condition $f(i) = i$. But the domain containing odd number of elements, at least for one element we must have $f(i) = i$. So, Q must be TRUE.

Since $f(i) = j$ and $f(j) = i$, and since $0 \leq i \leq 2014$ i must take all 2015 possible values (since f is a function, it must have a value for any element in the domain). We can easily see that $f(i)$ cannot be the same for two different is- because suppose $f(2) = 5$, and $f(3) = 5$. Now as per given condition, $f(5) = 2$ and $f(5) = 3$, which is not allowed in a function. So, all $f(i)$ values are unique $\implies$ co-domain = range as there are only 2015 values in co-domain. So, R is true.

An identity function satisfies the given conditions. But that alone cant prove that P is true. We can also have a different function where all even numbers maps to the next odd number and all odd numbers map to the previous even number which satisfies the given conditions, except the last one as we have odd number in set. i.e.,
$f(0) = 1, f(1) = 0, f(2) = 3, f(3) = 2 \ldots, f(2013) = 2012, f(2014) = 2014$

This proves, P is false.

So, (B) is the answer.

46 votes -- Arjun Suresh (348k points)

4.4.22 Functions: GATE2015-1-39 top

https://gateoverflow.in/8294

Selected Answer

g is preserving 0 as when all inputs are zero, output is always 0 and so g cannot be functionally complete.

f is not preserving 0.
f is not preserving 1. (when all inputs are 1, output is 0).
f is not linear as in XY' only one (odd) input $(X = 1, Y = Z = 0)$ needs to be 1 and in $X'YZ$ two inputs (even) $(X = 0, Y = Z = 1)$ need to be 1.
f is not monotone as changing Y from 0 to 1, can take f from 1 to 0.
f is not self dual as $f(X, Y, Z) \neq f(X, Y, Z)$

So, f satisfies all 5 conditions required for functional completeness and hence B is the answer.

http://cs.ucsb.edu/~victor/ta/cs40/posts-criterion.pdf

60 votes -- Arjun Suresh (348k points)

4.4.23 Functions: GATE2015-1-5 top

https://gateoverflow.in/8025

Selected Answer

option a) is correct.

$$g(h(x)) = g\left(\frac{x}{x-1}\right)$$
$$= 1 - \frac{x}{x-1}$$

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$= \frac{-1}{x-1}$

$h(g(x)) = h(1-x)$
$= \frac{1-x}{-x}$

$\frac{g(h(x))}{h(g(x))} = \frac{x}{(1-x)(x-1)} = \frac{h(x)}{g(x)}$

option A)

21 votes -- GateMaster Prime (1.5k points)

4.4.24 Functions: GATE2015-2-40 top

https://gateoverflow.in/8212

Selected Answer

We have 3 elements in set B and 4 elements in set A and surjection means every element in B must be mapped to. So, this problem reduces to distributing 4 distinct elements $(r=4)$ among 3 distinct bins $(n=3)$ such that no bin is empty, which is given by $n!S(r,n)$, where $S(r,n)$ is Stirling's number of 2nd kind. So, here we need $S(4,3)$.

We have $S(r+1,n) = n * S(r,n) + S(r,n-1)$

So, Stirling numbers of second kind can be generated as follows:

1

$1 \quad 1$

$1 \quad 3 \quad 1$

$1 \quad 7 \quad 6 \quad 1$

So, $S(4,3) = 6$ and $3! = 6$ giving, number of surjective functions $= 6 * 6 = 36$.

Ref: See Theorem 9:

http://www.cse.iitm.ac.in/~theory/tcslab/mfcs98page/mfcshtml/notes1/partset.html

---

Alternative approach ,

Answer is 36.
For onto function from a set A(m-element) to a set B(n-element), $m \geq n$.

Number of onto function
$= n^m -^n C_1(n-1)^m +^n C_2(n-2)^m -^n C_3(n-3)^m + \ldots +^n C_n(n-n)^m$

$(+, -$ alternative$)$

$$= \sum_{i=0}^{n} (-1)^i \, nC_i \, (n-i)^m$$

Here $m = 4$ and $n = 3$
So, number of onto functions

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$$= 3^4 -^3 C_1(3-1)^4 +^3 C_2(3-2)^4 -^3 C_3(3-3)^4$$
$$= 81 - 3*16 + 3*1 - 1*0$$
$$= 36.$$

ref@ http://www.cse.iitd.ac.in/~mittal/stirling.html

35 votes -- Arjun Suresh (348k points)

Alternatively this is equivalent to putting 4 different balls into 3 different boxe

Such that each box contain atleast one ball

So Possible arrangements as $(2,1,1)$ and its Permutation .

So Total no. of ways $= \binom{4}{2} \times \binom{2}{1} \times \binom{1}{1} \times 3 = 36$

28 votes -- Jagdish Singh (521 points)

4.4.25 Functions: GATE2015-2-54 top

https://gateoverflow.in/8257

Selected Answer

For a function, the first element in X has 20 choices (to map to) and the second element also has 20 choices. For a one-to-one function the second element has only 19 choices left after 1 being taken by the first. So, required probability

$$= \frac{(20\times 19)}{(20\times 20)} = 0.95$$

32 votes -- Vikrant Singh (13.5k points)

4.4.26 Functions: GATE2015-2-GA-3 top

https://gateoverflow.in/8030

Selected Answer

Here in diagram we can clearly see that,
At $x = 0$, $f(x)$ would be maximum which is 1.

Option C is correct.

Alternate Approach -
Put the value of x of all the options in $f(x)$ and find the value of $f(x)$.

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6 votes -- Ramjilal choudhary (347 points)

4.4.27 Functions: GATE2015-2-GA-9 top

https://gateoverflow.in/8040

Selected Answer

It is given that $h(p,q,r,s) =$ remainder of $\frac{(p\times q)}{(r\times s)}$ if $(p\times q) > (r\times s)$ or remainder of $\frac{(r\times s)}{(p\times q)}$ if $(r\times s) > (p\times q)$.

$h(2,5,7,3) =$ remainder of $\frac{(7\times 3)}{(2\times 5)}$, $\because (7\times 3) > (2\times 5)$

Thus, $h(2,5,7,3) = 1$

Again, it is given that $fg(p,q,r,s) = f(p,q,r,s) \times g(p,q,r,s)$

Also, $f(p,q,r,s) = \max(p,q,r,s)$, and $g(p,q,r,s) = \min(p,q,r,s)$

So we have:

$fg(1,4,6,8) = 8\times 1, \ \because \max(1,4,6,8) = 8 \ \& \ \min(1,4,6,8) = 1$

Thus, $fg(1,4,6,8) = 8$

Answer: 8

23 votes -- Shyam Singh (1.6k points)

4.4.28 Functions: GATE2016-1-28 top

https://gateoverflow.in/39717

Selected Answer

```
Let us assume: f(1) = x. Then,
 f(2) = f(2/2) = f(1) = x
 f(3) = f(3+5) = f(8) = f(8/2) = f(4/2) = f(2/1) = f(1) = x
Similarly, f(4) = x
  f(5) = f(5+5) = f(10/2) = f(5) = y.
```

So, it will have two values. All multiples of 5 will have value y and others will have value x.

70 votes -- Kumar Shikhar Deep (1.3k points)

4.4.29 Functions: ISI2017-MMA-24 top

https://gateoverflow.in/214015

Selected Answer

Consider the equality $(f(x))^2 = f(f(x))$.

Let $f(x) = a_0x^n + a_1x^{(n-1)} + a_2x^{(n-2)} + a_3x^{(n-3)} + \ldots + a_n$

Note that the degree of $(f(x))^2 = 2n$ and the degree of $f(f(x)) = n^2$

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Since $(f(x))^2 = f(f(x))$, we have $2n = n^2$.
$\implies n = 2.$

So, $f(x) = a_0x^2 + a_1x + a_2$

Now, we have the equality $f(x^2) = (f(x))^2$

Now, $f(x^2) = a_0x^4 + a_1x^2 + a_2$ and
$(f(x))^2 = a_0^2x^4 + 2a_0a_1x^3 + (2a_0a_2 + a_1^2)x^2 + 2a_1a_2x + a_2^2.$

Now we have,

$$a_0x^4 + a_1x^2 + a_2 = a_0^2x^4 + 2a_0a_1x^3 + (2a_0a_2 + a_1^2)x^2 + 2a_1a_2x + a_2^2 \quad \rightarrow (1)$$

Now the coefficient of x^3 in L.H.S. is 0 and the coefficient of x^3 in R.H.S. is $2a_0a_1$.

So, $2a_0a_1 = 0.$

Hence, $a_1 = 0$ (because a_0 can not be 0 since it is the leading coefficient)

Now we have by putting $a_1 = 0$ in equation (1)

$$a_0x^4 + a_2 = a_0^2x^4 + 2a_0a_2x^2 + a_2^2 \quad \rightarrow (2)$$

Similarly, by comparing the coefficients of x^2 on both sides we get $a_2 = 0.$

So, equation (2) becomes

$a_0x^4 = a_0^2x^4$
$\implies a_0 = a_0^2$
$\implies a_0 = 1.$

So, the required polynomial function is $f(x) = x^2$

Hence, there is only one polynomial which x^2

So, option B is correct.

2 votes -- Kushagra Chatterjee (8.2k points)

4.4.30 Functions: TIFR2012-B-1 top

https://gateoverflow.in/25046

Selected Answer

Take an example

Suppose the function f is from $\{0,1\}^3$ to $\{0,1\}$

Now, here we have to find the number of functions possible from $\{0,1\}^3$ to $\{0,1\}$ such that

$$f(x \oplus y) = f(x) \oplus f(y)$$

Now, observe that if we maintain the linearity of XOR then the values

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$f(000), f(101), f(110), f(011)$ and $f(111)$ are dependent on the values $f(100), f(010)$ and $f(001)$ (Reason given below)

So, if we fix the values of $f(100), f(010)$ and $f(001)$ then we will get the whole function.

Now, each of $f(100), f(010)$ and $f(001)$ has 2 options, either 0 or 1.

So, the number of linear functions possible is $2^3 = 8$.

Now, we can see how the values $f(000), f(101), f(110), f(011)$ and $f(111)$ are dependent on the values $f(100), f(010)$ and $f(001)$.

Let $f(100) = 0, f(010) = 0, f(001) = 0$

Now,

- $f(000) = f(100 \oplus 100) = f(100) \oplus f(100) = 0 \oplus 0 = 0$
- $f(101) = f(100 \oplus 001) = f(100) \oplus f(001) = 0 \oplus 0 = 0$
- $f(110) = f(100 \oplus 010) = f(100) \oplus f(010) = 0 \oplus 0 = 0$
- $f(011) = f(010 \oplus 001) = f(010) \oplus f(001) = 0 \oplus 0 = 0$
- $f(111) = f(100 \oplus 011) = f(100) \oplus f(011) = f(100) \oplus f(010) \oplus f(001) = 0$

So, we have seen how the values $f(000), f(101), f(110), f(011)$ and $f(111)$ are dependent on the values $f(100), f(010)$ and $f(001)$

Now, suppose the function is from $\{0,1\}^n$ to $\{0,1\}$

In this case if we fix the values of $f(100\ldots0), f(010\ldots0), f(001\ldots0), \ldots, f(000\ldots1)$ then we will get the whole function since rest of the values are dependent on the above n values.

Now, each of the n values $f(100\ldots0), f(010\ldots0), f(001\ldots0), \ldots, f(000\ldots1)$ has 2 options - either 0 or 1.

So, the number of linear functions possible is 2^n. (Answer)

2 votes -- Kushagra Chatterjee (8.2k points)

4.4.31 Functions: TIFR2013-B-16 top

https://gateoverflow.in/25859

Selected Answer

$T_{k,n} = 1$ iff there are at least k $1's$ in n bit binary string.

Similarly, $T_{k,n-1} = 1$ iff k $1's$ are there out of $(n-1)$ bits

It is given that $\langle y_1, y_2, y_3, y_4, y_5 \ldots y_n \rangle$ has exactly k $1's$ which means out of n bits, k bits are $1's$.

Now, suddenly one-bit y_i is removed (that one bit can be 1 or 0, we don't know). Removing one bit will make the number of bits as $(n-1)$.

So, $T_{k,n-1}$ really depends on whether i^{th} bit which we removed is a 0 or 1. If 0, then we have k $1's$ in our remaining $(n-1)$ bits and function will return 1.
If 1, then we have only $(k-1)$ $1's$ out of $(n-1)$ bits and so the function will return 0.

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So, $T_{k,n-1} = \neg y_i$ which depends on the value of y_i .

Option D is correct.

5 votes -- Vicky rix (6.9k points)

4.4.32 Functions: TIFR2014-B-17 top

https://gateoverflow.in/27344

Note: NOT is absent in function f.

for two boolean variables, $p_1 = 1$,$p_2 = 1$, neither $p_1 \wedge p_2$ nor $p_1 \vee p_2$ is 0. ie, $f(1,1)$ is never 0.

for $i = 1$ to n. $f(p_i)$ is a function of AND,OR operations on p_i. if all $p_i = 1$, then f can never be 0;

similarly, if all $p_i = 0$, f can never be 1;

Therefore A, B are not possible.

for $j < n$, if all $p_j = 0$ and $p_{n-j} = 1$,then $f(p_j, p_{n-j})$ =majority if each 0 is AND with each 1.The remaining $1's$ or $0's$ are OR with the result.

Hence, MAJORITY can be computed from f.

Option C is possible.

To check odd number of $1's$, for PARITY function, we have to get the result 0 for even number of $1's$ which is not possible with just AND and OR operations, how might we combine(since NOT is absent in f);

D is not possible.

For option E, we check by symmetry. When the inputs are complemented among $0's$ and $1's$, can f change to f'? f is not always fixed for a particular input,. example, $f(0,1) = 0 \vee 1 = 1$ $0 \wedge 1 = 0$,hence f can take multiple values for same input. Therefore E is also not right.

The only possible answer is C .'Hence A,B,D,E are not possible.

3 votes -- Vikranth Inti (537 points)

4.4.33 Functions: TIFR2014-B-18 top

https://gateoverflow.in/27351

Selected Answer

The function $g_z(Y)$ is defined as $[k] \rightarrow \{0,1\}$ where $[k]$ is the set of positive integers till k. That is, given a triplet (z_1, z_2, z_3), Y can take any value from 1 to k. If Y happen to be any of z_1, z_2, z_3, $g_z(Y) = 0$ due to the definition of f and g_z. Now even for different z, g_z may be the same. Otherwise, the answer would have been how many ways we can form a triplet z - which gives k^3 and for each z we get a function g_z.

For all unique combinations of z_1, z_2, z_3 are unique, we are guaranteed that we get a distinct function g_z. This is clear from the definition of g_z. For example, suppose $k = 4$. The triplets are

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$(1,2,3)$
$(1,2,4)$
$(1,3,4)$
$(2,3,4)$
For the triplet $(1,2,3)$, Y can be made in 4 ways as $(1,2,3,1),(1,2,3,2),(1,2,3,3)$and $(1,2,3,4)$. Now, as per definition of g_z, we get
$g_{(1,2,3)} = \{\{1 \to 0\},\{2 \to 0\},\{3 \to 0\},\{4 \to 1\}\}$.

Similarly, for the next three triplets, g_z are different as in second only 3 maps to 1, in third one only 2 maps to 1 and in fourth one only 1 maps to 1.

So, in general, for any given k, we have ${}^kC_3$ ways of forming distinct triplets and each of them guarantees a unique function g_z where exactly $k-3$ elements map to 1 and 3 elements map to 0. Now, if any of the elements in the triplet are same, then the function becomes $\{\{1 \to 0\},\{2 \to 0\},\ldots,\{k \to 0\}\}$, (all k elements mapping to 0) and this remains the same for any triplet. So, total number of possible functions are

$$ {}^kC_3 + 1 $$

6 votes -- Arjun Suresh (348k points)

4.4.34 Functions: TIFR2017-A-11 top

https://gateoverflow.in/95289

Selected Answer

$f : A \to B$ is injective if and only if, given any functions $g,h : B \to A$ whenever $f \circ g = f \circ h$, $f \circ g = f \circ h, then\ g = h$.

Refer to properties of Injective functions: https://en.wikipedia.org/wiki/Injective_function

Let us prove $(\forall g,h : f(g(x)) = f(h(x)) \to g(x) = h(x)) \to f\ is\ one-to-one$ is true.

This is equivalent to, $f\ is\ not\ one-to-one \to (\exists g,h : f(g(x)) = f(h(x)) \wedge g(x) \neq h(x))$

Let us assume LHS is true, i.e. $f\ is\ not\ one-to-one$.

Then there exists some $c,d \in A$ such that,

$$f(c) = f(d) = a, \text{ where } a \text{ is an arbitrary element which belongs to B}$$

Let g and h be some functions out of all possible functions from B to A such that $g \neq h$,

$$\text{i.e. } g(x) = c\ and\ h(x) = d\ \exists c,d \in A\ and\ \exists x \in B$$

$\therefore f(g(x)) = f(c) = a\ and\ f(h(x)) = f(d) = a$ and $g(x) \neq h(x)$, i.e. RHS is also true.

Thus, whenever $\forall g,h\ f \circ g = f \circ h \to g = h$ is true, $f\ is\ one-to-one$.

Domain of f need not be finite. Let $f : A \to B$ be identity function and A and B be infinite sets. Assume that $f \circ g = f \circ h$ is true,

then $f(g(x)) = f(h(x)) \to g(x) = h(x))$ will be true $\forall g,h$ since f is an identity function. So,

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even if domain of f is not finite, the condition holds true.

6 votes -- V S Silpa (4.4k points)

4.5 Groups(24) top

4.5.1 Groups: GATE1990-2-x top

https://gateoverflow.in/84039

Match the pairs in the following questions:

(a) Groups	(p) Associativity
(b) Semigroups	(q) Identity
(c) Monoids	(r) Commutativity
(d) Abelian groups	(s) Left inverse

gate1990 match-the-following set-theory&algebra groups

Answer

4.5.2 Groups: GATE1992-14a top

https://gateoverflow.in/593

If G is a group of even order, then show that there exists an element $a \neq e$, e, the identity in G, such that $a^2 = e$.

gate1992 set-theory&algebra groups normal descriptive proof

Answer

4.5.3 Groups: GATE1992-14b top

https://gateoverflow.in/43580

Consider the set of integers $\{1, 2, 3, 4, 6, 8, 12, 24\}$ together with the two binary operations LCM (lowest common multiple) and GCD (greatest common divisor). Which of the following algebraic structures does this represent?

A. group
B. ring
C. field
D. lattice

gate1992 set-theory&algebra groups normal

Answer

4.5.4 Groups: GATE1993-28 top

https://gateoverflow.in/2324

Let $(\{p, q\}, *)$ be a semigroup where $p * p = q$. Show that:

a. $p * q = q * p$ and
b. $q * q = q$

gate1993 set-theory&algebra groups normal descriptive

Answer

4.5.5 Groups: GATE1994-1.10 top

https://gateoverflow.in/2451

Some group (G, o) is known to be abelian. Then, which one of the following is true for G?

A. $g = g^{-1}$ for every $g \in G$

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B. $g = g^2$ for every $g \in G$

C. $(goh)^2 = g^2 oh^2$ for every $g, h \in G$

D. G is of finite order

gate1994 set-theory&algebra groups normal

Answer

4.5.6 Groups: GATE1995-2.17 top

https://gateoverflow.in/2629

Let A be the set of all non-singular matrices over real number and let $*$ be the matrix multiplication operation. Then

A. A is closed under $*$ but $\langle A, * \rangle$ is not a semigroup.
B. $\langle A, * \rangle$ is a semigroup but not a monoid.
C. $\langle A, * \rangle$ is a monoid but not a group.
D. $\langle A, * \rangle$ is a a group but not an abelian group.

gate1995 set-theory&algebra groups

Answer

4.5.7 Groups: GATE1995-21 top

https://gateoverflow.in/2659

Let G_1 and G_2 be subgroups of a group G.

a. Show that $G_1 \cap G_2$ is also a subgroup of G.
b. Is $G_1 \cup G_2$ always a subgroup of G?.

gate1995 set-theory&algebra groups normal descriptive proof

Answer

4.5.8 Groups: GATE1996-1.4 top

https://gateoverflow.in/2708

Which of the following statements is FALSE?

A. The set of rational numbers is an abelian group under addition
B. The set of integers in an abelian group under addition
C. The set of rational numbers form an abelian group under multiplication
D. The set of real numbers excluding zero is an abelian group under multiplication

gate1996 set-theory&algebra groups normal

Answer

4.5.9 Groups: GATE1997-3.1 top

https://gateoverflow.in/2232

Let $(Z, *)$ be an algebraic structure where Z is the set of integers and the operation $*$ is defined by $n * m = \max(n. m)$. Which of the following statements is true for $(Z, *)$?

A. $(Z, *)$ is a monoid
B. $(Z, *)$ is an Abelian group
C. $(Z, *)$ is a group
D. None of the above

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gate1997 set-theory&algebra groups normal

Answer

4.5.10 Groups: GATE1998-12 top

https://gateoverflow.in/1726

Let $(A, *)$ be a semigroup, Furthermore, for every a and b in A, if $a \neq b$, then $a * b \neq b * a$.

a. Show that for every a in A, $a * a = a$
b. Show that for every a, b in A, $a * b * a = a$
c. Show that for every a, b, c in A, $a * b * c = a * c$

gate1998 set-theory&algebra groups descriptive

Answer

4.5.11 Groups: GATE2000-4 top

https://gateoverflow.in/675

Let $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $\otimes$ denote multiplication modulo 8, that is, $x \otimes y = (xy) \mod 8$

Prove that $(\{0, 1\}, \otimes)$ is not a group.

Write three distinct groups $(G, \otimes)$ where $G \subset S$ and G has 2 elements.

gate2000 set-theory&algebra descriptive groups

Answer

4.5.12 Groups: GATE2002-1.6 top

https://gateoverflow.in/810

Which of the following is true?

A. The set of all rational negative numbers forms a group under multiplication.
B. The set of all non-singular matrices forms a group under multiplication.
C. The set of all matrices forms a group under multiplication.
D. Both B and C are true.

gate2002 set-theory&algebra groups normal

Answer

4.5.13 Groups: GATE2003-7 top

https://gateoverflow.in/898

Consider the set Σ^* of all strings over the alphabet $\Sigma = \{0, 1\}$. Σ^* with the concatenation operator for strings

A. does not form a group
B. forms a non-commutative group
C. does not have a right identity element
D. forms a group if the empty string is removed from Σ^*

gate2003 set-theory&algebra groups normal

Answer

4.5.14 Groups: GATE2004-72 top

https://gateoverflow.in/1066

The following is the incomplete operation table of a 4-element group.

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*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b				
c				

The last row of the table is

A. c a e b
B. c b a e
C. c b e a
D. c e a b

gate2004 set-theory&algebra groups normal

Answer

4.5.15 Groups: GATE2005-13 top https://gateoverflow.in/1163

The set $\{1, 2, 4, 7, 8, 11, 13, 14\}$ is a group under multiplication modulo 15. The inverses of 4 and 7 are respectively:

A. 3 and 13
B. 2 and 11
C. 4 and 13
D. 8 and 14

gate2005 set-theory&algebra normal groups

Answer

4.5.16 Groups: GATE2005-46 top https://gateoverflow.in/1171

Consider the set H of all $3 * 3$ matrices of the type

$$\begin{pmatrix} a & f & e \\ 0 & b & d \\ 0 & 0 & c \end{pmatrix}$$

where a, b, c, d, e and f are real numbers and $abc \neq 0$. Under the matrix multiplication operation, the set H is:

A. a group
B. a monoid but not a group
C. a semi group but not a monoid
D. neither a group nor a semi group

gate2005 set-theory&algebra groups normal

Answer

4.5.17 Groups: GATE2006-3 top https://gateoverflow.in/882

The set $\{1, 2, 3, 5, 7, 8, 9\}$ under multiplication modulo 10 is not a group. Given below are four possible reasons. Which one of them is false?

A. It is not closed

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B. 2 does not have an inverse
C. 3 does not have an inverse
D. 8 does not have an inverse

gate2006 set-theory&algebra groups normal

Answer

4.5.18 Groups: GATE2007-21 top

https://gateoverflow.in/1219

How many different non-isomorphic Abelian groups of order 4 are there?

A. 2
B. 3
C. 4
D. 5

gate2007 groups normal

Answer

4.5.19 Groups: GATE2009-1 top

https://gateoverflow.in/795

Which one of the following is **NOT** necessarily a property of a Group?

A. Commutativity
B. Associativity
C. Existence of inverse for every element
D. Existence of identity

gate2009 set-theory&algebra easy groups

Answer

4.5.20 Groups: GATE2009-22 top

https://gateoverflow.in/799

For the composition table of a cyclic group shown below:

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Which one of the following choices is correct?

A. a, b are generators
B. b, c are generators
C. c, d are generators
D. d, a are generators

gate2009 set-theory&algebra normal groups

Answer

4.5.21 Groups: GATE2014-3-3 top

https://gateoverflow.in/2037

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Let G be a group with 15 elements. Let L be a subgroup of G. It is known that $L \neq G$ and that the size of L is at least 4. The size of L is _____.

gate2014-3 set-theory&algebra groups numerical-answers normal

Answer

4.5.22 Groups: GATE2014-3-50 top

https://gateoverflow.in/2084

There are two elements x, y in a group $(G, *)$ such that every element in the group can be written as a product of some number of x's and y's in some order. It is known that

$$x * x = y * y = x * y * x * y = y * x * y * x = e$$

where e is the identity element. The maximum number of elements in such a group is ____.

gate2014-3 set-theory&algebra groups numerical-answers normal

Answer

4.5.23 Groups: GATE2018-19 top

https://gateoverflow.in/204093

Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is _____

gate2018 groups numerical-answers

Answer

4.5.24 Groups: ISI2017-MMA-28 top

https://gateoverflow.in/214013

Let H be a subgroup of group G and let N be a normal subgroup of G. Choose the correct statement :

A. $H \cap N$ is a normal subgroup of both H and N
B. $H \cap N$ is a normal subgroup of H but not necessarily of N
C. $H \cap N$ is a normal subgroup of N but not necessarily of H
D. $H \cap N$ need not to be a normal subgroup of either H or N

isi2017 groups

Answer

Answers: Groups

4.5.1 Groups: GATE1990-2-x top

https://gateoverflow.in/84039

Selected Answer

group $\longrightarrow$ left inverse

semigroup $\longrightarrow$ associative

monoid $\longrightarrow$ identity

abelian group $\longrightarrow$ commutative

11 votes -- kunal chalotra (20.4k points)

4.5.2 Groups: GATE1992-14a top

https://gateoverflow.in/593

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Selected Answer

I will proove it using contradiction. Assuming no element has order 2. i.e., $a^2 \neq e$ for any non-identity element a, means

$a \neq a^{-1}$ for any (non-identity element) a.

Rewriting the statement: the inverse of any element is not that element itself, it is something else.

But I want to somehow show that at least one element has inverse as its own. I am trying a method lets see if it works :).

I will select each element from the set and will check inverse of each element, and in this process, as soon as I encounter any element having $a^2 \neq e$ then I am done.

My Goal: to show G has at least 1 element as its own inverse.

Let $|G| = 2n$. Then we take out the identity and have $2n-1$ elements to choose from.

Step 1: Select an element, if it is its own inverse then I am done.

Step 2 (otherwise): If inverse of a is not a and is b, throw a and b out. $(\because$ if a inverse is b then b inverse is a, and inverse of an element is unique$)$

(Notice we always throw one pair)

In worst case I will end up throwing all but one element, because total number of elements is odd $(2n-1)$ and we always throw a pair of two (even).

Now question is, what is the inverse of that element?

It has to be its own inverse, it can not map to inverse of any other element because inverse is unique. And moreover it can not be inverse of identity element because inverse of identity is identity itself.

Finally, I can say there exist one non-identity element a of order 2.

Yes, it worked !

Hence Proved !

24 votes -- Sachin Mittal (15.6k points)

4.5.3 Groups: GATE1992-14b top

https://gateoverflow.in/43580

Selected Answer

Ans is lattice.

For **LCM** and **GCD:**

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16 votes -- Prashant Singh (59.8k points)

4.5.4 Groups: GATE1993-28 top

https://gateoverflow.in/2324

Selected Answer

a.
$p*p=q$
$p*p*p=p*q$ //left operation with p
$(p*p)*p=p*q$ //associative property
$q*p=p*q$ $//p*p=q$

b.
For a semi-group, two properties are known: associativity and closure. (Identity is not required).
Closure means that $p*q$ must be a part of the semi-group.
This means, either $p=p*q$ or $q=p*q$ as the semi-group is $(\{p,q\},*)$

CASE 1: $p=p*q$.
This means, $p=p*p*p$ as $p*p=q \quad \rightarrow (1)$
Then, $q*q=LHS=p*p*p*p=p*p=q=RHS.$ (From (1)).

CASE 2: $q=p*q$.
This means, $q=p*q=p*p*p \quad \rightarrow (2)$
Then, $q*q=LHS=p*p*p*p=p*q=q=RHS$ (based on Case 2's assumption).

13 votes -- meghashyamc (377 points)

4.5.5 Groups: GATE1994-1.10 top

https://gateoverflow.in/2451

Selected Answer

Associativity property of Group.

- For all a,b and c in G, the equation $(aob)oc=ao(boc)$ holds.

For an Abelian group, commutative property also holds.

- For all a,b in G, the equation $aob=boa$

From option C, using these two properties,

$$(goh)^2=(goh)o(goh)=(hog)o(goh)=((hog)og)oh=(ho(gog))oh=(hog^2)oh=(g^2oh)oh$$

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So, C is correct.

Integer addition $(Z, +)$ is an Abelian group.

Inverse of 1 is -1 and not 1. So, A is false.

$1^2 = 1 + 1 = 2 \neq 1$. So B also false.

Order of a group is the number of elements in it. Integer is an infinite set, so D is also false.
ref @ http://math.stackexchange.com/questions/40996/prove-that-if-abi-aibi-forall-a-b-in-g-for-three-consecutive-integers/41004#41004

@ http://math.stackexchange.com/questions/423745/a-group-g-is-abelian-iff-abn-an-bn-for-all-a-b-in-g-and-n-in-bb?lq=1

29 votes -- Arjun Suresh (348k points)

4.5.6 Groups: GATE1995-2.17 top

https://gateoverflow.in/2629

Selected Answer

Answer: D

As the matrices are non singular so their determinant $\neq 0$. Hence, the inverse matrix always exist.

But for a group to be abelian it should follow commutative property. As, matrix multiplication is not commutative, $\langle A, * \rangle$ is a group but not an abelian group.

Two nonsingular matrix can not give a singular matrix after product. Therefore it satisfies closure property also.

Proof: Let A and B are nonsingular and C is singular.

Claim: For any A, B and C :-
$AB = C$ is NOT possible.

I will proof using contradiction. Let it be possible to have $AB = C$.

$$\begin{aligned} & AB = C \\ \implies & det(AB) = det(C) \\ & det(AB) = 0 \; [\because \det(\mathrm{C})=0 \text{ as C is singular}] \\ \implies & \mid A \mid . \mid B \mid = 0 \\ \implies & \mid A \mid = 0 \text{ or } \mid B \mid = 0 \end{aligned}$$

which is contradiction as A and B both are non singular.

19 votes -- Rajarshi Sarkar (34.1k points)

4.5.7 Groups: GATE1995-21 top

https://gateoverflow.in/2659

Selected Answer

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Let G with the set $\{1,3,5,7\}$ be a group wrt $\otimes_8$ operation because all elements are less than 8 and co-prime to 8.

Let $G_1 = \{1,3\}, G_2 = \{1,5\}$

G_1 and G_2 are sub-groups of G having identity element 1. We can make composition table and cross verify.

a. $G_1 \cap G_2 = \{1\}$ is a subgroup containing only identity element (Trivial Subgroup).
b. $G_1 \cup G_2 = \{1,3,5\}$ which is not a subgroup.

Reason when we draw composition table $3 * 5 = 15 \mod 8 = 7$ which is not present in this subgroup so not satisfying closure property. Hence, not a subgroup.

11 votes -- Rajesh Pradhan (22.6k points)

4.5.8 Groups: GATE1996-1.4 top

https://gateoverflow.in/2708

Selected Answer

Answer: C

Rational numbers will include 0. As the group should be under multiplication we will not have any inverse element for 0. Thus, not even satisfying the group property.

14 votes -- Rajarshi Sarkar (34.1k points)

4.5.9 Groups: GATE1997-3.1 top

https://gateoverflow.in/2232

Selected Answer

Lets follow our checklist one by one to see what property this algebraic structure follows.

Closure -yes ($m * n = \max(m,n)$) Output is either m or n whichever is maximum and since m,n belongs to Z, the result of the binary operation also belongs to Z. So closure property is satisfied.

Associative-Yes the output is max among the elements and it is associative

Now for identity, we don't have a single unique element for all the elements which is less than all the elements. ie, $m * e = m \implies \max(m,e) = m$. We can't find a single unique e which is less than all possible integer m, such that comparison between the two would always give m itself.

If the set was for Natural numbers we could have had 1 as identity element as it is less than any other natural number and we would have got a monoid.

- Semi-group - Closed and associative
- Monoid - Closed, associative and has an identity
- Group - Monoid with inverse
- Abelian group- Group with commutative property.

Hence this is just a semigroup- D option.

Ans D)

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18 votes -- Sourav Roy (3.5k points)

4.5.10 Groups: GATE1998-12 top

https://gateoverflow.in/1726

Selected Answer

a. Let $a * a = b$. $(a * a) * a = b * a$. Since $(A, *)$ is a semigroup, $*$ is closed and associative. So, $(a * a) * a = a * (a * a) \implies a * b = b * a$, which is possible only if $a = b$. Thus we proved $a * a = a$.

b. Let $(a * b) * a = c \implies (a * b) * a * a = c * a \implies a * b * a = c * a \implies c * a = a$.

Similarly,
$a * (a * b * a) = a * c \implies a * a * (b * a) = a * c \implies a * (b * a) = a * c \implies a * c = a = c *$

So, $c = a$.

c. Let
$(a * b) * c = d. \implies (a * b) * c * c = d * c \implies a * b * c = d * c \implies d * c = d.$

Similarly,
$a * (a * b * c) = a * d \implies a * a * (b * c) = a * d \implies a * (b * c) = a * d \implies a * d = d.$

Thus
$d * c = a * d = d$

Now
$c * d * c = c * a * d = c * d \implies c = c * a * d = c * d$

and

$d * c * a = a * d * a = d * a \implies d * c * a = a = d * a$

So,

$a * c = (d * a) * (c * d) = d * (a * c) * d = d.$

Thus,
$a * b * c = a * c.$

12 votes -- Arjun Suresh (348k points)

4.5.11 Groups: GATE2000-4 top

https://gateoverflow.in/675

Selected Answer

A

$\otimes$	0	1
0	0	0
1	0	1

B

$\otimes$	1	3
1	1	3
3	3	1

$\otimes$	1	5
1	1	5
5	5	1

$\otimes$	1	7
1	1	7
7	7	1

A 1 is the identity element. Inverse does not exist for zero. So, it is not a group.

14 votes -- Anu (5.9k points)

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4.5.12 Groups: GATE2002-1.6 top

https://gateoverflow.in/810

Selected Answer

Answer: B

A: False. Multiplication of two negative rational numbers give positive number. So, closure property is not satisfied.

B: True. Matrices have to be non-singular (determinant !=0) for the inverse to exist.

C: False. Singular matrices do not form a group under multiplication.

D. False as C is false.

17 votes -- Rajarshi Sarkar (34.1k points)

4.5.13 Groups: GATE2003-7 top

https://gateoverflow.in/898

Selected Answer

Identity element for concatenation is empty string ϵ. Now, we cannot concatenate any string with a given string to get empty string $\implies$ there is no inverse for string concatenation. Only other 3 group properties -- closure, associative and existence of identity -- are satisfied and hence, ans should be (a).

26 votes -- Madhur Rawat (2.3k points)

4.5.14 Groups: GATE2004-72 top

https://gateoverflow.in/1066

Selected Answer

1. Group of order Prime Square (p^2) is always abelian. See
2. If the group is abelian, then $x * y = y * x$ for every x, y in it (Commutative). Therefore, the (i, j) entry is equal to the(j, i) entry in the Cayley table making the table is symmetric. $(\text{Informally, } 1^{st} \text{ row is same as } 1^{st} \text{ column, } 2^{nd} \text{ row is same as } 2^{nd} \text{ column and so on})$

Here, order $4 = p^2 = 2^2 (p = 2)$. Hence, it is abelian group.

Now abelian group's Cayley table is symmetric. So, 1st row will be same as 1st col and 2nd row will be same as 2nd column.

Matches with **option D** only.

14 votes -- Rajesh Pradhan (22.6k points)

4.5.15 Groups: GATE2005-13 top

https://gateoverflow.in/1163

Selected Answer

Option C.

Identity element here is 1.
$4 * 4 \mod 15 = 1$
$7 * 13 \mod 15 = 1.$

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17 votes -- anshu (3.3k points)

4.5.16 Groups: GATE2005-46 top

https://gateoverflow.in/1171

Selected Answer

Given Information: Matrix is upper triangular. It's determinant is multiplication of principle diagonal elements. i.e., abc.
It is given that $abc \neq 0$. So, Inverse for every such matrix exists.

Now this set is

1. Closed - You can see after multiplication matrix is in same format and $|AB| = |A||B| \neq 0$ as $|A|, |B| \neq 0$
2. Associative - Matrix multiplication is associative
3. Existence of Identity - Identity Matrix is present
4. Existence of Inverse - as determinant is non zero there exist inverse for every matrix

So, it is group.

21 votes -- Akash Kanase (42.5k points)

4.5.17 Groups: GATE2006-3 top

https://gateoverflow.in/882

Selected Answer

Answer: C

3 has an inverse, which is 7.
$3 * 7 \mod 10 = 1$.

19 votes -- Rajarshi Sarkar (34.1k points)

4.5.18 Groups: GATE2007-21 top

https://gateoverflow.in/1219

Selected Answer

The number of Abelian groups of order P^k (P is prime) is the number of partitions of k.
Here, order is 4 i.e. 2^2.
Partition of 2 are $\{1,1\},\{2,0\}$.
Total 2 partition so no. of different abelian groups are 2.

http://oeis.org/wiki/Number_of_groups_of_order_n.

---

1. First, find the prime factorization of n. For example, 4 has prime factorization as $2 * 2$. Also, 600 can be factorized as $2^3 * 3^1 * 5^2$
2. Now, find the number of partitions of all powers, and then multiply them. Number of Partitions of a number k is the number of ways k can be partitioned. For example, number of partitions of 3 is 3, because 3 can can be partitioned in 3 different ways: $\{1+1+1\},\{1+2\},\{3\}$. Similarly, 4 can be partitioned in 5 different ways: $\{1+1+1+1\},\{2+1+1\},\{2+2\},\{3+1\},\{4\}$. Note that order of elements in a partition does not matter, for example, partitions $\{2+1+1\}$ and $\{1+1+2\}$ are the same.
So for this question, we will find number of partitions of 2, which is $2 : \{1+1\},\{2\}$. There is

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no other power, so answer is 2 only.

3. Suppose, in question, order given is 600.. Then, different powers are $3, 1, 2$. Number of partitions for $3, 1, 2$ are $3, 1, 2$ respectively and result would have been $3 * 1 * 2 = 6$.
4. Group theorists view two isomorphic groups as follows: For every element *g* of a group *G*, there exists an element *h* of *H* group such that *h* 'behaves in the same way' as *g*(operates with other elements of the group in the same way as *g*). For instance, if *g* generates *G*, then so does *h*. This implies in particular that *G* and *H* are in bijective correspondence. Thus, the definition of an isomorphism is quite natural.

31 votes -- Digvijay (54.8k points)

4.5.19 Groups: GATE2009-1 top

https://gateoverflow.in/795

Selected Answer

- **Groupoid:** closure property.
- **Semigroup:** closure, associative.
- **Monoid-closure:** associative, identity.
- **Group:** closure, associative, identity, inverse.
- **Abelian group:** group properties + commutativity

So, ans should be A.

20 votes -- sonu (2.3k points)

4.5.20 Groups: GATE2009-22 top

https://gateoverflow.in/799

Selected Answer

An element is a generator for a cyclic group if on repeated applications of it upon itself, it can generate all elements of group.
For example here: $a * a = a$, then $(a * a) * a = a * a = a$, and so on. Here, we see that no matter how many times we apply a on itself, we cannot generate any other element except a. So, a is not a generator.

Now for $b, b * b = a$. Then, $(b * b) * b = a * b = b, (b * b * b) * b = b * b = a,$ and so on. Here again, we see that we can only generate a and b on repeated application of b on itself. So, b is not a generator.

Now for $c, c * c = b$. Then,
$(c * c) * c = b * c = d, (c * c * c) * c = d * c = a, (c * c * c * c) * c = a * c = c.$ So, we see that we have generated all elements of group. So, c is a generator.

For $d, d * d = b$. Then
$(d * d) * d = b * d = c, (d * d * d) * d = c * d = a, (d * d * d * d) * d = a * d = d.$ So, we have generated all elements of group from d. So, d is a generator.

c and d are generators. Option (C) is correct.

http://www.cse.iitd.ac.in/~mittal/gate/gate_math_2009.html

22 votes -- Anu (5.9k points)

4.5.21 Groups: GATE2014-3-3 top

https://gateoverflow.in/2037

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Selected Answer

Lagrange's theorem: For any finite group G, the order (number of elements) of every subgroup L of G divides the order of G.
G has 15 elements.
Factors of 15 are $1, 3, 5,$ and 15.
Since, the given size of L is at least 4 $(1$ and 3 eliminated$)$ and not equal to $G(15$ eliminated$)$, the only size left is 5.

Size of L is 5.

48 votes -- Srinath Jayachandran (3.7k points)

4.5.22 Groups: GATE2014-3-50 top

https://gateoverflow.in/2084

Selected Answer

It is given that:

- x is its own inverse.
- y is its own inverse.
- $x * y$ is its own inverse.
- $y * x$ is its own inverse.

Now I will show you that $x * y$ and $y * x$ are essentially same.
$x * y = x * e * y = x * (x * y * x * y) * y = (x * x) * y * x * (y * y) = e * y * x * e = y * x$

(Group is associative so I do not care about brackets)

This turns out to be abelian group. and $x * y$ is no different from $y * x$

Up to this point I have 4 elements - x, y, e, $x * y$. $(G$ is abelian therefore $x * y$ is same as $y * x)$
Now see if you can have a new element. It is given that every element is product of some numbers of x and y.
Lets try with x.
$x * \circ$, what u would like to put next to x ?
If you put x then there is no use and you have to start over again because of $x * x = e$ now you have to start all over again.
Put y next to x : $x * y$ (this element we already have, we want different element so try multiplying further.)
$x * y * \circ$, obviously you cannot put y, next to $x * y$ because it will be x again:
$x * y * y = x * e = x$
(you have to put alternate.)
Put x, next to $x * y$: $x * y * x$.
This is equal to $x * x * y$ because of commutative property. $x * y * x = x * x * y = y$.
I showed you that, once you get $x * y$ using x, you can not get next element by multiplying into $x * y$ further. Because of commutative property it will be again x or y.

Similarly, if we start with y, we have the same issue.

This concludes that we can not generate further element and only four element can be there at max.

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$\{x, y, x * y, e\}$.
There is a theorem for abelian group: If every element is its own inverse then Group G is abelian.
I am not sure if proof of that theorem relates to this problem somewhere, You can check it out. :)

52 votes -- Sachin Mittal (15.6k points)

4.5.23 Groups: GATE2018-19 top

https://gateoverflow.in/204093

Selected Answer

Order of a Subgroup always divides the order of Group.
Proper Subgroup of Group having order 84 would have order $1, 2, 4, 21, 42$.

So largest order would be 42.

15 votes -- Digvijay (54.8k points)

4.5.24 Groups: ISI2017-MMA-28 top

https://gateoverflow.in/214013

Selected Answer

$H \cap N$ is not necessarily a normal subgroup of N.

Take S_3 for example

S_3 is the normal subgroup of S_3.

and $(e, (1, 2))$ is the subgroup of S_3.

So, let $N = S_3$

and $H = (e, (1, 2))$

Then we find that $H \cap N = H = (e, (1, 2))$ which is not a normal subgroup of N. (Think)

So, $H \cap N$ need not necessarily be a normal subgroup of N.

So, either option B or option D is true.

Now, consider the theorem

N is a normal subgroup of G iff $gNg^{-1} \subseteq N$ for all $g \in G$.

Now, let x be an arbitrary element in N.

then $gxg^{-1} \in N$.

Now, H is a subgroup of G.

So, for all $g \in H$ and x an arbitrary element in H,

we have $gx \in H$.

Again from the definition of a subgroup we have

$gxg^{-1} \in H$.

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So, we get that for any arbitrary element x in $H \cap N$

$(gxg^{-1}) \in H \cap N$ for all $g \in H$.

Hence $(gH \cap Ng^{-1}) \subseteq H \cap N$, for all $g \in H$

So, $H \cap N$ is a normal subgroup of H.

Thus, Option B is the correct answer.

1 votes -- Kushagra Chatterjee (8.2k points)

4.6 Lattice(9) top

4.6.1 Lattice: GATE1988-1vii top

https://gateoverflow.in/91351

The complement(s) of the element 'a' in the lattice shown in below figure is (are) _____

gate1988 descriptive lattice set-theory&algebra

Answer

4.6.2 Lattice: GATE1994-2.9 top

https://gateoverflow.in/2476

The Hasse diagrams of all the lattices with up to four elements are _____ (write all the relevant Hasse diagrams)

gate1994 set-theory&algebra lattice normal descriptive

Answer

4.6.3 Lattice: GATE1997-3.3 top

https://gateoverflow.in/2234

In the lattice defined by the Hasse diagram given in following figure, how many complements does the element 'e' have?

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A. 2
B. 3
C. 0
D. 1

gate1997 set-theory&algebra lattice normal

Answer

4.6.4 Lattice: GATE2002-4 top

https://gateoverflow.in/857

$S = \{(1,2),(2,1)\}$ is binary relation on set $A = \{1,2,3\}$. Is it irreflexive? Add the minimum number of ordered pairs to S to make it an equivalence relation. Give the modified S.

Let $S = \{a,b\}$ and let $\square(S)$ be the powerset of S. Consider the binary relation '$\subseteq$ (set inclusion)' on $\square(S)$. Draw the Hasse diagram corresponding to the lattice $(\square(S), \subseteq)$

gate2002 set-theory&algebra normal lattice descriptive

Answer

4.6.5 Lattice: GATE2005-9 top

https://gateoverflow.in/1158

The following is the Hasse diagram of the poset $[\{a,b,c,d,e\}, \prec]$

The poset is :

A. not a lattice
B. a lattice but not a distributive lattice
C. a distributive lattice but not a Boolean algebra
D. a Boolean algebra

gate2005 set-theory&algebra lattice normal

Answer

4.6.6 Lattice: GATE2008-IT-28 top

https://gateoverflow.in/3318

Consider the following Hasse diagrams.

Which all of the above represent a lattice?

A. (i) and (iv) only
B. (ii) and (iii) only
C. (iii) only

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D. (i), (ii) and (iv) only

gate2008-it set-theory&algebra lattice normal

Answer

4.6.7 Lattice: GATE2015-1-34 top

https://gateoverflow.in/8281

Suppose $L = \{p, q, r, s, t\}$ is a lattice represented by the following Hasse diagram:

For any $x, y \in L$, not necessarily distinct , $x \vee y$ and $x \wedge y$ are join and meet of x, y, respectively. Let $L^3 = \{(x, y, z) : x, y, z \in L\}$ be the set of all ordered triplets of the elements of L. Let p_r be the probability that an element $(x, y, z) \in L^3$ chosen equiprobably satisfies $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$. Then

A. $p_r = 0$
B. $p_r = 1$
C. $0 < p_r \leq \frac{1}{5}$
D. $\frac{1}{5} < p_r < 1$

gate2015-1 set-theory&algebra normal lattice

Answer

4.6.8 Lattice: GATE2017-2-21 top

https://gateoverflow.in/118278

Consider the set $X = \{a, b, c, d, e\}$ under partial ordering $R = \{(a, a), (a, b), (a, c), (a, d), (a, e), (b, b), (b, c), (b, e), (c, c), (c, e), (d, d), (d, e), (e, e)\}$

The Hasse diagram of the partial order (X, R) is shown below.

The minimum number of ordered pairs that need to be added to R to make (X, R) a lattice is _____

gate2017-2 discrete-mathematics lattice numerical-answers normal

Answer

4.6.9 Lattice: TIFR2012-B-4 top

https://gateoverflow.in/25090

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Let $\wedge$, $\vee$ denote the meet and join operations of lattice. A lattice is called distributive if for all x, y, z,

$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

It is called complete if meet and join exist for every subset. It is called modular if for all x, y, z

$z \leq x \Rightarrow x \wedge (y \vee z) = (x \wedge y) \vee z$

The positive integers under divisibility ordering i.e. $p \leq q$ if p divides q forms a.

a. Complete lattice.
b. Modular, but not distributive lattice.
c. Distributive lattice.
d. Lattice but not a complete lattice.
e. Under the give ordering positive integers do not form a lattice.

tifr2012 set-theory&algebra lattice

Answer

Answers: Lattice

4.6.1 Lattice: GATE1988-1vii top

https://gateoverflow.in/91351

Selected Answer

- $lub(a, e) = lub(a, b) = lub(a, c) = lub(a, d) = I$ (Upper Bound of Lattice)
- $glb(a, e) = glb(a, b) = glb(a, c) = glb(a, d) = O$ (Lower Bound of Lattice)

So, e, b, c, d all are complement of a.

8 votes -- Ashwani Kumar (11.4k points)

4.6.2 Lattice: GATE1994-2.9 top

https://gateoverflow.in/2476

Selected Answer

1. Diamond structure
2. Straight line structure

15 votes -- Digvijay (54.8k points)

4.6.3 Lattice: GATE1997-3.3 top

https://gateoverflow.in/2234

Selected Answer

Answer: B

Complement of an element a is a' if:

- a ∧ a' = 0 (lowest vertex in the Hasse diagram)
- a ∨ a' = 1 (highest vertex in the Hasse diagram)

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g, c and d are the complements of e.

13 votes -- Rajarshi Sarkar (34.1k points)

4.6.4 Lattice: GATE2002-4 top

https://gateoverflow.in/857

Selected Answer

$S = \{(1,2),(2,1)\}$: This relation is Irreflexive, Symmetric, Not Transitive, Not Reflexive, Not Asymmetric, Not antisymmetric.

Equivalence Relation $\implies$ Symmetric, Transitive, Reflexive.

It is not transitive & Reflexive.

Reflexive closure of $S = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$

After taking transitive closure relation does not change.

Answer: $S = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$

$S = \{a,b\}$

$P(S) = \{\emptyset, a, b, a, b\}$

Related Hasse Diagram

13 votes -- Akash Kanase (42.5k points)

4.6.5 Lattice: GATE2005-9 top

https://gateoverflow.in/1158

Selected Answer

Option b ,a lattice has lub and glb but to be distributive it should have unique complement.

17 votes -- anshu (3.3k points)

4.6.6 Lattice: GATE2008-IT-28 top

https://gateoverflow.in/3318

Selected Answer

Answer is (A)
Hasse diagram is lattice when every pair of elements have a least upper bound and a greatest lower bound. In figures (ii) and (iii), every element is not having a least upper bound and a greatest lower bound. So, they are not lattices.

24 votes -- neha pawar (4.2k points)

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4.6.7 Lattice: GATE2015-1-34 top

https://gateoverflow.in/8281

Selected Answer

Number of elements in $L^3 =$ Number of ways in which we can choose 3 elements from 5 with repetition $= 5 * 5 * 5 = 125.$

Now, when we take $x = t$, then the given condition for L is satisfied for any y and z. Here, y and z can be taken in $5 * 5 = 25$ ways.

Take $x = r, y = p, z = p.$ Here also, the given condition is satisfied.
When $x = t$, we have $5 * 5 = 25$ cases $\big($for any y and $z\big)$ where the given conditions are satisfied.
Now, with $x = r, y = p, z = p,$ we have one more case. So, $26/125$ which means strictly greater than $1/5.$
So, this makes $p_r > \frac{25}{125}$

Also,
for $x = q, y = r, z = s,$ the given condition is **not satisfied** as $q \vee (r \wedge s) = q \vee p = q,$ while $(q \vee r) \wedge (q \vee s) = t \wedge t = t.$ So, $p_r \neq 1.$

These findings make option A, B, C as FALSE.

Hence, answer = **option D**

43 votes -- Arjun Suresh (348k points)

4.6.8 Lattice: GATE2017-2-21 top

https://gateoverflow.in/118278

Selected Answer

A Hasse Diagram is called a Lattice, if for every pair of elements there exists a LUB and GLB.

In the above Hasse Diagram, LUB and GUB exist for every two elements taken from $\{a, b, c, d, e\}.$ So, it is already a Lattice.

Hence, Minimum number of ordered pairs that need to be added $= 0$

35 votes -- Prashant Singh (59.8k points)

4.6.9 Lattice: TIFR2012-B-4 top

https://gateoverflow.in/25090

Consider the lattice with gcd as meet and lcm as join.

It is distributive

- $gcd(a, lcm(b, c)) = lcm(gcd(a, b), gcd(a, c))$
- $lcm(a, gcd(b, c)) = gcd(lcm(a, b), lcm(a, c))$

It is complete semi-meet lattice. As there is no upper bound it is not complete semi-join lattice.
ANS: C

4 votes -- pramod (3.3k points)

4.7 Mathematical Induction(2) top

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4.7.1 Mathematical Induction: GATE1995-23 top

https://gateoverflow.in/2661

Prove using mathematical induction for $n \geq 5, 2^n > n^2$

gate1995 set-theory&algebra proof mathematical-induction descriptive

Answer

4.7.2 Mathematical Induction: GATE2000-3 top

https://gateoverflow.in/674

Consider the following sequence:

$s_1 = s_2 = 1$ and $s_i = 1 + \min(s_{i-1}, s_{i-2})$ for $i > 2$.

Prove by induction on n that $s_n = \lceil \frac{n}{2} \rceil$.

gate2000 set-theory&algebra mathematical-induction descriptive

Answer

Answers: Mathematical Induction

4.7.1 Mathematical Induction: GATE1995-23 top

https://gateoverflow.in/2661

Selected Answer

Base case: $n = 1, 2^1 = 2 > 1^2$

Induction hypothesis: $2^n > n^2$

To prove: $2^{n+1} > (n+1)^2$

$LHS = 2.2^n > 2.n^2$ (Induction hypothesis)

$RHS = (n+1)^2 = n^2 + 2n + 1 < LHS$, hence proved.

10 votes -- Arjun Suresh (348k points)

4.7.2 Mathematical Induction: GATE2000-3 top

https://gateoverflow.in/674

Selected Answer

$s_3 = 1 + \min(s_1, s_2) = 1 + \min(1, 1) = 2 = \lceil \frac{3}{2} \rceil$.

So, base condition of induction satisfied.

Assume, $s_{n-2} = \lceil \frac{n-2}{2} \rceil$ and $s_{n-1} = \lceil \frac{n-1}{2} \rceil$ (Induction hypothesis)

Now, we have to prove,

$s_n = \lceil \frac{n}{2} \rceil$

$s_n = 1 + \min(s_{n-1}, s_{n-2}) = 1 + \lceil \frac{n-2}{2} \rceil = 1 + \lceil \frac{n}{2} \rceil - 1 = \lceil \frac{n}{2} \rceil$

(Hence, proved)

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11 votes -- Arjun Suresh (348k points)

4.8 Number Theory(9) top

4.8.1 Number Theory: GATE1991-15,a top

https://gateoverflow.in/542

Show that the product of the least common multiple and the greatest common divisor of two positive integers a and b is $a \times b$.

gate1991 set-theory&algebra normal number-theory proof descriptive

Answer

4.8.2 Number Theory: GATE1995-7 top

https://gateoverflow.in/2642

a. Determine the number of divisors of 600.

b. Compute without using power series expansion $\lim_{x \to 0} \frac{\sin x}{x}$

gate1995 normal number-theory combined-question

Answer

4.8.3 Number Theory: GATE2005-IT-34 top

https://gateoverflow.in/3780

Let $n = p^2 q$, where p and q are distinct prime numbers. How many numbers m satisfy $1 \leq m \leq n$ and $gcd\ (m, n) = 1$? Note that $gcd\ (m, n)$ is the greatest common divisor of m and n.

A. $p(q-1)$
B. pq
C. $\left(p^2 - 1\right)(q-1)$
D. $p(p-1)(q-1)$

gate2005-it set-theory&algebra normal number-theory

Answer

4.8.4 Number Theory: GATE2007-IT-16 top

https://gateoverflow.in/3449

The minimum positive integer p such that $3^p \pmod{17} = 1$ is

A. 5
B. 8
C. 12
D. 16

gate2007-it set-theory&algebra normal number-theory

Answer

4.8.5 Number Theory: GATE2008-IT-24 top

https://gateoverflow.in/3285

The exponent of 11 in the prime factorization of 300! is

A. 27
B. 28
C. 29

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D. 30

gate2008-it set-theory&algebra normal number-theory

Answer

4.8.6 Number Theory: GATE2014-2-49 top

https://gateoverflow.in/2015

The number of distinct positive integral factors of 2014 is _____

gate2014-2 set-theory&algebra easy numerical-answers number-theory

Answer

4.8.7 Number Theory: GATE2015-2-9 top

https://gateoverflow.in/8058

The number of divisors of 2100 is ____.

gate2015-2 set-theory&algebra number-theory easy numerical-answers

Answer

4.8.8 Number Theory: ISI2004-MIII-11 top

https://gateoverflow.in/123882

If $\alpha 1, \alpha 2, \ldots, \alpha n$ are the positive numbers then

$$\frac{a1}{a2} + \frac{a2}{a3} + \cdots + \frac{an-1}{an} + \frac{an}{a1}$$

is always

A. $\geq n$
B. $\leq n$
C. $\leq n^{\frac{1}{2}}$
D. None of the above

isi2004 set-theory&algebra number-theory

Answer

4.8.9 Number Theory: ISI2016 top

https://gateoverflow.in/45218

Find the number of positive integers n for which $n^2 + 96$ is a perfect square.

isi2016 set-theory&algebra number-theory numerical-answers

Answer

Answers: Number Theory

4.8.1 Number Theory: GATE1991-15,a top

https://gateoverflow.in/542

Selected Answer

x and y can be expressed as

$x = 2^{a_1}.3^{a_2} \ldots P^{a_n}$.

$y = 2^{b_1}.3^{b_2} \ldots P^{b_n}$.

where, a_i & $b_i \geq 0$ for $1 \leq i \leq n$, and P is a prime number.

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$x * y = 2^{a_1+b_1}.3^{a_2+b_2} \ldots P^{a_n+b_n}$

LCM(x,y)$= 2^{max(a_1+b_1)}.3^{max(a_2+b_2)} \ldots P^{max(a_n+b_n)}$

HCF(x,y)$= 2^{min(a_1+b_1)}.3^{min(a_2+b_2)} \ldots P^{min(a_n+b_n)}$

Since, $max(a_i+b_i)+min(a_i+b_i)=a_i+b_i$

So, LCM(x,y)*HCF(x,y) $= 2^{a_1+b_1}.3^{a_2+b_2} \ldots P^{a_n+b_n} = x * y.$

Proved!

6 votes -- ZAHID WAKEEL (1.2k points)

4.8.2 Number Theory: GATE1995-7 top

https://gateoverflow.in/2642

Answer for Part A.

Prime factorization of $600 = 2^3 \times 3 \times 5^2$.

Total no of divisors $= (3+1)(1+1)(2+1) = 4 \times 2 \times 3 = 24$ Divisors

Here, we are choosing either $0, 1, 2$ or 3, $2's$ so $3+1=4$ choices for 2 and so on for all.

In case we do not choose any of above factors, we get 1 as divisor !

7 votes -- Akash Kanase (42.5k points)

Answer for Part B.

$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1}$ (Applying L-Hospital's rule since 0/0 form)
$= 1.$

6 votes -- abhi18459 (251 points)

4.8.3 Number Theory: GATE2005-IT-34 top

https://gateoverflow.in/3780

Selected Answer

$n = p^2q$, where p and q are prime.

So, number of multiples of p in $n = pq$

Number of multiples of q in $n = p^2$

Number of multiples of pq in $n = p$

Since, prime factorization of n consists of only p and $q, gcd(m,n)$ will be a multiple of these or 1.
So, number of possible m such that $gcd(m,n)$ is 1 will be $n-$ number of multiples of either p or q

$$\begin{aligned}
&= n - p^2 - pq + p \\
&= p^2q - p^2 - pq + p \\
&= p(pq - p - q + 1) \\
&= p(p-1)(q-1)
\end{aligned}$$

20 votes -- Arjun Suresh (348k points)

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Euler's totient function $\phi(n)$ is being asked here :

Euler's totient function $\phi(n)$ = Number of positive integers which are $\leq n$ and relatively prime or co-prime to n. (ie. co-prime means if $\gcd(a,b)=1$)

It is given by $\phi(n) = n \times (\frac{(P_1-1)(P_2-1)\ldots(P_k-1)}{P_1P_2..P_k})$

where $P_1P_2\ldots P_k$ distinct prime divisors of n

We have $n = p^2q$.

Therefore,

$$\begin{aligned}\phi(n) &= n(\frac{(p-1)(q-1)}{pq}) \\ &= p^2q(\frac{(p-1)(q-1)}{pq}) \\ &= p(p-1)(q-1)\end{aligned}$$

18 votes -- pC (22k points)

4.8.4 Number Theory: GATE2007-IT-16 top

https://gateoverflow.in/3449

Selected Answer

Fermat's Little Theorem :

$$a^p \equiv a(\mod p)$$

According to Modular Arithmetic $a \equiv b(\mod n)$ if their difference $(a-b)$ is an integral multiple of n (n divides $(a-b)$)

So, $(a^p - a)$ is an integral multiple of p. Now as a is not divisible by p so definitely, $(a^{p-1}-1)$ is an integral multiple of p. This simply means if we divide a^{p-1} by p, the remainder would be 1. i.e., $a^{p-1} \mod p = 1$.

Put the values in the formula.
$p = 17$
So, $p-1 = 16$.

12 votes -- Rupie_c (655 points)

4.8.5 Number Theory: GATE2008-IT-24 top

https://gateoverflow.in/3285

Selected Answer

Simple Method:

$\lfloor 300/11 \rfloor = 27$

$\lfloor 27/11 \rfloor = 2$

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$\lfloor 2/11 \rfloor = 0$

Repeat this and when get 0 stop and sum all the results.

Ans: $27 + 2 + 0 = 29.$

34 votes -- papesh (25.7k points)

300! is 1*2*3*...*300

Now there are 27 multiples of 11 from 1 to 300, so they will include 11 as a prime factor atleast once.

121 and 242 will contain an extra 11, all other will contain 11 as a factor only once.

So total number of 11's = 27+2 = 29.

So exponent of 11 is 29 i.e. option C.

22 votes -- Happy Mittal (11.2k points)

4.8.6 Number Theory: GATE2014-2-49 top

https://gateoverflow.in/2015

Selected Answer

First do prime factorization of 2014 - 2^1 x 19^1 x 53^1

Now to get a factor of 2014, we can choose any combination of the prime factors including 0. i.e; 2^0 and 2^1 are possible and similarly for other prime factors also, there are 2 possibilities. So, total number of positive integral factors

$= 2 \times 2 \times 2 = 8$

(When all the powers of prime factors are 0, we get 1 and when all the powers are maximum, we get the given number.)

23 votes -- Arjun Suresh (348k points)

4.8.7 Number Theory: GATE2015-2-9 top

https://gateoverflow.in/8058

Selected Answer

Answer: 36

$2100 = 7 \times 3 \times 2^2 \times 5^2$

Hence, total number of factors will be
$= (1+1) \times (1+1) \times (2+1) \times (2+1) = 2 \times 2 \times 3 \times 3 = 36,$

because any factor is obtained by multiplying the prime factors zero or more times. (one extra for zero)

22 votes -- Rajarshi Sarkar (34.1k points)

4.8.8 Number Theory: ISI2004-MIII-11 top

https://gateoverflow.in/123882

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Selected Answer

Given, $\alpha 1, \alpha 2, \ldots, \alpha n$ are the positive numbers.

So, $\left(\frac{\alpha 1}{\alpha 2}\right), \left(\frac{\alpha 2}{\alpha 3}\right)$ etc will be also positive numbers.

Hence applying A.M. $\geq$ G.M,

$$\frac{\left(\left(\frac{\alpha 1}{\alpha 2}\right)+\left(\frac{\alpha 2}{\alpha 3}\right)+\ldots+\left(\frac{\alpha n}{\alpha 1}\right)\right)}{n}$$

$$\geq \left(\left(\frac{\alpha 1}{\alpha 2}\right).\left(\frac{\alpha 2}{\alpha 3}\right)\ldots\left(\frac{\alpha n}{\alpha 1}\right)\right)^{\frac{1}{n}}$$

$$\Rightarrow \frac{\left(\left(\frac{\alpha 1}{\alpha 2}\right)+\left(\frac{\alpha 2}{\alpha 3}\right)+\ldots+\left(\frac{\alpha n}{\alpha 1}\right)\right)}{n}$$

$$\geq n.\left(\left(\frac{\alpha 1}{\alpha 2}\right).\left(\frac{\alpha 2}{\alpha 3}\right)\ldots\left(\frac{\alpha n}{\alpha 1}\right)\right)^{\frac{1}{n}}$$

$$\Rightarrow \frac{\left(\left(\frac{\alpha 1}{\alpha 2}\right)+\left(\frac{\alpha 2}{\alpha 3}\right)+\ldots+\left(\frac{\alpha n}{\alpha 1}\right)\right)}{n}$$

$$\geq n.1^{\frac{1}{n}}$$

$$\Rightarrow \frac{\left(\left(\frac{\alpha 1}{\alpha 2}\right)+\left(\frac{\alpha 2}{\alpha 3}\right)+\ldots+\left(\frac{\alpha n}{\alpha 1}\right)\right)}{n}$$

$$\geq n$$

Hence A) is the correct answer..

10 votes -- HABIB MOHAMMAD KHAN (98.7k points)

4.8.9 Number Theory: ISI2016 top

https://gateoverflow.in/45218

Selected Answer

$n^2+96=x^2.$
$x^2-n^2=96.$
$(x-n)(x+n)=96.$

Since, x and n both should be positive integer. $(x-n)$ and $(x+n)$ will be divisors of 96.

By observation, $(x-n)$ should be smaller than $(x+n)$ because x and n are positive integers.
$(x-n)=k1 \rightarrow x=n+k1$

$(x+n)=k2 \rightarrow n+k1+n=k2 \rightarrow 2n+k1=k2 \rightarrow 2n=k2-k1 \rightarrow n=\left(\frac{k2-k1}{2}\right).$

As we have seen from above,
$n=\left(\frac{k2-k1}{2}\right)$

Therefore, for n to be a positive integer,$k2-k1$ should be even. **That is, both should be odd or both should be even.**

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There are 6 pairs of divisors when multiplied becomes
$96 = (1,96),(2,48),(3,32),(4,24),(6,16),(8,12).$

Therfore, there are **only 4** such possibilties. Phew!

14 votes -- Shyam Singh (1.5k points)

4.9 Partial Order(13) top

4.9.1 Partial Order: GATE1991-01,xiv top

https://gateoverflow.in/509

If the longest chain in a partial order is of length n, then the partial order can be written as a _____ of n antichains.

gate1991 set-theory&algebra partial-order normal descriptive

Answer

4.9.2 Partial Order: GATE1993-8.5 top

https://gateoverflow.in/2303

The less-than relation, $<$, on reals is

A. a partial ordering since it is asymmetric and reflexive
B. a partial ordering since it is antisymmetric and reflexive
C. not a partial ordering because it is not asymmetric and not reflexive
D. not a partial ordering because it is not antisymmetric and reflexive
E. none of the above

gate1993 set-theory&algebra partial-order easy

Answer

4.9.3 Partial Order: GATE1996-1.2 top

https://gateoverflow.in/2706

Let $X = \{2,3,6,12,24\}$, Let $\leq$ be the partial order defined by $X \leq Y$ if x divides y. Number of edges in the Hasse diagram of $(X, \leq)$ is

A. 3
B. 4
C. 9
D. None of the above

gate1996 set-theory&algebra partial-order normal

Answer

4.9.4 Partial Order: GATE1997-6.1 top

https://gateoverflow.in/2257

A partial order ≤ is defined on the set $S = \{x, a1, a2, \ldots an, y\}$ as $x \leq_i a_i$ for all i and $a_i \leq y$ for all i, where $n \geq 1$. The number of total orders on the set S which contain the partial order $\leq$ is

A. $n!$
B. $n+2$
C. n
D. 1

gate1997 set-theory&algebra partial-order normal

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Answer

4.9.5 Partial Order: GATE1998-11 top

https://gateoverflow.in/1725

Suppose $A = \{a, b, c, d\}$ and Π_1 is the following partition of A

$\Pi_1 = \{\{a, b, c\}\{d\}\}$

a. List the ordered pairs of the equivalence relations induced by Π_1.

b. Draw the graph of the above equivalence relation.

c. Let $\Pi_2 = \{\{a\}, \{b\}, \{C\}, \{d\}\}$

$\Pi_3 = \{\{a, b, c, d\}\}$

and $\Pi_4 = \{\{a, b\}, \{c, d\}\}$

Draw a Poset diagram of the poset, $\langle \{\Pi_1, \Pi_2, \Pi_3, \Pi_4\}, \text{ refines } \rangle$.

gate1998 set-theory&algebra normal partial-order descriptive

Answer

4.9.6 Partial Order: GATE2003-31 top

https://gateoverflow.in/921

Let $(S, \leq)$ be a partial order with two minimal elements a and b, and a maximum element c. Let P: S $\rightarrow$ {**True, False**} be a predicate defined on S. Suppose that P(a) = **True**, P(b) = **False** and P(x) $\implies$ P(y) for all $x, y \in S$ satisfying $x \leq y$, where $\implies$ stands for logical implication. Which of the following statements CANNOT be true?

A. P(x) = **True** for all x $\in$ S such that x $\neq$ b
B. P(x) = **False** for all x $\in$ S such that x $\neq$ a and x $\neq$ c
C. P(x) = **False** for all x $\in$ S such that b $\leq$ x and x $\neq$ c
D. P(x) = **False** for all x $\in$ S such that a $\leq$ x and b $\leq$ x

gate2003 set-theory&algebra partial-order normal

Answer

4.9.7 Partial Order: GATE2004-73 top

https://gateoverflow.in/1067

The inclusion of which of the following sets into

$S = \{\{1, 2\}, \{1, 2, 3\}, \{1, 3, 5\}, \{1, 2, 4\}, \{1, 2, 3, 4, 5\}\}$

is necessary and sufficient to make S a complete lattice under the partial order defined by set containment?

A. $\{1\}$
B. $\{1\}, \{2, 3\}$
C. $\{1\}, \{1, 3\}$
D. $\{1\}, \{1, 3\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}$

gate2004 set-theory&algebra partial-order normal

Answer

4.9.8 Partial Order: GATE2007-26 top

https://gateoverflow.in/1224

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Consider the set $S = \{a, b, c, d\}$. Consider the following 4 partitions $\pi_1, \pi_2, \pi_3, \pi_4$ on S : $\pi_1 = \{\overline{abcd}\}, \quad \pi_2 = \{\overline{ab}, \overline{cd}\}, \quad \pi_3 = \{\overline{abc}, \overline{d}\}, \quad \pi_4 = \{\bar{a}, \bar{b}, \bar{c}, \bar{d}\}$.
Let $\prec$ be the partial order on the set of partitions $S' = \{\pi_1, \pi_2, \pi_3, \pi_4\}$ defined as follows: $\pi_i \prec \pi_j$ if and only if π_i refines π_j. The poset diagram for $(S', \prec)$ is:

A.

B.

C.

D.

gate2007 set-theory&algebra normal partial-order descriptive

Answer

4.9.9 Partial Order: GATE2007-IT-23 top

https://gateoverflow.in/3456

A partial order P is defined on the set of natural numbers as follows. Here $\frac{x}{y}$ denotes integer division.

i. $(0, 0) \in P$.
ii. $(a, b) \in P$ if and only if $(a\%10) \leq (b\%10)$ and $(\frac{a}{10}, \frac{b}{10}) \in P$.

Consider the following ordered pairs:

i. $(101, 22)$
ii. $(22, 101)$
iii. $(145, 265)$
iv. $(0, 153)$

Which of these ordered pairs of natural numbers are contained in P?

A. (i) and (iii)
B. (ii) and (iv)
C. (i) and (iv)
D. (iii) and (iv)

gate2007-it set-theory&algebra partial-order normal

Answer

4.9.10 Partial Order: TIFR2012-B-5 top

https://gateoverflow.in/25092

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Let R be a binary relation over a set S. The binary relation R is called an equivalence relation if it is reflexive transitive and symmetric. The relation is called partial order if it is reflexive, transitive and anti symmetric. (Notation: Let aRb denote that order pair $(a,b) \in R$.) The relation R is called a well-order if R is a partial order and there does not exist an infinite descending chain (with respect to R) within S. An infinite sequence $x_1, x_2 \ldots$ of elements of S is called an infinite descending chain if for all i we have $x_{i+1} R x_i$ and $x_i \neq x_{i+1}$.

Take $S = \aleph \times \aleph$ and let the binary relation $\sqsubseteq$ over S be such that $(i_1, j_1) \sqsubseteq (i_2, j_2)$ if and only if either $(i_1 < i_2)$ or $((i_1 = i_2) \wedge (j_1 \leq j_2))$. Which statement is true of $\sqsubseteq$?

a. $\sqsubseteq$ is an equivalence relation but not a well order.
b. $\sqsubseteq$ is a partial order but not a well order.
c. $\sqsubseteq$ is a partial order and a well order.
d. $\sqsubseteq$ is an equivalence relation and a well order.
e. $\sqsubseteq$ is neither a partial order nor an equivalence relation.

tifr2012 set-theory&algebra partial-order

Answer

4.9.11 Partial Order: TIFR2013-B-4 top

https://gateoverflow.in/25664

A set S together with partial order $\ll$ is called a well order if it has no infinite descending chains, i.e. there is no infinite sequence $x_1, x_2, \ldots$ of elements from S such that $x_{i+1} \ll x_i$ and $x_{i+1} \neq x_i$ for all i.

Consider the set of all words (finite sequence of letters $a - z$), denoted by W, in dictionary order.

a. Between "aa" and "az" there are only 24 words.
b. Between "aa" and "az" there are only 2^{24} words.
c. W is not a partial order.
d. W is a partial order but not a well order.
e. W is a well order.

tifr2013 set-theory&algebra partial-order

Answer

4.9.12 Partial Order: TIFR2014-B-15 top

https://gateoverflow.in/27322

Consider the set N^* of finite sequences of natural numbers with $x \leq_p y$ denoting that sequence x is a prefix of sequence y. Then, which of the following is true?

a. N^* is uncountable.
b. $\leq_p$ is a total order.
c. Every non-empty subset of N^* has a least upper bound.
d. Every non-empty subset of N^* has a greatest lower bound.
e. Every non-empty finite subset of N^* has a least upper bound.

tifr2014 set-theory&algebra partial-order

Answer

4.9.13 Partial Order: TIFR2014-B-16 top

https://gateoverflow.in/27341

Consider the ordering relation $x \mid y \subseteq N \times N$ over natural numbers N such that $x \mid y$ if there exists $z \in N$ such that $x \bullet z = y$. A set is called lattice if every finite subset has a least upper

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bound and greatest lower bound. It is called a complete lattice if every subset has a least upper bound and greatest lower bound. Then,

a. $|$ is an equivalence relation.
b. Every subset of N has an upper bound under $|$.
c. $|$ is a total order.
d. $(N, |)$ is a complete lattice.
e. $(N, |)$ is a lattice but not a complete lattice.

tifr2014 set-theory&algebra partial-order

Answer

Answers: Partial Order

4.9.1 Partial Order: GATE1991-01,xiv top

https://gateoverflow.in/509

Selected Answer

Suppose the length of the longest chain in a partial order is n. Then the elements in the poset can be partitioned into n disjoint antichains.

11 votes -- Rajarshi Sarkar (34.1k points)

4.9.2 Partial Order: GATE1993-8.5 top

https://gateoverflow.in/2303

Selected Answer

Relation $<$ is :

a. not reflexive
b. Irreflexive
c. not symmetric
d. Asymmetric
e. Anti symmetric

Relation is not POSET because it is irreflexive.
Check AntiSymmetry: $aRb \neq bRa$ unless $a = b$.

A relation may be 'not Asymmetric and not reflexive' but still Antisymmetric.
as $\{(1,1),(1,2)\}$

Not Asymmetric and Irreflexive = Antisymmetric
Option E.

12 votes -- Digvijay (54.8k points)

4.9.3 Partial Order: GATE1996-1.2 top

https://gateoverflow.in/2706

Selected Answer

Answer: B

Hasse Diagram is:

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```
      24
     /
    12
   /
  6
 /  \
2    3
```

12 votes -- Rajarshi Sarkar (34.1k points)

4.9.4 Partial Order: GATE1997-6.1 top

https://gateoverflow.in/2257

Selected Answer

To make this partial order a total order, we need the relation to hold for every two element of the partial order. Currently between any a_i and a_j, there is no relation. So, for every a_i, a_j, we have to add either (a_i, a_j) or (a_j, a_i) in total order. So, this translates to giving an ordering for n elements between x and y, which can be done in $n!$ ways. So, answer is (a).

The bottom figure is for a total order. We can permute the a_i from $i = 1$ to n, and each permutation will also be a total order containing the given partial order.

22 votes -- Arjun Suresh (348k points)

4.9.5 Partial Order: GATE1998-11 top

https://gateoverflow.in/1725

(a) The ordered pairs of the equivalence relations induced
$= \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)\}$

PS: equivalence relations = each partition in power set - ϕ

6 votes -- Nitin Sharma (2.4k points)

4.9.6 Partial Order: GATE2003-31 top

https://gateoverflow.in/921

Selected Answer

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Maximum element is c: So, c is of higher order than any other element in S

Minimal elements are a and b: No other element in S is of lower order than either a or b.

We are given $P(a) = \text{TRUE}$. So, for all x such that $a \leq x$, $P(x)$ must be TRUE. We do have at least one such x, which is c as it is the maximum element. So, D **CANNOT** be true.

- A can be TRUE as all elements mapped to TRUE doesn't violate the given implication.
- B can be TRUE if a is related only to c.
- C can be TRUE as $b \leq x$ ensures $x \neq a$ and for all other elements $P(x)$ can be FALSE without violaing the given implication.

Ref: https://en.wikipedia.org/wiki/Partially_ordered_set

22 votes -- Arjun Suresh (348k points)

4.9.7 Partial Order: GATE2004-73 top

https://gateoverflow.in/1067

Selected Answer

Answer: A

A lattice is complete if every subset of partial order set has a supremum and infimum element.

For example, here we are given a partial order set S. Now it will be a complete lattice if whatever be the subset we choose, it has a supremum and infimum element. Here relation given is set containment, so supremum element will be just union of all sets in the subset we choose. Similarly, infimum element will be just intersection of all the sets in the subset we choose.

Now as we can see, S now is not complete lattice, because although it has a supremum for every subset we choose, but some subsets have no infimum. For example: if we take subset $\{1, 3, 5, 1, 2, 4\}$, then intersection of sets in this is $\{1\}$, which is not present in S. So clearly, if we add set $\{1\}$ in S, we will solve the problem. So, adding $\{1\}$ is necessary and sufficient condition for S to be complete lattice. Thus, option (A) is correct.

18 votes -- Rajarshi Sarkar (34.1k points)

4.9.8 Partial Order: GATE2007-26 top

https://gateoverflow.in/1224

Selected Answer

Answer is option C.

Suppose we have two partitions of a set S: $P_1 = \{A_1, A_2, \ldots\}$ and $P_2 = \{B_1, B_2, \ldots\}$.

- We say that P_1 is a refinement of P_2 if every A_i is a subset of some B_j.

Refer https://www.cs.sfu.ca/~ggbaker/zju/math/equiv-rel.html

$п_{4}$ refines all of them.

1. π_4 refining π_2, π_3.
2. π_4 & π_2 refining π_1. π_4 & π_3 refining π_1. (π_2 refines π_1, i.e., (ab) and (cd) in π_2 are joined as $(abcd)$ in π_1. And π_3 refining π_1 i.e., $(abc)(d)$ in π_3 are joined as $(abcd)$ in π_1)

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That symbol $(\overline{x})$ represents a **partition** . . . $\overline{x}$ means (x).

As it is poset we are not showing transitive dependency.

Partition concept :- https://en.wikipedia.org/wiki/Partition_of_a_set

25 votes -- Akash Kanase (42.5k points)

4.9.9 Partial Order: GATE2007-IT-23 top

https://gateoverflow.in/3456

Selected Answer

Ans. D
For ordered pair (a, b), to be in P, each digit in a starting from unit place must not be larger than the corresponding digit in b.

This condition is satisfied by options

- (iii) $(145, 265) \rightarrow 5 \leq 5, 4 < 6$ and $1 < 2$
- (iv) $(0, 153) \rightarrow 0 < 3$ and no need to examine further

24 votes -- Vikrant Singh (13.5k points)

4.9.10 Partial Order: TIFR2012-B-5 top

https://gateoverflow.in/25092

Answer: C

$S = (i_1 j_1) \sqsubseteq (i_2 j_2)$ iff $(i_1 < i_2)$ or $((i_1 = i_2) \wedge (j_1 \leq j_2))$

1. $(m, n) R (m, n)$?

yes, here $m \not< n$, so we go at second criteria.

Now, $m = n$ & $n = n$. So, this is reflexive.

2. Antisymmetric

$(1, 2) R (2, 3) \implies (2, 3) R (1, 2)?$
No, as $2 \not< 1$.

If you see the definition, it is clear that other than diagonal element no other element is related to itself. So, antisymmetric.

3. Transitivity
$(1, 2) R (2, 3)$ & $(2, 3) R (2, 4) \implies (1, 2) R (2, 4)?$
Yes. It can be seen easily from the definition of S. Same for any other pairs. Not going to prove this formally.

4. It is not reflexive $(1, 2) R (2, 3)$ but $(2, 3) R (3, 2)$

5. This is well ordered. We do not have infinite descending chain. As we have least element $(0, 0)$ our chain stops there.

Ref :-

https://en.wikipedia.org/wiki/Infinite_descending_chain
https://en.wikipedia.org/wiki/Well-order#Examples_and_counterexamples

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3 votes -- Akash Kanase (42.5k points)

4.9.11 Partial Order: TIFR2013-B-4 top

https://gateoverflow.in/25664

Answer: E. well order

Minimal Element is a. It is less than all the elements.

A. False, after aa, we can have ab. Then $aba, abb, abc, \ldots$ Not limited to 24
B. False. after aa, we can have $ab, aba, abc, \ldots$ In fact $ab(a-z)^*$. Not limited to $2^{24}.$
C. False. Why not partial order? Dictionary order is partial order as it is Reflexive, Antisymmetric & Transitive. Even definition in wikipedia says it is.
D. False. Dictionary order is well order.

Definition of lexicographical ordering:

Given two partially ordered sets *A* and *B*, the lexicographical order on the Cartesian product *A* × *B* is defined as

(*a*,*b*) ≤ (*a*′,*b*′) if and only if *a* < *a*′ or (*a* = *a*′ and *b* ≤ *b*′).

The result is a partial order. If *A* and *B* are each totally ordered, then the result is a total order as well. The lexicographical order of two totally ordered sets is thus a linear extension of their product order.

5 votes -- Akash Kanase (42.5k points)

4.9.12 Partial Order: TIFR2014-B-15 top

https://gateoverflow.in/27322

Selected Answer

Consider any sequence like $43, 9, 8, 2-$ it can have many (infinite) least upper bounds like $43, 9, 8, 2, 5, 43, 9, 8, 2, 1,$ but can have only 1 greatest lower bound $-43, 9, 8$ because we are using prefix relation. So, option D is true.

8 votes -- Arjun Suresh (348k points)

4.9.13 Partial Order: TIFR2014-B-16 top

https://gateoverflow.in/27341

Selected Answer

A. Taking an example, $\{4, 2\}$ is not symmetric as $4|2$ is not equal to $2|4$. So, it cannot be an equivalence.

B. It is not necessary that every subset will have an upper bound.

C. For being total every element in the subset should be comparable have it is not total as well. We have counter example: $\{1, 2, 3, 9, 15\}$. Here, $2\&3$ are incomparable. $9\&15$ are also not comparable in this division relation.

D. Not every subset is a complete lattice. Counter example: $\{1, 2, 3, 24, 30\}$ Here, $2\&3$ have no LUB.

So, E is the answer.

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4 votes -- bharti (3.1k points)

4.10 Polynomials(9) top

4.10.1 Polynomials: GATE1987-1-xxii top

https://gateoverflow.in/80379

The equation

$7x^7 + 14x^6 + 12x^5 + 3x^4 + 12x^3 + 10x^2 + 5x + 7 = 0$ has

A. All complex roots
B. At least one real root
C. Four pairs of imaginary roots
D. None of the above

gate1987 polynomials

Answer

4.10.2 Polynomials: GATE1995-2.8 top

https://gateoverflow.in/2620

If the cube roots of unity are $1, \omega$ and ω^2, then the roots of the following equation are

$$(x-1)^3 + 8 = 0$$

A. $-1, 1+2\omega, 1+2\omega^2$
B. $1, 1-2\omega, 1-2\omega^2$
C. $-1, 1-2\omega, 1-2\omega^2$
D. $-1, 1+2\omega, -1+2\omega^2$

gate1995 set-theory&algebra normal polynomials

Answer

4.10.3 Polynomials: GATE1997-4.4 top

https://gateoverflow.in/2245

A polynomial $p(x)$ is such that $p(0) = 5, p(1) = 4, p(2) = 9$ and $p(3) = 20$. The minimum degree it should have is

A. 1
B. 2
C. 3
D. 4

gate1997 set-theory&algebra normal polynomials

Answer

4.10.4 Polynomials: GATE2000-2.4 top

https://gateoverflow.in/651

A polynomial $p(x)$ satisfies the following:

- $p(1) = p(3) = p(5) = 1$
- $p(2) = p(4) = -1$

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The minimum degree of such a polynomial is

A. 1
B. 2
C. 3
D. 4

gate2000 set-theory&algebra normal polynomials

Answer

4.10.5 Polynomials: GATE2014-2-5 top

https://gateoverflow.in/1957

A non-zero polynomial $f(x)$ of degree 3 has roots at $x = 1$, $x = 2$ and $x = 3$. Which one of the following must be TRUE?

A. $f(0)f(4) < 0$
B. $f(0)f(4) > 0$
C. $f(0) + f(4) > 0$
D. $f(0) + f(4) < 0$

gate2014-2 set-theory&algebra polynomials numerical-answers normal

Answer

4.10.6 Polynomials: GATE2017-2-24 top

https://gateoverflow.in/118185

Consider the quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base b. The solutions of this equation in the same base b are $x = 5$ and $x = 6$. Then b= _____

gate2017-2 polynomials numerical-answers

Answer

4.10.7 Polynomials: ISI2004-MIII top

https://gateoverflow.in/123818

The equation

$$\frac{1}{3} + \frac{1}{2}s^2 + \frac{1}{6}s^3 = s$$

has

A. exactly three solution in $[0.1]$
B. exactly one solution in $[0, 1]$
C. exactly two solution in $[0, 1]$
D. no solution in $[0, 1]$

isi2004 polynomials

Answer

4.10.8 Polynomials: ISI2004-MIII-7 top

https://gateoverflow.in/123770

The equation $x^6 - 5x^4 + 16x^2 - 72x + 9 = 0$ has

A. exactly two distinct real roots
B. exactly three distinct real roots
C. exactly four distinct real roots
D. six different real roots

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isi2004 polynomials maxima-minima

Answer

4.10.9 Polynomials: TIFR2012-A-12 top

https://gateoverflow.in/25035

For the polynomial $p(x) = 8x^{10} - 7x^3 + x - 1$ consider the following statements (which may be true or false)

i. It has a root between $[0, 1]$.
ii. It has a root between $[0, -1]$.
iii. It has no roots outside $(-1, 1)$.

Which of the above statements are true?

A. Only (i).
B. Only (i) and (ii).
C. Only (i) and (iii).
D. Only (ii) and (iii).
E. All of (i), (ii) and (iii).

tifr2012 set-theory&algebra polynomials

Answer

Answers: Polynomials

4.10.1 Polynomials: GATE1987-1-xxii top

https://gateoverflow.in/80379

Selected Answer

Since, the polynomial has highest degree 7. So there are 7 roots possible for it

Now suppose if an imaginary number $a + bi$ is also root of this polynomial then $a - bi$ will also be the root of this polynomial

That means there must be even number of complex root possible becoz they occur in pair.

Now we will solve this question option wise

A) All complex root

This is not possible. The polynomial has 7 roots and as I mention a polynomial should have even number of complex root and 7 is not even. So this option is wrong

B) At least one real root

This is possible. Since polynomial has 7 roots and only even number of complex root is possible, that means this polynomial has max 6 complex roots and Hence minimum one real root. So this option is correct

C) Four pairs of imaginary roots

4 pair means 8 complex root. But this polynomial can have at most 7 roots. So this option is also wrong

Hence answer should be B)

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12 votes -- Digvijaysingh Gautam (9k points)

4.10.2 Polynomials: GATE1995-2.8 top

https://gateoverflow.in/2620

Selected Answer

The given equation is: $(x-1)^3 = -8$

or, $$((x-1)/-2)^3 = 1$$

Let $((x-1)/-2) = Z$

So, the equation will be changed to: $Z^3 = 1$
The roots will be $Z = 1, \omega, \omega^2$

Putting back the value of Z

$$(x-1)/-2 = 1, \omega, \omega^2$$
$$\implies x = -1, 1-2\omega, 1-2\omega^2$$

10 votes -- Vivek Srivastava (3.6k points)

Answer is C,
Just put values of C in place of x. iIt will satisfy the equation.

13 votes -- jayendra (8.3k points)

4.10.3 Polynomials: GATE1997-4.4 top

https://gateoverflow.in/2245

Selected Answer

Lets take $p(x) = ax + b$
Now, $p(0) = 5 \implies b = 5.$

$p(1) = 4 \implies a + b = 4, a = -1$
$p(2) = 9 \implies 4a + b = 9 \implies -4 + 5 = 9$, which is false. So, degree 1 is not possible.

Let $p(x) = ax^2 + bx + c$

$p(0) = 5 \implies c = 5$
$p(1) = 4 \implies a + b + c = 4 => a + b = -1 \quad \rightarrow (1)$
$p(2) = 9 \implies 4a + 2b + c = 9 => 2a + b = 2 \quad \rightarrow (2)$

$(2) - (1) \implies a = 3, b = -1 - 3 = -4$

$p(3) = 20 \implies 9a + 3b + c = 20, 27 - 12 + 5 = 20$, equation holds.

So, degree 2 also will suffice.

27 votes -- Arjun Suresh (348k points)

4.10.4 Polynomials: GATE2000-2.4 top

https://gateoverflow.in/651

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Selected Answer

yes, option D is the correct answer.

Here is how $p(x)$ should look like:

Value of $P(x)$ will be zero at green circled (O) points,So they will be the roots of the polynomial $P(x)$.
Hence, the minimum degree of $P(x)$ will be 4.

28 votes -- Anurag Pandey (13.6k points)

4.10.5 Polynomials: GATE2014-2-5 top

https://gateoverflow.in/1957

Selected Answer

The roots are $x = 1, x = 2$, and $x = 3$.

So polynomial is $f(x) = (x-1)(x-2)(x-3)$

$f(0) = -6, f(4) = 6$

So $f(0)f(4) < 0$.

43 votes -- Happy Mittal (11.2k points)

4.10.6 Polynomials: GATE2017-2-24 top

https://gateoverflow.in/118185

Selected Answer

Let $ax^2 + bx + c = 0$ be a quadratic equation, then

Sum of roots $= \frac{-b}{a}$ and product of roots $= \frac{c}{a}$

$(5)_b + (6)_b = (13)_b \Rightarrow b = 8$

and $(5)_b * (6)_b = (36)_b$ means $30 = 3b + 6$. So, $b = 8$

30 votes -- Manish Joshi (27.9k points)

4.10.7 Polynomials: ISI2004-MIII top

https://gateoverflow.in/123818

Selected Answer

$S^3 + 3S^2 - 6S + 2 = 0$
$(S-1)(S^2 + 4S - 2) = 0$

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$S_1 = 1 \quad S_2, S_3 = \frac{-4 \pm \sqrt{16+8}}{2}$

$= -2 \pm \sqrt{6}$

$S_2 = -2 - \sqrt{6} \qquad = -4.4 \qquad S_3 = -2 + \sqrt{6} \qquad = .44$

$S_1, S_2 \in [0, 1]$ **Hence,exactly two solutions.**

c is ans.

5 votes -- 2018 (6.8k points)

4.10.8 Polynomials: ISI2004-MIII-7 top

https://gateoverflow.in/123770

Selected Answer

P(x) = x^6 – $5x^4$+ $16x^2$ –$72x + 9$

- $P(0) = 9$
- $P(-1) = -ve$
- $P(4) = +ve$

Hence, at least 2 real roots can be clearly seen, but what about other 4 roots left ?

That's why we check $P^n\ (x)$,

$P'(x)$ = $6x^5$ - $20x^3$ + $32x$–72

$P''(x) = 30x^4$ - $60x^2$+$32 > 0$ for any real value of x.

comparing this with a quadratic eq taking x^2as y we get $30y^2 - 60y + 32$. The discriminant $(b^2 - 4ac)$ is negative implying $P''(x)$ has no real roots

Hence by Rolle's theorem $P'(x)$ can have at most 1 real root and $P(x)$ can have at most 2 real roots. Because if a function $f(x)$ has 2 roots x_1 and x_2 then there exists a point $x \in [x1, x2]$ where the curve becomes flat i.e its the root of $f'(x)$ meaning $f(x)$ has max 2 roots

=> **Exactly two distinct real roots**

Option A

5 votes -- Heisenberg (2.3k points)

4.10.9 Polynomials: TIFR2012-A-12 top

https://gateoverflow.in/25035

Selected Answer

At $f(0)$ it is negative, at $f(1)$ it is positive, and at $f(-1)$ it is positive, which means there will be roots between $(0, 1)$ and $(-1, 0)$. Any values below -1 and above 1 will always yield positive values for $f(x)$, which means no roots available.

9 votes -- Shaun Patel (6.9k points)

4.11 Relations(32) top

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4.11.1 Relations: CMI2015-A-02 top

https://gateoverflow.in/216454

A binary relation $R \subseteq (S \times S)$ is said to be Euclidean if for every $a, b, c \in S, (a, b) \in R$ and $(a, c) \in R$ implies $(b, c) \in R$. Which of the following statements is valid?

A. If R is Euclidean, $(b, a) \in R$ and $(c, a) \in R$, then $(b, c) \in R$, for every $a, b, c \in S$
B. If R is reflexive and Euclidean, $(a, b) \in R$ implies $(b, a) \in R$, for every $a, b \in S$
C. If R is Euclidean, $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for every $a, b, c \in S$
D. None of the above.

cmi2015 relations set-theory&algebra

Answer

4.11.2 Relations: GATE1987-2d top

https://gateoverflow.in/80583

State whether the following statements are TRUE or FALSE:

The union of two equivalence relations is also an equivalence relation.

gate1987 discrete-mathematics relations descriptive

Answer

4.11.3 Relations: GATE1987-9a top

https://gateoverflow.in/82436

How many binary relations are there on a set A with n elements?

gate1987 set-theory&algebra relations descriptive

Answer

4.11.4 Relations: GATE1989-1-iv top

https://gateoverflow.in/87048

The transitive closure of the relation $\{(1, 2), (2, 3), (3, 4), (5, 4)\}$ on the set $\{1, 2, 3, 4, 5\}$ is _____.

gate1989 set-theory&algebra relations descriptive

Answer

4.11.5 Relations: GATE1994-2.3 top

https://gateoverflow.in/2470

Amongst the properties $\{\text{reflexivity}, \text{symmetry}, \text{anti-symmetry}, \text{transitivity}\}$ the relation $R = \{(x, y) \in N^2 | x \neq y\}$ satisfies _____

gate1994 set-theory&algebra normal relations descriptive

Answer

4.11.6 Relations: GATE1995-1.19 top

https://gateoverflow.in/2606

Let R be a symmetric and transitive relation on a set A. Then

A. R is reflexive and hence an equivalence relation
B. R is reflexive and hence a partial order
C. R is reflexive and hence not an equivalence relation
D. None of the above

gate1995 set-theory&algebra relations normal

Answer

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4.11.7 Relations: GATE1996-2.2 top

https://gateoverflow.in/2731

Let R be a non-empty relation on a collection of sets defined by ${}_AR_B$ if and only if $A \cap B = \phi$. Then, (pick the true statement)

A. A is reflexive and transitive

B. R is symmetric and not transitive

C. R is an equivalence relation

D. R is not reflexive and not symmetric

gate1996 set-theory&algebra relations normal

Answer

4.11.8 Relations: GATE1996-8 top

https://gateoverflow.in/2760

Let F be the collection of all functions $f : \{1,2,3\} \rightarrow \{1,2,3\}$. If f and $g \in F$, define an equivalence relation $\sim$ by $f \sim g$ if and only if $f(3) = g(3)$.

a. Find the number of equivalence classes defined by $\sim$.
b. Find the number of elements in each equivalence class.

gate1996 set-theory&algebra relations functions normal descriptive

Answer

4.11.9 Relations: GATE1997-14 top

https://gateoverflow.in/2274

Let R be a reflexive and transitive relation on a set A. Define a new relation E on A as

$$E = \{(a,b) \mid (a,b) \in R \text{ and } (b,a) \in R\}$$

Prove that E is an equivalence relation on A.

Define a reason $\leq$ on the equivalence classes of E as $E_1 \leq E_2$ if $\exists a, b$ such that $a \in E_1, b \in E_2$ and $(a,b) \in R$. Prove that $\leq$ is a partal order.

gate1997 set-theory&algebra relations normal proof descriptive

Answer

4.11.10 Relations: GATE1997-6.3 top

https://gateoverflow.in/2259

The number of equivalence relations of the set $\{1,2,3,4\}$ is

A. 15
B. 16
C. 24
D. 4

gate1997 set-theory&algebra relations normal

Answer

4.11.11 Relations: GATE1998-1.6 top

https://gateoverflow.in/1643

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Suppose A is a finite set with n elements. The number of elements in the largest equivalence relation of A is

A. n
B. n^2
C. 1
D. $n+1$

gate1998 set-theory&algebra relations easy

Answer

4.11.12 Relations: GATE1998-1.7 top https://gateoverflow.in/1644

Let R_1 and R_2 be two equivalence relations on a set. Consider the following assertions:

i. $R_1 \cup R_2$ is an equivalence relation
ii. $R_1 \cap R_2$ is an equivalence relation

Which of the following is correct?

A. Both assertions are true
B. Assertions (i) is true but assertions (ii) is not true
C. Assertions (ii) is true but assertions (i) is not true
D. Neither (i) nor (ii) is true

gate1998 set-theory&algebra relations normal

Answer

4.11.13 Relations: GATE1998-10 top https://gateoverflow.in/1724

a. Prove by induction that the expression for the number of diagonals in a polygon of n sides is $\frac{n(n-3)}{2}$

b. Let R be a binary relation on $A = \{a, b, c, d, e, f, g, h\}$ represented by the following two component digraph. Find the smallest integers m and n such that $m < n$ and $R^m = R^n$.

gate1998 set-theory&algebra descriptive relations

Answer

4.11.14 Relations: GATE1998-2.3 top https://gateoverflow.in/1675

The binary relation $R = \{(1,1),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4)\}$on the set $A = \{1,2,3,4\}$ is

A. reflective, symmetric and transitive

B. neither reflective, nor irreflexive but transitive

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C. irreflexive, symmetric and transitive

D. irreflexive and antisymmetric

gate1998 set-theory&algebra easy relations

Answer

4.11.15 Relations: GATE1999-1.2 top https://gateoverflow.in/1456

The number of binary relations on a set with n elements is:

A. n^2

B. 2^n

C. 2^{n^2}

D. None of the above

gate1999 set-theory&algebra relations permutations-and-combinations easy

Answer

4.11.16 Relations: GATE1999-2.3 top https://gateoverflow.in/1481

Let L be a set with a relation R which is transitive, anti-symmetric and reflexive and for any two elements $a, b \in L$, let the least upper bound $lub(a, b)$ and the greatest lower bound $glb(a, b)$ exist. Which of the following is/are true?

A. L is a poset

B. L is a Boolean algebra

C. L is a lattice

D. None of the above

gate1999 set-theory&algebra normal relations

Answer

4.11.17 Relations: GATE1999-3 top https://gateoverflow.in/1522

a. Mr. X claims the following:

   If a relation R is both symmetric and transitive, then R is reflexive. For this, Mr. X offers the following proof:

   "From xRy, using symmetry we get yRx. Now because R is transitive xRy and yRx together imply xRx. Therefore, R is reflexive".

b. Give an example of a relation R which is symmetric and transitive but not reflexive.

gate1999 set-theory&algebra relations normal descriptive

Answer

4.11.18 Relations: GATE2000-2.5 top https://gateoverflow.in/652

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A relation R is defined on the set of integers as xRy iff $(x+y)$ is even. Which of the following statements is true?

A. R is not an equivalence relation
B. R is an equivalence relation having 1 equivalence class
C. R is an equivalence relation having 2 equivalence classes
D. R is an equivalence relation having 3 equivalence classes

gate2000 set-theory&algebra relations normal

Answer

4.11.19 Relations: GATE2001-1.2 top

https://gateoverflow.in/695

Consider the following relations:

- R1 (a,b) iff $(a+b)$ is even over the set of integers
- R2 (a,b) iff $(a+b)$ is odd over the set of integers
- R3 (a,b) iff $a.b>0$ over the set of non-zero rational numbers
- R4 (a,b) iff $|a-b|\leq 2$ over the set of natural numbers

Which of the following statements is correct?

A. R1 and R2 are equivalence relations, R3 and R4 are not
B. R1 and R3 are equivalence relations, R2 and R4 are not
C. R1 and R4 are equivalence relations, R2 and R3 are not
D. R1, R2, R3 and R4 all are equivalence relations

gate2001 set-theory&algebra normal relations

Answer

4.11.20 Relations: GATE2002-2.17 top

https://gateoverflow.in/847

The binary relation $S=\phi(\text{empty set})$ on a set $A=\{1,2,3\}$ is

A. Neither reflexive nor symmetric
B. Symmetric and reflexive
C. Transitive and reflexive
D. Transitive and symmetric

gate2002 set-theory&algebra normal relations

Answer

4.11.21 Relations: GATE2002-3 top

https://gateoverflow.in/856

Let A be a set of $n(>0)$ elements. Let N_r be the number of binary relations on A and let N_f be the number of functions from A to A

A. Give the expression for N_r, in terms of n.
B. Give the expression for N_f, terms of n.
C. Which is larger for all possible n, N_r or N_f

gate2002 set-theory&algebra normal descriptive relations

Answer

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4.11.22 Relations: GATE2004-24 top

https://gateoverflow.in/1021

Consider the binary relation:

$S = \{(x, y) \mid y = x + 1 \text{ and } x, y \in \{0, 1, 2\}\}$

The reflexive transitive closure is S is

A. $\{(x, y) \mid y > x \text{ and } x, y \in \{0, 1, 2\}\}$

B. $\{(x, y) \mid y \geq x \text{ and } x, y \in \{0, 1, 2\}\}$

C. $\{(x, y) \mid y < x \text{ and } x, y \in \{0, 1, 2\}\}$

D. $\{(x, y) \mid y \leq x \text{ and } x, y \in \{0, 1, 2\}\}$

gate2004 set-theory&algebra easy relations

Answer

4.11.23 Relations: GATE2004-IT-4 top

https://gateoverflow.in/3645

Let R_1 be a relation from $A = \{1, 3, 5, 7\}$ to $B = \{2, 4, 6, 8\}$ and R_2 be another relation from B to C = {1, 2, 3, 4} as defined below:

i. An element x in A is related to an element y in B (under R_1) if $x + y$ is divisible by 3.
ii. An element x in B is related to an element y in C (under R_2) if $x + y$ is even but not divisible by 3.

Which is the composite relation $R_1 R_2$ from A to C?

A. $R_1 R_2$ = {(1, 2), (1, 4), (3, 3), (5, 4), (7, 3)}
B. $R_1 R_2$ = {(1, 2), (1, 3), (3, 2), (5, 2), (7, 3)}
C. $R_1 R_2$ = {(1, 2), (3, 2), (3, 4), (5, 4), (7, 2)}
D. $R_1 R_2$ = {(3, 2), (3, 4), (5, 1), (5, 3), (7, 1)}

gate2004-it set-theory&algebra relations normal

Answer

4.11.24 Relations: GATE2005-42 top

https://gateoverflow.in/1167

Let R and S be any two equivalence relations on a non-empty set A. Which one of the following statements is TRUE?

A. $R \cup S$, $R \cap S$ are both equivalence relations
B. $R \cup S$ is an equivalence relation
C. $R \cap S$ is an equivalence relation
D. Neither $R \cup S$ nor $R \cap S$ are equivalence relations

gate2005 set-theory&algebra normal relations

Answer

4.11.25 Relations: GATE2005-7 top

https://gateoverflow.in/1349

The time complexity of computing the transitive closure of a binary relation on a set of n elements is known to be:

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A. $O(n)$

B. $O(n \log n)$

C. $O\left(n^{\frac{3}{2}}\right)$

D. $O\left(n^3\right)$

gate2005 set-theory&algebra normal relations

Answer

4.11.26 Relations: GATE2006-4 top https://gateoverflow.in/883

A relation R is defined on ordered pairs of integers as follows:

$$(x,y)R(u,v) \text{ if } x < u \text{ and } y > v$$

Then R is:

A. Neither a Partial Order nor an Equivalence Relation
B. A Partial Order but not a Total Order
C. A total Order
D. An Equivalence Relation

gate2006 set-theory&algebra normal relations

Answer

4.11.27 Relations: GATE2007-2 top https://gateoverflow.in/1201

Let S be a set of n elements. The number of ordered pairs in the largest and the smallest equivalence relations on S are:

A. n and n
B. n^2 and n
C. n^2 and 0
D. n and 1

gate2007 set-theory&algebra normal relations

Answer

4.11.28 Relations: GATE2009-4 top https://gateoverflow.in/797

Consider the binary relation $R = \{(x,y),(x,z),(z,x),(z,y)\}$ on the set $\{x,y,z\}$. Which one of the following is **TRUE**?

A. R is symmetric but NOT antisymmetric
B. R is NOT symmetric but antisymmetric
C. R is both symmetric and antisymmetric
D. R is neither symmetric nor antisymmetric

gate2009 set-theory&algebra easy relations

Answer

4.11.29 Relations: GATE2010-3 top https://gateoverflow.in/1149

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What is the possible number of reflexive relations on a set of 5 elements?

A. 2^{10}
B. 2^{15}
C. 2^{20}
D. 2^{25}

gate2010 set-theory&algebra easy relations

Answer

4.11.30 Relations: GATE2015-2-16 top

https://gateoverflow.in/8089

Let R be the relation on the set of positive integers such that aRb and only if a and b are distinct and let have a common divisor other than 1. Which one of the following statements about R is true?

R is symmetric and reflexive but not transitive

R is reflexive but not symmetric not transitive

R is transitive but not reflexive and not symmetric

R is symmetric but not reflexive and not transitive

gate2015-2 set-theory&algebra relations normal

Answer

4.11.31 Relations: GATE2015-3-41 top

https://gateoverflow.in/8500

Let R be a relation on the set of ordered pairs of positive integers such that $((p,q),(r,s)) \in R$ if and only if $p-s=q-r$. Which one of the following is true about R?

A. Both reflexive and symmetric
B. Reflexive but not symmetric
C. Not reflexive but symmetric
D. Neither reflexive nor symmetric

gate2015-3 set-theory&algebra relations normal

Answer

4.11.32 Relations: GATE2016-2-26 top

https://gateoverflow.in/39603

A binary relation R on $\mathbb{N} \times \mathbb{N}$ is defined as follows: $(a,b)R(c,d)$ if $a \leq c$ or $b \leq d$. Consider the following propositions:

P : R is reflexive.

Q : R is transitive.

Which one of the following statements is **TRUE**?

A. Both P and Q are true.
B. P is true and Q is false.
C. P is false and Q is true.
D. Both P and Q are false.

gate2016-2 set-theory&algebra relations normal

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Answer

Answers: Relations

4.11.1 Relations: CMI2015-A-02 top

https://gateoverflow.in/216454

Selected Answer

Lets see option B

If R is reflexive then (a,a) belongs to R

It is given that (a,b) belongs to R

So, $(a,b) \in R$ and $(a,a) \in R$ that implies $(b,a) \in R$ for all $a,b \in S$ [since R is euclidean]

Thus option B is the correct answer.

2 votes -- Kushagra Chatterjee (8.2k points)

4.11.2 Relations: GATE1987-2d top

https://gateoverflow.in/80583

Selected Answer

No union of two equivalence relation may not be equivalence relation because of transitive dependency.

equivalence relation : satisfy Reflexive, symmtric and transitive property

Reflexive $\cup$ Reflexive = Reflexive

Symmtric $\cup$ Symmtric = Symmtric

Transitive $\cup$ Transitive $\neq$ Transitive why

Example : $R = \{ (1,2) , (3,4))$.....$S = (2,3)$

Union = $(1,2),(3,4),(2,3)$ which is not transitive i.e. $(1,3)$ and $(2,4)$ is missing.

So, False is answer.

11 votes -- Prashant Singh (59.8k points)

4.11.3 Relations: GATE1987-9a top

https://gateoverflow.in/82436

Selected Answer

Total number of binary relation from n element set to itself is 2^{n^2} I.e.n^2 entries with two choice take it or not.

14 votes -- Prashant Singh (59.8k points)

4.11.4 Relations: GATE1989-1-iv top

https://gateoverflow.in/87048

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Selected Answer

Transitive closure of R

1. It is transitive.

2. It contains R.

3. It is minimal satisfies 1 and 2.

$R = (1,2),(2,3),(3,4),(5,4)$

The transitive closure of the relation R = **{(1,2),(2,3),(1,3) ,(3,4),(2,4),(1,4),(5,4)}**

4 votes -- Hemant Parihar (14.8k points)

4.11.5 Relations: GATE1994-2.3 top

https://gateoverflow.in/2470

Selected Answer

It is not reflexive as xRx is not possible.
It is symmetric as if xRy then yRx.
It not antisymmetric as xRy and yRx are possible and we can have $x \neq y$.
It is not transitive as if xRy and yRz then xRz need not be true. This is violated when $z = x$.

So, symmetry is the answer.

19 votes -- Rajarshi Sarkar (34.1k points)

4.11.6 Relations: GATE1995-1.19 top

https://gateoverflow.in/2606

Selected Answer

Answer is D.

Let $A = 1,2,3$ and relation $R = (1,2),(2,1),(1,1),(2,2)$. R is symmetric and transitive but not reflexive.($(3,3)$ is not there.)

10 votes -- Anu (5.9k points)

4.11.7 Relations: GATE1996-2.2 top

https://gateoverflow.in/2731

Selected Answer

Let $A = \{1,2,3\}$ and $B = \{4,5\}$ and $C = \{1,6,7\}$

now $A \cap B = \emptyset$ and $B \cap C = \emptyset$ but $A \cap C \neq \emptyset$, so R is non transitive.

$A \cap A = A$, so R is not reflexive.

$A \cap B = B \cap A$, so R is symmetric

So, A is false as R is not reflexive or transitive

B is true.

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C is false because R is not trasitive or reflexive

D is false because R is symmetric

17 votes -- akash (1k points)

4.11.8 Relations: GATE1996-8 top

https://gateoverflow.in/2760

Selected Answer

Total number of functions = $3 * 3 * 3 = 27$ as each of $1, 2$, and 3 has 3 choice to map to.

Now, for the equivalence relation, we need the mapping of 3 to be fixed. i.e., two functions f and g are related if and only if $f(3) = g(3)$. So, with 3 -> 1, we can get 3 * 3 = 9 functions as 2 and 3 have 3 choices to map to each, and similarly 9 each for 3 -> 2 and 3 -> 3.

a. So, total number of equivalence classes $= 3$, one each for $3 -> 1, 3 -> 2$, and $3 -> 3$.

b. Number of elements (elements here are functions) in each equivalence class $= 9$.

16 votes -- Arjun Suresh (348k points)

4.11.9 Relations: GATE1997-14 top

https://gateoverflow.in/2274

Selected Answer

1. Since it is given that relation R is reflexive and transitive...and the new defined relation is the defination of symmectric only

So, it is reflexive,symmetric and transitive => equivalance.

2. P**artial order** is a binary relation "≤" over a set *P* which is reflexive, antisymmetric, and transitive

- *$a ≤ a$* (reflexivity);
- if *a* ≤ *b* and *b* ≤ *a*, then *a* = *b* (antisymmetry);
- if *a* ≤ *b* and *b* ≤ *c*, then *a* ≤ *c* (transitivity).

4 votes -- asutosh kumar Biswal (10.8k points)

4.11.10 Relations: GATE1997-6.3 top

https://gateoverflow.in/2259

Selected Answer

No. of Equivalence Relations is given by BELL number.

The *n*th of these numbers,B_n, counts the number of different ways to partition a set that has exactly $*n$ elements, or equivalently, the number of equivalence relations on it.

Ref: https://en.wikipedia.org/wiki/Bell_number

1 - Equivalence relation with 1 element

1 **2** - Equivalence relation with 2 elements

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2 3 **5** - Equivalence relation with 3 elements

5 7 10 **15** - Equivalence relation with 4 elements

20 votes -- Akash Kanase (42.5k points)

4.11.11 Relations: GATE1998-1.6 top

https://gateoverflow.in/1643

Selected Answer

Answer is B.

The largest equivalence relation will be when every element is related to every other element. So, $n \times n = n^2$ possible ordered pairs.

12 votes -- Keith Kr (6.1k points)

4.11.12 Relations: GATE1998-1.7 top

https://gateoverflow.in/1644

Selected Answer

Answer: C

$R1$ intersection $R2$ is equivalence relation..
$R1$ union $R2$ is not equivalence relation because transitivity needn't hold. For example, (a, b) can be in $R1$ and (b, c) be in $R2$ and (a, c) not in either $R1$ or $R2$.

16 votes -- Digvijay (54.8k points)

4.11.13 Relations: GATE1998-10 top

https://gateoverflow.in/1724

Selected Answer

Part A

Statement:

no_of_diagonal : $D(n) = \frac{n*(n-3)}{2}$

Step 1) Basis : for $n = 4$, $\frac{4*(4-3)}{2} = 2$ is true.

Step 2) Inductive Step :

If $D(k)$ is true. we need to prove $D(k+1)$ is also true.

We add one more vertex to the set of k vertices. Assume added vertex is C.

Further assume C will connects vertex A and B to close the polygon. (A and B already exists in k sided polygon)

From C, no of pairs to each k vertices = k, out of these two will be used to close the polygon, i.e. used as sides of new $(k+1)$ sided polygon. Further more, initial connection (edge or side) between A and B is now becomes a diagonal.

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No of Diagonals in for $(k+1)$ sided polygon = diagonal from k sided polygon + $k-2+1$

$\frac{k*(k-3)}{3}+k-1=\frac{k^2-k-2}{2}=\frac{(k+1)(k-2)}{2}\frac{(k+1)((k+1)-3)}{2}$

$\implies D(k+1)$ holds.

Since, both the basis and the inductive step have been performed, by mathematical induction, the statement $D(n)$ holds for all natural numbers $n>3$.

8 votes -- Debashish Deka (56.6k points)

4.11.14 Relations: GATE1998-2.3 top

https://gateoverflow.in/1675

Selected Answer

Not reflexive - $(4,4)$ not present.

Not irreflexive - $(1,1)$ is present.

Not symmetric - $(2,1)$ is present but not $(1,2)$.

Not antisymmetric - $(2,3)$ and $(3,2)$ are present.

Not Asymmetric - asymmetry requires both antisymmetry and irreflexivity

It is transitive so the correct option is B.
transitive.

18 votes -- Digvijay (54.8k points)

4.11.15 Relations: GATE1999-1.2 top

https://gateoverflow.in/1456

Selected Answer

Answer: C

In a binary relation two elements are chosen from the set. So, with n elements n^2 pairings are possible. Now, a relation can be any subset of these n^2 pairings and thus we get 2^{n^2} binary relations.

13 votes -- Rajarshi Sarkar (34.1k points)

4.11.16 Relations: GATE1999-2.3 top

https://gateoverflow.in/1481

Selected Answer

Which of the following **is/are** true? This is question with Multiple answers.

As our Relation R on Set L is Reflexive , anti symmetric & Transitive it is poset.

Since LUB & GLB exists for any two elements it is lattice.

Answer -> A & C.

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B is not guaranteed to be true.

Ref: http://uosis.mif.vu.lt/~valdas/PhD/Kursinis2/Sasao99/Chapter2.pdf

8 votes -- Akash Kanase (42.5k points)

4.11.17 Relations: GATE1999-3 top

https://gateoverflow.in/1522

Selected Answer

Let set A be $1, 2, 3$, and let a relation R on A be

$\{(1,1),(1,2),(2,1),(2,2)\}$

R is both symmetric and transitive, but not reflexive. The key point here is that there may be some element in set A which is not related to any of the element in R, but to be reflexive, all elements must be related to themselves.

18 votes -- Happy Mittal (11.2k points)

4.11.18 Relations: GATE2000-2.5 top

https://gateoverflow.in/652

Selected Answer

R is reflexive as $(x+x)$ is even for any integer.

R is symmetric as if $(x+y)$ is even $(y+x)$ is also even.

R is transitive as if $(x+(y+z))$ is even, then $((x+y)+z)$ is also even.

So, R is an equivalence relation.

For set of natural numbers, sum of even numbers always give even, sum of odd numbers always give even and sum of any even and any odd number always give odd. So, R must have two equivalence classes -one for even and one for odd.

$\{\ldots,-4,-2,0,2,4,\ldots\},\{\ldots,-3,-1,1,3,\ldots,\}$

C choice.

27 votes -- Anu (5.9k points)

4.11.19 Relations: GATE2001-1.2 top

https://gateoverflow.in/695

Selected Answer

$R1)$ Reflexive : $a+a=2a$ always even

Symmetric: either (a,b) both must be odd or both must be even to have sum as even

Therefore, if(a,b) then definitely (b,a)

Transitive: if(a,b) and (b,c) , then both of them must be even pairs or odd pairs and therefore (a,c) is even

$R2)$ Reflexive : $a+a=2a$ cant be odd ever

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$R3)$ Reflexive: $a.a > 0$

Symmetric: if $a, b > 0$ then both must be +ve or -ve, which means $b.a > 0$ also exists

Transitive : if $a.b > 0$ and $b.c > 0$ then to have b as same number, both pairs must be +ve or -ve which implies $a.c > 0$

$R4)$ Reflexive: $|a - a| \leq 2$

Symmetric: if $|a - b| \leq 2$ definitely $|b - a| \leq 2$ when a, b are natural numbers

Transitive: $|a - b| \leq 2$ and $|b - c| \leq 2$, does not imply $|a - c| \leq 2$

Ex: $|4 - 2| \leq 2$ and $|2 - 0| \leq 2$, but $|4 - 0| > 2$,

Hence, $R2$ and $R4$ are not equivalence.

Answer is B.

12 votes -- confused_luck (901 points)

4.11.20 Relations: GATE2002-2.17 top https://gateoverflow.in/847

Selected Answer

Answer is **D.**

$S = \emptyset$ (empty set) on a set $A = \{1, 2, 3\}$ is Irreflexive, Symmetric, Anti Symmetric, Asymmetric, Transitive..
but it is not Reflexive.

21 votes -- Digvijay (54.8k points)

4.11.21 Relations: GATE2002-3 top https://gateoverflow.in/856

Selected Answer

A. Consider a $2D$ matrix of $n \times n$ size where each dimension represent the n elements of A. So, we get n^2 elements where each one can be viewed as an ordered pair $(i, j), 1 \leq i, j \leq n$. Now if we consider a set with these n^2 elements, any subset of it will be a binary relation from A to A.

So, no. of binary relations, $N_r = 2^{n^2}$

For a function from a set of n elements to a set of m elements we have m choice for each of the n elements of the domain. Thus we get m^n possibilities. Here, $m = n$. So,

No. of functions, $N_f = n^n$

Now, number of functions must be smaller than the number of relations because every function is also a relation. We can prove this formally as follows:

$$\lim_{n \to \infty} \frac{2^{n^2}}{n^n}$$

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$\frac{\infty}{\infty}$ form. So, applying L'Hopital's rule (Limit remains same after taking derivative of both numerator and denominator)

$$= \lim_{n\to\infty} \frac{2n.\log 2.2^{n^2}}{nn^{n-1}}$$

We can continue like this applying L'Hopital's rule and we should eventually get 1 in the denominator (as after each derivation power of n decreases by 1) which gives the limit value as ∞. So, 2^{n^2} is faster growing than n^n.

So, for sufficiently large n, $N_r > N_f$

21 votes -- Digvijay (54.8k points)

4.11.22 Relations: GATE2004-24 top https://gateoverflow.in/1021

Selected Answer

Option b. Transitive means, x is related to all greater y (as every x is related to $x+1$) and reflexive means x is related to x.

12 votes -- anshu (3.3k points)

4.11.23 Relations: GATE2004-IT-4 top https://gateoverflow.in/3645

Selected Answer

Answer is C.

Explanation:

$R1 = (1,2),(1,8),(3,6),(5,4),(7,2),(7,8)$

$R2 = (2,2),(4,4),(6,2),(6,4),(8,2)$

So, $R1R2 = (1,2),(3,2),(3,4),(5,4),(7,2)$

18 votes -- chetna (383 points)

4.11.24 Relations: GATE2005-42 top https://gateoverflow.in/1167

Selected Answer

RUS might not be transitive. Say (a,b) be present in R and (b,c) be present in S and (a,c) not present in either, RUS will contain, (a,b) and (b,c) but not (a,c) and hence not transitive.

option c.

14 votes -- anshu (3.3k points)

4.11.25 Relations: GATE2005-7 top https://gateoverflow.in/1349

Selected Answer

Answer D

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Calculating Transitive Closure boils down To Matrix Multiplication.

We can do Matrix Multiplication in $O(n^3)$. There are better algo that do less than cubic time , but we can not surely do matrix multiplication in

A) $O(N)$

B) $O(Nlogn)$

C) $O(N^{1.5})$

18 votes -- Akash Kanase (42.5k points)

4.11.26 Relations: GATE2006-4 top

https://gateoverflow.in/883

Selected Answer

Answer is (A). Because the relation is not reflexive which is a necessary condition for both partial order and equivalence relation..!!

PS: For a relation to be reflexive $R(a,a)$ must hold for all possible a.

20 votes -- Vicky Bajoria (5k points)

4.11.27 Relations: GATE2007-2 top

https://gateoverflow.in/1201

Selected Answer

Answer is B.

equivalence relation means it is reflexive, symmetric and transitive

and if the relation is reflexive then it must have all the pairs of diagonal elements. and relation with only diagonal elements is also symmetric and transitive. therefore smallest is of size n.

with diagonal elements we can include all the elements therefore largest is n^2

20 votes -- jayendra (8.3k points)

4.11.28 Relations: GATE2009-4 top

https://gateoverflow.in/797

Selected Answer

Answer is D.

A binary relation R over a set X is symmetric if it holds for all a and b in X that if a is related to a.

$\forall_{a,b} \in X, aRb \Rightarrow bRa.$

Here (x,y) is there in R but (y,x) is not there.

$\therefore$ Not Symmetric.

For Antisymmetric Relations: $\forall_{a,b} \in X, R(a,b) \ \& \ R(b,a) \Rightarrow a = b.$

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Here (x, z) is there in R also (z, x) is there violating the antisymmetric rule.

$\therefore$ Not AntiSymmetric.

17 votes -- Sona Praneeth Akula (4.2k points)

4.11.29 Relations: GATE2010-3 top

https://gateoverflow.in/1149

Selected Answer

A relation consists of set of ordered pairs (a, b). Here a can be chosen in n ways and similarly b can be chosen in n ways. So, totally n^2 possible ordered pairs are possible for a relation. Now each of these ordered pair can either be present in the relation or not- 2 possibilities for each of the n^2 pair. So, total number of possible relations =

$$2^{(n^2)}$$

Now, for a relation R to be reflexive, ordered pairs $\{(a, a) \mid a \in S\}$, must be present in R. i.e.; the relation set R must have n ordered pairs fixed. So, number of ordered pairs possible is $n^2 - n$ and hence total number of reflexive relations is equal to

$$2^{(n^2-n)}$$

for $n = 5$, answer will be, $2^{5^2-5} = 2^{20}$

therefore
$option\ C$ is correct

25 votes -- Arjun Suresh (348k points)

4.11.30 Relations: GATE2015-2-16 top

https://gateoverflow.in/8089

Selected Answer

Answer: D

Take $(3, 6)$ and $(6, 2)$ elements of R. For transitivity $(3, 2)$ must be element of R, but 3 and 2 don't have a common divisor and hence not in R.

For any positive integer n, (n, n) is not element of R as only distinct m and n are allowed for (m, n) in R. So, not reflexive also.

34 votes -- Rajarshi Sarkar (34.1k points)

4.11.31 Relations: GATE2015-3-41 top

https://gateoverflow.in/8500

Selected Answer

The key trick here is to realize that the relation is of the form :

{ordered pair, ordered pair} and not simply ordered pair.

Ok, so for reflexive

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$\forall_{a,b}\, if((a,b),(a,b)) \in R \rightarrow$ reflexive

$((a,b),(a,b)) \in R \leftrightarrow (a-b=b-a)$ (not possible for any postive integers b and a)

but that is a contradiction hence it is not reflexive.

now, for symmetric

$((a,b),(c,d)) \in R \rightarrow ((c,d),(a,b)) \in R$

$((a,b),(c,d)) \in R \rightarrow (a-d=b-c)$

$((c,d),(a,b)) \in R$

$\because (c-b=d-a) \leftrightarrow (d-a=c-b) \leftrightarrow (-(a-d)=-(b-c)) \leftrightarrow (a-d=b-c)$

So, it is symmetric.

Hence, **C is the correct option.**

48 votes -- Tamojit Chatterjee (2.3k points)

4.11.32 Relations: GATE2016-2-26 top

https://gateoverflow.in/39603

Selected Answer

(B) Reflexive, but not transitive.

it is "$a \leq c$ OR $b \leq d$",

NOT
"$a \leq c$ AND $b \leq d$"

$(2,5)R(6,3), \quad (6,3)R(1,4)$,but $(2,5)R(1,4)$

42 votes -- Ashish Deshmukh (1.5k points)

4.12 Ring(1) top

4.12.1 Ring: GATE2010-4 top

https://gateoverflow.in/1150

Consider the set $S=\{1,\omega,\omega^2\}$, where ω and ω^2 are cube roots of unity. If $*$ denotes the multiplication operation, the structure $(S,*)$ forms

A. A Group
B. A Ring
C. An integral domain
D. A field

gate2010 set-theory&algebra normal ring groups

Answer

Answers: Ring

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4.12.1 Ring: GATE2010-4 top

https://gateoverflow.in/1150

Selected Answer

Answer: A

Cayley Table

	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

The structure (S,*) satisfies closure property, associativity, commutativity. The structure also has an identity element (i.e. 1) and an inverse for each element. So, the structure is an abelian group.

22 votes -- Rajarshi Sarkar (34.1k points)

4.13 Sets(35) top

4.13.1 Sets: GATE1993-17 top

https://gateoverflow.in/2314

Out of a group of 21 persons, 9 eat vegetables, 10 eat fish and 7 eat eggs. 5 persons eat all three. How many persons eat at least two out of the three dishes?

gate1993 set-theory&algebra easy sets descriptive

Answer

4.13.2 Sets: GATE1993-8.3 top

https://gateoverflow.in/2301

Let S be an infinite set and $S_1 \ldots, S_n$ be sets such that $S_1 \cup S_2 \cup \cdots \cup S_n = S$. Then

A. at least one of the set S_i is a finite set
B. not more than one of the set S_i can be finite
C. at least one of the sets S_i is an infinite
D. not more than one of the sets S_i can be infinite
E. None of the above

gate1993 set-theory&algebra normal sets

Answer

4.13.3 Sets: GATE1993-8.4 top

https://gateoverflow.in/2302

Let A be a finite set of size n. The number of elements in the power set of $A \times A$ is:

A. 2^{2^n}
B. 2^{n^2}
C. $(2^n)^2$
D. $(2^2)^n$
E. None of the above

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Answer

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4.13.4 Sets: GATE1994-2.4 top

https://gateoverflow.in/2471

The number of subsets $\{1, 2, \ldots, n\}$ with odd cardinality is _____

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Answer

4.13.5 Sets: GATE1994-3.8 top

https://gateoverflow.in/2494

Give a relational algebra expression using only the minimum number of operators from $(\cup, -)$ which is equivalent to $R \cap S$.

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Answer

4.13.6 Sets: GATE1994-3.9 top

https://gateoverflow.in/2495

Every subset of a countable set is countable.

State whether the above statement is true or false with reason.

gate1994 set-theory&algebra normal sets descriptive

Answer

4.13.7 Sets: GATE1995-1.20 top

https://gateoverflow.in/2607

The number of elements in the power set $P(S)$ of the set $S = \{\{\phi\}, 1, \{2, 3\}\}$ is:

A. 2
B. 4
C. 8
D. None of the above

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Answer

4.13.8 Sets: GATE1996-1.1 top

https://gateoverflow.in/2705

Let A and B be sets and let A^c and B^c denote the complements of the sets A and B. The set $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to

A. $A \cup B$

B. $A^c \cup B^c$

C. $A \cap B$

D. $A^c \cap B^c$

gate1996 set-theory&algebra easy sets

Answer

4.13.9 Sets: GATE1996-2.4 top

https://gateoverflow.in/2733

Which one of the following is false?

A. The set of all bijective functions on a finite set forms a group under function composition.

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B. The set $\{1, 2, \ldots p-1\}$ forms a group under multiplication mod p, where p is a prime number.
C. The set of all strings over a finite alphabet forms a group under concatenation.
D. A subset $S \neq \emptyset$ of G is a subgroup of the group $\langle G, * \rangle$ if and only if for any pair of elements $a, b \in S, a * b^{-1} \in S$.

gate1996 set-theory&algebra normal sets

Answer

4.13.10 Sets: GATE1998-2.4 top

https://gateoverflow.in/1676

In a room containing 28 people, there are 18 people who speak English, 15, people who speak Hindi and 22 people who speak Kannada. 9 persons speak both English and Hindi, 11 persons speak both Hindi and Kannada whereas 13 persons speak both Kannada and English. How many speak all three languages?

A. 9
B. 8
C. 7
D. 6

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Answer

4.13.11 Sets: GATE2000-2.6 top

https://gateoverflow.in/653

Let $P(S)$ denotes the power set of set S. Which of the following is always true?

A. $P(P(S)) = P(S)$
B. $P(S) \cap P(P(S)) = \{\emptyset\}$
C. $P(S) \cap S = P(S)$
D. $S \notin P(S)$

gate2000 set-theory&algebra easy sets

Answer

4.13.12 Sets: GATE2000-6 top

https://gateoverflow.in/677

Let S be a set of n elements $\{1, 2, \ldots\ldots, n\}$ and G a graph with 2^n vertices, each vertex corresponding to a distinct subset of S. Two vertices are adjacent iff the symmetric difference of the corresponding sets has exactly 2 elements. Note: The symmetric difference of two sets R_1 and R_2 is defined as $(R_1 \setminus R_2) \cup (R_2 \setminus R_1)$

Every vertex in G has the same degree. What is the degree of a vertex in G?

How many connected components does G have?

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Answer

4.13.13 Sets: GATE2001-2.2 top

https://gateoverflow.in/720

Consider the following statements:

- $S1$: There exists infinite sets A, B, C such that $A \cap (B \cup C)$ is finite.
- $S2$: There exists two irrational numbers x and y such that $(x + y)$ is rational.

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Which of the following is true about $S1$ and $S2$?

A. Only $S1$ is correct
B. Only $S2$ is correct
C. Both $S1$ and $S2$ are correct
D. None of $S1$ and $S2$ is correct

gate2001 set-theory&algebra normal sets

Answer

4.13.14 Sets: GATE2001-3 top

https://gateoverflow.in/744

a. Prove that powerset $(A \cap B) = \text{powerset}(A) \cap \text{powerset}(B)$
b. Let $sum(n) = 0 + 1 + 2 + \ldots\ldots + n$ for all natural numbers n. Give an induction proof to show that the following equation is true for all natural numbers m and n:

$$sum(m+n) = sum(m) + sum(n) + mn$$

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Answer

4.13.15 Sets: GATE2004-IT-2 top

https://gateoverflow.in/3643

In a class of 200 students, 125 students have taken Programming Language course, 85 students have taken Data Structures course, 65 students have taken Computer Organization course; 50 students have taken both Programming Language and Data Structures, 35 students have taken both Programming Language and Computer Organization; 30 students have taken both Data Structures and Computer Organization, 15 students have taken all the three courses.

How many students have not taken any of the three courses?

A. 15
B. 20
C. 25
D. 30

gate2004-it set-theory&algebra easy sets

Answer

4.13.16 Sets: GATE2005-8 top

https://gateoverflow.in/1157

Let A, B and C be non-empty sets and let X = (A - B) - C and Y = (A - C) - (B - C). Which one of the following is TRUE?

A. X = Y
B. X ⊂ Y
C. Y ⊂ X
D. None of these

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Answer

4.13.17 Sets: GATE2005-IT-33 top

https://gateoverflow.in/3779

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Let A be a set with n elements. Let C be a collection of distinct subsets of A such that for any two subsets S_1 and S_2 in C, either $S_1 \subset S_2$ or $S_2 \subset S_1$. What is the maximum cardinality of C?

A. n
B. $n+1$
C. $2^{n-1}+1$
D. $n!$

gate2005-it set-theory&algebra normal sets

Answer

4.13.18 Sets: GATE2006-22 top

https://gateoverflow.in/983

Let E, F and G be finite sets. Let

$X = (E \cap F) - (F \cap G)$ and
$Y = (E - (E \cap G)) - (E - F)$.

Which one of the following is true?

A. $X \subset Y$
B. $X \supset Y$
C. $X = Y$
D. $X - Y \neq \phi$ and $Y - X \neq \phi$

gate2006 set-theory&algebra normal sets

Answer

4.13.19 Sets: GATE2006-24 top

https://gateoverflow.in/987

Given a set of elements N = {1, 2, ..., n} and two arbitrary subsets A⊆N and B⊆N, how many of the n! permutations π from N to N satisfy min(π(A)) = min(π(B)), where min(S) is the smallest integer in the set of integers S, and π(S) is the set of integers obtained by applying permutation π to each element of S?

A. $(n - |A \cup B|)|A||B|$
B. $(|A|^2 + |B|^2)n^2$
C. $n!\frac{|A \cap B|}{|A \cup B|}$
D. $\frac{|A \cap B|^2}{{}^n\mathrm{C}_{|A \cup B|}}$

gate2006 set-theory&algebra normal sets

Answer

4.13.20 Sets: GATE2006-IT-23 top

https://gateoverflow.in/3562

Let P, Q and R be sets let Δ denote the symmetric difference operator defined as $P \Delta Q = (PUQ) - (P \cap Q)$. Using Venn diagrams, determine which of the following is/are TRUE?

I. $P \Delta (Q \cap R) = (P \Delta Q) \cap (P \Delta R)$
II. $P \cap (Q \cap R) = (P \cap Q) \Delta (P \Delta R)$

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A. I only
B. II only
C. Neither I nor II
D. Both I and II

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Answer

4.13.21 Sets: GATE2006-IT-24 top

https://gateoverflow.in/3563

What is the cardinality of the set of integers X defined below?

$x = \{n \mid 1 \leq n \leq 123, n$ is not divisible by either $2, 3$ or $5\}$

A. 28
B. 33
C. 37
D. 44

gate2006-it set-theory&algebra normal sets

Answer

4.13.22 Sets: GATE2008-2 top

https://gateoverflow.in/400

If P, Q, R are subsets of the universal set U, then

$$(P \cap Q \cap R) \cup (P^c \cap Q \cap R) \cup Q^c \cup R^c$$

is

A. $Q^c \cup R^c$
B. $P \cup Q^c \cup R^c$
C. $P^c \cup Q^c \cup R^c$
D. U

gate2008 normal set-theory&algebra sets

Answer

4.13.23 Sets: GATE2014-2-50 top

https://gateoverflow.in/2016

Consider the following relation on subsets of the set S of integers between 1 and 2014. For two distinct subsets U and V of S we say $U < V$ if the minimum element in the symmetric difference of the two sets is in U.

Consider the following two statements:

- $S1$: There is a subset of S that is larger than every other subset.
- $S2$: There is a subset of S that is smaller than every other subset.

Which one of the following is CORRECT?

A. Both $S1$ and $S2$ are true
B. $S1$ is true and $S2$ is false
C. $S2$ is true and $S1$ is false
D. Neither $S1$ nor $S2$ is true

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gate2014-2 set-theory&algebra normal sets

Answer

4.13.24 Sets: GATE2014-3-16 top

https://gateoverflow.in/2050

Let Σ be a finite non-empty alphabet and let 2^{Σ^*} be the power set of Σ^*. Which one of the following is **TRUE**?

A. Both 2^{Σ^*} and Σ^* are countable
B. 2^{Σ^*} is countable and Σ^* is uncountable
C. 2^{Σ^*} is uncountable and Σ^* is countable
D. Both 2^{Σ^*} and Σ^* are uncountable

gate2014-3 set-theory&algebra sets normal

Answer

4.13.25 Sets: GATE2015-1-16 top

https://gateoverflow.in/8238

For a set A, the power set of A is denoted by 2^A. If $A = \{5, \{6\}, \{7\}\}$, which of the following options are TRUE?

I. $\phi \in 2^A$
II. $\phi \subseteq 2^A$
III. $\{5, \{6\}\} \in 2^A$
IV. $\{5, \{6\}\} \subseteq 2^A$

A. I and III only
B. II and III only
C. I, II and III only
D. I, II and IV only

gate2015-1 set-theory&algebra sets normal

Answer

4.13.26 Sets: GATE2015-2-18 top

https://gateoverflow.in/8092

The cardinality of the power set of $\{0, 1, 2, \ldots, 10\}$ is _____

gate2015-2 set-theory&algebra sets easy numerical-answers

Answer

4.13.27 Sets: GATE2015-3-23 top

https://gateoverflow.in/8426

Suppose U is the power set of the set $S = \{1, 2, 3, 4, 5, 6\}$. For any $T \in U$, let $|T|$ denote the number of elements in T and T' denote the complement of T. For any $T, R \in U$ let $T \backslash R$ be the set of all elements in T which are not in R. Which one of the following is true?

A. $\forall X \in U, (|X| = |X'|)$
B. $\exists X \in U, \exists Y \in U, (|X| = 5, |Y| = 5$ and $X \cap Y = \phi)$
C. $\forall X \in U, \forall Y \in U, (|X| = 2, |Y| = 3$ and $X \backslash Y = \phi)$
D. $\forall X \in U, \forall Y \in U, (X \backslash Y = Y' \backslash X')$

gate2015-3 set-theory&algebra sets normal

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Answer

4.13.28 Sets: GATE2016-2-28 top

https://gateoverflow.in/39595

Consider a set U of 23 different compounds in a chemistry lab. There is a subset S of U of 9 compounds, each of which reacts with exactly 3 compounds of U. Consider the following statements:

I. Each compound in U \ S reacts with an odd number of compounds.
II. At least one compound in U \ S reacts with an odd number of compounds.
III. Each compound in U \ S reacts with an even number of compounds.

Which one of the above statements is **ALWAYS TRUE**?

A. Only I
B. Only II
C. Only III
D. None.

gate2016-2 set-theory&algebra difficult sets

Answer

4.13.29 Sets: GATE2017-1-47 top

https://gateoverflow.in/118330

The number of integers between 1 and 500 (both inclusive) that are divisible by 3 or 5 or 7 is _____ .

gate2017-1 set-theory&algebra normal numerical-answers sets

Answer

4.13.30 Sets: TIFR2010-A-15 top

https://gateoverflow.in/18394

Let A, B be sets. Let $\bar{A}$ denote the compliment of set A (with respect to some fixed universe), and $(A-B)$ denote the set of elements in A which are not in B. Set $(A-(A-B))$ is equal to:

a. B
b. $A \cap \bar{B}$
c. $A-B$
d. $A \cap B$
e. $\bar{B}$

tifr2010 set-theory&algebra sets

Answer

4.13.31 Sets: TIFR2010-A-18 top

https://gateoverflow.in/18496

Let X be a set of size n. How many pairs of sets (A, B) are there that satisfy the condition $A \subseteq B \subseteq X$?

a. 2^{n+1}
b. 2^{2n}
c. 3^n
d. 2^n+1
e. 3^{n+1}

tifr2010 sets

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Answer

4.13.32 Sets: TIFR2011-A-10 top

https://gateoverflow.in/20039

Let m, n denote two integers from the set $\{1, 2, \ldots, 10\}$. The number of ordered pairs (m, n) such that $2^m + 2^n$ is divisible by 5 is.

a. 10
b. 14
c. 24
d. 8
e. None of the above.

tifr2011 set-theory&algebra sets

Answer

4.13.33 Sets: TIFR2012-A-8 top

https://gateoverflow.in/21007

How many pairs of sets (A, B) are there that satisfy the condition $A, B \subseteq \{1, 2, \ldots, 5\}, A \cap B = \{\}$?

a. 125
b. 127
c. 243
d. 257

tifr2012 set-theory&algebra sets

Answer

4.13.34 Sets: TIFR2016-A-8 top

https://gateoverflow.in/97234

Let A and B be finite sets such that $A \subseteq B$. Then, what is the value of the expression:

$$\Sigma_{C:A \subseteq C \subseteq B} (-1)^{|C \setminus A|},$$

Where $C \setminus A = \{x \in C : x \notin A\}$?

A. Always 0
B. Always 1
C. 0 if $A = B$ and 1 otherwise
D. 1 if $A = B$ and 0 otherwise
E. Depends on the soze of the universe

tifr2016 set-theory&algebra sets

Answer

4.13.35 Sets: TIFR2017-A-10 top

https://gateoverflow.in/95272

For a set A define $P(A)$ to be the set of all subsets of A. For example, if $A = \{1, 2\}$ then $P(A) = \{\emptyset, \{1, 2\}, \{1\}, \{2\}\}$. Let $A \to P(A)$ be a function and A is not empty. Which of the following must be TRUE?

1. f cannot be one-to-one (injective)
2. f cannot be onto (surjective)

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3. f is both one-to-one and onto (bijective)
4. there is no such f possible
5. if such a function f exists, then A is infinite

tifr2017 set-theory&algebra sets functions easy

Answer

Answers: Sets

4.13.1 Sets: GATE1993-17 top

https://gateoverflow.in/2314

Selected Answer

$N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) - N(A \cap C) - N(B \cap C) + N(A \cap B \cap$

Let Y be the no. of persons who eat at least one item. $21 - Y$ people do not eat anything.

$Y = 9 + 10 + 7 - [N(A \cap B) + N(A \cap C) + N(B \cap C)] + 5$

$[N(A \cap B) + N(A \cap C) + N(B \cap C)] = 31 - Y$.

Now, these include the no. of persons who eat all 3 items thrice. So, excluding those, we get, no. of persons who eat at least two items (by adding the no. of persons eating EXACTLY 2 dishes and the number of persons eating all 3 dishes) as

$31 - Y - 2 * 5 = 21 - Y$.

The minimum value of Y is 10 as 10 people eat fish. Is this possible? Yes.

The maximum value of Y is 21. Is this possible? No. Because 5 people eat all three items. So, the no. of persons eating at most 2 items $= (9 - 5) + (10 - 5) + (7 - 5) = 11$. And adding 5 we get 16 people who eat at least one item.

So, our required answer is $21 - 10 \geq X \geq 21 - 16 \implies 5 \leq X \leq 11$.

14 votes -- Rajarshi Sarkar (34.1k points)

4.13.2 Sets: GATE1993-8.3 top

https://gateoverflow.in/2301

Selected Answer

A) at least one of the set S_i is Finite Set. Well it is not said that$S_1, S_2 \ldots S_n$ whether they are finite or infinite. It is possible to break down infinite set into few sets (Some of which can be finite). This seems True, but I'm not able to prove it. **Please Give suitable counterexample here, if you think this is false.**

Ex-: $a^*, \text{this is infinite set. I can write it as} \{\} \cup \{a^*\}$,where $\{a^*\}$ is infinite.

B) Not more than one of set can be finite -> This is false.

ex : $a^*b^* \Rightarrow \{ab\} \cup \{\} \cup \{aa^+bb^+\}$.

C) at least one of the sets is Infinite -> This must be True. As this is finite union of sets, one of set must be infinite to make whole thing infinite. True.

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D) not more than one of the sets S_i can be infinite. This is false.

Ex : $a^*b^* = \{a^pb^q | p = q\} \cup \{a^mb^n | m \neq n\}$ such that $p, q, m, n \geq 0$.

Answer -> C is surely true.

ESi

19 votes -- Akash Kanase (42.5k points)

4.13.3 Sets: GATE1993-8.4 top

https://gateoverflow.in/2302

Selected Answer

Cardinality of $A \times A = n^2$
Cardinality of power set of $A \times A = 2^{n^2}$

18 votes -- Digvijay (54.8k points)

4.13.4 Sets: GATE1994-2.4 top

https://gateoverflow.in/2471

Selected Answer

Answer: 2^{n-1}

No. of subsets with cardinality $i = {}^nC_i$

So, no. of subsets with odd cardinality = $\sum_{i=1,3,\ldots,n-1} {}^nC_i = 2^{n-1}$ (Proof given below)

We have,

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$$

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = \begin{cases} {}^{n+1}C_1 + {}^{n+1}C_3 + \cdots + {}^{n+1}C_n, n \text{ is even} \\ {}^{n+1}C_1 + {}^{n+1}C_3 + \cdots + {}^{n+1}C_{n-1} + {}^nC_n, n \text{ is odd} \end{cases}$$

$$\left(\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\right) = 2^n$$

$$\implies \left.\begin{matrix} {}^nC_1 + {}^nC_3 + \cdots + {}^nC_{n-1}, n \text{ is even} \\ {}^nC_1 + {}^nC_3 + \cdots + {}^nC_n, n \text{ is odd} \end{matrix}\right\} = 2^{n-1} \left(\text{ replacing } n \text{ by } n-1, {}^nC_n = {}^{n-1}C_{n-1}\right)$$

Proof for ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$${}^nC_r + {}^nC_{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

$$= \frac{n!(n-r+1)+n!r}{r!(n-r+1)!}$$

$$= \frac{n!(n+1)}{r!(n-r+1)!}$$

$$= \frac{(n+1)!}{r!(n-r+1)!} = {}^{n+1}C_r$$

14 votes -- Rajarshi Sarkar (34.1k points)

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4.13.5 Sets: GATE1994-3.8 top

https://gateoverflow.in/2494

Selected Answer

$R-(R-S)$

There is no need to use Union operator here.

Just because they say you can use operators from $(\cup, -)$ we don't need to use both of them.

Also they are saying that **only the minimum number of operators from (∪, −)** which is equivalent to $R \cap S$.

My expression is Minimal.

26 votes -- Akash Kanase (42.5k points)

4.13.6 Sets: GATE1994-3.9 top

https://gateoverflow.in/2495

Selected Answer

True

https://proofwiki.org/wiki/Subset_of_Countable_Set_is_Countable

8 votes -- Anu (5.9k points)

4.13.7 Sets: GATE1995-1.20 top

https://gateoverflow.in/2607

Selected Answer

no of elements in power set is = $2^{\text{(no of elements in the set)}} = 2^3 = 8$

Elements are {∅,{{∅}},{1},{{2,3}},{{∅},1},{1,{2,3}},{{∅},{2,3}},{{∅},1,{2,3}}}

Hence, Option is (C) 8.

16 votes -- jayendra (8.3k points)

4.13.8 Sets: GATE1996-1.1 top

https://gateoverflow.in/2705

Selected Answer

$(A-B) \cup (B-A) \cup (A \cap B)$
$A-B$ is A but not B. i.e. only A
$B-A$ is B but not A. i.e. only B
$A \cap B$ is A and B both

Union of all is (only A) U (only B) U (both A and B)
$= A \cup B$

18 votes -- Digvijay (54.8k points)

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4.13.9 Sets: GATE1996-2.4 top

https://gateoverflow.in/2733

Selected Answer

(a) Let set = $1, 2, 3, 4$

We can have identity function as $\{(1,1), (2,2), (3,3), (4,4)\}$

Since function is bijective and mapping to same set, we can have an inverse for any function by inverting the relation (changing the mapping $a->b$ to $b->a$)

Since the function maps to the same set, it must be closed and associative also. So, all four properties of group satisfied. So, (a) is true.

(b) Let $p = 5$. So, set $= \{1, 2, 3, 4\}$

Identity element is 1.

*	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

This forms a group. Similarly for any p, we get a group. So, (b) is also true.

(c) is false as string concatenation operation is a monoid (doesn't have inverse to become a group).

http://en.wikipedia.org/wiki/Concatenation

(d) is True.

http://www.math.niu.edu/~beachy/abstract_algebra/study_guide/32.html

22 votes -- Arjun Suresh (348k points)

4.13.10 Sets: GATE1998-2.4 top

https://gateoverflow.in/1676

Selected Answer

Apply set formula of A union B union C ....
$28 = (18 + 15 + 22) - (9 + 11 + 13) + x$
$28 = 55 - 33 + x$
$x = 6$

12 votes -- Digvijay (54.8k points)

4.13.11 Sets: GATE2000-2.6 top

https://gateoverflow.in/653

Selected Answer

$S = \{1\}$

$P(S) = \{\{\}, \{1\}\}$

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$P(P(S)) = \{\{\},\{\{\}\},\{\{1\}\},\{\{\},\{1\}\}\}$

$A)\ P(P(S)) = P(S)$ – This is false. Counterexample given above.

$C)\ P(S) \cap S = P(S)$– This is false. This intersection is usually Empty set.

$D)\ S \notin P(S)$– This is false. S belongs to $P(S)$.

Edit:-

B. It seems like B is true, but there is counter-example for B too. (Given By @Pragy Below)

$$\begin{aligned} S &= \{\emptyset\} \\ P(S) &= \{\emptyset, \{\emptyset\}\} \\ P(P(S)) &= \left\{ \emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\} \right\} \\ P(S) \cap P(P(S)) &= \{\emptyset, \{\emptyset\}\} \\ &\neq \{\emptyset\} \end{aligned}$$

So, answer is none of the above, all options are false.

But if we consider Simple sets(Except Empty Set) only then best Suitable Option is B among the given Options.

28 votes -- Akash Kanase (42.5k points)

4.13.12 Sets: GATE2000-6 top

https://gateoverflow.in/677

Selected Answer

$S = 1, 2, 3, 4, 5, 6.....n$

Let us assume any two subset S_1 and S_2. We can simply assume $n(S_1 \cap S_2) = 0$ to consider the disconnected sets if we want.

Now there are three cases in which $(S_1 \backslash S_2) \cup (S_2 \backslash S_1)\ \ Or,\ \ (S_1 \oplus S_2)$ has only 2 element.

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1. Both green shaded area has one element each and in this case sizes of S_1 and S_2 are same.
2. The green area of S_1 contains 2 element and the green area of S_2 contains none. In this case size of S_1 is 2 more than that of S_2.
3. The green area of S_2 contains 2 element and the green area of S_1 contains none. In this case size of S_2 is 2 more than that of S_1.

So, if we are only interested in a **particular** set vertex corresponding to set S_1 of size $= m$, then S_1 is connected to **three types** of set vertices as shown below. We will use the words "**set**" and "**vertices**" synonymously.

In this above image, we have considered $m \geq 2$. The cases for $m = 1 \text{ and } m = 0$ will be **discussed later**.

Now, what we need to find is the **no of set vertices** in each of the **above** three types and sum them up to get the **degree** of the vertex corresponding to the set S_1.

For simplicity let us assume $S = \{1, 2, 3, 4, 5, 6, 7\}$ and set $S_1 = \{1, 2, 3, 4\}$. Our interest will be to find S_2 such that vertices corresponding to S_1 and S_2 are connected.

1. **CASE 1** : If we try to find another set S_2 having 4 elements and satisfying constraint $n(S_1 \oplus S_2) = 2$, then we will see that no of such set S_2 is $4 \cdot (7 - 4)$. Or in general if S_1 is an m element set then no of such S_2 sets with constraint $n(S_1 \oplus S_2) = 2$ will be equal to $m \cdot (n - m)$.
2. **CASE 2** : S_1 contains 4 element and If we try to find S_2 where S_2 contains 2 elements and satisfying constraint $n(S_1 \oplus S_2) = 2$, then no of such S_2 will be $4C2$ or in general, for m element set S_1, we have $mC2$ no of S_2 type sets all with $(m - 2)$ size.
3. **CASE 3**: S_1 contains 4 element and If we try to find S_2 where S_2 contains 6 element and satisfying constraint $n(S_1 \oplus S_2) = 2$, then no of such S_2 sets will be $3C2$ or $(7 - 4)C2$. In general, with S_1 being m element set, then $(n - m)C2$ no of S_2 sets will be possible.

Therefore, summing all three cases :

Degree of vertex S_1 (assuming general case of $n(S_1) = m$)

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$$= m \cdot (n-m) + \binom{m}{2} + \binom{n-m}{2}$$
$$= m \cdot n - m^2 + \frac{m^2}{2} - \frac{m}{2} + \frac{(n-m)\cdot(n-m-1)}{2}$$
$$= m \cdot n - m^2 + \frac{m^2}{2} - \frac{m}{2} + \frac{n\cdot(n-1)}{2}$$
$$- \frac{n\cdot m}{2} - \frac{n\cdot m}{2} + \frac{m^2}{2} + \frac{m}{2}$$
$$= \frac{n\cdot(n-1)}{2}$$
$$= \binom{n}{2}$$

This result is independent of m for $m \geq 2$ and $m \leq n$.

For $m = 0$ and $m = 1$ also we can show that degree of 0 and 1 size set vertices is nothing but $nC2$ only. (fairly straight forward cases).

So we can conclude that every vertex has the same degree and the degree is $nC2$.

Now we can guess one thing by looking at the following image:

i.e.for $m \geq 2$ if m is even the S_1 is connected to only even cardinality type of sets (**at least one**) or if m is odd then S_1 is connected to only odd cardinality type of sets (**at least one**). By this, we can almost say that there are two connected components in the graph.

But there is little more argument before we can proceed and have a valid proof.

if $m = 0$ then $S_1 = \phi$, Then S_1 will be connected to all $m = 2$ type of sets or 2 cardinality sets.

if $m = 1$ then S_1 will be one of all 1 element sets, Then S_1 will be connected to all other 1 cardinality sets and at least one 3 cardinality set.

We can argue that, one m (even) cardinality set is at least connected to one $(m-2)$ cardinality set. That particular $(m-2)$ cardinality set is at least connected to one $(m-4)$ cardinality set and so on till
ϕ set vertex. There for all even cardinality sets are connected to ϕ directly or indirectly.

A similar argument holds for odd cardinality set vertices till we reach some 1 cardinality set. Moreover all
1 cardinality sets are connected.

Therefore we have a situation now that all even cardinality sets form one connected component and all odd cardinality set form another component.

For example : $n = 4$:

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14 votes -- Debashish Deka (56.6k points)

4.13.13 Sets: GATE2001-2.2 top

https://gateoverflow.in/720

Selected Answer

$S1$:

CounterExample : $a^* \cap (b^* \cup c^*)$, Here a^*, b^*, c^* all are infinite,
$\text{Result is finite language} = \{\in\}$.

So S1 is True

$S2$->

Counterexample : $x = 1 + \sqrt{2}$, $y = 1 - \sqrt{2}$,
$x + y = 2$.

So S2 is True.

Answer -> C

30 votes -- Akash Kanase (42.5k points)

4.13.14 Sets: GATE2001-3 top

https://gateoverflow.in/744

For question (a):

To prove $P(A \cap B) = P(A) \cap P(B)$ we should show that $P(A \cap B) \subseteq P(A) \cap P(B)$ and $P(A) \cap P(B) \subseteq P(A \cap B)$.

for first part:

let take some subset $X \subseteq A \cap B$ then $X \in P(A \cap B)$. Also $X \subseteq A \wedge X \subseteq B$, means $X \in P(A) \wedge X \in P(B)$. Again this proves that $X \cap P(A) \cap P(B)$. This proves $P(A \cap B) \subseteq P(A) \cap P(B)$.....(1)

for second part:

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Take any X such that $X \subseteq A$ and $X \subseteq B$. This is $X \in P(A) \wedge X \in P(B)$. That means $X \in P(A) \cap P(B)$.Now also $X \subseteq (A \cap B)$. This also means that $X \in P(A \cap B)$. This proves $P(A) \cap P(B) \subseteq P(A \cap B)$ .....(2)

From result (1) and (2),

$$P(A) \cap P(B) = P(A \cap B)$$

For question (b):

as obvious $sum(n) = \frac{n(n+1)}{2}$

so for n=1
$sum(1) = \frac{1(1+1)}{2} = 1$, which is true.

let assume that for n=n+m,

$$sum(m+n) = sum(m) + sum(n) + mn$$

$$= \frac{m(m+1)}{2} + \frac{n(n+1)}{2} + mn$$

$$= \frac{(m+n)(m+n+1)}{2}$$ is true.

then for n=m+n+1,

$$sum(m+n+1) = sum(m+n) + sum(1) + (m+n) * 1$$

from above results,

$$sum(m+n+1) = \frac{(m+n)(m+n+1)}{2} + 1 + (m+n)$$

$$= \frac{(m+n+1)(m+n+2)}{2}$$

is proved by mathematical induction.

2 votes -- Sheshang M. Ajwalia (3k points)

4.13.15 Sets: GATE2004-IT-2 top

https://gateoverflow.in/3643

Selected Answer

The question has a slight misprint. It should be what Bhagirathi says in the comments.

Nevertheless,

$$\left|A \cup B \cup C\right| = |A| + |B| + |C| - \left|A \cap B\right| - \left|A \cap C\right| - \left|B \cap C\right| + \left|A \cap B \cap C\right|$$

$A \equiv$ Students who have taken Programming.
$B \equiv$ Students who have taken Data Structures.
$C \equiv$ Students who have taken Computer Organisation.

So, the number of students who have taken any of the 3 courses is given by:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= 125 + 85 + 65 - 50 - 35 - 30 + 15 = 175$$

Therefore, the number of students who haven't taken any of the 3 courses is: $200 - 175 = 25$

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Hence, the answer is Option C.

22 votes -- Pragy Agarwal (20.6k points)

4.13.16 Sets: GATE2005-8 top

https://gateoverflow.in/1157

Selected Answer

$x = (A - B) - C = 1, 4 - 4, 5, 6, 7 = 1$

$y = (A - C) - (B - C) = 1, 2 - 2, 3 = 1$

So $X = Y$. Answer A

12 votes -- Akash Kanase (42.5k points)

4.13.17 Sets: GATE2005-IT-33 top

https://gateoverflow.in/3779

Selected Answer

Let's take an example set a, b, c

Now lets try to create the required set of subsets, say S.

Let's start by adding sets of size 1 to S. We can only add one of the sets $\{a\}, \{b\}, \{c\}$

Lets say we add $\{a\}$, so S now becomes $\{\{a\}\}$

Now lets add sets of size 2 to S. Again we see that we can only add one of $\{a, b\}, \{a, c\}$ or $\{b, c\}$, and we cannot add {b,c} since we already added {a}.

Continuing this way we see we can add only one set for a all size till n.

So the answer should be $2)\ n + 1$ (include the empty set)

30 votes -- Omesh Pandita (2.5k points)

4.13.18 Sets: GATE2006-22 top

https://gateoverflow.in/983

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Selected Answer

Answer is C using **Venn diagram.**

15 votes -- Anu (5.9k points)

4.13.19 Sets: GATE2006-24 top

https://gateoverflow.in/987

Selected Answer

$min(\pi(N)) = 1$, since in a permutation of n elements from $1..n$, some element must get 1.

SImilarly, in any subsets A and B, $min(\pi(A)) = min(\pi(B))$ only if A and B has a common element and it is the smallest of all the other elements in A and B.

(With this understanding itself we can eliminate options A and B)

Now we have n! total permutations and we have to see the number of permutations satisfying the given condition. If $A = B$, all of the n! permutations satisfy the given condition. (This is enough to get the answer as C). Otherwise, the fraction of the n! permutations satisfying the given condition

= $|A \cap B|/|A \cup B|$

This is because without the given restriction, the smallest element (among the $|A \cap B|$ elements) can be any one of the $|A \cup B|$ elements, and with the restriction, the smallest element must be one of the $|A \cap B|$ elements.

So, answer is C.

19 votes -- Arjun Suresh (348k points)

4.13.20 Sets: GATE2006-IT-23 top

https://gateoverflow.in/3562

Selected Answer

I got answer c

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22 votes -- Anu (5.9k points)

4.13.21 Sets: GATE2006-IT-24 top

https://gateoverflow.in/3563

Selected Answer

No's divisible by 2 in $X = 61$ [= integer(123/2)]

No's divisible by 3 in $X = 41$

No's divisible by 5 in $X = 24$

No's divisible by 2 and 3 .i,e by $6 = 20$

No's divisible by 2 and 5 i.e by $10 = 12$

No's divisible by 3 and 5 , i.e by $15 = 8$

No's divisible by 2 and 3 and 5 ..ie by $30 = 4$

No's divisible by either 2 or 3 or 5 = N(AUBUC) = N(A) +N(B)+N(C) -N(A∩B)-N(B∩C)-N(A∩C)+ N(A∩B∩C)

$= 61 + 41 + 24 - 20 - 12 - 8 + 4 = 90$

$X = \{ n, 1 \leq n \leq 123, n$ is **not divisible** by either $2, 3$ or $5 \}$

Cardinality = $123 - 90$ = **33**

27 votes -- Praveen Saini (54.5k points)

4.13.22 Sets: GATE2008-2 top

https://gateoverflow.in/400

Selected Answer

Answer D

$$(P \cap Q \cap R) \cup (P^c \cap Q \cap R) \cup Q^c \cup R^c$$

$$= (P \cup P^c) \cap (Q \cap R) \cup Q^c \cup R^c$$

$$= (Q \cap R) \cup Q^c \cup R^c$$

$$= (Q \cap R) \cup (Q \cap R)^C$$

$$= U.$$

23 votes -- Anu (5.9k points)

4.13.23 Sets: GATE2014-2-50 top

https://gateoverflow.in/2016

Selected Answer

Symmetric difference (SD) - suppose A and B are 2 sets then symmetric difference of A and B is $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

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In question : U < V if the minimum element in the symmetric difference of the two sets is in U .
Example: $\{1,2,3\} < \{2,3,4,5,6\}$

Symmetric difference is $\{1\} \cup \{4,5,6\}$.

Now Consider a smaller set. Suppose $S = \{1,2,3,4\}$

Now the given 2 statements are about smallest and largest subset. So considering set S and $\emptyset$ (empty set) will be helpful.

First take $U = \{1,2,3,4\}$ and $V = \{1,2\}$ (we can take any set other than ∅ and S)

$SD = \{3,4\}$ (just exclude the elements which are common in the 2 sets)

Minimum element of SD is 3 which is in U and if we observe carefully minimum element will always be in U. Whatever the V is.

So acc. to the question $\{1,2,3,4\}$ is smaller than any other subset of S. **S2 is true.**

Now consider

$U = \emptyset$ and $V = \{1,2\}$ (we can take any subset of S)

$SD = \{1,2\}$

The symmetric difference will always be equal to V. So minimum element of SD will always exist in V when U is $\emptyset$.

So acc. to the que, $\emptyset$ is greater than any other subset of S. **S1 is also true.**

This is true even when $S = \{1,2,3,\ldots,2014\}$.

So answer is A. Both S1 and S2 are true

26 votes -- Soumya Jain (9.7k points)

4.13.24 Sets: GATE2014-3-16 top

https://gateoverflow.in/2050

Selected Answer

A set is countable means there exist a enumeration procedure to generate each of its elements and for a given element of set, it take finite step to generate it using the enumeration procedure.

Let $\sum = a, b$ and there exist a enumeration procedure to generate all the string of the language $\sum^*$.

$\sum^* . = \in \$, a, b, aa, ab, ba, bb, aaa, \ldots\ldots\ldots$

Here enumeration procedure is simply the generating string of the language by length for the fixed length string are in alphabetical order.

This way $\sum^*$. is countably infinite & $2^{\sum^*}$ will be uncountable set

Because the power set of countably infinite set are uncountable.

Ref: http://www.cs.xu.edu/csci250/06s/Theorems/powerSetuncountable.pdf

20 votes -- Sandeep Singh (7.8k points)

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4.13.25 Sets: GATE2015-1-16 top

https://gateoverflow.in/8238

Selected Answer

Power set of A consists of all subsets of A and from the definition of a subset, ϕ is a subset of any set. So, I and II are TRUE.

5 and $\{6\}$ are elements of A and hence $\{5, \{6\}\}$ is a subset of A and hence an element of 2^A. An element of a set is never a subset of the set. For that the element must be inside a set- i.e., a singleton set containing the element is a subset of the set, but the element itself is not. Here, option IV is false. To make IV true we have to do as follows:

$\{5, \{6\}\}$ is an element of 2^A. So, $\{\{5, \{6\}\}\} \subseteq 2^A$.

So, option C.

51 votes -- Arjun Suresh (348k points)

4.13.26 Sets: GATE2015-2-18 top

https://gateoverflow.in/8092

Selected Answer

Answer: 2048

Number of elements in set = 11.

Therefore, cardinality of power set $= 2^{11} = 2048$.

24 votes -- Rajarshi Sarkar (34.1k points)

4.13.27 Sets: GATE2015-3-23 top

https://gateoverflow.in/8426

Selected Answer

Answer IS D.

As X and Y elements of U, X and Y are subsets of S

Option A is wrong consider $X = \{1, 2\}$ therefore $X' = \{3, 4, 5, 6\}$, $|X| = 2$ and $|X'| = 4$

Option B is wrong as any two possible subsets of S with 5 elements should have atleast 4 elements in common (Pigeonhole principle). Hence X intersection Y cannot be null.

Option C is wrong, X and Y can have any number of elements from 0 to 5. Even for the given constraint, consider $X = \{1, 2\} Y = \{3, 4, 5\}$ and $X \backslash Y = \{1, 2\}$ which is not null.

34 votes -- overtomanu (1.2k points)

4.13.28 Sets: GATE2016-2-28 top

https://gateoverflow.in/39595

Selected Answer

Option B should be the correct answer.

It is given that the number of compounds in $U = 23$ and the number of compounds in $S = 9$, so

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the number of compounds in $U \backslash S = 23 - 9 = 14$.

Considering each of these compounds as nodes of a graph G.So vetex set of G is U and S is a subset of vertices of G.

The relation "A reacts with B" is a symmetric relation, that is A reacts with B is same as B reacts with A.

for example, consider the following reaction:

$$\mathrm{HCl} + \mathrm{NaOH} \rightarrow \mathrm{NaCl} + \mathrm{H_2O}$$

Here we can say either HCl reacts with NaOH to produce $\mathrm{NaCl} + \mathrm{H_2O}$ or we can say that NaOH reacts with HCl to produce $\mathrm{NaCl} + \mathrm{H_2O}$,so both of these statements are equivalent.

Since the relation based on which we are going to draw the edges is symmetric, we can use an undirected edge (A, B) between any two compounds to represent the fact that A reacts with B as well as B reacts with A.

Each compound in S reacts with exactly 3 compounds in U.

It means that the degree of every node(or compound) in S is 3.

So sum of all the degree in $S =$ number of nodes in $S \times$ degree of each node $= 9 \times 3 = 27$.

Now in $U \backslash S$ we have 14 nodes(or compounds), thus clearly $U \backslash S$ contains an even number of compounds.

Now if each compound in $U \backslash S$ reacts with an even number of compounds, the sum of degrees of all the node in $U \backslash S$ would be even, and consequently, the sum of degrees of all the nodes in our graph G would be odd as the sum of degrees of all the nodes in S is odd, and an odd number added with an even number produces an odd number.

But since in a graph, every edge corresponds to two degrees and the number of edges in a graph must be a (non-negative)integral value & not fractional value hence the sum of the degrees all the nodes of a graph must be even. (This is Handshaking Lemma).

So statement III should be false(always).

Also, adding fourteen odd numbers gives an even number.

Hence, if each compound in $U \backslash S$ reacts with an odd number of compounds, the sum of degrees of all the node in $U \backslash S$ would be even, and consequently, the sum of degrees of all the nodes in our graph G would be odd as the sum of degrees of all the nodes in S is odd, and an odd number added with an even number produces an odd number.

Again by using Handshaking Lemma, this is not possible.

So statement I should also be false(always).

Thus, from the previous two cases, it can be observed that to satisfy the Handshaking Lemma for G, the sum of the degrees of all the nodes $U \backslash S$ must be odd.To make this happen, we must assign at least one node of $U \backslash S$, an odd degree.

If at least, one node(or compound) in $U \backslash S$ would have an odd degree(or reacts with odd numbers of compounds) then we can assign degrees in such a way that the sum of the degrees of all the nodes $U \backslash S$ will be odd, & thus the Handshaking Lemma would be satisfied.

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Hence, statement II is the only statement which is guaranteed to be true always.

Moreover, we can also make some stronger claims from the given information like,

always an odd number of compounds in $U \backslash S$ reacts with an odd number of compounds and

at least, one compound in S reacts with a compound in $U \backslash S$ and so on.

68 votes -- Anurag Pandey (13.6k points)

4.13.29 Sets: GATE2017-1-47 top

https://gateoverflow.in/118330

Selected Answer

Here, we can apply the property of set. Let D_n denote divisibility by n, D_{n_1,n_2} denote divisibility by both n_1 and n_2 and so on.

$$\begin{aligned} N(D_3 \cup D_5 \cup D_7) &= N(D_3) + N(D_5) + N(D_7) - N(D_{3,5}) - N(D_{5,7}) - N(D_{3,7}) + N(D_{3,5,7}) \\ &= 166 + 100 + 71 - 33 - 14 - 23 + 4 \\ &= 271 \end{aligned}$$

19 votes -- sriv_shubham (3.3k points)

4.13.30 Sets: TIFR2010-A-15 top

https://gateoverflow.in/18394

Selected Answer

$(A - (A - B)) = A \cap (A \cap B')'$ Since $A - B = A \cap B'$

$= A \cap (A'UB)$ Since $(A \cap B)' = A'UB'$

$= A \cap B$ Option D

11 votes -- Umang Raman (15.7k points)

4.13.31 Sets: TIFR2010-A-18 top

https://gateoverflow.in/18496

Selected Answer

Option $C)$ i.e. 3^n must be the right answer.

It is given that there are n elements in the set X.

Consider an element p of set X.

What are the choices it will have,

1) Either it can be present in set A & set B both,

2) Or it can absent from set A & present in set B,

3) Or it can be absent from both set A & set B.

but since it is given that A must be a subset of B, it is not possible that it can be present in A & absent from B.

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So, it each of the n elements of set X have 3 choices available.

So total choices available for formation of sets A & $B = 3^n$, which will give 3^n such different (A, B) pairs.

25 votes -- Anurag Pandey (13.6k points)

4.13.32 Sets: TIFR2011-A-10 top

https://gateoverflow.in/20039

Selected Answer

Ending in $2 : \{2^1, 2^5, 2^9\}$

Ending in $4 : \{2^2, 2^6, 2^{10}\}$

Ending in $6 : \{2^4, 2^8\}$

Ending in $8 : \{2^3, 2^7\}$

To make $2^m + 2^n$ divisible by 5, it must end in either a 0 or a 5.

Since $m, n > 1$, all numbers $2^m, 2^n$ are even. Since sum of even numbers is even, $2^m + 2^n$ cannot end in a 5

Thus, $2^m + 2^n$ must end in a 0

Possible ways to achieve a number ending with 0 are:

$$\begin{array}{llll}
2^m + 2^n : & m \in \{1, 5, 9\}, & n \in \{3, 7\} & \implies 3 \times 2 = 6 \text{ pairs} \\
2^m + 2^n : & m \in \{3, 7\}, & n \in \{1, 5, 9\} & \implies 2 \times 3 = 6 \text{ pairs} \\
 & & & \\
2^m + 2^n : & m \in \{2, 6, 10\}, & n \in \{4, 8\} & \implies 3 \times 2 = 6 \text{ pairs} \\
2^m + 2^n : & m \in \{4, 8\}, & n \in \{2, 6, 10\} & \implies 2 \times 3 = 6 \text{ pairs}
\end{array}$$

$$\text{Total} = 6 + 6 + 6 + 6 = 24 \text{ ordered pairs}$$

Thus, option c is correct.

25 votes -- Pragy Agarwal (20.6k points)

4.13.33 Sets: TIFR2012-A-8 top

https://gateoverflow.in/21007

Selected Answer

$D)$ 243. First take A as ϕ and B as power set of $\{1, 2, 3, 4, 5\}$ which is 2^5 . then take A as set of one element ex: when A=$\{1\}$ then set B could be any of the 2^4 elements of power set. This will give us 16×5 . In similar fashion when A consist of 2 element set we get total pairs $5C2$x2^3; when A is of three elements we get 40; for 4 elements we get 10 pairs and when A is of 5

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elements we get one pair which is A=$\{1,2,3,4,5\}$ and $B=\{\}$.....so in total $=32+80+80+40+10+1=243$

Alternative Solution:

for each element in $[n]$, you have 3 choices:

1. Include it in A but not in B
2. Include it in B, but not in A
3. Include it in neither

so this gives 3^n pairs.

for set $\{1,2,,\ldots,5\}$, $n=5$, $3^5=243$

28 votes -- Shaun Patel (6.9k points)

4.13.34 Sets: TIFR2016-A-8 top

https://gateoverflow.in/97234

Selected Answer

Let set B be of cardinality n.

Total subsets(**A**) possible are : $nC_0+nC_1+nC_2+\ldots+nC_n$. i.e nC_r number of subsets exist with r cardinality.

Note that for each r, summation has 2^{n-r} terms to sum.

Case 1: $r=0$. which is $\emptyset$.

Total terms $=2^n$.

Total terms when $|C\backslash A|$ even $=nC_0+nC_2+nC_4+\ldots+nC(n-1)$ { if n is odd, nC_n otherwise }

Similarly for odd $=2^n-|C\backslash A|$

Even contributes to 1 whereas odd contributes to -1.

Therefore Summation $=0$ as 2^n terms are present with half as 1 & half as -1.

Case 2: $r=1$, total terms $=2^{n-1}$

Total terms when $|C\backslash A|$ even $=nC_r+nC_{r+2}+\ldots+nC_n$ { if n is odd $n-1$ otherwise }

Total terms when $|C\backslash A|$ odd $=nC_{r+1}+nC_{r+3}+\ldots$

Again we are end up with total even terms with half contributing to 1 & half -1.

same situation will arise for every $r\neq n$ (as for such r, 2^r is always even) i.e summation $=0$ for all $r, r\neq n$.

Case n: $r=n$, total terms $=2^{n-n}=1$. This is the case when $B=A$.

$\therefore |C\backslash A|=0$ as both are equal.

Summation $=1$.

therefore answer is : 1 if $A=B, 0$ otherwise.

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4 votes -- Aghori (6k points)

4.13.35 Sets: TIFR2017-A-10 top

https://gateoverflow.in/95272

Selected Answer

Even if it can be **one-to-one** in the following way,

But, It cannot be onto,because, the number of elements in domain (A) $<$ the number of elements in co-domain $(P(A))$. For a function to be **onto**, the domain should be able to cover all elements of co-domain with each element of the domain having exactly one image in co-domain.
so option(B)

14 votes -- Motamarri Anusha (12.3k points)

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5 Engineering Mathematics: Calculus (55) top

5.1 Area Under Curve(1) top

5.1.1 Area Under Curve: ISI 2017 MMA 1 top

https://gateoverflow.in/213886

The area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 1$ is given by

A. $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$
B. $\frac{\pi}{6} + \frac{\sqrt{3}}{4}$
C. $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$
D. $\frac{\pi}{6} + \frac{\sqrt{3}}{2}$

isi2017 calculus area-under-curve

Answer

Answers: Area Under Curve

5.1.1 Area Under Curve: ISI 2017 MMA 1 top

https://gateoverflow.in/213886

Selected Answer

We can write the equation as $y = \sqrt{4 - x^2}$

$\int_0^1 \sqrt{4 - x^2}\, dx$

Substituting $x = 2 \sin \theta$

$dx = 2 \cos \theta d\theta$

$\implies \theta = \sin^{-1}\left(\frac{x}{2}\right)$

$\int_0^{\frac{\pi}{6}} \sqrt{4 - 4 \sin^2 \theta} * 2 \cos \theta d\theta$

$\int_0^{\frac{\pi}{6}} 2 \cos \theta * 2 \cos \theta d\theta$

$4 \int_0^{\frac{\pi}{6}} \cos^2 \theta d\theta$

$4 \int_0^{\frac{\pi}{6}} \frac{1 + \cos 2\theta}{2} d\theta$

$2 \int_0^{\frac{\pi}{6}} 1 + \cos 2\theta d\theta$

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$2\int_0^{\frac{\pi}{6}} d\theta + 2\int_0^{\frac{\pi}{6}} \cos 2\theta d\theta$

$\frac{\pi}{3} + 2\int_0^{\frac{\pi}{6}} \cos 2\theta d\theta$

Substituting $2\theta = y$,

$d\theta = \frac{y}{2}$

$\frac{\pi}{3} + \int_0^{\frac{\pi}{3}} \cos y dy$

$\frac{\pi}{3} + \frac{\sqrt{3}}{2}$

This is total area of fourth and first quadrant combined so area of only first quadrant will be

$\frac{\pi}{6} + \frac{\sqrt{3}}{4}$

Option B

2 votes -- Keval Malde (16.1k points)

5.2 Continuity(4) top

5.2.1 Continuity: GATE2010-ME top

https://gateoverflow.in/41570

The function $y = |2 - 3x|$

A. **is continuous** $\forall x \in R$ and differentiable $\forall x \in R$
B. **is continuous** $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = \frac{3}{2}$
C. **is continuous** $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = \frac{2}{3}$
D. **is continuous** $\forall x \in R$ except $x = 3$ and differentiable $\forall x \in R$

calculus gate2010me engineering-mathematics continuity

Answer

5.2.2 Continuity: GATE2013-22 top

https://gateoverflow.in/1533

Which one of the following functions is continuous at $x = 3$?

A. $f(x) = \begin{cases} 2, & \text{if } x = 3 \\ x - 1 & \text{if } x > 3 \\ \frac{x+3}{3} & \text{if } x < 3 \end{cases}$

B. $f(x) = \begin{cases} 4, & \text{if } x = 3 \\ 8 - x & \text{if } x \neq 3 \end{cases}$

C. $f(x) = \begin{cases} x + 3, & \text{if } x \leq 3 \\ x - 4 & \text{if } x > 3 \end{cases}$

D. $f(x) = \{ \frac{1}{x^3 - 27} \quad \text{if } x \neq 3$

gate2013 calculus continuity normal

Answer

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5.2.3 Continuity: GATE2014-1-47 top

https://gateoverflow.in/1925

A function $f(x)$ is continuous in the interval $[0,2]$. It is known that $f(0)=f(2)=-1$ and $f(1)=1$. Which one of the following statements must be true?

A. There exists a y in the interval $(0,1)$ such that $f(y)=f(y+1)$
B. For every y in the interval $(0,1)$,$f(y)$ = $f(2-y)$
C. The maximum value of the function in the interval $(0,2)$ is 1
D. There exists a y in the interval $(0,1)$ such that $f(y)$ = $-f(2-y)$

gate2014-1 calculus continuity normal

Answer

5.2.4 Continuity: GATE2015-2-26 top

https://gateoverflow.in/8124

Let $f(x)=x^{-\left(\frac{1}{3}\right)}$ and A denote the area of region bounded by $f(x)$ and the X-axis, when x varies from -1 to 1. Which of the following statements is/are TRUE?

I. f is continuous in $[-1,1]$
II. f is not bounded in $[-1,1]$
III. A is nonzero and finite

A. II only
B. III only
C. II and III only
D. I, II and III

gate2015-2 continuity functions normal

Answer

Answers: Continuity

5.2.1 Continuity: GATE2010-ME top

https://gateoverflow.in/41570

Selected Answer

$$y=\begin{cases}2-3x, & 2-3x \geq 0\\ 3x-2, & 2-3x < 0\end{cases}$$

$$\implies y=\begin{cases}2-3x, & x \leq \frac{2}{3}\\ 3x-2, & x > \frac{2}{3}\end{cases}$$

As y is polynomial it is continuous and differentiable at all points but do not know at $x=\frac{2}{3}$

To check continuity at $x=\frac{2}{3}$
Left limit = $2-3\times\frac{2}{3}=0$
Right limit = $3\times\frac{2}{3}-2=0$
$f(a)=f(2/3)=2-3\times\frac{2}{3}=0$

$\because LL=RL=f(a), y$ is continuous $\forall_{x\epsilon R}$.

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To check differentiability at $x = \frac{2}{3}$
Left derivative $= 0 - 3 = -3$
Right derivative $= 3 - 0 = 3$

$\because LD \neq RD, y$ is not differentiable at $x = \frac{2}{3}$.

So Answer is option C

18 votes -- Lokesh Dafale (11.4k points)

5.2.2 Continuity: GATE2013-22 top

https://gateoverflow.in/1533

Selected Answer

For continuity, Left hand limit must be equal to right hand limit. For continuity at $x = 3$,

the value of $f(x)$ just above and just below 3 must be the same.

A. $f(3) = 2. f(3+) = x - 1 = 2. f(3-) = \frac{(x+3)}{3} = \frac{6}{3} = 2.$ Hence continuous.

B. $f(3) = 4. f(3+) = f(3-) = 8 - 3 = 5.$ So, not continuous.

C. $f(3) = f(3-) = x + 3 = 6. f(3+) = x - 4 = -1.$ So, not continuous.

D. $f(3)$ is not existing. So, not continuous.

24 votes -- Arjun Suresh (348k points)

5.2.3 Continuity: GATE2014-1-47 top

https://gateoverflow.in/1925

Let us define a new function g :

$g(y) = f(y) - f(y+1)$

Since function
f is continuous in $[0, 2]$, g would be continuous in $[0, 1]$

$g(0) = -2, g(1) = 2$

Since g is continuous and goes from negative to positive value in $[0, 1]$, at some point g would be 0 in $(0, 1)$.

$g = 0 \implies f(y) = f(y+1)$ for some $y \in (0, 1)$.

Therefore, correct answer would be (A).

43 votes -- suraj (5.6k points)

5.2.4 Continuity: GATE2015-2-26 top

https://gateoverflow.in/8124

Selected Answer

Answer: C

I. False.

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II. True.
III. True. An area is always positive, while the definite integral might be composed of several regions, some positive and some negative. A definite integral gets you the net area, because any part of the graph that is below the x-axis will give you a negative area. So, a definite integral is not necessarily the area under the curve, but the value of the area above the x-axis less the area under the x-axis. So, A is non-zero and finite.

17 votes -- Rajarshi Sarkar (34.1k points)

5.3 Differentiability(8) top

5.3.1 Differentiability: GATE1996-1.6 top

https://gateoverflow.in/2710

The formula used to compute an approximation for the second derivative of a function f at a point X_0 is

A. $\frac{f(x_0+h)+f(x_0-h)}{2}$

B. $\frac{f(x_0+h)-f(x_0-h)}{2h}$

C. $\frac{f(x_0+h)+2f(x_0)+f(x_0-h)}{h^2}$

D. $\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2}$

gate1996 calculus differentiability normal

Answer

5.3.2 Differentiability: GATE1996-3 top

https://gateoverflow.in/2755

Let f be a function defined by

$$f(x)=\begin{cases} x^2 & \text{for } x\leq 1 \\ ax^2+bx+c & \text{for } 1<x\leq 2 \\ x+d & \text{for } x>2 \end{cases}$$

Find the values for the constants a, b, c and d so that f is continuous and differentiable everywhere on the real line.

gate1996 calculus continuity differentiability normal descriptive

Answer

5.3.3 Differentiability: GATE1998-1.4 top

https://gateoverflow.in/1641

Consider the function $y=|x|$ in the interval $[-1,1]$. In this interval, the function is

A. continuous and differentiable
B. continuous but not differentiable
C. differentiable but not continuous
D. neither continuous nor differentiable

gate1998 calculus continuity differentiability easy

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Answer

5.3.4 Differentiability: GATE2007-1 top https://gateoverflow.in/1200

Consider the following two statements about the function $f(x) = |x|$:

- P. $f(x)$ is continuous for all real values of x.
- Q. $f(x)$ is differentiable for all real values of x .

Which of the following is **TRUE**?

A. P is true and Q is false.
B. P is false and Q is true.
C. Both P and Q are true.
D. Both P and Q are false.

gate2007 calculus continuity differentiability easy

Answer

5.3.5 Differentiability: GATE2014-1-46 top https://gateoverflow.in/1924

The function $f(x) = x \sin x$ satisfies the following equation:

$$f''(x) + f(x) + t \cos x = 0$$

. The value of t is_____.

gate2014-1 calculus easy numerical-answers differentiability

Answer

5.3.6 Differentiability: GATE2014-1-6 top https://gateoverflow.in/1763

Let the function

$$f(\theta) = \begin{vmatrix} \sin\theta & \cos\theta & \tan\theta \\ \sin(\frac{\pi}{6}) & \cos(\frac{\pi}{6}) & \tan(\frac{\pi}{6}) \\ \sin(\frac{\pi}{3}) & \cos(\frac{\pi}{3}) & \tan(\frac{\pi}{3}) \end{vmatrix}$$

where

$\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and $f'(\theta)$ denote the derivative of f with respect to θ. Which of the following statements is/are **TRUE**?

I. There exists $\theta \in (\frac{\pi}{6}, \frac{\pi}{3})$ such that $f'(\theta) = 0$
II. There exists $\theta \in (\frac{\pi}{6}, \frac{\pi}{3})$ such that $f'(\theta) \neq 0$

A. I only
B. II only
C. Both I and II
D. Neither I Nor II

gate2014-1 calculus differentiability normal

Answer

5.3.7 Differentiability: GATE2016-2-02 top https://gateoverflow.in/39571

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Let $f(x)$ be a polynomial and $g(x) = f'(x)$ be its derivative. If the degree of $(f(x) + f(-x))$ is 10, then the degree of $(g(x) - g(-x))$ is _____.

gate2016-2 calculus normal numerical-answers differentiability

Answer

5.3.8 Differentiability: GATE2017-2-10 top

https://gateoverflow.in/118262

If $f(x) = R\ \sin(\frac{\pi x}{2}) + S.\ f'(\frac{1}{2}) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2R}{\pi}$, then the constants R and S are

A. $\frac{2}{\pi}$ and $\frac{16}{\pi}$
B. $\frac{2}{\pi}$ and 0
C. $\frac{4}{\pi}$ and 0
D. $\frac{4}{\pi}$ and $\frac{16}{\pi}$

gate2017-2 engineering-mathematics calculus differentiability

Answer

Answers: Differentiability

5.3.1 Differentiability: GATE1996-1.6 top

https://gateoverflow.in/2710

Selected Answer

Option D.

Ref: http://en.wikipedia.org/wiki/Second_derivative

7 votes -- Arjun Suresh (348k points)

5.3.2 Differentiability: GATE1996-3 top

https://gateoverflow.in/2755

Selected Answer

f is differentiable at 1 if

$$\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h}$$

$$\Rightarrow 2 = 2a + b - (1)$$

f is differentiable at 2 if

$$\lim_{h\to 0^-} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h}$$

$$\Rightarrow 4a + b = 1 - (2)$$

Solving (1) and (2), we get

$$a = -0.5, b = 3$$

Now f has to be continous on 1 also, so

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$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} (x) = f(1)$$

$$\Rightarrow 1 = a + b + c$$

$$\Rightarrow c = -1.5$$

Similarly f has to be continous on 2 also, so

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} (x) = f(2)$$

$$\Rightarrow 4a + 2b + c = 2 + d$$

$$\Rightarrow d = 0.5$$

So $a = -0.5, b = 3, c = -1.5, d = 0.5$

12 votes -- Happy Mittal (11.2k points)

5.3.3 Differentiability: GATE1998-1.4 top

https://gateoverflow.in/1641

Selected Answer

(b) y is continuous but not differentiable at $x = 0$ as left hand limit will be negative while the right hand limit will be positive but for differentiation, both must be same.

10 votes -- Gate_15_isHere (583 points)

5.3.4 Differentiability: GATE2007-1 top

https://gateoverflow.in/1200

Selected Answer

Ans is A.

$f(x) =\mid x \mid$. here for all values of x, $f(x)$ exists. Therefore it is continuous for all real values of x.

At $x = 0$, $f(x)$ is not differentiable. Because if we take the left hand limit here, it is negative while the right hand limit is positive making $LHL \neq RHL$

Ref: http://math.stackexchange.com/questions/991475/why-is-the-absolute-value-function-not-differentiable-at-x-0

14 votes -- jayendra (8.3k points)

5.3.5 Differentiability: GATE2014-1-46 top

https://gateoverflow.in/1924

Selected Answer

$$f'(x) = x\cos(x) + \sin(x)$$

$$f''(x) = x(-\sin x) + \cos x + \cos x$$

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now $f''(x) + f(x) + t\cos x = 0$

$\Rightarrow x(-\sin x) + \cos x + \cos x + x\sin x + t\cos x = 0$

$\Rightarrow 2\cos x + t\cos x = 0$
$\Rightarrow \cos x(t+2) = 0$
$\Rightarrow t + 2 = 0, t = -2$

19 votes -- SAKET NANDAN (4.8k points)

5.3.6 Differentiability: GATE2014-1-6 top

https://gateoverflow.in/1763

Selected Answer

We need to solve this by Rolle's theorem. To apply Rolle's theorem following 3 conditions should be satisfied:

1. $f(x)$ should be continuous in interval $[a, b]$,
2. $f(x)$ should be differentiable in interval (a, b), and
3. $f(a) = f(b)$

If these 3 conditions are satisfied simultaneously then, there exists at least one $'x'$ such that $f'(x) = 0$

For the given question, it satisfies all the three conditions, so we can apply Rolle's theorem, i.e, there exists at least one θ that gives $f'(\theta) = 0$

Also, the given function is also not a constant function, i.e., for some θ, $f'(\theta) \neq 0$

So, answer is C.

28 votes -- Saurabh Sharma (579 points)

5.3.7 Differentiability: GATE2016-2-02 top

https://gateoverflow.in/39571

Selected Answer

Let $f(x) = x^{10}$ Degree$= 10$.

$$\begin{aligned} f(x) + f(-x) &= x^{10} + (-x)^{10} \\ &= x^{10} + x^{10} \\ &= 2.x^{10} \end{aligned}$$

$$\begin{aligned} g(x) - g(-x) &= 10.x^9 - \{-10x^9\} \\ &= 20.x^9 \end{aligned}$$

So answer is 9.

42 votes -- Akash Kanase (42.5k points)

5.3.8 Differentiability: GATE2017-2-10 top

https://gateoverflow.in/118262

Selected Answer

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C.

$f(x) = R * \sin(\frac{\pi * x}{2}) + S$

$f'(x) = R * \cos(\frac{\pi * x}{2}) * \frac{\pi}{2}$

$f'(\frac{1}{2}) = R * \cos(\frac{\pi}{4}) * \frac{\pi}{2} = \sqrt{2}$

$R = \frac{\sqrt{2} * \sqrt{2} * 2}{\pi} = \frac{4}{\pi}$

$f(x) = \frac{4}{\pi} * \sin(\frac{\pi * x}{2}) + S$

$\int_0^1 f(x) * dx = \int_0^1 (\frac{4}{\pi} * \sin(\frac{\pi * x}{2}) + S) * dx = \frac{2 * R}{\pi} = \frac{8}{\pi^2}$

$\frac{4}{\pi} \int_0^1 \sin(\frac{\pi * x}{2}) * dx + \int_0^1 S * dx = \frac{8}{\pi^2}$

$\frac{4}{\pi} \left[-\cos(\frac{\pi * x}{2}) * \frac{2}{\pi} \right]_0^1 + S[x]_0^1 = \frac{8}{\pi^2}$

$\frac{8}{\pi^2}[-0 + 1] + S = \frac{8}{\pi^2}$

$S = 0.$

17 votes -- Dhruv Patel (2k points)

5.4 Functions(5) top

5.4.1 Functions: GATE1987-1-xxvi top

https://gateoverflow.in/80571

If $f(x_i).f(x_{i+1}) < 0$ then

A. There must be a root of $f(x)$ between x_i and x_{i+1}
B. There need not be a root of $f(x)$ between x_i and x_{i+1}.
C. There fourth derivative of $f(x)$ with respect to x vanishes at x_i.
D. The fourth derivative of $f(x)$ with respect to x vanishes at x_{i+1}.

gate1987 calculus functions

Answer

5.4.2 Functions: GATE1995-1.21 top

https://gateoverflow.in/2608

In the interval $[0, \pi]$ the equation $x = \cos x$ has

A. No solution

B. Exactly one solution

C. Exactly two solutions

D. An infinite number of solutions

gate1995 calculus normal functions

Answer

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5.4.3 Functions: ISI2004-MIII-22 top

https://gateoverflow.in/124264

If $f(x) = \frac{\sqrt{3}\sin x}{2+\cos x}$ then the range of $f(x)$ is

A. the interval $\left[-1, \frac{\sqrt{3}}{2}\right]$
B. the interval $\left[\frac{-\sqrt{3}}{2}, 1\right]$
C. the interval $[-1, 1]$
D. none of the above

isi2004 engineering-mathematics functions

Answer

5.4.4 Functions: ISI2004-MIII-23 top

https://gateoverflow.in/124324

If $f(x) = x^2$ and $g(x) = x\sin x + \cos x$ then

A. f and g agree at no point
B. f and g agree at exactly one point
C. f and g agree at exactly two point
D. f and g agree at more then two point

isi2004 engineering-mathematics functions

Answer

5.4.5 Functions: ISI2017-13 top

https://gateoverflow.in/209635

An even function $f(x)$ has left derivative 5 at $x = 0$. Then

A. the right derivative of $f(x)$ at $x = 0$ need not exist
B. the right derivative of $f(x)$ at $x = 0$ exists and is equal to 5
C. the right derivative of $f(x)$ at $x = 0$ exists and is equal to -5
D. none of the above is necessarily true.

isi2017 functions calculus

Answer

Answers: Functions

5.4.1 Functions: GATE1987-1-xxvi top

https://gateoverflow.in/80571

Selected Answer

As $f(x_i).f(x_{i+1}) < 0$

means one of them is positive and one of them in negative . as their multiplication is negative.

so when you draw the graph for $f(x)$ where $x_i \leq x \leq x_{i+1}$. Definitely $F(x)$ will cut the X- axis.

so there will definitely a root of $F(x)$ between x_i and x_{i+1}.

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9 votes -- Amit Pal (4k points)

5.4.2 Functions: GATE1995-1.21 top

https://gateoverflow.in/2608

Selected Answer

Therefore Answer is B.

(Graph is not to exact scale. one point of intersection as our range is between 0 to pi, in case it is -pi to pi, we will have two point of intersections)

20 votes -- Akash Kanase (42.5k points)

5.4.3 Functions: ISI2004-MIII-22 top

https://gateoverflow.in/124264

Selected Answer

$\frac{\sqrt{3}sinx}{2+cosx}$ This is of the form $\frac{u}{v}$

Differentiating above equation we get

$\frac{(2+cosx)\sqrt{3}cosx-\sqrt{3}sinx(-sinx)}{(2+cosx)^2}$ = 0

$$2\sqrt{3}cosx+\sqrt{3}cos^2x+\sqrt{3}sin^2x=0$$

$$2\sqrt{3}cosx+\sqrt{3}=0$$

$$2cosx+1=0$$

x = $\frac{2\pi}{3}, \frac{-2\pi}{3}$

for x= $\frac{2\pi}{3}$, $\frac{\sqrt{3}sinx}{2+cosx}$ = 1 and

for x= $\frac{-2\pi}{3}$, $\frac{\sqrt{3}sinx}{2+cosx}$ = -1

Hence, option C is correct.

7 votes -- Heisenberg (2.3k points)

5.4.4 Functions: ISI2004-MIII-23 top

https://gateoverflow.in/124324

Selected Answer

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$$f(x) = x^2$$
$$g(x) = x\sin x + \cos x$$

We are asked to make this two function equal and see at how many points they meet.

$\Rightarrow f(x) = g(x)$
$\Rightarrow x^2 = x\sin x + \cos x$
$\Rightarrow x^2 - x\sin x - \cos x = 0$
$\Rightarrow F(x) = 0$

This $F(x)$ is even and we just need to check at how many points $F(x)$ crosses the X - axis for $x > 0$.

$F(0) = -1$

$F'(x) = 2x - x\cos x$

$F'(x) = x(2 - \cos x)$

Here $2 - \cos x$ is greater than 0 and $x(2 - \cos x)$ will be also greater than 0 for $x > 0$.

$\Rightarrow F(x)$ is strictly increasing function.

Therefore $F(x)$ will cut at only one point for $x > 0$.

As we have already seen that $F(x)$ is even, therefore $F(x)$ has only two real roots.

9 votes -- Debashish Deka (56.6k points)

5.4.5 Functions: ISI2017-13 top

https://gateoverflow.in/209635

Even function $f(x) = f(-x)$

$L.H.S. = R.H.S. = 5$

answer is (**B**)

2 votes -- Jayanta Mahata (1.5k points)

5.5 Integration(11) top

5.5.1 Integration: GATE2000-2.3 top

https://gateoverflow.in/650

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Let S = $\sum_{i=3}^{100} i \log_2 i$, and

T = $\int_2^{100} x \log_2 x dx$.

Which of the following statements is true?

A. $S > T$
B. $S = T$
C. $S < T$ and $2S > T$
D. $2S \leq T$

gate2000 calculus integration normal

Answer

5.5.2 Integration: GATE2005-IT-35 top

https://gateoverflow.in/3782

What is the value of $\int_0^{2\pi} (x-\pi)^2 (\sin x) dx$

A. -1
B. 0
C. 1
D. π

gate2005-it calculus integration normal

Answer

5.5.3 Integration: GATE2009-25 top

https://gateoverflow.in/802

$\int_0^{\pi/4} (1-\tan x)/(1+\tan x)\, dx$

A. 0
B. 1
C. $ln2$
D. $1/2ln2$

gate2009 calculus integration normal

Answer

5.5.4 Integration: GATE2011-31 top

https://gateoverflow.in/2133

Given $i = \sqrt{-1}$, what will be the evaluation of the definite integral $\int_0^{\pi/2} \frac{\cos x + i \sin x}{\cos x - i \sin x} dx$?

A. 0
B. 2
C. $-i$
D. i

gate2011 calculus integration normal

Answer

5.5.5 Integration: GATE2014-3-47 top

https://gateoverflow.in/2081

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The value of the integral given below is

$$\int_0^{\pi} x^2 \cos x \, dx$$

A. -2π
B. π
C. $-\pi$
D. 2π

gate2014-3 calculus limits integration normal

Answer

5.5.6 Integration: GATE2014-3-6 top

https://gateoverflow.in/2040

If $\int_0^{2\pi} |x \sin x| dx = k\pi$, then the value of k is equal to _____.

gate2014-3 calculus integration limits numerical-answers easy

Answer

5.5.7 Integration: GATE2015-1-44 top

https://gateoverflow.in/8314

Compute the value of:

$$\int_{\frac{1}{\pi}}^{\frac{2}{\pi}} \frac{\cos(1/x)}{x^2} dx$$

gate2015-1 calculus integration normal numerical-answers

Answer

5.5.8 Integration: GATE2015-3-45 top

https://gateoverflow.in/8554

If for non-zero x, $af(x) + bf(\frac{1}{x}) = \frac{1}{x} - 25$ where a $a \neq b$ then $\int_1^2 f(x)dx$ is

A. $\frac{1}{a^2-b^2}\left[a(\ln 2 - 25) + \frac{47b}{2}\right]$
B. $\frac{1}{a^2-b^2}\left[a(2\ln 2 - 25) - \frac{47b}{2}\right]$
C. $\frac{1}{a^2-b^2}\left[a(2\ln 2 - 25) + \frac{47b}{2}\right]$
D. $\frac{1}{a^2-b^2}\left[a(\ln 2 - 25) - \frac{47b}{2}\right]$

gate2015-3 calculus integration normal

Answer

5.5.9 Integration: GATE2018-16 top

https://gateoverflow.in/204090

The value of $\int_0^{\pi/4} x \cos(x^2) dx$ correct to three decimal places (assuming that $\pi = 3.14$) is ____

gate2018 calculus integration normal numerical-answers

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Answer

5.5.10 Integration: ISI2004-MIII-13 top

https://gateoverflow.in/123887

Let $X = \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + \ldots + \frac{1}{3001}$. Then

A. $X < 1$
B. $X > \frac{3}{2}$
C. $1 < X < \frac{3}{2}$
D. none of the above

isi2004 engineering-mathematics integration

Answer

5.5.11 Integration: TIFR2011-A-11 top

https://gateoverflow.in/20219

$$\int_0^1 \ln x \, \mathrm{d}x =$$

a. 1
b. -1
c. ∞
d. $-\infty$
e. None of the above.

tifr2011 calculus integration

Answer

Answers: Integration

5.5.1 Integration: GATE2000-2.3 top

https://gateoverflow.in/650

Selected Answer

$x \log_2 x$ is a continuously increasing function, and for a continuously increasing function $f(x)$,

$$\sum_{x=a}^{b} f(x) > \int_a^b f(x)dx$$

But in question, summation of L.H.S. above, $a = 3$ and in R.H.S, $a = 2$, so we don't know whether $S > T$. So we compute some initial values :

$\sum_{x=3}^{4} x \log_2 x \approx 12.754$, and $\int_2^4 x \log_2 x = 11$

Since $\sum_{x=3}^{4} x \log_2 x > \int_2^4 x \log_2 x$, and since we already know that

$\sum_{x=5}^{100} x \log_2 x > \int_5^{100} x \log_2 x$

So $\sum_{x=3}^{100} x \log_2 x > \int_2^{100} x \log_2 x$

So S > T, and option **(A) is correct.**

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15 votes -- Happy Mittal (11.2k points)

5.5.2 Integration: GATE2005-IT-35 top

https://gateoverflow.in/3782

Selected Answer

answer is (b)

Put $x-\pi=t$ then limit 0 changes to $-\pi$ and upper limit 2π changes to π.

$\frac{d}{dx}(x-\pi)=dt \implies dx=dt$

Integration of $t^2\sin t dt$ for limit $-\pi$ to π. One is an odd function and one is even and product of odd and even functions is odd function and integrating an odd function from the same negative value to positive value gives 0.

11 votes -- SAKET NANDAN (4.8k points)

5.5.3 Integration: GATE2009-25 top

https://gateoverflow.in/802

Selected Answer

Let $I=\int_0^{\frac{\pi}{4}}\frac{1-\tan x}{1+\tan x}dx=\int_0^{\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos x+\sin x}dx$

Now put
$\cos x+\sin x=t$, Then
$(-\sin x+\cos x)\,dx=dt$ and changing limit

So we get
$I=\int_1^{\sqrt{2}}\frac{1}{t}dt=[\ln t]=\ln(\sqrt{2})=\frac{\ln 2}{2}$

29 votes -- Jagdish Singh (521 points)

5.5.4 Integration: GATE2011-31 top

https://gateoverflow.in/2133

Selected Answer

Answer is **D.**

$\int_0^{\frac{\pi}{2}}\frac{e^{ix}}{e^{-ix}}dx=\int_0^{\frac{\pi}{2}}e^{2ix}dx=\frac{e^{2ix}}{2i}\Big|_0^{\frac{\pi}{2}}=\frac{-2}{2i}=\frac{i^2}{i}=i.$

23 votes -- Sona Praneeth Akula (4.2k points)

5.5.5 Integration: GATE2014-3-47 top

https://gateoverflow.in/2081

Selected Answer

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Answer is A.

$\int_0^\pi x^2 \cos x dx$

$= x^2 \sin x]_0^\pi - \int_0^\pi 2x \sin x$

$= x^2 \sin x]_0^\pi + 2x \cos x]_0^\pi - \int_0^\pi 2 \cos x dx$

$= x^2 \sin x]_0^\pi + 2x \cos x]_0^\pi - 2 \sin x]_0^\pi$

$= [\pi^2(0) - 0] + 2[\pi(-1) - 0] - 2[0 - 0]$

$= -2\pi$

Integral of a multiplied by b equals

a multiplied by integral of b

minus

integral of derivative of a multiplied by integral of b

13 votes -- Keith Kr (6.1k points)

5.5.6 Integration: GATE2014-3-6 top

https://gateoverflow.in/2040

Selected Answer

There is a mod term in the given integral. So, first we have to remove that. We know that x is always positive here and $\sin x$ is positive from 0 to π. From π to 2π, x is positive while $\sin x$ changes sign. So, we can write

$$\int_0^{2\pi} \mid x \sin x \mid dx = \int_0^\pi x \sin x dx + \left(-\int_\pi^{2\pi} x \sin x dx\right)$$

$$= \int_0^\pi x \sin x dx - \int_\pi^{2\pi} x \sin x dx.$$

$\int_0^\pi u dv = uv - \int_0^\pi v du$

$\text{Here } u = x, du = dx, dv = \sin x dx, \text{ so } v = -\cos x$

$\therefore \int_0^\pi x \sin x dx = [-x \cos x]_0^\pi + \int_0^\pi \cos x dx$

$= \pi + [\sin x]_0^\pi = \pi$

$\text{Now, } \int_\pi^{2\pi} x \sin x = [-x \cos x]_\pi^{2\pi} + \int_\pi^{2\pi} \cos x dx$

$= -3\pi + [\sin x]_\pi^{2\pi} = -3\pi$

So, given integral = $\pi - (-3\pi) = 4\pi$

So, k = 4.

27 votes -- Arjun Suresh (348k points)

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5.5.7 Integration: GATE2015-1-44 top

https://gateoverflow.in/8314

Selected Answer

For the integrand $\frac{\cos(1/x)}{x^2}$, substitute $u = \frac{1}{x}$ and $\mathrm{d}u = -\frac{1}{x^2}\,\mathrm{d}x$.

This gives a new lower bound $u = \frac{1}{1/\pi} = \pi$ and upper bound $u = \frac{1}{2/\pi} = \frac{\pi}{2}$. Now, our integral becomes:

$$I = -\int_{\pi}^{\pi/2} \cos(u)\,\mathrm{d}u$$

$$= \int_{\pi/2}^{\pi} \cos(u)\,\mathrm{d}u$$

Since the antiderivative of $\cos(u)$ is $\sin(u)$, applying the fundamental theorem of calculus, we get:

$$I = \sin(u)\Big|_{\pi/2}^{\pi}$$
$$= \sin(\pi) - \sin\left(\frac{\pi}{2}\right)$$
$$= 0 - 1$$
$$I = -1$$

34 votes -- Shyam Singh (1.6k points)

5.5.8 Integration: GATE2015-3-45 top

https://gateoverflow.in/8554

Selected Answer

$$af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 25 \text{ --- } (1)$$

Integrating both sides,

$$a\int_1^2 f(x)\,dx + b\int_1^2 f\left(\frac{1}{x}\right)dx = [\log(x) - 25x]_1^2 = \log 2 - 25 \text{ --- } (2)$$

Replacing x by $\frac{1}{x}$ in (1), we get

$$af\left(\frac{1}{x}\right) + bf(x) = x - 25$$

Integrating both sides, we get

$$a\int_1^2 f\left(\frac{1}{x}\right)dx + b\int_1^2 f(x)\,dx = \left[\frac{x^2}{2} - 25x\right]_1^2 = -\frac{47}{2} \text{ --- } (3)$$

Eliminate $\int_1^2 f\left(\frac{1}{x}\right)$ between (2) and (3) by multiplying (2) by a and (3) by b and subtracting

$$\therefore \left(a^2 - b^2\right)\int_1^2 f(x)\,dx = a(\log 2 - 25) + b \times \frac{47}{2}$$

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$\therefore \int_1^2 f(x)\,dx = \frac{1}{(a^2-b^2)}\left[a(\log 2 - 25) + \frac{47b}{2}\right]$

Answer: A. $\frac{1}{(a^2-b^2)}\left[a(\log 2 - 25) + \frac{47b}{2}\right]$

37 votes -- Shyam Singh (1.6k points)

5.5.9 Integration: GATE2018-16 top

https://gateoverflow.in/204090

Selected Answer

$$\int_0^{\frac{\pi}{4}} x cos(x^2) dx$$

put $x^2 = t$

$2xdx = dt$

t will range from 0 to $\frac{\pi^2}{16}$

Now our new Integral is : $\frac{1}{2}\int_0^{\frac{\pi^2}{16}} cos(t)dt$

= $\frac{1}{2}[sin(t)]_0^{\frac{\pi^2}{16}} = \frac{1}{2}[sin(.616225) - 0]$ = $\frac{0.5779}{2}$

sin(.616225 radians) =

0.5779586366

0.289 Answer

11 votes -- sumit_goyal (12.1k points)

5.5.10 Integration: ISI2004-MIII-13 top

https://gateoverflow.in/123887

Selected Answer

Take the series as $\frac{1}{x}$

and use **Riemann integral to evaluate the series**

integrate $\frac{1}{x}dx$ **from**
$x = 1001$ **to**
3001 **then we get**
1.0979 **which is in between**
1 **and**
$\frac{3}{2}$.

So C is correct option.

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$1 < X < \frac{3}{2}$

4 votes -- Shubham Sharma (5.6k points)

5.5.11 Integration: TIFR2011-A-11 top

https://gateoverflow.in/20219

Selected Answer

Use Integration by Parts

$$\int \ln(x)dx$$

set
$u = \ln(x), \quad dv = dx$
then we find
$du = \left(\frac{1}{x}\right) dx, \quad v = x$

substitute

$$\int \ln(x)dx = \int u\, dv$$

and use integration by parts

$$= uv - \int v\, du$$

substitute $u = \ln(x), v = x,$ and $du = \left(\frac{1}{x}\right) dx$

$$= \ln(x)\, x - \int x \left(\frac{1}{x}\right) dx$$
$$= \ln(x)x - \int dx$$
$$= \ln(x)x - x + C$$
$$= x\ln(x) - x + C.$$

Now Put Limits

$$[\ln(1) - 1 + C] - [0 - 0 + C] = -1$$

Note-

$$\lim_{x=0}[x \ln x] = 0.$$

7 votes -- sonu (2.3k points)

5.6 Limits(12) top

5.6.1 Limits: GATE1993-02.1 top

https://gateoverflow.in/605

In questions 2.1 to 2.10 below, each blank (____) is to be suitably filled in. $\lim_{x\to 0} \frac{x(e^x-1)+2(\cos x-1)}{x(1-\cos x)}$ is_____

gate1993 limits calculus normal numerical-answers

Answer

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5.6.2 Limits: GATE2008-1 top

https://gateoverflow.in/399

$\lim_{x\to\infty} \frac{x-\sin x}{x+\cos x}$ equals

A. 1
B. -1
C. ∞
D. $-\infty$

gate2008 calculus limits easy

Answer

5.6.3 Limits: GATE2010-5 top

https://gateoverflow.in/1151

What is the value of $\lim_{n\to\infty} \left(1-\frac{1}{n}\right)^{2n}$?

A. 0
B. e^{-2}
C. $e^{-1/2}$
D. 1

gate2010 calculus limits normal

Answer

5.6.4 Limits: GATE2015-1-4 top

https://gateoverflow.in/8021

$\lim_{x\to\infty} x^{\frac{1}{x}}$ is

A. ∞
B. 0
C. 1
D. Not defined

gate2015-1 calculus limits normal

Answer

5.6.5 Limits: GATE2015-3-9 top

https://gateoverflow.in/8403

The value of $\lim_{x\to\infty}(1+x^2)^{e^{-x}}$ is

A. 0
B. $\frac{1}{2}$
C. 1
D. ∞

gate2015-3 calculus limits normal

Answer

5.6.6 Limits: GATE2016-1-3 top

https://gateoverflow.in/39630

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$$\lim_{x \to 4} \frac{\sin(x-4)}{x-4}$$

=_____.

gate2016-1 calculus limits easy numerical-answers

Answer

5.6.7 Limits: GATE2017-1-28 top

https://gateoverflow.in/118309

The value of $\lim_{x \to 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

A. is 0
B. is -1
C. is 1
D. does not exist

gate2017-1 calculus limits normal

Answer

5.6.8 Limits: ISI2017-14 top

https://gateoverflow.in/209650

Let (v_n) be a sequence defined by $v_1 = 1$ and $v_{n+1} = \sqrt{v_n^2 + \left(\frac{1}{5}\right)^n}$ for $n \geq 1$. Then $\lim_{n \to \infty} v_n$ is

A. $\sqrt{5/3}$
B. $\sqrt{5/4}$
C. 1
D. nonexistent

isi2017 limits

Answer

5.6.9 Limits: TIFR2010-A-7 top

https://gateoverflow.in/18234

The limit of $\frac{10^n}{n!}$ as $n \to \infty$ is.

A. 0
B. 1
C. e
D. 10
E. ∞

tifr2010 calculus limits

Answer

5.6.10 Limits: TIFR2011-A-14 top

https://gateoverflow.in/20224

What is the value of the following limit?

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$$\lim_{x \to 0} \frac{d}{dx} \frac{\sin^2 x}{x}$$

a. 0
b. 2
c. 1
d. $\frac{1}{2}$
e. None of the above

tifr2011 calculus limits

Answer

5.6.11 Limits: TIFR2011-A-17 top

https://gateoverflow.in/20254

What is the value of the following limit?

$$\lim_{x \to 0} \frac{2^x - 1}{x}$$

a. 0
b. $\log_2(e)$
c. $\log_e(2)$
d. 1
e. None of the above.

tifr2011 limits

Answer

5.6.12 Limits: TIFR2012-A-14 top

https://gateoverflow.in/25037

The limit $\lim_{n \to \infty} \left(\sqrt{n^2 + n} - n\right)$ equals.

A. ∞
B. 1
C. $1/2$
D. 0
E. None of the above.

tifr2012 calculus limits

Answer

Answers: Limits

5.6.1 Limits: GATE1993-02.1 top

https://gateoverflow.in/605

Selected Answer

Use LH rule:

First Derivative: $\frac{[x(e^x) + (e^x - 1) - 2(\sin x)]}{[x \sin x + (1 - \cos x)]}$

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Second Derivative: $\frac{[xe^x + e^x + e^x - 2\cos x]}{[x\cos x + \sin x + \sin x]}$

Third Derivative: $\frac{[xe^x + e^x + e^x + e^x + 2\sin x]}{[-x\sin x + \cos x + \cos x + \cos x]}$

Put $x = 0$: $\frac{[0+1+1+1+0]}{[0+1+1+1]} = \frac{3}{3} = 1.$

7 votes -- Rajarshi Sarkar (34.1k points)

5.6.2 Limits: GATE2008-1 top

https://gateoverflow.in/399

Selected Answer

$\lim_{x\to\infty} \frac{x - \sin x}{x + \cos x}$

$= \lim_{x\to\infty} \frac{x(1 - \frac{\sin x}{x})}{x(1 + \frac{\cos x}{x})}$

$= \lim_{x\to\infty} \frac{1 - \frac{\sin x}{x}}{1 + \frac{\cos x}{x}}$

now to calculate values of $\frac{\sin x}{x}$ and $\frac{\cos x}{x}$ we use Squeezing Theorem.

$-1 \leq \sin x \leq +1 \frac{-1}{x} \leq \frac{\sin x}{x} \leq \frac{+1}{x}$

$-1 \leq \cos x \leq +1 \frac{-1}{x} \leq \frac{\cos x}{x} \leq \frac{+1}{x}$

now as $x \to \infty$ we get $\frac{1}{x} \to 0$, this implies that:

$0 \leq \frac{\sin x}{x} \leq 00 \leq \frac{\cos x}{x} \leq 0$

Hence,
$\lim_{x\to\infty} \frac{x - \sin x}{x + \cos x}$

$= \lim_{x\to\infty} \frac{1 - \frac{\sin x}{x}}{1 + \frac{\cos x}{x}}$

$= \lim_{x\to\infty} \frac{1-0}{1+0} = 1$

answer = **option A**

31 votes -- Amar Vashishth (30.5k points)

5.6.3 Limits: GATE2010-5 top

https://gateoverflow.in/1151

Selected Answer

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i will solve by two methods

method 1:

$y = \lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^{2n}$

Taking log

$\log y = \lim_{n\to\infty} 2n \log\left(1 - \frac{1}{n}\right)$

$= \lim_{n\to\infty} \frac{\log\left(1 - \frac{1}{n}\right)}{\left(\frac{1}{2n}\right)}$ ----------(converted this so as to have form $\left(\frac{0}{0}\right)$)

apply L' hospital rule

$$\log y = \lim_{n\to\infty} \frac{\left(\frac{1}{1 - \frac{1}{n}}\right) \cdot \frac{1}{n^2}}{\left(\frac{-1}{2n^2}\right)}$$

$\log y = -2$

$y = e^{-2}.$

---

method 2:

it takes 1 to power infinity form

$\lim_{x\to\infty} f(x)^{g(x)}$

$= e^{\lim_{x\to\infty} (f(x)-1)g(x)}$

where, $(f(x) - 1) * g(x) = \frac{-1}{n} * 2n = -2.$

ie -2 constant.

so we get final ans is $= e^{-2}.$

u can refer this link for second method

http://www.vitutor.com/calculus/limits/one_infinity.html

26 votes -- Pooja Palod (31.3k points)

5.6.4 Limits: GATE2015-1-4 top

https://gateoverflow.in/8021

Selected Answer

Apply an exponential of a logarithm to the expression.

$\lim_{x\to\infty} x^{\frac{1}{x}} = \lim_{x\to\infty} \exp\left(\log\left(x^{\frac{1}{x}}\right)\right) = \lim_{x\to\infty} \exp\left(\frac{\log(x)}{x}\right)$

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$\lim_{x\to\infty} \exp\left(\frac{\log(x)}{x}\right)$

Since the exponential function is continuous, we may factor it out of the limit.

$\lim_{x\to\infty} \exp\left(\frac{\log(x)}{x}\right) = \exp\left(\lim_{x\to\infty} \frac{\log(x)}{x}\right)$

$\exp\left(\lim_{x\to\infty} \frac{\log(x)}{x}\right)$

Logarithmic functions grow asymptotically slower than polynomials.

Since $\log(x)$ grows asymptotically slower than the polynomial x as x approaches ∞,

$\lim_{x\to\infty} \frac{\log(x)}{x} = 0:$

$e^0 = 1$

22 votes -- Shyam Singh (1.6k points)

5.6.5 Limits: GATE2015-3-9 top

https://gateoverflow.in/8403

Selected Answer

Apply an exponential of a logarithm to the expression.

$$\lim_{x\to\infty} (x^2+1)^{e^{-x}} = \lim_{x\to\infty} \exp\left(log((x^2+1)^{e^{-x}})\right)$$

$$= \lim_{x\to\infty} \exp\left(\frac{log(x^2+1)}{e^x}\right)$$

Since the exponential function is continuous, we may factor it out of the limit.

$$\lim_{x\to\infty} \exp\left(\frac{log(x^2+1)}{e^x}\right)$$

$$= \exp\left(\lim_{x\to\infty} \frac{log(x^2+1)}{e^x}\right)$$

The numerator of $e^{-x}\log(x^2+1)$ grows asymptotically slower than its denominator as x approaches ∞.

Since $\log(x^2+1)$ grows asymptotically slower than e^x as x approaches ∞,
$\lim_{x\to\infty} e^{-x}\log(x^2+1) = 0 : e^0$.

Evaluate e^0.

$e^0 = 1:$

Answer: 1.

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32 votes -- Shyam Singh (1.6k points)

5.6.6 Limits: GATE2016-1-3 top

https://gateoverflow.in/39630

Selected Answer

substitute h=x-4..so it becomes $\lim_{h\to 0}$ (sinh)/h ... which is a standard limit.. Ans would be 1.

17 votes -- Abhilash Panicker (9.4k points)

5.6.7 Limits: GATE2017-1-28 top

https://gateoverflow.in/118309

Selected Answer

Since substituting $x = 1$ we get $\frac{0}{0}$ which is indeterminate.

after applying L'Hospital rule, we get $\frac{(7x^6 - 10x^4)}{(3x^2 - 6x)}$

now substituting $x = 1$ we get $\left(\frac{-3}{-3}\right) = 1.$

Hence, answer is $1.$

13 votes -- sriv_shubham (3.3k points)

5.6.8 Limits: ISI2017-14 top

https://gateoverflow.in/209650

Selected Answer

$V_{n+1} = \sqrt{V_n^2 + (1/5)^n}$

on squaring both side, we get

$V_{n+1}^2 - V_n^2 = (1/5)^n$

$V_2^2 - V_1^2 = (1/5)$

$V_3^2 - V_2^2 = (1/5)^2$

$V_4^2 - V_3^2 = (1/5)^3$

$V_5^2 - V_4^2 = (1/5)^4$

$\vdots$

$V_n^2 - V_{n-1}^2 = (1/5)^n - 1$

On adding all we get,

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$$V_n^2 - V_1^2 = ((1/5) + (1/5)^2 + (1/5)^3 + (1/5)^4 + (1/5)^5 + \cdots + (1/5)^{n-1}$$

$$= (1 - (1/5)^{n-1})/4$$

$$= 1/4 - (1/5)^{n-1}/4$$

$$\implies V_n^2 = V_1^2 + 1/4 - (1/5)^{n-1}/4$$

$$= 1 + 1/4 - (1/5)^{n-1}/4$$

$$= 5/4 - (1/5)^{n-1}/4$$

on taking limit, we get

$$V_n = \sqrt{5/4}$$

$$(v) \lim_{x \to \infty} a^x = \begin{cases} 0, & 0 \le a < 1 \\ 1, & a = 1 \\ \infty, & a > 1 \\ \text{does not exist}, & a < 0 \end{cases}$$

4 votes -- Pankaj Kumar (8.5k points)

5.6.9 Limits: TIFR2010-A-7 top

https://gateoverflow.in/18234

Selected Answer

$$\lim_{n \to \infty} \frac{10^n}{n!}$$

$$= \frac{\overbrace{10 \times 10 \times 10 \times \ldots \times 10 \times 10}^{n \text{ times}}}{1 \times 2 \times 3 \times \cdots \times (n-1) \times n}$$

$$= \overbrace{\frac{10}{1} \cdot \frac{10}{2} \cdots \frac{10}{10}}^{\approx 2755} \cdot \underbrace{\frac{10}{11} \cdots \frac{10}{100} \cdot \frac{10}{101}}_{\ll 1} \cdots \overbrace{\frac{10}{10000} \cdot \frac{10}{10001}}^{\lll 1} \cdots \text{ goes forever}$$

Now we can see that after the $\frac{10}{10}$ term, all subsequent terms are < 1, and keep decreasing. As we increase the value of n it the product will get close to 0.

So as $n \to \infty$ $\frac{10^n}{n!} \to 0$.

Hence, the answer is option a) 0

17 votes -- sonam vyas (14.8k points)

5.6.10 Limits: TIFR2011-A-14 top

https://gateoverflow.in/20224

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Selected Answer

Answer is 1.

$$\lim_{x\to 0}\frac{d}{dx}\frac{sin^2x}{x}$$

Find $\frac{d}{dx}\left(\frac{\sin^2 x}{x}\right)$

$$=\frac{(\sin^2 x)^1 x-(x)^1\sin^2 x}{x^2}$$

$$=\frac{[2\sin x.\cos x]\,x-sin^2x}{x^2}$$

$$=\frac{x\sin 2x-\sin^2 x}{x^2}$$

$$\lim_{x\to 0}\left(\frac{x\sin 2x-\sin^2 x}{x^2}\right)$$

$$=\lim_{x\to 0}\left(\frac{x\sin 2x}{x^2}-\frac{\sin^2 x}{x^2}\right)$$

$$=\lim_{x\to 0}\frac{x\sin 2x}{x^2}-\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2$$

$$=\lim_{x\to 0}\frac{\cos 2x.2}{1}-1$$

$$=2[\cos 2(0)]-1$$

$$=2(1)-1\Rightarrow 1.$$

13 votes -- Prabhanjan_1 (12.4k points)

5.6.11 Limits: TIFR2011-A-17 top

https://gateoverflow.in/20254

Selected Answer

Since we have a $\frac{0}{0}$ form, we can apply the L'Hôpital's rule.

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$$L = \lim_{x \to 0} \frac{2^x - 1}{x}$$

$$= \lim_{x \to 0} \frac{\frac{d}{dx}(2^x - 1)}{\frac{d}{dx}x}$$

$$= \lim_{x \to 0} \frac{2^x \ln 2}{1}$$

$$= \ln 2$$

$$L = \log_e(2)$$

Hence, option c is correct.

12 votes -- Pragy Agarwal (20.6k points)

5.6.12 Limits: TIFR2012-A-14 top

https://gateoverflow.in/25037

Selected Answer

$$L = \lim_{n \to \infty} \sqrt{n^2 + n} - n$$

$$= \lim_{n \to \infty} \left(\sqrt{n^2 + n} - n\right) \times \left(\frac{\sqrt{n^2 + n} + n}{\sqrt{n^2 + n} + n}\right)$$

$$= \lim_{n \to \infty} \frac{n^2 + n - n^2}{\sqrt{n^2 + n} + n}$$

$$= \lim_{n \to \infty} \frac{n}{n\left(\sqrt{1 + \frac{1}{n}} + 1\right)}$$

$$= \lim_{n \to \infty} \frac{1}{\sqrt{1 + \frac{1}{n}} + 1}$$

$$= \frac{1}{\sqrt{1 + \frac{1}{\infty}} + 1}$$

$$L = \frac{1}{2}$$

Hence, option C is the correct answer.

8 votes -- Jagdish Singh (521 points)

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5.7 Maxima Minima(14) top

5.7.1 Maxima Minima: GATE1995-25 top

https://gateoverflow.in/2664

Find the minimum value of $3 - 4x + 2x^2$.

Determine the number of positive integers (< 720) which are not divisible by any of $2, 3$ or 5.

gate1995 calculus normal maxima-minima combined-question

Answer

5.7.2 Maxima Minima: GATE1997-4.1 top

https://gateoverflow.in/2242

What is the maximum value of the function $f(x) = 2x^2 - 2x + 6$ in the interval $[0, 2]$?

A. 6
B. 10
C. 12
D. 5.5

gate1997 calculus maxima-minima normal

Answer

5.7.3 Maxima Minima: GATE1998-8 top

https://gateoverflow.in/1722

(a) Find the points of local maxima and minima, if any, of the following function defined in $0 \leq x \leq 6$.

$$x^3 - 6x^2 + 9x + 15$$

(b) Integrate

$$\int_{-\pi}^{\pi} x \cos x dx$$

gate1998 calculus maxima-minima integration normal descriptive

Answer

5.7.4 Maxima Minima: GATE2008-25 top

https://gateoverflow.in/423

A point on a curve is said to be an extremum if it is a local minimum or a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 24x^2 + 37$ is

A. 0
B. 1
C. 2
D. 3

gate2008 calculus maxima-minima easy

Answer

5.7.5 Maxima Minima: GATE2008-IT-31 top

https://gateoverflow.in/3341

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If $f(x)$ is defined as follows, what is the minimum value of $f(x)$ for $x \in (0,2]$?

$$f(x) = \begin{cases} \frac{25}{8x} \text{ when } x \leq \frac{3}{2} \\ x + \frac{1}{x} \text{ otherwise} \end{cases}$$

A. 2
B. $2\left(\frac{1}{12}\right)$
C. $2\left(\frac{1}{6}\right)$
D. $2\left(\frac{1}{2}\right)$

gate2008-it calculus maxima-minima normal

Answer

5.7.6 Maxima Minima: GATE2012-9 top

https://gateoverflow.in/41

Consider the function $f(x) = \sin(x)$ in the interval $x = \left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$. The number and location(s) of the local minima of this function are

A. One, at $\frac{\pi}{2}$
B. One, at $\frac{3\pi}{2}$
C. Two, at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$
D. Two, at $\frac{\pi}{4}$ and $\frac{3\pi}{2}$

gate2012 calculus maxima-minima normal nielit

Answer

5.7.7 Maxima Minima: ISI2004-MIII-12 top

https://gateoverflow.in/123886

The maximum possible value of xy^2z^3 subjected to condition $x, y, z \geq 0$ and $x + y + z = 3$ is

A. 1
B. $\frac{9}{8}$
C. $\frac{9}{4}$
D. $\frac{27}{16}$

isi2004 engineering-mathematics maxima-minima

Answer

5.7.8 Maxima Minima: TIFR2010-A-3 top

https://gateoverflow.in/18209

The function $f(x) = 2.5 \log_e\left(2 + \exp\left(x^2 - 4x + 5\right)\right)$ attains a minimum at $x =$?

a. 0
b. 1
c. 2
d. 3

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e. 4

tifr2010 calculus maxima-minima

Answer

5.7.9 Maxima Minima: TIFR2011-A-4 top

https://gateoverflow.in/20002

Consider the problem of maximizing $x^2 - 2x + 5$ such that $0 < x < 2$. The value of x at which the maximum is achieved is:

a. 0.5
b. 1
c. 1.5
d. 1.75
e. None of the above.

tifr2011 calculus maxima-minima

Answer

5.7.10 Maxima Minima: TIFR2012-A-13 top

https://gateoverflow.in/25036

The maximum value of the function.

$$f(x, y, z) = (x - 1/3)^2 + (y - 1/3)^2 + (z - 1/3)^2$$

Subject to the constraints

$$x + y + z = 1, x \geq 0, y \geq 0, z \geq 0$$

is

a. $1/3$
b. $2/3$
c. 1
d. $4/3$
e. $4/9$

tifr2012 calculus maxima-minima

Answer

5.7.11 Maxima Minima: TIFR2012-A-15 top

https://gateoverflow.in/25040

Consider the differential equation $dx/dt = (1 - x)(2 - x)(3 - x)$. Which of its equilibria is unstable?

a. $x = 0$
b. $x = 1$
c. $x = 2$
d. $x = 3$
e. None of the above.

tifr2012 calculus maxima-minima

Answer

5.7.12 Maxima Minima: TIFR2013-A-16 top

https://gateoverflow.in/25496

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The minimum of the function $f(x) = x\log_e(x)$ over the interval $[\frac{1}{2}, \infty)$ is

a. 0
b. $-e$
c. $\frac{-\log_e(2)}{2}$
d. $\frac{-1}{e}$
e. None of the above

tifr2013 calculus maxima-minima

Answer

5.7.13 Maxima Minima: TIFR2014-A-9 top

https://gateoverflow.in/25996

Solve min $x^2 + y^2$

subject to

$$x + y \geq 10,$$
$$2x + 3y \geq 20,$$
$$x \geq 4,$$
$$y \geq 4.$$

a. 32
b. 50
c. 52
d. 100
e. None of the above

tifr2014 calculus maxima-minima

Answer

5.7.14 Maxima Minima: TIFR2015-A-11 top

https://gateoverflow.in/29581

Suppose that $f(x)$ is a continuous function such that $0.4 \leq f(x) \leq 0.6$ for $0 \leq x \leq 1$. Which of the following is always true?

A. $f(0.5) = 0.5$.
B. There exists x between 0 and 1 such that $f(x) = 0.8x$.
C. There exists x between 0 and 0.5 such that $f(x) = x$.
D. $f(0.5) > 0.5$.
E. None of the above statements are always true.

tifr2015 maxima-minima calculus

Answer

Answers: Maxima Minima

5.7.1 Maxima Minima: GATE1995-25 top

https://gateoverflow.in/2664

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Selected Answer

$f(x) = 3 - 4x + 2x^2$

$f'(x) = -4 + 4x = 0 \implies x = 1$

$f''(x) = 4$

$f''(1) = 4 > 0$, therefore at $x = 1$ we will get minimum value, which is : $3 - 4(1) + 2(1)^2 = 1$.

Ans for B:

10 votes -- jayendra (8.3k points)

5.7.2 Maxima Minima: GATE1997-4.1 top

https://gateoverflow.in/2242

Selected Answer

Answer: B

For $f(x)$ to be maximum

$f'(x) = 4x - 2 = 0 \implies x = \frac{1}{2}$

So at $x = \frac{1}{2}, f(x)$ is an extremum (either maximum or minimum).

$f(2) = 2(2)^2 - 2(2) + 6 = 8 - 4 + 6 = 10$

$f\left(\frac{1}{2}\right) = 2\frac{1}{2}^2 - 2\frac{1}{2} + 6 = 5.5$, so $x = \frac{1}{2}$ is a mimimum.

$f(0) = 6.$

So, the maximum value is at $x = 2$ which is 10 as there are no other extremum for the given function.

11 votes -- Rajarshi Sarkar (34.1k points)

5.7.3 Maxima Minima: GATE1998-8 top

https://gateoverflow.in/1722

Selected Answer

(a)$f(x) = x^3 - 6x^2 + 9x + 15$

so $f'(x) = 3x^2 - 12x + 9 = 0 \Rightarrow x = 1, 3$

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Now $f''(x) = 6x - 12$

$f''(1) < 0$, so $x = 1$ is point of local maxima, $f''(3) > 0$, so $x = 3$ is point of local minima.

Also the end points 0 and 6 are critical points. 0 is point of local minima, because it is to the left of $x = 1$ (which is point of maxima). Similarly $x = 6$ is point of local maxima.

(b) Since $x \cos x$ is an odd function, by the properties of definite integration, answer is 0.

12 votes -- Happy Mittal (11.2k points)

5.7.4 Maxima Minima: GATE2008-25 top

https://gateoverflow.in/423

Selected Answer

Answer is (b).

$f(x) = 3x^4 - 16x^3 + 24x^2 + 37$

$f'(x) = 12x^3 - 48x^2 + 48x = 0$

$\implies 12x\left(x^2 - 4x + 4\right) = 0$

$x(x-2)^2 = 0$

$\implies x = 0, 2$

$f''(x) = 36x^2 - 96x + 48$

at $x = 0, f''(x) = 48 > 0$ it means that $x = 0$ is local minima.

but at $x = 2, f''(x) = 0$ so we can't apply second derivative test. So, we can apply first derivative test.

$f'(1) = 12, f'(3) = 36$. So, $f'(x)$ is not changing sign on either side of 2. So, $x = 2$ is neither maxima nor minima.

So, only one extremum i.e. x=0.

Ref: https://cims.nyu.edu/~kiryl/Calculus/Section_4.3--Derivatives_and_the_Shapes_of_Graphs/Derivatives_and_the_Shapes_of_Graphs.pdf

23 votes -- SAKET NANDAN (4.8k points)

5.7.5 Maxima Minima: GATE2008-IT-31 top

https://gateoverflow.in/3341

Selected Answer

Answer: B

at x = $\frac{3}{2}$,

f(x) =$2\left(\frac{1}{12}\right)$.

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11 votes -- Rajarshi Sarkar (34.1k points)

5.7.6 Maxima Minima: GATE2012-9 top

https://gateoverflow.in/41

Selected Answer

Answer is (d)

f '(s) = cosx =0 gives root $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ which lie between given domain in question $[\frac{\pi}{4},\frac{7\pi}{4}]$

f ''(x)= - (sinx) at $\frac{\pi}{2}$ it gives -1<0 it means it is local maxima and at $\frac{3\pi}{2}$ it gives 1>0 which is local minima

and since, at $\frac{\pi}{2}$ it is local maxima so, before it, graph is strictly increasing, so $\frac{\pi}{4}$ is also local minima

So, there are two local minima $\frac{\pi}{4}$ and $\frac{3\pi}{2}$

21 votes -- SAKET NANDAN (4.8k points)

5.7.7 Maxima Minima: ISI2004-MIII-12 top

https://gateoverflow.in/123886

Selected Answer

Given ,

$x+y+z=3$

$\Rightarrow x+\left(\frac{y}{2}\right)+\left(\frac{y}{2}\right)+\left(\frac{z}{3}\right)+\left(\frac{z}{3}\right)+\left(\frac{z}{3}\right)=3.$

Now using A.M. G.M inequality , we have :

$$\frac{\left[x+\left(\frac{y}{2}\right)+\left(\frac{y}{2}\right)+\left(\frac{z}{3}\right)+\left(\frac{z}{3}\right)+\left(\frac{z}{3}\right)\right]}{6}$$

$$\geq \left[x.\left(\frac{y}{2}\right).\left(\frac{y}{2}\right).\left(\frac{z}{3}\right).\left(\frac{z}{3}\right).\left(\frac{z}{3}\right)\right]^{\frac{1}{6}}$$

$$\Rightarrow \left(x.\left(\frac{y}{2}\right).\left(\frac{y}{2}\right).\left(\frac{z}{3}\right).\left(\frac{z}{3}\right).\left(\frac{z}{3}\right)\right)^{\frac{1}{6}} \leq \frac{1}{2}$$

$$\Rightarrow \left(x.\left(\frac{y}{2}\right).\left(\frac{y}{2}\right).\left(\frac{z}{3}\right).\left(\frac{z}{3}\right).\left(\frac{z}{3}\right)\right) \leq \frac{1}{64}$$

$$\Rightarrow x.y^2.z^3 \leq \frac{108}{64}=\frac{27}{16}.$$

Hence D) is the correct answer

13 votes -- HABIB MOHAMMAD KHAN (98.7k points)

5.7.8 Maxima Minima: TIFR2010-A-3 top

https://gateoverflow.in/18209

Selected Answer

Digvijay is right, $f(x)$ will be minimum at $x=2$,

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Here is another approach.
Since log and exponent are monotonically increasing functions, the problem of minimizing $f(x)$ can be reduced to just minimizing the quadratic expression

$x^2 - 4x + 5,$
this quadratic expression can be written as $(x^2 - 4x + 4) + 1$ which is equal to $(x-2)^2 + 1.$
now since $(x-2)^2$ can not be less than 0, so $(x-2)^2 + 1$ can not be less than $1.$
Also $(x-2)^2 + 1$ will be at its minimum value $(= 1)$, when $x = 2.$

so value of $f(x)$ will be minimum at $x = 2.$

10 votes -- Anurag Pandey (13.6k points)

5.7.9 Maxima Minima: TIFR2011-A-4 top

https://gateoverflow.in/20002

Selected Answer

$$P(x) = x^2 - 2x + 5$$

Since a polynomial is defined and continuous everywhere, we only need to check the critical point and the boundaries.

$$\frac{d}{dx}P(x) = 2x - 2$$

Critical point: $2x - 2 = 0 \implies x = 1$ gives $P(x) = 4$, which is the minimum.

Boundaries: $\lim_{x \to 0^+} P(x) = \lim_{x \to 2^-} P(x) = 5$

Since $P(x)$ increases as x goes farther away from the 1, but
$P(x)$ **is defined on an open interval,**
$P(x)$ **never attains a maximum!**

Hence, **e. None of the above** is the correct answer.

11 votes -- Pragy Agarwal (20.6k points)

5.7.10 Maxima Minima: TIFR2012-A-13 top

https://gateoverflow.in/25036

Selected Answer

I think the answer is 2/3. Verification required.

Expanding given equation

$x^2 + y^2 + z^2 - \frac{2}{3}(x + y + z) + \frac{1}{3}$

$= x^2 + y^2 + z^2 - \frac{2}{3} + \frac{1}{3}$

$= (x + y + z)^2 - \frac{1}{3} - 2(xy + yz + xz)$

$= 1 - \frac{1}{3} - 2(xy + yz + xz)$

$= \frac{2}{3} - 2(xy + yz + xz)$

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Now to maximize it, we need to minimize $(xy + yz + xz)$. As all x,y and z are non-negative xy + yz + xz >= 0. so the maximum value is $\frac{2}{3}$.

3 votes -- Dhruv Patel (2k points)

5.7.11 Maxima Minima: TIFR2012-A-15 top

https://gateoverflow.in/25040

Selected Answer

Here, dx/dt is neither constant nor function of variable 't'. So on solving for 'x', we get exponential decay function something similar to radioactive decay dN/dt = -KN ;K>0.

For any equilibrium point, we have dx/dt = 0 => x = 1,2,3.

Now if we consider sign scheme of dx/dt for different ranges of x, we have :

dx/dt > 0 for x < 1 ;

dx/dt < 0 for 1 < x < 2;

dx/dt > 0 for 2 < x < 3;

dx/dt < 0 for x > 3;

For x -> 2 , dx/dt becomes -ve to +ve. So x = 2 is point of unstable equilibrium.

Since here dx/dt is function of 'x' not 't' , so condition for checking stable / unstable points (aka local minima / local maxima) is opposite to what we apply in case of dy/dx = f(x). Here we have dy/dx = f(y) which is actually exponential function in y and x.

We can understand equilibrium in terms of radioactive decay.

Let dN/dt = -KN ;K>0 its significance is that an element is loosing energy so it is getting stability because we know more energy an element gets,more de-stability it gains and vice versa.

4 votes -- Shashank Kumar (3.3k points)

5.7.12 Maxima Minima: TIFR2013-A-16 top

https://gateoverflow.in/25496

Minimum value of function occurs at end points or critical points

$$f'(x) = 1 + \log x$$

Equate it to 40

$$x = \frac{1}{e}$$

$$f''(x) = \frac{1}{x}$$

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Put $x = \frac{1}{e}$ $f''(x) = e$ so minima at $\frac{1}{e}$.

But $\frac{1}{e} = 0.36$

But $x \in \left[\frac{1}{2}, \infty\right]$

So min occurs at $\frac{1}{2}$

So min value=$\frac{1}{2}\log\frac{1}{2} = \frac{1}{2} * \log_e -2$

So, answer is C

6 votes -- Pooja Palod (31.3k points)

5.7.13 Maxima Minima: TIFR2014-A-9 top

https://gateoverflow.in/25996

Selected Answer

Answer -> Option $B)$ 50

$x >= 4$ and $y >= 4$, So we can take both $x = 5$ & $y = 5$

$x + y >= 10 =>$ Satisfied , $5 + 5 = 10$

$2x + 3y >= 20$. Satisfied.

This is infact minimum value.

Other options =>

$4, 4 => x + y$ constraint fail

$4, 5 => x + y$ fail

$6, 4 =>$ Still giving 52 as sum which is more than 50 !, This can not be answer.

$7, 3 => 49 + 9 > 58 > 50$.

6 votes -- Akash Kanase (42.5k points)

5.7.14 Maxima Minima: TIFR2015-A-11 top

https://gateoverflow.in/29581

This is a repeating question on continuity. Let me solve it a non-standard way -- which should be useful in GATE.

From the question f is a function mapping the set of real (or rational) numbers between [0,1] to [0.4,0.6]. So, clearly the co-domain here is smaller than the domain set. The function is not given as onto and so, there is no requirement that all elements in co-domain set be mapped to by the domain set. We are half done now. Lets see the options:

A. $f(0.5) = 0.5$. False, as we can have $f(0.5) = 0.4$, continuity does not imply anything other than all points being mapped being continuous.

C. Again false, we can have $f(x) = 0.6$ for all x.

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D. False, same reason as for A.

Only B option left- which needs to be proved as correct now since we also have E option. We know that for a function all elements in domain set must have a mapping. All these can map to either 1 or more elements but at least one element must be there in the range set. i.e., $f(x) = y$ is true for some y which is in $[0.4, 0.6]$. In the minimal case this is a single element say c. Now for $x = 1/0.8$, option B is true. In the other case, say the minimal value of $f(x) = a$ and the maximum value be $f(x) = b$. Now,

as per Intermediate Value theorem (see: *https://www.math.ucdavis.edu/~hunter/intro_analysis_pdf/ch7.pdf*), all points between a and b are also in the range set as f is continuous. Now, we need to consider x in the range $[0.5, 0.75]$ as then only $f(x)$ can be $0.8x$ and be in $[0.4, 0.6]$. In our case we have

$f(x_1) = a, f(x_2) = b$. Lets assume $a! = 0.8x_1$ and $b! = 0.8x_2$. Now, for all other points in $[0.5, 0.75]$, $f(x)$ must be between a and b and all points between a and b must be mapped by some x.

Moreover, for $x = 0.5$, $f(x) \geq 0.4$ aand for $x = 0.75$, $f(x) \leq 0.6$. So, if we plot $g(x) = 0.8x$, **this line should cross**
$f(x)$ **at some point between**
0.5 **and**
0.75 because at $x = 0.5$, $f(x)$ must be above or equal to the line $0.8x$ (shown below) and for $x = 0.75$ it must be below or equal which means an intersection must be there.

This shows there exist some x between 0.5 and 0.75 for which $f(x) = 0.8x$ a stronger case than option B. So, B option is true. Now please try for $f(x) = 0.9x$ and see if it is true.

7 votes -- Arjun Suresh (348k points)

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6 Engineering Mathematics: Linear Algebra (79) top

6.1 Determinant(5) top

6.1.1 Determinant: GATE1997-1.3 top

https://gateoverflow.in/2219

The determinant of the matrix $\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

A. 11
B. -48
C. 0
D. -24

gate1997 linear-algebra normal determinant

Answer

6.1.2 Determinant: GATE2000-1.3 top

https://gateoverflow.in/626

The determinant of the matrix

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix}$$

A. 4
B. 0
C. 15
D. 20

gate2000 linear-algebra easy determinant

Answer

6.1.3 Determinant: GATE2005-IT-3 top

https://gateoverflow.in/3747

The determinant of the matrix given below is

$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix}$$

A. -1

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B. 0
C. 1
D. 2

gate2005-it linear-algebra normal determinant

Answer

6.1.4 Determinant: GATE2013-3 top

https://gateoverflow.in/1412

Which one of the following does NOT equal

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

?

A.
$$\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$$

B.
$$\begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix}$$

C.
$$\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$$

D.
$$\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$$

gate2013 linear-algebra normal determinant

Answer

6.1.5 Determinant: GATE2014-2-4 top

https://gateoverflow.in/1956

If the matrix A is such that

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$$

then the determinant of A is equal to _____.

gate2014-2 linear-algebra numerical-answers easy determinant

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Answer

Answers: Determinant

6.1.1 Determinant: GATE1997-1.3 top

https://gateoverflow.in/2219

Selected Answer

As the given matrix upper traingular matrix, determinant will be the product of main diagonal elements.

$det(A) = 6 * 2 * 4 * -1 = -48.$

Similar concept can be appliead, if Matrix is lower triangular or Diagonal Matrix.

18 votes -- durgesh (233 points)

6.1.2 Determinant: GATE2000-1.3 top

https://gateoverflow.in/626

Selected Answer

Answer: A

$$2 * \begin{bmatrix} 1 & 7 & 2 \\ 0 & 2 & 0 \\ 0 & 6 & 1 \end{bmatrix} = 4$$

10 votes -- Rajarshi Sarkar (34.1k points)

6.1.3 Determinant: GATE2005-IT-3 top

https://gateoverflow.in/3747

Selected Answer

$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix}$$

reduce this matrix into Upper Triangular matrix using row and column transformations:
$R1 \leftrightarrow R2$
$R3 \leftrightarrow R4$

$$\begin{bmatrix} -1 & 1 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R3 \leftarrow R1 + R3$
$R3 \leftarrow R2 - R3$

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Resulteed Upper Triangular matrix will be:

$$\begin{bmatrix} -1 & 1 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Detereminant will be product of diagonal elements = $-1 * 1 * 1 * 1 = -1$

Hence, A is correct option!

9 votes -- Manu Thakur (39.6k points)

$$+0 \begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 1 \\ -2 & 0 & 1 \end{bmatrix} -1 \begin{bmatrix} -1 & 1 & 3 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} +0 \begin{bmatrix} -1 & 1 & 3 \\ 0 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix} -2 \begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & -2 & 0 \end{bmatrix}$$

$$=-1 \begin{bmatrix} -1 & 1 & 3 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} = -1$$

20 votes -- neha pawar (4.2k points)

6.1.4 Determinant: GATE2013-3 top

https://gateoverflow.in/1412

Selected Answer

Answer is **A**.

Operations are:

$C_3 \leftarrow C_3 + C_2$

$C_2 \leftarrow C_2 + C_1$

Swap $C_2 \,\&\, C_3$

The Swapping operations mske the determinant as $(-1) * |A|$

where as the other options have their determinant as $|A|$.

19 votes -- Sona Praneeth Akula (4.2k points)

6.1.5 Determinant: GATE2014-2-4 top

https://gateoverflow.in/1956

Selected Answer

Hi,

For this kind of matrices Determinant is zero.

A will be a 3x3 matrix where the first row will be 2 $[1\ 9\ 5]$, second row will be -4 $[1\ 9\ 5]$ and third

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will be $7\ [19\ 5]$. That is, all the rows of A are linearly dependent which means A is singular.

When matrix is singular $|A| = 0$.

References: https://www.youtube.com/watch?v=aKX5_DucNq8&list=PL221E2BBF13BECF6C&index=19

26 votes -- Prasanna Ranganathan (4.7k points)

6.2 Eigen Value(23) top

6.2.1 Eigen Value: GATE1993-01.1 top

https://gateoverflow.in/596

In questions 1.1 to 1.7 below, one or more of the alternatives are correct. Write the code letter(s) a, b, c, d corresponding to the correct alternative(s) in the answer book. Marks will be given only if all the correct alternatives have been selected and no incorrect alternative is picked up.

1.1). The eigen vector (s) of the matrix

$$\begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \alpha \neq 0$$

is (are)

A. $(0, 0, \alpha)$
B. $(\alpha, 0, 0)$
C. $(0, 0, 1)$
D. $(0, \alpha, 0)$

gate1993 eigen-value linear-algebra easy

Answer

6.2.2 Eigen Value: GATE2002-5a top

https://gateoverflow.in/858

a. Obtain the eigen values of the matrix

$$A = \begin{bmatrix} 1 & 2 & 34 & 49 \\ 0 & 2 & 43 & 94 \\ 0 & 0 & -2 & 104 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

gate2002 linear-algebra eigen-value normal descriptive

Answer

6.2.3 Eigen Value: GATE2005-49 top

https://gateoverflow.in/1174

What are the eigenvalues of the following 2×2 matrix?

$$\begin{pmatrix} 2 & -1 \\ -4 & 5 \end{pmatrix}$$

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A. -1 and 1
B. 1 and 6
C. 2 and 5
D. 4 and -1

gate2005 linear-algebra eigen-value easy

Answer

6.2.4 Eigen Value: GATE2006-IT-26 top

https://gateoverflow.in/3565

What are the eigenvalues of the matrix P given below

$$P = \begin{pmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & a \end{pmatrix}$$

A. $a, a-\sqrt{2}, a+\sqrt{2}$
B. a, a, a
C. $0, a, 2a$
D. $-a, 2a, 2a$

gate2006-it linear-algebra eigen-value normal

Answer

6.2.5 Eigen Value: GATE2007-25 top

https://gateoverflow.in/254

Let A be a 4×4 matrix with eigen values -5,-2,1,4. Which of the following is an eigen value of the matrix $\begin{bmatrix} A & I \\ I & A \end{bmatrix}$, where I is the 4×4 identity matrix?

A. -5
B. -7
C. 2
D. 1

gate2007 eigen-value linear-algebra difficult

Answer

6.2.6 Eigen Value: GATE2007-IT-2 top

https://gateoverflow.in/3433

Let A be the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$. What is the maximum value of x^T Ax where the maximum is taken over all x that are the unit eigenvectors of A?

A. 3
B. $\frac{(5+\sqrt{5})}{2}$
C. 3
D. $\frac{(5-\sqrt{5})}{2}$

gate2007-it linear-algebra eigen-value normal

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Answer

6.2.7 Eigen Value: GATE2008-28 top

https://gateoverflow.in/426

How many of the following matrices have an eigenvalue 1?

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$

A. one
B. two
C. three
D. four

gate2008 eigen-value linear-algebra

Answer

6.2.8 Eigen Value: GATE2010-29 top

https://gateoverflow.in/1155

Consider the following matrix

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

If the eigenvalues of A are 4 and 8, then

A. $x = 4, y = 10$
B. $x = 5, y = 8$
C. $x = 3, y = 9$
D. $x = -4, y = 10$

gate2010 linear-algebra eigen-value easy

Answer

6.2.9 Eigen Value: GATE2011-40 top

https://gateoverflow.in/2142

Consider the matrix as given below.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$$

Which one of the following options provides the **CORRECT** values of the eigenvalues of the matrix?

A. $1, 4, 3$
B. $3, 7, 3$
C. $7, 3, 2$
D. $1, 2, 3$

gate2011 linear-algebra eigen-value easy

Answer

6.2.10 Eigen Value: GATE2012-11 top

https://gateoverflow.in/43

Let A be the 2×2 matrix with elements $a_{11} = a_{12} = a_{21} = +1$ and $a_{22} = -1$. Then the

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eigenvalues of the matrix A^{19} are

A. 1024 and -1024
B. $1024\sqrt{2}$ and $-1024\sqrt{2}$
C. $4\sqrt{2}$ and $-4\sqrt{2}$
D. $512\sqrt{2}$ and $-512\sqrt{2}$

gate2012 linear-algebra eigen-value

Answer

6.2.11 Eigen Value: GATE2014-1-5 top

https://gateoverflow.in/1760

The value of the dot product of the eigenvectors corresponding to any pair of different eigenvalues of a $4-by-4$ symmetric positive definite matrix is _____

gate2014-1 linear-algebra eigen-value numerical-answers normal

Answer

6.2.12 Eigen Value: GATE2014-2-47 top

https://gateoverflow.in/2013

The product of the non-zero eigenvalues of the matrix is ____

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

gate2014-2 linear-algebra eigen-value normal numerical-answers

Answer

6.2.13 Eigen Value: GATE2014-3-4 top

https://gateoverflow.in/2038

Which one of the following statements is TRUE about every $n \times n$ matrix with only real eigenvalues?

A. If the trace of the matrix is positive and the determinant of the matrix is negative, at least one of its eigenvalues is negative.
B. If the trace of the matrix is positive, all its eigenvalues are positive.
C. If the determinant of the matrix is positive, all its eigenvalues are positive.
D. If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive.

gate2014-3 linear-algebra eigen-value normal

Answer

6.2.14 Eigen Value: GATE2015-1-36 top

https://gateoverflow.in/8285

Consider the following 2×2 matrix A where two elements are unknown and are marked by a and b. The eigenvalues of this matrix are -1 and 7. What are the values of a and b?

$$A = \left(\begin{pmatrix} 1 & 4 \\ b & a \end{pmatrix}\right)$$

A. $a = 6, b = 4$
B. $a = 4, b = 6$

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C. $a = 3, b = 5$
D. $a = 5, b = 3$

gate2015-1 linear-algebra eigen-value normal

Answer

6.2.15 Eigen Value: GATE2015-2-5 top

https://gateoverflow.in/8051

The larger of the two eigenvalues of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ is _____.

gate2015-2 linear-algebra eigen-value easy numerical-answers

Answer

6.2.16 Eigen Value: GATE2015-3-15 top

https://gateoverflow.in/8411

In the given matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$, one of the eigenvalues is 1. The eigenvectors corresponding to the eigenvalue 1 are

A. $\{a(4,2,1) \mid a \neq 0, a \in \mathbb{R}\}$
B. $\{a(-4,2,1) \mid a \neq 0, a \in \mathbb{R}\}$
C. $\{a(\sqrt{2},0,1) \mid a \neq 0, a \in \mathbb{R}\}$
D. $\{a(-\sqrt{2},0,1) \mid a \neq 0, a \in \mathbb{R}\}$

gate2015-3 linear-algebra eigen-value normal

Answer

6.2.17 Eigen Value: GATE2016-1-05 top

https://gateoverflow.in/39634

Two eigenvalues of a 3×3 real matrix P are $(2 + \sqrt{-1})$ and 3. The determinant of P is _____

gate2016-1 linear-algebra eigen-value numerical-answers normal

Answer

6.2.18 Eigen Value: GATE2016-2-06 top

https://gateoverflow.in/39549

Suppose that the eigenvalues of matrix A are $1, 2, 4$. The determinant of $\left(A^{-1}\right)^T$ is _____.

gate2016-2 linear-algebra eigen-value normal numerical-answers

Answer

6.2.19 Eigen Value: GATE2017-1-31 top

https://gateoverflow.in/118312

Let A be $n \times n$ real valued square symmetric matrix of rank 2 with $\sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij}^2 = 50$. Consider the following statements.

I. One eigenvalue must be in $[-5, 5]$
II. The eigenvalue with the largest magnitude must be strictly greater than 5

Which of the above statements about eigenvalues of A is/are necessarily CORRECT?

A. Both I and II

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B. I only
C. II only
D. Neither I nor II

gate2017-1 linear-algebra eigen-value normal

Answer

6.2.20 Eigen Value: GATE2017-2-22 top

https://gateoverflow.in/118363

Let $P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$ be two matrices.

Then the rank of $P+Q$ is _____ .

gate2017-2 linear-algebra eigen-value numerical-answers

Answer

6.2.21 Eigen Value: GATE2017-2-52 top

https://gateoverflow.in/118618

If the characteristic polynomial of a 3×3 matrix M over $\mathbb{R}$ (the set of real numbers) is $\lambda^3 - 4\lambda^2 + a\lambda + 30, \quad a \in \mathbb{R}$, and one eigenvalue of M is 2, then the largest among the absolute values of the eigenvalues of M is _____

gate2017-2 engineering-mathematics linear-algebra numerical-answers eigen-value

Answer

6.2.22 Eigen Value: GATE2018-17 top

https://gateoverflow.in/204091

Consider a matrix $A = uv^T$ where $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Note that v^T denotes the transpose of v. The largest eigenvalue of A is ____

gate2018 linear-algebra eigen-value normal numerical-answers

Answer

6.2.23 Eigen Value: GATE2018-26 top

https://gateoverflow.in/204100

Consider a matrix P whose only eigenvectors are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Consider the following statements.

I. P does not have an inverse
II. P has a repeated eigenvalue
III. P cannot be diagonalized

Which one of the following options is correct?

A. Only I and III are necessarily true
B. Only II is necessarily true
C. Only I and II are necessarily true
D. Only II and III are necessarily true

gate2018 linear-algebra matrices eigen-value normal

Answer

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Answers: Eigen Value

6.2.1 Eigen Value: GATE1993-01.1 top

https://gateoverflow.in/596

Selected Answer

Answer: B, D

Eigen values are: $0, 0, 0$

The eigen vector should satisfy the equation: $\alpha z = 0$

14 votes -- Rajarshi Sarkar (34.1k points)

6.2.2 Eigen Value: GATE2002-5a top

https://gateoverflow.in/858

Selected Answer

$5(a)$ the eigen value for upper triangular/lower triangular/diagonal matrices are the diagonal elements of the matrix

16 votes -- Madhur Rawat (2.3k points)

6.2.3 Eigen Value: GATE2005-49 top

https://gateoverflow.in/1174

Selected Answer

Let the eigen values be a, b

Sum of Eigen Values = Trace(Diagonal Sum)

$$\implies a + b = 2 + 5 = 7$$

Product of Eigen Values = Det(A)

$$\implies a \times b = 6$$

Solving these we get eigenvalues as **1 and 6**.

Option(B) is **Correct.**

14 votes -- Himanshu Agarwal (15.3k points)

6.2.4 Eigen Value: GATE2006-IT-26 top

https://gateoverflow.in/3565

Selected Answer

det(A-λI)=0

$$\begin{vmatrix} a-\lambda & 1 & 0 \\ 1 & a-\lambda & 1 \\ 0 & 1 & a-\lambda \end{vmatrix} = 0$$

$$(a-\lambda) * [(a-\lambda) * (a-\lambda) - 1 * 1] - 1 * [1 * (a-\lambda) - 0 * 1] + 0 * [1 * 1 - 0 * (a-\lambda)] = 0$$

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$(a-\lambda)^3 - 2(a-\lambda)) = 0$

$(a-\lambda)((a-\lambda)-2)^2 = 0$

$(a-\lambda)((a-\lambda)^2 - (\sqrt{2})^2) = 0$

$(a-\lambda)(a-\lambda+\sqrt{2})(a-\lambda-\sqrt{2}) = 0$

Eigen values , $\lambda = a, a+\sqrt{2}, a-\sqrt{2}.$

28 votes -- Praveen Saini (54.5k points)

6.2.5 Eigen Value: GATE2007-25 top

https://gateoverflow.in/254

Selected Answer

Ans is (C) 2

$Ax = \lambda x$, where λ is the eigen value of A . Hence $(A-\lambda I)x = 0$ or $|A-\lambda I| = 0$
So, for our given matrix, we have

$$\begin{bmatrix} A-\lambda I & I \\ I & A-\lambda I \end{bmatrix} = 0$$

This is a 2×2 block matrix where the first and last and the second and third elements are the same. So, applying the formula for determinant of a block matrix as given here

(second last case) https://en.wikipedia.org/wiki/Determinant#Block_matrices

we get,

$|A-\lambda I - I| \times |A-\lambda I + I| = 0$

$\implies |A-(\lambda+1)I| \times |A-(\lambda-1)I| = 0$

Each of the eigen value of A is the solution of the equation $|A-\alpha I| = 0$ (α being the eigen value of A). So, we can equate $\lambda+1$ and $\lambda-1$ to any of the eigen value of A, and that will get our value of λ. If we take $\alpha = 1$, we get $\lambda = 2$, and that is one of the choice. For no other choice, this equation holds. So, (c) 2 is the answer.

28 votes -- Keith Kr (6.1k points)

6.2.6 Eigen Value: GATE2007-IT-2 top

https://gateoverflow.in/3433

Selected Answer

x = [x1 , x2] be a unit eigen vector

Given $\sqrt{(x12+x22)} = 1$ i.e. x12 + x22 = 1 {$\because$ x is a unit Eigen vector}

Ax = Lx , where L = eigen value

x'Ax = x'Lx= Lx'x = L [x1,x2]' [x1,x2] = L [x12 + x22] = L(1) = L .

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The maximum value of L = $\frac{(5+\sqrt{5})}{2}$.

maximum value of $\mathbf{x}^T$ Ax is $\frac{(5+\sqrt{5})}{2}$. Option B

33 votes -- saloni (277 points)

6.2.7 Eigen Value: GATE2008-28 top

https://gateoverflow.in/426

Selected Answer

Characteristic equation is $|A - \lambda I| = 0$
(1)

$$\begin{vmatrix} 1-\lambda & 0 \\ 0 & -\lambda \end{vmatrix} = 0$$

$(1-\lambda)(-\lambda) = 0$

$\lambda = 0, 1$

Similarly, (2) $\lambda = 0, 0$

(3) $\lambda \neq 1$

(4) $\lambda = -1, -1$

Therefore, Answer is (A) one

16 votes -- Keith Kr (6.1k points)

6.2.8 Eigen Value: GATE2010-29 top

https://gateoverflow.in/1155

Selected Answer

Sum of eigenvalues is equal to trace (sum of diagonal elements) and product of eighen values is equal to the determinant of matrix

So, $2 + y = 8 + 4$ and $2y - 3x = 32$

Solving this we get $y = 10, x = -4$.

Option D is answer.

17 votes -- Pooja Palod (31.3k points)

6.2.9 Eigen Value: GATE2011-40 top

https://gateoverflow.in/2142

Selected Answer

The answer is A.

The given matrix is an upper triangular matrix and the eigenvalues of upper or lower triangular matrix are

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the diagonal values itself.(Property)

19 votes -- Sona Praneeth Akula (4.2k points)

6.2.10 Eigen Value: GATE2012-11 top

https://gateoverflow.in/43

Selected Answer

Characteristic Equation is $A - \mu I = 0$

$$\begin{vmatrix} 1-\mu & 1 \\ 1 & -1-\mu \end{vmatrix} = 0$$

$$\implies (1-\mu)(-1-\mu) - 1 = 0$$

$$\implies -1 - \mu + \mu + \mu^2 - 1 = 0$$

$$\implies \mu^2 - 2 = 0$$

$$\implies \mu = +\sqrt{2} \text{ and } -\sqrt{2}$$

so according to properties of Eigen values,

eigen values of A^{19} =(eigen value of A)^{19}

= $(\sqrt{2})^{19}$ and $(-\sqrt{2})^{19}$

Hence Ans is **option (D).**

27 votes -- Narayan Kunal (421 points)

6.2.11 Eigen Value: GATE2014-1-5 top

https://gateoverflow.in/1760

Selected Answer

Answer to this question is ZERO.

This is because eigen vectors corresponding to DIFFERENT eigen values of a REAL symmetric matrix are ORTHOGONAL to each other.

However, same eigen values they may not be.

And Dot -product of orthogonal vectors(perpendicular vectors) is 0 (ZERO)

For more info see the link: http://math.stackexchange.com/questions/82467/eigenvectors-of-real-symmetric-matrices-are-orthogonal

30 votes -- Sandeep_Uniyal (7.5k points)

Let λ_1 and λ_2 be two distinct eigenvalues of matrix A and u and v be their corresponding eigenvectors respectively.

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We know that an eigenvector X corresponding to an eigenvalue λ of matrix A satisfies

$AX = \lambda X$

$\therefore Au = \lambda_1 u \quad \rightarrow (1)$ and $Av = \lambda_2 v \quad \rightarrow (2)$

On pre-multiplying eqn (1) with v^T, we get

$v^T Au = v^T \lambda_1 u$

$\left(v^T A\right) u = v^T \lambda_1 u$

$\left(A^T v\right)^T u = v^T \lambda_1 u$

$(Av)^T u = v^T \lambda_1 u$ (since A is a symmetric matrix, we can write $A^T = A$)

But $Av = \lambda_2 v$ ... from (2)

$(\lambda_2 v)^T u = v^T \lambda_1 u$

$\lambda_2 v^T u = v^T \lambda_1 u$

$\lambda_2 v^T u = \lambda_1 v^T u$ (as λ_1 is a constant, we can write $v^T \lambda_1 = \lambda_1 v^T$)

$\lambda_2 v^T u - \lambda_1 v^T u = 0$

$(\lambda_2 - \lambda_1) v^T u = 0$

$\therefore$ Either $(\lambda_2 - \lambda_1) = 0$ or $v^T u = 0$

But since $\lambda_2 \neq \lambda_1$, $v^T u$ must be 0

$v^T u$ is nothing but the dot product of the eigenvectors u and v

Hence, we can conclude that the eigenvectors corresponding to distinct eigenvalues of a real symmetric matrix are orthogonal.

27 votes -- just_bhavana (11.9k points)

6.2.12 Eigen Value: GATE2014-2-47 top

https://gateoverflow.in/2013

Selected Answer

Since rank of matrix is 2, we can have maximum 2 non-zero eigen values.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$\because AX = \lambda X$$

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$x_1 + x_5 = \lambda x_1$ (from first row)
$x_1 + x_5 = \lambda x_5$ (from last row)
Adding these two,
$2(x_1 + x_5) = \lambda(x_1 + x_5)$
$\implies \lambda = 2.$

Similarly,
$x_2 + x_3 + x_4 = \lambda x_2$
$x_2 + x_3 + x_4 = \lambda x_3$
$x_2 + x_3 + x_4 = \lambda x_4$
Adding these three,
$3(x_2 + x_3 + x_4) = \lambda(x_2 + x_3 + x_4)$
$\implies \lambda = 3.$

So, product of non-zero eigenvalues of the matrix $= 2 \times 3 = 6.$

92 votes -- Subha (273 points)

We can see that the rank of the given matrix is 2 (since 3 rows are same, and other 2 rows are also same). Sum of eigen values = sum of diagonals. So, we have two eigen values which sum to 5. This information can be used to get answer in between the following solution.

Let Eigen value be X. Now, equating the determinant of the following to 0 gives us the values for X. To find X in the following matrix, we can equate the determinant to 0. For finding the determinant we can use row and column additions and make the matrix a triangular one. Then determinant will just be the product of the diagonals which should equate to 0.

1-X	0	0	0	1
0	1-X	1	1	0
0	1	1-X	1	0
0	1	1	1-X	0
1	0	0	0	1-X

R1 ← R1 + R5, R4 ← R4 - R3

2-X	0	0	0	2-X
0	1-X	1	1	0
0	1	1-X	1	0
0	0	X	-X	0
1	0	0	0	1-X

Taking X out from R4, 2-X from R1, (so, X = 2 is one eigen value)

1	0	0	0	1
0	1-X	1	1	0
0	1	1-X	1	0
0	0	1	-1	0
1	0	0	0	1-X

R2 ← R2 - R3, R5 ← R5 - R1

1	0	0	0	1
0	-X	X	0	0
0	1	1-X	1	0

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0	0	1	-1	0
0	0	0	0	-X

C3 ← C3 + C4

1	0	0	0	1
0	-X	X	0	0
0	1	2-X	1	0
0	0	0	-1	0
0	0	0	0	-X

Taking X out from R2

1	0	0	0	1
0	-1	1	0	0
0	1	2-X	1	0
0	0	0	-1	0
0	0	0	0	-X

R3 ← R3 + R2

1	0	0	0	1
0	-1	1	0	0
0	0	3-X	1	0
0	0	0	-1	0
0	0	0	0	-X

Now, we got a triangular matrix and determinant of a triangular matrix is product of the diagonal.

So (3-X) (-X) = 0 => X = 3 or X = 0. So, X = 3 is another eigen value and product of non-zero eigen values = 2 * 3 = 6.

https://people.richland.edu/james/lecture/m116/matrices/determinant.html

46 votes -- Arjun Suresh (348k points)

6.2.13 Eigen Value: GATE2014-3-4 top

https://gateoverflow.in/2038

Selected Answer

Trace is the sum of all diagonal elements of a square matrix.
Determinant of a matrix = Product of eigen values.

$A)$ Is the right answer. To have the determinant negative ,at least one eigen value has to be negative(but reverse may not be true).
{you can take simple example with upper or lower triangular matrices. In the case option (b),(c) and(d)$ reverse is always true .}

26 votes -- Srinath Jayachandran (3.7k points)

6.2.14 Eigen Value: GATE2015-1-36 top

https://gateoverflow.in/8285

Selected Answer

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$$\begin{aligned} \text{Sum of Eigenvalues} &= \text{trace of matrix} \\ -1+7 &= 1+a \\ \text{Product of Eigenvalues} &= \text{Determinant of matrix} \\ -1\times 7 &= a-4b \end{aligned}$$

This gives $a=5$ and $b=3$

answer = **option D**

28 votes -- Amar Vashishth (30.5k points)

6.2.15 Eigen Value: GATE2015-2-5 top

https://gateoverflow.in/8051

Selected Answer

For finding the Eigen Values of a Matrix we need to build the Characteristic equation which is of the form,

$A-\lambda I$

Where A is the given Matrix.

λ is a constant

I is the identity matrix.

We'll have a Linear equation after solving $A-\lambda I$. Which will give us 2 roots for λ.

(4-λ) (1-λ) - 10 = 0

4 - 5λ + λ^2 = 10

λ^2 -5λ - 6 = 0

(λ - 6) (λ + 1) = 0

So λ = -1 , 6.

6 is larger and hence is the Answer.

24 votes -- Gate Keeda (19.6k points)

6.2.16 Eigen Value: GATE2015-3-15 top

https://gateoverflow.in/8411

Selected Answer

$$\begin{bmatrix} 1-1 & -1 & 2 \\ 0 & 1-1 & 0 \\ 1 & 2 & 1-1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$-y+2z=0$

$x+2y=0$

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Now consider each of the triplets as the value of x, y, z and put in these equations the one which satisfies is the answer.

Why so because an eigen vector represents a vector which passes through all the points which can solve these equations.

So, we can observe that only option B is satisfying the equations.

35 votes -- Tamojit Chatterjee (2.3k points)

6.2.17 Eigen Value: GATE2016-1-05 top

https://gateoverflow.in/39634

Selected Answer

Eigen values are roots of Characterstic equation $|A - \lambda I| = 0$

For a 3×3 matrix, characterstic equation will be cubic, so will have 3 roots. Two roots are given as: $2 + i$ and 3 and We know that complex roots always occur in pairs so, if $2 + i$ is a root of characterstic equation, then $2 - i$ must be other root.

$\lambda_1 = 2 + i$, $\lambda_2 = 2 - i$ and $\lambda_3 = 3$

$\det(A) = \lambda_1 \lambda_2 \lambda_3 = (2 + i) * (2 - i) * 3 = (2^2 - i^2) * 3 = 5 * 3 = 15$

37 votes -- Manish Joshi (27.9k points)

6.2.18 Eigen Value: GATE2016-2-06 top

https://gateoverflow.in/39549

Selected Answer

Determinant of Matrix A = product of eigen values = $1 \times 2 \times 4 = 8$

Determinant of Inverse Matrix of A, $\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{8}$

Determinant remains same after the Transpose

So, determinant of $(A^{-1})^T$ = $\det(A^{-1}) = \frac{1}{8}$ = 0.125

34 votes -- Praveen Saini (54.5k points)

6.2.19 Eigen Value: GATE2017-1-31 top

https://gateoverflow.in/118312

Selected Answer

Eigen values of $\begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}$ are ± 5. Therefore second statement is false.

Since, the rank of matrix A is 2, therefore atleast one eigen value would be zero for n>3.

For n= 2, It can be proven that $\lambda_1^2 + \lambda_2^2 \leq \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij}^2$.

$\lambda_1^2 + \lambda_1^2 \leq 50$

Both λ_1 and λ_2 would be real because A is a real symmetric matrix. Which implies that atleast one eigen value would be in [-5,5].

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Hence, correct answer is (B)

Now, to prove $\lambda_1^2 + \lambda_2^2 \leq \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij}^2$ for 2×2 matrix, let us consider the matrix is $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ and λ is the eigen value of this matrix.

$$\begin{vmatrix} a-\lambda & c \\ b & d-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - (a+d)\lambda + ad - bc = 0$$

Let λ_1 and λ_2 are roots of this equation.

$$\lambda_1^2 + \lambda_2^2 = (\lambda_1 + \lambda_2)^2 - 2\lambda_1\lambda_2$$

= $(a+d)^2 - 2(ad - bc)$

=$\sum_{i=1}^{2} \sum_{j=1}^{2} A_{ij}^2 - (b-c)^2$
For real valued matrix,
$\leq \sum_{i=1}^{2} \sum_{j=1}^{2} A_{ij}^2$ (For real symmetric matrix, b=c and $\leq$ would be replaced by equal sign)

22 votes -- suraj (5.6k points)

6.2.20 Eigen Value: GATE2017-2-22 top

https://gateoverflow.in/118363

Selected Answer

$$P + Q = \begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix}$$

$det(P+Q) = 0$, So Rank cannot be 3, but there exists a $2 * 2$ submatrix such that determinant of submatrix is not 0.

So, $Rank(P+Q) = 2$

28 votes -- Manish Joshi (27.9k points)

6.2.21 Eigen Value: GATE2017-2-52 top

https://gateoverflow.in/118618

Selected Answer

Given that $\lambda = 2$ is an eigen value. So, it must satisfy characterstic equation.

$2^3 - 4 * 2^2 + 2a + 30 = 0 \Rightarrow a = -11$

Characterstic eq : $\lambda^3 - 4\lambda^2 - 11\lambda + 30$

$\Rightarrow (\lambda - 2)(\lambda - 5)(\lambda + 3) = 0$

$\lambda_1 = 2, \lambda_2 = 5$ and $\lambda_3 = -3$

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Max Eigen Value $= 5$

35 votes -- Manish Joshi (27.9k points)

6.2.22 Eigen Value: GATE2018-17 top

https://gateoverflow.in/204091

Selected Answer

u = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$v^T = \begin{bmatrix} 1 & 1 \end{bmatrix}$

A= uv^T

A= $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix}$

= $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

A - λ I = 0

=> $\begin{bmatrix} 1-\lambda & 1 \\ 2 & 2-\lambda \end{bmatrix}$ = 0

=> (1- λ) (2- λ) - 2 =0

=> $\lambda^2 - 3\lambda = 0$

=> λ = 0, 3

So maximum is 3.

6 votes -- Subham Mishra (6.2k points)

6.2.23 Eigen Value: GATE2018-26 top

https://gateoverflow.in/204100

Selected Answer

Eigenvectors are multiple of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$,

So, it has repeated eigen value.

Hence, It cannot be Diagonalizable since repeated eigenvalue, [we know if distinct eigen vector then diagonalizable]

D must be answer.

4 votes -- Prashant Singh (59.8k points)

6.3 Inequality(1) top

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6.3.1 Inequality: GATE1987-1-xxi top

https://gateoverflow.in/80377

If $a, b,$ and c are constants, which of the following is a linear inequality?

A. $ax + bcy = 0$
B. $ax^2 + cy^2 = 21$
C. $abx + a^2y \geq 15$
D. $xy + ax \geq 20$

gate1987 linear-algebra inequality

Answer

Answers: Inequality

6.3.1 Inequality: GATE1987-1-xxi top

https://gateoverflow.in/80377

Selected Answer

Linear inequalities with 2 variables are of the form $Px + Qy \leq K$ or $Px + Qy \geq K$ for some constants P and Q.

A. is an equality.
B. is not linear since the power of x and y are not 1.
C. it is a linear inequality.
D. not linear.

So correct answer is C)

8 votes -- air1 (3.5k points)

6.4 Lines Curves(1) top

6.4.1 Lines Curves: GATE2007-IT-80 top

https://gateoverflow.in/3532

Let $P_1, P_2, \ldots, P_n$ be n points in the $xy-$plane such that no three of them are collinear. For every pair of points P_i and P_j, let L_{ij} be the line passing through them. Let L_{ab} be the line with the steepest gradient amongst all $\frac{n(n-1)}{2}$ lines.

Which one of the following properties should necessarily be satisfied ?

A. P_a and P_b are adjacent to each other with respect to their x-coordinate
B. Either P_a or P_b has the largest or the smallest y-coordinate among all the points
C. The difference between x-coordinates P_a and P_b is minimum
D. None of the above

gate2007-it lines-curves

Answer

Answers: Lines Curves

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6.4.1 Lines Curves: GATE2007-IT-80 top

https://gateoverflow.in/3532

Answer: C

Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$

For gradient to be maximum $(x_2 - x_1)$ should be minimum.

4 votes -- Rajarshi Sarkar (34.1k points)

6.5 Matrices(29) top

6.5.1 Matrices: GATE1987-1-xxiii top

https://gateoverflow.in/80380

A square matrix is singular whenever

A. The rows are linearly independent
B. The columns are linearly independent
C. The row are linearly dependent
D. None of the above

gate1987 linear-algebra matrices

Answer

6.5.2 Matrices: GATE1993-02.7 top

https://gateoverflow.in/611

If $A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & i & i \\ 0 & 0 & 0 & -i \end{pmatrix}$ the matrix A^4, calculated by the use of Cayley-Hamilton theorem or otherwise, is _____

gate1993 linear-algebra normal matrices numerical-answers

Answer

6.5.3 Matrices: GATE1994-1.2 top

https://gateoverflow.in/2438

Let A and B be real symmetric matrices of size $n \times n$. Then which one of the following is true?

A. $AA' = I$
B. $A = A^{-1}$
C. $AB = BA$
D. $(AB)' = BA$

gate1994 linear-algebra normal matrices

Answer

6.5.4 Matrices: GATE1994-1.9 top

https://gateoverflow.in/2446

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The rank of matrix $\begin{bmatrix} 0 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{bmatrix}$ is:

A. 0
B. 1
C. 2
D. 3

gate1994 linear-algebra matrices easy

Answer

6.5.5 Matrices: GATE1994-3.12 top

https://gateoverflow.in/2498

Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

gate1994 linear-algebra matrices easy descriptive

Answer

6.5.6 Matrices: GATE1995-1.24 top

https://gateoverflow.in/2611

The rank of the following $(n+1) \times (n+1)$ matrix, where a is a real number is

$$\begin{bmatrix} 1 & a & a^2 & \dots & a^n \\ 1 & a & a^2 & \dots & a^n \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & a & a^2 & \dots & a^n \end{bmatrix}$$

A. 1
B. 2
C. n
D. Depends on the value of a

gate1995 linear-algebra matrices normal

Answer

6.5.7 Matrices: GATE1996-10 top

https://gateoverflow.in/2762

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ be two matrices such that $AB = I$. Let $C = A \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $CD = I$. Express the elements of D in terms of the elements of B.

gate1996 linear-algebra matrices normal descriptive

Answer

6.5.8 Matrices: GATE1996-2.6 top

https://gateoverflow.in/2735

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The matrices $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ commute under multiplication

A. if $a = b$ or $\theta = n\pi, n$ an integer
B. always
C. never
D. if $a\cos\theta = b\sin\theta$

gate1996 linear-algebra normal matrices

Answer

6.5.9 Matrices: GATE1997-4.2 top

https://gateoverflow.in/2243

Let $a = (a_{ij})$ be an n-rowed square matrix and I_{12} be the matrix obtained by interchanging the first and second rows of the n-rowed Identify matrix. Then AI_{12} is such that its first

A. Row is the same as its second row
B. Row is the same as the second row of A
C. Column is the same as the second column of A
D. Row is all zero

gate1997 linear-algebra easy matrices

Answer

6.5.10 Matrices: GATE1998-2.1 top

https://gateoverflow.in/1673

The rank of the matrix given below is:

$$\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix}$$

A. 3
B. 1
C. 2
D. 4

gate1998 linear-algebra matrices normal

Answer

6.5.11 Matrices: GATE1998-2.2 top

https://gateoverflow.in/1674

Consider the following determinant $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & C & ab \end{vmatrix}$

Which of the following is a factor of Δ?

A. $a + b$
B. $a - b$
C. $a + b + c$
D. abc

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gate1998 linear-algebra matrices normal

Answer

6.5.12 Matrices: GATE2001-1.1 top

https://gateoverflow.in/694

Consider the following statements:

- S1: The sum of two singular $n \times n$ matrices may be non-singular
- S2: The sum of two $n \times n$ non-singular matrices may be singular

Which one of the following statements is correct?

A. $S1$ and $S2$ both are true
B. $S1$ is true, $S2$ is false
C. $S1$ is false, $S2$ is true
D. $S1$ and $S2$ both are false

gate2001 linear-algebra normal matrices

Answer

6.5.13 Matrices: GATE2002-1.1 top

https://gateoverflow.in/805

The rank of the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is

A. 4
B. 2
C. 1
D. 0

gate2002 linear-algebra easy matrices

Answer

6.5.14 Matrices: GATE2004-26 top

https://gateoverflow.in/1023

The number of different $n \times n$ symmetric matrices with each element being either 0 or 1 is: (Note: $\text{power}(2, X)$ is same as 2^X)

A. $\text{power}(2, n)$
B. $\text{power}(2, n^2)$
C. $\text{power}\left(2, \frac{(n^2+n)}{2}\right)$
D. $\text{power}\left(2, \frac{(n^2-n)}{2}\right)$

gate2004 linear-algebra normal matrices

Answer

6.5.15 Matrices: GATE2004-27 top

https://gateoverflow.in/1024

Let A, B, C, D be $n \times n$ matrices, each with non-zero determinant. If $ABCD = I$, then B^{-1} is

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A. $D^{-1}C^{-1}A^{-1}$
B. CDA
C. ADC
D. Does not necessarily exist

gate2004 linear-algebra normal matrices

Answer

6.5.16 Matrices: GATE2004-76 top

https://gateoverflow.in/1070

In an $M \times N$ matrix all non-zero entries are covered in a rows and b columns. Then the maximum number of non-zero entries, such that no two are on the same row or column, is

A. $\leq a+b$
B. $\leq \max(a,b)$
C. $\leq \min(M-a, N-b)$
D. $\leq \min(a,b)$

gate2004 linear-algebra normal matrices

Answer

6.5.17 Matrices: GATE2004-IT-32 top

https://gateoverflow.in/3675

Let A be an n × n matrix of the following form.

$$A = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 3 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 & \dots & 0 & 0 & 0 \\ \dots & & & & & & & & \\ \dots & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 3 \end{bmatrix}_{n \times n}$$

What is the value of the determinant of A?

A. $\left(\frac{5+\sqrt{3}}{2}\right)^{n-1}\left(\frac{5\sqrt{3}+7}{2\sqrt{3}}\right)+\left(\frac{5-\sqrt{3}}{2}\right)^{n-1}\left(\frac{5\sqrt{3}-7}{2\sqrt{3}}\right)$
B. $\left(\frac{7+\sqrt{5}}{2}\right)^{n-1}\left(\frac{7\sqrt{5}+3}{2\sqrt{5}}\right)+\left(\frac{7-\sqrt{5}}{2}\right)^{n-1}\left(\frac{7\sqrt{5}-3}{2\sqrt{5}}\right)$
C. $\left(\frac{3+\sqrt{7}}{2}\right)^{n-1}\left(\frac{3\sqrt{7}+5}{2\sqrt{7}}\right)+\left(\frac{3-\sqrt{7}}{2}\right)^{n-1}\left(\frac{3\sqrt{7}-5}{2\sqrt{7}}\right)$
D. $\left(\frac{3+\sqrt{5}}{2}\right)^{n-1}\left(\frac{3\sqrt{5}+7}{2\sqrt{5}}\right)+\left(\frac{3-\sqrt{5}}{2}\right)^{n-1}\left(\frac{3\sqrt{5}-7}{2\sqrt{5}}\right)$

gate2004-it linear-algebra matrices normal

Answer

6.5.18 Matrices: GATE2004-IT-36 top

https://gateoverflow.in/3679

If matrix $X = \begin{bmatrix} a & 1 \\ -a^2+a-1 & 1-a \end{bmatrix}$ and $X^2 - X + I = O$ (I is the identity matrix and O is the zero matrix), then the inverse of X is

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A. $\begin{bmatrix} 1-a & -1 \\ a^2 & a \end{bmatrix}$

B. $\begin{bmatrix} 1-a & -1 \\ a^2-a+1 & a \end{bmatrix}$

C. $\begin{bmatrix} -a & 1 \\ -a^2+a-1 & 1-a \end{bmatrix}$

D. $\begin{bmatrix} a^2-a+1 & a \\ 1 & 1-a \end{bmatrix}$

gate2004-it linear-algebra matrices normal

Answer

6.5.19 Matrices: GATE2006-23 top

https://gateoverflow.in/984

F is an $n \times n$ real matrix. b is an $n \times 1$ real vector. Suppose there are two $n \times 1$ vectors, u and v such that, $u \neq v$ and $Fu = b, Fv = b$. Which one of the following statements is false?

A. Determinant of F is zero.
B. There are an infinite number of solutions to $Fx = b$
C. There is an $x \neq 0$ such that $Fx = 0$
D. F must have two identical rows

gate2006 linear-algebra normal matrices

Answer

6.5.20 Matrices: GATE2008-IT-29 top

https://gateoverflow.in/3319

If M is a square matrix with a zero determinant, which of the following assertion (s) is (are) correct?

S1: Each row of M can be represented as a linear combination of the other rows
S2: Each column of M can be represented as a linear combination of the other columns
S3: $MX = 0$ has a nontrivial solution
S4: M has an inverse

A. $S3$ and $S2$
B. $S1$ and $S4$
C. $S1$ and $S3$
D. $S1, S2$ and $S3$

gate2008-it linear-algebra normal matrices

Answer

6.5.21 Matrices: GATE2015-1-18 top

https://gateoverflow.in/8241

In the LU decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$, if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is_____.

gate2015-1 linear-algebra matrices numerical-answers

Answer

6.5.22 Matrices: GATE2015-2-27 top

https://gateoverflow.in/8131

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Perform the following operations on the matrix $\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}$

i. Add the third row to the second row
ii. Subtract the third column from the first column.

The determinant of the resultant matrix is _____.

gate2015-2 linear-algebra matrices easy numerical-answers

Answer

6.5.23 Matrices: ISI-2017-5 top

https://gateoverflow.in/209513

If A is a 2×2 matrix such that trace $A = det\ A = 3$, then what is the trace of A^{-1}?

A. 1
B. $\left(\frac{1}{3}\right)$
C. $\left(\frac{1}{6}\right)$
D. $\left(\frac{1}{2}\right)$

engineering-mathematics isi2017 matrices

Answer

6.5.24 Matrices: ISI2016 top

https://gateoverflow.in/45220

Let A be a matrix such that:

$$A = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$$

and $B = A + A^2 + A^3 + \ldots + A^{50}$. Then which of the following is true?

A. $B^2 = I$
B. $B^2 = 0$
C. $B^2 = B$
D. None of the above

isi2016 matrices

Answer

6.5.25 Matrices: TIFR2010-A-16 top

https://gateoverflow.in/18492

Let the characteristic equation of matrix M be $\lambda^2 - \lambda - 1 = 0$. Then.

a. M^{-1} does not exist.
b. M^{-1} exists but cannot be determined from the data.
c. $M^{-1} = M + I$

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d. $M^{-1} = M - I$
e. M^{-1} exists and can be determined from the data but the choices (c) and (d) are incorrect.

tifr2010 linear-algebra matrices

Answer

6.5.26 Matrices: TIFR2012-B-12 top

https://gateoverflow.in/25141

Let A be a matrix such that $A^k = 0$. What is the inverse of $I - A$?

a. 0
b. I
c. A
d. $1 + A + A^2 + \ldots + A^{k-1}$
e. Inverse is not guaranteed to exist.

tifr2012 linear-algebra matrices

Answer

6.5.27 Matrices: TIFR2013-B-3 top

https://gateoverflow.in/25659

How many 4×4 matrices with entries from $0, 1$ have odd determinant?

Hint: Use modulo 2 arithmetic.

a. 20160
b. 32767
c. 49152
d. 57343
e. 65520

tifr2013 linear-algebra matrices

Answer

6.5.28 Matrices: TIFR2015-A-14 top

https://gateoverflow.in/29588

Consider the following 3×3 matrices.

$$M_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

How may $0 - 1$ column vectors of the form

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

are there such that $M_1 X = M_2 X$ (modulo 2)? (modulo 2 means all operations are done modulo 2,

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i.e, $3 = 1$ (modulo 2), $4 = 0$ (modulo 2)).

a. None
b. Two
c. Three
d. Four
e. Eight

tifr2015 matrices

Answer

6.5.29 Matrices: TIFR2018-A-14 top

https://gateoverflow.in/179377

Let A be an $n \times n$ invertible matrix with real entries whose row sums ar all equal to c. Consider the following statements:

1. Every row in the matrix $2A$ sums to $2c$.
2. Every row in the matrix A^2 sums to c^2.
3. Every row in the matrix A^{-1} sums to c^{-1}.

Which of the following is **TRUE**?

A. none of the statements $(1), (2), (3)$ is correct
B. statement (1) is correct but not necessarily statements (2) or (3)
C. statement (2) is correct but not necessarily statements (1) or (3)
D. statement (1) and (2) are correct but not necessarily statement (3)
E. all the three statements $(1), (2),$ and (3) are correct

tifr2018 matrices linear-algebra

Answer

Answers: Matrices

6.5.1 Matrices: GATE1987-1-xxiii top

https://gateoverflow.in/80380

Selected Answer

When the rows are linearly dependent the determinant of the matrix becomes 0 hence in that case it will become singular matrix.

Hence, C is the correct option.

12 votes -- HABIB MOHAMMAD KHAN (98.7k points)

6.5.2 Matrices: GATE1993-02.7 top

https://gateoverflow.in/611

Selected Answer

Let λ be eighen value

Characteristic polynomial is

$$(1-\lambda)(-1-\lambda)(i-\lambda)(-i-\lambda)$$

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$= \left(\lambda^2 - 1\right)\left(\lambda^2 + 1\right)$

$= \lambda^4 - 1$

Characteristic equation is $\lambda^4 - 1 = 0$

According to Cayley Hamilton theorem every matrix matrix satisfies its own characteristic equation

So, $A^4 = I$

24 votes -- Pooja Palod (31.3k points)

6.5.3 Matrices: GATE1994-1.2 top

https://gateoverflow.in/2438

Selected Answer

A **symmetric matrix** is a **square matrix** that is equal to its transpose.

(AB)' = B'A' =BA as Both A and B are symmetric matrices, hence $B' = B$ and $A' = A$

So, **(D) is correct option!**

Why is (C) not correct option? see the following example:

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

There are two symmetric matrices given of size 2x2 and $AB! = BA$. **Therefore (C) is not a correct option!**

12 votes -- Manu Thakur (39.6k points)

6.5.4 Matrices: GATE1994-1.9 top

https://gateoverflow.in/2446

Selected Answer

Answer: C

Determinant comes out to be 0. So, rank cannot be 3. The minor $\begin{bmatrix} 3 & 5 \\ 1 & 1 \end{bmatrix} ! = 0$. So, rank is 2.

(or)

if we do elementary row operations on the given matrix then we get

$$\begin{bmatrix} 0 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & -3 \\ 0 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

As the number of non zero row is 2,then the rank of the matrix is also 2.

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16 votes -- Rajarshi Sarkar (34.1k points)

6.5.5 Matrices: GATE1994-3.12 top

https://gateoverflow.in/2498

Selected Answer

An Easy Procedure.

Using Eigen values, the characteristic equation we get is -

$$-\lambda^3 + 2\lambda^2 - 2 = 0$$

Using Cayley-Hamilton Theorem-

$$-A^3 + 2A^2 - 2I = 0$$

So, $A^{-1} = \frac{1}{2}(2A - A^2)$

Solving that we get,

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \end{bmatrix}$$

15 votes -- Shweta Nair (615 points)

6.5.6 Matrices: GATE1995-1.24 top

https://gateoverflow.in/2611

Selected Answer

$$\begin{bmatrix} 1 & a & a^2 & \dots & a^n \\ 1 & a & a^2 & \dots & a^n \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & a & a^2 & \dots & a^n \end{bmatrix}$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1, R_4 \to R_4 - R_1$,and so on

$$\begin{bmatrix} 1 & a & a^2 & \dots & a^n \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

Rank of the Matrix=1

Hence, option **(A)1** is the correct choice.

14 votes -- Leen Sharma (40k points)

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6.5.7 Matrices: GATE1996-10 top

https://gateoverflow.in/2762

Selected Answer

$AB = I$, B is equal to the inverse of A and vice versa.

So, $B = A^{-1}$

Now $CD = I$, C is equal to the inverse of D and vice versa.

So, $D = C^{-1}$

$$= \left(A.\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\right)^{-1}$$

Remark: $(AB)^{-1} = B^{-1}A^{-1}$

$$= \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}^{-1}.A^{-1}$$

$$= \begin{bmatrix}1 & 0\\-1 & 1\end{bmatrix}.B$$

$$= \begin{bmatrix}b_{11} & b_{12}\\b_{21}-b_{11} & b_{22}-b_{12}\end{bmatrix}$$

22 votes -- Sayantan Ganguly (6.4k points)

6.5.8 Matrices: GATE1996-2.6 top

https://gateoverflow.in/2735

Selected Answer

Answer: A

$$\begin{bmatrix}\cos(\theta) & -\sin(\theta)\\\sin(\theta) & \cos(\theta)\end{bmatrix} * \begin{bmatrix}a & 0\\0 & b\end{bmatrix} = \begin{bmatrix}a\cos(\theta) & -b\sin(\theta)\\a\sin(\theta) & b\cos(\theta)\end{bmatrix}$$

and

$$\begin{bmatrix}a & 0\\0 & b\end{bmatrix} * \begin{bmatrix}\cos(\theta) & -\sin(\theta)\\\sin(\theta) & \cos(\theta)\end{bmatrix} = \begin{bmatrix}a\cos(\theta) & -a\sin(\theta)\\b\sin(\theta) & b\cos(\theta)\end{bmatrix}$$

The multiplication will commute if

$a\sin(\theta) = b\sin(\theta)$ or a = b or $\theta = n\pi$.

12 votes -- Rajarshi Sarkar (34.1k points)

6.5.9 Matrices: GATE1997-4.2 top

https://gateoverflow.in/2243

Selected Answer

C is the answer, in $AI12$ matrix will result into the same matrix as A but first column will be

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exchanged with second column.
A matrix:
$a\ b\ c$
$d\ e\ f$
$g\ h\ i$

$I12$ matrix
$0\ 1\ 0$
$1\ 0\ 0$
$0\ 0\ 1$

resulted matrix
$b\ a\ c$
$e\ d\ f$
$h\ g\ i$

17 votes -- Manu Thakur (39.6k points)

6.5.10 Matrices: GATE1998-2.1 top

https://gateoverflow.in/1673

Selected Answer

It's an **non-singular** 4x4 matrix, hence its rank is 4. A non-singular matrix has non-zero determinant.

(D) is correct option!

17 votes -- Keith Kr (6.1k points)

6.5.11 Matrices: GATE1998-2.2 top

https://gateoverflow.in/1674

Selected Answer

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

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$$\begin{vmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & c-a & ab-bc \end{vmatrix} \qquad R_2 \to R_2 - R_1 \ , \ R_3 \to R_3 - R_1$$

$(b-a)(ab-bc)-(c-a)(ca-bc)$

$-(a-b)\ \ b\ \ (a-c)+(a-c)\ \ c\ \ (a-b)$

$(a-b)(a-c)(c-b)$

Option$(B)a-b$ is the correct choice.

11 votes -- Leen Sharma (40k points)

6.5.12 Matrices: GATE2001-1.1 top

https://gateoverflow.in/694

Selected Answer

Yes A is correct option!, Both statements are True

S_1 : True

$$A = \begin{matrix} 2 & 10 \\ 1 & 5 \end{matrix} \qquad B = \begin{matrix} 3 & 6 \\ 2 & 4 \end{matrix}$$

$|A| = 0 \quad |B| = 0$

$|A+B| = -3$

S_2 : True

$$A = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \qquad B = \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$$

$|A| = 1 \quad |B| = -1$

$|A+B| = 0$

14 votes -- Vijay Thakur (17.1k points)

6.5.13 Matrices: GATE2002-1.1 top

https://gateoverflow.in/805

Selected Answer

Rank of this matrix is 1 as the determint of 2nd order matrix is 0 and 1st order matrix is non zero so rank is 1.

15 votes -- Bhagirathi Nayak (14.1k points)

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6.5.14 Matrices: GATE2004-26 top

https://gateoverflow.in/1023

Selected Answer

In symmetric matrix, $A[i][j] = A[j][i]$. So, we have choice only for either the upper triangular elements or the lower triangular elements. Number of such elements will be $n + (n-1) + (n-2) + \cdots + 1 = n\frac{(n+1)}{2} = \frac{(n^2+n)}{2}$. Now, each element being either 0 or 1 means, we have 2 choices for each element and thus for $\frac{(n^2+n)}{2}$ elements we have $2^{\frac{(n^2+n)}{2}}$ possibilities.

Choice is C.

25 votes -- Arjun Suresh (348k points)

6.5.15 Matrices: GATE2004-27 top

https://gateoverflow.in/1024

Selected Answer

Given

$ABCD = I$

multiply LHS,RHS by A^{-1}

$A^{-1}ABCD = A^{-1}I$ (position of A^{-1} on both sides should be left)

$\Rightarrow \quad BCD = A^{-1}$

$\Rightarrow \quad BCDD^{-1} = A^{-1}D^{-1}$

$\Rightarrow \quad BC = A^{-1}D^{-1}$

$\Rightarrow \quad BCC^{-1} = A^{-1}D^{-1}C^{-1}$

$\Rightarrow \quad B = A^{-1}D^{-1}C^{-1}$

Now $B^{-1} = (A^{-1}D^{-1}C^{-1})^{-1}$

$B^{-1} = CDA$

30 votes -- Madhur Rawat (2.3k points)

6.5.16 Matrices: GATE2004-76 top

https://gateoverflow.in/1070

Selected Answer

maximum number of non-zero entries, such that no two are on the same row or column

Any entry will be a member of some row and some column. So, with a rows we can have maximum a elements such that no row has a repeated element. Same is applicable for b columns also. So, combining both, answer should be $\leq \min(a, b)$.

We can also apply pigeonhole principle here. Let $p = \min(a, b)$ be the number of holes. So, we can place up to p non-zero entries (pigeons) and as soon as

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$(p+1)^{th}$ entry comes it must be making two entries in some column or row.

16 votes -- Arjun Suresh (348k points)

6.5.17 Matrices: GATE2004-IT-32 top

https://gateoverflow.in/3675

Selected Answer

The best part of this question is don't solve by mathematical procedures. Verification is very easy in this question.
Just put $n=1$, you'll get a matrix like $[3]$.
Find its determinant.
Determinant $=3$. now check options.

by putting $n=1$, I am getting following results.
$A.5$
$B.7$
$C.3$
$D.3$

A, B cant be the answer.

Now check for $n=2$.
Determinant $=9-1=8$

Put $n=2$ in C, D.
$C.7$
$D.8$

So, D is the answer.

52 votes -- Digvijay (54.8k points)

6.5.18 Matrices: GATE2004-IT-36 top

https://gateoverflow.in/3679

Selected Answer

Given, $X^2 - X + I = O$

=> $X^2 = X - I$

=> $X^{-1}(X^2) = X^{-1}(X - I)$ {Multiplying X^{-1} both sides..}

=> $X = I - X^{-1}$

=> $X^{-1} = I - X$ Which gives **Option (B)..**

20 votes -- Himanshu Agarwal (15.3k points)

6.5.19 Matrices: GATE2006-23 top

https://gateoverflow.in/984

Selected Answer

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(A) : Correct. We are given

$$Fu = b$$

$$Fv = b$$

So $F(u-v) = 0$

Since $u \neq v$, so we have a non-zero solution $w = (u-v)$ to homogeneous equation $Fx = 0$. Now any vector λw is also a solution of $Fx = 0$, and so we have infinitely many solutions of $Fx = 0$, and so determinant of F is zero.

(B) : Correct. Consider a vector $u + \lambda w$.

$$F(u + \lambda w) = Fu + F(\lambda w) = b + 0 = b$$

So there are infinitely many vectors of the form $u + \lambda w$, which are solutions to equation $Fx = b$.

(C) : Correct. In option (a), we proved that vector $(u-v) \neq 0$ satisfies equation $Fx = 0$.

(D) : False. This is not necessary.

So, option (D) is the answer.

23 votes -- Happy Mittal (11.2k points)

6.5.20 Matrices: GATE2008-IT-29 top

https://gateoverflow.in/3319

Selected Answer

Since, M has zero determinant, its rank is not full i.e. if M is of size $3 * 3$, then its rank is not 3. So there is a linear combination of rows which evaluates to 0 i.e.

$$k_1R_1 + k_2R_2 + \ldots + k_nR_n = 0$$

and there is a linear combination of columns which evaluates to 0 i.e.

$$k_1C_1 + k_2C_2 + \ldots + k_nC_n = 0$$

Now any row R_i can be written as linear combination of other rows as :

$$R_i = -\frac{k_1}{k_i}R_1 - \frac{k_2}{k_i}R_2 - \ldots - \frac{k_{i-1}}{k_i}R_{i-1} - \frac{k_{i+1}}{k_i}R_{i+1} - \ldots - \frac{k_n}{k_i}R_n$$

Similar is the case for columns.

Now $MX = 0$ always has one solution : $X = 0$ (which is called trivial solution). Now if $|M| = 0$, then $MX = 0$ has non-trivial solutions also.

So, $(S1)$, $(S2)$, and $(S3)$ are true. So, option D is correct.

25 votes -- Happy Mittal (11.2k points)

6.5.21 Matrices: GATE2015-1-18 top

https://gateoverflow.in/8241

Selected Answer

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Given, $\begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} * \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$

$= \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} * \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$

$L11 = 2$
$L11 * U12 = 2$
Solving, $U12 = 1$
$L21 = 4$
$L21 * U12 + L22 = 9$
Solving, $L22 = 5$

31 votes -- GATERush (1.1k points)

6.5.22 Matrices: GATE2015-2-27 top

https://gateoverflow.in/8131

Selected Answer

Answer : -0, because it is easy to see that first column and third column are multiple of each other.

Third column = First column * 15.

So rank is < 3, so Determinant must be 0.

It stays zero as row & column transformations don't affect determinant.

19 votes -- Akash Kanase (42.5k points)

6.5.23 Matrices: ISI-2017-5 top

https://gateoverflow.in/209513

Selected Answer

Assuming,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Which is a 2×2 matrix.

Now, $det\, A$ will be $(ad - bc)$

Given $det\, A$ is 3

$\therefore (ad - bc) = 3$

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What is Trace of a matrix?

The trace of an $n \times n$ square matrix A is defined to be the sum of the elements on the main diagonal (the diagonal from the upper left to the lower right) of A, i.e.,

$$\operatorname{tr}(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \cdots + a_{nn}$$

where a_{ii} denotes the entry on the ith row and i^{th} column of A.

$\therefore$Trace of $A = a + d$

Given that Trace of $A = 3$

$\therefore a + d = 3$

Now, $A^{-1} = \dfrac{1}{det\, A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$= \frac{1}{3}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \begin{bmatrix} d\,/\,3 & -b\ / \\ -c\ /\ & 3a\,/\,3 \end{bmatrix} 3$$

$\therefore$ Trace of $A^{-1} = \dfrac{d}{3} + \dfrac{a}{3}$

$$= \frac{a+d}{3}$$

$$= \frac{3}{3} \qquad [\because a + d = 3]$$

$$= 1$$

$\therefore$ Answer is option A) 1

3 votes -- Sukanya Das (11.5k points)

6.5.24 Matrices: ISI2016 top

https://gateoverflow.in/45220

Selected Answer

Answer B)

$$B = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 6 \\ 0 & -1 \end{pmatrix} + \cdots$$

$$= \begin{pmatrix} 0 & (2-4) + (6-8)\cdots \\ 0 & 0 \end{pmatrix}$$

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$$= \begin{pmatrix} 0 & -2-2-2\cdots \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -2\times 25 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -50 \\ 0 & 0 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 0 & -50 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & -50 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

12 votes -- srestha (86.6k points)

6.5.25 Matrices: TIFR2010-A-16 top

https://gateoverflow.in/18492

Selected Answer

I think we can solve using Cayley- Hamilton Theorem

$$\lambda^2 - \lambda - 1 = 0$$

$$M^2 - M - I = 0$$

$$I = M^2 - M$$

NOW PRE MULTIPLYING M^{-1}

$$M^{-1}I = M^{-1}M^2 - M^{-1}M$$

$$M^{-1} = M - I$$

SHOULD BE D.

12 votes -- Umang Raman (15.7k points)

6.5.26 Matrices: TIFR2012-B-12 top

https://gateoverflow.in/25141

Selected Answer

Given $A^k = 0$

$$I \ - \ A^k = 1$$

$$I - A^k \ = (I - A)\ (\ I + A + A^2 + A^3 + \ldots\ldots\ldots A^{k-1}\ )$$

$$I \ = (I - A)\ (\ I + A + A^2 + A^3 + \ldots\ldots\ldots A^{k-1}\ )$$

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$(I-A)^{-1}$ = $(I+A+A^2+A^3+.....A^{k-1})$

Hence, (D) is the Answer.

23 votes -- Leen Sharma (40k points)

6.5.27 Matrices: TIFR2013-B-3 top

https://gateoverflow.in/25659

OPTION a is correct

---

whenever 1st row is 0 then its determent is 0 , and similarly if any 2 or more rows are linearly dependent then its $|det| = 0$

In order to find the odd determinant the

1st row must be non zero $\Rightarrow$ totally(2^4-1) possibilities $\left|\frac{0}{1}\ \frac{0}{1}\ \frac{0}{1}\ \frac{0}{1}\right|$ like totally=$16-1$

$2nd$ row must be non zero and not linearly depends on 1st row so$\Rightarrow$ totally (2^4-2) possibilities

for $3rd$ row it must be non-zero as well as not linearly depends on first 2 rows(not start with 0) $\Rightarrow$ totally (2^4-4)

for 4th row $\Rightarrow$ (2^4-8)

:: total possibilities=$(2^4-2^0)*(2^4-2^1)*(2^4-2^2)*(2^4-2^3) = 15*14*12*8 = 20160$ possible

11 votes -- venky.victory35 (749 points)

6.5.28 Matrices: TIFR2015-A-14 top

https://gateoverflow.in/29588

Selected Answer

$$M_1X = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_2+x_3 \\ x_1+x_3 \\ x_1+x_2 \end{pmatrix}$$

$$M_2X = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1+x_3 \\ 0 \\ x_1+x_3 \end{pmatrix}$$

Given $M_1X = M_2X(mod 2)$

So, by comparing both matrices , we can say

(X_2+X_3) mod 2 = (X_1+X_3) mod 2

(X_1+X_3) mod 2=0 // so either $X1 = X3 = 0$ OR $X1 = X3 = 1$

(X_1+X_2) mod 2 =(X_1+X_3)mod 2

So, we can see when $X1 = X3 = 0$ then $X_2 = 0$

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and when $X1 = X3 = 1$ then $X_2 = 1$

So eventually then can be two $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ 0-1 column vectors for X.

1 votes -- Rupendra Choudhary (11.5k points)

6.5.29 Matrices: TIFR2018-A-14 top

https://gateoverflow.in/179377

Take a matrix:

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

Sum of each row= a+b [matrix is invertible hence determinant is not 0 i.e. a and b are different]

Now lets prove:

$$2 \times \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 2b & 2a \end{bmatrix}$$ = sum of each row = 2(a+b)

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} \times \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & ab+ba \\ ab+ba & a^2+b^2 \end{bmatrix}$$ = sum is $(a^2+b^2+2ab)= (a+b)^2$

$$\frac{1}{a^2-b^2}\begin{bmatrix} a & -b \\ -b & a \end{bmatrix} = \frac{(a-b)}{a^2-b^2} = \frac{(a-b)}{(a+b)(a-b)} = \frac{1}{a+b}$$

Hence all are true

3 votes -- Anu007 (16.8k points)

6.6 Rank Of Matrix(1) top

6.6.1 Rank Of Matrix: ISI-2017-29 top

https://gateoverflow.in/209917

Suppose the rank of the matrix

$$\begin{pmatrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 3 \\ a & b & b & 1 \end{pmatrix}$$

is 2 for some real numbers a and b. Then b equals

A. 1
B. 3
C. $1/2$
D. $1/3$

isi2017 engineering-mathematics matrices rank-of-matrix

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Answer

Answers: Rank Of Matrix

6.6.1 Rank Of Matrix: ISI-2017-29 top

https://gateoverflow.in/209917

try to reduce the matrix in row echolen form $\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 0 & -1 & 1 \\ 0 & b-a & b-2a & 1-2a \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 0 & -1 & 1 \\ 0 & b-a & 0 & 1+b-4a \end{bmatrix}$$

since rank is 2 third row must be zero

implies b = a and 1 + b-4a=0

which gives a = b = 1/3

2 votes -- Kaluti (5.4k points)

6.7 System Of Equations(12) top

6.7.1 System Of Equations: GATE1996-1.7 top

https://gateoverflow.in/2711

Let $Ax = b$ be a system of linear equations where A is an $m \times n$ matrix and b is a $m \times 1$ column vector and X is an $n \times 1$ column vector of unknowns. Which of the following is false?

A. The system has a solution if and only if, both A and the augmented matrix $[Ab]$ have the same rank.

B. If $m < n$ and b is the zero vector, then the system has infinitely many solutions.

C. If $m = n$ and b is a non-zero vector, then the system has a unique solution.

D. The system will have only a trivial solution when $m = n$, b is the zero vector and $\text{rank}(A) = n$.

gate1996 linear-algebra system-of-equations normal

Answer

6.7.2 System Of Equations: GATE1998-1.2 top

https://gateoverflow.in/1639

Consider the following set of equations

$$x + 2y = 54x + 8y = 123x + 6y + 3z = 15$$

This set

A. has unique solution
B. has no solution
C. has finite number of solutions
D. has infinite number of solutions

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gate1998 linear-algebra system-of-equations easy

Answer

6.7.3 System Of Equations: GATE2003-41 top https://gateoverflow.in/932

Consider the following system of linear equations

$$\begin{pmatrix} 2 & 1 & -4 \\ 4 & 3 & -12 \\ 1 & 2 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha \\ 5 \\ 7 \end{pmatrix}$$

Notice that the second and the third columns of the coefficient matrix are linearly dependent. For how many values of α, does this system of equations have infinitely many solutions?

A. 0
B. 1
C. 2
D. 3

gate2003 linear-algebra system-of-equations normal

Answer

6.7.4 System Of Equations: GATE2004-71 top https://gateoverflow.in/1065

How many solutions does the following system of linear equations have?

$-x + 5y = -1$

$x - y = 2$

$x + 3y = 3$

A. infinitely many
B. two distinct solutions
C. unique
D. none

gate2004 linear-algebra system-of-equations normal

Answer

6.7.5 System Of Equations: GATE2004-IT-6 top https://gateoverflow.in/3647

What values of x, y and z satisfy the following system of linear equations?

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 12 \end{bmatrix}$$

A. $x = 6, y = 3, z = 2$
B. $x = 12, y = 3, z = -4$
C. $x = 6, y = 6, z = -4$
D. $x = 12, y = -3, z = 0$

gate2004-it linear-algebra system-of-equations easy

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Answer

6.7.6 System Of Equations: GATE2005-48 top

https://gateoverflow.in/1173

Consider the following system of linear equations :

$$2x_1 - x_2 + 3x_3 = 1$$
$$3x_1 + 2x_2 + 5x_3 = 2$$
$$-x_1 + 4x_2 + x_3 = 3$$

The system of equations has

A. no solution
B. a unique solution
C. more than one but a finite number of solutions
D. an infinite number of solutions

gate2005 linear-algebra system-of-equations normal

Answer

6.7.7 System Of Equations: GATE2008-3 top

https://gateoverflow.in/401

The following system of equations

$$x_1 + x_2 + 2x_3 = 1$$
$$x_1 + 2x_2 + 3x_3 = 2$$
$$x_1 + 4x_2 + \alpha x_3 = 4$$

has a unique solution. The only possible value(s) for α is/are

A. 0
B. either 0 or 1
C. one of $0, 1,$ or -1
D. any real number

gate2008 easy linear-algebra system-of-equations

Answer

6.7.8 System Of Equations: GATE2014-1-4 top

https://gateoverflow.in/1757

Consider the following system of equations:

$$3x + 2y = 1$$
$$4x + 7z = 1$$
$$x + y + z = 3$$
$$x - 2y + 7z = 0$$

The number of solutions for this system is _____

gate2014-1 linear-algebra system-of-equations numerical-answers normal

Answer

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6.7.9 System Of Equations: GATE2015-3-33 top https://gateoverflow.in/8490

If the following system has non-trivial solution,

$$px + qy + rz = 0$$

$$qx + ry + pz = 0$$

$$rx + py + qz = 0,$$

then which one of the following options is TRUE?

A. $p - q + r = 0$ or $p = q = -r$
B. $p + q - r = 0$ or $p = -q = r$
C. $p + q + r = 0$ or $p = q = r$
D. $p - q + r = 0$ or $p = -q = -r$

gate2015-3 linear-algebra system-of-equations normal

Answer

6.7.10 System Of Equations: GATE2016-2-04 top https://gateoverflow.in/39545

Consider the system, each consisting of m linear equations in n variables.

I. If $m < n$, then all such systems have a solution.
II. If $m > n$, then none of these systems has a solution.
III. If $m = n$, then there exists a system which has a solution.

Which one of the following is **CORRECT**?

a. I, II and III are true.
b. Only II and III are true.
c. Only III is true.
d. None of them is true.

gate2016-2 linear-algebra system-of-equations normal

Answer

6.7.11 System Of Equations: GATE2017-1-3 top https://gateoverflow.in/118282

Let $c_1 \ldots\ldots c_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i$ = 0 where a_i are column vectors in R^n.

Consider the set of linear equations

$$Ax = b$$

where $A = [a_1 \ldots\ldots a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has

A. a unique solution at $x = J_n$ where J_n denotes a n-dimensional vector of all 1.
B. no solution
C. infinitely many solutions
D. finitely many solutions

gate2017-1 linear-algebra system-of-equations normal

Answer

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6.7.12 System Of Equations: ISI-2016-04 top

https://gateoverflow.in/210190

If a, b, c and d satisfy the equations

$$a+7b+3c+5d=168a+4b+6c+2d=-162a+6b+4c+8d=165a+3b+7c+d=-1$$

Then $(a+d)(b+c)$ equals

A. -4
B. 0
C. 16
D. -16

isi2016 engineering-mathematics system-of-equations

Answer

Answers: System Of Equations

6.7.1 System Of Equations: GATE1996-1.7 top

https://gateoverflow.in/2711

Selected Answer

Answer would be C because it is a case of linear non-homogeneous equations so by having **m = n**, we can't say that it will have unique solution. Solution depends on rank of matrix **A** and matrix **[A B]**.

If **rank[A] = rank[A B]**, then it will have solution otherwise no solution

17 votes -- Vivek sharma (2.4k points)

6.7.2 System Of Equations: GATE1998-1.2 top

https://gateoverflow.in/1639

Selected Answer

There are no solutions.

If we multiply 1st equation by 4, we get

4x + 8y = 20

But 2nd equation says

4x + 8y = 12

Clearly, there can not be any pair of (x,y), which satisfies both equations.

24 votes -- Happy Mittal (11.2k points)

6.7.3 System Of Equations: GATE2003-41 top

https://gateoverflow.in/932

Determinant=0. Therefore apply reduction method on $(A|B)$

$R_2 \leftarrow R_2 - 2R_1, R_3 \leftarrow R_3 - 0.5R_1, R_3 \leftarrow R_3 - 1.5R_2,$

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obtain the resultant matrix

$$\begin{pmatrix} 2 & 1 & -4 & a \\ 4 & 3 & -12 & 5 \\ 1 & 2 & -8 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0.5 & -2 & 0.5a \\ 0 & 1 & -4 & 5-2a \\ 0 & 0 & -0 & 2+1.5a \end{pmatrix}$$

or infinitely many solutions, we must have $2+1.5a=0\ i.e., a=\frac{-4}{3}$ so for only 1 value of a, this system has infinitely many solutions. So, option **(B)** is correct.

10 votes -- Keith Kr (6.1k points)

6.7.4 System Of Equations: GATE2004-71 top

https://gateoverflow.in/1065

Selected Answer

answer = C

rank = r(A) = r(A|B) = 2

rank = total number of variables
Hence, unique solution

15 votes -- Amar Vashishth (30.5k points)

6.7.5 System Of Equations: GATE2004-IT-6 top

https://gateoverflow.in/3647

Selected Answer

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 12 \end{bmatrix}$$

$x+2y+3z=6.....(1)$

$x+3y+4z=8.....(2)$

$2x+2y+3z=12.....(3)$

Apply (3)-(1)

x=6

Put value of x in any of 2 equations.Let's take (1) and (2)

$2y+3z=0$

$3y+4z=2$

$y=6, z=-4$

Hence,Option(C) $x=6, y=6, z=-4.$

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11 votes -- Leen Sharma (40k points)

6.7.6 System Of Equations: GATE2005-48 top

https://gateoverflow.in/1173

Selected Answer

rank of matrix $=$ rank of augmented matrix $=$ no of unknown $= 3$
so unique solution..

12 votes -- Digvijay (54.8k points)

6.7.7 System Of Equations: GATE2008-3 top

https://gateoverflow.in/401

Selected Answer

$$A.X = B \implies \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 4 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

So, $X = A^{-1}B$

$$A^{-1} = \frac{adj(A)}{det(A)}$$

$$\because \left[adj(A) = Transporse(cofactor(A)), cofactor(A) = (-1)^{i+j} minor(A)\right]$$

$$adj(A) = \begin{bmatrix} 2\alpha - 12 & 8 - \alpha & -1 \\ 3 - \alpha & \alpha - 2 & -1 \\ 2 & -3 & 1 \end{bmatrix}$$

$$det(A) = 2\alpha - 12 + 3 - \alpha + 4 = \alpha - 5$$

Now, $A^{-1}B = \frac{1}{\alpha - 5} \begin{bmatrix} 0 \\ \alpha - 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

So, the solution is independent of α and any real value of α would suffice. (This can be seen even by observing the equations carefully- each equation value is dependent only on x_2). But a value of 5 would cause the determinant to be 0 and multiple solutions to the given equation. So, any real value except 5 should be the answer- none of the choices is correct.

Ref: http://www.mathwords.com/i/inverse_of_a_matrix.htm

18 votes -- Arjun Suresh (348k points)

6.7.8 System Of Equations: GATE2014-1-4 top

https://gateoverflow.in/1757

Selected Answer

Since, equation (2) - equation (1) produces equation (4), we have 3 independent equations in 3 variables, hence unique solution.

So, answer is 1.

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23 votes -- Happy Mittal (11.2k points)

6.7.9 System Of Equations: GATE2015-3-33 top

https://gateoverflow.in/8490

Selected Answer

for non-trivial solution

$$|A| = 0$$

where $|A| = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix} = p*(rq-p^2) - q*(q^2-pr) + r*(qp-r^2)$

$= prq - p^3 - q^3 + prq + prq - r^3$

$= 3prq - p^3 - q^3 - r^3$

$= -(p+q+r)^3 + 3(p+q+r)(pq+qr+pr)$

now if you check the options the only options where each individual condition can make $|A| = 0$ zero is C.

25 votes -- Tamojit Chatterjee (2.3k points)

6.7.10 System Of Equations: GATE2016-2-04 top

https://gateoverflow.in/39545

Selected Answer

Correct answer => $C)$

why ?

$I)$ This is false. Consider a system with m < n, which are incosistent like

$a+b+c=2$

$a+b+c=3$

Here $m < n$ but no solution because of inconsistency !

$II)$ $m > n$ but no solution for none of system $=>$ What if this system of equations have 2 equations which are dependent ?

ex $=> a+b=2$

$2a+2b=4$

$a-b=0$

Then $a=1, b=1$ is solutions . II) Is false.

$III)$ this is true, $M=2, N=2$

$a+b=2$

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$a-b=0$

Then $m=1, n=1$ Now there exists system which has solution . III) is correct. Answer is C !

29 votes -- Akash Kanase (42.5k points)

6.7.11 System Of Equations: GATE2017-1-3 top https://gateoverflow.in/118282

Selected Answer

$\sum_i c_i a_i = 0$ with $\exists i : c_i \neq 0$ indicates that column vectors of A are linearly dependent. Determinant of matrix A would be zero. Therefore either $Ax=b$ has no solution or infinitely many solutions. From $\sum_i a_i = b$, it is clear that a n-dimensional vector of all 1 is a solution of equation Ax=b.

Hence, $Ax=b$ will have infinitely many solutions. The correct answer is (C).

27 votes -- suraj (5.6k points)

6.7.12 System Of Equations: ISI-2016-04 top https://gateoverflow.in/210190

$a+7b+3c+5d=16$

$2a+6b+4c+8d=16$

Or, $(a+a)+(7b-b)+(3c+c)+(5d+3d)=16$

Or, $(a+7b+3c+5d)+(a-b+c+3d)=16$

Or, $a-b+c+3d=0 \qquad \because [a+7b+3c+5d=16]$ ---------- 1)

$8a+4b+6c+2d=-16$

$5a+3b+7c+d=-16$

Or, $(5a+3a)+(3b+b)+(7c-c)+(d+d)=-16$

Or, $(5a+3b+7c+d)+(3a+b-c+d)=-16$

Or, $3a+b-c+d=0 \qquad [\because 5a+3b+7c+d=-16]$ ---------- 2)

Adding 1) & 2)

$3a+b-c+d=0a-b+c+3d=0$

Or, $4a+4d=0$

Or, $a+d=0$

Now, we've to find $(a+d)(b+c)=?$

$(a+d)(b+c)=(0)\times(b+c) \qquad \because [a+d=0]$

$\therefore (a+d)(b+c)=0$

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Hence, option $B)$ is the right answer.

2 votes -- Sukanya Das (11.5k points)

6.8 Vector Space(7) top

6.8.1 Vector Space: GATE1995-2.13 top

https://gateoverflow.in/2625

A unit vector perpendicular to both the vectors $a = 2i - 2j + k$ and $b = 1 + j - 2k$ is:

A. $\frac{1}{\sqrt{3}}(i + j + k)$
B. $\frac{1}{3}(i + j - k)$
C. $\frac{1}{3}(i - j - k)$
D. $\frac{1}{\sqrt{3}}(i + j - k)$

gate1995 linear-algebra normal vector-space

Answer

6.8.2 Vector Space: GATE2007-27 top

https://gateoverflow.in/1225

Consider the set of (column) vectors defined by

$$X = \left\{ x \in R^3 \mid x_1 + x_2 + x_3 = 0, \text{ where } x^T = [x_1, x_2, x_3]^T \right\}$$

.

Which of the following is TRUE?

A. $\left\{ [1, -1, 0]^T, [1, 0, -1]^T \right\}$ is a basis for the subspace X.
B. $\left\{ [1, -1, 0]^T, [1, 0, -1]^T \right\}$ is a linearly independent set, but it does not span X and therefore is not a basis of X.
C. X is not a subspace of R^3.
D. None of the above

gate2007 linear-algebra normal vector-space

Answer

6.8.3 Vector Space: GATE2014-3-5 top

https://gateoverflow.in/2039

If V_1 and V_2 are 4-dimensional subspaces of a 6-dimensional vector space V, then the smallest possible dimension of $V_1 \cap V_2$ is _____.

gate2014-3 linear-algebra vector-space normal numerical-answers

Answer

6.8.4 Vector Space: GATE2017-1-30 top

https://gateoverflow.in/118311

Let u and v be two vectors in R2 whose Euclidean norms satisfy $\|u\| = 2\|v\|$. What is the value of α such that $w = u + \alpha v$ bisects the angle between u and v?

A. 2

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B. $\frac{1}{2}$
C. 1
D. $\frac{-1}{2}$

gate2017-1 linear-algebra normal vector-space

Answer

6.8.5 Vector Space: ISI 2018 MMA 2 top

https://gateoverflow.in/216667

The volume of the region $S = \{(x, y, z) : |x| + |y| + |z| \leq 1\}$ is

A. $\frac{1}{6}$
B. $\frac{1}{3}$
C. $\frac{2}{3}$
D. $\frac{4}{3}$

isi2018 vector-space

Answer

6.8.6 Vector Space: TIFR2010-A-11 top

https://gateoverflow.in/18503

The length of a vector $x = (x_1, \ldots, x_n)$ is defined as

$$\|x\| = \sqrt{\sum_{i=1}^{n} x_i^2}.$$

Given two vectors $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$, which of the following measures of discrepancy between x and y is insensitive to the length of the vectors?

A. $\|x - y\|$
B. $\|x - y\| / \|x\| \|y\|$
C. $\|x\| - \|y\|$
D. $\left\| \frac{X}{\|X\|} - \frac{Y}{\|Y\|} \right\|$
E. None of the above.

tifr2010 linear-algebra vector-space

Answer

6.8.7 Vector Space: TIFR2017-A-2 top

https://gateoverflow.in/94938

For vectors x, y in $\mathbb{R}^n$, define the inner product $\langle x, y \rangle = \Sigma_{i=1}^{n} x_i y_i$, and the length of x to be $\|x\| = \sqrt{\langle x, x \rangle}$. Let a, b be two vectors in $\mathbb{R}^n$ so that $\|b\| = 1$. Consider the following statements:

i. $\langle a, b \rangle \leq \|b\|$
ii. $\langle a, b \rangle \leq \|a\|$
iii. $\langle a, b \rangle = \|a\| \|b\|$
iv. $\langle a, b \rangle \geq \|b\|$
v. $\langle a, b \rangle \geq \|a\|$

Which of the above statements must be TRUE of a, b? Choose from the following options.

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A. ii only
B. i and ii
C. iii only
D. iv only
E. iv and v

tifr2017 linear-algebra vector-space

Answer

Answers: Vector Space

6.8.1 Vector Space: GATE1995-2.13 top

https://gateoverflow.in/2625

Selected Answer

Answer should be A.

dot product of two perpendicular vector is 0.

vector given in option A gives 0 dotproduct with vector b. while any other vector is not giving 0 in dotproduct. therefore ans should be A.

to find the perpendicular unit vector to two vectors the procedure is as follows:

http://www.leadinglesson.com/problem-on-finding-a-vector-perpendicular-to-two-vectors

6 votes -- jayendra (8.3k points)

6.8.2 Vector Space: GATE2007-27 top

https://gateoverflow.in/1225

Option (a), here is the answer,

http://math.stackexchange.com/a/1843452/153195

6 votes -- Pranav Kant Gaur (1.6k points)

6.8.3 Vector Space: GATE2014-3-5 top

https://gateoverflow.in/2039

Selected Answer

A 6-dimensional vector space $\{a_1, a_2, a_3, a_4, a_5, a_6\}$
Let V_1 be $\{a_1, a_2, a_3, a_4\}$
and V_2 be $\{a_3, a_4, a_5, a_6\}$
$V_1 \cap V_2 = \{a_3, a_4\}$
This is the smallest possible dimension, which is 2.

The largest possible dimension will be 4,when $V_1 = V_2$

31 votes -- Srinath Jayachandran (3.7k points)

6.8.4 Vector Space: GATE2017-1-30 top

https://gateoverflow.in/118311

Selected Answer

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Angle between u and $w =$ Angle between w and v

$$\frac{\vec{u}.\vec{w}}{\|u\|\|w\|} = \frac{\vec{w}.\vec{v}}{\|w\|\|v\|}$$

$$\vec{u}.\vec{w} = 2\vec{w}.\vec{v}$$

$$(\alpha - 2)\vec{u}.\vec{v} = 2(\alpha - 2)\|v\|^2$$

LHS and RHS would be equal for $\alpha = 2$. Hence, correct answer is (A).

15 votes -- suraj (5.6k points)

6.8.5 Vector Space: ISI 2018 MMA 2 top

https://gateoverflow.in/216667

Selected Answer

We know that $|x| = x$ when $x >= 0$ and $|x| = -x$ when $x < 0$.

So, region covered by the graph $|x| + |y| <= 1$ is bounded by the lines $(x + y = 1)$, $(-x + y = 1)$, $(x - y = 1)$, $(-x - y = 1)$ [As shown in Fig A below]

The region covered by the graph |x| + |y| <= 1

Fig A

So, the region covered will be the square $[(0,1),(1,0),(0,-1),(-1,0)]$[As shown in Fig A]

The side length of the above square is $\sqrt{2}$.

Now, in the $3D$ version of the above scenario the space will be covered by the lines $(x + z = 1)$, $(-x + z = 1)$, $(-y + z = 1)$,

$(y + z = 1)$, $(-y - z = 1)$, $(-y + z = 1)$, $(x - z = 1)$, $(-x - z = 1)$. [As shown in the figure B]

The region covered by the graph |x| + |y| + |z| <= 1

Fig B

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This will create two pyramids $[(0,0,1),(0,1,0),(1,0,0),(0,-1,0),(-1,0,0)]$and $[(0,0,-1),(0,1,0),(1,0,0),(0,-1,0),(-1,0,0)]$

[As shown in the above figure B]

Note that the base of the above two pyramids are same and it is the square formed in the $2D$ version.

Now, we know that the volume of the pyramid is $1/3$ (area of the base) (height)

Here the area of the base $\left(\sqrt{2}\right)^2 = 2$ square units. and the height = 1 unit.

Thus the volume of the pyramids formed $= 2 *$ ($1/3 * 2 * 1$) cubic units = $4/3$ cubic units

which matches ith option D.

Hence, D will be the correct answer.

2 votes -- Kushagra Chatterjee (8.2k points)

6.8.6 Vector Space: TIFR2010-A-11 top

https://gateoverflow.in/18503

Selected Answer

Option D is the correct answer.

The following statement

discrepancy between x and y is insensitive to the length of the vectors

means that The discrepancy, as measured by the formula, between two vectors x and y is same as the discrepancy between the vectors c_1x and c_2y, for any **constant scalars** c_1, c_2. That is,

$$D(x,y) = D(c_1x, c_2y), \quad \forall c_1, c_2 \in \mathbb{R}$$

Now, lets think about which formula achieves that.

Let us also define two pairs of vectors as follows:

$$x_1 = (0.1067, 0.9619, 0.0046, 0.7749, 0.8173)$$

$$y_1 = (0.8687, 0.0844, 0.3998, 0.2599, 0.8001)$$

$$x_2 = 0.4314 \times x_1 = (0.0460, 0.4150, 0.0020, 0.3343, 0.3526)$$

$$y_2 = 0.9106 \times y_1 = (0.7911, 0.0769, 0.3641, 0.2367, 0.7286)$$

A) $\|x-y\|$

Since the definition of $\|x\|$ is sensitive to scaling, option A won't be insensitive to scaling either.

For example,

$$D(x_1,y_1) = \|x_1 - y_1\| \approx 1.3313$$

$$D(x_2,y_2) = \|x_2 - y_2\| \approx 0.9754$$

$$D(x_1,y_1) \neq D(x_2,y_2)$$

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B) $\frac{\|x-y\|}{\|x\|\|y\|}$

Once we've subtracted the vectors, scaling them according to their original lengths won't help at all.

For example,

$$D(x_1, y_1) = \frac{\|x_1-y_1\|}{\|x_1\|\|y_1\|} \approx 0.7024$$

$$D(x_2, y_2) = \frac{\|x_2-y_2\|}{\|x_2\|\|y_2\|} \approx 1.3099$$

$$D(x_1, y_1) \neq D(x_2, y_2)$$

C) $\|x\| - \|y\|$

We aren't doing any scaling in this definition of discrepancy. So, this definition is certainly sensitive to scaling, and thus, not the correct answer.

For example,

$$D(x_1, y_1) = \|x_1\| - \|y_1\| \approx 0.2086$$

$$D(x_2, y_2) = \|x_2\| - \|y_2\| \approx -0.5217$$

$$D(x_1, y_1) \neq D(x_2, y_2)$$

D) $\left\| \frac{x}{\|x\|} - \frac{y}{\|y\|} \right\|$

In this, we first scale each vector x and y down to their unit vectors, and then calculate the discrepancy.

Since $x_2 = c_1 x_1$, x_2 will have the same unit vector as x_1.
Similarly, y_2 will have the same unit vector as y_1.

Thus, no matter how we scale x_2 and y_2, as long as they are derived from x_1 and y_1, their discrepancy will be the same.

Therefore, our formula will be insensitive to scaling, which is exactly what we want!

For example,

$$D(x_1, y_1) = \left\| \frac{x_1}{\|x_1\|} - \frac{y_1}{\|y_1\|} \right\| \approx 0.9551$$

$$D(x_2, y_2) = \left\| \frac{x_2}{\|x_2\|} - \frac{y_2}{\|y_2\|} \right\| \approx 0.9551$$

$$D(x_1, y_1) = D(x_2, y_2)$$

Thus, **option D** is the correct answer.

You can use this matlab code to test the options with randomly generated vectors.

```
%% Get two random vectors x1 and y1, each of length 5
x1 = rand(5,1);
y1 = rand(5,1);
%% Create two more vectors x2 and y2, which are multiples of x1 and y1
x2 = rand()*x1;
y2 = rand()*y1;
%% Define the modd function
```

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```
modd = @(z) sqrt(sum(z.^2));
%% Define the answers function that computes the values
%  obtained from options A, B, C and D
answers = @(x,y) [
                modd(x-y);
                modd(x-y)/(modd(x)*modd(y));
                modd(x) - modd(y);
                modd(x/modd(x)  - y/modd(y))
            ];
%% Define function to perform floating point comparision
%  Copied from stackoverflow.com/a/22034483/2570622
isequalRel = @(x,y,tol) ( abs(x-y) <= ( tol*max(abs(x),abs(y)) + eps) );
%% Calculate the answers for (x1, y1) and (x2, y2) and see which option
%  remains unaffected.
isequalRel(answers(x1,y1), answers(x2,y2), 1e-6)
```

9 votes -- Pragy Agarwal (20.6k points)

6.8.7 Vector Space: TIFR2017-A-2 top

https://gateoverflow.in/94938

Let $n = 1$ then,

if $a = \{2\}, b = \{1\}$

$< a, b > = 2$

$|| b || = 1$

Hence, i is incorrect.

Let a = {-1} , b ={1}

$< a, b >= -1$

$|| a || \, || b || = 1$ x 1 = 1

Hence, iii and iv is incorrect, if iv is incorrect d and e both can't be right.

So, by elimination (a) is correct.

2 votes -- tarun_svbk (1.5k points)

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7 Engineering Mathematics: Probability (103) top

7.1 Binomial Distribution(7) top

7.1.1 Binomial Distribution: GATE2005-52 top https://gateoverflow.in/1177

A random bit string of length n is constructed by tossing a fair coin n times and setting a bit to 0 or 1 depending on outcomes head and tail, respectively. The probability that two such randomly generated strings are not identical is:

A. $\frac{1}{2^n}$
B. $1 - \frac{1}{n}$
C. $\frac{1}{n!}$
D. $1 - \frac{1}{2^n}$

gate2005 probability binomial-distribution easy

Answer

7.1.2 Binomial Distribution: GATE2005-IT-32 top https://gateoverflow.in/3778

An unbiased coin is tossed repeatedly until the outcome of two successive tosses is the same. Assuming that the trials are independent, the expected number of tosses is

A. 3
B. 4
C. 5
D. 6

gate2005-it probability binomial-distribution expectation normal

Answer

7.1.3 Binomial Distribution: GATE2006-21 top https://gateoverflow.in/982

For each element in a set of size $2n$, an unbiased coin is tossed. The $2n$ coin tosses are independent. An element is chosen if the corresponding coin toss was a head. The probability that exactly n elements are chosen is

A. $\frac{{}^{2n}C_n}{4^n}$
B. $\frac{{}^{2n}C_n}{2^n}$
C. $\frac{1}{{}^{2n}C_n}$
D. $\frac{1}{2}$

gate2006 probability binomial-distribution normal

Answer

7.1.4 Binomial Distribution: GATE2006-IT-22 top https://gateoverflow.in/3561

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When a coin is tossed, the probability of getting a Head is $p, 0 < p < 1$. Let N be the random variable denoting the number of tosses till the first Head appears, including the toss where the Head appears. Assuming that successive tosses are independent, the expected value of N is

A. $\frac{1}{p}$
B. $\frac{1}{(1-p)}$
C. $\frac{1}{p^2}$
D. $\frac{1}{(1-p^2)}$

gate2006-it probability binomial-distribution expectation normal

Answer

7.1.5 Binomial Distribution: TIFR2010-A-6 top https://gateoverflow.in/18222

Given 10 tosses of a coin with probability of head = $.4$ = (1 - the probability of tail), the probability of at least one head is?

a. $(.4)^{10}$
b. $1-(.4)^{10}$
c. $1-(.6)^{10}$
d. $(.6)^{10}$
e. $10(.4)(.6)^9$

tifr2010 probability binomial-distribution

Answer

7.1.6 Binomial Distribution: TIFR2010-B-38 top https://gateoverflow.in/19050

Suppose three coins are lying on a table, two of them with heads facing up and one with tails facing up. One coin is chosen at random and flipped. What is the probability that after the flip the majority of the coins(i.e., at least two of them) will have heads facing up?

a. $\left(\frac{1}{3}\right)$
b. $\left(\frac{1}{8}\right)$
c. $\left(\frac{1}{4}\right)$
d. $\left(\frac{1}{4}+\frac{1}{8}\right)$
e. $\left(\frac{2}{3}\right)$

tifr2010 probability binomial-distribution

Answer

7.1.7 Binomial Distribution: TIFR2011-A-3 top https://gateoverflow.in/20000

The probability of three consecutive heads in four tosses of a fair coin is.

a. $\left(\frac{1}{4}\right)$

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b. $\left(\frac{1}{8}\right)$
c. $\left(\frac{1}{16}\right)$
d. $\left(\frac{3}{16}\right)$
e. None of the above.

tifr2011 probability binomial-distribution

Answer

Answers: Binomial Distribution

7.1.1 Binomial Distribution: GATE2005-52 top

https://gateoverflow.in/1177

Selected Answer

Answer - D

Suppose there are k places within n bit string where mismatch has occoured

Probability of this occurring is

$${}^nC_k.(\text{prob. of mismatch})^k(\text{prob. of match})^{(n-k)}$$

$$={}^n C_k\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{(n-k)}$$

$$={}^n C_k\left(\frac{1}{2}\right)^n.$$

k can range from 1 to n, hence the required probability $\sum\left({}^nC_k\left(\frac{1}{2}\right)^n\right)$ where k ranges from 1 to n

is $\left(\frac{1}{2^n}\right)(2^n-1)$.

Alternatively

Probability of matching at given place $\frac{1}{2}$.

there are n places hence probability of matching $\frac{1}{2^n}$.

hence probability of mismatch $1-\frac{1}{2^n}$.

25 votes -- Ankit Rokde (9k points)

7.1.2 Binomial Distribution: GATE2005-IT-32 top

https://gateoverflow.in/3778

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Selected Answer

Probability on each branch is = x = $\frac{1}{2}$

2nd toss onwards, each toss layer gives us two success. (i.e. HH event or TT event)

$$\begin{aligned} E &= \sum k.\,p(k) \\ &= 2.(2x^2) + 3.(2x^3) + 4.(2x^4) + 5.(2x^5) + \ldots \\ &= 2.\left[2x^2 + 3x^3 + 4x^4 + 5x^5 + \ldots.\right] \\ &= 2.\left[\frac{x}{(1-x)^2} - x\right] \end{aligned}$$

$$\text{putting x} = \frac{1}{2} \quad ;$$

$$\begin{aligned} &= 2.\left[\frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^2} - \frac{1}{2}\right] \\ &= 3 \end{aligned}$$

A very similar QS :

An unbiased coin is tossed repeatedly and outcomes are recorded. What is the expected no of toss to get HT (one head and one tail consecutively) ?

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Probability in each branch is = 0.5. I double circled the satisfying toss events.
While observing the diagram I noticed that, from 2nd toss onward our required event starts showing up. Additionally,

1. in the 2nd toss (or the 3rd level) we have one satisfying case.
2. in the 3rd toss (or the 4th level) we have two satisfying case.
3. in the 4th toss (or the 5th level) we have three satisfying case.
4. in the 5th toss (or the 6th level) we have four satisfying case.
5. etc.

i.e. in the kth toss we would have $(k-1)$ satisfying case.
So,

$$\begin{aligned} E(x) &= \sum_{k=2}^{\infty} k.\,P(k) \\ &= \sum_{k=2}^{\infty} k.\left\{(k-1)*(0.5)^k\right\} \\ &= \sum_{k=2}^{\infty} \left\{(k^2-k)*(0.5)^k\right\} \end{aligned}$$

Using geometric series identity
: https://en.wikipedia.org/wiki/Geometric_series#Geometric_power_series

$$\sum_{k=2}^{\infty} k(k-1)x^{k-2} = \frac{2}{(1-x)^3} \quad \text{for}$$

In our case : $x = 0.5$ So,

$$E = \sum_{k=2}^{\infty} k(k-1)x^k = x^2 \sum_{k=2}^{\infty} k(k-1)x^{k-2} = x^2.\frac{2}{(1-x)^3}$$

putting $x = \frac{1}{2}$; we get $E = 4$

More example:

For consecutive two heads ; HH

By drawing the tree diagram we can find the following series :

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$$E = \sum k.P(k)$$
$$= 2.(1.x^2) + 3.(1.x^3) + 4.(2.x^4) + 5.(3.x^5) + 6.(5.x^6) + 7.(8.x^7) + \ldots\ldots\infty$$

Above series is a nice combination of AP , generating function and Fibonacci numbers !!!!

- AP terms can be handled by integration or differentiation
- Fibanacci Generating function is = $\dfrac{1}{1-x-x^2}$

$$\Rightarrow \frac{E}{x} = 2.(1.x^1) + 3.(1.x^2) + 4.(2.x^3) + 5.(3.x^4) + 6.(5.x^5) + 7.(8.x^6) + \ldots\ldots\infty$$
$$\Rightarrow \int \frac{E}{x}.dx = 1.x^2 + 1.x^3 + 2.x^4 + 3.x^5 + 5.x^6 + \ldots\ldots\infty$$
$$\Rightarrow \int \frac{E}{x}.dx = x^2.\left(1.x^0 + 1.x^1 + 2.x^2 + 3.x^3 + 5.x^4 + \ldots\ldots\infty\right)$$
$$\Rightarrow \int \frac{E}{x}.dx = \frac{x^2}{1-x-x^2}$$
$$\Rightarrow \frac{E}{x} = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x^2}{1-x-x^2}\right]$$
$$\Rightarrow \frac{E}{x} = \frac{2x(1-x-x^2) + (1+2x)x^2}{(1-x-x^2)^2}$$
$$\Rightarrow E = x.\left\{\frac{2x(1-x-x^2) + (1+2x)x^2}{(1-x-x^2)^2}\right\}$$
$$\Rightarrow E = \frac{1}{2}.\left\{\frac{2.\frac{1}{2}(1-\frac{1}{2}-\frac{1}{4}) + (1+2.\frac{1}{2}).\frac{1}{4}}{(1-\frac{1}{2}-\frac{1}{4})^2}\right\}$$
$$\Rightarrow E = 6$$

Infact 2nd QS on HT can also be done in the above way using integration.

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40 votes -- Debashish Deka (56.6k points)

Answer is (A)

$E(X) = \sum X_i \times P_i$

Where X=no of tosses when you get successive HEAD/TAIL(only one is possible at a time though).

P_i=Probability that you get in X_i tosses.

Now see solution:

You need atleast 2 tosses to get 2 heads/tails. Now see if you throw twice probability to get 2 heads/tails is $\frac{1}{2}$ out of 4 outcomes $[HT, HH, TH, TT]$.

Similarly if you get result in $3rd$ toss that means you did not get in $2nd$ toss so favourable cases for this can be THH and HTT only out of total 8 outcomes. So probability is $\frac{2}{8} = \frac{1}{2^2}$.

To generalize ,you can see that in every case you will have only two favourable cases and 2^n sample space. So for n th throw probability is $\frac{1}{(2^{n-1})}$.

Now coming to $E(X) = 2 \times \frac{1}{2} + 3 \times \frac{1}{4} + 4 \times \frac{1}{8} + \ldots \text{till infinity}.$

See this is combined AP-GP, So multiplying E(X) by $\frac{1}{2}$ and subtracting from E(X).

$E(X) = 2 \times \frac{1}{2} + 3 \times 14 + 4 \times \frac{1}{8} + \ldots$

$0.5 \times E(X) = 2 \times \frac{1}{4} + 3 \times \frac{1}{8} + \ldots$

subtracting, we get $\frac{1}{2} \times E(X) = 1 + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$

$0.5 \times E(X) = 1 + \left(\frac{1}{4}\right) \div (1 - 0.5) = 1 + \frac{1}{2} = \frac{3}{2} \qquad \left(\frac{a}{1-r}\right)$

$E(x) = 3.$

31 votes -- Sandeep_Uniyal (7.5k points)

7.1.3 Binomial Distribution: GATE2006-21 top

https://gateoverflow.in/982

Selected Answer

Answer - A

Ways of getting n heads out of $2n$ tries $={}^{2n}C_n$.

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Probability of getting exactly n-heads and n-tails $= \left(\frac{1}{2^n}\right) . \left(\frac{1}{2^n}\right)$

Number of ways $= \frac{{}^{2n}C_n}{4^n}$.

16 votes -- Ankit Rokde (9k points)

7.1.4 Binomial Distribution: GATE2006-IT-22 top https://gateoverflow.in/3561

Selected Answer

$E = 1 \times p + 2 \times (1-p)p + 3 \times (1-p)(1-p)p + \ldots$

multiply both side with $(1-p)$ and subtract:

$E - (1-p)E = 1 \times p + (1-p)p + (1-p)(1-p)p + \ldots$

$= p/(1-(1-p)) = 1$ (because it is now forming a GP)

$\implies (1-1+p)E = 1$

$\implies E = 1/p$

So, option (A).

20 votes -- Vicky Bajoria (5k points)

7.1.5 Binomial Distribution: TIFR2010-A-6 top https://gateoverflow.in/18222

Selected Answer

10 tosses of coin are there.

Probability of head = 0.4

Probability of tail = 0.6

Probability of at least one head $= 1 - P_{\text{no head occur}} = 1 - (0.6)^{10}$.

6 votes -- Digvijay (54.8k points)

7.1.6 Binomial Distribution: TIFR2010-B-38 top https://gateoverflow.in/19050

Selected Answer

(e) is correct

Table has 3 coins with H, H, T facing up.

Now, probability of choosing any coin is $\frac{1}{3}$, as we can chose any of the three coins.

Case A: 1st coin : either H or T can come.

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so, HHT, THT possible.only HHT is favorable.
which gives $\left(\frac{1}{3}\right) \times \left(\frac{1}{2}\right) = \frac{1}{6}$.

Case B: $2nd$ coin : either H or T can come.
so, HHT, HTT possible.only HHT is favourable.
which gives $\left(\frac{1}{3}\right) \times \left(\frac{1}{2}\right) = \frac{1}{6}$.

case C: $3rd$ coin : Table already contains two $H's$ so, whatever comes is favourable.
which gives $\left(\frac{1}{3}\right) \times 1 = \frac{1}{3}$.

Summing up the total gives $\frac{1}{6} + \frac{1}{6} + \frac{1}{3} = \frac{2}{3}$.

13 votes -- Himanshu Agarwal (15.3k points)

7.1.7 Binomial Distribution: TIFR2011-A-3 top

https://gateoverflow.in/20000

Selected Answer

Let the 4 tosses be named P, Q, R and S

To have 3 consecutive heads:

- Q, R must be both heads.
- At least one of $\{P, S\}$ must be a head.

Thus, the probability of getting 3 consecutive heads is given by:

$$P = P_q \times P_r \times \underbrace{(P_p + P_s - P_p P_s)}_{\text{atleast one}}$$

$$= \frac{1}{2} \times \frac{1}{2} \times \left(\frac{1}{2} + \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2}\right)$$

$$P = \frac{3}{16}.$$

Hence, option D is the correct answer.

Another way of looking at it is:

$$P = P_{HHHT} + P_{THHH} + P_{HHHH} = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$$

10 votes -- Pragy Agarwal (20.6k points)

7.2 Conditional Probability(11) top

7.2.1 Conditional Probability: CMI2013-A-02 top

https://gateoverflow.in/46592

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10% of all email you receive is spam. Your spam filter is 90% reliable: that is, 90% of the mails it marks as spam are indeed spam and 90% of spam mails are correctly labelled as spam. If you see a mail marked spam by your filter, what is the probability that it really is spam?

A. 10%
B. 50%
C. 70%
D. 90%

cmi2013 probability conditional-probability

Answer

7.2.2 Conditional Probability: GATE1994-1.4, ISRO2017-2 top

https://gateoverflow.in/2441

Let A and B be any two arbitrary events, then, which one of the following is TRUE?

A. $P(A \cap B) = P(A)P(B)$
B. $P(A \cup B) = P(A) + P(B)$
C. $P(A \mid B) = P(A \cap B)P(B)$
D. $P(A \cup B) \leq P(A) + P(B)$

gate1994 probability conditional-probability normal isro2017

Answer

7.2.3 Conditional Probability: GATE2003-3 top

https://gateoverflow.in/894

Let $P(E)$ denote the probability of the event E. Given $P(A) = 1$, $P(B) = \frac{1}{2}$, the values of $P(A \mid B)$ and $P(B \mid A)$ respectively are

A. $\left(\frac{1}{4}\right), \left(\frac{1}{2}\right)$
B. $\left(\frac{1}{2}\right), \left(\frac{1}{4}\right)$
C. $\left(\frac{1}{2}\right), 1$
D. $1, \left(\frac{1}{2}\right)$

gate2003 probability easy conditional-probability

Answer

7.2.4 Conditional Probability: GATE2005-51 top

https://gateoverflow.in/1176

Box P has 2 red balls and 3 blue balls and box Q has 3 red balls and 1 blue ball. A ball is selected as follows: (i) select a box (ii) choose a ball from the selected box such that each ball in the box is equally likely to be chosen. The probabilities of selecting boxes P and Q are $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Given that a ball selected in the above process is a red ball, the probability that it came from the box P is:

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A. $\frac{4}{19}$

B. $\frac{5}{19}$

C. $\frac{2}{9}$

D. $\frac{19}{30}$

gate2005 probability conditional-probability normal

Answer

7.2.5 Conditional Probability: GATE2012-33 top

https://gateoverflow.in/1751

Suppose a fair six-sided die is rolled once. If the value on the die is $1, 2,$ or $3,$ the die is rolled a second time. What is the probability that the sum total of values that turn up is at least 6 ?

A. $\frac{10}{21}$

B. $\frac{5}{12}$

C. $\frac{2}{3}$

D. $\frac{1}{6}$

gate2012 probability conditional-probability normal

Answer

7.2.6 Conditional Probability: GATE2014-EC01-GA10 top

https://gateoverflow.in/41499

You are given three coins: one has heads on both faces, the second has tails on both faces, and the third has a head on one face and a tail on the other. You choose a coin at random and toss it, and it comes up heads. The probability that the other face is tails is

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

gate2014-ec01 numerical-ability probability conditional-probability

Answer

7.2.7 Conditional Probability: GATE2016-2-05 top

https://gateoverflow.in/39541

Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2

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is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is _____.

gate2016-2 probability conditional-probability normal numerical-answers

Answer

7.2.8 Conditional Probability: GATE2017-2-26 top https://gateoverflow.in/118368

P and Q are considering to apply for a job. The probability that P applies for the job is $\frac{1}{4}$, the probability that P applies for the job given that Q applies for the job is $\frac{1}{2}$, and the probability that Q applies for the job given that P applies for the job is $\frac{1}{3}$. Then the probability that P does not apply for the job given that Q does not apply for this job is

A. $\left(\frac{4}{5}\right)$

B. $\left(\frac{5}{6}\right)$

C. $\left(\frac{7}{8}\right)$

D. $\left(\frac{11}{12}\right)$

gate2017-2 probability conditional-probability

Answer

7.2.9 Conditional Probability: GATE2018-44 top https://gateoverflow.in/204119

Consider Guwahati, (G) and Delhi (D) whose temperatures can be classified as high (H), medium (M) and low (L). Let $P(H_G)$ denote the probability that Guwahati has high temperature. Similarly, $P(M_G)$ and $P(L_G)$ denotes the probability of Guwahati having medium and low temperatures respectively. Similarly, we use $P(H_D)$, $P(M_D)$ and $P(L_D)$ for Delhi.

The following table gives the conditional probabilities for Delhi's temperature given Guwahati's temperature.

	H_D	M_D	L_D
H_G	0.40	0.48	0.12
M_G	0.10	0.65	0.25
L_G	0.01	0.50	0.49

Consider the first row in the table above. The first entry denotes that if Guwahati has high temperature (H_G) then the probability of Delhi also having a high temperature (H_D) is 0.40; i.e., $P(H_D \mid H_G) = 0.40$. Similarly, the next two entries are $P(M_D \mid H_G) = 0.48$ and $P(L_D \mid H_G) = 0.12$. Similarly for the other rows.

If it is known that $P(H_G) = 0.2$, $P(M_G) = 0.5$, and $P(L_G) = 0.3$, then the probability (correct to two decimal places) that Guwahati has high temperature given that Delhi has high temperature is _____

gate2018 probability conditional-probability numerical-answers

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Answer

7.2.10 Conditional Probability: TIFR2010-A-19, TIFR2014-A-6

top https://gateoverflow.in/18499

Karan tells truth with probability $\frac{1}{3}$ and lies with probability $\frac{2}{3}$. Independently, Arjun tells truth with probability $\frac{3}{4}$ and lies with probability $\frac{1}{4}$. Both watch a cricket match. Arjun tells you that India won, Karan tells you that India lost. What probability will you assign to India's win?

a. $\left(\frac{1}{2}\right)$
b. $\left(\frac{2}{3}\right)$
c. $\left(\frac{3}{4}\right)$
d. $\left(\frac{5}{6}\right)$
e. $\left(\frac{6}{7}\right)$

tifr2010 probability conditional-probability tifr2014

Answer

7.2.11 Conditional Probability: TIFR2012-A-1 top https://gateoverflow.in/20938

Amar and Akbar both tell the truth with probability $\frac{3}{4}$ and lie with probability $\frac{1}{4}$. Amar watches a test match and talks to Akbar about the outcome. Akbar, in turn, tells Anthony, "Amar told me that India won". What probability should Anthony assign to India's win?

a. $\left(\frac{9}{16}\right)$
b. $\left(\frac{6}{16}\right)$
c. $\left(\frac{7}{16}\right)$
d. $\left(\frac{10}{16}\right)$
e. None of the above

tifr2012 probability conditional-probability

Answer

Answers: Conditional Probability

7.2.1 Conditional Probability: CMI2013-A-02 top https://gateoverflow.in/46592

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Selected Answer

10% email are spam, i.e. 90% email are not spam

90% of mail marked as spam is spam, 10% mail marked as spam are not spam

By Bayes theorem the probability that a mail marked spam is really a spam

$$= \frac{\text{Probability of being spam and being detected as spam}}{\text{Probability of being detected as spam}}$$
$$= \frac{0.1 \times 0.9}{(0.1 \times 0.9) + (0.9 \times 0.1)}$$
$$= 50\%$$

8 votes -- srestha (86.6k points)

7.2.2 Conditional Probability: GATE1994-1.4, ISRO2017-2 top

https://gateoverflow.in/2441

Selected Answer

(a) is true only if events are independent.

(b) is true only if events are mutually exclusive i.e. $P(A \cap B) = 0$

(c) is false everywhere

(d) is always true as $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Since, $P(A \cap B) > 0$, $P(A \cup B) \leq P(A) + P(B)$

25 votes -- Happy Mittal (11.2k points)

7.2.3 Conditional Probability: GATE2003-3 top

https://gateoverflow.in/894

Selected Answer

It immediately follows from the monotonicity property that,
$0 \leq P(E) \leq 1,$

Probability of at least one means union of the probability of events, i.e.,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
here, $P(A \cup B) = 1$, because it can not be more than 1 and if at least one of the event has probability 1 (here , $P(A) = 1$), then union of both should be 1.
So,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$1 = 1 + \frac{1}{2} - P(A \cap B),$$
$$P(A \cap B) = \frac{1}{2},$$

Now,

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$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)} = 1,$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{\left(\frac{1}{2}\right)}{1} = \frac{1}{2}.$$

Hence, option is (D).

NOTE :- if at least one of the two events has probability 1, then both events should be independent but vise versa is not true.

23 votes -- Mithlesh Upadhyay (5.8k points)

7.2.4 Conditional Probability: GATE2005-51 top https://gateoverflow.in/1176

Selected Answer

The probability of selecting a red ball,

$$= \left(\frac{1}{3}\right) * \left(\frac{2}{5}\right) + \left(\frac{2}{3}\right) * \left(\frac{3}{4}\right)$$

$$= \frac{2}{15} + \frac{1}{2} = \frac{19}{30}$$

Probability of selecting a red ball from box,

$$P = \left(\frac{1}{3}\right) * \left(\frac{2}{5}\right) = \frac{2}{15}$$

Given that a ball selected in the above process is a red ball, the probability that it came from the box P is $= \left(\frac{2}{15} \div \frac{19}{30}\right) = \frac{4}{19}$

14 votes -- akash (1k points)

7.2.5 Conditional Probability: GATE2012-33 top https://gateoverflow.in/1751

Selected Answer

Here our sample space consists of $3 + 3 \times 6 = 21$ events- $(4), (5), (6), (1,1), (1,2) \ldots (3,6)$.

Favorable cases $= (6), (1,5), (1,6), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6)$.

Required Probability $= \frac{\text{No. of favorable cases}}{\text{Total cases}} = \frac{10}{21}$

But this is wrong way of doing. Because due to 2 tosses for some and 1 for some, individual probabilities are not the same. i.e., while (6) has $\frac{1}{6}$ probability of occurrence, $(1,5)$ has only $\frac{1}{36}$

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probability. So, our required probability

$$\Rightarrow \frac{1}{6} + \left(9 \times \frac{1}{36}\right) = \frac{5}{12}.$$

54 votes -- Arjun Suresh (348k points)

7.2.6 Conditional Probability: GATE2014-EC01-GA10 top

https://gateoverflow.in/41499

Selected Answer

We are given 3 coins out of which 1 coin (TT) is automatically removed from the total cases as we have already got a H.

Now when we toss any of two coins (HH, HT), the cases for the front side are: (H, H, H, T) and each of these are **equally likely**. It is given that the tossed side is H, so fourth option is also removed from the total cases.

Now as we have one coin with HH and another coin with HT and one H side is present, the choices for another side are (H, H, T).

Now we want the T side as favourable outcome out of total 3 outcomes.

According to Bayes theorem or conditional probability

$$p\,(\text{another side is T}) = \frac{\#\text{ favourable outcomes}}{\#\text{total outcomes}}$$

$$\therefore\ p\ = \frac{1}{3}$$

So, answer is **B.**

Reference link: https://www.khanacademy.org/math/precalculus/prob-comb/dependent-events-precalc/v/bayes-theorem-visualized

15 votes -- Sheshang M. Ajwalia (3k points)

7.2.7 Conditional Probability: GATE2016-2-05 top

https://gateoverflow.in/39541

Selected Answer

Given that the shop has an equal number of LED bulbs of two different types. Therefore,

Probability of Taking Type 1 Bulb $= 0.5$

Probability of Taking Type 2 Bulb $= 0.5$

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The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. i.e.,

$$Prob(100+ \mid Type1) = 0.7$$

$$Prob(100+ \mid Type2) = 0.4$$

$$Prob(100+) = Prob(100+ \mid Type1) \times Prob(Type1) + Prob(100+ \mid Type2) \times Prob(Type2)$$

$$= 0.7 \times 0.5 + 0.4 \times 0.5 = 0.55.$$

29 votes -- Akash Kanase (42.5k points)

7.2.8 Conditional Probability: GATE2017-2-26 top https://gateoverflow.in/118368

Selected Answer

Let,

$P =$ P applies for the job

$Q =$ Q applies for the job

$P(P) = \frac{1}{4} \rightarrow (1)$

$P(P \mid Q) = \frac{1}{2} \rightarrow (2)$

$P(Q \mid P) = \frac{1}{3} \rightarrow (3)$

Now, we need to find $P(P' \mid Q')$

From (2)

$P(P \mid Q) = \frac{P(P \cap Q)}{P(Q)} = \frac{1}{2} \rightarrow (4)$

From (1) and (3),

$P(Q \mid P) = \frac{P(P \cap Q)}{P(P)} = \frac{P(P \cap Q)}{\frac{1}{4}} = \frac{1}{3}$
$\therefore P(P \cap Q) = \frac{1}{12} \rightarrow (5)$

From (4) and (5),

$P(Q) = \frac{1}{6} \rightarrow (6)$

Now, $P(P' \mid Q') = \frac{P(P' \cap Q')}{P(Q')} \rightarrow (7)$

From (6)
$P(Q') = 1 - 1/6 = 5/6 \rightarrow (8)$

Also, $P(P' \cap Q') = 1 - [P(P \cup Q)]$

$$= 1 - [P(P) + P(Q) - P(P \cap Q)]$$
$$= 1 - [1/4 + 1/6 - 1/12]$$
$$= 1 - [1/3]$$

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$= 2/3 \rightarrow (9)$

Hence, from $(7), (8)$ and (9)

$P(P' \mid Q') = \frac{\frac{2}{3}}{\frac{5}{6}} = \frac{4}{5}.$

34 votes -- Sushant Gokhale (18.1k points)

7.2.9 Conditional Probability: GATE2018-44 top https://gateoverflow.in/204119

Selected Answer

$P(H_G \mid H_D) = \frac{P(H_G \wedge H_D)}{P(H_D)} = P(H_D \mid H_G) \times \frac{P(H_G)}{P(H_D)}$

$= 0.40 \times 0.2 \times \frac{1}{P(H_D)}.$

$P(H_D) = P(H_D \mid H_G).P(H_G) + P(H_D \mid M_G).P(M_G) + P(H_D \mid L_G).P(L_G)$

$= 0.4 \times 0.2 + 0.1 \times 0.5 + 0.01 \times 0.3 = 0.133$

$\therefore P(H_G \mid H_D) = 0.40 \times 0.2 \times \frac{1}{P(H_D)} = \frac{0.08}{0.133} = 0.6015$

Ref: https://en.wikipedia.org/wiki/Bayes%27_theorem

10 votes -- Arjun Suresh (348k points)

7.2.10 Conditional Probability: TIFR2010-A-19, TIFR2014-A-6 top https://gateoverflow.in/18499

Selected Answer

Probability of India win $= \frac{1}{2}$

Probability of India lost $= \frac{1}{2}$

If really India wins, then Karan lies i.e. $= \frac{2}{3}$ and Arjun tells truth $= \frac{3}{4}$

Now prob. of Karan lies and Arjun tells truth $= \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

Now prob. of Arjun lies and Karan tells truth $= \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

So, by Bayes theorem,

$$\frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{12}} = \frac{6}{7}$$

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So, answer is (e)

PS: Assuming superover in case of tie.

9 votes -- srestha (86.6k points)

7.2.11 Conditional Probability: TIFR2012-A-1 top https://gateoverflow.in/20938

Selected Answer

Option D should be the correct answer.

Consider the following events,

W : India wins,

$W\neg$: India does not wins (India Lost/ Match Draw/ Match Tie/ Match Suspended etc.)

X : Akbar tells Anthony, "Amar told me that India won"

$X\neg$: Akbar tells Anthony, "Amar told me that India did not won"

Given X, we have to find W, that is we have to calculate $P\left(\frac{W}{X}\right)$.

$P\left(\frac{W}{X}\right)$ can be calculated using Bayes's theorem as follow:

$$P\left(\frac{\text{India Wins}}{\text{Akbar tells Anthony "Amar told me that India won"}}\right)$$

$$= \frac{P\left(\frac{\text{Akbar tells Anthony "Amar told me that India won"}}{\text{India Wins}}\right)}{P\left(\frac{\text{Akbar tells Anthony "Amar told me that India won"}}{\text{India Won}}\right) \cup P\left(\frac{\text{Akbar tells Anthony "Amar told me that India}}{\text{India didn't won}}\right.}$$

rewriting same equation using the events defined:

$$P\left(\frac{W}{X}\right) = \frac{P(\frac{X}{W})}{P(\frac{X}{W})+P(\frac{X}{W\neg})}$$

Calculation of $P\left(\frac{X}{W}\right) and\ P\left(\frac{X}{W\neg}\right)$ **:**

$$P\left(\frac{X}{W}\right) = P\left(\frac{Case\ 1}{W}\right) \cup\ P\left(\frac{Case\ 4}{W}\right)$$

$$P\left(\frac{Case\ 1}{W}\right) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

$$P\left(\frac{Case\ 4}{W}\right) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$So\ P\left(\frac{X}{W}\right) = \frac{9}{16} + \frac{1}{16} = \frac{10}{16}$$

$$P\left(\frac{X}{W\neg}\right) = P\left(\frac{Case\ 6}{W\neg}\right) \cup\ P\left(\frac{Case\ 7}{W\neg}\right)$$

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$P\left(\frac{Case\ 6}{W\neg}\right) = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$

$P\left(\frac{Case\ 7}{W\neg}\right) = \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$

$So\ P\left(\frac{X}{W\neg}\right) = \frac{3}{16} + \frac{3}{16} = \frac{6}{16}$

$Hence\ P\left(\frac{W}{X}\right) = \frac{\frac{10}{16}}{\frac{10}{16}+\frac{6}{16}} = \frac{10}{16}.$

7 votes -- Anurag Pandey (13.6k points)

7.3 Expectation(10) top

7.3.1 Expectation: GATE1999-1.1 top

https://gateoverflow.in/1455

Suppose that the expectation of a random variable X is 5. Which of the following statements is true?

A. There is a sample point at which X has the value 5.
B. There is a sample point at which X has value greater than 5.
C. There is a sample point at which X has a value greater than equal to 5.
D. None of the above

gate1999 probability expectation easy

Answer

7.3.2 Expectation: GATE2004-74 top

https://gateoverflow.in/1068

An examination paper has 150 multiple choice questions of one mark each, with each question having four choices. Each incorrect answer fetches -0.25 marks. Suppose 1000 students choose all their answers randomly with uniform probability. The sum total of the expected marks obtained by all these students is

A. 0
B. 2550
C. 7525
D. 9375

gate2004 probability expectation normal

Answer

7.3.3 Expectation: GATE2006-18 top

https://gateoverflow.in/979

We are given a set $X = \{X_1, \ldots, X_n\}$ where $X_i = 2^i$. A sample $S \subseteq X$ is drawn by selecting each X_i independently with probability $P_i = \frac{1}{2}$. The expected value of the smallest number in sample S is:

A. $\left(\frac{1}{n}\right)$
B. 2
C. $\sqrt{n}$
D. n

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gate2006 probability expectation normal

Answer

7.3.4 Expectation: GATE2011-18 top

https://gateoverflow.in/2120

If the difference between the expectation of the square of a random variable $\left(E\left[X^2\right]\right)$ and the square of the expectation of the random variable $(E[X])^2$ is denoted by R, then

A. $R = 0$
B. $R < 0$
C. $R \geq 0$
D. $R > 0$

gate2011 probability random-variable expectation normal

Answer

7.3.5 Expectation: GATE2013-24 top

https://gateoverflow.in/1535

Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is $\frac{1}{2}$. What is the expected number of unordered cycles of length three?

A. $\frac{1}{8}$
B. 1
C. 7
D. 8

gate2013 probability expectation normal

Answer

7.3.6 Expectation: GATE2014-2-2 top

https://gateoverflow.in/1954

Each of the nine words in the sentence "The quick brown fox jumps over the lazy dog" is written on a separate piece of paper. These nine pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The expected length of the word drawn is _____. (The answer should be rounded to one decimal place.)

gate2014-2 probability expectation numerical-answers easy

Answer

7.3.7 Expectation: TIFR2011-A-6 top

https://gateoverflow.in/20011

Assume that you are flipping a fair coin, i.e. probability of heads or tails is equal. Then the expected number of coin flips required to obtain two consecutive heads for the first time is.

a. 4
b. 3
c. 6
d. 10
e. 5

tifr2011 probability expectation

Answer

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7.3.8 Expectation: TIFR2012-B-7 top

https://gateoverflow.in/25107

A bag contains 16 balls of the following colors: 8 red, 4 blue, 2 green, 1 black, and 1 white. Anisha picks a ball randomly from the bag, and messages Babu its color using a string of zeros and ones. She replaces the ball in the bag, and repeats this experiment, many times. What is the minimum expected length of the message she has to convey to Babu per experiment?

a. $\frac{3}{2}$
b. $\log 5$
c. $\frac{15}{8}$
d. $\frac{31}{16}$
e. 2

tifr2012 probability expectation

Answer

7.3.9 Expectation: TIFR2014-A-17 top

https://gateoverflow.in/27111

A fair dice (with faces numbered $1, \ldots, 6$) is independently rolled repeatedly. Let X denote the number of rolls till an even number is seen and let Y denote the number of rolls till 3 is seen. Evaluate $E(Y|X=2)$.

A. $6\frac{5}{6}$
B. 6
C. $5\frac{1}{2}$
D. $6\frac{1}{3}$
E. $5\frac{2}{3}$

tifr2014 expectation

Answer

7.3.10 Expectation: TIFR2015-A-6 top

https://gateoverflow.in/29567

Ram has a fair coin, i.e., a toss of the coin results in either head or tail and each event happens with probability exactly half $(1/2)$. He repeatedly tosses the coin until he gets heads in two consecutive tosses. The expected number of coin tosses that Ram does is.

A. 2
B. 4
C. 6
D. 8
E. None of the above.

tifr2015 expectation

Answer

Answers: Expectation

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7.3.1 Expectation: GATE1999-1.1 top

https://gateoverflow.in/1455

Selected Answer

Expectation of discrete random variable (finite case)

$E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$

$E(X) = 5, 0 \leq pi \leq 1$

$p_1 + p_2 + \cdots + p_n = 1$

Therefore, $E(X) = 5$ is possible only if at-least one of the $x_i \geq 5$

18 votes -- suraj (5.6k points)

7.3.2 Expectation: GATE2004-74 top

https://gateoverflow.in/1068

Selected Answer

Answer is **option D.**

Probability of choosing the correct option = $\frac{1}{4}$
Probability of choosing a wrong option = $\frac{3}{4}$

So, expected mark for a question for a student = $\frac{1}{4} \times 1 + \frac{3}{4} \times (-0.25) = 0.0625$

Expected mark for a student for 150 questions = $0.0625 \times 150 = 9.375$

So, sum total of the expected marks obtained by all 1000 students = $9.375 \times 1000 = 9375.$

33 votes -- Arjun Suresh (348k points)

7.3.3 Expectation: GATE2006-18 top

https://gateoverflow.in/979

Selected Answer

The smallest element in sample S would be X_i for which i is smallest.

The given probability is for selection of each item of X. Independent selection means each item is selected with probability $\frac{1}{2}$.

Probability for X_1 to be smallest in $S = \frac{1}{2}$.
Value of $X_1 = 2$.
Probability for X_2 to be smallest in S = Probability of X_1 not being in $S \times$ Probability of X_2 being in $S = \frac{1}{2}.\frac{1}{2}$.
Value of $X_2 = 2^2 = 4$.
Similarly, Probability for X_i to be smallest in $S = (1/2)^i$.

Value of $X_i = 2^i$.

Now Required Expectation= $\sum_{i=1}^{n} 2^i \times \left(\frac{1}{2}\right)^i = \sum_{i=1}^{n} 1 = n$.

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The answer is option D.

32 votes -- Mari Ganesh Kumar (2k points)

7.3.4 Expectation: GATE2011-18 top

https://gateoverflow.in/2120

Selected Answer

The difference between $(E[X^2])$ and $(E[X])^2$ is called variance of a random variable.
Variance measures how far a set of numbers is spread out. (A variance of zero indicates that all the values are identical.) A non-zero variance is always positive:

13 votes -- Divya Bharti (8.1k points)

7.3.5 Expectation: GATE2013-24 top

https://gateoverflow.in/1535

Selected Answer

A cycle of length 3 requires 3 vertices.

Number of ways in which we can choose 3 vertices from 8 = ${}^8C_3 = 56$.

Probability that 3 vertices form a cycle
$= \text{ Probability of edge between vertices 1 and 2} \times \text{ Probability of edge between vertices 2 and 3} \times \text{Probability of edge between vertices 1 and 3}$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

So, expected number of cycles of length $3 = 56 \times \frac{1}{8} = 7$

ref@ http://stackoverflow.com/questions/14801072/number-of-cycles-in-a-random-graph

57 votes -- Arjun Suresh (348k points)

7.3.6 Expectation: GATE2014-2-2 top

https://gateoverflow.in/1954

Selected Answer

Each of the nine words have equal probability. So, expected length
$= 3 \times \frac{1}{9} + 5 \times \frac{1}{9} + 5 \times \frac{1}{9} + 3 \times \frac{1}{9} + 5 \times \frac{1}{9} + 4 \times \frac{1}{9} + 3 \times \frac{1}{9} + 4 \times \frac{1}{9} + 3 \times \frac{1}{9}$
$= \frac{35}{9}$
$= 3.9$

28 votes -- Arjun Suresh (348k points)

7.3.7 Expectation: TIFR2011-A-6 top

https://gateoverflow.in/20011

Selected Answer

Let the expected number of coin flips be X. The case analysis goes as follows:

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a. If the first flip is a tails, then we have wasted one flip. The probability of this event is $\frac{1}{2}$ and the total number of flips required is $X+1$.
b. If the first flip is a heads and second flip is a tails, then we have wasted two flips. The probability of this event is $\frac{1}{4}$ and the total number of flips required is $X+2$. as the same scenario as beginning is there even after 2 tosses.
c. If the first flip is a heads and second flip is also heads, then we are done. The probability of this event is $\frac{1}{4}$ and the total number of flips required is 2.

Adding, the equation that we get is -
$X = \frac{1}{2}(X+1) + \frac{1}{4}(X+2) + \frac{1}{4}2$

Solving, we get $X = 6$.

Thus, the expected number of coin flips for getting two consecutive heads is 6.

22 votes -- Avdhesh Singh Rana (3.2k points)

7.3.8 Expectation: TIFR2012-B-7 top

https://gateoverflow.in/25107

Selected Answer

using static huffman compression you can encode the more common colours in fewer bits than the rare colours, that being the case on can expect that common colours will usually be chosen.

eg:

```
red    1
blue   01
green  001
white  0001
black  0000
```

on average from 16 draws there will be

```
8 reds   = 8 bits
4 blues  = 8 bits
2 greens = 6 bits
1 white  = 4 bits
1 black  = 4 bits
```

for a total of $\frac{30}{16} = \frac{15}{8}$ bits on average.

10 votes -- sudipta roy (423 points)

7.3.9 Expectation: TIFR2014-A-17 top

https://gateoverflow.in/27111

X: The value of X denotes the number of rolls till an even number is seen.

Y: The value of Y denotes the number of rolls till a 3 is seen.

For example:

$X = 2$ implies an even number first time occurred on second roll, or outcome of the first roll is odd & outcome of the second roll is even.

$Y = 4$ implies 3 appeared for first time in the 4th die roll.

Ranges of Random Variables X & Y

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$X : \{1, 2, 3, \ldots, \infty\}$

$Y : \{1, 2, 3, \ldots, \infty\}$

$E[Y \mid X = 2]$: Expected number of rolls till a 3 is seen given that an even number appeared for the first time in the second roll.

It is sure that 3 cannot appear on 2nd toss, i.e. $P[Y = 2 \mid X = 2] = 0$ and henceforth $E[Y = 2 \mid X = 2] = 0.$

Now, there are two cases possible:

Case 1: 3 appears on the first toss given that outcome of first toss is odd.

i.e., $E[Y = 1 \mid X = 2]$

Here we need not to concern about outcomes of rolls other than the first roll.

Probability of getting 3 in first toss given that o/c of the first toss is odd
$= P(Y = 1 \mid X = 2) = \frac{1}{3} = 0.33$

So, Expectation $E[Y = 1 \mid X = 2] = y \times P(Y = 1 \mid X = 2) = 1 \times 0.33 = 0.33$

Case 2: 3 appears on any toss after the second toss given that outcome of first toss is odd, & that of second toss is even

$P[Y = y \mid X = 2]$ = given that 1st roll is an odd number and $2nd$ roll is an even number, Probability that out of y rolls,

None of the first $(y–1)$ roll's outcome is 3 &

Outcome of the yth roll is 3.

So $P[Y = y \mid X = 2]$

$$= \left(\frac{2}{3}\right) (\text{for first o/c odd but not } 3)$$
$$\times \left(\frac{5}{6}\right)^{(y–3)} (\text{for not getting a } 3 \ from\ 3^{rd}\ to\ 0(y–1)^{th} rolls)$$
$$\times \left(\frac{1}{6}\right) (\text{for } y^{th} \text{o/c to be } 3).$$

$$P[Y = y \mid X = 2] = \left(\frac{2}{3}\right) \times \left(\frac{5}{6}\right)^{(y–3)} \times \left(\frac{1}{6}\right)$$

So $E[Y = y \mid X = 2] =$ Summation from $y = 3$ to $\text{infinity}(y \times P(Y = y \mid X = 2)) = 5.33$ (where $y \geq 3$)

This summation will give sum of all the expectations from $Y = 3$ to $\text{infinity}.$

Now:

Net Expectation is given as:

$E[Y = y \mid X = 2] = E[Y = 1 \mid X = 2] + E[Y = 2 \mid X = 2] + E[Y = y' \mid X = 2]$ where $y' \geq 3.$

Putting all the values,

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$E[Y = y \mid X = 2] = 0.33 + 0 + 5.33$

So, $E[Y = y \mid X = 2] = 5.66 = \frac{17}{3}$.

10 votes -- Anurag Pandey (13.6k points)

7.3.10 Expectation: TIFR2015-A-6 top

https://gateoverflow.in/29567

Selected Answer

By drawing the tree diagram we can find the following series :

$$E = \sum k.P(k)$$
$$= 2.(1.x^2) + 3.(1.x^3) + 4.(2.x^4) + 5.(3.x^5) + 6.(5.x^6) + 7.(8.x^7) + \ldots \infty$$

Above series is a nice combination of AP , generating function and Fibonacci numbers !!!!

- AP terms can be handled by integration or differentiation
- Fibonacci generating function is $= \frac{1}{1 - x - x^2}$
- probability on each branch is $x = \frac{1}{2}$

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$$\Rightarrow \frac{E}{x} = 2.(1.x^1) + 3.(1.x^2) + 4.(2.x^3) + 5.(3.x^4) + 6.(5.x^5) + 7.(8.x^6) + \ldots\infty$$

$$\Rightarrow \int \frac{E}{x}.dx = 1.x^2 + 1.x^3 + 2.x^4 + 3.x^5 + 5.x^6 + \ldots\ldots\infty$$

$$\Rightarrow \int \frac{E}{x}.dx = x^2.\left(1.x^0 + 1.x^1 + 2.x^2 + 3.x^3 + 5.x^4 + \ldots\infty\right)$$

$$\Rightarrow \int \frac{E}{x}.dx = \frac{x^2}{1-x-x^2}$$

$$\Rightarrow \frac{E}{x} = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x^2}{1-x-x^2}\right]$$

$$\Rightarrow \frac{E}{x} = \frac{2x(1-x-x^2) + (1+2x)x^2}{(1-x-x^2)^2}$$

$$\Rightarrow E = x.\left\{\frac{2x(1-x-x^2) + (1+2x)x^2}{(1-x-x^2)^2}\right\}$$

$$\Rightarrow E = \frac{1}{2}.\left\{\frac{2.\frac{1}{2}(1-\frac{1}{2}-\frac{1}{4}) + (1+2.\frac{1}{2}).\frac{1}{4}}{(1-\frac{1}{2}-\frac{1}{4})^2}\right\}$$

$$\Rightarrow E = 6$$

Similar Kind of Question as a Reference

- https://gateoverflow.in/3778/gate2005-it-32

7 votes -- Debashish Deka (56.6k points)

7.4 Exponential Distribution(1) top

7.4.1 Exponential Distribution: GATE2004-IT-33 top https://gateoverflow.in/3676

Let X and Y be two exponentially distributed and independent random variables with mean α and β, respectively. If Z = min (X, Y), then the mean of Z is given by

A. $\left(\frac{1}{\alpha+\beta}\right)$
B. $\min(\alpha, \beta)$
C. $\left(\frac{\alpha\beta}{\alpha+\beta}\right)$
D. $\alpha+\beta$

gate2004-it probability exponential-distribution random-variable normal

Answer

Answers: Exponential Distribution

7.4.1 Exponential Distribution: GATE2004-IT-33 top https://gateoverflow.in/3676

Selected Answer

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X is an exponential random variable of parameter λ when its probability distribution function is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

For a > 0, we have the cumulative distribution function

$$F_x(a) = \int_0^a f(x)dx = \int_0^a \lambda e^{-\lambda x} dx = -e^{-\lambda x} \mid_0^a = 1 - e^{-\lambda a}$$

So,

$$P\{X < a\} = 1 - e^{-\lambda a}$$

and

$$P\{X > a\} = e^{-\lambda a}$$

Now, we use $P\{X > a\}$ for our problem because our concerned variable Z is **min** of X and Y.

For exponential distribution with parameter λ, mean is given by $\frac{1}{\lambda}$.
We have,

$P\{X > a\} = e^{-\frac{1}{\alpha}a}$

$P\{Y > a\} = e^{-\frac{1}{\beta}a}$

So,

$$P\{Z > a\} = P\{X > a\}P\{Y > a\} \, (\because \text{X and Y are independent events and } Z > \min(X, Y)$$
$$= e^{-\frac{1}{\alpha}a} e^{-\frac{1}{\beta}a}$$
$$= e^{-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)a}$$
$$= e^{-\left(\frac{\alpha+\beta}{\alpha\beta}\right)a}$$

This shows that Z is also exponentially distributed with parameter $\frac{\alpha+\beta}{\alpha\beta}$ and mean $\frac{\alpha\beta}{\alpha+\beta}$.

Ref: http://ocw.mit.edu/courses/mathematics/18-440-probability-and-random-variables-spring-2011/lecture-notes/MIT18_440S11_Lecture20.pdf

20 votes -- Arjun Suresh (348k points)

7.5 Normal Distribution(1) top

7.5.1 Normal Distribution: GATE2008-29 top

https://gateoverflow.in/427

Let X be a random variable following normal distribution with mean $+1$ and variance 4. Let Y be another normal variable with mean -1 and variance unknown. If $P(X \leq -1) = P(Y \geq 2)$, the standard deviation of Y is

A. 3

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B. 2
C. $\sqrt{2}$
D. 1

gate2008 random-variable normal-distribution probability normal

Answer

Answers: Normal Distribution

7.5.1 Normal Distribution: GATE2008-29 top

https://gateoverflow.in/427

Selected Answer

$$\underbrace{P(X \leq -1)} = P(Y \geq 2)$$

We can compare their values using standard normal distributions.

$$\begin{array}{ll} X & Y \\ \mu_X = +1 & \mu_Y = -1 \\ {\sigma_X}^2 = 4 & {\sigma_Y}^2 = ? \end{array}$$

$$Z_X = \frac{X-1}{\sqrt{4}} \quad Z_Y = \frac{Y-(-1)}{\sigma_Y}$$

$$2Z_X + 1 = X \quad Y = \sigma_Y Z_Y - 1$$

$$P(2Z_X + 1 \leq -1) = P(\sigma_Y Z_Y - 1 \geq 2)$$

$$P(Z_X \leq -1) = P(Z_Y \geq \frac{3}{\sigma_Y})$$

$$-(-1) = \frac{3}{\sigma_Y}$$

$$\sigma_Y = 3$$

The random variables X and Y can be written as $X = 2U + 1$ and $Y = \sigma V - 1$ where U and V both have standard normal distribution.

$$P\{U \geq 1\} = P\{U \leq -1\} = P\{X \leq -1\} = P\{Y \geq 2\} = P\left\{V \geq \frac{3}{\sigma}\right\} = P\left\{U \geq \frac{3}{\sigma}\right\}$$

The first equality is a consequence of the fact that the standard normal distribution is symmetric. The last equality is a consequence of the fact that U and V have the same distribution.

So $1 = \frac{3}{\sigma}$, so that $\sigma^2 = 9$.

ANswer is **option A**

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21 votes -- Amar Vashishth (30.5k points)

7.6 Poisson Distribution(4) top

7.6.1 Poisson Distribution: GATE2007-IT-57 top

https://gateoverflow.in/3499

In a multi-user operating system on an average, 20 requests are made to use a particular resource per hour. The arrival of requests follows a Poisson distribution. The probability that either one, three or five requests are made in 45 minutes is given by :

A. $6.9 \times 10^6 \times e^{-20}$
B. $1.02 \times 10^6 \times e^{-20}$
C. $6.9 \times 10^3 \times e^{-20}$
D. $1.02 \times 10^3 \times e^{-20}$

gate2007-it probability poisson-distribution normal

Answer

7.6.2 Poisson Distribution: GATE2013-2 top

https://gateoverflow.in/62

Suppose p is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and p has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?

A. $\frac{8}{(2e^3)}$
B. $\frac{9}{(2e^3)}$
C. $\frac{17}{(2e^3)}$
D. $\frac{26}{(2e^3)}$

gate2013 probability poisson-distribution normal

Answer

7.6.3 Poisson Distribution: GATE2017-2-48 top

https://gateoverflow.in/118513

If a random variable X has a Poisson distribution with mean 5, then the expectation $E\left[(x+2)^2\right]$ equals ____.

gate2017-2 expectation poisson-distribution numerical-answers

Answer

7.6.4 Poisson Distribution: ISI2015-MMA-7 top

https://gateoverflow.in/129269

Suppose X is distributed as Poisson with mean λ. Then $E(1/(X+1))$ is

A. $\frac{e^{\lambda}-1}{\lambda}$

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B. $\frac{e^{\lambda}-1}{\lambda+1}$
C. $\frac{1-e^{-\lambda}}{\lambda}$
D. $\frac{1-e^{-\lambda}}{\lambda+1}$

isi2015 engineering-mathematics poisson-distribution

Answer

Answers: Poisson Distribution

7.6.1 Poisson Distribution: GATE2007-IT-57 top

https://gateoverflow.in/3499

Selected Answer

20 request in 1 hour.. so we can expect 15 request in 45 minutes...

So, $\lambda = 15$ (expected value)

poission distribution formula: $f(x,\lambda) = p(X=x) = \frac{e^{-\lambda} * \lambda^x}{x!}$

Therefore $Prob(1\ request) + Prob(3\ request) + Prob(5\ request)$ is:

= $p(1;15) + p(3;15) + p(5;15)$

= $6.9 \times 10^3 \times e^{-15}$

= $6.9 \times 10^3 \times e^{-15} = 6.9 \times 10^3 \times e^5 \times e^{-20} = 1.02 \times 106 \times e^{-20}$.

Answer is (B).

19 votes -- Vicky Bajoria (5k points)

7.6.2 Poisson Distribution: GATE2013-2 top

https://gateoverflow.in/62

Selected Answer

Poisson Probability Density Function (with mean λ) = $\frac{\lambda^k}{(e^{\lambda}k!)}$,

We have to sum the probability density function for $k = 0, 1$ and 2 and $\lambda = 3$(thus finding the cumulative mass function)

= $\left(\frac{1}{e^3}\right) + \left(\frac{3}{e^3}\right) + \left(\frac{9}{2e^3}\right)$

= $\frac{17}{(2e^3)}$

24 votes -- Arjun Suresh (348k points)

7.6.3 Poisson Distribution: GATE2017-2-48 top

https://gateoverflow.in/118513

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Selected Answer

In Poisson distribution :

Mean = Variance as n is large and p is small

And we know:

$$\text{Variance} = E\left(X^2\right) - [E(X)]^2$$

$$\Rightarrow E(X^2) = [E(X)]^2 + \text{Variance}$$

$$\Rightarrow E(X^2) = 5^2 + 5$$

$$\Rightarrow E(X^2) = 30$$

So by linearity of expectation,

$$E[(X+2)^2] = E[X^2 + 4X + 4]$$

$$= E(X^2) + 4E(X) + 4$$

$$= 30 + (4 \times 5) + 4$$

$$= 54$$

Hence 54 should be the right answer..

40 votes -- HABIB MOHAMMAD KHAN (98.7k points)

7.6.4 Poisson Distribution: ISI2015-MMA-7 top

https://gateoverflow.in/129269

Selected Answer

C. $\frac{1-e^{-\lambda}}{\lambda}$

$$E\left(\frac{1}{1+x}\right) = \sum_{k=0}^{\infty}\left(\frac{1}{1+k}\right) * \frac{\lambda^k * e^{-\lambda}}{k!}$$

$$= \frac{1}{\lambda} * \sum_{k=0}^{\infty} \frac{\lambda^{k+1} * e^{-\lambda}}{(k+1)!}$$

$$= \frac{e^{-\lambda}}{\lambda} * \sum_{k=1}^{\infty} \frac{\lambda^k}{k!}$$

$$= \frac{e^{-\lambda}}{\lambda} * \left(\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} - 1\right)$$

$$= \frac{e^{-\lambda}}{\lambda} * \left(e^{\lambda} - 1\right)$$

$$= \frac{1-e^{-\lambda}}{\lambda}$$

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11 votes -- Dhruv Patel (2k points)

7.7 Probability(59) top

7.7.1 Probability: CMI2012-A-07 top

https://gateoverflow.in/46536

A man has three cats. At least one is male. What is the probability that all three are male?

A. $\frac{1}{2}$
B. $\frac{1}{7}$
C. $\frac{1}{8}$
D. $\frac{3}{8}$

cmi2012 probability

Answer

7.7.2 Probability: CMI2014-A-02 top

https://gateoverflow.in/46965

The 12 houses on one side of a street are numbered with even numbers starting at 2 and going up to 24. A free newspaper is delivered on Monday to 3 different houses chosen at random from these 12. Find the probability that at least 2 of these newspapers are delivered to houses with numbers strictly greater than 14.

A. $\frac{7}{11}$
B. $\frac{5}{12}$
C. $\frac{4}{11}$
D. $\frac{5}{22}$

cmi2014 probability

Answer

7.7.3 Probability: CMI2015-A-07 top

https://gateoverflow.in/47043

You arrive at a snack bar and you can't decide whether to order a lime juice or a lassi. You decide to throw a fair 6-sided die to make the choice, as follows.

- If you throw 2 or 6 you order a lime juice.
- If you throw a 4, you order a lassi.
- Otherwise, you throw the die again and follow the same algorithm.

What is the probability that you end up ordering a lime juice?

A. $\frac{1}{3}$
B. $\frac{1}{2}$
C. $\frac{2}{3}$
D. $\frac{3}{4}$

cmi2015 probability

Answer

7.7.4 Probability: CMI2017-A-02 top

https://gateoverflow.in/203259

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An FM radio channel has a repository of 10 songs. Each day, the channel plays 3 distinct songs that are chosen randomly from the repository.

Mary decides to tune in to the radio channel on the weekend after her exams. What is the probability that no song gets repeated during these 2 days?

A. $\binom{10}{3}^{2} * \binom{10}{6}^{-1}$

B. $\binom{10}{6} * \binom{10}{3}^{-2}$

C. $\binom{10}{3} * \binom{7}{3} * \binom{10}{3}^{-2}$

D. $\binom{10}{3} * \binom{7}{3} * \binom{10}{6}^{-1}$

probability cmi2017

Answer

7.7.5 Probability: GATE-2014-2-1 top

https://gateoverflow.in/1953

The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by p. Then $100p$ = _____.

gate2014-2 probability numerical-answers normal

Answer

7.7.6 Probability: GATE1994-2.6 top

https://gateoverflow.in/2473

The probability of an event B is P_1. The probability that events A and B occur together is P_2 while the probability that A and $\bar{B}$ occur together is P_3. The probability of the event A in terms of P_1, P_2 and P_3 is _____

gate1994 probability normal descriptive

Answer

7.7.7 Probability: GATE1994-2.8 top

https://gateoverflow.in/2475

Let A, B and C be independent events which occur with probabilities $0.8, 0.5$ and 0.3 respectively. The probability of occurrence of at least one of the event is _____

gate1994 probability normal numerical-answers

Answer

7.7.8 Probability: GATE1995-1.18 top

https://gateoverflow.in/780

The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is:

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A. $\frac{16}{25}$

B. $\left(\frac{9}{10}\right)^3$

C. $\frac{27}{75}$

D. $\frac{18}{25}$

gate1995 probability normal

Answer

7.7.9 Probability: GATE1995-2.14 top

https://gateoverflow.in/2626

A bag contains 10 white balls and 15 black balls. Two balls are drawn in succession. The probability that one of them is black and the other is white is:

A. $\frac{2}{3}$

B. $\frac{4}{5}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

gate1995 probability normal

Answer

7.7.10 Probability: GATE1996-1.5 top

https://gateoverflow.in/2709

Two dice are thrown simultaneously. The probability that at least one of them will have 6 facing up is

A. $\frac{1}{36}$

B. $\frac{1}{3}$

C. $\frac{25}{36}$

D. $\frac{11}{36}$

gate1996 probability easy

Answer

7.7.11 Probability: GATE1996-2.7 top

https://gateoverflow.in/2736

The probability that top and bottom cards of a randomly shuffled deck are both aces is

A. $\frac{4}{52} \times \frac{4}{52}$

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B. $\frac{4}{52} \times \frac{3}{52}$

C. $\frac{4}{52} \times \frac{3}{51}$

D. $\frac{4}{52} \times \frac{4}{51}$

gate1996 probability easy

Answer

7.7.12 Probability: GATE1997-1.1 top

https://gateoverflow.in/2217

The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and tomorrow?

A. 0.3
B. 0.25
C. 0.35
D. 0.4

gate1997 probability easy

Answer

7.7.13 Probability: GATE1998-1.1 top

https://gateoverflow.in/1638

A die is rolled three times. The probability that exactly one odd number turns up among the three outcomes is

A. $\frac{1}{6}$
B. $\frac{3}{8}$
C. $\frac{1}{8}$
D. $\frac{1}{2}$

gate1998 probability easy

Answer

7.7.14 Probability: GATE1998-3a top

https://gateoverflow.in/1694

Two friends agree to meet at a park with the following conditions. Each will reach the park between 4:00 pm and 5:00 pm and will see if the other has already arrived. If not, they will wait for 10 minutes or the end of the hour whichever is earlier and leave. What is the probability that the two will not meet?

gate1998 probability normal numerical-answers

Answer

7.7.15 Probability: GATE1999-2.1 top

https://gateoverflow.in/1479

Consider two events E_1 and E_2 such that probability of E_1, $P_r[E_1] = \frac{1}{2}$, probability of E_2, $P_r[E_2] = \frac{1}{3}$, and probability of E_1, and E_2, $P_r[E_1 \; and \; E_2] = \frac{1}{5}$. Which of the following

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statements is/are true?

A. $P_r[E_1 \text{ or } E_2]$ is $\frac{2}{3}$

B. Events E_1 and E_2 are independent

C. Events E_1 and E_2 are not independent

D. $P_r[E_1 \mid E_2] = \frac{4}{5}$

gate1999 probability normal

Answer

7.7.16 Probability: GATE2000-2.2 top https://gateoverflow.in/649

E_1 and E_2 are events in a probability space satisfying the following constraints:

- $Pr(E_1) = Pr(E_2)$
- $Pr(E_1 \cup E_2) = 1$
- E_1 and E_2 are independent

The value of $Pr(E_1)$, the probability of the event E_1, is

A. 0

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. 1

gate2000 probability easy

Answer

7.7.17 Probability: GATE2001-2.4 top https://gateoverflow.in/722

Seven (distinct) car accidents occurred in a week. What is the probability that they all occurred on the same day?

A. $\frac{1}{7^7}$

B. $\frac{1}{7^6}$

C. $\frac{1}{2^7}$

D. $\frac{7}{2^7}$

gate2001 probability normal

Answer

7.7.18 Probability: GATE2002-2.16 top https://gateoverflow.in/846

Four fair coins are tossed simultaneously. The probability that at least one head and one tail turn up is

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A. $\frac{1}{16}$
B. $\frac{1}{8}$
C. $\frac{7}{8}$
D. $\frac{15}{16}$

gate2002 probability easy

Answer

7.7.19 Probability: GATE2003-60, ISRO2007-45 top https://gateoverflow.in/948

A program consists of two modules executed sequentially. Let $f_1(t)$ and $f_2(t)$ respectively denote the probability density functions of time taken to execute the two modules. The probability density function of the overall time taken to execute the program is given by

A. $f_1(t) + f_2(t)$

B. $\int_0^t f_1(x) f_2(x) dx$

C. $\int_0^t f_1(x) f_2(t-x) dx$

D. $\max\{f_1(t), f_2(t)\}$

gate2003 probability normal isro2007

Answer

7.7.20 Probability: GATE2004-25 top https://gateoverflow.in/1022

If a fair coin is tossed four times. What is the probability that two heads and two tails will result?

A. $\frac{3}{8}$

B. $\frac{1}{2}$

C. $\frac{5}{8}$

D. $\frac{3}{4}$

gate2004 probability easy

Answer

7.7.21 Probability: GATE2004-78 top https://gateoverflow.in/1072

Two n bit binary strings, S_1 and S_2 are chosen randomly with uniform probability. The probability that the Hamming distance between these strings (the number of bit positions where the two strings differ) is equal to d is

A. $\frac{{}^nC_d}{2^n}$

B. $\frac{{}^nC_d}{2^d}$

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C. $\frac{d}{2^n}$

D. $\frac{1}{2^d}$

gate2004 probability normal

Answer

7.7.22 Probability: GATE2004-IT-1 top

https://gateoverflow.in/3642

In a population of N families, 50% of the families have three children, 30% of the families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children?

A. $\left(\frac{3}{23}\right)$

B. $\left(\frac{6}{23}\right)$

C. $\left(\frac{3}{10}\right)$

D. $\left(\frac{3}{5}\right)$

gate2004-it probability normal

Answer

7.7.23 Probability: GATE2005-IT-1 top

https://gateoverflow.in/3745

A bag contains 10 blue marbles, 20 green marbles and 30 red marbles. A marble is drawn from the bag, its colour recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same colour is

A. $\left(\frac{1}{36}\right)$

B. $\left(\frac{1}{6}\right)$

C. $\left(\frac{1}{4}\right)$

D. $\left(\frac{1}{3}\right)$

gate2005-it probability normal

Answer

7.7.24 Probability: GATE2006-IT-1 top

https://gateoverflow.in/3538

In a certain town, the probability that it will rain in the afternoon is known to be 0.6. Moreover, meteorological data indicates that if the temperature at noon is less than or equal to $25^{\circ}C$, the probability that it will rain in the afternoon is 0.4. The temperature at noon is equally likely to be above $25^{\circ}C$, or at/below $25^{\circ}C$. What is the probability that it will rain in the afternoon on a day

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when the temperature at noon is above $25^{\circ}C$?

A. 0.4
B. 0.6
C. 0.8
D. 0.9

gate2006-it probability normal

Answer

7.7.25 Probability: GATE2007-24 top

https://gateoverflow.in/1222

Suppose we uniformly and randomly select a permutation from the $20!$ permutations of $1, 2, 3 \ldots, 20$. What is the probability that 2 appears at an earlier position than any other even number in the selected permutation?

A. $\left(\frac{1}{2}\right)$
B. $\left(\frac{1}{10}\right)$
C. $\left(\frac{9!}{20!}\right)$
D. None of these

gate2007 probability easy

Answer

7.7.26 Probability: GATE2007-IT-1 top

https://gateoverflow.in/3432

Suppose there are two coins. The first coin gives heads with probability $\frac{5}{8}$ when tossed, while the second coin gives heads with probability $\frac{1}{4}$. One of the two coins is picked up at random with equal probability and tossed. What is the probability of obtaining heads ?

A. $\left(\frac{7}{8}\right)$
B. $\left(\frac{1}{2}\right)$
C. $\left(\frac{7}{16}\right)$
D. $\left(\frac{5}{32}\right)$

gate2007-it probability normal

Answer

7.7.27 Probability: GATE2008-27 top

https://gateoverflow.in/425

Aishwarya studies either computer science or mathematics everyday. If she studies computer science on a day, then the probability that she studies mathematics the next day is 0.6. If she

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studies mathematics on a day, then the probability that she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probability that she studies computer science on Wednesday?

A. 0.24
B. 0.36
C. 0.4
D. 0.6

gate2008 probability normal

Answer

7.7.28 Probability: GATE2008-IT-2 top

https://gateoverflow.in/3224

A sample space has two events A and B such that probabilities $P(A \cap B) = \frac{1}{2}, P(A') = \frac{1}{3}, P(B') = \frac{1}{3}$. What is $P(A \cup B)$?

A. $\left(\frac{11}{12}\right)$
B. $\left(\frac{10}{12}\right)$
C. $\left(\frac{9}{12}\right)$
D. $\left(\frac{8}{12}\right)$

gate2008-it probability easy

Answer

7.7.29 Probability: GATE2008-IT-23 top

https://gateoverflow.in/3284

What is the probability that in a randomly chosen group of r people at least three people have the same birthday?

A. $1 - \frac{365 - 364 \ldots (365 - r + 1)}{365^r}$
B. $\frac{365 \cdot 364 \ldots (365 - r + 1)}{365^r} +^r C_1 \cdot 365 \cdot \frac{364.363 \ldots (364 - (r-2) + 1)}{364^{r+2}}$
C. $1 - \frac{365 \cdot 364 \ldots (365 - r + 1)}{365^r} -^r C_2 \cdot 365 \cdot \frac{364 \cdot 363 \ldots (364 - (r-2) + 1)}{364^{r-2}}$
D. $\frac{365 \cdot 364 \ldots (365 - r + 1)}{365^r}$

gate2008-it probability normal

Answer

7.7.30 Probability: GATE2009-21 top

https://gateoverflow.in/798

An unbalanced dice (with 6 faces, numbered from 1 to 6) is thrown. The probability that the face value is odd is 90% of the probability that the face value is even. The probability of getting any even numbered face is the same. If the probability that the face is even given that it is greater than 3 is

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0.75, which one of the following options is closest to the probability that the face value exceeds 3?

A. 0.453
B. 0.468
C. 0.485
D. 0.492

gate2009 probability normal

Answer

7.7.31 Probability: GATE2010-26 top

https://gateoverflow.in/1152

Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty?

A. $pq + (1-p)(1-q)$
B. $(1-q)p$
C. $(1-q)p$
D. pq

gate2010 probability easy

Answer

7.7.32 Probability: GATE2010-27 top

https://gateoverflow.in/1153

What is the probability that divisor of 10^{99} is a multiple of 10^{96}?

A. $\left(\frac{1}{625}\right)$
B. $\left(\frac{4}{625}\right)$
C. $\left(\frac{12}{625}\right)$
D. $\left(\frac{16}{625}\right)$

gate2010 probability normal

Answer

7.7.33 Probability: GATE2011-3 top

https://gateoverflow.in/2105

If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads?

A. $\left(\frac{1}{3}\right)$
B. $\left(\frac{1}{4}\right)$

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C. $\left(\frac{1}{2}\right)$

D. $\left(\frac{2}{3}\right)$

gate2011 probability easy

Answer

7.7.34 Probability: GATE2011-34 top

https://gateoverflow.in/2136

A deck of 5 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card?

A. $\left(\frac{1}{5}\right)$

B. $\left(\frac{4}{25}\right)$

C. $\left(\frac{1}{4}\right)$

D. $\left(\frac{2}{5}\right)$

gate2011 probability normal

Answer

7.7.35 Probability: GATE2012-XE top

https://gateoverflow.in/161893

Ten chocolates are distributed randomly among three children standing in a row. The probability that the first child receives exactly three chocolates is

A. $\frac{5\times 2^{11}}{3^9}$

B. $\frac{5\times 2^{10}}{3^9}$

C. $\frac{1}{3^9}$

D. $\frac{1}{3}$

gate2012-xe probability numerical-ability

Answer

7.7.36 Probability: GATE2014-1-48 top

https://gateoverflow.in/1927

Four fair six-sided dice are rolled. The probability that the sum of the results being 22 is $\frac{X}{1296}$. The value of X is _____

gate2014-1 probability numerical-answers normal

Answer

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7.7.37 Probability: GATE2014-2-48 top

https://gateoverflow.in/2014

The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is _____ .

gate2014-2 probability numerical-answers normal

Answer

7.7.38 Probability: GATE2014-3-48 top

https://gateoverflow.in/2082

Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If $P(.)$ denotes the probability of the event, the maximum value of $P(A)P(B)$ is_____.

gate2014-3 probability numerical-answers normal

Answer

7.7.39 Probability: GATE2016-1-04 top

https://gateoverflow.in/39661

A probability density function on the interval $[a, 1]$ is given by $1/x^2$ and outside this interval the value of the function is zero. The value of a is _____.

gate2016-1 probability normal numerical-ability numerical-answers

Answer

7.7.40 Probability: GATE2016-1-29 top

https://gateoverflow.in/39709

Consider the following experiment.

Step 1. Flip a fair coin twice.

Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.

Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is Y is (up to two decimal places)

gate2016-1 probability normal numerical-answers

Answer

7.7.41 Probability: GATE2018-15 top

https://gateoverflow.in/204089

Two people, P and Q, decide to independently roll two identical dice, each with 6 faces, numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is ____

gate2018 probability normal numerical-answers

Answer

7.7.42 Probability: ISI-2017-21 top

https://gateoverflow.in/209735

There are four machines and it is known that exactly two of them are faulty. They are tested one by one in a random order till both the faulty machines are identified. The probability that only two tests are required is

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A. $\left(\frac{1}{2}\right)$
B. $\left(\frac{1}{3}\right)$
C. $\left(\frac{1}{4}\right)$
D. $\left(\frac{1}{6}\right)$

engineering-mathematics isi2017 probability

Answer

7.7.43 Probability: ISI2017-MMA-27 top

https://gateoverflow.in/214012

A box contains 5 fair and 5 biased coins. Each biased coin has a probability of head $\frac{4}{5}$. A coin is drawn at random from the box and tossed. Then the second coin is drawn at random from the box (without replacing the first one). Given that the first coin has shown head, the conditional probability that the second coin is fair, is

A. $\frac{20}{39}$
B. $\frac{20}{37}$
C. $\frac{1}{2}$
D. $\frac{7}{13}$

isi2017 probability

Answer

7.7.44 Probability: TIFR2010-A-10 top

https://gateoverflow.in/26481

A drawer contains 2 Blue, 4 Red and 2 Yellow balls. No two balls have the same radius. If two balls are randomly selected from the drawer, what is the probability that they will be of the same colour?

A. $\left(\frac{2}{7}\right)$
B. $\left(\frac{2}{5}\right)$
C. $\left(\frac{3}{7}\right)$
D. $\left(\frac{1}{2}\right)$
E. $\left(\frac{3}{5}\right)$

tifr2010 probability

Answer

7.7.45 Probability: TIFR2010-A-13 top

https://gateoverflow.in/18392

A cube whose faces are colored is split into 1000 small cubes of equal size. The cubes thus obtained

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are mixed thoroughly. The probability that a cube drawn at random will have exactly two colored faces is:

a. 0.096
b. 0.12
c. 0.104
d. 0.24
e. None of the above

tifr2010 probability

Answer

7.7.46 Probability: TIFR2011-A-19 top

https://gateoverflow.in/26479

Three dice are rolled independently. What is the probability that the highest and the lowest value differ by 4?

A. $\left(\frac{1}{3}\right)$
B. $\left(\frac{1}{6}\right)$
C. $\left(\frac{1}{9}\right)$
D. $\left(\frac{5}{18}\right)$
E. $\left(\frac{2}{9}\right)$

tifr2011 probability

Answer

7.7.47 Probability: TIFR2011-A-9 top

https://gateoverflow.in/20020

You have to play three games with opponents A and B in a specified sequence. You win the series if you win two consecutive games. A is a stronger player than B. Which sequence maximizes your chance of winning the series?

a. AAB
b. ABA
c. BAB
d. BAA
e. All are the same.

tifr2011 probability

Answer

7.7.48 Probability: TIFR2012-A-19 top

https://gateoverflow.in/25044

An electric circuit between two terminals A and B is shown in the figure below, where the numbers indicate the probabilities of failure for the various links, which are all independent.

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What is the probability that A and B are connected?

a. $\left(\frac{6}{25}\right)$
b. $\left(\frac{379}{400}\right)$
c. $\left(\frac{1}{1200}\right)$
d. $\left(\frac{1199}{1200}\right)$
e. $\left(\frac{59}{60}\right)$

tifr2012 probability

Answer

7.7.49 Probability: TIFR2012-A-20 top

https://gateoverflow.in/25045

There are 1000 balls in a bag, of which 900 are black and 100 are white. I randomly draw 100 balls from the bag. What is the probability that the 101st ball will be black?

a. $9/10$
b. More than $9/10$ but less than 1.
c. Less than $9/10$ but more than 0.
d. 0
e. 1

tifr2012 probability

Answer

7.7.50 Probability: TIFR2012-A-9 top

https://gateoverflow.in/21008

The probability of throwing six perfect dices and getting six different faces is

a. $1 - \frac{6!}{6^6}$
b. $\frac{6!}{6^6}$
c. 6^{-6}
d. $1 - 6^{-6}$
e. None of the above.

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tifr2012 probability

Answer

7.7.51 Probability: TIFR2013-A-13 top

https://gateoverflow.in/25435

Doctors A and B perform surgery on patients in stages III and IV of a disease. Doctor A has performed a 100 surgeries (on 80 stage III and 20 stage IV patients) and 80 out of her 100 patients have survived (78 stage III and 2 stage IV survivors). Doctor B has also performed 100 surgeries (on 50 stage III and 50 stage IV patients). Her success rate is $\frac{600}{100}$ (49 stage III survivors and 11 stage IV survivors).A patient has been advised that she is equally likely to be suffering from stage III or stage IV of this disease. Which doctor would you recommend to this patient and why?

a. Doctor A since she has a higher success rate
b. Doctor A since she specializes in stage III patients and the success of surgery in stage IV patients is anyway too low
c. Doctor B since she has performed more stage IV surgeries
d. Doctor B since she appears to be more successful
e. There is not enough data since the choice depends on the stage of the disease the patient is suffering from.

tifr2013 probability

Answer

7.7.52 Probability: TIFR2013-A-14 top

https://gateoverflow.in/25437

An unbiased die is thrown n times. The probability that the product of numbers would be even is

a. $\frac{1}{(2n)}$
b. $\frac{1}{[(6n)!]}$
c. $1-6^{-n}$
d. 6^{-n}
e. None of the above.

tifr2013 probability

Answer

7.7.53 Probability: TIFR2013-A-4 top

https://gateoverflow.in/25386

A biased coin is tossed repeatedly. Assume that the outcomes of different tosses are independent and probability of heads is $\frac{2}{3}$ in each toss. What is the probability of obtaining an even number of heads in 5 tosses, zero being treated as an even number?

a. $\left(\frac{121}{243}\right)$
b. $\left(\frac{122}{243}\right)$
c. $\left(\frac{124}{243}\right)$

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d. $\left(\frac{125}{243}\right)$

e. $\left(\frac{128}{243}\right)$

tifr2013 probability

Answer

7.7.54 Probability: TIFR2013-A-6 top

https://gateoverflow.in/25390

You are lost in the National park of Kabrastan. The park population consists of tourists and Kabrastanis. Tourists comprise two-thirds of the population the park and give a correct answer to requests for directions with probability $\frac{3}{4}$. The air of Kabrastan has an amnesaic quality, however, and so the answers to repeated questions to tourists are independent, even if the question and the person are the same. If you ask a Kabrastani for directions, the answer is always wrong.

Suppose you ask a randomly chosen passer-by whether the exit from the park is East or West. The answer is East. You then ask the same person again, and the reply is again East. What is the probability of East being correct?

A. $\left(\frac{1}{4}\right)$

B. $\left(\frac{1}{3}\right)$

C. $\left(\frac{1}{2}\right)$

D. $\left(\frac{2}{3}\right)$

E. $\left(\frac{3}{4}\right)$

tifr2013 probability

Answer

7.7.55 Probability: TIFR2015-A-1 top

https://gateoverflow.in/29156

Consider a 6-sided die with all sides not necessarily equally likely such that probability of an even number is $P(\{2,4,6\}) = \frac{1}{2}$, probability of a multiple of 3 is $P(\{3,6\}) = 1/3$ and probability of 1 is $P(\{1\}) = \frac{1}{6}$. Given the above conditions, choose the strongest (most stringent) condition of the following that must always hold about $P(\{5\})$, the probability of 5.

A. $P(\{5\}) = \frac{1}{6}$

B. $P(\{5\}) \geq \frac{1}{6}$

C. $P(\{5\}) \leq \frac{1}{6}$

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D. $P(\{5\}) \leq \frac{1}{3}$
E. None of the above.

tifr2015 probability

Answer

7.7.56 Probability: TIFR2016-A-12 top

https://gateoverflow.in/73498

There are two rocks A and B, located close to each other, in a lily pond. There is a frog that jumps randomly between the two rocks at time $t = 0; 1; 2, \ldots$. The location of the frog is determined as follows. Initially, at time $t = 0$, the frog is at A. From then on, the frog's location is determined as follows. If the frog is at A at time t, then at time $t + 1$, with probability $2/3$ it jumps to B and with probability $1/3$, it jumps on the spot and stays at A. If the frog is at B at time t, then at time $t + 1$, with probability $1/2$ it jumps to A and with probability $1/2$ it jumps on the spot and stays at B. What is the probability that the frog is at B at time 3 (just after its third jump)?

A. $\frac{1}{2}$
B. $\frac{31}{54}$
C. $\frac{14}{27}$
D. $\frac{61}{108}$
E. $\frac{2}{3}$

tifr2016 probability

Answer

7.7.57 Probability: TIFR2017-A-9 top

https://gateoverflow.in/95042

Consider the *majority* function on three bits, $\mathbf{maj} : \{0,1\}^3 \rightarrow \{0,1\}$ where $\mathbf{maj}(x_1, x_2, x_3) = 1$ if and only if $x_1 + x_2 + x_3 \geq 2$. Let $p(\alpha)$ be the probability that the output is 1 when each input is set to 1 independently with probability α. What is $p'(\alpha) = \frac{d}{d\alpha}p(\alpha)$?

A. 3α
B. α^2
C. $6\alpha(1-\alpha)$
D. $3\alpha^2(1-\alpha)$
E. $6\alpha(1-\alpha) + \alpha^2$

tifr2017 probability

Answer

7.7.58 Probability: TIFR2018-A-13 top

https://gateoverflow.in/179371

A hacker knows that the password to the TIFR server is 10-letter string consisting of lower-case letters from the English alphabet. He guesses a set of 5 distinct 10-letter strings (with lower-case letters) uniformly at random. What is the probability that one of the guesses of the hacker is correct password?

A. $\frac{5}{(26)^{10}}$
B. $1 - \left(1 - \frac{1}{(26)^{10}}\right)^5$

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C. $1-\left\{\left(\frac{(26)^{10}-1}{(26)^{10}}\right)\left(\frac{(26)^{10}-2}{(26)^{10}}\right)\left(\frac{(26)^{10}-3}{(26)^{10}}\right)\left(\frac{(26)^{10}-4}{(26)^{10}}\right)\left(\frac{(26)^{10}-5}{(26)^{10}}\right)\right\}$

D. $\frac{1}{(26)^{10}}$

E. None of the above

tifr2018 probability

Answer

7.7.59 Probability: TIFR2018-A-15 top

https://gateoverflow.in/179366

Suppose a box contains 20 balls: each ball has a distinct number in $\{1,\ldots,20\}$ written on it. We pick 10 balls (without replacement) uniformly at random and throw them out of the box. Then we check if the ball with number "1" on it is present in the box. If it is present, then we throw it out of the box; else we pick a ball from the box uniformly at random and throw it out of the box.

What is the probability that the ball with number "2" on it is present in the box?

A. $9/20$
B. $9/19$
C. $1/2$
D. $10/19$
E. None of the above

tifr2018 probability

Answer

Answers: Probability

7.7.1 Probability: CMI2012-A-07 top

https://gateoverflow.in/46536

Selected Answer

Given that A man has three cats and At least one is male.

Possible combination for at least one cat is male=
$(M,F,F),(F,M,F),(F,F,M),(M,F,M),(M,M,F),(F,M,M),(M,M,M)$

Probability that all three are male=$\frac{1}{7}$

Hence, option**(B)**
$\frac{1}{7}$ is the correct Choice.

7 votes -- Leen Sharma (40k points)

7.7.2 Probability: CMI2014-A-02 top

https://gateoverflow.in/46965

Selected Answer

There are 12 houses on one side of a street are numbered with even numbers.

In which 5 houses are strictly greater than Number 14.

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And remaining 7 houses are numbered smaller than 14 (i.e. including 14)

No of way of choosing at least 2 of these newspapers are delivered to houses with numbers strictly greater than 14.

$^5C_3 +^5 C_2 \times^7 C_1 = 80$

Total way of choosing 3 houses$=^{12} C_3 = 220$

So Required probability= $\frac{80}{220} = \frac{4}{11}$.

8 votes -- Manoj Kumar (38.7k points)

7.7.3 Probability: CMI2015-A-07 top

https://gateoverflow.in/47043

Selected Answer

If we want lime juice then we need to throw $\{2$ or $6\}$. And If we don't get $\{2$ or $6\}$ in first go, then we need to throw $\{1$ or 3 or $5\}$ in first go and again we will try to get $\{2$ or $6\}$ in $2nd$ throw and so on.

Note - If we throw 4 in any go then we will end up getting lassi. But we want lime juice.

So, probability of getting lime juice $= \frac{2}{6} + \frac{3}{6} \times \frac{2}{6} + \left(\frac{3}{6}\right)^2 \times \frac{2}{6} + \ldots, +\infty.$

$= \frac{2}{6} + \frac{3}{6} \times \frac{2}{6}\{1 + \frac{3}{6} + \left(\frac{3}{6}\right)^2 + \ldots\}$

$= \frac{2}{6} + \frac{3}{6} \times \frac{2}{6} \times 2$

$= \frac{2}{3}.$

6 votes -- Dhananjay Kumar Sharma (25.7k points)

7.7.4 Probability: CMI2017-A-02 top

https://gateoverflow.in/203259

Mary listen to song on weekend $-> 2$ days

Selecting 3 songs from 10 can be done by $10C3$ ways

Now we select 3 songs from remaining 7 song in $7C3$

If we select this way we are sure that Mary will listen new song only

Probability can be given by $= \frac{^{10}C_3 * ^7C_3}{^{10}C_3 * ^{10}C_3}$

Which is option C.

3 votes -- Keval Malde (16.1k points)

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7.7.5 Probability: GATE-2014-2-1 top

https://gateoverflow.in/1953

Selected Answer

Initially P (working computer) $=\frac{4}{10}$, P (non-working computer) $=\frac{6}{10}$.

Case 1 : three computers are functional : There are 4 sub-cases $WWWN, WWNW, WNWW, NWWW,$ where W means working, N means non-working, but $P(WWWN) = P(WWNW) = P(WNWW) = P(NWWW),$

because for example $P(WWWN) = \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} \times \frac{6}{7} = \frac{144}{5040}$

In all other 3 sub-cases, we get same numerators and denominators (in different order), so total prob in this case is

$$\frac{4 \times 144}{5040} = \frac{576}{5040}.$$

Case 2 : all 4 are working

$$P(WWWW) = \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} = \frac{24}{5040}$$

P(atleast 3 are working) $=\frac{600}{5040}$

So $100 \times p = 11.90$

23 votes -- Happy Mittal (11.2k points)

all are working $+$ 3 Working and 1 not working

$$\Rightarrow \left(\frac{1}{{}^{10}C_4}\right) + \frac{\left({}^4C_3 \times {}^6C_1\right)}{{}^{10}C_4}$$

34 votes -- Saumya Bhattacharya (813 points)

7.7.6 Probability: GATE1994-2.6 top

https://gateoverflow.in/2473

Selected Answer

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$\implies P(A) = P_2 + P_3$$

13 votes -- Saumya Bhattacharya (813 points)

7.7.7 Probability: GATE1994-2.8 top

https://gateoverflow.in/2475

Selected Answer

$$P(A) = 0.8 \implies P(A') = 1 - 0.8 = 0.2$$

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$P(B) = 0.5 \implies P(B') = 1 - 0.5 = 0.5$

$P(C) = 0.3 \implies P(C') = 1 - 0.3 = 0.7$

P(No event will occur): $=0.2 * 0.5 * 0.7 = 0.07$

P(at least 1 event will occur): $1 - 0.07 = 0.93$

11 votes -- Manu Thakur (39.6k points)

7.7.8 Probability: GATE1995-1.18 top

https://gateoverflow.in/780

Selected Answer

First digit can be chosen in 8 ways from $1 - 9$ excluding 7

Second digit can be chosen in 9 ways from $0 - 9$ excluding 7 and similarly the third digit in 9 ways.

So, total no. of ways excluding $7 = 8 \times 9 \times 9$

Total no. of ways including $7 = 9 \times 10 \times 10$

So, answer $= \dfrac{(8 \times 9 \times 9)}{(9 \times 10 \times 10)} = \dfrac{18}{25}$

25 votes -- gatecse (18k points)

7.7.9 Probability: GATE1995-2.14 top

https://gateoverflow.in/2626

Selected Answer

Answer - C

probability of first ball white and second one black $= \left(\dfrac{10}{25}\right) \times \left(\dfrac{15}{24}\right)$

probability of first ball black and second one white$= \left(\dfrac{15}{25}\right) \times \left(\dfrac{10}{24}\right)$

probabilty = sum of above two probabilities $= \dfrac{1}{2}$.

14 votes -- Ankit Rokde (9k points)

7.7.10 Probability: GATE1996-1.5 top

https://gateoverflow.in/2709

Selected Answer

$1-$(no. 6 in both the dice)$= 1 - \left(\dfrac{5}{6} \times \dfrac{5}{6}\right) = \dfrac{11}{36}$.

12 votes -- Bhagirathi Nayak (14.1k points)

7.7.11 Probability: GATE1996-2.7 top

https://gateoverflow.in/2736

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Selected Answer

There are 52 cards including 4 aces so the probability must be $\frac{4}{52} \times \frac{3}{51}$.

13 votes -- Bhagirathi Nayak (14.1k points)

7.7.12 Probability: GATE1997-1.1 top

https://gateoverflow.in/2217

Selected Answer

Answer: D

P(it will rain today either today or tomorrow) = P(it will rain today) $+P$(it will rain tomorrow) $-P$(it will rain today and tomorrow)

So, $0.7 = 0.5 + 0.6 - P$(it will rain today and tomorrow)

$\Rightarrow P$ (it will rain today and tomorrow) =0.4

14 votes -- Rajarshi Sarkar (34.1k points)

7.7.13 Probability: GATE1998-1.1 top

https://gateoverflow.in/1638

Selected Answer

Answer - B

There are 6 possible outcomes for a die roll. Out of these 3 are even and 3 are odd. So, when we consider odd/even a die roll has only 2 possible outcomes. So, for three rolls of the die we have 8 possible outcomes.

Out of them only 3 will have exactly one odd number$\{OEE, EOE, EEO\}$

Probability = $3/8$.

14 votes -- Ankit Rokde (9k points)

7.7.14 Probability: GATE1998-3a top

https://gateoverflow.in/1694

Selected Answer

We are given that both will be reaching the park between $4:00$ and $5:00$.

Probability that one friend arrives between $4:00$ and $4:50 = \frac{5}{6}$
Probability that one friend arrives between $4:00$ and $4:50$ and meets the other arriving in the next 10 minutes $= \frac{5}{6} \times \frac{1}{6} \times 2 = \frac{10}{36} = \frac{5}{18}$.
(For any time of arrival between $4:00$ and $4:50$, we have a 10 minute interval possible for the second friend to arrive, and 2 cases as for choosing which friend arrives first)

Probability that both friend arrives between $4:50$ and $5:00 = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

This covers all possibility of a meet. So, required probability of non-meet

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$$= 1 - \left(\frac{5}{18} + \frac{1}{36}\right)$$
$$= 1 - \frac{11}{36}$$
$$= \frac{25}{36}.$$

13 votes -- Arjun Suresh (348k points)

7.7.15 Probability: GATE1999-2.1 top

https://gateoverflow.in/1479

Selected Answer

Answer - C

If events E_1 and E_2 are independent then $P[E_1 \text{ and } E_2]$ = $P[E_1] \times P[E_2]$ which is not the case here.

8 votes -- Ankit Rokde (9k points)

7.7.16 Probability: GATE2000-2.2 top

https://gateoverflow.in/649

Selected Answer

Answer -D

let probability of Event $E1 = x =$ prob of $E2$

prob$(E1$ union $E2)$ = prob$(E1)$ + prob$(E2)$ - prob$(E1$ intersect $E2)$

$1 = x + x$ -x^2 (prob$(E1$ intersect $E2) =$ prob$(E1)$ * prob$(E2)$ as events are independent)

$x = 1$

20 votes -- Ankit Rokde (9k points)

7.7.17 Probability: GATE2001-2.4 top

https://gateoverflow.in/722

Selected Answer

Answer - B

for every car accident we can pick a day in 7 ways

total number of ways in which accidents can be assigned to days $= 7^7$

probability of accidents happening on a particular day $= \frac{1}{7^7}$

we can choose a day in 7 ways.

hence probability $= \frac{7}{7^7} = \frac{1}{7^6}$.

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20 votes -- Ankit Rokde (9k points)

7.7.18 Probability: GATE2002-2.16 top

https://gateoverflow.in/846

Selected Answer

Answer - C

probability of getting all heads $= \frac{1}{16}$

probability of getting all tails $= \frac{1}{16}$

probability of getting at least one head and one tail $= 1 - \frac{1}{16} - \frac{1}{16} = \frac{7}{8}$.

15 votes -- Ankit Rokde (9k points)

7.7.19 Probability: GATE2003-60, ISRO2007-45 top

https://gateoverflow.in/948

Selected Answer

We assume the total time to be 't' units and $f1$ executes for 'x' units.

Since, $f1(t)$ and $f2(t)$ are executed sequentially.
So, $f2$ is executed for '$t-x$' units.

We apply convolution on the sum of two independent random variables to get probability density function of the overall time taken to execute the program.

$f1(x)*f2(t-x)$

17 votes -- Pranabesh Ghosh (4k points)

7.7.20 Probability: GATE2004-25 top

https://gateoverflow.in/1022

Selected Answer

Answer - A

Out of 4 times 2 times head should be present

No. of ways of selecting these 2 places $= {}^4C_2$

probability of getting 2 heads and 2 tails $= \left(\frac{1}{2^2}\right).\left(\frac{1}{2^2}\right)$

probability $= \frac{{}^4C_2}{2^4} = \frac{3}{8}$.

9 votes -- Ankit Rokde (9k points)

7.7.21 Probability: GATE2004-78 top

https://gateoverflow.in/1072

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Selected Answer

Answer - A

there n binary bits that can differ but only d should differ in this case,

ways of choosing these d bits $= {}^nC_d$

probability of d bits differ but, $n-d$ bits do not differ $= \left(\frac{1}{2}\right)^d . \left(\frac{1}{2}\right)^{(n-d)}$

no of ways $= \frac{{}^nC_d}{2^n}$.

21 votes -- Ankit Rokde (9k points)

7.7.22 Probability: GATE2004-IT-1 top

https://gateoverflow.in/3642

Selected Answer

Answer is B) $\frac{6}{23}$

Let N be the total number of families.

Number of children in a family of 3 children $= \left(\frac{N}{2}\right) \times 3$

Number of children in a family of 2 children $= \left(\frac{3N}{10}\right) \times 2$

Number of children in a family of 1 child $= \left(\frac{N}{5}\right) \times 1$

Probability $= \frac{\text{Favorable case}}{\text{Total cases}}$

$$= \frac{\left(\left(\frac{3}{10}\right) \times 2\right)}{\left(\left(\frac{1}{2}\right) \times 3 + \left(\frac{3}{10}\right) \times 2 + \frac{1}{5}\right)}$$

$$= \frac{6}{23}.$$

13 votes -- Prateeksha Keshari (2.2k points)

7.7.23 Probability: GATE2005-IT-1 top

https://gateoverflow.in/3745

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Selected Answer

No two marbles have the same color means, the final outcome of the three draws must be a permutation of

Blue, Green, Red

There are $3! = 6$ such permutations possible.

Now, probability of getting a Blue first, Green second and Red third $= \frac{10}{60} \times \frac{20}{60} \times \frac{30}{60}$

Required probability $= 6 \times \frac{10}{60} \times \frac{20}{60} \times \frac{30}{60} = \frac{1}{6}$.

14 votes -- Arjun Suresh (348k points)

7.7.24 Probability: GATE2006-IT-1 top

https://gateoverflow.in/3538

Selected Answer

Answer is C) 0.8

P(rain in afternoon) $= 0.5 \times P($rain when temp $\leq 25) + 0.5 \times P($ rain when temp $> 25)$
$0.6 = 0.5 \times 0.4 + 0.5 \times P($ rain when temp $> 25)$
so,
$P($ rain when temp > 25 $) = 0.8$

(Answer courtesy- Pradeep Pandey sir - https://gateetude.wordpress.com/category/gate-computer-science/information-technology-solutions/)

19 votes -- Prateeksha Keshari (2.2k points)

7.7.25 Probability: GATE2007-24 top

https://gateoverflow.in/1222

Selected Answer

There are 10 even numbers $(2, 4 \ldots 20)$ possible as the one in the earliest position and all of these are equally likely. So, the probability of 2 becoming the earliest is simply $\frac{1}{10}$.

35 votes -- Arjun Suresh (348k points)

7.7.26 Probability: GATE2007-IT-1 top

https://gateoverflow.in/3432

Selected Answer

Answer is C) $\frac{7}{16}$

Probability of obtaining head
$= \text{Probability of picking first coin} \times \text{Probability of getting head on first coin}$

$+\text{Probability of picking second coin} \times \text{Probability of getting head on second coin}$

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$$= \left(\frac{1}{2} \times \frac{5}{8}\right) + \left(\frac{1}{2} \times \frac{1}{4}\right) = \frac{7}{16}.$$

14 votes -- Prateeksha Keshari (2.2k points)

7.7.27 Probability: GATE2008-27 top

https://gateoverflow.in/425

Selected Answer

on **Wed**nesday we want **cs**

required probability = $0.6 \times 0.4 + 0.4 \times 0.4 = 0.4$

answer = **option C**

26 votes -- Amar Vashishth (30.5k points)

7.7.28 Probability: GATE2008-IT-2 top

https://gateoverflow.in/3224

Selected Answer

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= (1 - P(A')) + (1 - P(B')) - P(A \cap B)$$

$$= \left(1 - \frac{1}{3}\right) + \left(1 - \frac{1}{3}\right) - \frac{1}{2}$$

$$= \frac{4}{3} - \frac{1}{2}$$

$$= \frac{5}{6}$$

$$= \frac{10}{12}.$$

13 votes -- Arjun Suresh (348k points)

7.7.29 Probability: GATE2008-IT-23 top

https://gateoverflow.in/3284

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Selected Answer

Answer: C

Case 1: Among 365 people If all r have birthdays on different days.
Then first one can have his birthday in 365 ways. Second one can have in 364 ways, and so on up to r^{th} person, who can have his birthday in $(365-(r-1))$ ways.

Case 2: Among 365 people If exactly 2 persons have birthdays on same day.
Then we can consider these 2 persons as single entity and Selecting these 2 persons in $C(r,2)$ ways.Then these two (assumed as first person) can have their birthday in 365 ways.
Third person can have in 364 ways, and so on up to r^{th} person, who can have his birthday in $(365-(r-2))$ ways (since 1 person is less now).

As we know,
$P_{\text{at least 3 with same birthday}} = 1 - [P_{\text{no two having same birthday}} + P_{\text{exactly 2 having same birthday}}$
Hence,
$P_{\text{at least 3 with same birthday}} \Rightarrow$

$$= 1 - \left[\frac{365.364\cdots(365-(r-1))}{365^r} + C(r,2).365.\frac{364.363\ldots(365-(r-2))}{364^{r-2}}\right]$$

$$= 1 - \frac{365.364\cdots(365-(r-1))}{365^r} - C(r,2).365.\frac{364.363\cdots(365-(r-2))}{364^{r-2}}$$

21 votes -- Rajarshi Sarkar (34.1k points)

7.7.30 Probability: GATE2009-21 top

https://gateoverflow.in/798

Selected Answer

$P(\{1,3,5\}) = 0.9P(\{2,4,6\})$ and their sum must be 1. So,

$P(\{1,3,5\}) = \frac{0.9}{1.9} = 0.4736$ and

$P(\{2,4,6\}) = \frac{1}{1.9} = 0.5263$

Given that probability of getting 2 or 4 or 6 is same.

So, $P(2) = P(4) = P(6) = \frac{0.5263}{3} = 0.1754$

$P(\{4,6\}) \mid x > 3) = 0.75$

$\Rightarrow P(5 \mid x > 3) = 0.25$

$\Rightarrow P(5) = \frac{1}{3}(P(4) + P(6)) \because x > 3$

$\Rightarrow x \in \{4,5,6\}$

$\Rightarrow P(5) = \frac{2.P(4)}{3} \because P(4) = P(6).$

So, $P(x > 3) = P(4) + P(5) + P(6) = \frac{8}{3} \times 0.1754 = 0.468$

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31 votes -- Arjun Suresh (348k points)

7.7.31 Probability: GATE2010-26 top

https://gateoverflow.in/1152

Selected Answer

answer = **option A**

in image below the ticks shows those branch where the result is declared as faulty.

so required probability = sum of those two branches $= pq + (1-p)(1-q)$

21 votes -- Amar Vashishth (30.5k points)

7.7.32 Probability: GATE2010-27 top

https://gateoverflow.in/1153

Selected Answer

Prime factorization of $10 = 2 \times 5$.
So, $10^{99} = 2^{99} \times 5^{99}$ and

No. of possible factors for $10^{99} =$ No. of ways in which prime factors can be combined
$= 100 \times 100$ (1 extra possibility for each prime number as prime factor raised to 0 is also possible for a factor)

$10^{99} = 10^{96} \times 1000$
So, no. of multiples of 10^{96} which divides $10^{99} =$ No. of possible factors of 1000

$= 4 \times 4 \left(\because 1000 = 2^3 \times 5^3\right)$ (See below)

$= 16$

So, required probability $= \frac{16}{10000}$

$= \frac{1}{625}$

How number of possible factors of 1000 = 16?

Here we can prime factorize 1000 as $2^3 \times 5^3$. Now, any factor of 1000 will be some combination of these prime factors. For 2, a factor has 4 options - $2^0, 2^1, 2^2$ or 2^4. Similarly 4 options for 5 also. This is true for any number n, if n can be prime factorized as $a_1^{m_1}.a_2^{m_2}.....a_n^{m_n}$, number of factors of $n = (m_1+1) \times (m_2+1) \times \cdots \times (m_n+1)$,

the extra one in each factor term coming for power being 0.

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48 votes -- Arjun Suresh (348k points)

7.7.33 Probability: GATE2011-3 top

https://gateoverflow.in/2105

Selected Answer

Answer - A

prob(at least one head) = $\frac{3}{4}$

prob(both heads) = $\frac{1}{4}$

using bayes' theorem = $\frac{\left(\frac{1}{4}\right)}{\left(\frac{3}{4}\right)} = \frac{1}{3}.$

19 votes -- Ankit Rokde (9k points)

7.7.34 Probability: GATE2011-34 top

https://gateoverflow.in/2136

Selected Answer

The number on the first card needs to be **<u>One</u> higher** than that on the second card, so possibilities are :

1st card	2nd card
1	–
2	1
3	2
4	3
5	4
–	5

$\text{Total : 4 possibilities}$

Total possible ways of picking up the cards $= 5 \times 4 = 20$

Thus, the required Probability $= \frac{\text{favorable ways}}{\text{total possible ways}} = \frac{4}{20} = \frac{1}{5}$

Option A is correct

31 votes -- Amar Vashishth (30.5k points)

7.7.35 Probability: GATE2012-XE top

https://gateoverflow.in/161893

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Selected Answer

Let the three children be A, B and C where A is the first child

Calculating the total number of case

For each chocolate (assuming chocolates are distinguishable) we have 3 choices, give it to A or B or to C.

So for ten chocolates we have total no. of choices $= \underbrace{3 \times 3 \times 3 \times \ldots \times 3}_{\text{ten times}} = 3^{10}$.

Calculating the number of favorable cases

Firstly we have to select three chocolates out of ten that have to be given to A and number of ways for this $= {}^{10}C_3$.

For remaining 7 chocolates, for each we have two choices i,e either to give it to B or to C.

So, no of favorable cases $= {}^{10}C_3 \times 2^7$.

Our required probability $= \frac{\text{No. of favorable cases}}{\text{Total no. of cases}}$

$= \frac{{}^{10}C_3 \times 2^7}{3^{10}}$

$= \frac{5 \times 2^{10}}{3^9}$

i.e., option B

5 votes -- Akshay Bhatia (3.1k points)

7.7.36 Probability: GATE2014-1-48 top

https://gateoverflow.in/1927

Selected Answer

There are only two possible sets whose elements sum to 22 : $\{6,6,6,4\},\{6,6,5,5\}$

Number of permutations for 1st set : $\frac{4!}{3!} = 4$

Number of permutations for $2nd$ set : $\frac{4!}{(2! * 2!)} = 6$

So total number of ways to sum $22 = 10$

So $X = 10$.

33 votes -- Happy Mittal (11.2k points)

7.7.37 Probability: GATE2014-2-48 top

https://gateoverflow.in/2014

Selected Answer

Answer - 0.26

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no of integers divisible by $2 = 50$

no of integers divisible by $3 = 33$

no of integers divisible by $5 = 20$

no of integers divisible by 2 and 3 = 16

no of integers divisible by 2 and 5 = 10
no of integers divisible by 3 and 5 = 6

no of integers divisible by 2 and 3 and 5 = 3

total numbers divisible by 2 or 3 or $5 = 50$ + 33 + 20 -16 -10 - 6 + 3 = 74

total number not divisible by 2 or 3 or 5 = 26

probability = 0.26 [EDIT]

19 votes -- Ankit Rokde (9k points)

7.7.38 Probability: GATE2014-3-48 top

https://gateoverflow.in/2082

Selected Answer

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$P(A) + P(B) = 1$, since both are mutually exclusive and $A \cup B = S$.
When sum is a constant, product of two numbers becomes maximum when they are equal.

So,$P(A) = P(B) = \frac{1}{2}$.

15 votes -- Saumya Bhattacharya (813 points)

7.7.39 Probability: GATE2016-1-04 top

https://gateoverflow.in/39661

Selected Answer

We know that the sum of all the probabilities is 1

Therefore, on integrating $\frac{1}{x^2}$ with limits a to 1, the result should be 1.

Hence, $\int_a^1 \frac{1}{x^2} dx = 1$

$$\left[-\frac{1}{x}\right]_a^1 = 1$$

$$-1 + \frac{1}{a} = 1$$

Hence, **a = 0.5**

27 votes -- ryan sequeira (3.4k points)

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7.7.40 Probability: GATE2016-1-29 top

https://gateoverflow.in/39709

Selected Answer

Answer is 0.33

1st time it is $0.25\left(\frac{1}{4}\right)$, when tail tail comes, entire process gets repeated, so next time probability of Y to happen is $0.25 \times 0.25\left(\frac{1}{4} \times \frac{1}{4}\right)$, likewise it goes on as infinite GP

Sum of infinite GP $= \frac{a}{(1-r)}$

here, $a = \frac{1}{4}$ and $r = \frac{1}{4}$

so answer becomes $\frac{1}{3}$ i.e 0.33

26 votes -- Sreyas S (1.9k points)

7.7.41 Probability: GATE2018-15 top

https://gateoverflow.in/204089

Selected Answer

One of them win in the third trial i.e. first two trial would be Tie and third should not be Tie.

Probability of Tie $= \frac{6}{36} = \frac{1}{6}$
Probability of NO Tie $= 1 - \frac{1}{6} = \frac{5}{6}$

Winning in the third Tie
$= (First\ Tie) * (Second\ Tie) * (No\ Tie) = \frac{1}{6} * \frac{1}{6} * \frac{5}{6} = \frac{5}{216} = 0.023$

11 votes -- Digvijay (54.8k points)

7.7.42 Probability: ISI-2017-21 top

https://gateoverflow.in/209735

Selected Answer

There will be two ways -

1. When the first test & the second test both identified the faulty machines

OR

2. When the first test & the second test both identified the non-faulty machines (because then we know that the remaining two machines are faulty)

1. Probability of the first machine tested is faulty = $\frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}} = \frac{2}{4}$
$[\because \text{as there are 2 faulty machines out of 4 machines}]$

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Now, there are 3 machines left as 1 faulty machine has already been identified.

Probability (second machine tested is faulty in the second test) =
$$\frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}} = \frac{1}{3}$$

$$\text{Probability that only two test is needed} = \text{Probability of the first machine tested is faulty} \times \text{Probability of the second machine tested is faulty in the second test}$$

$$= \frac{2}{4} \times \frac{1}{3}$$

$$= \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{1}{6}$$

2. Probability of the first machine tested is **Not-faulty** =
$$\frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}} = \frac{2}{4}$$
$$[\because \text{as there are 2 Non-faulty machines out of 4 machines}]$$

Now, there are 3 machines left as 1 Non-faulty machine has already been identified.

Probability (second machine tested is **Not-faulty** in the second test) =
$$\frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}} = \frac{1}{3}$$

$$\text{Probability that only two test is needed} = \text{Probability of the first machine tested is Not-faulty} \times \text{Probability of the second machine tested is Not-faulty in the second test}$$

$$= \frac{2}{4} \times \frac{1}{3}$$

$$= \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{1}{6}$$

Now, Required Probability = Case 1) OR Case 2)

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

6 votes -- Sukanya Das (11.5k points)

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7.7.43 Probability: ISI2017-MMA-27 top

https://gateoverflow.in/214012

Selected Answer

prob of head from fair coin= (5*1/2)/((5*1/2)+(5*4/5))=5/13

prob of head from biased coin= (5*4/5)/((5*1/2)+(5*4/5))=8/13

now probability of second coin to be fair is given as = (head from baised coin*probability of fair coin if biased coin was selected at first)+(head from fair coin*probability of fair coin if fair coin was selected at first)= 8/13*5/9 + 5/13*4/9 = 20/39.

Ans-A

2 votes -- Tarun kushwaha (2k points)

7.7.44 Probability: TIFR2010-A-10 top

https://gateoverflow.in/26481

Selected Answer

If any 2 balls selected from 8 balls then we can choose ${}^8C_2$ ways=28 ways

If selected 2 balls are same color then${}^2C_2 + {}^4C_2 + {}^2C_2$ ways=$1+6+1$ ways=8 ways

So, required probability=$\frac{8}{28} = \frac{2}{7}$

Answerps (A)

8 votes -- srestha (86.6k points)

7.7.45 Probability: TIFR2010-A-13 top

https://gateoverflow.in/18392

Selected Answer

0.096 should be the correct answer, i.e. option $a)$

Suppose that the side of larger cube is $10\ m$ then volume of the larger cube will be $10 \times 10 \times 10 = 1000\ m^3$.

After dividing the cube into 1000 equal sized small cubes, volume of each smaller cube will be

$$\left(\frac{10 \times 10 \times 10}{1000}\right) m^3 = 1\, m^3.$$

So the sides of the each of the smaller cube will be $1\ m$, which is 10 times less than the side length of larger cube.

So, each EDGE of the larger larger cube will contain 10 smaller cube edges.

each FACE of the larger cube will contain $10 \times 10 = 100$ smaller cube faces.

each CORNER of the larger cube will contain 1 smaller cube corner.

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Position of each of the smaller cube can be as follows:

$A)$ It can be in the corners of the larger cube, In this case it would have three of its faces colored.

There are total

$8(\text{number of corners}) \times 1(\text{number of smaller cubes per corner}) = 8 \text{ such cubes.}$

$B)$ It can be in the edges of the larger cube, In this case it would have two of its faces colored.

There are total

$12(\text{number of edges}) \times 8(\text{number of smaller cubes per edge excluding corner cubes of the ed}$

$C)$ It can be on the face of the larger cube but not in the edges of a face, in this case it would have one face colored.

There are total

$6(\text{number of faces}) \times 64(\text{number of smaller cubes per face excluding the edge \& corner cube}$

$D)$ It can be inside the core of the larger cube, in this case it will be uncolored.

There will be $512 = (1000 - (384 + 96 + 8))$ cubes.

Now since there are 96 cubes out of 1000 which have 2 colored faces, so required probability = $\frac{96}{1000} = 0.096$

Now, since total number of edges in the larger cube $= 12,$

So, total number of smaller cubes with two colored faces $= 12 \times 8 = 96.$

9 votes -- Anurag Pandey (13.6k points)

7.7.46 Probability: TIFR2011-A-19 top

https://gateoverflow.in/26479

Selected Answer

suppose three dices are there A,B,C, three are rolled independently, see we get 4 difference between max and min when we get either (1,5) or (2,6) in any of the two dices. okay?

1st case: we get (1,5), 1 and 5 can be the result of any of the two dice in 3C2=3 ways, either (A-B) or (B-C) and (C-A) and also they can be permutated between them,so 3C2*2! ways possible, now for each way 3rd result can come either 1 or 2 or 3 or 4 or 5(6 cant be possible as min:1 and max:5) But we cant do 3C2*2!*5 because then (1,1,5), (1,5,1),(5,1,1),(1,5,5),(5,1,5),(5,5,1) all these 6 ways are counted two times.So we do like below:
When 3rd result is 1, no of ways=3!/2!=3 because its no of permutation of 1,1 and 5

When 3rd result is 5,no of ways=3!/2!=3 because its no of permutation of 1,5 and 5

when 3rd result is 2 or 3 or 4, 3C2*2!*3=18 ways possible.so totally 18+3+3=24

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2nd case: we get (2,6), 2 and 6 can be the result of any of the two dice in 3C2=3 ways, either (A-B) or (B-C) and (C-A) and also they can permutate between them,so 3C2*2! ways possible, now for each way 3rd dice result can come either 2 or 3 or 4 or 5 or 6(min:2 and max:6) But we cant do 3C2*2!*5, So we do like below:

Similarly as shown above, when 3rd result is 2 or 6,no of ways=2*3!/2=6 ways possible,when 3rd result is 3 or 4 or 5,3C2*2!*3 =18 ways possible.So totally 18+6=24 ways possible.

Finally total=24+24=48 ways possible
sample space= 6^3=216
so probability= 48/216=2/9

5 votes -- aritra nayak (607 points)

7.7.47 Probability: TIFR2011-A-9 top

https://gateoverflow.in/20020

Selected Answer

Let the three games in a series be called G_1, G_2 and G_3 respectively, and the probability of winning the game x be denoted as $P(x)$

You can win the series if and only if: You win G_2 and you win atleast one of $\{G_1, G_3\}$.

$$\boxed{P\begin{pmatrix}\text{winning}\\ \text{the series}\\ G_1G_2G_3\end{pmatrix} = P(G_2) \times \Big(\big(P(G_1) + P(G_3) - P(G_1) \times P(G_3)\big)\Big)}$$

Let the probability of winning against player A be a and the probability of winning against player B be b.

Then, $a < b. (A \text{ is a stronger player than} B, \text{ so probability of winning against } A$

$\text{is smaller compared to } B)$

Let $P(xyz)$ be the probability of winning the series in which the games played are against x, y and z in order.

a. $P(AAB) = a(a + b - ab) = a^2 + ab - a^2b$
b. $P(ABA) = b(a + a - aa) = 2ab - a^2b$
c. $P(BAB) = a(b + b - b^2) = 2ab - ab^2$
d. $P(BAA) = a(b + a - ba) = a^2 + ab - a^2b$

We can see that **not all probabilities are equal, so option E is not correct**.

We can also see that options **A and D result in the same value, so they are not correct either**.

Comparing option B and option C.

Since $a < b$ and $a, b \geq 0$, we have that $2ab - a^2b > 2ab - ab^2$

Hence, option B is the correct answer.

15 votes -- Pragy Agarwal (20.6k points)

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7.7.48 Probability: TIFR2012-A-19 top

https://gateoverflow.in/25044

Selected Answer

Lets define the following events:

P : Uppermost link is working.

Q: Middle link is working.

R: Lowermost link is working.

W : Terminals A & B are connected.

From the given information, we can calculate the probabilities of events P, Q and R as follow:

(as I already consumed letter (capital) P denoting an event so here by I will use letter (small) p to denote probabilities,

e.g. $p(X)$ denotes probability of happening of event X).

$$p(P) = \frac{4}{5} \times \frac{4}{5} = \frac{16}{25},$$

$$\Rightarrow p\left(\widehat{P}\right) = 1 - \frac{16}{25} = \frac{9}{25},$$

$$p(Q) = \frac{2}{3},$$

$$\Rightarrow p\left(\widehat{Q}\right) = \frac{1}{3},$$

$$p(R) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16},$$

$$\Rightarrow p\left(\widehat{R}\right) = 1 - \frac{9}{16} = \frac{7}{16}.$$

The situation here can be represented using the following Venn Diagram:

Here Red region denotes the event $\widehat{W}$ in which $A\&B$ are disconnected.

Different gradients of Green colour represent the W in which terminals $A\&B$ are connected.

We have to find $p(W)$

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Events W and $\widehat{W}$ are mutually exclusive & totally exhaustive,

since either terminals A and B will be connected or they will be disconnected.

So we can write $p(W)$ in terms of $p\left(\widehat{W}\right)$ as follow:

$$p(W) = 1 - p\left(\widehat{W}\right).$$

Also it can be seen that

$p\left(\widehat{W}\right) = p\left(\widehat{P}\right) \cap \left(\widehat{Q}\right) \cap \left(\widehat{R}\right)$ where $\widehat{P}$, $\widehat{Q}$ and $\widehat{R}$ are independent events.

That is terminals A & B will be disconnected only when all of the links will fail simultaneously.

Using independence we can write,

$$p\left(\widehat{W}\right) = p(W) = p\left(\widehat{P}\right) \times \left(\widehat{Q}\right) \times \left(\widehat{R}\right),$$

$$\Rightarrow p\left(\widehat{W}\right) = \frac{9}{25} \times \frac{1}{3} \times \frac{7}{16} = \frac{21}{400}$$

$$\Rightarrow p(W) = 1 - \frac{21}{400} = \frac{379}{400}.$$

16 votes -- Anurag Pandey (13.6k points)

7.7.49 Probability: TIFR2012-A-20 top

https://gateoverflow.in/25045

Selected Answer

Here we are having a **total** of 1000 Balls**,** out of which we firstly **draw** 100 **balls** , and **then** 101st **ball..**

Firstly we have to **find expected number of white and black balls in** drawn 100 **balls** , as both can occur in 100 balls..

We have a situation like this:

Expected number of **white balls** $= n \times \frac{W}{N} = 100 \times \left(\frac{100}{1000}\right) = 10$

Expected number of **black balls** $= n \times \frac{B}{N} = 100 \times \left(\frac{900}{1000}\right) = 90$

So, **we have drawn** 100 **balls(**90 **black,** 10 **white)**

Left balls = (810 Black , 90 White) = 900 **total**

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Now,

probability for 101st **ball to be black** = $\frac{810}{900} = \frac{9}{10}$.

So, option (A) is Correct

13 votes -- Himanshu Agarwal (15.3k points)

7.7.50 Probability: TIFR2012-A-9 top

https://gateoverflow.in/21008

Selected Answer

$B)$ for all the 6 different faces $1, 2, 3, 4, 5, 6$

the probability is $\frac{1}{6^6}$ and then for $6!$ different permutations : $\frac{6!}{6^6}$.

6 votes -- Shaun Patel (6.9k points)

7.7.51 Probability: TIFR2013-A-13 top

https://gateoverflow.in/25435

Selected Answer

Answer will be $d)$

As, $\%$ of Doctor A successful for stage III $= \frac{78}{80} \times 100 = 97.5$

For stage IV$= \frac{2}{20} \times 100 = 10$

As, $\%$ of Doctor B successful for stage III $= \frac{49}{50} \times 100 = 98$

For stage IV$= \frac{11}{50} \times 100 = 22$

In both cases Doc B is more successful.

7 votes -- srestha (86.6k points)

7.7.52 Probability: TIFR2013-A-14 top

https://gateoverflow.in/25437

Selected Answer

Even number $= 2, 4, 6$
odd number $= 1, 3, 5$
Product will come even when even one time even number comes
odd product will be if every time odd comes

so, $P(Even) = 1 - P(odd)$

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$= 1 - {}^nC_n \times \left(\frac{3}{6}\right)^0 \times \left(\frac{3}{6}\right)^n$

$= 1 - \left(\frac{1}{2}\right)^n$

So i think option E.

14 votes -- Umang Raman (15.7k points)

7.7.53 Probability: TIFR2013-A-4 top

https://gateoverflow.in/25386

Selected Answer

probability of obtaining an even number of heads in 5 tosses, zero being treated as an even number

number of event = 0 head or 2 head or 4 head

Probability of head $= \frac{2}{3}$

Probability of tail = $\frac{1}{3}$

Probability = ${}^5C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^5 + {}^5C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3 + {}^5C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^1$

$= \frac{121}{243}.$

Option A

10 votes -- Umang Raman (15.7k points)

7.7.54 Probability: TIFR2013-A-6 top

https://gateoverflow.in/25390

Selected Answer

Ans should be $\frac{1}{2}$

Tourists are $\frac{2}{3}$ of population of the park

Among them probability of correct answer is $\frac{3}{4}$

so, $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}.$

3 votes -- srestha (86.6k points)

7.7.55 Probability: TIFR2015-A-1 top

https://gateoverflow.in/29156

Selected Answer

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$P\{3,5\} = 1 - P\{2,4,6\} - P\{1\} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

Can $P3 = 0$? then $P\{6\} = \frac{1}{3}$ and $P\{2,4\} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.

And

$P\{5\} = \frac{1}{3}$. Possible.

So, option D.

6 votes -- Arjun Suresh (348k points)

7.7.56 Probability: TIFR2016-A-12 top

https://gateoverflow.in/73498

Selected Answer

In such cases where the events are mutually exclusive and collectively exhaustive and consists of a sequence of such events , then it is always preferrable to use tree diagram to find the final outcome.The tree diagram is shown as :

Hence we have P(frog is at B at t = 3) = $31/54$..I hope I have cleared your doubt.

2 votes -- HABIB MOHAMMAD KHAN (98.7k points)

7.7.57 Probability: TIFR2017-A-9 top

https://gateoverflow.in/95042

Selected Answer

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$$\Rightarrow [\mathbf{maj}\{x_i\} = 1] \Leftrightarrow \left(\left[\sum_{i=1}^{3} x_i\right] \geq 2\right)$$

$$\Rightarrow Prob\left(\left[\sum_{i=1}^{3} x_i\right] \geq 2\right) = Prob\left(\left[\sum_{i=1}^{3} x_i\right] = 2\right) + Prob\left(\left[\sum_{i=1}^{3} x_i\right] = 3\right)$$

$$\Rightarrow Prob\left(\left[\sum_{i=1}^{3} x_i\right] \geq 2\right) = \binom{3}{2} * \{\alpha.\alpha.(1-\alpha)\} + \alpha^3$$

$$\Rightarrow Prob\left(\left[\sum_{i=1}^{3} x_i\right] \geq 2\right) = 3.\alpha^2(1-\alpha) + \alpha^3$$

$$\Rightarrow Prob\left(\left[\sum_{i=1}^{3} x_i\right] \geq 2\right) = 3.\alpha^2 - 2.\alpha^3$$

$$\Rightarrow P^{'}(\alpha) = \frac{\mathrm{d}}{\mathrm{d}\alpha}\left[3.\alpha^2 - 2.\alpha^3\right]$$

$$\Rightarrow P^{'}(\alpha) = 6.\alpha(1-\alpha)$$

12 votes -- Debashish Deka (56.6k points)

7.7.58 Probability: TIFR2018-A-13 top

https://gateoverflow.in/179371

Total number of 5 string sets is $\binom{26^{10}}{5}$

Out of which, total $\binom{26^{10}-1}{4}$ sets will contain password.

So required probability that his guessed set contains the password will be $\frac{\binom{26^{10}-1}{4}}{\binom{26^{10}}{5}}$.

And if you simplify $\frac{\binom{n-1}{r-1}}{\binom{n}{r}}$, it will be $\frac{r}{n} \rightarrow \frac{5}{26^{10}}$.

$\rightarrow$ So, Option A

6 votes -- Shivansh Gupta (2.4k points)

7.7.59 Probability: TIFR2018-A-15 top

https://gateoverflow.in/179366

Required probability has 2 parts.

a) When both 1 and 2 remain after 10 are chosen. Later 1 will be chosen with probability 1 and hence, 2 will remain.

b) When 1 is thrown along with some other 9, and 2 remains. Later we have to multiply with probability that 2 will not be thrown.

P(2 remains after all the events) = P(1 and 2 are not thrown out) + P(1 is thrown out , 2 remains) * P(from remaining 10 2 is not chosen)

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$\rightarrow \frac{{}^{18}_{10}C + {}^{18}_{9}C * \frac{9}{10}}{{}^{20}_{10}C}$

$\rightarrow \frac{9}{19}$

Hence, B.

2 votes -- Shivansh Gupta (2.4k points)

7.8 Random Variable(7) top

7.8.1 Random Variable: GATE2005-12, ISRO2009-64 top

https://gateoverflow.in/1162

Let $f(x)$ be the continuous probability density function of a random variable x, the probability that $a < x \leq b$, is :

A. $f(b-a)$
B. $f(b) - f(a)$
C. $\int_a^b f(x)dx$
D. $\int_a^b xf(x)dx$

gate2005 probability random-variable easy isro2009

Answer

7.8.2 Random Variable: GATE2011-33 top

https://gateoverflow.in/2135

Consider a finite sequence of random values $X = [x_1, x_2, \ldots x_n]$. Let μ_x be the mean and σ_x be the standard deviation of X. Let another finite sequence Y of equal length be derived from this as $y_i = a * x_i + b$, where a and b are positive constants. Let μ_y be the mean and σ_y be the standard deviation of this sequence.

Which one of the following statements is **INCORRECT**?

A. Index position of mode of X in X is the same as the index position of mode of Y in Y
B. Index position of median of X in X is the same as the index position of median of Y in Y
C. $\mu_y = a\mu_x + b$
D. $\sigma_y = a\sigma_x + b$

gate2011 probability random-variable normal

Answer

7.8.3 Random Variable: GATE2012-21 top

https://gateoverflow.in/1577

Consider a random variable X that takes values $+1$ and -1 with probability 0.5 each. The values of the cumulative distribution function $F(x)$ at $x = -1$ and $+1$ are

A. 0 and 0.5
B. 0 and 1
C. 0.5 and 1
D. 0.25 and 0.75

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gate2012 probability random-variable easy

Answer

7.8.4 Random Variable: GATE2015-3-37 top

https://gateoverflow.in/8496

Suppose X_i for $i = 1, 2, 3$ are independent and identically distributed random variables whose probability mass functions are $Pr[X_i = 0] = Pr[X_i = 1] = \frac{1}{2}$ for $i = 1, 2, 3$. Define another random variable $Y = X_1 X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then $Pr[Y = 0 \mid X_3 = 0] =$_____.

gate2015-3 probability random-variable normal numerical-answers

Answer

7.8.5 Random Variable: GATE2017-1-19 top

https://gateoverflow.in/118299

Let X be a Gaussian random variable with mean 0 and variance σ^2. Let $Y = max(X, 0)$ where $max(a, b)$ is the maximum of a and b. The median of Y is _____ .

gate2017-1 probability random-variable numerical-answers

Answer

7.8.6 Random Variable: GATE2017-2-31 top

https://gateoverflow.in/118373

For any discrete random variable X, with probability mass function

$P(X = j) = p_j, p_j \geq 0, j \in \{0, \ldots, N\}$, and $\Sigma_{j=0}^{N} p_j = 1$, define the polynomial function $g_x(z) = \Sigma_{j=0}^{N} p_j z^j$. For a certain discrete random variable Y, there exists a scalar $\beta \in [0, 1]$ such that $g_y(z) = (1 - \beta + \beta z)^N$. The expectation of Y is

A. $N\beta(1 - \beta)$
B. $N\beta$
C. $N(1 - \beta)$
D. Not expressible in terms of N and β alone

gate2017-2 probability random-variable

Answer

7.8.7 Random Variable: TIFR2011-A-7 top

https://gateoverflow.in/20012

Let X and Y be two independent and identically distributed random variables. Then $P(X > Y)$ is.

a. $\frac{1}{2}$
b. 1
c. 0
d. $\frac{1}{3}$
e. Information is insufficient.

tifr2011 probability random-variable

Answer

Answers: Random Variable

7.8.1 Random Variable: GATE2005-12, ISRO2009-64 top

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Selected Answer

https://gateoverflow.in/1162

A. This gives the probability at the point of $b-a$ which is not having any significant w.r.t a and b.

B. This gives the difference of the probabilities at b and a. Note: This is different from cumulative distribution function $F(b)-F(a)$.
Ref: https://en.wikipedia.org/wiki/Cumulative_distribution_function

C. This is Probability Density Function.
Ref: https://en.wikipedia.org/wiki/Probability_density_function

D. This is expected value of continuous random variable. Ref: https://en.wikipedia.org/wiki/Expected_value
Answer is C.

11 votes -- Akash Kanase (42.5k points)

7.8.2 Random Variable: GATE2011-33 top

https://gateoverflow.in/2135

Selected Answer

Answer - D.

Mean, median and mode are linear functions over a random vaiable.

So, multiplying by constants or adding constants wont change their relative position.

Standard deviation is not a linear function over a random variable.

21 votes -- Ankit Rokde (9k points)

7.8.3 Random Variable: GATE2012-21 top

https://gateoverflow.in/1577

Selected Answer

Given $P(-1)=0.5$ and $P(1)=0.5$. So, at all other points P must be zero as the sum of all probabilities must be 1.

$So, F(-1)=0.5$ and

$F(1)=P(-1)+0+0+\ldots+P(1)$

= $0.5+0.5=1$

27 votes -- Arjun Suresh (348k points)

7.8.4 Random Variable: GATE2015-3-37 top

https://gateoverflow.in/8496

Selected Answer

Answer is 0.75

As $X_3=0$ is given, to have Y = 0, X_1X_2 should be 0, meaning (X_1,X_2) should be one of $\{(0,0)(0,1)(1,0)\}$

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So, required probability $= 3 \times \frac{1}{2} \times \frac{1}{2} = 0.75 \because$ we can choose any of the 3 possibilities in 3 ways and then probability of each set of two combination is $\frac{1}{2} \times \frac{1}{2}$.

We can also do like follows:

There are totally 4 possibilities - $\{(0,0)(0,1)(1,0),(1,1)\}$ out of which 3 are favourable cases.

So, required probability $= \frac{3}{4} = 0.75$.

32 votes -- Srijay Deshpande (389 points)

7.8.5 Random Variable: GATE2017-1-19 top https://gateoverflow.in/118299

Selected Answer

Variable Y can take only non-negative values. Median of a distribution is a value c such that

$$P(0 < Y < c) = P(c < Y < \infty)$$

Now for L.H.S., Y will lie between 0 and c only when $X < c$ i.e $P(0 < Y < c) = P(X < c)$.

For R.H.S, $Y > c$ only when $X > c$ i.e. $P(c < Y < \infty) = P(X > c) = 1 - P(X < c)$

Equating both sides, we get $P(X < c) = 1 - P(X < c) \implies P(X < c) = 0.5 \implies c = 0$.

Hence 0 is the answer.

17 votes -- Happy Mittal (11.2k points)

7.8.6 Random Variable: GATE2017-2-31 top https://gateoverflow.in/118373

Selected Answer

Notice that the derivative of $g_x(z)$ evaluated at $z = 1$ gives expectation $E(X)$ of X

Therefore, take derivative of $g_y(z)$ with respect to z, and plug in $z = 1$

Derivative is $N.\beta.(1 - \beta + \beta_z)(N - 1)$, plug in $z = 1$, gives $N\beta$

So, Answer is option (B)

12 votes -- madan123456789 (277 points)

7.8.7 Random Variable: TIFR2011-A-7 top https://gateoverflow.in/20012

Selected Answer

Let the probability $P(X = Y) > 0$. This can happen if X and Y are discrete random variables.

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Also, if X and Y are continuous random variables, it could be that some values have a non-zero probability of getting selected.

Then $P(X > Y) = P(Y > X) = \frac{1 - P(X = Y)}{2}$

Since nothing is said about the value of $P(X = Y)$, the correct answer will be option e. Information is insufficient.

6 votes -- Pragy Agarwal (20.6k points)

7.9 Uniform Distribution(3) top

7.9.1 Uniform Distribution: GATE2004-80 top

https://gateoverflow.in/1074

A point is randomly selected with uniform probability in the $X - Y$ plane within the rectangle with corners at $(0,0), (1,0), (1,2)$ and $(0,2)$. If p is the length of the position vector of the point, the expected value of p^2 is

A. $\left(\frac{2}{3}\right)$
B. 1
C. $\left(\frac{4}{3}\right)$
D. $\left(\frac{5}{3}\right)$

gate2004 probability uniform-distribution expectation normal

Answer

7.9.2 Uniform Distribution: GATE2014-1-2 top

https://gateoverflow.in/1717

Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is _____ .

gate2014-1 probability uniform-distribution expectation numerical-answers normal

Answer

7.9.3 Uniform Distribution: ISI2017-17 top

https://gateoverflow.in/209670

Suppose that X is chosen uniformly from $\{1, 2, \ldots, 100\}$ and given $X = x$, Y is chosen uniformly from $\{1, 2, \ldots, x\}$.Then $P(Y = 30) =$

A. $\frac{1}{100}$
B. $\frac{1}{100} \times \left(\frac{1}{30} + \ldots + \frac{1}{100}\right)$
C. $\frac{1}{30}$
D. $\frac{1}{100} \times \left(\frac{1}{1} + \ldots + \frac{1}{30}\right)$

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isi2017 probability uniform-distribution

Answer

Answers: Uniform Distribution

7.9.1 Uniform Distribution: GATE2004-80 top

https://gateoverflow.in/1074

Selected Answer

Above diagram depicts the scenario of our Question.

The length P of our point (x, y) selected randomly in XY plane, from origin is given by

$P = \sqrt{x^2 + y^2}$

$P^2 = x^2 + y^2$

Expected length of P^2 is given by

$E[P^2] = E[x^2 + y^2]$

By linearity of expectation

$E[x^2 + y^2] = E[x^2] + E[y^2]$

Now we need to calculate the probability density function of X and Y.

Since, distribution is Uniform

X goes from 0 to 1, so $PDF(x) = \frac{1}{1-0} = 1$

Y goes from 0 to 2 so $PDF(y) = \frac{1}{2-0} = \frac{1}{2}$

Now we evaluate

$E[X^2] = \int_0^1 x^2.1dx = \frac{1}{3}$

$E[Y^2] = \int_0^2 y^2 \times (1/2)dy = \frac{4}{3}$

$E[P^2] = E[X^2] + E[Y^2] = \frac{5}{3}$

3 votes -- Ayush Upadhyaya (8.7k points)

7.9.2 Uniform Distribution: GATE2014-1-2 top

https://gateoverflow.in/1717

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Selected Answer

The length of the shorter stick can be from 0 to 0.5 (because if it is greater than 0.5, it is no longer a shorter stick).

This random variable L (length of shorter stick) follows a uniform distribution, and hence probability density function of L is $\frac{1}{0.5-0} = 2$ for all lengths in range 0 to 0.5

Now expected value of $L = \int_0^{0.5} L * p(L)dL = \int_0^{0.5} L * 2dL = 2 * \left[\frac{L^2}{2}\right]_0^{0.5} = 0.25$

39 votes -- Happy Mittal (11.2k points)

7.9.3 Uniform Distribution: ISI2017-17 top

https://gateoverflow.in/209670

Selected Answer

At first X is chosen uniformly from $1-100$, $P(\text{choosing } X) = \frac{1}{100}$

Now, Y is chosen from the set$\{1, 2....x\}$, so this set should definitely contain atleast 30 elements from $1-30$, then only $Y = 30$ can be chosen, $P(Y = 30) = \frac{1}{30}$

We can expand the set to contain 31 elements, then $P(Y = 30) = \frac{1}{31}$

.....

We can expand the set till ut contains all the 100 elements, then $P(Y = 30) = \frac{1}{100}$

So, $P(Y = 30) = \frac{1}{100} * (\frac{1}{30} + \frac{1}{31} + \frac{1}{32} + ... + \frac{1}{100})$

3 votes -- Sukannya Purkayastha (2.6k points)

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8 General Aptitude: Numerical Ability (226) top

8.1 Absolute Value(5) top

8.1.1 Absolute Value: GATE-2013-AE-GA-8 top https://gateoverflow.in/40249

If $|-2X+9|=3$ then the possible value of $|-X|-X^2$ would be:

A. 30
B. -30
C. -42
D. 42

gate2013-ae numerical-ability absolute-value

Answer

8.1.2 Absolute Value: GATE2013-CE-7 top https://gateoverflow.in/40275

If $|4X-7|=5$ then the values of $2|X|-|-X|$ is:

A. $2, \left(\frac{1}{3}\right)$
B. $\left(\frac{1}{2}\right), 3$
C. $\left(\frac{3}{2}\right), 9$
D. $\left(\frac{2}{3}\right), 9$

gate2013-ce numerical-ability absolute-value

Answer

8.1.3 Absolute Value: GATE2014-2-GA-8 top https://gateoverflow.in/1950

If x is real and $|x^2-2x+3|=11$, then possible values of $|-x^3+x^2-x|$ include

A. $2, 4$
B. $2, 14$
C. $4, 52$
D. $14, 52$

gate2014-2 numerical-ability normal absolute-value

Answer

8.1.4 Absolute Value: GATE2016-Session-8-GA-5 top https://gateoverflow.in/111286

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If $|9y-6|=3$, then $y^2-\frac{4}{3}y$ is .

A. 0
B. $+\frac{1}{3}$
C. $-\frac{1}{3}$
D. undefined

gate2016session8aptitude numerical-ability absolute-value

Answer

8.1.5 Absolute Value: GATE2017-1-GA-8 top

https://gateoverflow.in/118411

The expression $\frac{(x+y)-|x-y|}{2}$ is equal to :

A. The maximum of x and y
B. The minimum of x and y
C. 1
D. None of the above

gate2017-1 numerical-ability maxima-minima absolute-value

Answer

Answers: Absolute Value

8.1.1 Absolute Value: GATE-2013-AE-GA-8 top

https://gateoverflow.in/40249

Selected Answer

Given $|-2X+9|=3,$

$\Rightarrow -2X+9=3$ or $-(-2X+9)=3,$

$\Rightarrow X=3$ or $X=6,$

$\Rightarrow |-X|-X^2=|-3|-3^2=-6$ or $|-X|-X^2=|-6|-6^2=-30,$

Thus B is the correct option.

7 votes -- Anurag Pandey (13.6k points)

8.1.2 Absolute Value: GATE2013-CE-7 top

https://gateoverflow.in/40275

Selected Answer

$A).\ |4X-7|=5$,since it is in absolute form the regular form of this equation is given as

$(4X-7)=5$

$(4X-7)=-5$

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By solving the above equations we get the following answer

$X = \frac{1}{2}, X = 3$

Now $2 \mid X \mid - \mid -X \mid = 2 \mid X \mid - \mid X \mid = \mid X \mid$.

So, our answer will be B.

7 votes -- ibia (3.5k points)

8.1.3 Absolute Value: GATE2014-2-GA-8 top https://gateoverflow.in/1950

Selected Answer

$x^2 - 2x + 3 = 11$ OR $x^2 - 2x + 3 = -11$
(Any one of them can be correct because of mod)

Lets take first one:
$x^2 - 2x + 3 = 11$
$\implies x^2 - 2x - 8 = 0$
$\implies (x-4)(x+2) = 0$
$\implies x = 4 \text{ or } x = -2.$

Now put these values of x in the given equation $\left|-x^3 + x^2 - x\right|$

for $x = 4$, we will get $\left|64 + 16 - 4\right| = 52.$
for $x = -2$, we will get $\left|8 + 4 + 2\right| = 14.$

So, answer is D.

18 votes -- Akash Kanase (42.5k points)

8.1.4 Absolute Value: GATE2016-Session-8-GA-5 top https://gateoverflow.in/111286

Selected Answer

$\left|9y - 6\right| = 3$

squaring both side

$81y^2 - 108y + 36 = 9$

dividing both side by 9

$9y^2 - 12y + 4 = 1$

again dividing by 9

$$y^2 - \frac{4}{3}y + \frac{4}{9} = \frac{1}{9}$$

$$\Rightarrow y^2 - \frac{4}{3}y = \frac{1}{9} - \frac{4}{9} = -\frac{1}{3}.$$

Hence answer is option C

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6 votes -- Aboveallplayer (18.5k points)

8.1.5 Absolute Value: GATE2017-1-GA-8 top

https://gateoverflow.in/118411

Selected Answer

When $x > y, \left| x - y \right| \quad = x - y,$ if we substitute in expression we get y.
When $x < y, \left| x - y \right| \quad = -(x - y),$ if we substitute in expression we get x.

Therefore in both the case we get minimum of (x, y).
ANS: B

23 votes -- Vinay Rachapalli (1.4k points)

8.2 Age Relation(1) top

8.2.1 Age Relation: GATE2013-ce-10 top

https://gateoverflow.in/40280

Abhishek is elder to Savar. Savar is younger to Anshul. Which of the given conclusions is logically valid and is inferred from the above statements?

A. Abhishek is elder to Anshul
B. Anshul is elder to Abhishek
C. Abhishek and Anshul are of the same age
D. No conclusion follows

gate2013-ce logical-reasoning age-relation

Answer

Answers: Age Relation

8.2.1 Age Relation: GATE2013-ce-10 top

https://gateoverflow.in/40280

Selected Answer

$D)$ No conclusion follows as we cannot derive any relationship between Abhishek and Anshul with the given data.

7 votes -- vamsi2376 (3.6k points)

8.3 Algebra(1) top

8.3.1 Algebra: GATE2011-MN2011-61 top

https://gateoverflow.in/31536

If $\frac{(2y+1)}{(y+2)} < 1$, then which of the following alternatives gives the CORRECT range of y ?

A. $-2 < y < 2$
B. $-2 < y < 1$
C. $-3 < y < 1$
D. $-4 < y < 1$

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numerical-ability gate2011-mn algebra

Answer

Answers: Algebra

8.3.1 Algebra: GATE2011-MN2011-61 top

https://gateoverflow.in/31536

Selected Answer

$$\frac{2y+1}{y+2} < 1$$

$$y < 1$$

1. For any value of $y < -2$, Numerator become greater than Denominator.

2. But since in LHS, denominator cannot be zero

i.e., $y > -2$.

Therefore, $-2 < y < 1$
Option B.

8 votes -- Monanshi Jain (9.3k points)

8.4 Arithmetic Series(2) top

8.4.1 Arithmetic Series: GATE2015-2-GA-6 top

https://gateoverflow.in/8035

If the list of letters, P, R, S, T, U is an arithmetic sequence, which of the following are also in arithmetic sequence?

I. $2P, 2R, 2S, 2T, 2U$
II. $P-3, R-3, S-3, T-3, U-3$
III. P^2, R^2, S^2, T^2, U^2

A. I only
B. I and II
C. II and III
D. I and III

gate2015-2 numerical-ability normal arithmetic-series

Answer

8.4.2 Arithmetic Series: ISI-MTECH-CSE-2018 top

https://gateoverflow.in/216598

The number of common terms in the two sequences $\{3, 7, 11, \ldots, 407\}$ and $\{2, 9, 16, \ldots, 709\}$ is

A. 13
B. 14
C. 15
D. 16

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isi2018 arithmetic-series numerical-ability

Answer

Answers: Arithmetic Series

8.4.1 Arithmetic Series: GATE2015-2-GA-6 top

https://gateoverflow.in/8035

Selected Answer

Answer is B because If we multiply the terms of an AP with a constant, the common difference will get multiplied by the constant and if we subtract the terms of an AP with a constant, the common difference will be same but if we make square of AP element then difference will not be same.

7 votes -- Anoop Sonkar (5k points)

8.4.2 Arithmetic Series: ISI-MTECH-CSE-2018 top

https://gateoverflow.in/216598

Selected Answer

$$S_1 = \{3, 7, 11, 15, 19, 23, \ldots, 407\}$$
$$S_2 = \{2, 9, 16, 23, \ldots, 709\}$$

$$\text{common diff}(d_1) \text{ for } S_1 = 4$$
$$\text{common diff}(d_2) \text{ for } S_2 = 7$$

Now, we try to make a new sequence which contains the common elements of the given two arithmetic progression (AP) sequences.

$\text{first common term}(a) = 23$, this will be the first term in our new sequence which will also be an AP sequence and its common difference,

$$d = LCM(d_1, d_2) = LCM(4, 7) = 28.$$

n^{th} term of our new sequence is given by,

$$\begin{aligned} T_n &= a + (n-1) * d \\ &= 23 + (n-1) * 28 \\ &= 23 + 28n - 28 \\ &= 28n - 5 \end{aligned}$$

For the last term of our sequence we get,

$$28n - 5 = \min(407, 709) = 407$$

$$\implies 28n = 407 + 5 = 412$$

$$n = \frac{412}{28} = 14.741$$

n must be integer and since we are finding number of present terms we must take $floor$ meaning the last term is not 407 but some value lower than it. So, $n = 14$.

There are 14 common terms.

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5 votes -- Pankaj Kumar (8.5k points)

8.5 Bar Charts(2) top

8.5.1 Bar Charts: GATE2014-EC01-GA9 top

https://gateoverflow.in/41498

The exports and imports (in crores of Rs.) of a country from 2000 to 2007 are given in the following bar chart. If the trade deficit is defined as excess of imports over exports, in which year is the trade deficit $1/5$th of the exports?

A. 2005
B. 2004
C. 2007
D. 2006

gate2014-ec01 numerical-ability data-interpretation bar-charts normal

Answer

8.5.2 Bar Charts: GATE2016-Session-8-GA-6 top

https://gateoverflow.in/111289

The following graph represents the installed capacity for cement production (in tonnes) and the actual production (in tonnes) of nine cement plants of a cement company. Capacity utilization of a plant is defined as ratio of actual production of cement to installed capacity. A plant with installed capacity of at least 200 tonnes is called a large plant and a plant with lesser capacity is called a small plant. The difference between total production of large plants and small plants, in tonnes is _____.

gate2016session8aptitude numerical-ability numerical-answers data-interpretation bar-charts

Answer

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Answers: Bar Charts

8.5.1 Bar Charts: GATE2014-EC01-GA9 top

https://gateoverflow.in/41498

Selected Answer

$\left(\frac{1}{5}\right)^{th}$ excess of the exports means
$(1+\frac{1}{5})\times$ Export = Import

1. Option A
$(1+\frac{1}{5})\times 70 = 84 \neq$ Import

2. Option B
$(1+\frac{1}{5})\times 70 = 84 \neq$ Import

3. Option C
$(1+\frac{1}{5})\times 110 = 132 \neq$ Import

4. Option D
$(1+\frac{1}{5})\times 100 = 120$ = Import

So, answers is option D.

6 votes -- Lokesh Dafale (11.4k points)

8.5.2 Bar Charts: GATE2016-Session-8-GA-6 top

https://gateoverflow.in/111289

Selected Answer

Total Production of large plants as evident from the graph(those with capacity 200 or more) is $(160+190+230+190) = 770.$

Total Production of small plants $= 150+160+120+100+120 = 650.$

Difference $= 770 - 650 = 120.$

6 votes -- vishal76 (1.7k points)

8.6 Binomial Distribution(1) top

8.6.1 Binomial Distribution: GATE2018-GA-10 top

https://gateoverflow.in/204071

A six sided unbiased die with four green faces and two red faces is rolled seven times. Which of the following combinations is the most likely outcome of the experiment?

A. Three green faces and four red faces.
B. Four green faces and three red faces.
C. Five green faces and two red faces.
D. Six green faces and one red face

gate2018 numerical-ability probability binomial-distribution normal

Answer

© Copyright GATE Overflow. All rights reserved.

Answers: Binomial Distribution

8.6.1 Binomial Distribution: GATE2018-GA-10 top

https://gateoverflow.in/204071

Selected Answer

We can calculate the probability of each of the options but by logic we can easily eliminate the ones with more number of red faces - option A can be avoided without any calculation.

$$A = P(3G, 4R) = {}^7C_3 . \left(\frac{4}{6}\right)^3 \left(\frac{2}{6}\right)^4$$

$$B = P(4G, 3R) = {}^7C_3 . \left(\frac{4}{6}\right)^4 \left(\frac{2}{6}\right)^3$$

$$C = P(3G, 4R) = {}^7C_5 . \left(\frac{4}{6}\right)^5 \left(\frac{2}{6}\right)^2$$

$$D = P(3G, 4R) = {}^7C_6 . \left(\frac{4}{6}\right)^6 \left(\frac{2}{6}\right)^1$$

Option A is clearly smaller and hence eliminated.

$$\frac{C}{B} = \frac{{}^7C_5}{{}^7C_3} . \frac{4}{6} . \frac{6}{2} = \frac{21}{35} . 2 > 1.$$

So, $C > B$.

$$\frac{C}{D} = \frac{{}^7C_5}{{}^7C_6} . \frac{6}{4} . \frac{2}{6} = \frac{21}{7} . 0.5 > 1.$$

So, $C > D$.

Hence, C is the most favourable outcome.

17 votes -- Arjun Suresh (348k points)

8.7 Cartesian Coordinates(3) top

8.7.1 Cartesian Coordinates: GATE-2012-AE-9 top

https://gateoverflow.in/40220

Two points $(4, p)$ and $(0, q)$ lie on a straight line having a slope of $3/4$. The value of $(p - q)$ is

A. -3
B. 0
C. 3
D. 4

gate2012-ae numerical-ability cartesian-coordinates geometry

Answer

8.7.2 Cartesian Coordinates: GATE2014-ae-4 top

https://gateoverflow.in/40303

If $y = 5x^2 + 3$, then the tangent at $x = 0$, $y = 3$

A. passes through $x = 0, y = 0$
B. has a slope of $+1$

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C. is parallel to the x-axis
D. has a slope of -1

gate-2014-ae numerical-ability geometry cartesian-coordinates

Answer

8.7.3 Cartesian Coordinates: TIFR2015-A-13 top https://gateoverflow.in/29586

Imagine the first quadrant of the real plane as consisting of unit squares. A typical square has 4 corners: $(i,j),(i+1,j),(i+1,j+1)$,and $(i,j+1)$, where (i,j) is a pair of non-negative integers. Suppose a line segment l connecting $(0,0)$ to $(90,1100)$ is drawn. We say that l passes through a unit square if it passes through a point in the interior of the square. How many unit squares does l pass through?

a. $98,990$
b. $9,900$
c. $1,190$
d. $1,180$
e. $1,010$

tifr2015 numerical-ability cartesian-coordinates

Answer

Answers: Cartesian Coordinates

8.7.1 Cartesian Coordinates: GATE-2012-AE-9 top https://gateoverflow.in/40220

Selected Answer

For two points (x_1,y_1) and (x_2,y_2) on a line, Slope of line $= \dfrac{y_2-y_1}{x_2-x_1}$

so , $\dfrac{q-p}{0-4} = \dfrac{3}{4}$

$\therefore (p-q) = 3$

7 votes -- Praveen Saini (54.5k points)

8.7.2 Cartesian Coordinates: GATE2014-ae-4 top https://gateoverflow.in/40303

Selected Answer

Given, $y = 5x^2+3$
Slope $dy/dx = 10x$
Given point $x=0, y=3 : (0,3)$
Now, at $(0,3)$ we have the slope as $dy/dx = 10\times 0 = 0$, which means line is parallel to X-axis.
Options B and D eliminated as slope is 0.
Option A is eliminated as given the tangent is at point $(0,3)$ and we found $slope = 0$, which means it is parallel to X axis. The equation of line is nothing but $y=3$, which never passes through $(0,0)$. Hence A too eliminated.
Answer: Option $C)$

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7 votes -- Abhilash Panicker (9.4k points)

8.7.3 Cartesian Coordinates: TIFR2015-A-13 top https://gateoverflow.in/29586

Selected Answer

Answer will be (d) 1180

If a line segment passes through unit square from $(0,0)$ to (i,j) the line intersects $(i+j-gcd(i,j))$

no. of squares $=(90+1100-10)=1180.$

9 votes -- srestha (86.6k points)

8.8 Circle(2) top

8.8.1 Circle: GATE2018-GA-3 top https://gateoverflow.in/204064

The area of a square is d. What is the area of the circle which has the diagonal of the square as its diameter?

A. πd
B. πd^2
C. $\frac{1}{4}\pi d^2$
D. $\frac{1}{2}\pi d$

gate2018 numerical-ability geometry circle normal

Answer

8.8.2 Circle: TIFR2011-A-18 top https://gateoverflow.in/20255

The equation of the tangent to the unit circle at point $(\cos\alpha, \sin\alpha)$ is

a. $x\cos\alpha - y\sin\alpha = 1$
b. $x\sin\alpha - y\cos\alpha = 1$
c. $x\cos\alpha + y\sin\alpha = 1$
d. $x\sin\alpha - y\cos\alpha = 1$
e. None of the above.

tifr2011 numerical-ability geometry circle

Answer

Answers: Circle

8.8.1 Circle: GATE2018-GA-3 top https://gateoverflow.in/204064

Selected Answer

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The area of a square is d

We know that, if a square has a units as the length of its side, then area of the square will be a^2.

$\Rightarrow$ **every side of the square will be** $\sqrt{d}$

Now, the diagonal of the square will be $\sqrt{(\sqrt{d})^2 + (\sqrt{d})^2} = \sqrt{2d}$

Now, $\sqrt{2d}$ is the perimeter of a circle.

$\therefore$ Radius of the circle would be $\dfrac{\sqrt{2d}}{2}$

We know **Area of the circle** is πr^2, where, $r = \text{radius of the circle}$

$$= \frac{22}{7} \times \frac{\sqrt{2d}}{2} \times \frac{\sqrt{2d}}{2}$$

$$= \frac{22}{7} \times \frac{2d}{2 \times 2}$$

$$= \frac{22}{7} \times \frac{d}{2}$$

$$= \pi . \frac{d}{2} \Rightarrow \text{Option D)}$$

2 votes -- Subarna Das (10.5k points)

8.8.2 Circle: TIFR2011-A-18 top

https://gateoverflow.in/20255

Selected Answer

Assuming that the unit circle is centered at the origin, the equation of the unit circle is:
$x^2 + y^2 = 1$

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The slope of the tangent to the unit circle at point (x, y) can be derived by implicit differentiation as follows:

$$\begin{aligned} x^2 + y^2 &= 1 \\ \frac{d}{dx}\left(x^2 + y^2\right) &= \frac{d}{dx}1 \\ 2x + 2y\frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= -\frac{x}{y} \end{aligned}$$

Thus, the slope of the tangent at the point $(\cos\alpha, \sin\alpha)$ is $-\frac{\cos\alpha}{\sin\alpha}$

The equation of the tangent line then will be:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - \sin\alpha &= -\frac{\cos\alpha}{\sin\alpha}(x - \cos\alpha) \\ y\sin\alpha - \sin^2\alpha &= -x\cos\alpha + \cos^2\alpha \\ y\sin\alpha + x\cos\alpha &= \sin^2\alpha + \cos^2\alpha = 1 \end{aligned}$$

Hence, option c is correct.

8 votes -- Pragy Agarwal (20.6k points)

8.9 Clock Time(5) top

8.9.1 Clock Time: GATE2014-2-GA-10 top

https://gateoverflow.in/1952

At what time between 6 a. m. and 7 a. m. will the minute hand and hour hand of a clock make an angle closest to 60°?

A. $6:22$ a.m.
B. $6:27$ a.m.
C. $6:38$ a.m.
D. $6:45$ a.m.

gate2014-2 numerical-ability normal clock-time

Answer

8.9.2 Clock Time: GATE2016-Session-3-GA-8 top

https://gateoverflow.in/108724

Two and quarter hours back, when seen in a mirror, the reflection of a wall clock without number markings seemed to show $1:30$. What is the actual current time shown by the clock?

A. $8:15$
B. $11:15$
C. $12:15$
D. $12:45$

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gate2016session3aptitude clock-time

Answer

8.9.3 Clock Time: TIFR2010-A-2 top

https://gateoverflow.in/18206

The hour hand and the minute hands of a clock meet at noon and again at mid-night. In between they meet N times, where N is.:

a. 6
b. 11
c. 12
d. 13
e. None of the above.

tifr2010 numerical-ability clock-time

Answer

8.9.4 Clock Time: TIFR2013-A-20 top

https://gateoverflow.in/25502

Consider a well functioning clock where the hour, minute and the seconds needles are exactly at zero. How much time later will the minutes needle be exactly one minute ahead ($1/60$ th of the circumference) of the hours needle and the seconds needle again exactly at zero?

Hint: When the desired event happens both the hour needle and the minute needle have moved an integer multiple of $1/60$ th of the circumference.

a. 144 minutes
b. 66 minutes
c. 96 minutes
d. 72 minutes
e. 132 minutes

tifr2013 numerical-ability clock-time

Answer

8.9.5 Clock Time: TIFR2014-A-10 top

https://gateoverflow.in/25998

A person went out between 4pm and 5pm to chat with her friend and returned between 5pm and 6pm. On her return, she found that the hour-hand and the minute-hand of her (well-functioning) clock had just exchanged their positions with respect to their earlier positions at the time of her leaving. The person must have gone out to chat at

a. Twenty five minutes past 4pm.
b. Twenty six and $\frac{122}{143}$ minutes past 4pm.
c. Twenty seven and $\frac{1}{3}$ minutes past 4pm.
d. Twenty eight minutes past 4pm.
e. None of the above.

tifr2014 numerical-ability clock-time

Answer

Answers: Clock Time

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8.9.1 Clock Time: GATE2014-2-GA-10 top

https://gateoverflow.in/1952

Selected Answer

At 6 a.m. the hour hand and minute hand are separated by 180 degree. Now,

Speed of hour hand = 360 degree/12 hour (clock is $12\ hrs$ as am/pm is given) = 30 degrees /hr = 0.5 degree per minute

Speed of minute hand = 360 degree per 60 minutes = 6 degrees per minute.

So, we want the relative distance between minute and hour hand to be 60 degree as per question which would mean a relative distance traversal of $180 - 60 = 120$ degrees. This happens after x minutes such that

$$6x - 0.5x = 120 \implies x = \frac{120}{5.5} = 21.81$$

So, closest time is $6:22$ a.m.

18 votes -- Arjun Suresh (348k points)

8.9.2 Clock Time: GATE2016-Session-3-GA-8 top

https://gateoverflow.in/108724

Selected Answer

Reflection of time is 1:30

so actual time is (11:60 - 1:30) = 10:30

here mentioned 2:15 hrs back

so add it to 10:30

we got 12:45

the actual current time shown by the clock is 12:45

3 votes -- rajukalyadapu (137 points)

8.9.3 Clock Time: TIFR2010-A-2 top

https://gateoverflow.in/18206

Selected Answer

Let δ be the difference in minutes between hour and minute hand at $1:05$. So, the meeting times are

$1:05+\delta$

$2:10+2\delta$

$3:15+3\delta$

$4:20+4\delta$

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$5 : 25 + 5\delta$

$6 : 30 + 6\delta$

$7 : 35 + 7\delta$

$8 : 40 + 8\delta$

$9 : 45 + 9\delta$

$10 : 50 + 10\delta$

$11 : 55 + 11\delta$

We have $11\delta = 5$

$$\implies \delta = \frac{5}{11}$$

minutes as the meeting time is $0 : 00$. So, we have $N = 10$.

Alternatively,

Speed of minute hand = 360 degrees per 60 minutes = 6 degrees per minute.

Speed of hour hand = 360 degree per $12 * 60$ minutes = 0.5 degree per minute.

For first meeting, distance traveled by minute hand = $360 +$ distance traveled by hour hand

Let, x be the minutes after which the hands intersect.

So, $6x = 360 + 0.5x$

$$\implies 5.5x = 360$$

$$\implies x = \frac{720}{11}$$

In 12 hours we have 12×60 minutes.

So, no. of intersections $= \dfrac{12 \times 60}{720/11} = 11$.

But the last intersection is at midnight and must be excluded as er given question. So, $N = 10$.

3 votes -- Arjun Suresh (348k points)

8.9.4 Clock Time: TIFR2013-A-20 top

https://gateoverflow.in/25502

Let the event takes place at H hr M min

Given the condition We can see $11M$ - $60H$ = 12.

Only **e** satisfies this.

2 votes -- sudipta roy (423 points)

8.9.5 Clock Time: TIFR2014-A-10 top

https://gateoverflow.in/25998

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Selected Answer

Let the time of departure be $4:x$ and time of arrival be $5:y$.

Angle made by hour hand during departure $= \frac{4}{12} \times 360 + x\frac{360}{12 \times 60} = 120 + \frac{x}{2}$.

This angle is equal to the angle made by the minute hand on arrival which is $6y$. So,

$$240 + x = 12y \rightarrow (1)$$

Similarly, the angle made by the hour hand on arrival is equal to the angle made by the minute hand on departure, which gives

$$\frac{5}{12} \times 360 + \frac{y}{2} = 6x \implies 300 + y = 12x \rightarrow (2).$$

Eliminating y from (1) and (2),

$$240 + x = 12(12x - 300) \implies 143x = 3840 \implies x = 26\frac{122}{143}$$

6 votes -- Arjun Suresh (348k points)

8.10 Complex Number(1) top

8.10.1 Complex Number: TIFR2011-A-13 top

https://gateoverflow.in/20223

If $z = \frac{\sqrt{3} - i}{2}$ and $\left(z^{95} + i^{67}\right)^{97} = z^n$, then the smallest value of n is?

a. 1
b. 10
c. 11
d. 12
e. None of the above.

tifr2011 numerical-ability complex-number

Answer

Answers: Complex Number

8.10.1 Complex Number: TIFR2011-A-13 top

https://gateoverflow.in/20223

Selected Answer

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$$z = \frac{1}{2}\left(\sqrt{3} - i\right)$$

$$z^2 = \frac{1}{4}\left(3 - 1 - 2\sqrt{3}\,i\right) = \frac{1}{2}\left(1 - \sqrt{3}\,i\right)$$

$$z^4 = \frac{1}{4}\left(1 - 3 - 2\sqrt{3}\,i\right) = \frac{1}{2}\left(-1 - \sqrt{3}\,i\right)$$

$$z^8 = \frac{1}{4}\left(1 - 3 + 2\sqrt{3}\,i\right) = \frac{1}{2}\left(-1 + \sqrt{3}\,i\right)$$

$$z^{16} = \frac{1}{4}\left(1 - 3 - 2\sqrt{3}\,i\right) = \frac{1}{2}\left(-1 - \sqrt{3}\,i\right) = z^4$$

$$z^{32} = z^{16} \times z^{16} = z^4 \times z^4 = z^8$$

$$z^{64} = z^{32} \times z^{32} = z^8 \times z^8 = z^{16} = z^4$$

$$z^{95} = z^{64} \times z^{16} \times z^{15}$$

$$= z^4 \times z^4 \times z^{15}$$

$$= z^{16} \times z^7$$

$$= z^4 \times z^7$$

$$= z^8 \times z^2 \times z$$

$$= \frac{1}{2}\left(-1 + \sqrt{3}\,i\right) \times \frac{1}{2}\left(1 - \sqrt{3}\,i\right) \times \frac{1}{2}\left(\sqrt{3} - i\right)$$

$$= \frac{1}{2}\left(\sqrt{3} + i\right)$$

$$i^{67} = i^{64} \times i^3$$

$$= 1 \times (-i)$$

$$= -i$$

$$z^{95} + i^{67} = \frac{1}{2}\left(\sqrt{3} + i\right) - i$$

$$= \frac{1}{2}\left(\sqrt{3} - i\right)$$

$$= z$$

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$$\left(z^{95}+i^{67}\right)^{97}=z^{97}=z^{95}\times z^{2}$$

$$=\frac{1}{2}\left(\sqrt{3}+i\right)\times\frac{1}{2}\left(1-\sqrt{3}i\right)$$

$$=\frac{1}{2}\left(\sqrt{3}-i\right)$$

$$=z$$

Hence, option a is the correct answer.

11 votes -- Pragy Agarwal (20.6k points)

8.11 Compound Interest(1) top

8.11.1 Compound Interest: GATE2014-AG-GA5 top https://gateoverflow.in/41669

The population of a new city is 5 million and is growing at 20% annually. How many years would it take to double at this growth rate?

A. $3-4$ years
B. $4-5$ years
C. $5-6$ years
D. $6-7$ years

gate2014-ag numerical-ability growth-rate compound-interest normal

Answer

Answers: Compound Interest

8.11.1 Compound Interest: GATE2014-AG-GA5 top https://gateoverflow.in/41669

Selected Answer

Initial population $P=5m$

After 1 year $P=5m*1.2=6m$

Now $2nd$ year Current $P=6m$

Now after inc=$6m$*1.2=7.2

After 3 year =$8.65m$

After 4 year=$10.38m$

So, answer should be A.

7 votes -- Manoj Kumar (38.7k points)

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8.12 Conditional Probability(3) top

8.12.1 Conditional Probability: GATE2012-63 top

https://gateoverflow.in/2211

An automobile plant contracted to buy shock absorbers from two suppliers X and Y . X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of $X's$ shock absorbers, 96% are reliable. Of $Y's$ shock absorbers, 72% are reliable.

The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is

A. 0.288
B. 0.334
C. 0.667
D. 0.720

gate2012 numerical-ability probability normal conditional-probability

Answer

8.12.2 Conditional Probability: GATE2013-AE-GA-10 top

https://gateoverflow.in/40251

In a factory, two machines $M1$ and $M2$ manufacture 60% and 40% of the autocomponents respectively. Out of the total production, 2% of $M1$ and 3% of $M2$ are found to be defective. If a randomly drawn autocomponent from the combined lot is found defective, what is the probability that it was manufactured by $M2$?

A. 0.35
B. 0.45
C. 0.5
D. 0.4

gate2013-ae numerical-ability conditional-probability

Answer

8.12.3 Conditional Probability: GATE2014-AG-GA10 top

https://gateoverflow.in/41874

10% of the population in a town is HIV^+. A new diagnostic kit for HIV detection is available; this kit correctly identifies HIV^+ individuals 95% of the time, and HIV^- individuals 89% of the time. A particular patient is tested using this kit and is found to be positive. The probability that the individual is actually positive is _____.

gate2014-ag numerical-ability probability conditional-probability normal numerical-answers

Answer

Answers: Conditional Probability

8.12.1 Conditional Probability: GATE2012-63 top

https://gateoverflow.in/2211

Selected Answer

B.

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Then by using Bayes' Theorem :

$$\text{Probability of Y given R} = \frac{\text{Probability of Y and R}}{\text{Probability of R}}$$

$$= \frac{0.4 \times 0.72}{0.4 \times 0.72 + 0.6 \times 0.96}$$

$$= \frac{1}{3} = 0.33$$

19 votes -- shreya ghosh (3.5k points)

8.12.2 Conditional Probability: GATE2013-AE-GA-10 top

https://gateoverflow.in/40251

Selected Answer

$C)\ 0.5$

Let $P(M_i)$ denote the probability that the component is manufactured by machine M_i, and $P(def)$ denote the probability that the component is *defective*.

We have to find $P(M_2|def)$.

$$P(M_2|def) = P(M_2 \bigcap def)/P(def) = \frac{0.4 \times 0.03}{(0.4 \times 0.03) + (0.6 \times 0.02)} = 0.5$$

Drawing probability tree diagram for such questions makes them easier to solve. Please refer link1 and link2 for more details.

6 votes -- Gaurav Sharma (2.4k points)

8.12.3 Conditional Probability: GATE2014-AG-GA10 top

https://gateoverflow.in/41674

Selected Answer

If you dont like Bayes theorem then use this **TREE METHOD:**

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$$\text{Probability} = \frac{0.10 \times 0.95}{0.10 \times 0.95 + 0.90 \times 0.11}$$
$$= 0.4896$$

13 votes -- Bhagirathi Nayak (14.1k points)

8.13 Cost Market Price(4) top

8.13.1 Cost Market Price: GATE2011-63 top

https://gateoverflow.in/2173

The variable cost (V) of manufacturing a product varies according to the equation $V = 4q$, where q is the quantity produced. The fixed cost (F) of production of same product reduces with q according to the equation $F = \frac{100}{q}$. How many units should be produced to minimize the total cost $(V + F)$?

A. 5
B. 4
C. 7
D. 6

gate2011 numerical-ability cost-market-price normal

Answer

8.13.2 Cost Market Price: GATE2012-56 top

https://gateoverflow.in/2193

The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of ₹50 per unit. The number of units to be produced by the firm such that the profit is maximized is

A. 5
B. 10
C. 15
D. 25

gate2012 numerical-ability cost-market-price normal

Answer

8.13.3 Cost Market Price: TIFR2012-A-6 top

https://gateoverflow.in/21002

A certain pair of used shoes can be repaired for $Rs.1250$ and will last for 1 year. A pair of the same kind of shoes can be purchased new for $Rs.2800$ and will last for 2 years. The average cost per year of the new shoes is what percent greater than the cost of repairing the used shoes?

a. 5
b. 12
c. 15
d. 3
e. 24

tifr2012 cost-market-price

Answer

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8.13.4 Cost Market Price: gate-2014-ae-5 top

https://gateoverflow.in/40304

A foundry has a fixed daily cost of $\text{Rs } 50,000$ whenever it operates and a variable cost of $\text{RS } 800Q$,where Q is the daily production in tonnes. What is the cost of production in Rs per tonne for a daily production of 100 tonnes.

gate-2014-ae numerical-ability cost-market-price numerical-answers

Answer

Answers: Cost Market Price

8.13.1 Cost Market Price: GATE2011-63 top

https://gateoverflow.in/2173

Selected Answer

Total Cost, $T = 4q + \frac{100}{q}$

When total cost becomes minimum, first derivative of T becomes 0 and second derivative at the minimum point will be positive.

Differentiating T with respect to q and equating to 0,

$4 - \frac{100}{q^2} = 0 \Rightarrow q = +5$ or -5. Since, we can't have negative number of product, $q = 5$.

Taking second derivative, at $q = 5$ gives $\frac{200}{125} = \frac{8}{5} > 0$, and hence 5 is the minimum point.

13 votes -- Arjun Suresh (348k points)

8.13.2 Cost Market Price: GATE2012-56 top

https://gateoverflow.in/2193

Selected Answer

Answer is A.

The equation for profit is Profit=$SP - CP$,

here $SP = Q \times 50$ and $CP = 5Q^2$.

So when a function attains its maximum value its first order differentiation is zero.

Hence $50 - 5 \times 2 \times Q = 0 \therefore Q = 5$.

For example :

5 units = $CP = 125\ SP = 250\ \therefore Profit = 125$

10 units = $CP = 500\ SP = 500 \therefore Profit = 0$ and so forth..

Therefore its maximum at unit
$= 5$

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21 votes -- kireeti (1.2k points)

8.13.3 Cost Market Price: TIFR2012-A-6 top https://gateoverflow.in/21002

Selected Answer

Repaired shoes lasts 1 year worth $Rs. 1250$

New shoes lasts 2 year worth $Rs. 2800$

So, for one year new shoes costs $Rs. 1400$

% greater of new shoes cost than the cost of repaired shoes $(1400 - 1250/1250)$ = 12% (option b)

1 votes -- khush tak (7.5k points)

8.13.4 Cost Market Price: gate-2014-ae-5 top https://gateoverflow.in/40304

Selected Answer

Answer: 1300

For a daily production of 100 tonnes, the daily cost of foundry would be Rs $50,000 + (800 \times 100) =$ Rs $130,000$.

Hence cost of production in Rs per tonne $= \frac{130,000}{100} =$ Rs $1,300$.

5 votes -- Anurag Pandey (13.6k points)

8.14 Currency Money(1) top

8.14.1 Currency Money: GATE2012-CY-GA-10 top https://gateoverflow.in/40241

Raju has 14 currency notes in his pocket consisting of only Rs. 20 notes and Rs. 10 notes. The total money value of the notes is Rs. 230. The number of Rs. 10 notes that Raju has is

A. 5
B. 6
C. 9
D. 10

gate2012-cy numerical-ability numerical-computation currency-money

Answer

Answers: Currency Money

8.14.1 Currency Money: GATE2012-CY-GA-10 top https://gateoverflow.in/40241

Selected Answer

No. of Rs 20 notes $-x$

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No. of Rs 10 notes $-y$
Given,
$x+y=14$
$20x+10y=230$

Solving the two equations, we get.
$x=9, y=5$
So, the number of Rs. 10 notes is 5.

Answer A.

6 votes -- Abhilash Panicker (9.4k points)

8.15 Data Interpretation(17) top

8.15.1 Data Interpretation: GATE-2013-AE-GA-7 top https://gateoverflow.in/40248

Following table gives data on tourist from different countries visiting India in the year 2011

Country	Number of tourists
USA	2000
England	3500
Germany	1200
Italy	1100
Japan	2400
Australia	2300
France	1000

Which two countries contributed to the one third of the total number of tourists who visited India in 2011?

A. USA and Japan
B. USA and Australia
C. England and France
D. Japan and Australia

gate2013-ae numerical-ability data-interpretation normal

Answer

8.15.2 Data Interpretation: GATE2011-62 top https://gateoverflow.in/2172

P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below:

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A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt?

A. P
B. Q
C. R
D. S

gate2011 numerical-ability data-interpretation normal

Answer

8.15.3 Data Interpretation: GATE2011-GG-GA_9 top https://gateoverflow.in/40210

The quality of services delivered by a company consists of six factors as shown below in the radar diagram. The dots in the figure indicate the score for each factor on a scale of 0 to 10. The standardized coefficient for each factor is given in the parentheses. The contribution of each factor to the overall service quality is directly proportional to the factor score and its standardized coefficient.

The lowest contribution among all the above factors to the overall quality of services delivered by the company is

A. 10%
B. 20%
C. 24%
D. 40%

gate2011-gg difficult numerical-ability data-interpretation

Answer

8.15.4 Data Interpretation: GATE2013-CE-8 top https://gateoverflow.in/40276

Following table provides figures(in rupees) on annual expenditure of a firm for two years -2010 and 2011.

Category	2010	2011

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Raw material	5200	6240
Power & fuel	7000	9450
Salary & wages	9000	12600
Plant & machinery	20000	25000
Advertising	15000	19500
Research & Development	22000	26400

In 2011 , which of the two categories have registered increase by same percentage?

A. Raw material and Salary & wages.
B. Salary & wages and Advertising.
C. Power & fuel and Advertising.
D. Raw material and research & Development.

numerical-ability gate2013-ce data-interpretation normal

Answer

8.15.5 Data Interpretation: GATE2014-1-GA-9 top https://gateoverflow.in/777

In a survey, 300 respondents were asked whether they own a vehicle or not. If yes, they were further asked to mention whether they own a car or scooter or both. Their responses are tabulated below. What percent of respondents do not own a scooter?

		Men	Women
Own vehicle	Car	40	34
	Scooter	30	20
	Both	60	46
Do not own vehicle		20	50

gate2014-1 numerical-ability normal numerical-answers data-interpretation

Answer

8.15.6 Data Interpretation: GATE2014-2-GA-9 top https://gateoverflow.in/1951

The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students doubled in 2009, by what percent did the number of male students increase in 2009?

gate2014-2 numerical-ability data-interpretation numerical-answers normal

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Answer

8.15.7 Data Interpretation: GATE2014-3-GA-5 top https://gateoverflow.in/2028

The table below has question-wise data on the performance of students in an examination. The marks for each question are also listed. There is no negative or partial marking in the examination.

Q No.	Marks	Answered Correctly	Answered Wrongly	Not Attempted
1	2	21	17	6
2	3	15	27	2
3	2	23	18	3

What is the average of the marks obtained by the class in the examination?

A. 1.34
B. 1.74
C. 3.02
D. 3.91

gate2014-3 numerical-ability normal data-interpretation

Answer

8.15.8 Data Interpretation: GATE2014-3-GA-9 top https://gateoverflow.in/2033

The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students in 2011 and 2012 is equal, what is the ratio of male students in 2012 to male students in 2011?

A. $1:1$
B. $2:1$
C. $1.5:1$
D. $2.5:1$

gate2014-3 numerical-ability data-interpretation normal

Answer

8.15.9 Data Interpretation: GATE2014-AE-10 top https://gateoverflow.in/40310

The monthly rainfall chart based on 50 years of rainfall in Agra is shown in the following figure.

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Which of the following are true? (k percentile is the value such that k percent of the data fall below that value)

i. On average, it rains more in July than in December
ii. Every year, the amount of rainfall in August is more than that in January
iii. July rainfall can be estimated with better confidence than February rainfall
iv. In August, there is at least 500 mm of rainfall

A. (i) and (ii)
B. (i) and (iii)
C. (ii) and (iii)
D. (iii) and (iv)

gate-2014-ae numerical-ability data-interpretation

Answer

8.15.10 Data Interpretation: GATE2015-1-GA-6 top https://gateoverflow.in/8010

The number of students in a class who have answered correctly, wrongly, or not attempted each question in an exam, are listed in the table below. The marks for each question are also listed. There is no negative or partial marking.

Q No.	Marks	Answered Correctly	Answered Wrongly	Not Attempted
1	2	21	17	6
2	3	15	27	2
3	1	11	29	4
4	2	23	18	3
5	5	31	12	1

What is the average of the marks obtained by the class in the examination?

A. 2.290
B. 2.970
C. 6.795
D. 8.795

gate2015-1 numerical-ability easy data-interpretation

Answer

8.15.11 Data Interpretation: GATE2015-1-GA-9 top https://gateoverflow.in/8013

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The pie chart below has the breakup of the number of students from different departments in an engineering college for the year 2012. The proportion of male to female students in each department is $5:4$. There are 40 males in Electrical Engineering. What is the difference between the numbers of female students in the civil department and the female students in the Mechanical department?

gate2015-1 numerical-ability data-interpretation numerical-answers

Answer

8.15.12 Data Interpretation: GATE2015-3-GA-10 top https://gateoverflow.in/8389

The exports and imports (in crores of $Rs.$) of a country from the year 2000 to 2007 are given in the following bar chart. In which year is the combined percentage increase in imports and exports the highest?

gate2015-3 numerical-ability data-interpretation normal numerical-answers

Answer

8.15.13 Data Interpretation: GATE2016-1-GA06 top https://gateoverflow.in/39616

A shaving set company sells 4 different types of razors- Elegance, Smooth, Soft and Executive.

Elegance sells at $\text{Rs. } 48$, Smooth at $\text{Rs. } 63$, Soft at $\text{Rs. } 78$ and Executive at $\text{Rs. } 173$ per piece. The table below shows the numbers of each razor sold in each quarter of a year.

Quarter/ Product	Elegance	Smooth	Soft	Executive
$Q1$	27300	20009	17602	9999
$Q2$	25222	19392	18445	8942
$Q3$	28976	22429	19544	10234
$Q4$	21012	18229	16595	10109

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Which product contributes the greatest fraction to the revenue of the company in that year?

A. Elegance
B. Executive
C. Smooth
D. Soft

gate2016-1 numerical-ability data-interpretation easy

Answer

8.15.14 Data Interpretation: GATE2016-2-GA-10 top https://gateoverflow.in/39535

A. $f(x) = 1 - |x - 1|$
B. $f(x) = 1 + |x - 1|$
C. $f(x) = 2 - |x - 1|$
D. $f(x) = 2 + |x - 1|$

gate2016-2 numerical-ability data-interpretation normal

Answer

8.15.15 Data Interpretation: GATE2017-1-GA-10 top https://gateoverflow.in/118413

A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot. If in a flood, the water level rises to 525 m, which of the villages P, Q, R, S, T get submerged?

A. P, Q
B. P, Q, T
C. R, S, T
D. Q, R, S

gate2017-1 numerical-ability data-interpretation normal

Answer

8.15.16 Data Interpretation: GATE2017-2-GA-10 top https://gateoverflow.in/118424

An air pressure contour line joins locations in a region having the same atmospheric pressure. The following is an air pressure contour plot of a geographical religion. Contour lines are shown at 0.05

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bar intervals in this plot.

If the possibility of a thunderstorm is given by how fast air pressure rises or drops over a region, which of the following regions is most likely to have a thunderstorm?

A. P
B. Q
C. R
D. S

gate2017-2 numerical-ability data-interpretation normal

Answer

8.15.17 Data Interpretation: GATE_IN_61 top

https://gateoverflow.in/31386

The fuel consumed by a motor cycle during a journey while travelling at various speeds is indicated in the graph below:

The distance covered during four laps of the journey are listed in the table below:

Lap	Distance (Kilometeres)	Average Speed (kilometeres per hour)
P	15	15
Q	74	45
R	40	75
S	10	10

From the given data, we can conclude that the fuel consumed per kilometre was least during the lap

A. P
B. Q
C. R
D. S

gate2011-in numerical-ability data-interpretation normal

Answer

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Answers: Data Interpretation

8.15.1 Data Interpretation: GATE-2013-AE-GA-7 top https://gateoverflow.in/40248

Selected Answer

Total number of tourists who visited India in $2011 = 13500$

One third of total tourists $= 13500 \div 3 = 4500$

OPTION A: USA and Japan $= 4400$

OPTION B: USA and Australia $= 4300$

OPTION D: Japan and Australia $= 4700$

OPTION C: England and France $= 4500$ which is one third of total tourists. **Hence C is the answer.**

6 votes -- sonam vyas (14.8k points)

8.15.2 Data Interpretation: GATE2011-62 top https://gateoverflow.in/2172

Selected Answer

Answer is D.

As per the question, it is quite clear that the danger of a microbe to human being will be directly proportional to potency and growth. At the same time it is inversely proportional to toxicity, defined as(more dangerous will a microbe be if lesser of its milligram is required).

So, $\text{Level Of Danger (D)} \propto \text{Growth (G)} \propto \text{Potency (P)} \frac{1}{\propto} \text{Toxicity (T)}$

$$D = \frac{KGP}{T}$$

where K is contant of proportionality.

So level of danger will be maximum for S.

Given by,

$$DS = \frac{0.8 \times \pi(10)^2}{200}$$
$$= 1.256$$

Similar calculations for D_P, D_Q, D_R can be done. Which will consequently lead to D_S being the most dangerous and hence will be targeted first.

23 votes -- Gate Keeda (19.6k points)

8.15.3 Data Interpretation: GATE2011-GG-GA_9 top https://gateoverflow.in/40210

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Selected Answer

Answer should be A.

As given in question

> The contribution of each factor to the overall service quality is directly proportional to the factor score and its standardized coefficient.

So, $\text{contribution} \propto \text{factor score} \times \text{standardized coefficient}$

1. Contribution of outcome factor
$= Q \times \text{factor score} \times \text{standardized coefficient} = Q \times 6 \times 0.8$
$= 4.8Q = 4.8$ (Here, Q the proportional constant assumed to be $1 \ldots$)
Similarly,
2. Contribution of tangibles $= 5 \times 0.4 = 2$
3. Contribution of reliability $= 6 \times 0.7 = 4.2$
4. Contribution of responsiveness $= 6 \times 0.6 = 3.6$
5. Contribution of empathy $= 4 \times 0.6 = 2.4$
6. Contribution of assurance $= 6 \times 0.5 = 3$

So, we can see here lowest contribution is "2" and total contribution is "
$4.8 + 2 + 4.2 + 3.6 + 2.4 + 3 = 20$"

The lowest contribution among all the above factors to the overall quality of services delivered by the company $= \left(\frac{2}{20}\right) \times 100 = 10\%$

4 votes -- sonam vyas (14.8k points)

8.15.4 Data Interpretation: GATE2013-CE-8 top

https://gateoverflow.in/40276

Selected Answer

Answer is D.

$\%$ increase in Raw material $= \frac{6240 - 5200}{5200} = 20\%$

$\%$ increase in Power & fuel $= \frac{9450 - 7000}{7000} = 35\%$

$\%$ increase in Salary & wages $= \frac{9000 - 12600}{9000} = 40\%$

$\%$ increase in Plant & machinery $= \frac{20000 - 25000}{20000} = 25\%$

$\%$ increase in Advertising $= \frac{19500 - 15000}{15000} = 30\%$

$\%$ increase in Research & Developement $= \frac{26400 - 22000}{22000} = 20\%$

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3 votes -- richa07 (957 points)

8.15.5 Data Interpretation: GATE2014-1-GA-9 top https://gateoverflow.in/777

Selected Answer

Not having scooter from Men $(40\ (\text{car owner}) + 20\ (\text{nothing owns})) = 60$

Not having scooter from Women $(34\ (\text{car owner}) + 50\ (\text{nothing owns})) = 84$

percentage = $\frac{(60+84)}{300} = 48$ i.e. 48%

19 votes -- Palash Nandi (1.5k points)

8.15.6 Data Interpretation: GATE2014-2-GA-9 top https://gateoverflow.in/1951

Selected Answer

In 2008 $\frac{M}{F}$ ratio is 2.5

Assume 250 Males, 100 Females.

In 2009 $\frac{M}{F}$ ratio is 3. Also total no of females doubled

Females $=100 \times 2 = 200.$

So,

$$\frac{M}{F} = 3$$

$$\frac{M}{200} = 3$$

$$M = 200 \times 3 = 600.$$

Increase in Male Students $=600 - 250 = 350$

increase = $\left(\frac{350}{250}\right) \times 100\% = 140\%$

12 votes -- Akash Kanase (42.5k points)

8.15.7 Data Interpretation: GATE2014-3-GA-5 top https://gateoverflow.in/2028

Selected Answer

The question paper has only 3 questions. There are 3 ways a student can attempt a question.

1. Attempt correctly
2. Attempt wrongly
3. Not attempted

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Each row lists number of students who attempted the question correctly , wrongly and who did not attempt.
Sum of these, of any row gives the total strength of class , $21+17+6=44$.

Average marks obtained by the class in the examination,

$$= \frac{(\text{Total marks(Q1)} + \text{Total marks(Q2)} + \text{Total marks(Q3)})}{\text{Total strength of class}}$$
$$= \frac{((2\times 21)+(3\times 15)+(2\times 23))}{44}$$
$$= \frac{133}{44} = 3.02$$

Note : There is no negative or partial marking for the wrongly attempted and the non-attempted questions.

13 votes -- Srinath Jayachandran (3.7k points)

8.15.8 Data Interpretation: GATE2014-3-GA-9 top https://gateoverflow.in/2033

Selected Answer

C) 1.5 : 1

2011

m_1 = no of male students
f_1 = no of female students

$$\frac{m_1}{f_1} = 1$$

2012

m_2 = no of male students
f_2 = no of female students

$$\frac{m_2}{f_2} = 1.5$$

Given : $f_1 = f_2 = f$

So , $\frac{m_1}{f} = 1$ and $\frac{m_2}{f} = 1.5$
$f = \frac{m_1}{1}$ and $f = \frac{m_2}{1.5}$

Both the equations can be equated..

$$\frac{m_1}{1} = \frac{m_2}{1.5}$$

Ratio of male students in 2012 to male students in 2011 $\Rightarrow \frac{m_2}{m_1} = \frac{1.5}{1} = 1.5 : 1$

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12 votes -- Srinath Jayachandran (3.7k points)

8.15.9 Data Interpretation: GATE2014-AE-10 top https://gateoverflow.in/40310

Selected Answer

1st statement true since average of July is more than that of December in the Average plot

2nd statement - not true since, we have data till 95% only -- in all data of 50 years, amount of rainfall is below the value presented by graph in 95% cases. So in the remaining 5% of years, there can be a year where it rains more in January than August

3rd statement true- since difference between $5-\text{percentile}$ plot and $95-\text{percentile}$ plot indicates that this was range for 90% of years. In February this range is bigger compared to July.

4th statement - not necessarily true. Here, the $5-\text{percentile}$ plot is touching $500mm$ level. But any year in this 5% could have got a lower level of rainfall.

So, answer is B.

8 votes -- Pavan Dongare (187 points)

8.15.10 Data Interpretation: GATE2015-1-GA-6 top https://gateoverflow.in/8010

Selected Answer

Avg. mark $=\dfrac{(21\times 2+15\times 3+11\times 1+23\times 2+31\times 5)}{(21+17+6)}$

$=\dfrac{(42+45+11+46+155)}{44}$

$=\dfrac{299}{44}$

$=6.795$

11 votes -- Arjun Suresh (348k points)

8.15.11 Data Interpretation: GATE2015-1-GA-9 top https://gateoverflow.in/8013

Selected Answer

Number of female students in Electrical = $40\times\dfrac{4}{5}=32$

Number of female students in Civil = $32\times\dfrac{30}{20}=48$ (Since proportion of male students to female students is same, the breakup chart is the same for number of female students)

Number of female students in Mechanical = $32\times\dfrac{10}{20}=16$

So, answer = $48-16=32$

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16 votes -- Arjun Suresh (348k points)

8.15.12 Data Interpretation: GATE2015-3-GA-10 top https://gateoverflow.in/8389

Selected Answer

Answer is 2006

No need to do any calculations ,just observe the bar chart.

Here %increase is asked, so just check for previous year & next year pair, where you can find maximum positive change.

Consider year 2006.

Red Bar is above 3 steps than 2005 AND Blue Bar is 3 steps more than 2005, total $= 3 + 3 = 6$, it's maximum.

17 votes -- Rohan Mundhey (3.5k points)

8.15.13 Data Interpretation: GATE2016-1-GA06 top https://gateoverflow.in/39616

Selected Answer

To calculate the revenue generated by each product in a year we need to first calculate total units of each category sold, and then multiply it with the cost per unit.

Revenue generated by products

	Total units sold	Price per unit	Total revenue
Elegance	102,510	48	4,920,480
Smooth	80,059	63	5,043,717
Soft	72,186	78	5,630,508
Executive	39,284	173	6,796,132

From the table above we can see that the max revenue is generated by Executive.

Hence **answer is B**.

11 votes -- ryan sequeira (3.4k points)

8.15.14 Data Interpretation: GATE2016-2-GA-10 top https://gateoverflow.in/39535

Selected Answer

Answer is Option C

The equation of line, from coordinates $(1, 2)$ to $(3, 0)$, where $|x - 1| = (x - 1)$

$$(y - 2) = \frac{(0 - 2)}{(3 - 1)}(x - 1)$$

$$y = 2 - (x - 1)$$

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$y = 2 - |x - 1|$

The equation of line, from coordinates $(-3, -2)$ to $(1, 2)$, where $|x - 1| = -(x - 1)$

$$(y - (-2)) = \frac{(2 - (-2))}{(1 - (-3))}(x - (-3))$$

$y = x + 1$

$y = 2 - (-(x - 1))$

$y = 2 - |x - 1|$

Note : Equation of line when two coordinates (x_2, y_2) and (x_1, y_1) are given is

$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

15 votes -- Praveen Saini (54.5k points)

8.15.15 Data Interpretation: GATE2017-1-GA-10 top https://gateoverflow.in/118413

Selected Answer

Height of each place will be (in meters)

$$600 > P > 550, \quad 550 > Q > 525, \quad 500 > R > 450, \quad 450 > S > 425, \quad 525 > T > 500$$

Only $R, S, T < 525$

Hence most suitable answer will be (c) R,S,T

These places will definitely be submerged if water level rises to $525m$

20 votes -- VIPIN NARAYAN (34.1k points)

8.15.16 Data Interpretation: GATE2017-2-GA-10 top https://gateoverflow.in/118424

Selected Answer

More Pressure gradient (variation of pressure) causes more thunderstorm possibility. In the region, R contour lines are very dense and giving more pressure gradient (notice two of them are very close). ($\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}$, one of these values will be very high in comparison to other regions).

R is the answer.

14 votes -- Debashish Deka (56.6k points)

8.15.17 Data Interpretation: GATE_IN_61 top https://gateoverflow.in/31386

Selected Answer

This is not very hard question, You just got to do table like the below (It is 2 mark question)

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Lap	Distance	Speed	Mileage	Total Petrol Used	Fuel Consumed Per KM
P	15	15	60	250ml	16.66ml
Q	75	45	90	833ml	11.11ml
R	40	75	75	533ml	13.33ml
S	10	10	30	333ml	33.33ml

So, answer is Q

You get speed, Mileage, Distance from given diagram. You can easily calculate

Total Petrol used in ltr = Distance / Mileage

Per Km => Petrol Used / Mileage.

5 votes -- Akash Kanase (42.5k points)

8.16 Direction Sense(4) top

8.16.1 Direction Sense: GATE2014-AG-GA9 top

https://gateoverflow.in/41673

X is 1 km northeast of Y. Y is 1 km southeast of Z. W is 1 km west of Z. P is 1 km south of W. Q is 1 km east of P. What is the distance between X and Q in km?

A. 1
B. $\sqrt{2}$
C. $\sqrt{3}$
D. 2

gate2014-ag numerical-ability direction-sense normal

Answer

8.16.2 Direction Sense: GATE2015-2-GA-7 top

https://gateoverflow.in/8036

Four branches of a company are located at M, N, O and P. M is north of N at a distance of $4km$; P is south of O at a distance of $2\ km$; N is southeast of O by $1km$. What is the distance between M and P in km?

A. 5.34
B. 6.74
C. 28.5
D. 45.49

gate2015-2 numerical-ability normal direction-sense

Answer

8.16.3 Direction Sense: GATE2016-Session-3-GA-9 top

https://gateoverflow.in/108726

M and N start from the same location. M travels 10 km East and then 10 km North-East. N travels 5 km South and then 4 km South-East. What is the shortest distance (in km) between M and N at the end of their travel?

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A. 18.60
B. 22.50
C. 20.61
D. 25.00

gate2016session3aptitude direction-sense numerical-ability

Answer

8.16.4 Direction Sense: GATE2017-2-GA-3 top

https://gateoverflow.in/118417

There are five buildings called V, W, X, Y and Z in a row (not necessarily in that order). V is to the West of W. Z is to the East of X and the West of V. W is to the West of Y$. Which is the building in the middle?

A. V
B. W
C. X
D. Y

gate2017-2 numerical-ability direction-sense normal

Answer

Answers: Direction Sense

8.16.1 Direction Sense: GATE2014-AG-GA9 top

https://gateoverflow.in/41673

Selected Answer

distance between XQ is $\sqrt{(1^2 + (\sqrt{2})^2)} = \sqrt{3}$

9 votes -- srestha (86.6k points)

8.16.2 Direction Sense: GATE2015-2-GA-7 top

https://gateoverflow.in/8036

Selected Answer

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$$a = \frac{1}{\sqrt{2}}$$

$$x = \sqrt{[4 + (2 - a)]^2 + a^2}$$

Solving we get x = 5.34.

15 votes -- Arjun Suresh (348k points)

8.16.3 Direction Sense: GATE2016-Session-3-GA-9 top

Selected Answer

https://gateoverflow.in/108726

$\underline{\text{In } \triangle \text{ DEH}}$

$$\sin 45 = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{EH}{4}$$

$$\Rightarrow EH = 2\sqrt{2}$$

$$\cos 45 = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{DE}{4}$$

$$\Rightarrow DE = 2\sqrt{2}$$

$\underline{\text{In } \triangle \text{ BJC}}$

$$\sin 45 = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{CJ}{10}$$

$$\Rightarrow CJ = 5\sqrt{2}$$

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$$\cos 45 = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{BJ}{10}$$

$$\Rightarrow BJ = 5\sqrt{2}$$

Required Shortest Distance HC

Using Pythagoras's theorem in Triangle $\triangle HIC$

$$HC = \sqrt{(HI)^2 + (CI)^2}$$

Where as ,

$$HI = HL + LI$$

From Figure, $HL = 10 - 2\sqrt{2}$ and $LI = 5\sqrt{2}$

$$HI = 10 - 2\sqrt{2} + 5\sqrt{2} = 10 + 3\sqrt{2}$$

$$CI = CJ + JG + GI$$

Fom Figure , $CJ = 5\sqrt{2}$, $JG = 5$, $GI = 2\sqrt{2}$

$$CI = 5\sqrt{2} + 5 + 2\sqrt{2} = 5 + 7\sqrt{2}$$

Therefore ,

$$HC = \sqrt{(10 + 3\sqrt{2})^2 + (5 + 7\sqrt{2})^2} = 20.61\text{km}$$

4 votes -- pC (22k points)

8.16.4 Direction Sense: GATE2017-2-GA-3 top

https://gateoverflow.in/118417

Selected Answer

Ans: **A. V**

Given order: (West on left)
$VW, \quad XZ, \quad ZV, \quad WY$

Considering them together:
$XZ, \quad ZV \implies XZV$
$VW, \quad WY \implies VWY$
$XZV, \quad VWY \implies XZVWY$

So, V is the building in the middle.

16 votes -- Orochimaru (581 points)

8.17 Factors(7) top

8.17.1 Factors: GATE2013-62 top

https://gateoverflow.in/1566

Out of all the 2-digit integers between 1 and 100, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7 ?

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A. $\left(\frac{13}{90}\right)$
B. $\left(\frac{12}{90}\right)$
C. $\left(\frac{78}{90}\right)$
D. $\left(\frac{77}{90}\right)$

gate2013 numerical-ability easy probability factors

Answer

8.17.2 Factors: GATE2014-2-GA-4 top

https://gateoverflow.in/1941

What is the average of all multiples of 10 from 2 to 198?

A. 90
B. 100
C. 110
D. 120

gate2014-2 numerical-ability easy numerical-computation factors

Answer

8.17.3 Factors: GATE2018-GA-4 top

https://gateoverflow.in/204065

What would be the smallest natural number which when divided either by 20 or by 42 or by 76 leaves a remainder of 7 in each case?

A. 3047
B. 6047
C. 7987
D. 63847

gate2018 numerical-ability factors

Answer

8.17.4 Factors: ISI2012-A-1b top

https://gateoverflow.in/47823

How many 0's are there at the end of $50!$?

descriptive isi2012 numerical-ability factors numerical-computation numerical-answers

Answer

8.17.5 Factors: TIFR2010-A-20 top

https://gateoverflow.in/18500

How many integers from 1 to 1000 are divisible by 30 but not by 16?

A. 29
B. 31
C. 32
D. 33
E. 25

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tifr2010 numerical-ability factors

Answer

8.17.6 Factors: TIFR2011-A-15 top

https://gateoverflow.in/20226

The exponent of 3 in the product $100!$ is

a. 27
b. 33
c. 44
d. 48
e. None of the above.

tifr2011 numerical-ability factors tricky

Answer

8.17.7 Factors: TIFR2013-A-12 top

https://gateoverflow.in/25434

Among numbers 1 to 1000 how many are divisible by 3 or 7?

a. 333
b. 142
c. 475
d. 428
e. None of the above.

tifr2013 numerical-ability factors normal

Answer

Answers: Factors

8.17.1 Factors: GATE2013-62 top

https://gateoverflow.in/1566

Selected Answer

The number of 2 digit multiples of $7 = 13$

not divisible by 7=\dfrac{(90-13)}{90}=\dfrac{77}{90}$

14 votes -- Bhagirathi Nayak (14.1k points)

8.17.2 Factors: GATE2014-2-GA-4 top

https://gateoverflow.in/1941

Selected Answer

$a = 10, l = 190$

$$s = \frac{n(a+l)}{2} = \frac{19 \times (200)}{2} = 1900$$

$$\text{Average} = \frac{1900}{19} = 100$$

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Ans. is 100

13 votes -- aditi (319 points)

8.17.3 Factors: GATE2018-GA-4 top

https://gateoverflow.in/204065

Selected Answer

LCM$(20, 42, 76)$ = 7980

Remainder is given 7

NUMBER=7980+7=7987

Option - C
7987

7 votes -- Sandip Shaw (1.3k points)

8.17.4 Factors: ISI2012-A-1b top

https://gateoverflow.in/47823

Selected Answer

We get a 0 at end of a number if the number is divisible by 10. In terms of prime numbers this means a number being divisible by each pair of (2, 5) we get a 0 at end of it. When we consider factorial, number of 2's will always be greater than number of 5's. So, we just need to count the no. of powers of 5.

Number of 0 at the end of $50!$ will be = $\frac{50}{5}$ + $\frac{50}{25}$ + $\frac{50}{125}$

= 10 + 2 + 0

= 12

9 votes -- Muktinath Vishwakarma (35.3k points)

8.17.5 Factors: TIFR2010-A-20 top

https://gateoverflow.in/18500

Selected Answer

Option A. i.e., 29 should be the answer.

Number of integers between 1 to 1000 that are divisible by $30 = \lfloor \frac{1000}{30} \rfloor = 33.$

Now, since LCM of $30, 16 = 240$, only the numbers that are divisible by 240 between 1 to 1000 will divisible by both 30 & 16.
So, number of integers between 1 to 1000 that are divisible by 240 (i.e., divisible by both 30 and $16 = \lfloor \frac{1000}{240} \rfloor = 4.$

So, total number of integers that are divisible by 30 but not divisible by $16 = 33 - 4 = 29.$

10 votes -- Anurag Pandey (13.6k points)

8.17.6 Factors: TIFR2011-A-15 top

https://gateoverflow.in/20226

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Selected Answer

Exponent of p in $n!$, where p is a prime number, and n is an integer greater than p is:

$$E_p(n!) = \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \cdots + \left\lfloor \frac{n}{p^S} \right\rfloor$$

where S is the largest positive integer such that $p^s \leq n \leq p^{s+1}$

So,

$$E_3(100!) = \left\lfloor \frac{100}{3} \right\rfloor + \left\lfloor \frac{100}{3^2} \right\rfloor + \left\lfloor \frac{100}{3^3} \right\rfloor + \left\lfloor \frac{100}{3^4} \right\rfloor$$

$$= 33 + 11 + 3 + 1$$
$$= 48$$

8 votes -- sonu (2.3k points)

8.17.7 Factors: TIFR2013-A-12 top

https://gateoverflow.in/25434

Selected Answer

Divisible by $3 = \frac{1000}{3} = 333$

Divisible by $7 = \frac{1000}{7} = 142$

Divisible by both = $\frac{1000}{\text{LCM OF } 3 \,\&\, 7} = \frac{1000}{21} = 47$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 333 + 142 - 47 = 428.$$

5 votes -- Umang Raman (15.7k points)

8.18 Fractions(4) top

8.18.1 Fractions: GATE2016-Session-1-GA-9 top

https://gateoverflow.in/108093

If $q^{-a} = \frac{1}{r}$ and $r^{-b} = \frac{1}{s}$ and $s^{-c} = \frac{1}{q}$, the value of abc is _____.

A. $(rqs)^{-1}$
B. 0
C. 1
D. $r + q + s$

gate2016session1aptitude numerical-ability fractions

Answer

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8.18.2 Fractions: GATE2018-GA-6 top

https://gateoverflow.in/204067

In appreciation of the social improvements completed in a town, a wealthy philanthropist decided to gift $Rs\ 750$ to each male senior citizen in the town and $Rs\ 1000$ to each female senior citizen. Altogether, there were 300 senior citizens eligible for this gift. However, only $\frac{8}{9}^{th}$ of the eligible men and $\frac{2}{3}^{rd}$ of the eligible women claimed the gift.How much money (in Rupees) did the philanthropist give away in total?

A. $1,50,000$
B. $2,00,000$
C. $1,75,000$
D. $1,51,000$

gate2018 numerical-ability fractions normal

Answer

8.18.3 Fractions: TIFR2014-A-11 top

https://gateoverflow.in/26329

A large community practices birth control in the following peculiar fashion. Each set of parents continues having children until a son is born; then they stop. What is the ratio of boys to girls in the community if, in the absence of birth control, 51% of the babies are born male?

a. $51:49$
b. $1:1$
c. $49:51$
d. $51:98$
e. $98:51$

tifr2014 numerical-ability fractions tricky

Answer

8.18.4 Fractions: TIFR2017-A-1 top

https://gateoverflow.in/94931

A suitcase weighs one kilogram plus half of its weight. How much does the suitcase weigh?

A. $1.3333...$ kilograms
B. 1.5 kilograms
C. $1.666...$ kilograms
D. 2 kilograms
E. cannot be determined from the given data

tifr2017 numerical-ability fractions normal

Answer

Answers: Fractions

8.18.1 Fractions: GATE2016-Session-1-GA-9 top

https://gateoverflow.in/108093

Selected Answer

Given that,

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$q^{-a} = \frac{1}{r}, r^{-b} = \frac{1}{s}, s^{-c} = \frac{1}{q}$

Now, $q^{-a} = \frac{1}{r}$

$\Rightarrow \frac{1}{q^a} = \frac{1}{r}$

$\Rightarrow q^a = r$

Similarly, $r^b = s$ and $s^c = q$

In $q^a = r$, we replace q by s^c and r by $s^{\frac{1}{b}}$

We get, $s^{ca} = s^{\frac{1}{b}}$

Since, the bases are equal, we can work with the exponents.

So, we get, $ac = \frac{1}{b} \Rightarrow abc = 1$

2 votes -- Samujjal Das (9.6k points)

8.18.2 Fractions: GATE2018-GA-6 top

https://gateoverflow.in/204067

Selected Answer

Let no. of senior male be x
Let no. of senior female be y
$x + y = 300$

Total money given $= \left(\frac{8x}{9} \times 750 = \frac{2000x}{3}\right) + \left(\frac{2y}{3} \times 1000 = \frac{2000y}{2}\right)$

$= \frac{2000}{3} \times (x + y)$
$= \frac{2000}{3} \times 300$
$= 200000.$

So, B is correct.

PS: Some people are assuming the number of senior male and female is same, which is wrong. Do not assume anything not mentioned.

17 votes -- Subham Mishra (6.2k points)

8.18.3 Fractions: TIFR2014-A-11 top

https://gateoverflow.in/26329

Selected Answer

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A) should be the correct choice.

In the community we know that each set of parents will have exactly 1 boy.

The number of girls might differ.

To find the ratio of boys to girls in the community, we are going to find the the expected number of girls that each parent set can have.

Henceforth in this question we are going to use the word "family" to denote a "parent set".

Calculation of expected number of girls in any family

Let X be a random variable that denotes the number of girls that any family.Each family will have exactly 1 boy.

$P(X=0)$ will denote : Probability that a family has 0 girls and 1 boy.

$P(X=0)=(0.49)^0(0.51)$

$P(X=3)$ will denote : Probability that a family has
3 girls and
1 boy.

$P(X=3)=(0.49)^3(0.51)$.

and so on.

In general we can say that

$P(X=i)$ will denote the probability that a family has
i girls and
1 boy, and

$P(X=i)=(0.49)^i(0.51)$.

Now the expected number of girls in any family will be denoted by
$E[X]$.

Here

$E[X]=\sum_{i=0}^{\infty} i\cdot P(X=i)$.

but
$P(X=i)=(0.49)^i(0.51)$. so we get,

$E[X]=\sum_{i=0}^{\infty} i\cdot(0.49)^i(0.51)$.

This implies
$E[X]=(0.51)\sum_{i=0}^{\infty} i\cdot(0.49)^i$.

The formula for summation of series of type
$\sum_{k=0}^{\infty} k\cdot x^k$ can be found by differentiating

$\sum_{k=0}^{\infty} x^k$ with respect to
x.(See the reference below).

This gives

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$\sum_{k=0}^{\infty} k \cdot x^k = \frac{x}{(x-1)^2}$

So $\sum_{i=0}^{\infty} i \cdot (0.49)^i = \frac{0.49}{(0.49-1)^2}$

Hence

$E[X] = (0.51) \cdot \frac{0.49}{(0.51)^2} = \frac{0.49}{0.51}$

Now the ratio of boys to girls can be given by number of boys in each family/expected number of girls in each family.

that is $Ratio\,(\text{B to G}) = \frac{1}{E[X]}$

So $Ratio\,(\text{B to G}) = \frac{1}{\frac{0.49}{0.51}} = \frac{51}{49}$

Reference for series summation:

http://math.stackexchange.com/questions/629589/converge-of-the-sum-sum-k-1n-k-xk

8 votes -- Anurag Pandey (13.6k points)

8.18.4 Fractions: TIFR2017-A-1 top

https://gateoverflow.in/94931

Selected Answer

D. 2 Kg

Let x be the weight of suitcase.

A suitcase weighs one kilogram plus half of its weight. So,

$(1 + x/2 = x) \implies x = 2.$

10 votes -- Prajwal Bhat (11.3k points)

8.19 Functions(3) top

8.19.1 Functions: GATE2012-AR-7 top

https://gateoverflow.in/40228

Let $f(x) = x - [x]$, where $x \geq 0$ and $[x]$ is the greatest integer not larger than x. Then $f(x)$ is a

A. monotonically increasing function
B. monotonically decreasing function
C. linearly increasing function between two integers
D. linearly decreasing function between two integers

gate2012-ar numerical-ability functions normal

Answer

8.19.2 Functions: GATE2015-3-GA-5 top

https://gateoverflow.in/8303

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A function $f(x)$ is linear and has a value of 29 at $x = -2$ and 39 at $x = 3$. Find its value at $x = 5$.

A. 59
B. 45
C. 43
D. 35

gate2015-3 numerical-ability normal functions

Answer

8.19.3 Functions: GATE2015-3-GA-8 top

https://gateoverflow.in/8385

Choose the most appropriate equation for the function drawn as thick line, in the plot below.

A. $x = y - |y|$
B. $x = -(y - |y|)$
C. $x = y + |y|$
D. $x = -(y + |y|)$

gate2015-3 numerical-ability normal functions

Answer

Answers: Functions

8.19.1 Functions: GATE2012-AR-7 top

https://gateoverflow.in/40228

Selected Answer

$f(x) = x - [x]$
$f(x)$ gives nothing but the fractional part of x.
Now, as we move up the number line between two consecutive integers say a and $a+1$.
The value of $f(a)$ starts from 0 and grows linearly till the consecutive integer and just before $x = a+1$(left neighbourhood of $a+1$) it tends to the value 1.

Between a and $a+1$, the graph grows linearly from 0 to 1.
But, when at $x = a+1$, the value comes back to 0 and same linear graph continues between the next two consecutive integers and so on.
So, **Answer C) linearly increasing function between two integers.**

12 votes -- Abhilash Panicker (9.4k points)

8.19.2 Functions: GATE2015-3-GA-5 top

https://gateoverflow.in/8303

Selected Answer

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$f(x)$ is linear means it is of the form $ax+b$

given $f(-2)$ and $f(3)$

solve the equation and find out value for a and b then find $f(5)$. it will be 43

15 votes -- Sankaranarayanan P.N (11.5k points)

8.19.3 Functions: GATE2015-3-GA-8 top

https://gateoverflow.in/8385

Selected Answer

When y is -1, x is 2.

When y is positive x is 0.

So, $x=-(y-|y|)$

19 votes -- Arjun Suresh (348k points)

8.20 Geometry(12) top

8.20.1 Geometry: GATE2014-1-GA-10 top

https://gateoverflow.in/778

When a point inside of a tetrahedron (a solid with four triangular surfaces) is connected by straight lines to its corners, how many (new) internal planes are created with these lines?

gate2014-1 numerical-ability geometry permutations-and-combinations normal numerical-answers

Answer

8.20.2 Geometry: GATE2015-2-GA-8 top

https://gateoverflow.in/8039

In a triangle PQR, PS is the angle bisector of $\angle QPR$ and $\angle QPS = 60^\circ$. What is the length of PS ?

A. $\left(\frac{(q+r)}{qr}\right)$

B. $\left(\frac{qr}{q+r}\right)$

C. $\sqrt{(q^2+r^2)}$

D. $\left(\frac{(q+r)^2}{qr}\right)$

gate2015-2 numerical-ability geometry difficult

Answer

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8.20.3 Geometry: GATE2016-1-GA05 top https://gateoverflow.in/39610

A cube is built using 64 cubic blocks of side one unit. After it is built, one cubic block is removed from every corner of the cube. The resulting surface area of the body (in square units) after the removal is _____.

a. 56
b. 64
c. 72
d. 96

gate2016-1 numerical-ability geometry normal

Answer

8.20.4 Geometry: GATE2016-Session-2-GA-1 top https://gateoverflow.in/108240

The volume of a sphere of diameter 1 unit is _____ than the volume of a cube of side 1 unit.

A. Least
B. Less
C. Lesser
D. Low

gate2016session2aptitude geometry numerical-ability

Answer

8.20.5 Geometry: GATE2016-Session-2-GA-5 top https://gateoverflow.in/108289

A window is made up of a square portion and an equilateral triangle portion above it. The base of the triangular portion coincides with the upper side of the square. If the perimeter of the window is 6 m, the area of the window in m^2 is _____.

A. 1.43
B. 2.06
C. 2.68
D. 2.88

gate2016session2aptitude numerical-ability geometry

Answer

8.20.6 Geometry: GATE2016-Session-3-GA-10 top https://gateoverflow.in/108729

A wire of length 340 mm is to be cut into two parts. One of the parts is to be made into a square and the other into a rectangle where sides are in the ratio of $1:2$. What is the length of the side of the square (in mm) such that the combined area of the square and the rectangle is a **MINIMUM**?

A. 30
B. 40
C. 120
D. 180

gate2016session3aptitude geometry numerical-ability

Answer

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8.20.7 Geometry: GATE2016-Session-7-GA-9 top https://gateoverflow.in/110921

A square pyramid has a base perimeter x, and the slant height is half of the perimeter. What is the lateral surface area of the pyramid

A. x^2
B. $0.75x^2$
C. $0.50x^2$
D. $0.25x^2$

gate2016session7aptitude geometry numerical-ability

Answer

8.20.8 Geometry: GATE2018-GA-9 top https://gateoverflow.in/204070

In the figure below, $\angle DEC + \angle BFC$ is equal to _____

A. $\angle BCD - \angle BAD$
B. $\angle BAD + \angle BCF$
C. $\angle BAD + \angle BCD$
D. $\angle CBA + \angle ADC$

gate2018 numerical-ability geometry normal

Answer

8.20.9 Geometry: TIFR2010-A-17 top https://gateoverflow.in/18493

Suppose there is a sphere with diameter **at least** 6 inches. Through this sphere we drill a hole along a diameter. The part of the sphere lost in the process of drilling the hole looks like two caps joined to a cylinder, where the cylindrical part has length 6 inches. It turns out that the volume of the remaining portion of the sphere does not depend on the diameter of the sphere. Using this fact, determine the volume of the remaining part.

1. 24π cu. inches
2. 36π cu. inches
3. 27π cu. inches
4. 32π cu. inches
5. 35π cu. inches

tifr2010 numerical-ability geometry

Answer

8.20.10 Geometry: TIFR2012-A-4 top https://gateoverflow.in/20984

Let ABC be a triangle with n distinct points inside. A triangulation of ABC with respect to the n

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points is obtained by connecting as many points as possible, such that no more line segments can be added without intersecting other line segments. In other words ABC has been partitioned into triangles with end points at the n points or at the vertices $\mathrm{A,B,C}$. For example, the following figure gives one possible triangulation of ABC with two points inside it.

Although there are many different ways to triangulate ABC with the n points inside, the number of triangles depends only on n. In the above figure it is five. How many triangles are there in a triangulation of ABC with n points inside it?

A. $3n-1$
B. n^2+1
C. $n+3$
D. $2n+1$
E. $4n-3$

tifr2012 numerical-ability geometry

Answer

8.20.11 Geometry: TIFR2015-A-2 top

https://gateoverflow.in/29158

Consider a circle with a circumference of one unit length. Let $d<\frac{1}{6}$. Suppose that we independently throw two arcs, each of length d, randomly on this circumference so that each arc is uniformly distributed along the circle circumference. The arc attaches itself exactly to the circumference so that arc of length d exactly covers length d of the circumference. What can be said about the probability that the two arcs do not intersect each other?

A. It equals $(1-d)$
B. It equals $(1-3d)$
C. It equals $(1-2d)$
D. It equals 1
E. It equals $(1-d)\ (1-d)$

tifr2015 geometry

Answer

8.20.12 Geometry: TIFR2017-A-8 top

https://gateoverflow.in/95039

In a tutorial on geometrical constructions, the teacher asks a student to construct a right-angled triangle ABC where the hypotenuse BC is 8 inches and the length of the perpendicular dropped from A onto the hypotenuse is h inches, and offers various choices for teh value of h. For which value of h can such a triangle NOT exist?

A. 3.90 inches
B. $2\sqrt{2}$ inches
C. $2\sqrt{3}$ inches

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D. 4.1 inches
E. none of the above

tifr2017 numerical-ability geometry

Answer

Answers: Geometry

8.20.1 Geometry: GATE2014-1-GA-10 top

https://gateoverflow.in/778

Selected Answer

It is 6.

Tetrahedron has 4 corner points. So, it forms 4 planes. Now, we add an internal point making a total of 5 points. Any three combination of points leads to a plane and thus we can get ${}^5C_3 = 10$ planes. So, newly created planes = $10 - 4 = 6$.

29 votes -- Palash Nandi (1.5k points)

8.20.2 Geometry: GATE2015-2-GA-8 top

https://gateoverflow.in/8039

Selected Answer

As per **Angle Bisector theorem,**

$$\frac{QS}{SR} = \frac{r}{q}$$
$$\frac{QS}{(p - QS)} = \frac{r}{q}$$
$$QS = \frac{pr}{(q+r)} \qquad \longrightarrow (1)$$

We have in a triangle $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

So, from $\triangle QPS$, $\frac{QS}{\sin 60} = \frac{PS}{\sin Q}$

$$PS = \frac{QS \times \sin Q}{\sin 60} \qquad \longrightarrow (2)$$

From $\triangle PQR$, $\frac{p}{\sin 120} = \frac{q}{\sin Q}$

$$p = \frac{q \times \sin 120}{\sin Q} = \frac{\sin 60}{\sin Q} \qquad \longrightarrow (3)$$

So, from $(1), (2)$ and (3),

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$$PS = \frac{qr}{(q+r)}$$

B choice.

http://en.wikipedia.org/wiki/Angle_bisector_theorem

23 votes -- Arjun Suresh (348k points)

Area of a $\triangle = \frac{1}{2} \times ac \sin B = \frac{1}{2} \times bc \sin A = \frac{1}{2} ab \sin C$

so, Here area $(\triangle PQR) = \text{area } (\triangle PQS) + \text{area } (\triangle PRS)$

$$\frac{1}{2} rq \sin 120 = \frac{1}{2} PS \times r \sin 60 + \frac{1}{2} PS \times q \sin 60$$

$$\Rightarrow PS = \frac{rq}{(r+q)}$$

so, choice **(B)** is **correct**..

33 votes -- Himanshu Agarwal (15.3k points)

8.20.3 Geometry: GATE2016-1-GA05 top https://gateoverflow.in/39610

Selected Answer

Cube is built using 64 cubic blocks of side one unit. then it means it's a $4 * 4 * 4$ Cube.

Surface area of each face $= 4 * 4$sq. units

And number of faces in a cube $= 6$, Thus total surface area $= 6 * 4 * 4 = 96$sq. units.

A cube contains 8 corner pieces (containing 3 visible sides), and if these are removed then inner ones are visible, So, When a corner piece is removed 3 new faces of 1sq. unit are visible and thus removal doesn't change surface area.

Hence, Option (D) is correct.

33 votes -- The_cake (227 points)

8.20.4 Geometry: GATE2016-Session-2-GA-1 top https://gateoverflow.in/108240

Selected Answer

The volume of a sphere of diameter 11 unit is **less** than the volume of a cube of side 11 unit. option B..

2 votes -- Sudhanshu Srivastava (19.6k points)

8.20.5 Geometry: GATE2016-Session-2-GA-5 top https://gateoverflow.in/108289

Selected Answer

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Perimeter is 6 so side length is $\frac{6}{5} = 1.2m$

So area of square is $1.2 \times 1.2 = 1.44$

Area of equilateral triangle $= \frac{\sqrt{3}}{4} \times 1.2 \times 1.2 = 0.62352$

So, total area is $0.62352 + 1.44 = 2.06352$ so answer is B

2 votes -- Pavan Kumar Munnam (10.9k points)

8.20.6 Geometry: GATE2016-Session-3-GA-10 top https://gateoverflow.in/108729

Selected Answer

Let two parts be x and y and part x is used to make square and part y is used to make rectangle.

$x + y = 340$

If x part is used to make square, means it is the parameter of square and each side of square is $\frac{x}{4}$

Similarly, for rectangle $2 * (l + b) = y$ and $l = 2b$

$2 * 3b = y \Rightarrow b = \frac{y}{6}$ and $l = \frac{y}{3}$ (we know $y = 340 - x$)

Question says the combined area of the square and the rectangle is a MINIMUM

$A = (\frac{x}{4})^2 + \frac{340-x}{6} * \frac{340-x}{3}$

For combined area to be minimum $\frac{\mathrm{d}A}{\mathrm{d}y} = 0$

$\Rightarrow \frac{2x}{16} - \frac{2(340-x)}{18} = 0$

$\Rightarrow x = 160$

But question asks for MINIMUM length of side of square that is $\frac{x}{4} = 40mm$

2 votes -- Manish Joshi (27.9k points)

8.20.7 Geometry: GATE2016-Session-7-GA-9 top https://gateoverflow.in/110921

Selected Answer

Given, perimeter of a pyramid is x, so slant height is $\frac{x}{2}$.

Lateral Surface Area of a pyramid

$= \frac{1}{2} \times \text{perimeter} \times \text{slant height}$
$= \frac{1}{2} \times x \times x2$
$= 0.25x^2$

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so option D is correct .

1 votes -- Bikram (67k points)

8.20.8 Geometry: GATE2018-GA-9 top

https://gateoverflow.in/204070

Selected Answer

$\angle z = \angle x + \angle y$

This is triangle property that **"the exterior angle of a triangle is the sum of the two opposite interior angle."**

Similarly,

$\angle y = \angle a + \angle b$

By subsituting we get,

$\angle Z = \angle x + \angle a + \angle b.$

Now $\angle Z = \angle BCD$
$\angle b = \angle BAD.$

$\angle BCD - \angle BAD = \angle Z - \angle b$

$= \angle x + \angle a + \angle b - \angle b$

$= \angle x + \angle a$

$= \angle DEC + \angle BFC$

So **option A** is the answer.

9 votes -- Hemant Parihar (14.8k points)

8.20.9 Geometry: TIFR2010-A-17 top

https://gateoverflow.in/18493

Selected Answer

This is the Napkin ring problem.

The volume of the remaining part is

$$V = \frac{\pi h^3}{6}$$

For $h = 6, V = 36\pi$

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Hence, the answer is option (2).

1 votes -- admin (2.6k points)

8.20.10 Geometry: TIFR2012-A-4 top

https://gateoverflow.in/20984

Selected Answer

Any polygon can be split into triangles, so any n—triangulate for any n will always be composed of triangles.

Given an $(n-1)$ triangulate, we can add the point in the following three ways:

I. **The point lies on a point that is already there.** In this case, the point has already been connected to all possible vertices that it can be connected to (since we started with a $(n-1)$ triangulate). Example:

II. **The point lies on a line that is already there, but not on a point.** In this case, since the line is the common edge of at most 2 triangles, the point can only be connected to 2 vertices (the opposite ends of the triangles). For example:

This creates 4 new triangles, but destroys the original 2 triangles. Thus, the number of triangles increase by 2. This is an optimal case.

III. **The point lies inside of a triangle.** The new point can then be connected to exactly 3 vertices of the bounding triangle. Example:

This creates 3 new triangles, but destroys the original triangle. So, the number of triangles increase by 2. So, this is also an optimal case.

We can see that the nth triangulate has exactly 2 more triangles than the $(n-1)^{th}$ triangulate. **This gives us the following recurrence:**

$$T(n) = T(n-1) + 2, T(2) = 5$$

Which solves to:

$$\boxed{T(n) = 2n + 1}$$

Hence, option d is the correct answer.

3 votes -- Pragy Agarwal (20.6k points)

8.20.11 Geometry: TIFR2015-A-2 top

https://gateoverflow.in/29158

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$(1-2d)$ will be the correct answer.

Two points on the circumference of any circle divides the circle in two arcs, the length of smaller arc must be less than or equal to half of the circumference, & length of the larger arc must be greater than or equal to half the circumference.

but since here given length of the arc under consideration is strictly less than $\frac{1}{2}$, so henceforth in this answer, whenever I will use the term "arc", I'll be referring to the smaller of those two arcs.

Process of Arc drawing: I am going to follow a specific procedure for drawing any arc of length d, which is as follows:

1) Pick any point on the circumference of the circle, this will be the starting point.

2) Move d units CLOCKWISE on the circumference of the circle & mark that point as the finishing point.

Suppose we choose our first arc AB of length d, randomly anywhere on the circumference on the circle.

Here A is the starting point & B is the end point.

After drawing the arc AB, we have to draw another arc PQ on the circle of length d, where P will be the starting point & Q will be the end point.

Now if we have to make sure that arc PQ does not intersects with arc AB, we have to keep following things in mind hile choosing our starting point P:

1) P can not lie within arc AB, otherwise AB and PQ will intersect each other.

2) P can not lie anywhere within the anticlockwise distance d from the point A other wise, end part of arc PQ will intersect with starting part of arc AB.

So we can conclude that **If P lies anywhere on the circumference of the circle within a distance d from A then the arc PQ & AB will intersect.**

So $Probability(\text{Arc PQ does not intersects with Arc AB}) = \frac{\text{P lies atleast d distance away from A}}{\text{P lies anywhere in the circumference}}$

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$$\Rightarrow \textit{Probability}\left(\text{Arc PQ does not intersects with Arc AB}\right) = \frac{1-2d}{1} = (1-2d).$$

4 votes -- Anurag Pandey (13.6k points)

8.20.12 Geometry: TIFR2017-A-8 top

https://gateoverflow.in/95039

D. 4.1 inches

Theorem: Length of Perpendicular drawn to the hypotenuse can't be **greater than half of length of hypotenuse.**

Ref: https://goo.gl/CaL4sc

4 votes -- Prajwal Bhat (11.3k points)

8.21 Limits(1) top

8.21.1 Limits: GATE2016-Session-2-GA-10 top

https://gateoverflow.in/108309

Which of the following curves represents the function $y = In\left(\left|e^{[|\sin(|x|)|]}\right|\right)$ for $|x| < 2\pi$? Here, x represents the abscissa and y represents the ordinate.

A.

B.

C.

D.

gate2016session2aptitude limits calculus numerical-ability

Answer

Answers: Limits

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8.21.1 Limits: GATE2016-Session-2-GA-10 top

https://gateoverflow.in/108309

Selected Answer

$$f(x) = \ln\left(\left|e^{\left[\ |\sin(|x|)|\ \right]}\right|\right)$$

1. $|\mathbf{x}| \rightarrow \ f(x)$ is Even $\rightarrow$ option b not possible

2. $m = |\sin(|\mathbf{x}|)| \ \geq \ 0 \ \rightarrow e^{\mathbf{m}} \ \geq \ 1$

3. $\Rightarrow f(x) = \ln(|e^{\mathbf{m}}|) = \ln(e^{\mathbf{m}}) \geq 0 \quad \{\text{from } 2\} \ \rightarrow$ option a not possible

4. and $f(x)_{x=0} = 0 \ \rightarrow$ option d not possible

$\Rightarrow$ answer C

4 votes -- Debashish Deka (56.6k points)

8.22 Logarithms(2) top

8.22.1 Logarithms: GATE2012-AR-6 top

https://gateoverflow.in/40227

A value of x that satisfies the equation $\log x + \log(x-7) = \log(x+11) + \log 2$ is

A. 1
B. 2
C. 7
D. 11

gate2012-ar numerical-ability numerical-computation logarithms

Answer

8.22.2 Logarithms: TIFR2010-A-9 top

https://gateoverflow.in/18385

A table contains 287 entries. When any one of the entries is requested, it is encoded into a binary string and transmitted. The number of bits required is.

a. 8
b. 9
c. 10
d. Cannot be determined from the given information.
e. None of the above.

tifr2010 numerical-ability theory-of-computation logarithms

Answer

Answers: Logarithms

8.22.1 Logarithms: GATE2012-AR-6 top

https://gateoverflow.in/40227

Selected Answer

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$\log m + \log n = \log mn$

So,
$\log x + \log(x-7) = \log(x+11) + \log 2$

$\Rightarrow \log x(x-7) = \log 2(x+11)$

$\Rightarrow x(x-7) = 2(x+11)$

$\Rightarrow x^2 - 9x - 22 = 0$

$\Rightarrow (x-11)(x+2) = 0$

$\therefore x = 11$

$\because x \neq -2$, log is undefined for negative number.

9 votes -- Praveen Saini (54.5k points)

8.22.2 Logarithms: TIFR2010-A-9 top

https://gateoverflow.in/18385

Selected Answer

D should be the correct answer.

Each Entry represents a row of table,in general.

In order to encode all the information of a row/entry, we must know what is the size of that information.But it is not mentioned in the question.

We can ADDRESS 287 entries by ceiling of $\log 2$(287) = 9 bits.

but we CAN NOT ENCODE an entry, without knowing the size of that entry

So I don't think it is possible to tell how many bits are required to encode & transmit an entry unless any information is given about the data contained in entries.

4 votes -- Anurag Pandey (13.6k points)

8.23 Logical Reasoning(25) top

8.23.1 Logical Reasoning: CMI2012-A-05 top

https://gateoverflow.in/46534

Amma baked a cake and left it on the table to cool. Before going for her bath, she told her four children that they should not touch the cake as it was to be cut only the next day. However when she got back from her bath she found that someone had eaten a large piece of the cake. Since only her four children were present at home when this happened, she questioned them about who ate a piece of the cake. The four answers she got were:

- Lakshmi: Aruna ate the piece of cake.
- Ram: I did not eat the piece of cake.
- Aruna: Varun ate the cake.
- Varun: Aruna lied when she said I had eaten the piece of cake.

If exactly one of them was telling the truth and exactly one of them actually ate the piece of cake, who is the culprit that Amma is going to punish?

A. Lakshmi

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B. Ram
C. Aruna
D. Varun

cmi2012 logical-reasoning

Answer

8.23.2 Logical Reasoning: CMI2014-A-10 top https://gateoverflow.in/46978

Avinash is taller than Abhay. Bharat is taller than Vinu and Vinay is taller than Bharat. Which of the following is a minimal set of additional information that can determine the tallest person?

A. Vinay is taller than Avinash and Abhay is taller than Bharat.
B. Avinash is taller than Vinay.
C. Abhay is shorter than Vinay.
D. None of the above.

cmi2014 logical-reasoning

Answer

8.23.3 Logical Reasoning: GATE2010-61 top https://gateoverflow.in/2369

If $137 + 276 = 435$ how much is $731 + 672$?

A. 534
B. 1403
C. 1623
D. 1513

gate2010 numerical-ability normal logical-reasoning

Answer

8.23.4 Logical Reasoning: GATE2010-62 top https://gateoverflow.in/2370

Hari(H), Gita(G), Irfan(I) and Saira(S) are siblings (i.e., brothers and sisters). All were born on 1st January. The age difference between any two successive siblings (that is born one after another) is less than three years. Given the following facts:

i. Hari's age + Gita's age > Irfan's age + Saira's age
ii. The age difference between Gita and Saira is one year. However Gita is not the oldest and Saira is not the youngest.
iii. There are no twins.

In what order they were born (oldest first)?

A. $HSIG$
B. $SGHI$
C. $IGSH$
D. $IHSG$

gate2010 numerical-ability logical-reasoning normal

Answer

8.23.5 Logical Reasoning: GATE2011-GG-GA-8 top https://gateoverflow.in/40209

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Three sisters $(R, S,$ and $T)$ received a total of 24 toys during Christmas. The toys were initially divided among them in a certain proportion. Subsequently, R gave some toys to S which doubled the share of S. Then S in turn gave some of her toys to T, which doubled T's share. Next, some of T's toys were given to R, which doubled the number of toys that R currently had. As a result of all such exchanges, the three sisters were left with equal number of toys. How many toys did R have originally?

A. 8
B. 9
C. 11
D. 12

gate2011-gg logical-reasoning numerical-ability

Answer

8.23.6 Logical Reasoning: GATE2011-MN-63 top https://gateoverflow.in/31543

L, M and N are waiting in a queue meant for children to enter the zoo. There are 5 children between L and M, and 8 children between M and N. If there are 3 children ahead of N and 21 children behind L, then what is the minimum number of children in the queue?

A. 28
B. 27
C. 41
D. 40

numerical-ability gate2011-mn logical-reasoning

Answer

8.23.7 Logical Reasoning: GATE2012-AR-8 top https://gateoverflow.in/40229

Ravi is taller than Arun but shorter than Iqbal. Sam is shorter than Ravi. Mohan is shorter than Arun. Balu is taller than Mohan and Sam. The tallest person can be

A. Mohan
B. Ravi
C. Balu
D. Arun

gate2012-ar logical-reasoning

Answer

8.23.8 Logical Reasoning: GATE2012-CY-GA-9 top https://gateoverflow.in/40240

There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, the minimum number of weighing required to identify the heavier bag is

A. 2
B. 3
C. 4
D. 8

gate2012-cy numerical-ability logical-reasoning

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Answer

8.23.9 Logical Reasoning: GATE2014-AE-GA-7 top https://gateoverflow.in/40307

Anuj, Bhola, Chandan, Dilip, Eswar and Faisal live on different floors in a six-storeyed building (the ground floor is numbered 1, the floor above it 2, and so on) Anuj lives on an even-numbered floor, Bhola does not live on an odd numbered floor. Chandan does not live on any of the floors below Faisal's floor. Dilip does not live on floor number 2. Eswar does not live on a floor immediately above or immediately below Bhola. Faisal lives three floors above Dilip. Which of the following floor-person combinations is correct?

	Anuj	Bhola	Chandan	Dilip	Eswar	Faisal
(A)	6	2	5	1	3	4
(B)	2	6	5	1	3	4
(C)	4	2	6	3	1	5
(D)	2	4	6	1	3	5

gate-2014-ae logical-reasoning numerical-ability descriptive

Answer

8.23.10 Logical Reasoning: GATE2014-AG-GA6 top https://gateoverflow.in/41670

In a group of four children, Som is younger to Riaz. Shiv is elder to Ansu. Ansu is youngest in the group. Which of the following statements is/are required to find the eldest child in the group?

Statements

1. Shiv is younger to Riaz.
2. Shiv is elder to Som.

A. Statement 1 by itself determines the eldest child.
B. Statement 2 by itself determines the eldest child.
C. Statements 1 and 2 are both required to determine the eldest child.
D. Statements 1 and 2 are not sufficient to determine the eldest child.

gate2014-ag numerical-ability logical-reasoning normal

Answer

8.23.11 Logical Reasoning: GATE2014-EC01-GA7 top https://gateoverflow.in/41496

For submitting tax returns, all resident males with annual income below Rs 10 lakh should fill up Form P and all resident females with income below Rs 8 lakh should fill up Form Q. All people with incomes above Rs 10 lakh should fill up Form R, except non residents with income above Rs 15 lakhs, who should fill up Form S. All others should fill Form T. An example of a person who should fill Form T is

A. A resident male with annual income Rs 9 lakh
B. A resident female with annual income Rs 9 lakh
C. A non-resident male with annual income Rs 16 lakh
D. A non-resident female with annual income Rs 16 lakh

gate2014-ec01 numerical-ability logical-reasoning normal

Answer

8.23.12 Logical Reasoning: GATE2014-EC03-GA5 top https://gateoverflow.in/41144

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In which of the following options will the expression $P < M$ be definitely true?

A. $M < R > P > S$
B. $M > S < P < F$
C. $Q < M < F = P$
D. $P = A < R < M$

gate2014-ec03 logical-reasoning numerical-ability

Answer

8.23.13 Logical Reasoning: GATE2015-3-GA-1 top

https://gateoverflow.in/8298

If $ROAD$ is written as $URDG$, then $SWAN$should be written as:

A. $VXDQ$
B. $VZDQ$
C. $VZDP$
D. $UXDQ$

gate2015-3 numerical-ability easy logical-reasoning

Answer

8.23.14 Logical Reasoning: GATE2015-3-GA-7 top

https://gateoverflow.in/8308

The head of newly formed government desires to appoint five of the six selected members P, Q, R, S, T and U to portfolios of Home, Power, Defense, Telecom, and Finance. U does not want any portfolio if S gets one of the five. R wants either Home or Finance or no portfolio. Q says that if S gets Power or Telecom, then she must get the other one. T insists on a portfolio if P gets one.

Which is the valid distribution of portfolios?

A. P-Home, Q-Power, R-Defense, S-Telecom, T-Finance
B. R-Home, S-Power, P-Defense, Q-Telecom, T-Finance
C. P-Home, Q-Power, T-Defense, S-Telecom, U-Finance
D. Q-Home, U-Power, T-Defense, R-Telecom, P-Finance

gate2015-3 numerical-ability normal logical-reasoning

Answer

8.23.15 Logical Reasoning: GATE2016-Session-1-GA-5 top

https://gateoverflow.in/108077

Michael lives 10 km away from where I live. Ahmed lives 5 km away and Susan lives 7 km away from where I live. Arun is farther away than Ahmed but closer than Susan from where I live. From the information provided here, what is one possible distance (in km) at which I live from Arun's place?

A. 3.00
B. 4.99
C. 6.02
D. 7.01

gate2016session1aptitude logical-reasoning numerical-ability

Answer

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8.23.16 Logical Reasoning: GATE2016-Session-7-GA-8 top

https://gateoverflow.in/110918

Fact 1: Humans are mammals.

Fact 2: Some humans are engineers.

Fact 3: Engineers build houses.

If the above statements are facts, which of the following can be logically inferred?

I. All mammals build houses.

II. Engineers are mammals.

III. Some humans are not engineers.

A. II only.
B. III only.
C. I, II and III.
D. I only.

gate2016session7aptitude logical-reasoning

Answer

8.23.17 Logical Reasoning: GATE2017-1-GA-3 top https://gateoverflow.in/118406

Rahul, Murali, Srinivas and Arul are seated around a square table. Rahul is sitting to the left of Murali. Srinivas is sitting to the right of Arul. Which of the following pairs are seated opposite each other?

A. Rahul and Murali
B. Srinivas and Arul
C. Srinvas and Murali
D. Srinivas and Rahul

gate2017-1 numerical-ability logical-reasoning

Answer

8.23.18 Logical Reasoning: GATE2017-1-GA-7 top https://gateoverflow.in/118410

Six people are seated around a circular table. There are at least two men and two women. There are at least three right-handed persons. Every woman has a left-handed person to her immediate right. None of the women are right-handed. The number of women at the table is

A. 2
B. 3
C. 4
D. Cannot be determined

gate2017-1 numerical-ability logical-reasoning

Answer

8.23.19 Logical Reasoning: GATE2017-2-GA-7 top https://gateoverflow.in/118421

There are three boxes. One contains apples, another contains oranges and the last one contains both apples and oranges. All three are known to be incorrectly labeled. If you are permitted to open

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just one box and then pull out and inspect only one fruit, which box would you open to determine the contents of all three boxes?

A. The box labeled 'Apples'
B. The box labeled 'Apples and Oranges'
C. The box labeled 'Oranges'
D. Cannot be determined

gate2017-2 numerical-ability normal tricky logical-reasoning

Answer

8.23.20 Logical Reasoning: TIFR2010-A-1 top https://gateoverflow.in/18202

A box contains 731 black balls and 2000 white balls. The following process is to be repeated as long as possible. Arbitrarily select two balls from the box. If they are of the same color, throw them out and put a black ball into the box (enough extra black balls are available to do this). If they are of different color, place the white ball back into the box and throw the black ball away. Which of the following is correct?

A. The process can be applied indefinitely without any prior bound
B. The process will stop with a single white ball in the box
C. The process will stop with a single black ball in the box
D. The process will stop with the box empty
E. None of the above

tifr2010 numerical-ability logical-reasoning

Answer

8.23.21 Logical Reasoning: TIFR2013-A-10 top https://gateoverflow.in/25432

Three men and three rakhsasas arrive together at a ferry crossing to find a boat with an oar, but no boatman. The boat can carry one or at the most two persons, for example, one man and one rakhsasas, and each man or rakhsasas can row. But if at any time, on any bank, (including those who maybe are in the boat as it touches the bank) rakhsasas outnumber men, the former will eat up the latter. If all have to go to the other side without any mishap, what is the minimum number of times that the boat must cross the river?

a. 7
b. 9
c. 11
d. 13
e. 15

tifr2013 numerical-ability logical-reasoning

Answer

8.23.22 Logical Reasoning: TIFR2013-A-11 top https://gateoverflow.in/25433

Let there be a pack of 100 cards numbered 1 to 100. The ith card states: "There are at most $i-1$ true cards in this pack". Then how many cards of the pack contain TRUE statements?

a. 0
b. 1
c. 100
d. 50
e. None of the above.

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tifr2013 logical-reasoning

Answer

8.23.23 Logical Reasoning: TIFR2013-A-2 top

https://gateoverflow.in/25383

Consider the following two types of elections to determine which of two parties A and B forms the next government in the 2014 Indian elections. Assume for simplicity an Indian population of size $545545 (= 545 * 1001)$. There are only two parties A and B and every citizen votes.

TYPE C: The country is divided into 545 constituencies and each constituency has 1001 voters. Elections are held for each constituency and a party is said to win a constituency if it receive a majority of the vote in that constituency. The party that wins the most constituencies forms the next government.

TYPE P: There are no constituencies in this model. Elections are held throughout the country and the party that wins the most votes (among 545545 voters forms the government.

Which of the following is true?

a. If the party forms the govt. by election TYPE C winning at least two-third of the constituencies, then it will also forms the govt. by election TYPE P.
b. If a party forms govt. by election TYPE C, then it will also form the govt. by election TYPE P.
c. If a party forms govt. by election TYPE P, then it will also form the govt. by election TYPE C.
d. All of the above
e. None of the above.

tifr2013 logical-reasoning

Answer

8.23.24 Logical Reasoning: TIFR2016-A-1 top

https://gateoverflow.in/96822

Suppose the following statements about three persons in a room are true.

Chandni, Sooraj and Tara are in a room. Nobody else is in the room. Chandni is looking at Sooraj. Sooraj is looking at Tara. Chandni is married. Tara is not married. A married person in the room is looking at an unmarried person.

Which of the following is necessarily true?

A. Sooraj is married
B. Sooraj is unmarried
C. The situation described is impossible
D. There is insufficient information to conclude if Sooraj is married or unmarried
E. None of the above

tifr2016 logical-reasoning

Answer

8.23.25 Logical Reasoning: TIFR2017-A-14 top

https://gateoverflow.in/95657

Consider the following game with two players, Aditi and Bharat. There are n tokens in a bag. The two players know n, and take turns removing tokens from the bag. In each turn, a player can either remove one token or two tokens. The player that removes the last token from the bag loses. Assume that Aditi always goes first. Further, we say that a player has a winning strategy if she or he can win the game, no matter what other player does. Which of the following statements is TRUE?

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A. For $n = 3$, Bharath has a winning strategy. For $n = 4$, Aditi has a winning strategy.
B. For $n = 7$, Bharath has a winning strategy. For $n = 8$, Aditi has a winning strategy.
C. For both $n = 3$ and $n = 4$, Aditi has a winning strategy.
D. For both $n = 7$ and $n = 8$, Aditi has a winning strategy.
E. Bharat never has a winning strategy.

tifr2017 numerical-ability logical-reasoning

Answer

Answers: Logical Reasoning

8.23.1 Logical Reasoning: CMI2012-A-05 top

https://gateoverflow.in/46534

Selected Answer

All telling lie except ,Varun telling truth

Ram ate the piece of cake

Answer is $B)$

3 votes -- srestha (86.6k points)

8.23.2 Logical Reasoning: CMI2014-A-10 top

https://gateoverflow.in/46978

Selected Answer

It is given that

1. $\text{Avinash} > \text{Abhay}$
2. $\text{Vinay} > \text{Bharat} > \text{Vinu}$

Now come to Options

1. Vinay is taller than Avinash and Abhay is taller than Bharat.

$\implies$ If we add Vinay is taller than Avinash, then it will complete our purpose of getting tallest person - i.e., Vinay is tallest.

2. Avinash is taller than Vinay.

$\implies$ It also complete our purpose. Because we can see then Avinash is tallest now.

3. Abhay is shorter than Vinay.

$\implies$ This will not help you to get the tallest person, Because you can not understand that which one is tallest either Vinay or Avinash.

4. None of these.

$\implies$ This can not be the answer, because options (1) and (2) already help us to get the result.

Final Answer: Since in the question minimal set of information is mentioned, option B will be the answer.

4 votes -- Muktinath Vishwakarma (35.3k points)

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8.23.3 Logical Reasoning: GATE2010-61 top

https://gateoverflow.in/2369

Selected Answer

Answer. C.

$(137)_8 + (276)_8 = (435)_8$

So basically the number are given in Octal base.

Similarly, addition of 731,672 gives 1623 in octal.

19 votes -- shreya ghosh (3.5k points)

8.23.4 Logical Reasoning: GATE2010-62 top

https://gateoverflow.in/2370

Selected Answer

Given that,

- $H + G > S + I$
- No twins,
- And either $G = S + 1$ or $S = G + 1$
- G is not the oldest and S is not the youngest.

Lets analyze each case,

Case 1:

$G = S + 1,$

It implies, $H > I - 1$, Since twins are not possible $H > I$

Case 2:

$G = S - 1,$

It implies $H > I + 1$

Therefore, both cases imply H is older than I, also G and S must be placed successively.

Only **option B** satisfies these implications.

8 votes -- Pranav Kant Gaur (1.6k points)

8.23.5 Logical Reasoning: GATE2011-GG-GA-8 top

https://gateoverflow.in/40209

Selected Answer

We can proceed in reverse order:

$$\begin{array}{ccc} R & S & T \\ 8 & 8 & 8 \end{array}$$

and apply the given steps in reverse. We will get,

$$\begin{array}{ccc} R & S & T \end{array}$$

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$$\begin{array}{ccc} 4 & 8 & 12 \\ 4 & 14 & 6 \\ 11 & 7 & 6 \end{array}$$

9 votes -- Deepesh Kataria (2k points)

8.23.6 Logical Reasoning: GATE2011-MN-63 top https://gateoverflow.in/31543

Selected Answer

L, M and N are waiting in queue that are meant for children, so they are also counted as children.

13 votes -- Praveen Saini (54.5k points)

8.23.7 Logical Reasoning: GATE2012-AR-8 top https://gateoverflow.in/40229

Selected Answer

Balu **can be** the TALLEST.

Ravi cannot be the answer since it is clearly mentioned that he is shorter than Iqbal.

Iqbal and Balu could be the tallest as no comparison is provided where these two are smaller than anyone.

But Iqbal is not mentioned in options.

Hence, Balu (see the usage of 'can' in the question)

4 votes -- Govind Krishna Tewari (2.2k points)

8.23.8 Logical Reasoning: GATE2012-CY-GA-9 top https://gateoverflow.in/40240

Selected Answer

Answer is $a)$ i.e 2

Divide in to $3-3-2$ and then take $3-3$ pair and then if they are equal then weigh the remaining $1-1$.

if out of $3-3$ one is heavier then take the heavier group and divide it $1-1-1$ take any two of these group and weigh again to get the heavier.

14 votes -- Piyush Kapoor (905 points)

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8.23.9 Logical Reasoning: GATE2014-AE-GA-7 top https://gateoverflow.in/40307

Selected Answer

Time is less hence, calculating and applying permutations and combinations would be a layman's choice.

Question says that

Eswar does not live on a floor immediately above or immediately below Bhola

Option (A): Bhola's floor number $-$ 2 and Eswar's floor number $-$ 3

Option (B): Bhola's floor number $-$ 6 and Eswar's floor number $-$ 3

Option (C): Bhola's floor number $-$ 2 and Eswar's floor number $-$ 1

Option (D): Bhola's floor number $-$ 4 and Eswar's floor number $-$ 3

Here, we can clearly see that in options A, C and D the statement is not followed.

Therefore, the answer is option(B).

4 votes -- Sumit Kumar Patel (659 points)

8.23.10 Logical Reasoning: GATE2014-AG-GA6 top https://gateoverflow.in/41670

Selected Answer

I think answer should be A. GIVEN

- Som is younger to Riaz .
- Shiv is elder to Ansu.
- Ansu is youngest.

First Statement :

Shiv is younger to RIAZ, SOM is also younger to RIAZ,ANZU is the youngest. **So RIAZ would be the eldest.**

4 votes -- richa07 (957 points)

8.23.11 Logical Reasoning: GATE2014-EC01-GA7 top https://gateoverflow.in/41496

Selected Answer

I think its Option B

3 votes -- saif ahmed (4.5k points)

8.23.12 Logical Reasoning: GATE2014-EC03-GA5 top https://gateoverflow.in/41144

Selected Answer

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Answer D.
As $P = A$
and $A < R$ and $R < M \implies A < M \implies P < M$

3 votes -- Abhilash Panicker (9.4k points)

8.23.13 Logical Reasoning: GATE2015-3-GA-1 top

https://gateoverflow.in/8298

Selected Answer

Option B. VZDQ

(every letter is replaced by third letter in alphabetical order)

13 votes -- Anoop Sonkar (5k points)

8.23.14 Logical Reasoning: GATE2015-3-GA-7 top

https://gateoverflow.in/8308

Selected Answer

"U does not want any portfolio if S gets one of the five"

So, S and U cannot come together. Option C eliminated.

"R wants either Home or Finance or no portfolio"

So, options A and D eliminated.

So, answer is B.

Just to confirm:

Q says that if S gets Power or Telecom, then she must get the other one

In B, S gets Power and Q gets Telecom

"T insists on a portfolio if P gets one"

In B, T is getting a portfolio.

15 votes -- Arjun Suresh (348k points)

8.23.15 Logical Reasoning: GATE2016-Session-1-GA-5 top

https://gateoverflow.in/108077

Selected Answer

$$\text{Michael} \quad \underset{10\ kms}{----} \quad I$$

$$\text{Ahmed} \quad \underset{5\ kms}{----} \quad I$$

$$\text{Susan} \quad \underset{7\ kms}{----} \quad I$$

$$\text{Susan} \quad ---- \quad \text{Arun} \quad ---- \quad \text{Ahmed} \quad ---- \quad I$$

So, the distance between Arun and I is between $5\ kms$ and $7\ kms$.

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Answer C. 6.02

3 votes -- AKANKSHA DEWANGAN (3.9k points)

8.23.16 Logical Reasoning: GATE2016-Session-7-GA-8 top

We can draw 2 venn diagrams, one for Mammals another for Build Houses.

Mammals consists of Humans - as fact 1 said Humans are mammals means all humans are mammals.

fact 2 Some humans are engineers , see the intersect part, it represents this statement.

fact 3 says Engineers build houses means all engineers build houses . so Build Houses consists of Engineers.

Now From this diagram we try to get the meaning of given sentences

1. All mammals build houses.- clearly False , diagram says some mammals build houses.
2. Engineers are mammals - False again, diagram says not all engineers are mammals rather some engineers are mammals.
3. Some humans are not engineers - diagram says this is Correct . some part of H is not belongs to circle E .

so only statement III is correct , which is option B .

2 votes -- Bikram (67k points)

8.23.17 Logical Reasoning: GATE2017-1-GA-3 top https://gateoverflow.in/118406

Selected Answer

Arup and rahul will sit opposite to each other.

Murali and srinivas will sit opposite to each other in the square table.

Answer: C

5 votes -- Arnabi Bej (7.9k points)

8.23.18 Logical Reasoning: GATE2017-1-GA-7 top https://gateoverflow.in/118410

- 3 Right handed person.
- Every Person is left handed.
- Immediate right to woman is left handed person.

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- As already given every woman is left handed.So,given 3 Right handed Person actually man.

So at this moment we have $\begin{cases} 3 \text{ Right handed man} \\ 2 \text{ Left handed man} \end{cases}$

We need 1 more person to make total 6 person and that person may be either

$\rightarrow$ Woman with left handed.or $\rightarrow$ Man with either left handed or right handed.

$Case-1$ Let us suppose that person is woman. So, 3 Right handed Men 3 Left handed Women	 Now we have no more left handed person to put here.So, this case is not possible.
$Case-2$ 3 Right handed Men. 2 Left handed Women. 1 Left handed Man.	
$Case-3$ 3 right handed person (men) 2 left handed women 1 right handed man this case is also not possible, as w __ w__(for this place we don't hv any left handed person) so this arrangement is not valid.	

9 votes -- 2018 (6.8k points)

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8.23.19 Logical Reasoning: GATE2017-2-GA-7 top https://gateoverflow.in/118421

Selected Answer

B. the box labelled 'Apples and Oranges'.

Reason.

We know that the box labeled "Apples and Oranges" can't contain both, so whatever we pick will be the correct label.

Without the loss of generality say picked item was an orange, what that means is that the box that is labeled "Apples" can't contain just oranges. It also can't contain just apples as it is known to be wrong. So it only can contain combination of oranges and apples. Now the third box labeld "Oranges" contains only apples.

19 votes -- Dhruv Patel (2k points)

8.23.20 Logical Reasoning: TIFR2010-A-1 top https://gateoverflow.in/18202

Selected Answer

The total number of balls is 2371 and in each round 1 ball is removed. So after 2370 steps, we will have 1 ball in the box and we must stop.

Here, white balls are even and every time either it gets removed in a pair or remains unchanged.

So, the last ball remaining can never be a white ball.

Since at each step the number of black balls changes from odd to even or even to odd, so at the end only 1 black ball will be in the box.

Hence, option C is the correct answer.

3 votes -- Umang Raman (15.7k points)

8.23.21 Logical Reasoning: TIFR2013-A-10 top https://gateoverflow.in/25432

Selected Answer

I am denoting M for man and R for rakhsas.

Step 1 : Two rakhsasas row the boat from one side to another. So, one rakshas will reach other side. and one rakhsas will have to row back. So, first side now $(3M + 2R)$ and second side $(1R)$.

So, boat crosses 2 times.

Step 2 : $2R$ row to other side and leaves one R there and comes back to first side. So, now first side has $(3M + 1R)$ and second side has $(2R)$.

So, boat crosses 2 times.

Step 3 : 2 M row to other side and gets down at other side. So, first side has $(1R + 1M)$ and second side has $(2M + 2R)$.
So, boat crosses 1 time.

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Step 4: Now, $1M$ and $1R$ will have to row back from second side, and they will take $1M + 1M$ to other side. So, in the first side, we now have $2R$.

So, boat crosses 2 times.

Step 5: Last step , $2R$ will row to the other side.

So, boat crosses 1 time.

Total time boat crosses $= 8 + 1 = 9$ times.

1 votes -- worst_engineer (4.1k points)

8.23.22 Logical Reasoning: TIFR2013-A-11 top https://gateoverflow.in/25433

Selected Answer

Option D should be the correct one.

that is 50 cards of the pack contain true statements.

Why?

Because if the statement written on card number x is true then all the statements written in card numbers $x + 1$ to 100 must be true.

For Example if Card number 3 is true, then according to the statement written in this card, "There are at most $3 - 1 (= 2)$ true cards in the pack".

This implies number of true cards must be less than 3.

Now the statement on card number 4 will imply that the number of true cards must be less than 4.

similarly the statement on card number 100 will imply that the number of true cards must be less than 100.

So if statement written on card number 3 is true then all the statements written on the card numbers 4 to 100 will vacuously be true.

But now the the number of true cards will be 98 (from card number 3 to 100) hence the statement on the card number 3 must be false.

Clearly this is inconsistent so card number 3 can not be a true card.

Conclusion:

If card number x is a true card then:

1. There are at least $(100 - x) + 1$ true cards. and

2.Total Number of true cards must be less then or equal to $x - 1$

(where x belongs to Integers between 1 and 100).

For any value of $x \leq 50$ both of the above statements can not be true simultaneously, so none of the cards from 1 to 50 is a true card.

For any value of $x \geq 51$ both of the above statements can be true at the same time, so all of the cards from 51 to 100 must be true cards & the total number of true cards will be 50.

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It can be observed that comparing one of the boundary cases of each of the above two statements will give us of the boundary cases of our answer.

that is, on solving:

$$(100 - x) + 1 = x - 1$$

we get $x = 51$ which indeed is our smallest true card.

10 votes -- Anurag Pandey (13.6k points)

8.23.23 Logical Reasoning: TIFR2013-A-2 top

https://gateoverflow.in/25383

Selected Answer

Minimum condition for Type C winning: if any party wins 273 constituencies out of 545 and with vote 501 out of 1001 for each.
Minimum condition for Type P winning: if any party wins 272773 votes out of 545545

Option A:

- Type C : Let party A wins $2/3$ of constituencies i.e., 364 wins by 1 vote and 181 loss by all vote $364 \times 501 + 181 \times 0 = 182364$
- Type P : If A wins it should have more than half of vote i.e., 272773

  So, it is FALSE since A got only 182364 votes

Option B : Similar to option A

Option C:

- Type P: Let A win by 272773 votes
- Type C: If A wins 272 constituencies with 1001 votes and loss 272 with 1001 and 1 constituency with 1000 votes then, total received votes $= 272 \times 1001 + 272 \times 0 + 1 \times 1 = 272773$. But A lose the election since A won only 272 constituencies out of 545

So option E must be correct.

4 votes -- Umang Raman (15.7k points)

8.23.24 Logical Reasoning: TIFR2016-A-1 top

https://gateoverflow.in/96822

Selected Answer

*"**A married person in the room is looking at an unmarried person**" ---this is satisfied when Sooraj is made **married***

$$\underset{married}{\text{Chandni}} \longrightarrow \underset{married}{\text{Sooraj}} \longrightarrow \underset{unmarried}{\text{Tara}}$$

*"**A married person in the room is looking at an unmarried person**" ---this is satisfied when Sooraj is made **unmarried***

$$\underset{married}{\text{Chandni}} \longrightarrow \underset{unmarried}{\text{Sooraj}} \longrightarrow \underset{unmarried}{\text{Tara}}$$

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As both are possible we cannot finalize it to A or B --- we need extra information **so answer should be D**

7 votes -- Pavan Kumar Munnam (10.9k points)

8.23.25 Logical Reasoning: TIFR2017-A-14 top

https://gateoverflow.in/95657

Selected Answer

Point to be noted-

1. Always Aditi starts the game.
2. A player loses the game if he/she has to remove the last item.
3. A player always removes 1 or 2 items with a winning strategy.

Method 1: shortcut- time complexity- O(1)

n = total no of tokens.

If **n mod 3 - 1 = 0, then Bharat has a winning strategy (1 is the last token being taken by the loser)**

if **n mod 3 - 1 = 1 or 2, then Aditi has a winning strategy.**

The trick here is with 4 tokens whoever plays next is not having a winning startergy. So, the stratergy must be to ensure that the opponent always takes his turn with $3n+1$ tokens. This is possible because if the opponent takes 1, I take 2 and vice verse.

Let's check each option-
[Here my strategy of solving this would be just contradicting the each option by considering all the possibilities].

A. **For n =3.** // Here I am trying to make Bharat win, Let's see if he can win or not.
Aditi - removes 2 item , n = 3-2 = 1
Now only 1 item remains, so Bharat is forced to remove the last item and loses the game.
Hence for n=3, Aditi has a winning strategy.
The first part of A - False.

For n =4. // Here I am trying to make Aditi win, Let's see if she can win or not.

Case -1:	**case 2:**
Aditi - removes 1 item , n = 4-1 = 3	Aditi - removes 2 item , n = 4-2 = 1
Bharat- removes 2,item , n = 3 -2 =1	Bharat- removes 1,item , n = 2 -1 =1

Now only 1 item remains in both the case, so Aditi is forced to remove the last item and loses the game. **Hence for n=4, Bharat has a winning strategy.**
The 2nd part of A - True.

Hence, A- False.

B. For n =7. // Here I am trying to make Aditi win, Let's see if she can win or not.

Case 1:	Case 2:
Aditi - removes 1 item , n = 7- 1 = 6	Aditi - removes 2 item , n = 7- 2 = 5
Bharat- removes 2 item , n = 6 -2 = 4.	Bharat- removes 1 item , n = 5 -1 = 4.

Now remaining tokens in both the case, n = 4.

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case 1:

Aditi - removes 1 item ,n=4-1 = 3

Bharat- removes 2,item ,n=3-2=**1**,

case 2:

Aditi - removes 2 item , n = 4-2= 2.

Bharat- removes 1,item , n = 2-1 =**1.**

Now in both the case, only 1 item remains, so Aditi is forced to remove the last item and loses the game. **Hence for n=7, Bharat has a winning strategy. Aditi could not win the game even taking all possibility of Aditi's turn.**

For n = 8. // Here I am trying to make Aditi win taking all possibility of Bharat's turn, Let's see if she can win or not.

Aditi - removes 1 item , remaining item n = 8 - 1 = 7

Case 1:

Bharat- removes 1 item , n = 7 - 1 = 6.

Aditi - removes 2 item , n = 6 - 2 = 4 ,

case 2:

Bharat- removes 2 item ,n = 7 - 2 = 5.

Aditi - removes 1 item , n = 5-1 = 4.

N**ow In both cases, remaining items = 4.**

case 1

Bharat- removes 1,item , n = 4-1 = **3**

Aditi - removes 2 item , n = 3-2 = 1 ,

case 2

Bharat- removes 2,item , n = 4-2 = 2**.**

Aditi - removes 1 item , n = 2-1 = 1.

Now in both the case, only 1 item is remaining so Bharat is forced to remove the last item and loses the game. **Hence for n=8, Aditi has a winning strategy.**

B- True.

C. For both n=3 and n=4, Aditi has a winning strategy.

See explanation for A. where Bharat has a winning strategy for both n=3 and n=4.

Hence, False.

D. For both n=7 and n=8, Aditi has a winning strategy.

See explanation for B. where Bharat has a winning strategy for both n=7.

Hence, False.

E. Bharat never has a winning strategy.

See explanation for B where Aditi has a winning strategy for both n=8.

Hence, False.

Ans - B.

11 votes -- Dhananjay Kumar Sharma (25.7k points)

8.24 Maxima Minima(3) top

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8.24.1 Maxima Minima: GATE2012-62 top

https://gateoverflow.in/2210

A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

A. 8 meters
B. 10 meters
C. 12 meters
D. 14 meters

gate2012 numerical-ability normal maxima-minima

Answer

8.24.2 Maxima Minima: GATE2013-CE-6 top

https://gateoverflow.in/40274

X and Y are two positive real numbers such that $2X + Y \leq 6$ and $X + 2Y \leq 8$.For which of the following values of

(X, Y) the function $f(X, Y) = 3X + 6Y$ will give maximum value ?

A. $\left(\frac{4}{3}, \frac{10}{3}\right)$
B. $\left(\frac{8}{3}, \frac{20}{3}\right)$
C. $\left(\frac{8}{3}, \frac{10}{3}\right)$
D. $\left(\frac{4}{3}, \frac{20}{3}\right)$

gate2013-ce numerical-ability maxima-minima

Answer

8.24.3 Maxima Minima: GATE2017-2-GA-9 top

https://gateoverflow.in/118423

The number of roots of $e^x + 0.5x^2 - 2 = 0$ in the range [-5,5] is

A. 0
B. 1
C. 2
D. 3

gate2017-2 numerical-ability normal maxima-minima calculus

Answer

Answers: Maxima Minima

8.24.1 Maxima Minima: GATE2012-62 top

https://gateoverflow.in/2210

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Selected Answer

B. 10

$y = 2x - 0.1x^2$

$dy/dx = 2 - 2*0.1\ x => dy/dx = 0 => x = 10$

So, $y = 20 - 10 = 10$

9 votes -- shreya ghosh (3.5k points)

8.24.2 Maxima Minima: GATE2013-CE-6 top

https://gateoverflow.in/40274

Selected Answer

$f(x, y) = 3x + 6y = 3(x + 2y)$

maximum value of $x + 2y$ is given by equation 2 which is 8 $(x + 2y <= 8)$

i.e maximum value of $f(x, y) = 3 * (8) = 24$

$f(x, y) = 24$ can be obtained by putting $x = 4/3$ & $y = 10/3$

So, answer is (A).

5 votes -- Vivek Srivastava (3.6k points)

8.24.3 Maxima Minima: GATE2017-2-GA-9 top

https://gateoverflow.in/118423

Selected Answer

let $f(x) = e^x + 0.5x^2 - 2$, and we want to see if $f(x) = 0$ has any solution in $[-5, 5]$

$f'(x)$ = $e^x + x$ and we definitely have solution of $f'(x) = 0$ in $[-5, 5]$ because $f'(x)$ is continuous and $f'(-5) < 0,\ \ f'(5) > 0$

(Moreover $f'(x)$ has only one solution as it is strictly increasing function.)

Let k be one of the solution for $f'(x)$ hence $f'(k) = 0$ where $k \in [-5, 5]$, for "$< k$", $f'(x)$ is -ve and for "$> k$", $f'(x)$ is +ve. means for $< k$, $f(x)$ is decreasing and for $> k$, $f(x)$ is increasing.

rough plot of $f(x)$ may go like this

Plot of f(x)

Now it is clear that $f(x)$ has two solutions.

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BUT, How can i sure that these solutions are in between $[-5,5]$?

Yes i agree that $k \in [-5,5]$, but there are many possibilities of solution to be in $[-5,5]$

(in all three images $k \in [-5,5]$)

U wanna know which image is correct ? :O

just check sign of $f(-5)$ and $f(5)$, u will get to know "Image-1" is correct one.

Hence, there are 2 solutions in between of $[-5,5]$. **C** is correct answer.

18 votes -- Sachin Mittal (15.6k points)

$e^x = -0.5x^2 + 2$

Plot e^x and Downward Parabola $-0.5x^2 + 2$, And find intersection.

As, these two intersect at two points. So, (C) is correct answer.

43 votes -- Manish Joshi (27.9k points)

8.25 Modular Arithmetic(1) top

8.25.1 Modular Arithmetic: GATE2012-CY-GA-1 top https://gateoverflow.in/40232

If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$

A. 2.23
B. 4.33
C. 11.37
D. 27.64

gate2012-cy numerical-ability modular-arithmetic

Answer

Answers: Modular Arithmetic

8.25.1 Modular Arithmetic: GATE2012-CY-GA-1 top https://gateoverflow.in/40232

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Selected Answer

$1259 + 2062 = 3321$

So, Answer is $3.52*7.85$ = **27.64 (Ans)**

Option (d) is correct.

6 votes -- Himanshu Agarwal (15.3k points)

8.26 Number Representation(2) top

8.26.1 Number Representation: GATE2014-3-GA-10 top

https://gateoverflow.in/2034

Consider the equation: $(7526)_8 - (Y)_8 = (4364)_8$, where $(X)_N$ stands for X to the base N. Find Y.

A. 1634
B. 1737
C. 3142
D. 3162

gate2014-3 numerical-ability number-representation normal digital-logic

Answer

8.26.2 Number Representation: TIFR2012-A-11 top

https://gateoverflow.in/25015

Let N be the sum of all numbers from 1 to 1023 except the five primes numbers: $2, 3, 11, 17, 31$. Suppose all numbers are represented using two bytes (sixteen bits). What is the value of the least significant byte (the least significant eight bits) of N?

a. 00000000
b. 10101110
c. 01000000
d. 10000000
e. 11000000

tifr2012 numerical-ability number-representation

Answer

Answers: Number Representation

8.26.1 Number Representation: GATE2014-3-GA-10 top

https://gateoverflow.in/2034

Selected Answer

C.
The given numbers are in octal representation. Digits range from 0 to 7.
$Y = 7526 - 4364$

$$\begin{array}{r} 7526 \\ -4364 \end{array}$$

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$\overline{3142}$

steps : 1. $(6-4)=2$
2. $(2-6)$, borrow a 8. Now $(8+2-6)=4$
3. $(5-1-3)=1$ (Subtracted by 1 because ONE 8 was borrowed)
4. $(7-4)=3$

18 votes -- Srinath Jayachandran (3.7k points)

8.26.2 Number Representation: TIFR2012-A-11 top https://gateoverflow.in/25015

Selected Answer

This is another way of saying , what will be the remainder when N is divided by $2^8=256$?

Here $N=1023\times\frac{1024}{2}-(2+3+11+17+31)$

$=1023\times 512-64$

$=1022\times 512+(512-64)$

$=1022\times 512+448$

Now $448\%256=192=11000000$

So option e) is correct.

6 votes -- sudipta roy (423 points)

8.27 Number Series(11) top

8.27.1 Number Series: GATE2013-58 top https://gateoverflow.in/1562

What will be the maximum sum of $44, 42, 40, \ldots$?

A. 502
B. 504
C. 506
D. 500

gate2013 numerical-ability easy number-series

Answer

8.27.2 Number Series: GATE2013-61 top https://gateoverflow.in/1565

Find the sum of the expression

$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots\ldots\ldots\ldots+\frac{1}{\sqrt{80}+\sqrt{81}}$

A. 7
B. 8

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C. 9
D. 10

gate2013 numerical-ability normal number-series

Answer

8.27.3 Number Series: GATE2013-EE-10 top

https://gateoverflow.in/40297

Find the sum to $'n'$ terms of the series $10 + 84 + 734 + \ldots$

A. $\frac{9(9^n+1)}{10} + 1$
B. $\frac{9(9^n-1)}{8} + 1$
C. $\frac{9(9^n-1)}{8} + n$
D. $\frac{9(9^n-1)}{8} + n^2$

gate2013-ee numerical-ability number-series

Answer

8.27.4 Number Series: GATE2014-2-GA-5 top

https://gateoverflow.in/1942

The value of $\sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}}$ is

A. 3.464
B. 3.932
C. 4.000
D. 4.444

gate2014-2 numerical-ability easy number-series

Answer

8.27.5 Number Series: GATE2014-3-GA-4 top

https://gateoverflow.in/2027

Which number does not belong in the series below?

$2, 5, 10, 17, 26, 37, 50, 64$

A. 17
B. 37
C. 64
D. 26

gate2014-3 numerical-ability number-series easy

Answer

8.27.6 Number Series: GATE2014-EC01-GA5 top

https://gateoverflow.in/41494

What is the next number in the series?

$12 \quad 35 \quad 81 \quad 173 \quad 357$ _____.

gate2014-ec01 number-series numerical-ability numerical-answers

Answer

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8.27.7 Number Series: GATE2014-EC02-GA5 top https://gateoverflow.in/41512

Fill in the missing number in the series.

$2 \quad 3 \quad 6 \quad 15 \quad ___ \quad 157.5 \quad 630$

gate2014-ec02 number-series numerical-ability numerical-answers

Answer

8.27.8 Number Series: GATE2014-EC03-GA4 top https://gateoverflow.in/41143

The next term in the series $81, 54, 36, 24, \ldots$ is_____.

gate2014-ec03 number-series numerical-ability numerical-answers

Answer

8.27.9 Number Series: GATE2014-EC03-GA6 top https://gateoverflow.in/41145

Find the next term in the sequence: $7G, 11K, 13M,$ _____.

A. $15Q$
B. $17Q$
C. $15P$
D. $17P$

gate2014-ec03 number-series logical-reasoning numerical-ability

Answer

8.27.10 Number Series: TIFR2011-A-8 top https://gateoverflow.in/20014

The sum of the first n terms of the series $1, 11, 111, 1111, \ldots,$ is.

a. $\frac{1}{81}\left(10^{n+1} - 9n - 10\right)$
b. $\frac{1}{81}\left(10^{n} - 9n\right)$
c. $\frac{1}{9}\left(10^{n+1} - 1\right)$
d. $\frac{1}{9}\left(10^{n+1} - n10^{n}\right)$
e. None of the above.

tifr2011 numerical-ability number-series

Answer

8.27.11 Number Series: TIFR2013-A-8 top https://gateoverflow.in/25430

Find the sum of the infinite series

$$\frac{1}{1\times 3\times 5}+\frac{1}{3\times 5\times 7}+\frac{1}{5\times 7\times 9}+\frac{1}{7\times 9\times 11}+\ldots\ldots$$

A. ∞
B. $\left(\frac{1}{2}\right)$
C. $\left(\frac{1}{6}\right)$

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D. $\left(\frac{1}{12}\right)$

E. $\left(\frac{1}{14}\right)$

tifr2013 numerical-ability number-series

Answer

Answers: Number Series

8.27.1 Number Series: GATE2013-58 top

https://gateoverflow.in/1562

Selected Answer

This is in AP.

Maximum sum means we do not need to consider negative numbers and can stop at 0.

First find number of terms using the formula $a_n = a + (n-1)d$
Here,
$a = 44,$
$d = 42 - 44 = -2,$
$a_n = 0.$

Therefore, $0 = 44 + (n-1)(-2)$
$\Rightarrow n = 23.$

Now, sum of n terms of AP is given by: $S_n = \frac{n}{2}[a + a_n] = \frac{23}{2}[44 + 0] = 506.$

Option C is correct!

6 votes -- Monanshi Jain (9.3k points)

8.27.2 Number Series: GATE2013-61 top

https://gateoverflow.in/1565

Selected Answer

when you such overlapping expressions just rationalise it and add in most of the case you will be left with lesser number of terms ..in this case i am left with $\sqrt{81} - \sqrt{1} = 8.$

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots\ldots\ldots + \frac{1}{\sqrt{80}+\sqrt{81}}$$

$$= \frac{1}{\sqrt{1}+\sqrt{2}} \times \left(\frac{\sqrt{1}-\sqrt{2}}{\sqrt{1}-\sqrt{2}}\right) + \frac{1}{\sqrt{2}+\sqrt{3}} \times \left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}\right) + \frac{1}{\sqrt{3}+\sqrt{4}} \times \left(\frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}-\sqrt{4}}\right) + \ldots.$$

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$$= \frac{\sqrt{1}-\sqrt{2}}{(\sqrt{1})^2-(\sqrt{2})^2} + \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2} + \frac{\sqrt{3}-\sqrt{4}}{(\sqrt{3})^2-(\sqrt{4})^2} + \ldots\ldots\ldots\ldots + \frac{\sqrt{80}-\sqrt{81}}{(\sqrt{80})^2-(\sqrt{81})^2}$$

$$= -\left(\sqrt{1}-\sqrt{2}+\sqrt{2}-\sqrt{3}+\sqrt{3}-\sqrt{4}+\ldots\ldots\ldots+\sqrt{80}-\sqrt{81}\right)$$

$$= \sqrt{81}-\sqrt{1}$$

$$= 8$$

24 votes -- Bhagirathi Nayak (14.1k points)

8.27.3 Number Series: GATE2013-EE-10 top

https://gateoverflow.in/40297

Selected Answer

Ans). This is a arithmetico -geometric series . and the break down of each number in the sequence is as follows

$9^1+1, 9^2+3, 9^3+5, \ldots\ldots$

Now if we use substitution method we can find the right summation formula for the above sequence.

Take for example the first two terms of the sequence and add them we will get the sum as $10+84=94$

Now consider the option D ,here we have $n=2$, So ,we are trying to find the sum for first two numbers of the sequence.

$\frac{9(9^2-1)}{8}+2^2=94$. Same procedure can used to find the sum of first three numbers in the sequence

The answer is option D

5 votes -- ibia (3.5k points)

8.27.4 Number Series: GATE2014-2-GA-5 top

https://gateoverflow.in/1942

Selected Answer

$x=\sqrt{12+x}$

$\implies x^2=12+x$

$\implies x^2-x-12=0$

$\implies (x-4)(x+3)=0$

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$\implies x = 4 \text{ or } x = -3$

18 votes -- Arjun Suresh (348k points)

8.27.5 Number Series: GATE2014-3-GA-4 top

https://gateoverflow.in/2027

Selected Answer

If $a_1, a_2, a_3 \ldots a_n$ is the series and $i = 1$ to n, then the series is defined as $a_i = i^2 + 1$.
i.e the ith term is 1 plus the square of i.
Series will be as follows : $1^2 + 1, 2^2 + 1, 3^2 + 1, 4^2 + 1 \ldots n^2 + 1$
$2, 5, 10, 17, 26, 37, 50, 65$

Hence 64 does not belong to the series.

11 votes -- Srinath Jayachandran (3.7k points)

8.27.6 Number Series: GATE2014-EC01-GA5 top

https://gateoverflow.in/41494

Selected Answer

one way is this to getting answer as 725 bcz their diffrence is each time double

and other way is $12 \times 2 + 11 = 35$

$35 \times 2 + 11 = 81$

$81 \times 2 + 11 = 173$

so mul by 2 and add 11 and lastly we do $357 \times 2 + 11 = 725$

5 votes -- rajan (5k points)

8.27.7 Number Series: GATE2014-EC02-GA5 top

https://gateoverflow.in/41512

Selected Answer

$2 \times 1.5 = 3$

$3 \times 2 = 6$

$6 \times 2.5 = 15$

$15 \times 3 = 45$ is answer

$45 \times 3.5 = 157.5$ means each time increase multplication factor by 0.5

8 votes -- rajan (5k points)

8.27.8 Number Series: GATE2014-EC03-GA4 top

https://gateoverflow.in/41143

Selected Answer

We get next term in sequence by $\left(\frac{X}{3}\right) \times 2$

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$$\frac{81}{3} = 27 \times 2 = 54$$

$$\frac{54}{3} = 18 \times 2 = 36$$

$$\frac{36}{3} = 12 \times 2 = 24$$

$$\frac{24}{3} = 8 \times 2 = 16$$

Answer)16

7 votes -- Abhilash Panicker (9.4k points)

8.27.9 Number Series: GATE2014-EC03-GA6 top

https://gateoverflow.in/41145

Selected Answer

7G - G is the 7th alphabet
11K - K is the 11th alphabet
13M - M is the 13th alphabet

15th alphabet is O
17th alphabet is Q
Therefore A,C,D eliminated
Also, 7,11,13 are prime numbers, next prime in the sequence would be 17

Answer **Option B) 17Q**

6 votes -- Abhilash Panicker (9.4k points)

8.27.10 Number Series: TIFR2011-A-8 top

https://gateoverflow.in/20014

Selected Answer

$$S = (1 + 11 + 111 + \cdots n \text{ terms})$$

$$= \frac{1}{9} \times (9 + 99 + 999 + \cdots n \text{ terms})$$

$$= \frac{1}{9}\Big(\big((10-1) + (100-1) + (1000-1) + \cdots n \text{ terms}\big)\Big)$$

$$= \frac{1}{9}\Big(\big((10 + 100 + \cdots 10^n) - (1 + 1 + \cdots n \text{ terms})\big)\Big)$$

$$= \frac{1}{9}\left(\frac{10^{n+1} - 10}{10-1} - n\right)$$

$$= \frac{1}{9}\left(\frac{10^{n+1} - 10 - 9n}{9}\right)$$

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$$= \frac{10^{n+1} - 9n - 10}{81}$$

So, the correct answer is option A.

8 votes -- Pooja Palod (31.3k points)

8.27.11 Number Series: TIFR2013-A-8 top

https://gateoverflow.in/25430

Selected Answer

We note that:

$$\frac{1}{1 \times 3 \times 5} = \frac{1}{4} \cdot \left(\frac{5-1}{1 \times 3 \times 5} \right)$$
$$= \frac{1}{4} \cdot \left(\frac{1}{1 \times 3} - \frac{1}{3 \times 5} \right)$$

Now, we can rewrite the original series as a Telescoping series and simplify as follows:

$$\left(\frac{1}{1 \times 3 \times 5} + \frac{1}{3 \times 5 \times 7} + \frac{1}{5 \times 7 \times 9} + \cdots \right)$$

$$= \frac{1}{4} \cdot \left(\frac{1}{1 \times 3} - \frac{1}{3 \times 5} + \frac{1}{3 \times 5} - \frac{1}{5 \times 7} + \frac{1}{5 \times 7} - \frac{1}{7 \times 9} + \cdots \right)$$

$$= \frac{1}{4} \cdot \left(\frac{1}{1 \times 3} - \cancel{\frac{1}{3 \times 5}} + \cancel{\frac{1}{3 \times 5}} - \cancel{\frac{1}{5 \times 7}} + \cancel{\frac{1}{5 \times 7}} - \cancel{\frac{1}{7 \times 9}} + \cdots \right)$$

$$= \frac{1}{4} \cdot \left(\frac{1}{1 \times 3} \right)$$

$$= \frac{1}{12}$$

Hence (D) is the Answer.

15 votes -- Leen Sharma (40k points)

8.28 Numerical Computation(16) top

8.28.1 Numerical Computation: CMI2010-A-07 top

https://gateoverflow.in/46138

For integer values of n, the expression $\frac{n(5n+1)(10n+1)}{6}$

A. Is always divisible by 5.
B. Is always divisible by 3.
C. Is always an integer.
D. None of the above

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cmi2010 numerical-ability numerical-computation

Answer

8.28.2 Numerical Computation: GATE2011-57 top https://gateoverflow.in/2166

If $\log(\mathrm{P}) = (1/2)\log(\mathrm{Q}) = (1/3)\log(\mathrm{R})$, then which of the following options is **TRUE**?

A. $\mathrm{P}^2 = \mathrm{Q}^3\mathrm{R}^2$
B. $\mathrm{Q}^2 = \mathrm{PR}$
C. $\mathrm{Q}^2 = \mathrm{R}^3\mathrm{P}$
D. $\mathrm{R} = \mathrm{P}^2\mathrm{Q}^2$

gate2011 numerical-ability normal numerical-computation

Answer

8.28.3 Numerical Computation: GATE2011-65 top https://gateoverflow.in/2175

A container originally contains 10 litres of pure spirit. From this container, 1 litre of spirit replaced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this process is repeated one more time. How much spirit is now left in the container?

A. 7.58 litres
B. 7.84 litres
C. 7 litres
D. 7.29 litres

gate2011 numerical-ability normal numerical-computation

Answer

8.28.4 Numerical Computation: GATE2013-CE-1 top https://gateoverflow.in/40268

A number is as much greater than 75 as it is smaller than 117. The number is:

A. 91
B. 93
C. 89
D. 96

gate2013-ce numerical-ability numerical-computation

Answer

8.28.5 Numerical Computation: GATE2014-1-GA-4 top https://gateoverflow.in/773

If $\left(z + \frac{1}{z}\right)^2 = 98$, compute $\left(z^2 + \frac{1}{z^2}\right)$.

gate2014-1 numerical-ability easy numerical-answers numerical-computation

Answer

8.28.6 Numerical Computation: GATE2014-EC02-GA8 top

The sum of eight consecutive odd numbers is 656. The average of four consecutive even numbers is 87. What is the sum of the smallest odd number and second largest even number?

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gate2014-ec02 numerical-ability numerical-answers numerical-computation

Answer

8.28.7 Numerical Computation: GATE2014-EC04-GA4 top

https://gateoverflow.in/41466

Let $f(x,y) = x^n y^m = P$. If x is doubled and y is halved, the new value of f is

A. $2^{n-m}P$
B. $2^{m-n}P$
C. $2(n-m)P$
D. $2(m-n)P$

gate2014-ec04 numerical-ability easy numerical-computation

Answer

8.28.8 Numerical Computation: GATE2016-Session-7-GA-5 top

https://gateoverflow.in/110889

The sum of the digits of a two digit number is 12. If the new number formed by reversing the digits is greater than the original number by 54, find the original number.

A. 39
B. 57
C. 66
D. 93

gate2016session7aptitude numerical-ability numerical-computation

Answer

8.28.9 Numerical Computation: GATE2016-Session-8-GA-10 top

https://gateoverflow.in/111306

The numeral in the units position of $211^{870} + 146^{127} \times 3^{424}$ is _____.

gate2016session8aptitude numerical-ability numerical-answers numerical-computation

Answer

8.28.10 Numerical Computation: GATE2017-1-GA-4 top

https://gateoverflow.in/118407

Find the smallest number y such that $y \times 162$ is a perfect cube.

A. 24
B. 27
C. 32
D. 36

gate2017-1 numerical-ability numerical-computation

Answer

8.28.11 Numerical Computation: GATE2017-2-GA-4 top

https://gateoverflow.in/118416

A test has twenty questions worth 100 marks in total. There are two types of questions. Multiple choice questions are worth 3 marks each and essay questions are worth 11 marks each. How many multiple choice questions does the exam have?

A. 12

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B. 15
C. 18
D. 19

gate2017-2 numerical-ability numerical-computation

Answer

8.28.12 Numerical Computation: GATE2017-2-GA-8 top

https://gateoverflow.in/118422

X is a 30 digit number starting with the digit 4 followed by the digit 7. Then the number X^3 will have

A. 90 digits
B. 91 digits
C. 92 digits
D. 93 digits

gate2017-2 numerical-ability numerical-computation number-representation

Answer

8.28.13 Numerical Computation: ISI-2017-12 top https://gateoverflow.in/209610

Which of the following statements is true?

A. There are three consecutive integers with sum 2015
B. There are four consecutive integers with sum 2015
C. There are five consecutive integers with sum 2015
D. There are three consecutive integers with product 2015

isi2017 numerical-ability numerical-computation

Answer

8.28.14 Numerical Computation: TIFR2010-A-14 top https://gateoverflow.in/18393

A marine biologist wanted to estimate the number of fish in a large lake. He threw a net and found 30 fish in the net. He marked all these fish and released them into the lake. The next morning he again threw the net and this time caught 40 fish, of which two were found to be marked. The (approximate) number of fish in the lake is:

a. 600
b. 1200
c. 68
d. 800
e. 120

tifr2010 numerical-ability numerical-computation

Answer

8.28.15 Numerical Computation: TIFR2015-A-3 top https://gateoverflow.in/29159

Let $|z| < 1$. Define $M_n(z) = \sum_{i=1}^{10} z^{10^n(i-1)}$? what is

$$\prod_{i=0}^{\infty} M_i(z) = M_0(z) \times M_1(z) \times M_2(z) \times \ldots ?$$

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A. Can't be determined.
B. $1/(1-z)$
C. $1/(1+z)$
D. $1-z^9$
E. None of the above.

tifr2015 numerical-ability numerical-computation number-series

Answer

8.28.16 Numerical Computation: TIFR2015-B-12 top https://gateoverflow.in/30046

Let t_n be the sum of the first n natural numbers, for $n > 0$. A number is called triangular if it is equal to t_n for some n. Which of the following statements are true:

(i) There exists three successive triangular numbers whose product is a perfect square.

(ii) If the triangular number t_n is a perfect square, then so is $t_{4n(n+1)}$.

(iii) The sum of the reciprocals of the first n triangular numbers is less than 2, i.e.

$$\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\ldots.+\frac{1}{t_n}<2$$

a. (i) only.
b. (ii) only.
c. (iii) only.
d. All of the above.
e. None of the above.

tifr2015 numerical-ability normal numerical-computation

Answer

Answers: Numerical Computation

8.28.1 Numerical Computation: CMI2010-A-07 top https://gateoverflow.in/46138

Selected Answer

Let $X=\frac{n(5n+1)(10n+1)}{6}$

For $n=1, X=\frac{6*11}{6}=11$

For $n=2, X=\frac{2*11*21}{6}=77$

For $n=3, X=\frac{3*16*31}{6}=248$

For $n=4, X=\frac{4*21*41}{6}=574$

Here, we can see X is not divisible by 3 and 5 but X is always an Integer.

Hence, **Option(C) Is always an integer.**

4 votes -- Leen Sharma (40k points)

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8.28.2 Numerical Computation: GATE2011-57 top https://gateoverflow.in/2166

Selected Answer

B. is the answer.

Following logarithm formula, we get:
$P = Q^{\frac{1}{2}} = R^{\frac{1}{3}}$
So, $Q^2 = P^4 = P \times P^3 = PR$.

14 votes -- shreya ghosh (3.5k points)

8.28.3 Numerical Computation: GATE2011-65 top https://gateoverflow.in/2175

Selected Answer

Quantity left after n operations $= x\left(1 - \frac{y}{x}\right)^n$
where, $x =$ initial quantity
$y =$ amount of mixture withdrawn each time (this should be same every time)
$n =$ no. of times operation performed

$$= 10\left(1 - \frac{1}{10}\right)^n$$
$$= 10\left(\frac{9}{10}\right)^3$$
$$= 10 \times 0.9 \times 0.9 \times 0.9$$
$$= 10 \times 0.729 = 7.29 \text{ litres}$$

Hence, option D is correct.

18 votes -- Manu Thakur (39.6k points)

8.28.4 Numerical Computation: GATE2013-CE-1 top https://gateoverflow.in/40268

Selected Answer

D. 96

Let the number be x. Writing equation,

$x - 75 = 117 - x$.

$2 \times x = 192$.

5 votes -- Gaurav Sharma (2.4k points)

8.28.5 Numerical Computation: GATE2014-1-GA-4 top https://gateoverflow.in/773

Selected Answer

$$\left(Z + \frac{1}{Z}\right)^2$$

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$$= \left(z^2 + 2(z)\left(\frac{1}{z}\right) + \left(\frac{1}{z}\right)^2\right) = \left(z^2 + \frac{1}{z^2}\right) + 2 = 98$$

$\Rightarrow 98 - 2 = 96$ is answer..

17 votes -- Jay (1.2k points)

8.28.6 Numerical Computation: GATE2014-EC02-GA8 top

https://gateoverflow.in/41516

Selected Answer

Let the eight consecutive odd numbers be
$n, n+2, n+4, n+6, n+8, n+10, n+12$ and $n+14$
Sum of these numbers is $656, \implies 8n + 56 = 656,$ so, $n = 75.$

Let the four consecutive even numbers be $m, m+2, m+4,$ and $m+6$

Average of these numbers is $87, \implies \frac{(4m+12)}{4} = 87,$ so, $m = 84$

Sum of smallest odd number and second largest even number is $n + (m+4) = 75 + 88 = 163$

10 votes -- Praveen Saini (54.5k points)

8.28.7 Numerical Computation: GATE2014-EC04-GA4 top

https://gateoverflow.in/41466

Selected Answer

$f(x, y) = x^n y^m = P$

Now, x is doubled so we substitute $2x$ for x AND y is halved so we substitute $y/2$ for y

$f(x, y) = (2x)^n (\frac{y}{2})^m$

We get,

$f(x, y) = (2)^n (x)^n (y)^m (2)^{-m}$

$f(x, y) = 2^{n-m} x^n y^m = 2^{n-m} P$

Answer A) $2^{n-m} P$

7 votes -- Abhilash Panicker (9.4k points)

8.28.8 Numerical Computation: GATE2016-Session-7-GA-5 top

https://gateoverflow.in/110889

Selected Answer

Let the two digits be x and y.

$x + y = 12 \quad \rightarrow (1)$

Given that new number formed by reversing the digits is greater than the original number by 54,

$y * 10 + x = 10 * x + y - 54$

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$10y + x = 10x + y - 54$
$9x - 9y = 54$
$x - y = 6 \quad \rightarrow (2)$

From (1) and (2)

$2x = 18$
$\implies x = 9$ and
$y = 3.$

Hence, Original number $= 39.$

4 votes -- Muktinath Vishwakarma (35.3k points)

8.28.9 Numerical Computation: GATE2016-Session-8-GA-10 top

https://gateoverflow.in/111306

Selected Answer

211^{870} has unit digit 1 and unit digit of 146^{127} is 6 and unit digit of 3^{424} is 1

because 424 repeats remainder pattern $3, 9, 7, 1$ so on remainder divisible by 4 cases unit digit is $1.$

So in multiplication part multiplication unit digit = 6 as $6 * 1$ which is added to 1 hence answer is $7.$

5 votes -- robin sharma (243 points)

8.28.10 Numerical Computation: GATE2017-1-GA-4 top

https://gateoverflow.in/118407

Selected Answer

$y \times 162 = y \times 3^4 \times 2$

So, for perfect cube we need to add two $3s$ and two $2s.$

Answer is: $3^2 \times 2^2 = 36.$

15 votes -- KAUSHAL DUBEY (341 points)

8.28.11 Numerical Computation: GATE2017-2-GA-4 top

https://gateoverflow.in/118418

Selected Answer

Ans: **B.15**

Let the number of MCQs be x and number of essay questions be y.
Given MCQs are worth 3 marks each and essay questions are worth 11 marks each.
Also, total number of questions is 20 worth 100 marks in total.

$\Rightarrow x + y = 20$ and $3x + 11y = 100.$
$\Rightarrow 3x + 11(20 - x) = 100$
$\Rightarrow 8x = 120$
$\Rightarrow x = \dfrac{120}{8} = 15$

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8 votes -- Orochimaru (581 points)

8.28.12 Numerical Computation: GATE2017-2-GA-8 top

https://gateoverflow.in/118422

Selected Answer

$X = 4777\ldots$ (7 29 times)

It can be written as $X = 4.7777\cdots * 10^{29}$

$X^3 = (4.777\cdots * 10^{29})^3 = (4.777\ldots)^3 * 10^{87}$

Now, even if we round up $4.777\ldots$ to 5, we could represent $5^3 = 125$ in 3 digits. So, We can say $(4.77\ldots)^3$ also has 3 digits before decimal point.

So, X^3 requires $3 + 87 = 90$ digits.

50 votes -- Manish Joshi (27.9k points)

8.28.13 Numerical Computation: ISI-2017-12 top

https://gateoverflow.in/209610

Selected Answer

i) There are 3 consecutive integers with sum 2015

3 integers are $(x-1), x, (x+1)$

$(x-1) + x + (x+1) = 2015$

$\implies 3x = 2015$

$\implies x = 2015/3$

$= 671.66$

$670.66, 671.66, 672.66$ *are not integers.*

ii) There are 4 consecutive integers with sum 2015

4 integers are $(x+1), (x+2), (x+3), (x+4)$

$(x+1) + (x+2) + (x+3) + (x+4) = 2015$

$\implies 4x + 10 = 2015$

$\implies 4x = 2015 - 10 = 2005$

$\implies x = \dfrac{2005}{4}$

$= 501.25$

$502.25, 503.25, 504.25, 505.25$ *are not integers.*

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iii) There are 5 consecutive integers with sum 2015

5 integers are $(x-2),(x-1),x,(x+1),(x+2)$

$$(x-2)+(x-1)+x+(x+1)+(x+2)=2015$$

$$\implies 5x=2015$$

$$\implies x=2015/5$$

$$=403$$

$401,402,403,404,405$ *are integers.*

The integers are $(403-2),(403-1),403,(403+1),(403+2) \implies 401,402,403,404,405$

Therefore,sum = $401+402+403+404+405=2015$

iv) There are 3 consecutive integers with product 2015

3 integers are $(x-1),x,(x+1)$

$$(x-1)\times x\times(x+1)=2015$$

$$\implies (x^2-x)(x+1)=2015$$

$$\implies x^3-x^2+x^2-x=2015$$

$$\implies x^3-x=2015$$

$$\implies x^3-x-2015=0$$

it clearly shows that we cannot get integer from this equation.

$\therefore$ Correct option is III) There are 5 consecutive integers with sum 2015

2 votes -- Sukanya Das (11.5k points)

8.28.14 Numerical Computation: TIFR2010-A-14 top https://gateoverflow.in/18393

Selected Answer

Answer should be 600, Option A.

The problem given is equivalent to the problem in which an urn contains some number of white balls in it. We take 30 balls out of it, mark them and put them back into the urn. Now, we randomly take 40 balls out of the urn, 2 of them are found to be marked. What is the approximate number of balls that were present in the urn initially?

Solution: Suppose the urn contained X balls initially. Then
if we take n ball out of urn, probably $n\times(30/X)$ balls will be marked out of n balls.
Here, $n=40$.
So, Probably $40*(30/X)$ out of 40 balls will be marked.

But it is given that there are 2 marked balls,

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So, $2 = 40 \times (30/X)$, which gives

$X = (40 \times 30)/2 = 600.$

9 votes -- Anurag Pandey (13.6k points)

8.28.15 Numerical Computation: TIFR2015-A-3 top https://gateoverflow.in/29159

Selected Answer

$$M_n(z) = \sum_{i=1}^{10} z^{10^n(i-1)}$$

$$= z^{0*10^n} + z^{1*10^n} + z^{2*10^n} + \ldots + z^{9*10^n}$$

$$= 1 + z^{1*10^n} + z^{2*10^n} + \ldots + z^{9*10^n}$$

$$= \frac{1 - (z^{10^n})^{10}}{1 - z^{10^n}}$$

$$= \frac{1 - z^{10^{(n+1)}}}{1 - z^{10^n}}$$

$$M_n(z) = \frac{1 - z^{10^{(n+1)}}}{1 - z^{10^n}}$$

$Now,$

$$\prod_{i=0}^{\infty} M_i(z) = M_0(z) \times M_1(z) \times M_2(z) \times \ldots$$

$$= \left(\frac{1 - z^{10^1}}{1 - z^{10^0}}\right) \times \left(\frac{1 - z^{10^2}}{1 - z^{10^1}}\right) \times \left(\frac{1 - z^{10^3}}{1 - z^{10^2}}\right) \times \ldots \times \left(\frac{1 - z^{10^k}}{1 - z^{10^{(k-1)}}}\right) \times \left(\frac{1 - z^{10^{(k+1)}}}{1 - z^{10^k}}\right) \times \ldots$$

$$= \frac{1}{1 - z}$$

for ending terms, as $|z| < 1, z^{\infty}$ tends to $0, 1 - z^{\infty}$ tends to 1.

5 votes -- Praveen Saini (54.5k points)

8.28.16 Numerical Computation: TIFR2015-B-12 top https://gateoverflow.in/30046

Selected Answer

Triangular number, $t_n = \dfrac{n(n+1)}{2}$

Product of three consecutive Triangular numbers , $t_m \times t_{m+1} \times t_{m+2}$

$$= \frac{m(m+1)}{2} \times \frac{(m+1)(m+2)}{2} \times \frac{(m+2)(m+3)}{2}$$

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$$= \left(\frac{m(m+1)}{2}\right)^2 \times \left(\frac{m(m+3)}{2}\right)$$

At $m = 3, t_3 \times t_4 \times t_5$ is a perfect square.

(i) is True.

$$t_{4n(n+1)} = t_{4n^2+4n} = \frac{(4n^2+4n)(4n^2+4n+1)}{2}$$

$$= 4 \times (2n+1)^2 \times \frac{n(n+1)}{2} = 2^2 \times (2n+1)^2 \times t_n$$

If t_n is a perfect square, then $t_{4n(n+1)}$ is also a perfect square

(ii) is True

$$\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \ldots + \frac{1}{t_n}$$

$$= \frac{2}{1.(1+1)} + \frac{2}{2.(2+1)} + \frac{2}{3.(3+1)} + \frac{2}{4.(4+1)} + \ldots + \frac{2}{n.(n+1)}$$

$$= 2 \times \left(\frac{1}{1.(1+1)} + \frac{1}{2.(2+1)} + \frac{1}{3.(3+1)} + \frac{1}{4.(4+1)} + \ldots + \frac{1}{n.(n+1)}\right)$$

$$= 2 \times \left(\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \ldots + \frac{1}{n.(n+1)}\right)$$

$$= 2 \times \left(\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \ldots + \left(\frac{1}{n} - \frac{1}{(n+1)}\right)\right)$$

$$= 2 \times \left(1 - \frac{1}{(n+1)}\right)$$

$$= 2 \times \frac{n}{n+1}$$

for any $n > 0$, $\frac{n}{n+1}$ will be < 1, so $2 \times \frac{n}{n+1}$ will be < 2.

So, $\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \ldots + \frac{1}{t_n} < 2$

(iii) is also True.

17 votes -- Praveen Saini (54.5k points)

8.29 Odd One(3) top

8.29.1 Odd One: GATE2016-2-GA-04 top

https://gateoverflow.in/39528

Pick the odd one from the following options.

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A. $CADBE$
B. $JHKIL$
C. $XVYWZ$
D. $ONPMQ$

gate2016-2 verbal-ability odd-one normal

Answer

8.29.2 Odd One: GATE2016-Session-4-GA-4 top https://gateoverflow.in/110830

The number that least fits this set: $(324, 441, 97$ and $64)$ is _____.

A. 324
B. 441
C. 97
D. 64

gate2016session4aptitude odd-one numerical-ability

Answer

8.29.3 Odd One: GATE2016-Session-8-GA-3 top https://gateoverflow.in/111280

Pick the odd one out in the following:

$13, 23, 33, 43, 53$

A. 23
B. 33
C. 43
D. 53

gate2016session8aptitude odd-one logical-reasoning

Answer

Answers: Odd One

8.29.1 Odd One: GATE2016-2-GA-04 top https://gateoverflow.in/39528

Selected Answer

Option D.

A) $\underbrace{CADB\ E}_{3\ \ 1\ \ 4\ \ 2\ \ 5}$

B) $\underbrace{JHKI\ L}_{3\ \ 1\ \ 4\ \ 2\ \ 5}$

C) $\underbrace{XVYW\ Z}_{3\ \ 1\ \ 4\ \ 2\ \ 5}$

They are all making sequence of $1-2-3-4-5$ and as in alphabetical order

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17 votes -- Praveen Saini (54.5k points)

8.29.2 Odd One: GATE2016-Session-4-GA-4 top

https://gateoverflow.in/110830

Selected Answer

Ans. (C)

$324 = (18)^2$

$441 = (21)^2$

$64 = (8)^2$

Where as 97 is not perfect square of any number.

Therefore, ans is (c)

6 votes -- targate2018 (1.8k points)

8.29.3 Odd One: GATE2016-Session-8-GA-3 top

https://gateoverflow.in/111280

Selected Answer

33 is odd one among others because it is only composite number while others are prime numbers.

So,answer is B.

5 votes -- Arnabi Bej (7.9k points)

8.30 Percentage(9) top

8.30.1 Percentage: GATE-2012-AE-7 top

https://gateoverflow.in/40218

The total runs scored by four cricketers $P, Q, R\ and\ S$ in years 2009 and 2010 are given in the following table;

Player	2009	2010
P	802	1008
Q	765	912
R	429	619
S	501	701

The player with the lowest percentage increase in total runs is

A. P
B. Q
C. R
D. S

gate2012-ae numerical-ability percentage

Answer

8.30.2 Percentage: GATE2012-CY-GA-8 top

https://gateoverflow.in/40239

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The data given in the following table summarizes the monthly budget of an average household.

Category	Amount (Rs)
Food	4000
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget **NOT** spent on savings is

A. 10
B. 14
C. 81
D. 86

gate2012-cy numerical-ability percentage

Answer

8.30.3 Percentage: GATE2013-ee-2 top

https://gateoverflow.in/40289

In the summer of 2012, in New Delhi, the mean temperature of Monday to Wednesday was $41^{\circ}C$ and of Tuesday to Thursday was $43^{\circ}C$. If the temperature on Thursday was 15 higher than that of Monday, then the temperature in $^{\circ}C$ on Thursday was

A. 40
B. 43
C. 46
D. 49

gate2013-ee numerical-ability percentage

Answer

8.30.4 Percentage: GATE2014-1-GA-8 top

https://gateoverflow.in/776

Round-trip tickets to a tourist destination are eligible for a discount of 10 on the total fare. In addition, groups of 4 or more get a discount of 5 on the total fare. If the one way single person fare is $Rs\ 100$, a group of 5 tourists purchasing round-trip tickets will be charged Rs _____

gate2014-1 numerical-ability easy numerical-answers percentage

Answer

8.30.5 Percentage: GATE2014-3-GA-8 top

https://gateoverflow.in/2032

The Gross Domestic Product (GDP) in Rupees grew at 7 during $2012-2013$. For international comparison, the GDP is compared in US Dollars (USD) after conversion based on the market exchange rate. During the period $2012-2013$ the exchange rate for the USD increased from $Rs.$ $50/USD$ to $Rs.60/\ USD$. India's GDP in USD during the period $2012-2013$

A. increased by 5
B. decreased by 13
C. decreased by 20
D. decreased by 11

gate2014-3 numerical-ability normal percentage

Answer

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8.30.6 Percentage: GATE2014-AE-9 top https://gateoverflow.in/40309

One percent of the people of country X are taller than 6 ft. Two percent of the people of country Y are taller than 6 ft. There are thrice as many people in country X as in country Y. Taking both countries together, what is the percentage of people taller than 6 ft?

A. 3.0
B. 2.5
C. 1.5
D. 1.25

gate-2014-ae percentage numerical-ability

Answer

8.30.7 Percentage: GATE2014-EC04-GA8 top https://gateoverflow.in/41470

Industrial consumption of power doubled from $2000-2001$ to $2010-2011$. Find the annual rate of increase in percent assuming it to be uniform over the years.

A. 5.6
B. 7.2
C. 10.0
D. 12.2

gate2014-ec04 percentage normal numerical-ability

Answer

8.30.8 Percentage: GATE2016-Session-1-GA-4 top https://gateoverflow.in/108074

In a huge pile of apples and oranges, both ripe and unripe mixed together, 15% are unripe fruits. Of the unripe fruits, 45% are apples. Of the ripe ones, 66% are oranges. If the pile contains a total of 5692000 fruits, how many of them are apples?

A. 2029198
B. 2467482
C. 2789080
D. 3577422

gate2016session1aptitude percentage numerical-ability

Answer

8.30.9 Percentage: GATE2016-Session-7-GA-4 top https://gateoverflow.in/110885

$(x\% \text{ of } y) + (y\% \text{ of } x)$ is equivalent to _____ .

A. 2% of xy
B. 2% of $(xy/100)$
C. $xy\%$ of 100
D. 100% of xy

gate2016session7aptitude numerical-ability percentage

Answer

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Answers: Percentage

8.30.1 Percentage: GATE-2012-AE-7 top

https://gateoverflow.in/40218

Selected Answer

Total number of runs:

- $p = 1810$
- $q = 1677$
- $r = 1048$
- $s = 1202$

Now $\%$ increase is defined as: $\left(\frac{\text{final value - intital value}}{\text{initial value}}\right) \times 100$

$$p = \frac{206}{802} \times 100 = 25.68\%$$

$$q = \frac{147}{765} \times 100 = 19.21$$

$$r = \frac{190}{429} \times 100 = 44.28$$

$$s = \frac{200}{501} \times 100 = 39.42$$

So, the lowest increase is for Q.

B is the answer

3 votes -- Joker (1.9k points)

8.30.2 Percentage: GATE2012-CY-GA-8 top

https://gateoverflow.in/40239

Selected Answer

$$\frac{9000}{10500} \times 100 = 85.7\%$$

6 votes -- srestha (86.6k points)

8.30.3 Percentage: GATE2013-ee-2 top

https://gateoverflow.in/40289

Selected Answer

Let the temperatures on Monday, Tuesday, Wednesday and Thursday be $x^\circ, y^\circ, z^\circ$ and $w^\circ C$ respectively.

Given that,

- $x + y + z = 3 \times 41$

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- $y + z + w = 3 \times 43$

Thursday temperature is 15% higher than Monday temperature. So,

- $w = 1.15x$

After solving above equations, we get $w = 46$

Therefore, correct answer is (C).

4 votes -- suraj (5.6k points)

8.30.4 Percentage: GATE2014-1-GA-8 top

https://gateoverflow.in/776

Selected Answer

For individual , Round-trip discount 10% on TOTAL fare . So for each person (200 X 10%)=20 .

So for 5 member 100 rupees .

For 5 member group they will get 5% discount on TOTAL fare i.e. (5x200 X 5%)= 50 rupess.

Total discount is (100+50)=150 . They have to pay 850 rupees

12 votes -- Palash Nandi (1.5k points)

8.30.5 Percentage: GATE2014-3-GA-8 top

https://gateoverflow.in/2032

Selected Answer

Let India's $GDP = Rs.\, x$

$Rs.\ 50 = 1\ USD$

$Rs.\ x = \frac{1}{50} \times x\ USD$

New $GDP = GDP + 0.07 \times GDP$

$= Rs\ 1.07x$

$Rs\ 60 = 1\ USD$

$Rs\ 1.07x = \frac{1}{60} \times 1.07x$

$$\text{Change in GDP} = \frac{new - old}{old} \times 100$$

$$= \frac{\left(\frac{1.07x}{60}\right) - \left(\frac{x}{50}\right)}{\frac{x}{50}} \times 100$$

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$$= \frac{\left(\frac{-13x}{6000}\right)}{\left(\frac{x}{50}\right)} \times 100$$

$$= \left(\frac{-13}{120}\right) \times 100$$

$$= -0.10833 \times 100$$

$$= -10.833$$

$$\approx -11$$

So, there is an 11% decrease

Answer = **option D**

17 votes -- Amar Vashishth (30.5k points)

8.30.6 Percentage: GATE2014-AE-9 top

https://gateoverflow.in/40309

Selected Answer

Let the no. people in $Y = a$.

No. of people in Y greater than 6 feet $= \frac{2a}{100}$

No. of people in $X = 3a$

No. of people in X greater than 6 feet $= \frac{3a}{100}$

Total number of people greater than 6 feet $= \frac{2a}{100} + \frac{3a}{100} = \frac{5a}{100}$

Total number of people $= a + 3a = 4a$.

Percentage of people in $X + Y$ greater than 6 feet $= \left(\frac{5a}{100} \div 4a\right) \times 100 = 1.25\%$

5 votes -- Sandip Shaw (1.3k points)

8.30.7 Percentage: GATE2014-EC04-GA8 top

https://gateoverflow.in/41470

Selected Answer

Here we need to consider compound interest
consumption is doubled in 10 years

$$2P = P\left[1 + \frac{R}{100}\right]^{10}$$

$$2 = \left[1 + \frac{R}{100}\right]^{10}$$

$$\sqrt[10]{2} \times 100 = 100 + R$$

$$1.07177 \times 100 = 100 + R$$

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$R = 7.177 \approx 7.2$

So, ans is option B

11 votes -- Lokesh Dafale (11.4k points)

8.30.8 Percentage: GATE2016-Session-1-GA-4 top

https://gateoverflow.in/108074

Selected Answer

Total fruits = 5692000

$$= \begin{Bmatrix} Unripe(15\%) \rightarrow 853800 \\ Ripe(85\%) \rightarrow 4838200 \end{Bmatrix}$$

$$= \begin{Bmatrix} Unripe(15\%) \begin{Bmatrix} Apples(45\%) \rightarrow 384210 \\ Orange \rightarrow Don't\ care \end{Bmatrix} \\ Ripe(85\%) \begin{Bmatrix} Apples(34\%) \rightarrow 1644988 \\ Orange \rightarrow Don't\ care \end{Bmatrix} \end{Bmatrix}$$

So, Total Apples = 384210 + 1644988 = 2029198

3 votes -- Lokesh Dafale (11.4k points)

8.30.9 Percentage: GATE2016-Session-7-GA-4 top

https://gateoverflow.in/110885

Selected Answer

$X\%$ of $Y = \frac{X}{100} \times Y = X \times \frac{Y}{100} \quad \rightarrow (1)$

and

$Y\%$ of $X = \frac{Y}{100} \times X = Y \times \frac{X}{100} \quad \rightarrow (2)$

From equations (1) and (2) it is clear that $X\%$ of $Y+$ as $Y\%$ of X

$= 2 \times \frac{XY}{100}$ which is equivalent to 2% of XY

Hence, Answer is option A

5 votes -- Arnabi Bej (7.9k points)

8.31 Permutations And Combinations(9) top

8.31.1 Permutations And Combinations: GATE-MN-2011-59 top

https://gateoverflow.in/31531

In how many ways 3 scholarships can be awarded to 4 applicants,

when each applicant can receive any number of scholarships?

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A. 4
B. 12
C. 64
D. 81

numerical-ability gate2011-mn permutations-and-combinations

Answer

8.31.2 Permutations And Combinations: GATE2010-65 top

https://gateoverflow.in/2373

Given digits $2, 2, 3, 3, 3, 4, 4, 4, 4$ how many distinct 4 digit numbers greater than 3000 can be formed?

A. 50
B. 51
C. 52
D. 54

gate2010 numerical-ability permutations-and-combinations normal

Answer

8.31.3 Permutations And Combinations: GATE2012-AR-5 top

https://gateoverflow.in/40226

Ten teams participate in a tournament. Every team plays each of the other teams twice. The total number of matches to be played is

A. 20
B. 45
C. 60
D. 90

gate2012-ar numerical-ability permutations-and-combinations

Answer

8.31.4 Permutations And Combinations: GATE2014-EC04-GA10 top

https://gateoverflow.in/41472

A five digit number is formed using the digits $1, 3, 5, 7$ and 9 without repeating any of them. What is the sum of all such possible five digit numbers?

A. 6666660
B. 6666600
C. 6666666
D. 6666606

gate2014-ec04 numerical-ability normal permutations-and-combinations

Answer

8.31.5 Permutations And Combinations: GATE2016-2-GA-09 top

https://gateoverflow.in/39537

In a 2×4 rectangle grid shown below, each cell is rectangle. How many rectangles can be observed in the grid?

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A. 21
B. 27
C. 30
D. 36

gate2016-2 numerical-ability normal permutations-and-combinations

Answer

8.31.6 Permutations And Combinations: GATE2017-1-GA-9 top

https://gateoverflow.in/118412

Arun, Gulab, Neel and Shweta must choose one shirt each from a pile of four shirts coloured red, pink, blue and white respectively. Arun dislikes the colour red and Shweta dislikes the colour white. Gulab and Neel like all the colours. In how many different ways can they choose the shirts so that no one has a shirt with a colour he or she dislikes?

A. 21
B. 18
C. 16
D. 14

gate2017-1 permutations-and-combinations numerical-ability

Answer

8.31.7 Permutations And Combinations: ISI Sample Paper Question top

https://gateoverflow.in/125949

A club with n members is organized into four committees so that each member belongs to exactly two committees and each pair of committees has exactly one member in common. Then

A. $n = 4$
B. $n = 6$
C. $n = 8$
D. n cannot be determined from the given information

isisamplepapers numerical-ability permutations-and-combinations

Answer

8.31.8 Permutations And Combinations: TIFR2011-A-20 top

https://gateoverflow.in/20260

Let $n > 1$ be an odd integer. The number of zeros at the end of the number $99^n + 1$ is.

a. 1
b. 2
c. 3
d. 4
e. None of the above.

tifr2011 numerical-ability permutations-and-combinations

Answer

8.31.9 Permutations And Combinations: TIFR2013-A-1 top

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https://gateoverflow.in/25382

An infinite two-dimensional pattern is indicated below.

The smallest closed figure made by the lines is called a unit triangle. Within every unit triangle, there is a mouse.

At every vertex there is a laddoo. What is the average number of laddoos per mouse?

A. 3
B. 2
C. 1
D. $\left(\frac{1}{2}\right)$
E. $\left(\frac{1}{3}\right)$

tifr2013 numerical-ability permutations-and-combinations

Answer

Answers: Permutations And Combinations

8.31.1 Permutations And Combinations: GATE-MN-2011-59 top

https://gateoverflow.in/31531

Selected Answer

One scholarship can be awarded to 4 applicants in 4 ways. Three scholarship can be awarded to

4 applicants in $4^3 = 64$ ways.

(All scholarship are independent, any student can get any no of scholarships between 0 and 3)

7 votes -- Akash Kanase (42.5k points)

8.31.2 Permutations And Combinations: GATE2010-65 top

https://gateoverflow.in/2373

Selected Answer

first place should be occupied by either 3 or 4.

Case 1 : First place is occupied by the digit 4
4 _ _ _
now in the set from where we can pick numbers is left with $= \{2,2,3,3,3,4,4,4\}$
if we got 3 of each digit(which are $2,3$ and 4) then number of ways by **each** of those blanks can be

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filled in are 3 coz we have 3 choices of digits: $\text{pick } 2, 3$ or 4.
But we do not have just enough $2's$ to fill all those 3 spaces with the digit 2.

$\therefore$ we need to subtract this case where number would be 4222.
So, total numbers obtained using the numbers in our current set $= 1 \times 3 \times 3 \times 3 - 1 = 26$.

The first one is for the digit 4, coz its fixed for this case; the subtracted one is for the case 4222 that can't be made possible.

Case 2: First place is occupied by the digit 3
3 _ _ _
now in the set from where we can pick numbers is left with $= \{2, 2, 3, 3, 4, 4, 4, 4\}$
we have enough $4's$ here but lack $3's$ and $2's$ $\therefore$, the cases we need to subtract are 3222 and 3333
So, total numbers obtained using the numbers in our current set $= 1 \times 3 \times 3 \times 3 - 2 = 25$

both cases are independently capable of giving us the answer, we have $=26 + 25 = 51$.

Hence answer is **option B.**

44 votes -- Amar Vashishth (30.5k points)

8.31.3 Permutations And Combinations: GATE2012-AR-5 top

https://gateoverflow.in/40226

Selected Answer

Let $1.2.3.4.5.6.7.8.9.10$ be teams now

first team can play with any $=9$

similarly second can play with any $=8$ ...similarly $3rd$ team will play 7 games ...

So, $9 + 8 + 7 \ldots + 1 = \frac{n(n-1)}{2} = \frac{10 \times 9}{2} = 45$ ..we are said ..to have 2 matches= $45 \times 2 = 90$

8 votes -- Deepesh Kataria (2k points)

8.31.4 Permutations And Combinations: GATE2014-EC04-GA10

top https://gateoverflow.in/41472

Selected Answer

(B)

Consider the digits $1, 2, 3$.

The possible numbers are $\{123, 132, 213, 231, 312, 321\}$, count $= 3! = 6$.

- Consider the cases where the digit 3 is at the unit position. The number of such numbers (fix 3 at unit, permute the rest) will be $(3-1)! = 2!$. In all such cases, the 3 digit will contribute a 3 to the final sum. Total = $2! \times 3$
- Consider the cases where the digit 3 is at the decimal position. The number of such numbers (fix 3 at decimal, permute the rest) will be $(3-1)! = 2!$. In all such cases, the 3 digit will contribute a 30 to the final sum. Total = $2! \times 30$

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- Consider the cases where the digit 3 is at the hundreds position. The number of such numbers (fix 3 at hundreds, permute the rest) will be $(3-1)! = 2!$. In all such cases, the 3 digit will contribute a 300 to the final sum. Total = $2! \times 300$

In total, the digit 3 contributes $2! \times (3+30+300) = 2! \times 333$ to the final sum.

The same happens for all other digits.

Hence, the net sum (for the original question) will be:

$(5-1)! \times (11111 + 33333 + 55555 + 77777 + 99999)$

Note: $(5-1)!$ = the number of permutations after fixing 1 digit. $11111\ldots$ because the digit 1 contributes a $1, 10, 100, 1000, 10000$ to the final sum.

Required Answer = 6666600.

Hence, option B is correct.

14 votes -- Pragy Agarwal (20.6k points)

8.31.5 Permutations And Combinations: GATE2016-2-GA-09 top

https://gateoverflow.in/39537

Selected Answer

To form a rectangle, we must choose two horizontal sides and two vertical sides. Since there are three horizontal lines,
We can choose the horizontal sides in ${}^3C_2$ ways.
Similarly, to choose 2 vertical lines out of 5 vertical lines is ${}^5C_2$.

So, answer is $\binom{5}{2} \times \binom{3}{2}$.

43 votes -- rajan (5k points)

8.31.6 Permutations And Combinations: GATE2017-1-GA-9 top

https://gateoverflow.in/118412

Selected Answer

Total possibilities $= 4! = 24$.

No. of ways 'Arun chooses red' or 'Shwetha chooses white'

$$= (\text{no.of ways } `\text{Arun chooses red'})$$
$$+(\text{no.of ways } `\text{Shwetha chooses white'})$$
$$-(\text{no.of ways } `\text{Arun chooses red' and } `\text{Shwetha chooses white'})$$

$$= 6 + 6 - 2 = 10$$

required $= 24 - 10 = 14$.

31 votes -- Vinay Rachapalli (1.4k points)

8.31.7 Permutations And Combinations: ISI Sample Paper Question top

https://gateoverflow.in/125949

Selected Answer

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Let the first committee $= \{1, 2, 3\}$

Second $= \{3, 4, 5\}$

Third $= \{5, 6, 1\}$

Fourth $= \{2, 4, 6\}$

$n = 6$

Now, if we take intersection of any two committees we will get only one member in common, and also each member is part of exactly two groups.

We can also imagine it as an graph theory problem. 4 committees $= 4$ nodes and solution is to draw edges such that each edge is part of exactly two nodes and answer we will get is complete graph with 4 nodes (having total six edges.)

Hence, **answer B**

3 votes -- Keval Malde (16.1k points)

8.31.8 Permutations And Combinations: TIFR2011-A-20 top

Selected Answer

https://gateoverflow.in/20260

For odd n,

$$99^n = (100-1)^n$$

$$= 100^n - \binom{n}{1}100^{n-1} + \cdots - \binom{n}{n-2}100^2 + \binom{n}{n-1}100^1 - 1$$

$$99^n + 1 = 100^n - \binom{n}{1}100^{n-1} + \cdots - \binom{n}{n-2}100^2 + \binom{n}{n-1}100^1 - 1 + 1$$

$$= 100\left(100^{n-1} - \binom{n}{1}100^{n-2} + \cdots - \binom{n}{n-2}100 + n\right)$$

Since n is odd, it cannot end in a 0

Thus, $99^n + 1 = 100\,(\ldots \text{ doesn't end with } 0)$

which means $99^n + 1$ ends with exactly 2 zeros

Hence, option b) is correct.

Alternative way:

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$$\begin{array}{ccccc}
99 & \times & 01 & = & 99 \\
99 & \times & 099 & = & 9801 \\
99 & \times & (\ldots)801 & = & (\ldots)299 \\
99 & \times & (\ldots)299 & = & (\ldots)601 \\
99 & \times & (\ldots)601 & = & (\ldots)499 \\
99 & \times & (\ldots)499 & = & (\ldots)401 \\
99 & \times & (\ldots)401 & = & (\ldots)699 \\
99 & \times & (\ldots)699 & = & (\ldots)201 \\
99 & \times & (\ldots)201 & = & (\ldots)899 \\
99 & \times & (\ldots)899 & = & (\ldots)001
\end{array}$$

Thus, 99^n always ends in a 99 when n is odd, but never in a 999.

Hence, $99^n + 1$ will always end with exactly 2 zeros.

Note: We couldn't just say that $99^3 + 1$ ends with exactly 2 zeros, so b must be correct. This is because we also have an option e which says None of the above. Had it not been there, we could have marked b without having to prove that the pattern will continue.

6 votes -- Pragy Agarwal (20.6k points)

8.31.9 Permutations And Combinations: TIFR2013-A-1 top

https://gateoverflow.in/25382

Selected Answer

Let the number of lines per direction be 1 as shown bellow:

Here x, y, z depict the directions of the line.

$\eta_{laddoo} = 1$

$\eta_{mouse} = 0$

Add one more parallel line to each dimension x, y, z as shown bellow:

Encircled points represent laddoo $\Rightarrow \eta_{laddoo} = 3$

and triangle enclosed by them represent mouse $\Longrightarrow \eta_{mouse} = 1$

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Similarly for 3 lines in each direction

$\eta_{laddoo} = 6 \quad (1+2+3)$

$\eta_{mouse} = 4 \quad (2^2)$

As we continue we get a series which depends upon the no. of lines per direction $(\text{let say } l)$

So, $\eta_{laddoo} = \frac{l(l+1)}{2}$

$\eta_{mouse} = (l-1)^2$

$\lim_{l\to\infty} \frac{\eta_{laddoo}}{\eta_{mouse}} = 1/2.$

So, $D : 1/2$ is the correct answer.

3 votes -- Ritesh Kumar (229 points)

8.32 Pie Chart(4) top

8.32.1 Pie Chart: GATE2014-AG-GA8 top

https://gateoverflow.in/41672

The total exports and revenues from the exports of a country are given in the two pie charts below. The pie chart for exports shows the quantity of each item as a percentage of the total quantity of exports. The pie chart for the revenues shows the percentage of the total revenue generated through export of each item. The total quantity of exports of all the items is 5 lakh tonnes and the total revenues are 250 crore rupees. What is the ratio of the revenue generated through export of Item 1 per kilogram to the revenue generated through export of Item 4 per kilogram?

A. $1:2$
B. $2:1$
C. $1:4$
D. $4:1$

gate2014-ag numerical-ability data-interpretation pie-chart ratio-proportions normal

Answer

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8.32.2 Pie Chart: GATE2014-EC03-GA7 top

https://gateoverflow.in/41459

The multi-level hierarchical pie chart shows the population of animals in a reserve forest. The correct conclusions from this information are:

(i) Butterflies are birds

(ii) There are more tigers in this forest than red ants

(iii) All reptiles in this forest are either snakes or crocodiles

(iv) Elephants are the largest mammals in this forest

A. (i) and (ii) only
B. (i), (ii), (iii) and (iv)
C. (i), (iii) and (iv) only
D. (i), (ii) and (iii) only

gate2014-ec03 numerical-ability data-interpretation pie-chart normal

Answer

8.32.3 Pie Chart: GATE2014-EC03-GA9 top

https://gateoverflow.in/41461

A firm producing air purifiers sold 200 units in 2012. The following pie chart presents the share of raw material, labour, energy, plant & machinery, and transportation costs in the total manufacturing cost of the firm in 2012. The expenditure on labour in 2012 is Rs.4,50,000. In 2013, the raw material expenses increased by 30% and all other expenses increased by 20%. If the company registered a profit of Rs.10 lakhs in 2012, at what price (in Rs) was each air purifier sold?

gate2014-ec03 numerical-ability data-interpretation pie-chart numerical-answers

Answer

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8.32.4 Pie Chart: GATE2014-EC04-GA9 top

https://gateoverflow.in/41471

A firm producing air purifiers sold 200 units in 2012. The following pie chart presents the share of raw material, labour, energy, plant & machinery, and transportation costs in the total manufacturing cost of the firm in 2012. The expenditure on labour in 2012 is Rs. 4,50,000. In 2013, the raw material expenses increased by 30% and all other expenses increased by 20%. What is the percentage increase in total cost for the company in 2013?

gate2014-ec04 numerical-ability data-interpretation pie-chart numerical-answers

Answer

Answers: Pie Chart

8.32.1 Pie Chart: GATE2014-AG-GA8 top

https://gateoverflow.in/41672

Item 1: $11 \times x = 12$

Item 4: $22 \times y = 6$

$$\frac{x}{y} = \frac{12 \times 22}{11 \times 6} = 4:1$$

3 votes -- Arjun Suresh (348k points)

8.32.2 Pie Chart: GATE2014-EC03-GA7 top

https://gateoverflow.in/41459

Selected Answer

Ans will be (D)

The given pie chart is about population of animals, not about size of animals
So, (iv) is not true

(i) is true as butterfly comes under the pie birds.
(ii) is true as the pie chart portion of tigers is more than that of red ants.
(iii) is true as the pie portion corresponding to reptiles consists of only snakes and crocodiles.

7 votes -- srestha (86.6k points)

8.32.3 Pie Chart: GATE2014-EC03-GA9 top

https://gateoverflow.in/41461

Selected Answer

Expenditure on labour $= Rs. 4,50,000$
So, 15% of total expenditure $= Rs. 4,50,000$

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Then, total (100%) of expenditure $= Rs.\,4,50,000 \times \frac{100\%}{15\%} = Rs.\,30,00,000$

Now, Profit $= Rs.10,00,000$

Total selling cost $= Rs.30,00,000 + 10,00,000 = Rs.40,00,000.$

So, 200 air purifier selling cost $Rs.\,40,00,0001$ air purifier selling cost
$= \frac{Rs.40.00,000}{200} = Rs.\,20,000$

5 votes -- srestha (86.6k points)

8.32.4 Pie Chart: GATE2014-EC04-GA9 top

https://gateoverflow.in/41471

Selected Answer

$\%$ increase $= 30\%$ of raw material costs $+ 20\%$ of (labor costs + transportation costs + plant and machine costs + energy costs)

$= 30\%$ of 20% of total costs $+20\%$ of $(15\% + 10\% + 30\% + 25\%)$ of total cost

$= 30\%$ of 20% of total costs $+20\%$ of 80% of total costs

$= 110\%$ of 20% of total costs

$= 1.1 \times 20\%$ of total cost

$= 22\%$ of total cost

5 votes -- Aksh Singh (445 points)

8.33 Pigeonhole Principle(1) top

8.33.1 Pigeonhole Principle: CMI2012-A-06 top

https://gateoverflow.in/46535

A basket of fruit is being arranged out of apples, bananas, and oranges. What is the smallest number of pieces of fruit that should be put in the basket in order to guarantee that either there are at least 8 apples or at least 6 bananas or at least 9 oranges?

A. 9
B. 10
C. 20
D. 21

cmi2012 numerical-ability pigeonhole-principle

Answer

Answers: Pigeonhole Principle

8.33.1 Pigeonhole Principle: CMI2012-A-06 top

https://gateoverflow.in/46535

Selected Answer

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First, put **7 apples**, **5 bananas**, **and 8 oranges**, now put one more fruit to fulfill the mentioned condition either orange or banana or apple. Note that condition is given with OR.

Answer is $(7+5+8)+1=21$

3 votes -- Manu Thakur (39.6k points)

8.34 Polynomials(1) top

8.34.1 Polynomials: GATE2016-1-GA09 top

https://gateoverflow.in/39611

If $f(x)=2x^7+3x-5$, which of the following is a factor of $f(x)$?

A. (x^3+8)
B. $(x-1)$
C. $(2x-5)$
D. $(x+1)$

gate2016-1 numerical-ability polynomials normal

Answer

Answers: Polynomials

8.34.1 Polynomials: GATE2016-1-GA09 top

https://gateoverflow.in/39611

Selected Answer

for
$x=1,$

$f(1)=2(1)^7+3(1)-5=0\,,$

so
$x=1$ is a root for
$f(x)$

so $(x-1)$ is factor

31 votes -- Pooja Palod (31.3k points)

8.35 Probability(13) top

8.35.1 Probability: GATE-2012-AE-6 top

https://gateoverflow.in/40217

Two policemen, A and B, fire once each at the same time at an escaping convict. The probability that A hits the convict is three times the probability that B hits the convict. If the probability of the convict not getting injured is 0.5, the probability that B hits the convict is

A. 0.14
B. 0.22

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C. 0.33
D. 0.40

gate2012-ae numerical-ability probability

Answer

8.35.2 Probability: GATE2012-AR-9 top

https://gateoverflow.in/40230

A smuggler has 10 capsules in which five are filled with narcotic drugs and the rest contain the original medicine. All the 10 capsules are mixed in a single box, from which the customs officials picked two capsules at random and tested for the presence of narcotic drugs. The probability that the smuggler will be caught is

A. 0.50
B. 0.67
C. 0.78
D. 0.82

gate2012-ar numerical-ability probability

Answer

8.35.3 Probability: GATE2012-CY-GA-7 top

https://gateoverflow.in/4173

A and B are friends. They decide to meet between 1:00 pm and 2:00 pm on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is

A. $1/4$
B. $1/16$
C. $7/16$
D. $9/16$

gate2012-cy numerical-ability probability

Answer

8.35.4 Probability: GATE2013-ee-6 top

https://gateoverflow.in/40293

What is the chance that a leap year, selected at random, will contain 53 Saturdays?

A. $2/7$
B. $3/7$
C. $1/7$
D. $5/7$

gate2013-ee numerical-ability probability

Answer

8.35.5 Probability: GATE2014-AG-GA4 top

https://gateoverflow.in/41668

In any given year, the probability of an earthquake greater than Magnitude 6 occurring in the Garhwal Himalayas is 0.04. The average time between successive occurrences of such earthquakes is _____ years.

gate2014-ag numerical-ability probability numerical-answers normal

Answer

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8.35.6 Probability: GATE2014-EC02-GA4 top

https://gateoverflow.in/41511

A regular die has six sides with numbers 1 to 6 marked on its sides. If a very large number of throws show the following frequencies of occurrence: $1 \rightarrow 0.167; 2 \rightarrow 0.167; 3 \rightarrow 0.152; 4 \rightarrow 0.166; 5 \rightarrow 0.168; 6 \rightarrow 0.180.$ We call this die:

A. Irregular
B. Biased
C. Gaussian
D. Insufficient

gate2014-ec02 numerical-ability probability normal

Answer

8.35.7 Probability: GATE2014-EC03-GA10 top

https://gateoverflow.in/41462

A batch of one hundred bulbs is inspected by testing four randomly chosen bulbs. The batch is rejected if even one of the bulbs is defective. A batch typically has five defective bulbs. The probability that the current batch is accepted is_____.

gate2014-ec03 numerical-ability probability numerical-answers normal

Answer

8.35.8 Probability: GATE2015-1-GA-10 top

https://gateoverflow.in/8014

The probabilities that a student passes in mathematics, physics and chemistry are m, p and c respectively. Of these subjects, the student has 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Following relations are drawn in m, p, c:

I. $p + m + c = 27/20$
II. $p + m + c = 13/20$
III. $(p) \times (m) \times (c) = 1/10$

A. Only relation I is true.
B. Only relation II is true.
C. Relations II and III are true.
D. Relations I and III are true.

gate2015-1 numerical-ability probability

Answer

8.35.9 Probability: GATE2015-1-GA-3 top

https://gateoverflow.in/8004

Given Set $A = 2, 3, 4, 5$ and Set $B = 11, 12, 13, 14, 15$, two numbers are randomly selected, one from each set. What is the probability that the sum of the two numbers equals 16?

A. 0.20
B. 0.25
C. 0.30
D. 0.33

gate2015-1 numerical-ability probability normal

Answer

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8.35.10 Probability: GATE2016-Session-1-GA-6 top https://gateoverflow.in/108086

A person moving through a tuberculosis prone zone has a 50% probability of becoming infected. However, only 30% of infected people develop the disease. What percentage of people moving through a tuberculosis prone zone remains infected but does not show symptoms of disease?

A. 15
B. 33
C. 35
D. 37

gate2016session1aptitude numerical-ability probability

Answer

8.35.11 Probability: GATE2016-Session-8-GA-9 top https://gateoverflow.in/107575

Shaquille O' Neal is a 60% career free throw shooter, meaning that he successfully makes 60 free throws out of 100 attempts on average. What is the probability that he will successfully make exactly 6 free throws in 10 attempts?

A. 0.2508
B. 0.2816
C. 0.2934
D. 0.6000

gate2016session8aptitude probability

Answer

8.35.12 Probability: GATE2017-1-GA-5 top https://gateoverflow.in/118408

The probability that a k-digit number does NOT contain the digits $0, 5,$ or 9 is

A. 0.3^k
B. 0.6^k
C. 0.7^k
D. 0.9^k

gate2017-1 numerical-ability probability easy

Answer

8.35.13 Probability: GATE2017-2-GA-5 top https://gateoverflow.in/118419

There are 3 red socks, 4 green socks and 3 blue socks.You choose 2 socks. The probability that they are of the same colour is

A. $\frac{1}{5}$

B. $\frac{7}{30}$

C. $\frac{1}{4}$

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D. $\frac{4}{15}$

gate2017-2 numerical-ability probability

Answer

Answers: Probability

8.35.1 Probability: GATE-2012-AE-6 top

https://gateoverflow.in/40217

Selected Answer

$X-$ A hits the convict
$Y-$ B hits the convict
Given, $P(X) = 3 \times P(Y)$

$Z-$ Convict is injured
$Z'-$ Convict is not injured

Given, $P(Z') = 0.5$
$\implies P(Z) = 1 - P(Z') = 1 - 0.5 = 0.5$

Now,
$P(Z) = P(X) \times P(Y') + P(X') \times P(Y) + P(X) \times P(Y)$
Let $P(Y) = t$
$P(X) = 3t$
$P(Y') = 1 - t$
$P(X') = 1 - 3t$

Substituting in above equation,
$0.5 = (3t \times (1-t)) + ((1-3t) \times t) + (t \times 3t)$
$\implies 3t - 3t^2 + t - 3t^2 + 3t^2 = 0.5$
$\implies 3t^2 - 4t + 0.5 = 0$
$\implies 6t^2 - 8t + 1 = 0$

Solving, we get $t = 1.193$ (eliminated as probability cannot be greater than 1) OR $t = 0.1396$

Therefore. $P(Y) = t = 0.1396$
Answer (A) 0.14

Alternative Method: by Joker
$P(Z') = 0.5$
Now,
$P(Z') = (P(X') \times P(Y'))$
$\implies 0.5 = (1-t) \times (1-3t)$

Solving this gives the same equation as above.
$6t^2 - 8t + 1 = 0$
and the same answer.

6 votes -- Abhilash Panicker (9.4k points)

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8.35.2 Probability: GATE2012-AR-9 top

https://gateoverflow.in/40230

Selected Answer

The smuggler would be caught if any one or both the randomly picked capsule contains drugs.
M represents Medicine, D represents Drugs

- $A-$ The smuggler will be caught
- $B-$ The randomly picked capsules contains M, D
- $C-$ The randomly picked capsules contains D, M
- $D-$ The randomly picked capsules contains D, D

$$P(A) = P(B) + P(C) + P(D)$$

$$\begin{aligned} P(A) &= (5/10) \times (5/9) + (5/10) \times (5/9) + (5/10) \times (4/9) \\ &= 0.278 + 0.278 + 0.222 \\ &= 0.778 \end{aligned}$$

Hence, Answer is Option (C) 0.78

8 votes -- Abhilash Panicker (9.4k points)

8.35.3 Probability: GATE2012-CY-GA-7 top

https://gateoverflow.in/4173

Selected Answer

Meeting occurs if the first person arrives between $1:00$ and $1:45$ and the second person arrives in the next 15 minutes or if both the persons arrive between $1:45$ and $2:00$.

Case 1:

- $45/60$ are favourable cases and hence probability of first person arriving between $1:00$ and $1:45$ is $3/4$.
- Probability of second person arriving in the next 15 minutes $= 15/60 = 1/4$
- So, probability of one person arriving between $1:00$ and $1:45$ and meeting the other $= 3/4 \times 1/4 \times 2 = 3/8$ (2 for choosing the first arriving friend)

Case 2:

- Both friends must arrive between $1:45$ and $2:00$. Probability $= 1/4 \times 1/4 = 1/16$.

So, probability of a meet $= 3/8 + 1/16 = 7/16$

16 votes -- Arjun Suresh (348k points)

8.35.4 Probability: GATE2013-ee-6 top

https://gateoverflow.in/40293

Selected Answer

Sample space is
$S: \{\text{Monday-Tuesday, Tuesday-Wednesday, Wednesday-Thursday} \ldots \text{Sunday-Monday}\}$
Number of elements in $S = n(S) = 7$
What we want is a set A (say) that comprises of the elements
$\text{Saturday-Sunday and Friday-Saturday}$
Number of elements in set $A = n(A) = 2$

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By definition, probability of occurrence of $A = \frac{n(A)}{n(S)} = 27$

Therefore, probability that a leap year has 53 Saturdays is $\frac{2}{7}$.

7 votes -- Pooja Palod (31.3k points)

8.35.5 Probability: GATE2014-AG-GA4 top

https://gateoverflow.in/41668

Selected Answer

Probability of an earthquake greater than magnitude 6 is 0.04.

This means 4 out of 100 years will face an earthquake greater than magnitude 6. So average time between successive earthquakes will be $100/4 = 25$ years.

8 votes -- Aditya Sharma (585 points)

8.35.6 Probability: GATE2014-EC02-GA4 top

https://gateoverflow.in/41511

Selected Answer

For a very large number of throws, the frequency should be same for an unbiased die.

But given frequencies are not same, hence the die is biased.Hence, option B.

3 votes -- Ashish Gupta (1k points)

8.35.7 Probability: GATE2014-EC03-GA10 top

https://gateoverflow.in/41462

Selected Answer

In a batch there are total 100 bulbs of which
95 are working
5 are defective.

P(Selecting 4 working bulbs)$= \frac{95}{100} \times \frac{94}{99} \times \frac{93}{98} \times \frac{92}{97} = 0.81$

6 votes -- Abhilash Panicker (9.4k points)

8.35.8 Probability: GATE2015-1-GA-10 top

https://gateoverflow.in/8014

Selected Answer

Probability of non pass $= 1-$ Probability of at least one pass $= 1 - 0.75 = 0.25$

$(1-m)(1-p)(1-c) = 0.25$

$(1+mp-m-p)(1-c) = 0.25$

$1+mp-m-p-c-mpc+mc+pc = 0.25$

$m+p+c-mp-pc-mc+mpc = 0.75 \quad \rightarrow (1)$

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Probability of exactly 2 pass $= 0.4$

$mp(1-c) + pc(1-m) + mc(1-p) = 0.4$

$mp + pc + mc - 3mpc = 0.4$

$mp + pc + mc - 2mpc = 0.5 \quad \rightarrow (2)$
(Adding the probability of all pass to probability of exactly 2 pass gives probability of at least 2 pass)

So, $mpc = 0.1 \quad \rightarrow (3)$

From (2) and (3),
$mp + pc + mc - mpc = 0.6 \quad \rightarrow (4)$

From (1) and (4)

$m + p + c = 0.75 + 0.6$

$m + p + c = 1.35 = 135/100 = 27/20$

So, D option

25 votes -- Arjun Suresh (348k points)

8.35.9 Probability: GATE2015-1-GA-3 top https://gateoverflow.in/8004

Selected Answer

option A because total combinations are $5 \times 4 = 20$ and out of 20 we have only 4 combinations which have sum 16

1. $2, 14$
2. 3.13
3. 4.12
4. $5, 11$

11 votes -- Anoop Sonkar (5k points)

8.35.10 Probability: GATE2016-Session-1-GA-6 top https://gateoverflow.in/108086

Selected Answer

Option c) 35

$P(\text{infected}) = 0.5,$

$$P\left(\frac{No.\,of\ disease}{Infected}\right) = 0.7 \times 0.5 = 0.35$$

3 votes -- Riya Khandelwal (225 points)

8.35.11 Probability: GATE2016-Session-8-GA-9 top https://gateoverflow.in/107575

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Selected Answer

Let p be the probability of success cases and q be the probability of failure cases.

Given, $p = 0.6, q = 0.4$

The probability of 6 successful throws out of 10 can be given as:

$$= {}^{10}C_6 p^6 q^{10-6} = {}^{10}C_6 (0.6)^6 (0.4)^4 = 210 \times 0.046656 \times 0.0256 = 0.2508$$

2 votes -- Lokesh Dafale (11.4k points)

8.35.12 Probability: GATE2017-1-GA-5 top

https://gateoverflow.in/118408

Selected Answer

Total possibilities $=(10)^k$, because every digit has 10 options from $0\ to\ 9$.

Possibility of not containing any digit $0, 5, 9 = (7)^k$, now every digit has 7 options.

Asked probability $= \dfrac{(7)^k}{(10)^k} = (0.7)^k$

So C is the answer.

16 votes -- KAUSHAL DUBEY (341 points)

8.35.13 Probability: GATE2017-2-GA-5 top

https://gateoverflow.in/118419

Selected Answer

$$\frac{{}^3C_2 + {}^4C_2 + {}^3C_2}{{}^{10}C_2} = \frac{4}{15}.$$

assuming ball are not identical

24 votes -- 2018 (6.8k points)

8.36 Profit Loss(1) top

8.36.1 Profit Loss: GATE2013-CE-9 top

https://gateoverflow.in/40278

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A firm is selling its product at Rs. 60 per unit. The total cost of production is Rs. 100 and firm is earning total profit of Rs. 500. Later, the total cost increased by 30%. By what percentage the price should be increased to maintained the same profit level.

A. 5
B. 10
C. 15
D. 30

numerical-ability gate2013-ce profit-loss

Answer

Answers: Profit Loss

8.36.1 Profit Loss: GATE2013-CE-9 top

https://gateoverflow.in/40278

Selected Answer

Let N units be made.

Selling price per item $= Rs.60$

Let X be the cost per unit

We are given

$$500(\text{as profit}) = \underbrace{60 - X + 60 - X + 60 - X + 60 - X + \ldots 60 - X}_{N\ times}$$

$$N(60 - X) = 500 \implies 60N - NX = 500$$

Total cost is $Rs.100 \implies (60N - 100) = 500 \implies N = 10$

Now the cost has increased by 30%

New cost is $Rs.130$ and the extra 30 has to be equally divided among 10 units to maintain same profit level.

So, selling price should increase by 3 from $60 \implies \frac{3}{60} \times 100 = 5\%$ increase.

2 votes -- Deepesh Kataria (2k points)

8.37 Quadratic Equations(4) top

8.37.1 Quadratic Equations: GATE2011-MN-62 top

https://gateoverflow.in/31540

A student attempted to solve a quadratic equation in x twice. However, in the first attempt, he incorrectly wrote the constant term and ended up with the roots as $(4, 3)$. In the second attempt, he incorrectly wrote down the coefficient of x and got the roots as $(3, 2)$. Based on the above information, the roots of the correct quadratic equation are

A. $(-3, 4)$
B. $(3, -4)$
C. $(6, 1)$

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D. $(4, 2)$

gate2011-mn quadratic-equations numerical-ability

Answer

8.37.2 Quadratic Equations: GATE2013-ee-8 top https://gateoverflow.in/40295

The set of values of p for which the roots of the equation $3x^2 + 2x + p(p-1) = 0$ are of opposite sign is

A. $(-\infty, 0)$
B. $(0, 1)$
C. $(1, \infty)$
D. $(0, \infty)$

gate2013-ee numerical-ability quadratic-equations

Answer

8.37.3 Quadratic Equations: GATE2014-1-GA-5 top https://gateoverflow.in/770

The roots of $ax^2+bx+c=0$ are real and positive. a, b and c are real. Then $ax^2+b|x|+c=0$ has

A. no roots
B. 2 real roots
C. 3 real roots
D. 4 real roots

gate2014-1 numerical-ability quadratic-equations normal

Answer

8.37.4 Quadratic Equations: GATE2016-2-GA-05 top https://gateoverflow.in/39532

In a quadratic function, the value of the product of the roots (α, β) is 4. Find the value of

$$\frac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}}$$

A. n^4
B. 4^n
C. 2^{2n-1}
D. 4^{n-1}

gate2016-2 numerical-ability quadratic-equations normal

Answer

Answers: Quadratic Equations

8.37.1 Quadratic Equations: GATE2011-MN-62 top https://gateoverflow.in/31540

Selected Answer

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If equation is $ax^2 + bx + c = 0$ and roots of the equation are x_1 and x_2

then $x_1 + x_2 = -b/a$
and $x_1 . x_2 = c/a$

in 1st attempt c is incorrect, So, we can say $x_1 + x_2 = 4 + 3 = 7 \quad \rightarrow (i)$ [as here $-b/a$ is correct]

in 2nd attempt b is incorrect. So, we can say $x_1 . x_2 = 3.2 = 6 \quad \rightarrow (ii)$ [as here c/a is correct]

Now solving (ii) we get,

$x_2 = 6/x_1 \quad \rightarrow (iii)$

Putting it in eqn. (i)

$x_1 + \frac{6}{x_1} = 7$

$\implies x_1^2 - 7x_1 + 6 = 0$

$\implies x_1 = 1$ or $x_1 = 6$

Answer is (C)

7 votes -- srestha (86.6k points)

8.37.2 Quadratic Equations: GATE2013-ee-8 top https://gateoverflow.in/40295

Selected Answer

Roots of equation are of opposite sign then for the equation, $ax^2 + bx + c = 0$,

product of roots, $\frac{c}{a}$, should be (negative number) less than 0

$$\frac{p(p-1)}{3} < 0$$

$$p(p-1) < 0$$

so p must be less than 1 and greater than 0

Option B

14 votes -- Praveen Saini (54.5k points)

8.37.3 Quadratic Equations: GATE2014-1-GA-5 top https://gateoverflow.in/770

Selected Answer

Let the positive roots be m and n. Now, $-m$ and $-n$ will also satisfy the equation $ax^2 + b|x| + c = 0$ and hence we have 4 real roots.

25 votes -- Arjun Suresh (348k points)

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8.37.4 Quadratic Equations: GATE2016-2-GA-05 top https://gateoverflow.in/39532

Selected Answer

$$\frac{\alpha^n+\beta^n}{\alpha^{-n}+\beta^{-n}}$$

$$=\frac{\alpha^n+\beta^n}{\left(\frac{1}{\alpha^n}+\frac{1}{\beta^n}\right)}$$

$$=\frac{\alpha^n+\beta^n}{\left(\frac{\alpha^n+\beta^n}{\alpha^n\beta^n}\right)}$$

$$=(\alpha\beta)^n$$

$=4^n$

as product of roots,
$\alpha\beta=4$

26 votes -- Praveen Saini (54.5k points)

8.38 Ratio Proportions(8) top

8.38.1 Ratio Proportions: GATE2011-GG-GA-4 top https://gateoverflow.in/40205

If m students require a total of m pages of stationery in m days, then 100 students will require 100 pages of stationery in

A. 100 days
B. $m/100$ days
C. $100/m$ days
D. m days

gate2011_gg numerical-ability ratio-proportions

Answer

8.38.2 Ratio Proportions: GATE2013-AE-GA-1 top https://gateoverflow.in/40242

If $3 \leq X \leq 5$ and $8 \leq Y \leq 11$ then which of the following options is TRUE?

A. $\left(\frac{3}{5} \leq \frac{X}{Y} \leq \frac{8}{5}\right)$
B. $\left(\frac{3}{11} \leq \frac{X}{Y} \leq \frac{5}{8}\right)$
C. $\left(\frac{3}{11} \leq \frac{X}{Y} \leq \frac{8}{5}\right)$

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D. $\left(\frac{3}{5} \leq \frac{X}{Y} \leq \frac{8}{11}\right)$

gate2013-ae numerical-ability ratio-proportions normal

Answer

8.38.3 Ratio Proportions: GATE2014-AE-8 top https://gateoverflow.in/40308

The smallest angle of a triangle is equal to two thirds of the smallest angle of a quadrilateral. The ratio between the angles of the quadrilateral is $3:4:5:6$. The largest angle of the triangle is twice its smallest angle. What is the sum, in degrees, of the second largest angle of the triangle and the largest angle of the quadrilateral?

gate-2014-ae numerical-ability ratio-proportions numerical-answers

Answer

8.38.4 Ratio Proportions: GATE2016-1-GA10 top https://gateoverflow.in/39612

In a process, the number of cycles to failure decreases exponentially with an increase in load. At a load of
80 units, it takes
100 cycles for failure. When the load is halved, it takes
10000 cycles for failure.The load for which the failure will happen in
5000 cycles is _____.

A. 40.00
B. 46.02
C. 60.01
D. 92.02

gate2016-1 numerical-ability ratio-proportions normal

Answer

8.38.5 Ratio Proportions: GATE2018-GA-7 top https://gateoverflow.in/204068

If $pqr \neq 0$ and $p^{-x} = \frac{1}{q}, q^{-y} = \frac{1}{r}, r^{-z} = \frac{1}{p},$ what is the value of the product xyz ?

A. -1
B. $\frac{1}{pqr}$
C. 1
D. pqr

gate2018 numerical-ability ratio-proportions

Answer

8.38.6 Ratio Proportions: GATE2018-GA-8 top https://gateoverflow.in/204069

In a party, 60% of the invited guests are male and 40% are female.If 80% of the invited guests attended the party and if all the invited female guests attended, what would be the ratio of males to females among the attendees in the party?

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A. $2:3$
B. $1:1$
C. $3:2$
D. $2:1$

gate2018 numerical-ability ratio-proportions

Answer

8.38.7 Ratio Proportions: TIFR2012-A-18 top

https://gateoverflow.in/25043

A large community practices birth control in the following peculiar fashion. Each set of parents continues having children until a son is born; then they stop. What is the ratio of boys to girls in the community if, in the absence of birth control, 51% of the babies are born male?

a. $51:49$
b. $1:1$
c. $49:51$
d. $51:98$
e. $98:51$

tifr2012 numerical-ability ratio-proportions

Answer

8.38.8 Ratio Proportions: TIFR2014-A-2 top

https://gateoverflow.in/25987

A body at a temperature of 30 Celsius is immersed into a heat bath at 0 Celsius at time $t=0$. The body starts cooling at a rate proportional to the temperature difference. Assuming that the heat bath does not change in temperature throughout the process, calculate the ratio of the time taken for the body to reach 1 Celsius divided by the time taken for the body to reach 5 Celsius.

A. $\log 5.$

B. $\dfrac{\log 29}{\log 25}.$

C. $e^5.$

D. $1+\log_6 5.$

E. None of the above.

tifr2014 numerical-ability ratio-proportions

Answer

Answers: Ratio Proportions

8.38.1 Ratio Proportions: GATE2011-GG-GA-4 top

https://gateoverflow.in/40205

Selected Answer

Ans should be D)

as we know Men and days are inversely proportional and work and days are directly proportional

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so we can write $\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2}$

According to given data in question

$$\frac{m \times m}{m} = \frac{100 \times D_2}{100}$$

$D_2 = m$ days

7 votes -- sonam vyas (14.8k points)

8.38.2 Ratio Proportions: GATE2013-AE-GA-1 top https://gateoverflow.in/40242

Selected Answer

$\frac{X}{Y}$ is minimum for the given range when X is minimum possible and Y is maximum possible i.e., $3/11$

$\frac{X}{Y}$ is maximum for the given range when X is maximum possible and Y is minimum possible i.e., $5/8$

So, answer is (B)

9 votes -- Vivek Srivastava (3.6k points)

8.38.3 Ratio Proportions: GATE2014-AE-8 top https://gateoverflow.in/40308

Clearly, the angles of quadrilateral are 60, 80, 100, 120.

That makes smallest angle of triangle 60*(2/3) = 40.

That makes largest angle of triangle 40*(2) = 80.

so third angle of triangle = 60.

The required value is = 120 + 60 =180

3 votes -- Akshay Patil (41 points)

8.38.4 Ratio Proportions: GATE2016-1-GA10 top https://gateoverflow.in/39612

Selected Answer

The number of cycles to failure decrease exponentially with an increase in load.

So, we have general equation

$y = ae^{-bx}$

where y is number of cycles to failure, and x is load.

At load of 80 units , it takes 100 cycles for failure.

$100 = ae^{-80b} \qquad \rightarrow (1)$

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when load is halved it takes 10000 cycles for failure.

$10000 = ae^{-40b} \qquad \rightarrow (2)$

Divide (2) by (1)

$\implies e^{40b} = 100$

$\implies b = \dfrac{\log_e 100}{40} \qquad \rightarrow (3)$

At 5000 cycles to failure

$5000 = ae^{-xb} \qquad \rightarrow (4)$

divide (2) by (4)

$\implies e^{b(x-40)} = 2$

$\implies b(x-40) = \log_e 2$

$\implies \dfrac{\log_e 100}{40} \times (x-40) = \log_e 2$ (using (3))

$\implies x = 40 \times \dfrac{(\log_e 2 + \log_e 100)}{\log_e 100}$

$= 40 \times \dfrac{\log_e 200}{\log_e 100}$

$= 46.02$

43 votes -- Praveen Saini (54.5k points)

8.38.5 Ratio Proportions: GATE2018-GA-7 top

https://gateoverflow.in/204068

Selected Answer

$pqr \neq 0$

$p^{-x} = \dfrac{1}{q}$

Or,$\log(p^{-x}) = \log(\dfrac{1}{q})$

Or, $-x\log(p) = \log(1) - \log(q)$

$\left[\left[\because \log_b(m^n) = n.\log_b(m) \qquad \log_b(\dfrac{m}{n}) = \log_b(m) - \log_b(n)\right]\right]$

Or, $x\log(p) = \log(q) \qquad [\because \log(1) = 0]$

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Or, $x = \frac{\log(q)}{\log(p)}$

$q^{-y} = \frac{1}{r}$

Or,$\log(q^{-y}) = \log(\frac{1}{r})$

Or, $-y\log(q) = \log(1) - \log(r)$
$\left[\left[\because \log_b(m^n) = n.\log_b(m) \qquad \log_b(\frac{m}{n}) = \log_b(m) - \log_b(n)\right]\right]$

Or, $y\log(q) = \log(r) \qquad [\because \log(1) = 0]$

Or, $y = \frac{\log(r)}{\log(q)}$

$r^{-z} = \frac{1}{p}$

Or,$\log(r^{-z}) = \log(\frac{1}{p})$

Or, $-z\log(r) = \log(1) - \log(p)$
$\left[\left[\because \log_b(m^n) = n.\log_b(m) \qquad \log_b(\frac{m}{n}) = \log_b(m) - \log_b(n)\right]\right]$

Or, $z\log(r) = \log(p) \qquad [\because \log(1) = 0]$

Or, $z = \frac{\log(p)}{\log(r)}$

$\therefore x \times y \times z = \frac{\log(q)}{\log(p)} \times \frac{\log(r)}{\log(q)} \times \frac{\log(p)}{\log(r)}$

$= 1$

1 votes -- Subarna Das (10.5k points)

8.38.6 Ratio Proportions: GATE2018-GA-8 top

https://gateoverflow.in/204069

Selected Answer

Assume 100 people are invited, so 60 men and 40 women invited.

During the party only 80 people showed up, and since all women attended => 80 = 40 + men

Therefore, men = 40

men : women = 1:1

12 votes -- Wantei Warjri (333 points)

8.38.7 Ratio Proportions: TIFR2012-A-18 top

https://gateoverflow.in/25043

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Selected Answer

Let, X be the expected no of child a parent has.

So, no of boys $= 1.$
No. of girls $= X - 1.$

The probability of having a baby boy $= 0.51.$

And the probability of having a baby girl $= 0.49.$

So,

$$X = 1 \times (0.51) + 2 \times (0.49) \times (0.51) + 3 \times (0.49)^2 \times (0.51) + 4 \times (0.49)^3 \times (0.51)$$

$$0.49X = 1 \times (0.49) \times (0.51) + 2 \times (0.49)^2 \times (0.51) + 3 \times (0.49)^3 \times (0.51)$$

$$X - 0.49X = (0.51)[1 + (0.49) \times (0.51) + (0.49)^2 \times (0.51) + (0.49)^3 \times (0.51) + \ldots]$$

$$0.51X = (0.51)[1/0.51]$$

$$\implies X = 100/51.$$

So, No of girl children $= X - 1.$

$$= [100/51] - 1 = 49/51.$$

No. of boy children $= 1.$

Hence, Ratio, Boys : Girls $= 51 : 49.$

[Ans - A]

5 votes -- Dhananjay Kumar Sharma (25.7k points)

8.38.8 Ratio Proportions: TIFR2014-A-2 top https://gateoverflow.in/25987

Selected Answer

Let's say the Temperature of body at time t be T_t, which means $T_0 = 30.$

Now let the time at which temperature is 5 be t_1, which means $T_{t_1} = 5$

and the time at which temperature is 1 be t_2 , which means $T_{t_2} = 1.$

Question Asks us to find $\frac{t_2}{t_1}$.

Now, $\text{Temperature decrease(D) at time} t \propto \text{Body Temperature} - \text{Bath Temperature}$

$$D \propto T_t \ \{\text{As Bath Temp. is 0 \& Body Temp. is Tt}\}$$

$$D = k \times T_t \ \{\text{where } k \text{ is proportionality constant}\}$$

now, it gives $\quad T_{t+1} = T_t - D = T_t - k \times T_t = (1-k)T_t$

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Now, $T_0 = 30$

$$T_1 = 30(1-k)$$

$$T_2 = 30(1-k)^2$$

$$T_{t_1} = 30(1-k)^{t_1} \qquad \& \qquad T_{t_2} = 30(1-k)^{t_2}$$

$$\Rightarrow 30(1-k)^{t_1} = 5$$

$$\Rightarrow t_1 \times \log(1-k) = \log\left(\frac{5}{30}\right) = \log\left(\frac{1}{6}\right)$$

Similarly $\Rightarrow t_2 \times \log(1-k) = \log\left(\frac{1}{30}\right)$

$$\frac{t_2}{t_1} = \frac{\log\left(\frac{1}{30}\right)}{\log\left(\frac{1}{6}\right)}$$

$$= \frac{\log(30-1)}{\log(6-1)} = \frac{\log(30)}{\log(6)} = \log_6 30$$

$$= \log_6(6 \times 5) = \log_6 6 + \log_6 5 = 1 + \log_6 5$$

which is OPTION (D) ..

3 votes -- Himanshu Agarwal (15.3k points)

8.39 Sequence Series(6) top

8.39.1 Sequence Series: GATE2012-65 top

https://gateoverflow.in/2213

Given the sequence of terms, $AD\ CG\ FK\ JP$, the next term is

A. OV
B. OW
C. PV
D. PW

gate2012 numerical-ability sequence-series easy

Answer

8.39.2 Sequence Series: GATE2014-EC04-GA5 top

https://gateoverflow.in/41467

In a sequence of 12 consecutive odd numbers, the sum of the first 5 numbers is 425. What is the sum of the last 5 numbers in the sequence?

gate2014-ec04 numerical-ability sequence-series normal numerical-answers

Answer

8.39.3 Sequence Series: GATE2014-EC04-GA6 top

https://gateoverflow.in/41468

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Find the next term in the sequence: 13M, 17Q, 19S,_____.

A. 21W
B. 21V
C. 23W
D. 23V

gate2014-ec04 numerical-ability sequence-series normal

Answer

8.39.4 Sequence Series: GATE2016-Session-2-GA-8 top

https://gateoverflow.in/108300

Find the missing sequence in the letter series. $B, FH, LNP,$ _____.

A. $SUWY$
B. $TUVW$
C. $TVXZ$
D. $TWXZ$

gate2016session2aptitude sequence-series numerical-ability

Answer

8.39.5 Sequence Series: GATE2018-GA-5 top

https://gateoverflow.in/204066

What is the missing number in the following sequence?

$$2,\ 12,\ 60,\ 240,\ 720,\ 1440,\ ____,0$$

A. 2880
B. 1440
C. 720
D. 0

gate2018 numerical-ability sequence-series easy

Answer

8.39.6 Sequence Series: TIFR2013-A-19 top

https://gateoverflow.in/25500

Consider a sequence of numbers $(\epsilon_n : n = 1, 2, \ldots)$, such that $\epsilon_1 = 10$ and

$$\epsilon_{n+1} = \frac{20\epsilon_n}{20 + \epsilon_n}$$

for $n \geq 1$. Which of the following statements is true?

Hint: Consider the sequence of reciprocals.

1. The sequence $(\epsilon_n : n = 1, 2, \ldots)$ converges to zero.
2. $\epsilon_n \geq 1$ for all n
3. The sequence $(\epsilon_n : n = 1, 2, \ldots)$ is decreasing and converges to 1.
4. The sequence $(\epsilon_n : n = 1, 2, \ldots)$ is decreasing and then increasing. Finally it converges to 1.
5. None of the above.

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tifr2013 numerical-ability sequence-series

Answer

Answers: Sequence Series

8.39.1 Sequence Series: GATE2012-65 top

https://gateoverflow.in/2213

Selected Answer

A. OV

AD - difference 2 (B,C)

CG - difference 3 (D,E,F)

FK - difference 4 and JP - difference 5

so next term will have 6 differnce

again each term starts with preceding term's 2nd last letter

so JKLMNOP , next term will start with O and having 6 difference it will be OV

11 votes -- shreya ghosh (3.5k points)

8.39.2 Sequence Series: GATE2014-EC04-GA5 top

https://gateoverflow.in/41467

Selected Answer

Let a be the first odd number.
So the terms of sequence will be
$a, a+2, a+4, a+6, \ldots, a+20, a+22$

Sum of first 5 terms $= a+a+2+a+4+a+6+a+8 = 5a+20 = 425$
We get, $5a = 405$
$\implies a = 81$

Sum of last 5 terms $= a+22+a+20+a+18+a+16+a+14$
$= 5a+90$

Now, we have $a = 81$. Substituting it we get,
Answer as $5 \times 81 + 90 = 405 + 90 = 495$

Answer: 495

5 votes -- Abhilash Panicker (9.4k points)

8.39.3 Sequence Series: GATE2014-EC04-GA6 top

https://gateoverflow.in/41468

Selected Answer

$13, 17, 19, 23$ all are prime numbers

13th alphabet M

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17th alphabet Q
19th alphabet S
23th alphabet W

Answer C. $23W$

6 votes -- Abhilash Panicker (9.4k points)

8.39.4 Sequence Series: GATE2016-Session-2-GA-8 top

https://gateoverflow.in/108300

Selected Answer

Answer is C

2 votes -- Pavan Kumar Munnam (10.9k points)

8.39.5 Sequence Series: GATE2018-GA-5 top

https://gateoverflow.in/204066

Selected Answer

$2 \times 6 = 12$
$12 \times 5 = 60$
$60 \times 4 = 240$
$240 \times 3 = 720$
$720 \times 2 = 1440$
$1440 \times 1 = 1440$

So, option B,
1440 **is the next term.**

6 votes -- Rakesh Kumar (3.1k points)

8.39.6 Sequence Series: TIFR2013-A-19 top

https://gateoverflow.in/25500

Selected Answer

ϵ_1 is positive.

In the formula for ϵ_{n+1}, we only add, multiply and divide positive numbers. Thus, all ϵ_n are positive.

Also, $\epsilon_{n+1} < \epsilon_n$

Proof:

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$$\begin{aligned}
\epsilon_{n+1} - \epsilon_n &= \frac{20 \cdot \epsilon_n}{20 + \epsilon_n} - \epsilon_n \\
&= \frac{20 \cdot \epsilon_n - 20 \cdot \epsilon_n - (\epsilon_n)^2}{20 + \epsilon_n} \\
&= \frac{-(\epsilon_n)^2}{20 + \epsilon_n} \\
&< 0
\end{aligned}$$

$$\begin{aligned}
\epsilon_{n+1} - \epsilon_n &< 0 \\
\epsilon_{n+1} &< \epsilon_n
\end{aligned}$$

Thus, the sequence is decreasing.

Since the sequence is decreasing and is bounded below by 0, we know that the sequence converges (Monotone Convergence Theorem).

The only fixed point of the sequence can be found as follows:

$$\epsilon_f = \frac{20 \cdot \epsilon_f}{20 + \epsilon_f}$$

$$20 \cdot \epsilon_f + (\epsilon_f)^2 = 20 \cdot \epsilon_f$$

$$(\epsilon_f)^2 = 0$$

$$\epsilon_f = 0$$

Hence, the sequence converges to 0.

Option a is correct.

1 votes -- Pragy Agarwal (20.6k points)

8.40 Sets(2) top

8.40.1 Sets: CMI2013-A-08 top

https://gateoverflow.in/46598

In the passing out batch, 54 students know Java, 39 know Python and 43 know C++. Of these, 15 know both Java and Python, 17 know both Python and C++ and 23 know both Java and C++ and 11 know all three languages. If there are 100 students in the class, how many know none of these three languages?

A. 3
B. 8
C. 17
D. 19

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cmi2013 numerical-ability sets

Answer

8.40.2 Sets: GATE2016-2-GA-06 top

https://gateoverflow.in/39536

Among 150 faculty members in an institute, 55 are connected with each other through Facebook and 85 are connected through Whatsapp. 30 faculty members do not have Facebook or Whatsapp accounts. The numbers of faculty members connected only through Facebook accounts is _____.

A. 35
B. 45
C. 65
D. 90

gate2016-2 numerical-ability sets easy

Answer

Answers: Sets

8.40.1 Sets: CMI2013-A-08 top

https://gateoverflow.in/46598

Selected Answer

Let J denote the set of of students knowing Java and similarly P and C. Now,

$$JPC = J + P + C - JP - PC - JC + JPC = 54 + 39 + 43 - 15 - 17 - 23 + 11 = 92.$$

JPC gives the no. of students knowing at least one of Java, Python or C++. So, no. of students not knowing any of these languages
$= 100 - 92 = 8.$

2 votes -- Arjun Suresh (348k points)

8.40.2 Sets: GATE2016-2-GA-06 top

https://gateoverflow.in/39536

Selected Answer

Let F denotes Facebook users,

W denotes Whatsapp users ,

F' denotes those not using Facebook, and

W' denotes those not using Whatsapp.

U denotes universal set.

Total faculties in institute, $n(U) = 150$

Facebook users, $n(F) = 55$

Whatsapp users, $n(W) = 85$

Faculty members not using Facebook or Whatsapp $n(F' \cap W') = 30$

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Faculty members using either Facebook or Whatsapp

$$n(F \cup W) = n(U) - n(F' \cap W') = 150 - 30 = 120$$

Faculty members using both Facebook and Whatsapp

$$n(F \cap W) = n(F) + n(W) - n(F \cup W) = 55 + 85 - 120 = 20$$

Faculty members using Facebook only

$$n(F \cap W') = n(F) - n(F \cap W) = 55 - 20 = 35$$

24 votes -- Praveen Saini (54.5k points)

8.41 Speed Time Distance(9) top

8.41.1 Speed Time Distance: GATE-2013-AE-GA-6 top https://gateoverflow.in/40247

Velocity of an object fired directly in upward direction is given by $V = 80 - 32t$, where t (time) is in seconds. When will the velocity be between $32\ m/sec$ and $64\ m/sec$?

A. $\left(1, \frac{3}{2}\right)$
B. $\left(\frac{1}{2}, 1\right)$
C. $\left(\frac{1}{2}, \frac{3}{2}\right)$
D. $(1, 3)$

gate2013-ae numerical-ability speed-time-distance

Answer

8.41.2 Speed Time Distance: GATE2013-64 top https://gateoverflow.in/1568

A tourist covers half of his journey by train at $60\ km/h$, half of the remainder by bus at $30\ km/h$ and the rest by cycle at $10\ km/h$. The average speed of the tourist in km/h during his entire journey is

A. 36
B. 30
C. 24
D. 18

gate2013 numerical-ability easy speed-time-distance

Answer

8.41.3 Speed Time Distance: GATE2013-ee-9 top https://gateoverflow.in/40296

A car travels $8\ km$ in the first quarter of an hour, $6\ km$ in the second quarter and $16\ km$ in the third quarter. The average speed of the car in km per hour over the entire journey is

A. 30

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B. 36
C. 40
D. 24

gate2013-ee speed-time-distance numerical-ability

Answer

8.41.4 Speed Time Distance: GATE2014-EC01-GA8 top https://gateoverflow.in/41497

A train that is 280 metres long, travelling at a uniform speed, crosses a platform in 60 seconds and passes a man standing on the platform in 20 seconds. What is the length of the platform in metres?

gate2014-ec01 numerical-ability speed-time-distance normal numerical-answers

Answer

8.41.5 Speed Time Distance: GATE2014-EC02-GA10 top

https://gateoverflow.in/41518

It takes 30 minutes to empty a half-full tank by draining it at a constant rate. It is decided to simultaneously pump water into the half-full tank while draining it. What is the rate at which water has to be pumped in so that it gets fully filled in 10 minutes?

A. 4 times the draining rate
B. 3 times the draining rate
C. 2.5 times the draining rate
D. 2 times the draining rate

gate2014-ec02 numerical-ability speed-time-distance normal

Answer

8.41.6 Speed Time Distance: GATE2014-EC03-GA8 top https://gateoverflow.in/41460

A man can row at 8 km per hour in still water. If it takes him thrice as long to row upstream, as to row downstream, then find the stream velocity in km per hour.

gate2014-ec03 numerical-ability speed-time-distance normal numerical-answers

Answer

8.41.7 Speed Time Distance: GATE2016-Session-4-GA-6 top

https://gateoverflow.in/110634

The velocity V of a vehicle along a straight line is measured in m/s and plotted as shown with respect to time in seconds. At the end of the 7 seconds, how much will the odometer reading increase by (in m)?

A. 0
B. 3

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C. 4
D. 5

gate2016session4aptitude numerical-ability speed-time-distance data-interpretation

Answer

8.41.8 Speed Time Distance: TIFR2012-A-16 top https://gateoverflow.in/25041

Walking at $4/5$ is normal speed a man is 10 minute too late. Find his usual time in minutes.

a. 81
b. 64
c. 52
d. 40
e. It is not possible to determine the usual time from given data.

tifr2012 numerical-ability speed-time-distance

Answer

8.41.9 Speed Time Distance: TIFR2017-A-3 top https://gateoverflow.in/94941

On planet TIFR, the acceleration of an object due to gravity is half that on planet earth. An object on planet earth dropped from a height h takes time t to reach the ground. On planet TIFR, how much time would an object dropped from height h take to reach the ground?

A. $\left(\frac{t}{\sqrt{2}}\right)$
B. $\sqrt{2t}$
C. $2t$
D. $\left(\frac{h}{t}\right)$
E. $\left(\frac{h}{2t}\right)$

tifr2017 numerical-ability speed-time-distance

Answer

Answers: Speed Time Distance

8.41.1 Speed Time Distance: GATE-2013-AE-GA-6 top https://gateoverflow.in/40247

Selected Answer

Given

$32 < 80 - 32t < 64$

$-48 < -32t < -16$

$48 > 32t > 16$

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$$\frac{3}{2} > t > \frac{1}{2}$$

so ans is option c) $\left(\frac{1}{2}, \frac{3}{2}\right)$

3 votes -- sonam vyas (14.8k points)

8.41.2 Speed Time Distance: GATE2013-64 top https://gateoverflow.in/1568

Selected Answer

Let the total distance be D then

Avg speed $= \dfrac{D}{\text{total time taken}}$

Total time taken $= \left(\dfrac{D}{2} \times 60\right) + \left(\dfrac{D}{4} \times 30\right) + \left(\dfrac{D}{4} \times 10\right)$

$\text{Avg speed} - \dfrac{120}{5} = 24.$

11 votes -- Bhagirathi Nayak (14.1k points)

8.41.3 Speed Time Distance: GATE2013-ee-9 top https://gateoverflow.in/40296

Selected Answer

8 km in the first quarter of an hour, 6 km in the second quarter and 16 km in the third quarter.

Total distance $= 30$ km and total time $= 3/4$. $(1/4 + 1/4 + 1/4 = 3/4)$

$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{30}{3/4} = 30 \times (4/3) = 40\ km/hr$

8 votes -- learner_geek (3.5k points)

8.41.4 Speed Time Distance: GATE2014-EC01-GA8 top https://gateoverflow.in/41497

Selected Answer

Speed of train$= 280/20 = 14m/s$

Let L be the length of platform

$280 + L$ covered in $60s$

$\frac{280+L}{14} = 60$

$280 + L = 840$

$L = 840 - 280$

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$L = 560m$

7 votes -- Pooja Palod (31.3k points)

8.41.5 Speed Time Distance: GATE2014-EC02-GA10 top

Lets Say capacity of tank is 1 litre.

draining rate=$\frac{0.5 \text{ litre}}{30 \text{ minutes}} = \frac{1}{60} \text{ litre/min}$

let filling rate = $x \text{ litre/min}$

in 1 min tank gets $x - \left(\frac{1}{60}\right)$ litre filled.

to fill the remaining half part we need 10 mins

$x - \frac{1}{60} \text{ litre} = 1 \text{ min}$

$0.5 \text{ litre} = 10 \text{ mins}$

$$\frac{0.5}{\left(\frac{x-1}{60}\right)} = 10$$

solving, we get $x = \frac{4}{60}$
which is 4 times more than draining rate.
so option A

8 votes -- Motamarri Anusha (12.3k points)

8.41.6 Speed Time Distance: GATE2014-EC03-GA8 top https://gateoverflow.in/41460

Selected Answer

Speed of man $(m) = 8 \ km/h$
Let the speed of stream be s

According to the question:
Speed of man upstream $= S_1 = m - s$
Speed of man downstream $= S2 = m + s$

Speed = Distance/Time
Here, since the distance D is same,

$D = S_1 \times T_1$
$D = S_2 \times T_2$

$S_1 \times T_1 = S_2 \times T2$

$\frac{S1}{S2} = \frac{T2}{T1} = 3$

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$m + s = 3(m - s)$

or, $8 + s = 3(8 - s)$

$\implies s = 4km/h$

5 votes -- srestha (86.6k points)

8.41.7 Speed Time Distance: GATE2016-Session-4-GA-6 top

https://gateoverflow.in/110834

Selected Answer

Odometer is a device used to measure distance traveled by an object.
Area under Velocity-Time graph represents the distance traveled.

So, answer $= 0.5 + 1 + 1.5 + 1 + 0.5 + 0.5 = 5m.$

1 votes -- Akash Sheoran (2k points)

8.41.8 Speed Time Distance: TIFR2012-A-16 top

https://gateoverflow.in/25041

Selected Answer

Let the man travel distance d with speed s in time t.

$d = st \quad \rightarrow (1)$

According to the question we get second equation,

$d = \frac{4}{5} \times s(t + 10) \quad \rightarrow (2)$

From (1) and (2)

$\frac{5}{4}t - t = 10$

So, $t = 40$

Ans is d.

4 votes -- Pooja Palod (31.3k points)

8.41.9 Speed Time Distance: TIFR2017-A-3 top

https://gateoverflow.in/94941

Selected Answer

Let, The acceleration due to gravity on earth $=g$.

and the acceleration due to gravity on $TIFR = G = \frac{g}{2}$.

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Time taken to reach the ground on earth $= t = \sqrt{\frac{2h}{g}}$.

Similarly, on TIFR planet, time taken = T = $\sqrt{\frac{2h}{G}}$.= $\sqrt{\frac{4h}{g}}$.

$\Rightarrow T = \sqrt{2}t.$

Ans - Option (b)

7 votes -- Dhananjay Kumar Sharma (25.7k points)

8.42 Statement Argument(1) top

8.42.1 Statement Argument: GATE2015-2-GA-5 top https://gateoverflow.in/8033

Based on the given statements, select the most appropriate option to solve the given question.

What will be the total weight of 10 poles each of same weight?

Statements:

I. One fourth of the weight of the pole is $5\ Kg$.
II. The total weight of these poles is $160\ Kg$ more than the total weight of two poles.

A. Statement I alone is not sufficient.
B. Statement II alone is not sufficient.
C. Either I or II alone is sufficient.
D. Both statements I and II together are not sufficient.

gate2015-2 numerical-ability normal logical-reasoning statement-argument

Answer

Answers: Statement Argument

8.42.1 Statement Argument: GATE2015-2-GA-5 top https://gateoverflow.in/8033

Selected Answer

Ans C

4 votes -- Vikrant Singh (13.5k points)

8.43 Statement Sufficiency(1) top

8.43.1 Statement Sufficiency: GATE2015-1-GA-4 top https://gateoverflow.in/8006

Based on the given statements, select the most appropriate option to solve the given question.

If two floors in a certain building are 9 feet apart, how many steps are there in a set of stairs that extends from the first floor to the second floor of the building?

Statements:

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(I) Each step is $3/4$ foot high.

(II) Each step is 1 foot wide.

A. Statements I alone is sufficient, but statement II alone is not sufficient.
B. Statements II alone is sufficient, but statement I alone is not sufficient.
C. Both statements together are sufficient, but neither statement alone is sufficient.
D. Statements I and II together are not sufficient.

gate2015-1 numerical-ability easy statement-sufficiency

Answer

Answers: Statement Sufficiency

8.43.1 Statement Sufficiency: GATE2015-1-GA-4 top https://gateoverflow.in/8006

Selected Answer

A. Statements I alone is sufficient, but statement II alone is not sufficient.

13 votes -- Anoop Sonkar (5k points)

8.44 Statistics(3) top

8.44.1 Statistics: GATE-2012-AE-5 top https://gateoverflow.in/40216

The arithmetic mean of five different natural numbers is 12. The largest possible value among the numbers is

A. 12
B. 40
C. 50
D. 60

gate2012-ae statistics numerical-ability

Answer

8.44.2 Statistics: GATE2012-64 top https://gateoverflow.in/2212

Which of the following assertions are **CORRECT**?

P: Adding 7 to each entry in a list adds 7 to the mean of the list

Q: Adding 7 to each entry in a list adds 7 to the standard deviation of the list

R: Doubling each entry in a list doubles the mean of the list

S: Doubling each entry in a list leaves the standard deviation of the list unchanged

A. P, Q
B. Q, R
C. P, R
D. R, S

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gate2012 numerical-ability statistics normal

Answer

8.44.3 Statistics: GATE2016-Session-2-GA-6 top

https://gateoverflow.in/108297

Students taking an exam are divided into two groups, **P** and **Q** such that each group has the same number of students. The performance of each of the students in a test was evaluated out of 200 marks. It was observed that the mean of group **P** was 105, while that of group **Q** was 85. The standard deviation of group **P** was 25, while that of group **Q** was 5. Assuming that the marks were distributed on a normal distribution, which of the following statements will have the highest probability of being **TRUE**?

A. No student in group **Q** scored less marks than any student in group **P**.
B. No student in group **P** scored less marks than any student in group **Q**.
C. Most students of group **Q** scored marks in a narrower range than students in group **P**.
D. The median of the marks of group **P** is 100.

gate2016session2aptitude probability statistics

Answer

Answers: Statistics

8.44.1 Statistics: GATE-2012-AE-5 top

https://gateoverflow.in/40216

Selected Answer

It is 50 obviously.

Total sum of 5 natural numbers = $12*5 = 60$

If 60 is one natural number , then the other 4 numbers must be 0. As 0 is not a natural number(it is a whole number), 60 is not right,so next option remaining is 50.
and also in question they have told as different natural numbers. so 50, 1 ,2 , 3, 4 are the numbers.
Answer is 50, option C

4 votes -- Sreyas S (1.9k points)

8.44.2 Statistics: GATE2012-64 top

https://gateoverflow.in/2212

Selected Answer

Suppose we double each entry of a list

Initial Mean $(M_I) = \frac{\sum_{i=1}^{n} x_i}{n}$

New Mean $(M_N) = \frac{\sum_{i=1}^{n} 2 \times x_i}{n}$

$= \frac{2}{n} \sum_{i=1}^{n} x_i$

So, when each entry in the list is doubled, mean also gets doubled.

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Standard Deviation $\sigma_I = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(M_I - x_i)^2}$

New Standard Deviation $\sigma_N = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(M_N - 2 \times x_i)^2}$

$$= \sqrt{\frac{1}{N}\sum_{i=1}^{n}(2 \times (M_I - x_i))^2}$$

$$= 2\sigma_I$$

So, when each entry is doubled, standard deviation also gets doubled.

When we add a constant to each element of the list, it gets added to the mean as well. This can be seen from the formula of mean.

When we add a constant to each element of the list, the standard deviation (or variance) remains unchanged. This is because, the mean also gets added by the same constant and hence the deviation from the mean remains the same for each element.

So, here P and R are correct.

17 votes -- Arjun Suresh (348k points)

8.44.3 Statistics: GATE2016-Session-2-GA-6 top

https://gateoverflow.in/108297

Selected Answer

Group Q students have less standard deviation than group P , means most students in group Q got less marks than group P but Not all students in group Q got less marks than group P .

That makes statement A and B incorrect .

Mean, Median and Mode of Normal Distribution is same , so option D is wrong .

Only option C is correct.

1 votes -- Bikram (67k points)

8.45 Summation(1) top

8.45.1 Summation: ISI2013-A-2 top

https://gateoverflow.in/47496

Find the value of Σij, where the summation is over all integers i and j such that $1 \leq i < j \leq 10$.

isi2013 numerical-ability summation numerical-answers

Answer

Answers: Summation

8.45.1 Summation: ISI2013-A-2 top

https://gateoverflow.in/47496

Selected Answer

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Given: $1 \leq i < j \leq 10$

$$\sum ij = (1.2 + 1.3 + \cdots + 1.10) + (2.3 + 2.4 + \cdots + 2.10) \\ +(3.4 + 3.5 + \cdots + 3.10) \ldots (8.9 + 8.10) + 9.10$$

$$= 1.(2 + 3 + 4 \cdots + 10) + 2.(3 + 4 + \cdots + 10) + 3.(4 + 5 + \cdots + 10) + 4.(5 + 6 + \cdots + 10) \\ +5.(6 + 7 + \cdots + 10) + 6.(7 + 8 + \cdots + 10) + 7(8 + 9 + 10) + 8(9 + 10) + (9.10)$$

We knows Sum of first Natural Numbers= $\frac{n(n+1)}{2}$

now,

$$= \left[1.\left[\frac{n(n+1)}{2} - 1\right] + 2.\left[\frac{n(n+1)}{2} - 3\right] + 3.\left[\frac{n(n+1)}{2} - 6\right]\right. \\ +4.\left[\frac{n(n+1)}{2} - 10\right] + 5.\left[\frac{n(n+1)}{2} - 15\right] + 6.\left[\frac{n(n+1)}{2} - 21\right] \\ \left.+7.\left[\frac{n(n+1)}{2} - 28\right] + 8.\left[\frac{n(n+1)}{2} - 36\right] + 9.\left[\frac{n(n+1)}{2} - 45\right]\right]$$

$$= 55.[1 + 2 + \ldots 9] - \sum_{i=1}^{9} i.i.\frac{(i+1)}{2} \\ = 55.45 - \frac{1}{2}\sum_{i=1}^{10} i^3 + i^2 \\ = 2475 - \frac{1}{2}\left(45^2 + 9.10.\frac{19}{6}\right) \\ = 1320.$$

4 votes -- Leen Sharma (40k points)

8.46 System Of Equations(1) top

8.46.1 System Of Equations: GATE2011-GG-GA-6 top https://gateoverflow.in/40207

The number of solutions for the following system of inequalities is

- $X_1 \geq 0$
- $X_2 \geq 0$
- $X_1 + X_2 \leq 10$
- $2X_1 + 2X_2 \geq 22$

A. 0
B. infinite
C. 1
D. 2

gate2011_gg numerical-ability system-of-equations

Answer

Answers: System Of Equations

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8.46.1 System Of Equations: GATE2011-GG-GA-6 top https://gateoverflow.in/40207

Selected Answer

$X_1 \geq 0 \quad \rightarrow (1)$
$X_2 \geq 0 \quad \rightarrow (2)$
$X_1 + X_2 \leq 10 \quad \rightarrow (3)$
$2X_1 + 2X_2 \geq 22 \quad \rightarrow (4)$

Now the equation (4) can be written as
$X_1 + X_2 \geq 11 \quad \rightarrow (5)$

Now, equations (3) and (5) cannot hold true together since $X_1 \geq 0$ and $X_2 \geq 0$.
Hence, system of inequalities can never be satisfied.

Answer A. 0

6 votes -- Abhilash Panicker (9.4k points)

8.47 Variance(1) top

8.47.1 Variance: GATE2014-EC01-GA4 top https://gateoverflow.in/41493

The statistics of runs scored in a series by four batsmen are provided in the following table. Who is the most consistent batsman of these four?

Batsman	Average	Standard deviation
K	31.2	5.21
L	46.0	6.35
M	54.4	6.22
N	17.9	5.90

A. K
B. L
C. M
D. N

gate2014-ec01 statistics variance numerical-ability

Answer

Answers: Variance

8.47.1 Variance: GATE2014-EC01-GA4 top https://gateoverflow.in/41493

Selected Answer

Most consistent is the least dispersion in the scores. We have a parameter called Coefficient of Variance which is used to measure relative measure of dispersion between two sets.

Coefficient of Variance(CV) = $\frac{StandardDeviation}{Mean}$

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Calculating it for all the four players.

K's CV = 0.167

L's CV = 0.138

M's CV = 0.114

N's CV = 0.329

Larger the value of CV, less consistent the player is.

So the most consistent player is M here

Option C

4 votes -- Randomeshwar (317 points)

8.48 Venn Diagrams(3) top

8.48.1 Venn Diagrams: GATE2010-59 top https://gateoverflow.in/2367

25 persons are in a room. 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. Then the number of persons playing neither hockey nor football is:

A. 2
B. 17
C. 13
D. 3

gate2010 numerical-ability easy set-theory&algebra venn-diagrams

Answer

8.48.2 Venn Diagrams: GATE2011-GG-GA-7 top https://gateoverflow.in/40208

In a class of 300 students in an M.Tech programme, each student is required to take at least one subject from the following three:

- M600: Advanced Engineering Mathematics
- C600: Computational Methods for Engineers
- E600: Experimental Techniques for Engineers

The registration data for the M.Tech class shows that 100 students have taken M600, 200 students have taken C600, and 60 students have taken E600. What is the maximum possible number of students in the class who have taken all the above three subjects?

A. 20
B. 30
C. 40
D. 50

gate2011_gg numerical-ability set-theory&algebra venn-diagrams

Answer

8.48.3 Venn Diagrams: GATE2016-Session-3-GA-6 top https://gateoverflow.in/108484

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The Venn diagram shows the preference of the student population for leisure activities.

From the data given, the number of students who like to read books or play sports is _____.

A. 44
B. 51
C. 79
D. 108

gate2016session3aptitude venn-diagrams logical-reasoning

Answer

Answers: Venn Diagrams

8.48.1 Venn Diagrams: GATE2010-59 top https://gateoverflow.in/2367

Selected Answer

D. 3
No. of persons who play either football or hockey $= 15 + 17 - 10 = 22$
No. of persons playing neither hockey nor football $= 25 - 22 = 3$

11 votes -- shreya ghosh (3.5k points)

8.48.2 Venn Diagrams: GATE2011-GG-GA-7 top https://gateoverflow.in/40208

Selected Answer

Let the no. of students who took all courses be x.

Since every one must take at least 1 course, we have

$$200 - x + 100 - x + 60 - x \geq 300 - x \implies 360 - 2x \geq 300 \implies x \leq 30.$$

9 votes -- Arjun Suresh (348k points)

8.48.3 Venn Diagrams: GATE2016-Session-3-GA-6 top https://gateoverflow.in/108484

Selected Answer

The number of students who like to read books **or** play sports will be the sum of students who belong to both sets $= 13 + 12 + 44 + 7 + 17 + 15 = 108.$

Answer will be D

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2 votes -- ਜਗਮੀਤ (1.4k points)

8.49 Work Time(5) top

8.49.1 Work Time: GATE2010-64 top

https://gateoverflow.in/2372

5 skilled workers can build a wall in 20 days; 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long it will take to build the wall?

A. 20 days
B. 18 days
C. 16 days
D. 15 days

gate2010 numerical-ability normal work-time

Answer

8.49.2 Work Time: GATE2011-64 top

https://gateoverflow.in/2174

A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4th day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10th day. What is the minimum number of trucks required so that there will be no pending order at the end of 5th day?

A. 4
B. 5
C. 6
D. 7

gate2011 numerical-ability normal work-time

Answer

8.49.3 Work Time: GATE2013-65 top

https://gateoverflow.in/1569

The current erection cost of a structure is Rs. 13,200. If the labour wages per day increase by 1/5 of the current wages and the working hours decrease by 1/24 of the current period, then the new cost of erection in Rs. is

A. 16,500
B. 15,180
C. 11,000
D. 10,120

gate2013 numerical-ability normal work-time

Answer

8.49.4 Work Time: GATE2016-Session-3-GA-5 top

https://gateoverflow.in/108483

S, M, E and **F** are working in shifts in a team to finish a project. **M** works with twice the efficiency of others but for half as many days as **E** worked. **S** and **M** have 6 hour shifts in a day, whereas **E** and **F** have 12 hours shifts. What is the ratio of contribution of **M** to contribution of **E** in the project?

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A. $1:1$
B. $1:2$
C. $1:4$
D. $2:1$

gate2016session3aptitude numerical-ability work-time

Answer

8.49.5 Work Time: GATE2016-Session-7-GA-10 top https://gateoverflow.in/110924

Ananth takes 6 hours and Bharath takes 4 hours to read a book. Both started reading copies of the book at the same time. After how many hours is the number of pages to be read by Ananth, twice that to be read by Bharath? Assume Ananth and Bharath read all the pages with constant pace.

A. 1
B. 2
C. 3
D. 4

gate2016session7aptitude work-time numerical-ability

Answer

Answers: Work Time

8.49.1 Work Time: GATE2010-64 top https://gateoverflow.in/2372

Selected Answer

D. 15 days

1 skilled person can do $\frac{1}{20\times 5} = 1/100$ of work in 1 day, so 2 skilled person do $2/100$ of work in a day.

Similarly, 6 semi-skilled and 5 unskilled person can do $6/200$ and $5/300$ of work respectively in 1 day.

So, together they do $\frac{2}{100} + \frac{6}{200} + 5300 = \frac{1}{15}$ of work together in 1 day, which gives required number of day to complete the work $= 15$.

14 votes -- shreya ghosh (3.5k points)

8.49.2 Work Time: GATE2011-64 top https://gateoverflow.in/2174

Selected Answer

Let the amount of orders received per day be x, the amount of pending orders be y and the amount of orders carried by a truck each day be z.

$7z \times 4 = 4x + y \quad \rightarrow (1)$

$3z \times 10 = 10x + y \quad \rightarrow (2)$

$(2) - (1) \implies 2z = 6x, z = 3x, y = 80x$

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We want to find the number of trucks to finish the orders in 5 days. Let it be A.

$Az \times 5 = 5x + y$

$15Ax = 5x + 80x$

$A = \lceil 85/15 \rceil = \lceil 17/3 \rceil = 6$

So, minimum 6 trucks must be used.

28 votes -- Arjun Suresh (348k points)

8.49.3 Work Time: GATE2013-65 top

https://gateoverflow.in/1569

Selected Answer

Since wages per day increase by $\frac{1}{5}$ of current wages, new wages per day becomes $\frac{6}{5}$ of current wages.

Similarly, new working hours are $\frac{23}{24}$ of current working hours.

So new erection cost becomes $13200 \times \frac{6}{5} \times \frac{23}{24} = 15180.$

So option (B) is correct.

7 votes -- Happy Mittal (11.2k points)

8.49.4 Work Time: GATE2016-Session-3-GA-5 top

https://gateoverflow.in/108483

Selected Answer

Let the efficiency of S, E and F be
r units/hour.
So the efficiency of $M = 2r$ units/hour.

Let the number of days E works be d.

The amount of work done by M, $W_m = 2r \times (d/2) \times 6.$
(As M is working 6 hours per day)

The amount of work done by E, $W_e = r \times d \times 12.$

$W_m : W_e = \frac{2r \times (d/2) \times 6}{r \times d \times 12} = 1 : 2.$

3 votes -- Hemant Parihar (14.8k points)

8.49.5 Work Time: GATE2016-Session-7-GA-10 top

https://gateoverflow.in/110924

Selected Answer

Assume the book is of 240 pages.

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Ananth reads 40 pages per hour and Bharath reads 60 pages per hour.

Number of pages Ananth will complete at the end of each hour $-40, 80, 120, 160, 200, 240.$

Number of pages Bharath will complete at the end of each hour $-60, 120, 180, 240.$

At the end of 3rd hour, Bharath is left with 60 pages and Ananth is left with 120 pages.

So, 3 is the answer.

6 votes -- Adithya Kasarla (171 points)

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9 General Aptitude: Verbal Ability (146) top

9.1 Closest Word(3) top

9.1.1 Closest Word: GATE2013-CE-3 top https://gateoverflow.in/40270

Which of the following options is the closest in meaning to the word given below: Primeval

A. Modern
B. Historic
C. Primitive
D. Antique

gate2013-ce closest-word most-appropriate-word

Answer

9.1.2 Closest Word: GATE2013-ee-1 top https://gateoverflow.in/40288

They were requested not to **quarrel** with others.

Which one of the following options is the closest in meaning to the word **quarrel**?

A. make out
B. call out
C. dig out
D. fall out

gate2013-ee verbal-ability closest-word

Answer

9.1.3 Closest Word: GATE2014-AE-1 top https://gateoverflow.in/40300

A student is required to demonstrate a high level of comprehension of the subject, especially in the social sciences.
The word closest in meaning to **comprehension** is

A. understanding
B. meaning
C. concentration
D. stability

gate-2014-ae closest-word verbal-ability

Answer

Answers: Closest Word

9.1.1 Closest Word: GATE2013-CE-3 top https://gateoverflow.in/40270

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Selected Answer

Ans C. Primitive, it is synonym for Primeval

2 votes -- UK (1.7k points)

9.1.2 Closest Word: GATE2013-ee-1 top https://gateoverflow.in/40288

Selected Answer

quarrel means having an argument.
option D) **fall out** means the same.

6 votes -- Monanshi Jain (9.3k points)

9.1.3 Closest Word: GATE2014-AE-1 top https://gateoverflow.in/40300

Selected Answer

comprehension means understanding.
option A.

4 votes -- Monanshi Jain (9.3k points)

9.2 English Grammar(20) top

9.2.1 English Grammar: GATE-2013-AE-GA-2 top https://gateoverflow.in/40243

The Headmaster _____ to speak to you. Which of the following options is incorrect to complete the above sentence?

A. is wanting
B. wants
C. want
D. was wanting

gate2013-ae verbal-ability english-grammar

Answer

9.2.2 English Grammar: GATE-2013-AE-GA-4 top https://gateoverflow.in/40245

All engineering students should learn mechanics, mathematics and how to do computation.

I II III IV

Which of the above underlined parts of the sentence is not appropriate?

a. I
b. II
c. III
d. IV

gate2013-ae english-grammar verbal-ability

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Answer

9.2.3 English Grammar: GATE2012-AR-3 top

https://gateoverflow.in/40224

Choose the grammatically CORRECT sentence:

A. He laid in bed till 8 o'clock in the morning.
B. He layed in bed till 8 o'clock in the morning.
C. He lain in bed till 8 o'clock in the morning.
D. He lay in bed till 8 o'clock in the morning.

gate2012-ar verbal-ability english-grammar easy

Answer

9.2.4 English Grammar: GATE2012-AR-4 top

https://gateoverflow.in/40225

Which one of the parts (A, B, C, D) in the sentence contains an **ERROR**?

No sooner had the doctor seen the results of the blood test, than he suggested the patient to see the specialist.

A. no sooner had
B. results of the blood test
C. suggested the patient
D. see the specialist

gate2012-ar verbal-ability english-grammar

Answer

9.2.5 English Grammar: GATE2013-CE-2 top

https://gateoverflow.in/40269

The professor ordered to the students to go out of the class.

I II III IV

Which of the above underlined parts of the sentence is grammatically incorrect?

A. I
B. II
C. III
D. IV

gate2013-ce english-grammar verbal-ability

Answer

9.2.6 English Grammar: GATE2013-ee-4 top

https://gateoverflow.in/40291

Choose the grammatically **CORRECT** sentence:

A. Two and two add four.
B. Two and two become four.
C. Two and two are four.
D. Two and two make four.

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gate2013-ee english-grammar verbal-ability

Answer

9.2.7 English Grammar: GATE2014-3-GA-1 top https://gateoverflow.in/2024

While trying to collect an envelope from under the table, Mr. X fell down and
I II III
was losing consciousness.
IV
Which one of the above underlined parts of the sentence is NOT appropriate?

A. I
B. II
C. III
D. IV

gate2014-3 verbal-ability easy english-grammar

Answer

9.2.8 English Grammar: GATE2014-3-GA-2 top https://gateoverflow.in/2025

If she _____ how to calibrate the instrument, she _____ done the experiment.

A. knows, will have
B. knew, had
C. had known, could have
D. should have known, would have

gate2014-3 verbal-ability easy english-grammar

Answer

9.2.9 English Grammar: GATE2014-EC02-GA2 top https://gateoverflow.in/41509

Which of the options given below best completes the following sentence?

She will feel much better if she _____.

A. Will get some rest
B. Gets some rest
C. Will be getting some rest
D. Is getting some rest

gate2014-ec02 verbal-ability english-grammar normal

Answer

9.2.10 English Grammar: GATE2015-1-GA-1 top https://gateoverflow.in/7995

Didn't you buy _____ when you went shopping?

A. any paper
B. much paper
C. no paper
D. a few paper

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gate2015-1 verbal-ability easy english-grammar

Answer

9.2.11 English Grammar: GATE2015-2-GA-1 top https://gateoverflow.in/8028

We _____ our friends's birthday and we _____ how to make it up to him.

A. completely forgot --- don't just know
B. forgot completely --- don't just know
C. completely forgot --- just don't know
D. forgot completely --- just don't know

gate2015-2 verbal-ability easy english-grammar

Answer

9.2.12 English Grammar: GATE2015-2-GA-10 top https://gateoverflow.in/8041

Out of the following 4 sentences, select the most suitable sentence with respect to grammar and usage:

A. Since the report lacked needed information, it was of no use to them.
B. The report was useless to them because there were no needed information in it.
C. Since the report did not contain the needed information, it was not real useful to them.
D. Since the report lacked needed information, it would not had been useful to them.

gate2015-2 verbal-ability normal english-grammar

Answer

9.2.13 English Grammar: GATE2015-3-GA-2 top https://gateoverflow.in/8300

The Tamil version of _____ John Abraham-starrer *Madras Cafe* _____ cleared by the Censor Board with no cuts last week, but the film's distributor _____ no takers among the exhibitors for a release in Tamilnadu _____ this Friday.

A. Mr., was, found, on
B. a, was, found, at
C. the, was, found, on
D. a, being, find at

gate2015-3 verbal-ability normal english-grammar

Answer

9.2.14 English Grammar: GATE2016-1-GA01 top https://gateoverflow.in/39608

Out of the following four sentences, select the most suitable sentence with respect to grammar and usage.

A. I will not leave the place until the minister does not meet me.
B. I will not leave the place until the minister doesn't meet me.
C. I will not leave the place until the minister meet me.
D. I will not leave the place until the minister meets me.

gate2016-1 verbal-ability english-grammar easy

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Answer

9.2.15 English Grammar: GATE2016-1-GA03 top

https://gateoverflow.in/39606

Archimedes said, "Give me a lever long enough and a fulcrum on which to place it, and I will move the world."

The sentence above is an example of a _____ statement.

A. figurative
B. collateral
C. literal
D. figurine

gate2016-1 verbal-ability normal english-grammar

Answer

9.2.16 English Grammar: GATE2016-2-GA-01 top

https://gateoverflow.in/39529

The man who is now Municipal Commissioner worked as _____.

A. the security guard at a university
B. a security guard at the university
C. a security guard at university
D. the security guard at the university

gate2016-2 verbal-ability english-grammar normal

Answer

9.2.17 English Grammar: GATE2016-Session-1-GA-1 top

https://gateoverflow.in/108061

Which of the following is CORRECT with respect to grammar and usage?

Mount Everest is _____.

A. The highest peak in the world
B. Highest peak in the world
C. One of highest peak in the world
D. One of the highest peak in the world

gate2016session1aptitude verbal-ability english-grammar

Answer

9.2.18 English Grammar: GATE2016-Session-3-GA-1 top

https://gateoverflow.in/108474

Based on the given statements, select the appropriate option with respect to grammar and usage.

Statements

(i) The height of Mr. X is 6 feet.

(ii) The height of Mr. Y is 5 feet.

A. Mr. X is longer than Mr. Y.
B. Mr. X is more elongated than Mr. Y.
C. Mr. X is taller than Mr. Y.

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D. Mr. X is lengthier than Mr. Y.

gate2016session3aptitude verbal-ability english-grammar

Answer

9.2.19 English Grammar: GATE2017-1-GA-2 top https://gateoverflow.in/118405

Research in the workplace reveals that people work for many reasons _____ .

A. money beside
B. beside money
C. money besides
D. besides money

gate2017-1 verbal-ability english-grammar

Answer

9.2.20 English Grammar: GATE2017-2-GA-2 top https://gateoverflow.in/118416

Saturn is _____ to be seen on a clear night with the naked eye.

A. enough bright
B. bright enough
C. as enough bright
D. bright as enough

gate2017-2 verbal-ability english-grammar

Answer

Answers: English Grammar

9.2.1 English Grammar: GATE-2013-AE-GA-2 top https://gateoverflow.in/40243

Selected Answer

option C is correct.

Rule: Subjects and verbs must AGREE with one another in number (singular or plural). Thus, if a subject is singular, its verb must also be singular; if a subject **is plural**, its verb must also be plural.

Here **headmaste**r is singular subject so we don't use **plural verb 'want'** with Headmaster.

3 votes -- Akash Verma (2.6k points)

9.2.2 English Grammar: GATE-2013-AE-GA-4 top https://gateoverflow.in/40245

Selected Answer

I will have to go with D on this one.

All engineering students should learn mechanics, mathematics and computation.

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or possibly

All engineering students should learn how to do mechanics, mathematics and computation.

7 votes -- Kathleen Bankson (59.4k points)

9.2.3 English Grammar: GATE2012-AR-3 top

https://gateoverflow.in/40224

Selected Answer

Answer should be "D"

Lay vs Lie Chart					
Infinitive	**Defintiion**	**Present**	**Past**	**Past Participle**	**Present Participle**
to lay	to put or place something down	lay(s)	laid	laid	laying
to lie	to rest or recline	lie(s)	lay	lain	lying

Lay means **to put or set something down**,

Lie, on the other hand, is defined as, "**to be, to stay or to assume rest in a horizontal position**

9 votes -- Prateek kumar (7.7k points)

9.2.4 English Grammar: GATE2012-AR-4 top

https://gateoverflow.in/40225

Selected Answer

"**to see the specialist**"

Who is the specialist here? Not clear- it must be one of the specialist for that particular disease. So, "to see a specialist" is correct.

Rest all are fine in the sentence. "No sooner had- than" means the first part happened and the second part followed immediately. The first part uses past perfect tense and the second part uses simple past tense - which is as required.

http://www.englishgrammar.org/rewrite-sooner/

9 votes -- Arjun Suresh (348k points)

9.2.5 English Grammar: GATE2013-CE-2 top

https://gateoverflow.in/40269

Selected Answer

According to the question, in given sentence part II is a grammatical error. it should be '**ordered**' instead of '***ordered to***'.

The professor **ordered** the students to go out of the class.

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Explanation:

Use of '**ordered**' http://sentence.yourdictionary.com/ordered

You should use '***ordered to***' when someone is issuing a command. For example, *I was ordered to order a pizza*. This means that someone instructed you to order a pizza.

3 votes -- Akash Verma (2.6k points)

9.2.6 English Grammar: GATE2013-ee-4 top

https://gateoverflow.in/40291

Selected Answer

D) seems to be correct ans.

8 votes -- Manashi Sarkar (413 points)

9.2.7 English Grammar: GATE2014-3-GA-1 top

https://gateoverflow.in/2024

Selected Answer

Answer D.
While trying to collect an envelope from under the table, Mr. X fell down and was losing consciousness.

A :Mr. X is trying to collect an envelope from under the table (Present continuous tense)
B :Mr. X fell down (Simple past tense)
C :Mr. X was losing consciousness. (Past continuous tense)

While A, B and C

What is wrong is the usage of sentences B and C together.

"We use the past continuous tense with the simple past tense when we want to show that one thing happened in the middle of another thing."
eg-I was taking a bath and the telephone rang.

Here it says..
"Mr. X fell down and was losing consciousness."

i.e B is done.. and C starts. Where as the usage says.. C should have happened in the middle of B.

It should have been Mr. X fell down and lost consciousness.

Source - http://www.5minuteenglish.com/nov12.htm

9 votes -- Srinath Jayachandran (3.7k points)

9.2.8 English Grammar: GATE2014-3-GA-2 top

https://gateoverflow.in/2025

Selected Answer

A. We can use "will have done" to talk about what will have been achieved by a certain moment in time.
e.g. I'll have finished this project by Friday.

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So, (A) is not the correct option.

B. If + simple past $\rightarrow$ would/ would be
e.g.,
1. If It rained, you would get wet.
2. If I spoke Italian, I would be working in Italy.
So, (B) is not the correct option.

C. If + past perfect $\rightarrow$ would have/could have/ would have been
e.g.,
If you had studied harder, you could have passed the exam.

So, (C) is the correct option.

3 votes -- Manu Thakur (39.6k points)

9.2.9 English Grammar: GATE2014-EC02-GA2 top https://gateoverflow.in/41509

Selected Answer

She will feel much better if she gets some rest.
If clause (condition) - If + simple present - if she gets some rest
Main clause (result) - simple future - She will feel much better
As in all conditional sentences, the order of the clauses is not fixed. You may have to rearrange the pronouns and adjust punctuation when you change the order of the clauses, but the meaning is identical.
More on Conditional Sentences - http://www.ef.com/english-resources/english-grammar/type-1-conditional/

Hence, option B is correct.

2 votes -- Akash Verma (2.6k points)

9.2.10 English Grammar: GATE2015-1-GA-1 top https://gateoverflow.in/7995

Selected Answer

We can use "**any**" in negative sentences with plural ***countable*** nouns

e.g., Aren't there any **books?**

We can use "**any**" in negative sentences with **uncountable** nouns

e.g., Isn't there any **paper?** - here paper is uncountable

4 votes -- Prateek kumar (7.7k points)

9.2.11 English Grammar: GATE2015-2-GA-1 top https://gateoverflow.in/8028

Selected Answer

answer is option c

8 votes -- naresh1845 (1.6k points)

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9.2.12 English Grammar: GATE2015-2-GA-10 top https://gateoverflow.in/8041

Selected Answer

A.

Correct versions of other 3:

B. The report was useless to them because there **was** no needed information in it.

C. Since the report did not contain the needed information, it was not **really** useful to them.

D. Since the report lacked needed information, it would not **have** been useful to them.

13 votes -- Arjun Suresh (348k points)

9.2.13 English Grammar: GATE2015-3-GA-2 top https://gateoverflow.in/8300

Selected Answer

answer is c

14 votes -- naresh1845 (1.6k points)

9.2.14 English Grammar: GATE2016-1-GA01 top https://gateoverflow.in/39608

Selected Answer

Not is already embedded in until. So, A and B are incorrect. Also, the minister is a single person, and with a singular subject, singular verb follows(ending in 's'). ***Thus, C is incorrect and D is the right answer.***

5 votes -- Akash Verma (2.6k points)

9.2.15 English Grammar: GATE2016-1-GA03 top https://gateoverflow.in/39606

Selected Answer

A) Figurative

12 votes -- Abhilash Panicker (9.4k points)

9.2.16 English Grammar: GATE2016-2-GA-01 top https://gateoverflow.in/39529

Selected Answer

University considered as an organization that's why article **the** used before university.
And Post of security is a general post, so article **a** has been used for security guard.

So, option (B) is true.

3 votes -- Akash Verma (2.6k points)

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9.2.17 English Grammar: GATE2016-Session-1-GA-1 top

https://gateoverflow.in/108061

Selected Answer

An adjective can exist in three forms – positive, **comparative** and **superlative**. The positive **form** is the base **form** of the adjective. The **comparative form** expresses a higher **degree** of some quality. The **superlative form** expresses the highest **degree.**

Rules for superlative degree- https://www.englishgrammar.org/positive-comparative-superlative-degrees/

Here, Mount Everest is **the highest** peak in the world**.** Option (A) is correct.

2 votes -- Akash Verma (2.6k points)

9.2.18 English Grammar: GATE2016-Session-3-GA-1 top

https://gateoverflow.in/108474

Selected Answer

taller*. comparative form of tall: more tall.*

Taller *definition, having a relatively great height; of more than average stature: a tall woman; tall grass.*

Mr. X is **taller** than Mr. Y. So option **C** is correct.

2 votes -- Akash Verma (2.6k points)

9.2.19 English Grammar: GATE2017-1-GA-2 top

https://gateoverflow.in/118405

Selected Answer

The spelling difference between *beside* and *besides* is a small one, but their meanings and uses are quite different!

BESIDE, with no "s" at the end, is used only as a preposition, which means there is always a noun following it. It means either "next to" or "compared to," as shown in the examples below. The first meaning, "next to," is more common.

- She sat *beside* (= next to) him during dinner.
- These problems seem unimportant *beside* (= compared to) the potential benefits of the new system.

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BESIDES with an "s" at the end can also be used as a preposition, but it means "except" or "in addition to," as in these examples:

- There's no one here *besides* (= except) me.
- She wants to learn other languages *besides* (= in addition to) English and French.
- *Besides* (= in addition to) its famous cakes, the bakery also makes delicious breads and cookies.

so ans is D.

11 votes -- 2018 (6.8k points)

9.2.20 English Grammar: GATE2017-2-GA-2 top

https://gateoverflow.in/118416

Selected Answer

With adjectives and adverbs, **enough** comes after adjectives and adverbs.
With nouns, **enough** comes before noun.

In the given question, enough is used with bright which is an adjective, so enough will come after the adjective.
So, **bright enough** is the **correct option(B)**.

7 votes -- Akash Verma (2.6k points)

9.3 Grammatically Incorrect Sentence(2) top

9.3.1 Grammatically Incorrect Sentence: GATE2012-59 top

https://gateoverflow.in/2198

Choose the grammatically **INCORRECT** sentence:

A. They gave us the money back less the service charges of Three Hundred rupees.
B. This country's expenditure is not less than that of Bangladesh.
C. The committee initially asked for a funding of Fifty Lakh rupees, but later settled for a lesser sum.
D. This country's expenditure on educational reforms is very less.

gate2012 verbal-ability grammatically-incorrect-sentence normal

Answer

9.3.2 Grammatically Incorrect Sentence: GATE2013-60 top

https://gateoverflow.in/1564

Choose the grammatically **INCORRECT** sentence:

A. He is of Asian origin.
B. They belonged to Africa.
C. She is an European.
D. They migrated from India to Australia.

gate2013 verbal-ability grammatically-incorrect-sentence normal

Answer

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Answers: Grammatically Incorrect Sentence

9.3.1 Grammatically Incorrect Sentence: GATE2012-59 top

https://gateoverflow.in/2198

Selected Answer

D. "is very less" should be "is much less".

6 votes -- Kathleen Bankson (59.4k points)

9.3.2 Grammatically Incorrect Sentence: GATE2013-60 top

https://gateoverflow.in/1564

Selected Answer

C. She is a European

"The sound of a word's first letter determines which to use. If the word starts with a vowel sound, you should use an. If it starts with a consonant sound, you should use a."

The word "European" does not start with a vowel sound, it starts with the syllable "you". The "y"-sound is in this case a consonant (or at least a half-consonant), so the indefinite article is "a".

14 votes -- Kathleen Bankson (59.4k points)

9.4 Logical Reasoning(5) top

9.4.1 Logical Reasoning: GATE-2013-AE-GA-9 top

https://gateoverflow.in/40250

All professors are researchers Some scientists are professors Which of the given conclusions is logically valid and is inferred from the above arguments:

A. All scientists are researchers
B. All professors are scientists
C. Some researchers are scientists
D. No conclusion follows

gate2013-ae verbal-ability logical-reasoning

Answer

9.4.2 Logical Reasoning: GATE2016-1-GA04 top

https://gateoverflow.in/39609

If 'relftaga' means carefree, 'otaga' means careful and 'fertaga' means careless, which of the following could mean 'aftercare'?

A. zentaga
B. tagafer.
C. tagazen.
D. relffer.

gate2016-1 verbal-ability logical-reasoning normal

Answer

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9.4.3 Logical Reasoning: GATE2016-1-GA08 top

https://gateoverflow.in/39617

Consider the following statements relating to the level of poker play of four players P, Q, R and S.

I. P always beats Q
II. R always beats S
III. S loses to P only sometimes.
IV. R always loses to Q

Which of the following can be logically inferred from the above statements?

i. P is likely to beat all the three other players
ii. S is the absolute worst player in the set

A. (i). only
B. (ii) only
C. (i) and (ii) only'
D. neither (i) nor (ii)

gate2016-1 logical-reasoning normal

Answer

9.4.4 Logical Reasoning: GATE2016-2-GA-08 top

https://gateoverflow.in/39534

All hill-stations have a lake. Ooty has two lakes.

Which of the statement(s) below is/are logically valid and can be inferred from the above sentences?

(i) Ooty is not a hill-station.

(ii) No hill-station can have more than one lake.

A. (i) only.
B. (ii) only.
C. Both (i) and (ii)
D. Neither (i) nor (ii)

gate2016-2 verbal-ability logical-reasoning easy

Answer

9.4.5 Logical Reasoning: GATE2016-Session-2-GA-4 top

https://gateoverflow.in/108215

Fact: If it rains, then the field is wet.

Read the following statements:

(i) It rains

(ii) The field is not wet

(iii) The field is wet

(iv) It did not rain

Which one of the options given below is **NOT** logically possible, based on the given fact?

A. If (iii), then (iv).
B. If (i), then (iii).

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C. If (i), then (ii).
D. If (ii), then (iv).

gate2016session2aptitude logical-reasoning

Answer

Answers: Logical Reasoning

9.4.1 Logical Reasoning: GATE-2013-AE-GA-9 top https://gateoverflow.in/40250

Selected Answer

Answer C: If some scientists are professors and all professors are researchers, then that means that some scientists are researchers.

2 votes -- Kathleen Bankson (59.4k points)

9.4.2 Logical Reasoning: GATE2016-1-GA04 top https://gateoverflow.in/39609

Selected Answer

relf taga = care free

o taga = care full

fer taga = care less

Hence taga means care, and if present in the second part, care comes in the first part in English translation.

For after care, "taga-fer" and "taga-zen" are the options. But "fer" means less.

Hence **answer is C**

33 votes -- ryan sequeira (3.4k points)

9.4.3 Logical Reasoning: GATE2016-1-GA08 top https://gateoverflow.in/39617

Selected Answer

Answer is (d) because

i) p is not likely to beat s because s only loses sometimes to p

ii) s is not worst player because he is likely to beat p

36 votes -- Vaibhav Singh (507 points)

9.4.4 Logical Reasoning: GATE2016-2-GA-08 top https://gateoverflow.in/39534

Selected Answer

All hill stations have a lake $\Rightarrow \forall x(h(x) \rightarrow (\exists y, l(y) \wedge has(x,y)))$

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Ooty has two lakes $\Rightarrow \exists x(o(x) \wedge \exists y, z(has(x,y,z) \wedge l(y) \wedge l(z) \wedge (z! = y))$

Here, $h(x) \rightarrow x$ is hill station

- $l(x) \rightarrow x$ is lake
- $has(x,y) \rightarrow x$ has y
- $has(x,y,z) \rightarrow x$ has y, z
- $o(x) \rightarrow x$ is Ooty

i. Ooty is not a hill station $\implies$ we can not derive this above arguments, Ooty has two lakes already, if Ooty had 0 lakes only then this can become true.
ii. No hill station can have more than one lake

All arguments here are saying are if we have hill station, it can have lake. It is nowhere told that how many lakes it has. So, this is false.

Answer: (D) neither (i) nor (ii)

17 votes -- Akash Kanase (42.5k points)

9.4.5 Logical Reasoning: GATE2016-Session-2-GA-4 top

https://gateoverflow.in/108275

Selected Answer

Option B and D are wrong due to the fact that question asks about which of the given option NOT logically possible .

Option B is given fact .

In Option D, statement (ii) is direct result of statement (iv)

Option A, field may be wet but that does not mean there is a rain . So this logically possible.

Option C, it is logically not possible, if it rains then field must be wet. Thus option C is only correct option.

1 votes -- Bikram (67k points)

9.5 Meaning(17) top

9.5.1 Meaning: GATE2010-57 top

https://gateoverflow.in/2365

Which of the following options is the closest in meaning to the word given below:

Circuitous

A. cyclic
B. indirect
C. confusing
D. crooked

gate2010 verbal-ability meaning normal

Answer

9.5.2 Meaning: GATE2011-56 top

https://gateoverflow.in/2165

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Which of the following options is the closest in the meaning to the word below:

Inexplicable

A. Incomprehensible
B. Indelible
C. Inextricable
D. Infallible

gate2011 verbal-ability meaning normal

Answer

9.5.3 Meaning: GATE2012-58 top

https://gateoverflow.in/2197

Which one of the following options is the closest in meaning to the word given below?

Mitigate

A. Diminish
B. Divulge
C. Dedicate
D. Denote

gate2012 verbal-ability meaning easy

Answer

9.5.4 Meaning: GATE2013-56 top

https://gateoverflow.in/1559

Which one of the following options is the closest in meaning to the word given below?

Nadir

A. Highest
B. Lowest
C. Medium
D. Integration

gate2013 verbal-ability meaning normal

Answer

9.5.5 Meaning: GATE2014-1-GA-1 top

https://gateoverflow.in/56

Which of the following options is the closest in meaning to the phrase in bold in the sentence below?

It is fascinating to see life forms **cope with** varied environmental conditions.

A. Adopt to
B. Adapt to
C. Adept in
D. Accept with

verbal-ability gate2014-1 meaning easy

Answer

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9.5.6 Meaning: GATE2014-1-GA-3 top

https://gateoverflow.in/772

In a press meet on the recent scam, the minister said, "The buck stops here". What did the minister convey by the statement?

A. He wants all the money
B. He will return the money
C. He will assume final responsibility
D. He will resist all enquiries

gate2014-1 verbal-ability normal meaning

Answer

9.5.7 Meaning: GATE2014-2-GA-3 top

https://gateoverflow.in/1940

Match the columns.

Column 1	**Column 2**
1. eradicate	P. misrepresent
2. distort	Q. soak completely
3. saturate	R. use
4. utilize	S. destroy utterly

A. 1:S, 2:P, 3:Q, 4:R
B. 1:P, 2:Q, 3:R, 4:S
C. 1:Q, 2:R, 3:S, 4:P
D. 1:S, 2:P, 3:R, 4:Q

gate2014-2 verbal-ability meaning normal

Answer

9.5.8 Meaning: GATE2014-EC04-GA1 top

https://gateoverflow.in/41463

Which of the following options is the closest in meaning to the word underlined in the sentence below?

In a democracy, everybody has the freedom to ***disagree*** with the government.

A. Dissent
B. Descent
C. Decent
D. Decadent

gate2014-ec04 verbal-ability meaning normal

Answer

9.5.9 Meaning: GATE2014-EC04-GA2 top

https://gateoverflow.in/41464

After the discussion, Tom said to me, 'Please revert!'. He expects me to _____.

A. Retract
B. Get back to him
C. Move in reverse
D. Retreat

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gate2014-ec04 verbal-ability meaning easy

Answer

9.5.10 Meaning: GATE2014-EC04-GA3 top

https://gateoverflow.in/41465

While receiving the award, the scientist said, "I feel vindicated". Which of the following is closest in meaning to the word 'vindicated'?

A. Punished
B. Substantiated
C. Appreciated
D. Chastened

gate2014-ec04 verbal-ability meaning normal

Answer

9.5.11 Meaning: GATE2015-1-GA-2 top

https://gateoverflow.in/8003

Which of the following options is the closest in meaning of the sentence below?

She enjoyed herself immensely at the party.

A. She had a terrible time at the party
B. She had a horrible time at the party
C. She had a terrific time at the party
D. She had a terrifying time at the party

gate2015-1 verbal-ability easy meaning

Answer

9.5.12 Meaning: GATE2015-1-GA-7 top

https://gateoverflow.in/8011

Select the alternative meaning of the underlined part of the sentence.

The chain snatchers took to their heels when the police party arrived.

A. Took shelter in a thick jungle
B. Open indiscriminate fire
C. Took to flight
D. Unconditionally surrendered

gate2015-1 verbal-ability meaning easy

Answer

9.5.13 Meaning: GATE2015-2-GA-2 top

https://gateoverflow.in/8029

Choose the statement where underlined word is used correctly.

A. The industrialist had a personnel jet.
B. I write my experience in my personnel diary.
C. All personnel are being given the day off.
D. Being religious is a personnel aspect.

gate2015-2 verbal-ability meaning normal

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Answer

9.5.14 Meaning: GATE2016-1-GA02 top

https://gateoverflow.in/39607

A rewording of something written or spoken is a _____.

A. paraphrase
B. paradox
C. paradigm
D. paraffin

gate2016-1 verbal-ability meaning normal

Answer

9.5.15 Meaning: GATE2016-2-GA-02 top

https://gateoverflow.in/39531

Nobody knows how the Indian cricket team is going to cope with the difficult and seamer-friendly wickets in Australia.

Choose the option which is closest in meaning to the underlined phrase in the above sentence.

A. Put up with.
B. Put in with.
C. Put down to.
D. Put up against.

gate2016-2 verbal-ability meaning normal

Answer

9.5.16 Meaning: GATE2016-Session-1-GA-2 top

https://gateoverflow.in/108064

The policeman asked the victim of a theft, "What did you ___ ?"

A. Loose
B. Lose
C. Loss
D. Louse

gate2016session1aptitude verbal-ability meaning

Answer

9.5.17 Meaning: GATE2016-Session-3-GA-3 top

https://gateoverflow.in/108481

After India's cricket world cup victory in 1985, Shrotria who was playing both tennis and cricket till then, decided to concentrate only on cricket. And the rest is history.

What does the underlined phrase mean in this context?

A. History will rest in peace
B. Rest is recorded in history books
C. Rest is well known
D. Rest is archaic

gate2016session3aptitude verbal-ability meaning

Answer

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Answers: Meaning

9.5.1 Meaning: GATE2010-57 top

https://gateoverflow.in/2365

Selected Answer

B Indirect

Synonyms for circuitous

adj going around, indirect

7 votes -- Kathleen Bankson (59.4k points)

9.5.2 Meaning: GATE2011-56 top

https://gateoverflow.in/2165

Selected Answer

Answer is (a)

Inexplicable - difficult or impossible to explain

Incomprehensible - difficult or impossible to understand or comprehend **-** Most appropriate

Indelible - impossible to remove, erase or wash away **-** Not appropriate

Inextricable - unavoidable , inescapable **-** Not appropriate

Infallible - completely dependable or trustworthy **-** irrelevant

9 votes -- Kalpna Bhargav (3.3k points)

9.5.3 Meaning: GATE2012-58 top

https://gateoverflow.in/2197

Selected Answer

A. Diminish

6 votes -- Kathleen Bankson (59.4k points)

9.5.4 Meaning: GATE2013-56 top

https://gateoverflow.in/1559

Selected Answer

B, the lowest point

6 votes -- Kathleen Bankson (59.4k points)

9.5.5 Meaning: GATE2014-1-GA-1 top

https://gateoverflow.in/56

Selected Answer

Answer is Adapt to. Often seen in newspaper "Indian players could not adapt to foreign conditions".

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Adopt - means legally take care of. Also means to take up and use as in "He adopted my point of view."

Adept in - means smart in. Example- "Sachin is adept in batting."

13 votes -- Arjun Suresh (348k points)

9.5.6 Meaning: GATE2014-1-GA-3 top

https://gateoverflow.in/772

Selected Answer

C. The buck stops here is a term meaning to put an end to something, not continue, stop it.

(idiomatic) A statement that no excuses will be made, that the speaker is going to take direct responsibility for matters, rather than pass the responsibility to higher authorities.

12 votes -- Kathleen Bankson (59.4k points)

9.5.7 Meaning: GATE2014-2-GA-3 top

https://gateoverflow.in/1940

Selected Answer

Answer is A

8 votes -- Poshita Shrivastava (437 points)

9.5.8 Meaning: GATE2014-EC04-GA1 top

https://gateoverflow.in/41463

Selected Answer

Ans will be (A) Dissent , which means disagree

3 votes -- srestha (86.6k points)

9.5.9 Meaning: GATE2014-EC04-GA2 top

https://gateoverflow.in/41464

Selected Answer

Revert means to get back..
Answer B) Get back to him

5 votes -- Abhilash Panicker (9.4k points)

9.5.10 Meaning: GATE2014-EC04-GA3 top

https://gateoverflow.in/41465

Selected Answer

option B

Vindicated means to free from allegation(unproved) and substantiate means prove the truth of.

2 votes -- Prateek Banra (167 points)

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9.5.11 Meaning: GATE2015-1-GA-2 top

https://gateoverflow.in/8003

Selected Answer

C. She had a terrific time at the party

9 votes -- Kathleen Bankson (59.4k points)

9.5.12 Meaning: GATE2015-1-GA-7 top

https://gateoverflow.in/8011

Selected Answer

C Took to flight

9 votes -- Kathleen Bankson (59.4k points)

9.5.13 Meaning: GATE2015-2-GA-2 top

https://gateoverflow.in/8029

Selected Answer

Answer: C

Personnel: People employed in an organization or engaged in an organized undertaking such as military service.

Option A,B,D should use the word personal.

7 votes -- Rajarshi Sarkar (34.1k points)

9.5.14 Meaning: GATE2016-1-GA02 top

https://gateoverflow.in/39607

Selected Answer

paraphrase

paradox: a statement that apparently contradicts itself and yet might be true
paradigm: a typical example or pattern of something
paraffin: a white or colorless, tasteless, odorless, water-insoluble, solid substance not easily acted upon by reagents

14 votes -- Pooja Palod (31.3k points)

9.5.15 Meaning: GATE2016-2-GA-02 top

https://gateoverflow.in/39531

Selected Answer

but the closest meaning to cope with is put up with which means to tolerate. Cope with does not mean to place someone into competition with someone else.

9 votes -- UK (1.7k points)

9.5.16 Meaning: GATE2016-Session-1-GA-2 top

https://gateoverflow.in/108064

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Selected Answer

1. Loose means not firmly or tightly hold .
2. Lose means something lost in past .
3. Loss is present form of lose.
4. Louse means spoil or ruin something.

so going through the meaning of this sentence only option B is correct , Lose .

4 votes -- Bikram (67k points)

9.5.17 Meaning: GATE2016-Session-3-GA-3 top

https://gateoverflow.in/108481

Selected Answer

Ans: (C) rest is well known

2 votes -- Arnabi Bej (7.9k points)

9.6 Most Appropriate Alternative(4) top

9.6.1 Most Appropriate Alternative: GATE-2012-AE-1 top

https://gateoverflow.in/40212

Choose the most appropriate alternative from the options given below to complete the following sentence:

I ____ to have bought a diamond ring.

A. have a liking
B. should have liked
C. would like
D. may like

gate2012-ae most-appropriate-alternative verbal-ability

Answer

9.6.2 Most Appropriate Alternative: GATE-2012-AE-2 top

https://gateoverflow.in/40213

Choose the most appropriate alternative from the options given below to complete the following sentence:

Food prices ____ again this month.

A. have raised
B. have been raising
C. have been rising
D. have arose

gate2012-ae verbal-ability most-appropriate-alternative

Answer

9.6.3 Most Appropriate Alternative: GATE2012-57 top

https://gateoverflow.in/2195

Choose the most appropriate alternative from the options given below to complete the following

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sentence:

Despite several ---------- the mission succeeded in its attempt to resolve the conflict.

A. attempts
B. setbacks
C. meetings
D. delegations

gate2012 verbal-ability easy most-appropriate-alternative

Answer

9.6.4 Most Appropriate Alternative: GATE2012-CY-GA-5 top

https://gateoverflow.in/40236

Choose the most appropriate alternative from the options given below to complete the following sentence:
If the tired soldier wanted to lie down, he ___ the mattress out on the balcony.

1. should take
2. shall take
3. should have taken
4. will have taken

gate2012-cy most-appropriate-alternative english-grammar verbal-ability

Answer

Answers: Most Appropriate Alternative

9.6.1 Most Appropriate Alternative: GATE-2012-AE-1 top

https://gateoverflow.in/40212

Selected Answer

Ans C
I **would like** to have bought a diamond ring.

1 votes -- Abhilash Panicker (9.4k points)

9.6.2 Most Appropriate Alternative: GATE-2012-AE-2 top

https://gateoverflow.in/40213

Selected Answer

Food price **have been rising** *again* this month.
Answer C.

A) wrong - have risen
B) wrong
D)wrong - have risen

0 votes -- Abhilash Panicker (9.4k points)

9.6.3 Most Appropriate Alternative: GATE2012-57 top https://gateoverflow.in/2195

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Selected Answer

SETBACK: something that happens that delays or prevents a process from developing:

EX: Sally had been recovering well from her operation, but yesterday she experienced/suffered a setback.

EX: There has been a temporary/slight setback in our plans .

Reffer : link

3 votes -- avadh (287 points)

9.6.4 Most Appropriate Alternative: GATE2012-CY-GA-5 top

https://gateoverflow.in/40236

Selected Answer

Answer C: If the tired soldier wanted to lie down, he should have taken the mattress out on the balcony.

5 votes -- Kathleen Bankson (59.4k points)

9.7 Most Appropriate Word(32) top

9.7.1 Most Appropriate Word: GATE-2012-AE-3 top

https://gateoverflow.in/40214

Choose the most appropriate alternative from the options given below to complete the following sentence:

The administrators went on to implement yet another unreasonable measure, arguing that the measures were already ____ and one more would hardly make a difference.

A. reflective
B. utopian
C. luxuriant
D. unpopular

gate2012-ae most-appropriate-word verbal-ability

Answer

9.7.2 Most Appropriate Word: GATE-2012-AE-4 top

https://gateoverflow.in/40215

Choose the most appropriate alternative from the options given below to complete the following sentence:
To those of us who had always thought him timid, his ____ came as a surprise.

A. intrepidity
B. inevitability
C. inability
D. inertness

gate2012-ae verbal-ability most-appropriate-word

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Answer

9.7.3 Most Appropriate Word: GATE-MN-2011-58 top https://gateoverflow.in/31529

Choose the word or phrase that best completes the sentence below.
_____ in the frozen wastes of Arctic takes special equipment.

A. To survive
B. Surviving
C. Survival
D. That survival

verbal-ability gate2011-mn most-appropriate-word

Answer

9.7.4 Most Appropriate Word: GATE2010-56 top https://gateoverflow.in/2364

Choose the most appropriate word from the options given below to complete the following sentence:

His rather casual remarks on politics _____ his lack of seriousness about the subject.

A. masked
B. belied
C. betrayed
D. suppressed

gate2010 verbal-ability most-appropriate-word normal

Answer

9.7.5 Most Appropriate Word: GATE2010-58 top https://gateoverflow.in/2366

Choose the most appropriate word from the options given below to complete the following sentence:

If we manage to _____ our natural resources, we would leave a better planet for our children.

A. uphold
B. restrain
C. cherish
D. conserve

gate2010 verbal-ability most-appropriate-word easy

Answer

9.7.6 Most Appropriate Word: GATE2011-58 top https://gateoverflow.in/2167

Choose the most appropriate word(s) from the options given below to complete the following sentence.

I contemplated _____ Singapore for my vacation but decided against it.

A. to visit
B. having to visit
C. visiting
D. for a visit

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gate2011 verbal-ability most-appropriate-word easy

Answer

9.7.7 Most Appropriate Word: GATE2011-59 top https://gateoverflow.in/2169

Choose the most appropriate word from the options given below to complete the following sentence.

If you are trying to make a strong impression on your audience, you cannot do so by being understated, tentative or _____.

A. hyperbolic
B. restrained
C. argumentative
D. indifferent

gate2011 verbal-ability most-appropriate-word normal

Answer

9.7.8 Most Appropriate Word: GATE2011-GG-GA-1 top https://gateoverflow.in/40202

Choose the most appropriate word or phrase from the options given below to complete the following sentence.

The environmentalists hope _____ the lake to its pristine condition.

A. in restoring
B. in the restoration of
C. to restore
D. restoring

gate2011_gg verbal-ability most-appropriate-word

Answer

9.7.9 Most Appropriate Word: GATE2011-GG-GA-5 top https://gateoverflow.in/40206

Choose the most appropriate words from the options given below to complete the following sentence.
Because she had a reputation for _____ we were surprised and pleased when she greeted us so _____.

A. insolence …… irately
B. insouciance …… curtly
C. graciousness …… amiably
D. querulousness …… affably

gate2011_gg most-appropriate-word verbal-ability

Answer

9.7.10 Most Appropriate Word: GATE2011-MN-57 top https://gateoverflow.in/31522

Choose the most appropriate word(s) from the options given below to complete the following sentence.

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We lost confidence in him because he never _____ the grandiose promises he had made.

A. delivered
B. delivered on
C. forgot
D. reneged on

gate2011-mn verbal-ability most-appropriate-word

Answer

9.7.11 Most Appropriate Word: GATE2011-MN-60 top https://gateoverflow.in/31533

Choose the most appropriate word from the options given below to complete the following sentence. The _____ of evidence was on the side of the plaintiff since all but one witness testified that his story was correct.

A. paucity
B. propensity
C. preponderance
D. accuracy

verbal-ability gate2011-mn most-appropriate-word

Answer

9.7.12 Most Appropriate Word: GATE2012-60 top https://gateoverflow.in/2200

Choose the most appropriate alternative from the options given below to complete the following sentence:

Suresh's dog is the one ––––––––– was hurt in the stampede.

A. that
B. which
C. who
D. whom

gate2012 verbal-ability most-appropriate-word normal

Answer

9.7.13 Most Appropriate Word: GATE2012-AR-2 top https://gateoverflow.in/40223

Choose the most appropriate pair of words from the options given below to complete the following sentence:

The high level of ___ of the questions in the test was ___ by an increase in the period of time allotted for answering them.

A. difficulty, compensated
B. exactitude, magnified
C. aptitude, decreased
D. attitude, mitigated

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gate2012-ar most-appropriate-word verbal-ability normal

Answer

9.7.14 Most Appropriate Word: GATE2012-CY-GA-4 top

https://gateoverflow.in/40235

Choose the most appropriate word from the options given below to complete the following sentence:
Given the seriousness of the situation that he had to face, his ___ was impressive.

A. beggary
B. nomenclature
C. jealousy
D. nonchalance

gate2012-cy most-appropriate-word

Answer

9.7.15 Most Appropriate Word: GATE2013-CE-4 top https://gateoverflow.in/40271

Friendship, no matter how _____it is, has its limitations.

A. cordial
B. intimate
C. secret
D. pleasant

gate2013-ce most-appropriate-word verbal-ability

Answer

9.7.16 Most Appropriate Word: GATE2013-ee-3 top https://gateoverflow.in/40290

Complete the sentence:

Dare _____ mistakes.

A. commit
B. to commit
C. committed
D. committing

gate2013-ee most-appropriate-word easy verbal-ability

Answer

9.7.17 Most Appropriate Word: GATE2014-1-GA-2 top https://gateoverflow.in/771

Choose the most appropriate word from the options given below to complete the following sentence.

He could not understand the judges awarding her the first prize, because he thought that her performance was quite _____.

A. superb
B. medium
C. mediocre

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D. exhilarating

gate2014-1 verbal-ability most-appropriate-word easy

Answer

9.7.18 Most Appropriate Word: GATE2014-2-GA-1 top https://gateoverflow.in/1938

Choose the most appropriate phrase from the options given below to complete the following sentence.

India is a post-colonial country because

A. it was a former British colony
B. Indian Information Technology professionals have colonized the world
C. India does not follow any colonial practices
D. India has helped other countries gain freedom

gate2014-2 verbal-ability most-appropriate-word easy

Answer

9.7.19 Most Appropriate Word: GATE2014-AG-GA1 top https://gateoverflow.in/41660

Choose the most appropriate word from the options given below to complete the following sentence. A person suffering from Alzheimer's disease _____ short-term memory loss.

A. Experienced
B. Has experienced
C. Is experiencing
D. Experiences

gate2014-ag verbal-ability most-appropriate-word normal

Answer

9.7.20 Most Appropriate Word: GATE2014-AG-GA2 top https://gateoverflow.in/41665

Choose the most appropriate word from the options given below to complete the following sentence. _____ is the key to their happiness; they are satisfied with what they have.

A. Contentment
B. Ambition
C. Perseverance
D. Hunger

gate2014-ag verbal-ability most-appropriate-word easy

Answer

9.7.21 Most Appropriate Word: GATE2014-EC01-GA1 top

https://gateoverflow.in/41490

Choose the most appropriate phrase from the options given below to complete the following sentence.

The aircraft _____ take off as soon as its flight plan was filed.

A. Is allowed to
B. Will be allowed to

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C. Was allowed to
D. Has been allowed to

gate2014-ec01 verbal-ability most-appropriate-word easy

Answer

9.7.22 Most Appropriate Word: GATE2014-EC01-GA3 top

https://gateoverflow.in/41492

Choose the most appropriate word from the options given below to complete the following sentence.

Many ancient cultures attributed disease to supernatural causes. However, modern science has largely helped _____ such notions.

A. Impel
B. Dispel
C. Propel
D. Repel

gate2014-ec01 most-appropriate-word verbal-ability

Answer

9.7.23 Most Appropriate Word: GATE2014-EC02-GA3 top

https://gateoverflow.in/41510

Choose the most appropriate pair of words from the options given below to complete the following sentence.

She could not _____ the thought of _____ the election to her bitter rival.

A. Bear, loosing
B. Bare, loosing
C. Bear, losing
D. Bare, losing

gate2014-ec02 most-appropriate-word

Answer

9.7.24 Most Appropriate Word: GATE2014-EC03-GA2 top

https://gateoverflow.in/41141

The value of one U.S. dollar is 65 Indian Rupees today, compared to 60 last year. The Indian Rupee has _____.

A. Depressed
B. Depreciated
C. Appreciated
D. Stabilized

gate2014-ec03 most-appropriate-word verbal-ability

Answer

9.7.25 Most Appropriate Word: GATE2015-2-GA-4 top https://gateoverflow.in/8032

A generic term that includes various items of clothing such as a skirt, a pair of trousers and a shirt is

A. fabric
B. textile

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C. fiber
D. apparel

gate2015-2 verbal-ability easy most-appropriate-word

Answer

9.7.26 Most Appropriate Word: GATE2015-3-GA-3 top https://gateoverflow.in/8301

Extreme focus on syllabus and studying for tests has become such a dominant concern of Indian student that they close their minds to anything _____ to the requirements of the exam.

A. related
B. extraneous
C. outside
D. useful

gate2015-3 verbal-ability normal most-appropriate-word

Answer

9.7.27 Most Appropriate Word: GATE2016-Session-2-GA-2 top

https://gateoverflow.in/108249

The unruly crowd demanded that the accused be _____ without trial.

A. Hanged
B. Hanging
C. Hankering
D. Hung

gate2016session2aptitude most-appropriate-word verbal-ability

Answer

9.7.28 Most Appropriate Word: GATE2016-Session-3-GA-2 top

https://gateoverflow.in/108478

The students _____ the teacher on teachers' day for twenty years of dedicated teaching.

A. Facilitated
B. Felicitated
C. Fantasized
D. Facillitated

gate2016session3aptitude verbal-ability most-appropriate-word

Answer

9.7.29 Most Appropriate Word: GATE2016-Session-7-GA-1 top

https://gateoverflow.in/110868

If I were you, I _____ that laptop. It's much too expensive.

A. Won't buy
B. Shan't buy
C. Wouldn't buy
D. Would buy

gate2016session7aptitude verbal-ability english-grammar most-appropriate-word

Answer

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9.7.30 Most Appropriate Word: GATE2018-GA-1 top https://gateoverflow.in/204062

"From where are they bringing their books? _____ bringing _____ books from _____"

The words that best fill the blanks in the above sentence are

A. Their, they're, there
B. They're, their, there
C. There, their, they're
D. They're, there,there

gate2018 verbal-ability most-appropriate-word easy

Answer

9.7.31 Most Appropriate Word: GATE2018-GA-2 top https://gateoverflow.in/204063

A _____ investigation can sometimes yield new facts, but typically organized ones are more successful.

The word that best fills the blank in the above sentence is

A. meandering
B. timely
C. consistent
D. systematic

gate2018 verbal-ability most-appropriate-word normal

Answer

9.7.32 Most Appropriate Word: gate-2014-ae-2 top https://gateoverflow.in/40301

Choose the most appropriate word from the options given below to complete the following sentence.
One of his biggest _____ was his ability to forgive.

A. vice
B. virtues
C. choices
D. strength

gate-2014-ae most-appropriate-word

Answer

Answers: Most Appropriate Word

9.7.1 Most Appropriate Word: GATE-2012-AE-3 top https://gateoverflow.in/40214

(D)

unpopular seems to be the most suitable alternative to unreasonable

2 votes -- Sourav Mishra (सौरव मिश्रा) (1.9k points)

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9.7.2 Most Appropriate Word: GATE-2012-AE-4 top https://gateoverflow.in/40215

Selected Answer

(A)

Explanation : Timid means fearful and Intrepid means fearless which is the most suitable contrast.

3 votes -- Sourav Mishra (सौरव मिश्रा) (1.9k points)

9.7.3 Most Appropriate Word: GATE-MN-2011-58 top https://gateoverflow.in/31529

Official GATE answer: (A) To survive

2 votes -- Akash Kanase (42.5k points)

9.7.4 Most Appropriate Word: GATE2010-56 top https://gateoverflow.in/2364

Selected Answer

answer is option (c)

(a) Masked : Hide under a false appearance - **opposite**

(b) Belied : Be in contradiction with - **not appropriate**

(c) Betrayed : Reveal unintentionally - **most appropriate**

(d) Suppressed: To put down by force or authority - **irrelevant**

11 votes -- Kalpna Bhargav (3.3k points)

9.7.5 Most Appropriate Word: GATE2010-58 top https://gateoverflow.in/2366

Selected Answer

answer is (d)

Uphold : cause to remain - **Not appropriate**

Restrain : keep under control - **Not appropriate**

Cherish : be fond of - **Not related**

Conserve : keep in safety and protect from harm , decay, loss or destruction - **most appropriate**

7 votes -- Kalpna Bhargav (3.3k points)

9.7.6 Most Appropriate Word: GATE2011-58 top https://gateoverflow.in/2167

Selected Answer

Answer **"Visiting"**
some Verbs are Followed by Gerunds, not infinitives

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e.g.,

- I recommend leaving while we can.
- I have quit smoking
- Did I mention reading that novel last summer?
- He avoided talking to her.

Some verb can be followed by either an infinitive or a gerund, but there will be a difference in meaning.

e.g.,

- I stopped smoking - means something quite different, for instance,
- I stopped to smoke - the infinitive form will usually describe a potential action.

http://grammar.ccc.commnet.edu/grammar/gerunds.htm

4 votes -- Prateek kumar (7.7k points)

9.7.7 Most Appropriate Word: GATE2011-59 top https://gateoverflow.in/2169

Selected Answer

restrained -showing careful self-control

synonyms: - sober, quite, calm, steady

3 votes -- Prateek kumar (7.7k points)

9.7.8 Most Appropriate Word: GATE2011-GG-GA-1 top https://gateoverflow.in/40202

(C)

The environmentalists hope to restore the lake to its pristine condition.

Pristine means original condition.

2 votes -- Sourav Mishra (सौरव मिश्रा) (1.9k points)

9.7.9 Most Appropriate Word: GATE2011-GG-GA-5 top https://gateoverflow.in/40206

(D)

Explanation : As the sentence reveals both the words should be contrasting in nature.

Among the options given (querulous,affable) is the only contrasting pair.

2 votes -- Sourav Mishra (सौरव मिश्रा) (1.9k points)

9.7.10 Most Appropriate Word: GATE2011-MN-57 top https://gateoverflow.in/31522

Answer B

(B) Delivered on is most appropriate.

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(A) Delivered this is not correct because of "The" in next part. Does not sound appropriate.

(C) forgot: Does not fit properly

(D) reneged on: This does not seem appropriate, As he never go back on a promise then we should not loose confidence !

3 votes -- Akash Kanase (42.5k points)

9.7.11 Most Appropriate Word: GATE2011-MN-60 top https://gateoverflow.in/31533

Ans: (C)

Preponderance: the quality or fact of being greater in number, quantity, or importance.

"the preponderance of women among older people" C is correct option.

Here, plaintiff means a person who brings a case against another in a court of law.

"the plaintiff commenced an action for damages"

3 votes -- Akash Kanase (42.5k points)

9.7.12 Most Appropriate Word: GATE2012-60 top https://gateoverflow.in/2200

Selected Answer

A. that

Who and whom are people, not dogs.

Regarding that and which...

Restrictive Clause—That

A restrictive clause is just part of a sentence that you can't get rid of because it specifically restricts some other part of the sentence. Here's an example:

- Gems that sparkle often elicit forgiveness.

The words *that sparkle* restrict the kind of gems you're talking about. Without them, the meaning of the sentence would change. Without them, you'd be saying that all gems elicit forgiveness, not just the gems that sparkle. (And note that you don't need commas around the words *that sparkle*.)

Nonrestrictive Clause—Which

A nonrestrictive clause is something that can be left off without changing the meaning of the sentence. You can think of a nonrestrictive clause as simply additional information. Here's an example:

- Diamonds, which are expensive, often elicit forgiveness.

13 votes -- Kathleen Bankson (59.4k points)

9.7.13 Most Appropriate Word: GATE2012-AR-2 top https://gateoverflow.in/40223

Selected Answer

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(A) difficulty, compensated

3 votes -- Sourav Mishra (सौरव मिश्रा) (1.9k points)

9.7.14 Most Appropriate Word: GATE2012-CY-GA-4 top

Beggary - means poverty which is not suiting here
Nomenclature - is choosing names for things which also seems wrong
Jealousy - Wrong
So the Correct **Option is D**
Nonchalance - means casualness or the state of being relaxed.

3 votes -- saif ahmed (4.5k points)

9.7.15 Most Appropriate Word: GATE2013-CE-4 top https://gateoverflow.in/40271

Selected Answer

B. Intimate

Statement says about limitations of friendship and our required word follows "no matter how" meaning it should be something which increases the friendship bond.

3 votes -- vamsi2376 (3.6k points)

9.7.16 Most Appropriate Word: GATE2013-ee-3 top https://gateoverflow.in/40290

Selected Answer

commit

Dare commit mistakes. Option A

6 votes -- Sreyas S (1.9k points)

9.7.17 Most Appropriate Word: GATE2014-1-GA-2 top https://gateoverflow.in/771

Selected Answer

C. Mediocre meaning not very good, not up to par, average. Her performance was average and not worthy of 1st prize.

10 votes -- Kathleen Bankson (59.4k points)

9.7.18 Most Appropriate Word: GATE2014-2-GA-1 top https://gateoverflow.in/1938

Selected Answer

Ans is A

4 votes -- Keith Kr (6.1k points)

9.7.19 Most Appropriate Word: GATE2014-AG-GA1 top https://gateoverflow.in/41660

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Selected Answer

D. experiences.

Reason: We should generally use simple present tense for describing FACTUAL information.

3 votes -- ashishkr73 (239 points)

9.7.20 Most Appropriate Word: GATE2014-AG-GA2 top https://gateoverflow.in/41665

Selected Answer

Ans : A

contentment means $\rightarrow$ A state of happiness and satisfaction.

9 votes -- Marylyn Joseph (221 points)

9.7.21 Most Appropriate Word: GATE2014-EC01-GA1 top

Option C

4 votes -- saif ahmed (4.5k points)

9.7.22 Most Appropriate Word: GATE2014-EC01-GA3 top

https://gateoverflow.in/41492

Selected Answer

Propel :- motivate,stimulate,to push forward
Dispel :- do away,banish,discard,delete
repel: - to push or thrust away
Impel: – urged; persuaded

option "B"

2 votes -- Prateek kumar (7.7k points)

9.7.23 Most Appropriate Word: GATE2014-EC02-GA3 top

https://gateoverflow.in/41510

Selected Answer

Ans : Option C

bear means to carry *(in this context)*

losing OR loosing ?

loose is opposite of tight. Hence losing

4 votes -- Desert_Warrior (8.5k points)

9.7.24 Most Appropriate Word: GATE2014-EC03-GA2 top

https://gateoverflow.in/41141

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Selected Answer

Ans: B) **Depreciated** meaning diminish in value over a period of time.

2 votes -- biranchi (1.5k points)

9.7.25 Most Appropriate Word: GATE2015-2-GA-4 top https://gateoverflow.in/8032

Selected Answer

its 'D' apparel

8 votes -- Anoop Sonkar (5k points)

9.7.26 Most Appropriate Word: GATE2015-3-GA-3 top https://gateoverflow.in/8301

Selected Answer

answer is b

9 votes -- naresh1845 (1.6k points)

9.7.27 Most Appropriate Word: GATE2016-Session-2-GA-2 top

https://gateoverflow.in/108249

Selected Answer

The unruly crowd demanded that the accused be **hanged** without trial.
option A.

2 votes -- Sudhanshu Srivastava (19.6k points)

9.7.28 Most Appropriate Word: GATE2016-Session-3-GA-2 top

https://gateoverflow.in/108478

Selected Answer

A. **Facilitated:** to make easier or less difficult; help forward (an action, a process, etc.)
B. **Felicitated**: to compliment upon a happy event; congratulate.
C. **Fantasized**: to create in one's fancy, daydreams, or the like; imagine
D. **Facillitated:** incorrect word

So only suitable option is B.

3 votes -- ਜਗਮੀਤ (1.4k points)

9.7.29 Most Appropriate Word: GATE2016-Session-7-GA-1 top

https://gateoverflow.in/110868

Selected Answer

If I were you, I **wouldn't buy** that laptop. It’s much too expensive .

you can visit this site for more knowledge about conditional

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sentence.http://www.myenglishpages.com/site_php_files/grammar-lesson-conditionals.php

2 votes -- Akash Verma (2.6k points)

9.7.30 Most Appropriate Word: GATE2018-GA-1 top https://gateoverflow.in/204062

Selected Answer

They're for pointing group

Their for pointing people

There for place

B is answer

6 votes -- Prashant Singh (59.8k points)

9.7.31 Most Appropriate Word: GATE2018-GA-2 top https://gateoverflow.in/204063

Selected Answer

The correct word is **meandering**. It means '*proceeding in a convoluted or undirected fashion.*'
All other options are similar to the word organized.

9 votes -- Rakesh Kumar (3.1k points)

9.7.32 Most Appropriate Word: gate-2014-ae-2 top https://gateoverflow.in/40301

Selected Answer

B) Virtues

Virtues is related to moral standards while strength is related to physical ability.

3 votes -- Monanshi Jain (9.3k points)

9.8 Noun Verb Adjective(1) top

9.8.1 Noun Verb Adjective: GATE2014-EC03-GA3 top https://gateoverflow.in/41142

'Advice' is _____.

A. A verb
B. A noun
C. An adjective
D. Both a verb and a noun

gate2014-ec03 verbal-ability noun-verb-adjective

Answer

Answers: Noun Verb Adjective

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9.8.1 Noun Verb Adjective: GATE2014-EC03-GA3 top https://gateoverflow.in/41142

Selected Answer

Advise [verb] :- give advice to ,offer an opinion or suggestion

eg. I advised him rent was due

I would advise you to go to the hospital immediately

Advice [noun] :- a proposal for appropriate cause of action

eg plz gives us some advice about better planning for preparation for GATE

His advice made me think twice about how to handle different subject parallelly

1 votes -- Prateek kumar (7.7k points)

9.9 Odd One(3) top

9.9.1 Odd One: GATE2014-AE-6 top https://gateoverflow.in/40306

Find the odd one in the following group: ALRVX, EPVZB, ITZDF,OYEIK

A. ALRVX
B. EPVZB
C. ITZDF
D. OYEIK

gate2014-ae odd-one verbal-reasoning verbal-ability

Answer

9.9.2 Odd One: GATE2014-EC02-GA6 top https://gateoverflow.in/41513

Find the odd one in the following group

Q,W,Z,B B,H,K,M W,C,G,J M,S,V,X

A. Q,W,Z,B
B. B,H,K,M
C. W,C,G,J
D. M,S,V,X

gate2014-ec02 verbal-ability verbal-reasoning odd-one normal

Answer

9.9.3 Odd One: GATE2016-2-GA-03 top https://gateoverflow.in/39530

Find the odd one in the following group of words.

mock, deride, praise, jeer

A. Mock

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B. Deride
C. Praise
D. Jeer

gate2016-2 verbal-ability odd-one easy

Answer

Answers: Odd One

9.9.1 Odd One: GATE2014-AE-6 top

https://gateoverflow.in/40306

Selected Answer

here, ans is D

for all words except D, i th letter of word and i th letter of its previous word is differing by 3 letters.

1 votes -- Sanju Rakonde (437 points)

9.9.2 Odd One: GATE2014-EC02-GA6 top

https://gateoverflow.in/41513

Selected Answer

Here in each option

1st letter and $2nd$ letter difference $= 6$ letters

$2nd$ and 3rd letter diff $= 3$ letters

3rd and $4th$ letter diff $= 2$ letters

but option (C) is not matching these all criteria

So,Ans is (C)

6 votes -- srestha (86.6k points)

9.9.3 Odd One: GATE2016-2-GA-03 top

https://gateoverflow.in/39530

Selected Answer

Ans C , praise. Rest all have same meaning.

9 votes -- UK (1.7k points)

9.10 Opposite(2) top

9.10.1 Opposite: GATE2011-60 top

https://gateoverflow.in/2170

Choose the word from the options given below that is most nearly opposite in the meaning to the given word

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Amalgamate

A. merge
B. split
C. collect
D. separate

gate2011 verbal-ability opposite normal

Answer

9.10.2 Opposite: GATE2014-3-GA-3 top

https://gateoverflow.in/2026

Choose the word that is opposite in meaning to the word "coherent".

A. sticky
B. well-connected
C. rambling
D. friendly

gate2014-3 verbal-ability opposite easy

Answer

Answers: Opposite

9.10.1 Opposite: GATE2011-60 top

https://gateoverflow.in/2170

Selected Answer

(D) separate

http://ref:https://www.google.co.in/search?q=Amalgamate&rlz=1C1GIWA_enIN597IN597&oq=Amalgamate&aqs=chrome..69i57&sourceid=chrome8

5 votes -- Kathleen Bankson (59.4k points)

9.10.2 Opposite: GATE2014-3-GA-3 top

https://gateoverflow.in/2026

Selected Answer

C)Rambling

Coherent = Logical and clear,
Rambling = Confused,

5 votes -- Srinath Jayachandran (3.7k points)

9.11 Passage Reading(21) top

9.11.1 Passage Reading: GATE-2012-AE-10 top

https://gateoverflow.in/40221

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In the early nineteenth century, theories of social evolution were inspired less by Biology than by the conviction of social scientists that there was a growing improvement in social institutions. Progress was taken for granted and social scientists attempted to discover its laws and phases.

Which one of the following inferences may be drawn with the greatest accuracy from the above passage?

Social scientists

A. did not question that progress was a fact.
B. did not approve of Biology.
C. framed the laws of progress.
D. emphasized Biology over Social Sciences.

gate2012-ae verbal-ability passage-reading

Answer

9.11.2 Passage Reading: GATE2010-63 top

https://gateoverflow.in/2371

Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.

Which of the following statements best sums up the meaning of the above passage:

A. Modern warfare has resulted in civil strife.
B. Chemical agents are useful in modern warfare.
C. Use of chemical agents in warfare would be undesirable.
D. People in military establishments like to use chemical agents in war.

gate2010 verbal-ability passage-reading normal

Answer

9.11.3 Passage Reading: GATE2011-61 top

https://gateoverflow.in/2171

Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.

Based on the above passage which topic would not be included in a unit on bereavement?

A. how to write a letter of condolence
B. what emotional stages are passed through in the healing process
C. what the leading causes of death are
D. how to give support to a grieving friend

gate2011 verbal-ability passage-reading normal

Answer

9.11.4 Passage Reading: GATE2011-GG-GA-10 top

https://gateoverflow.in/40211

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In order to develop to full potential, a baby needs to be physically able to respond to the environment.

It can be inferred from the passage that

A. Full physical potential is needed in order for a baby to be able to respond to the environment.
B. It is necessary for a baby to be able to physically respond to the environment for it to develop its full potential.
C. Response to the environment of physically able babies needs to be developed to its full potential.
D. A physically able baby needs to develop its full potential in order to respond to its environment.

gate2011-gg logical-reasoning passage-reading

Answer

9.11.5 Passage Reading: GATE2012-AR-10 top https://gateoverflow.in/40231

The documents expose the cynicism of the government officials – and yet as the media website reflects, not a single newspaper has reported on their existence.

Which one of the following inferences may be drawn with the greatest accuracy from the above passage?

A. Nobody other than the government officials knew about the existence of the documents.
B. Newspapers did report about the documents but nobody cared.
C. Media reports did not show the existence of the documents.
D. The documents reveal the attitude of the government officials.

gate2012-ar verbal-ability passage-reading

Answer

9.11.6 Passage Reading: GATE2012-CY-GA-6 top https://gateoverflow.in/40237

One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting,even when the odds and conditions were against them.

Which one of the following statements best sums up the meaning of the above passage?

A. Thorough regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.
B. The legions were treated inhumanly as if the men were animals.
C. Discipline was the armies' inheritance from their seniors.
D. The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

gate2012-cy verbal-ability passage-reading

Answer

9.11.7 Passage Reading: GATE2013-63 top https://gateoverflow.in/1567

After several defeats in wars, Robert Bruce went in exile and wanted to commit suicide. Just before committing suicide, he came across a spider attempting tirelessly to have its net. Time and again, the spider failed but that did not deter it to refrain from making attempts. Such attempts by the

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spider made Bruce curious. Thus, Bruce started observing the near-impossible goal of the spider to have the net. Ultimately, the spider succeeded in having its net despite several failures. Such act of the spider encouraged Bruce not to commit suicide. And then, Bruce went back again and won many a battle, and the rest is history.

Which one of the following assertions is best supported by the above information?

A. Failure is the pillar of success.
B. Honesty is the best policy.
C. Life begins and ends with adventures.
D. No adversity justifies giving up hope.

gate2013 verbal-ability passage-reading normal

Answer

9.11.8 Passage Reading: GATE2014-1-GA-6 top https://gateoverflow.in/774

The Palghat Gap (or Palakkad Gap) , a region about $30\ km$ wide in the southern part of the Western Ghats in India, is lower than the hilly terrain to its north and south. The exact reasons for the formation of this gap are not clear. It results in the neighbouring regions of Tamil Nadu getting more rainfall from the South West monsoon and the neighbouring regions of Kerala having higher summer temperatures.

What can be inferred from this passage?

Select one:

A. The Palghat gap is caused by high rainfall and high temperatures in southern Tamil Nadu and Kerala
B. The regions in Tamil Nadu and Kerala that are near the Palghat Gap are low-lying
C. The low terrain of the Palghat Gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala
D. Higher summer temperatures result in higher rainfall near the Palghat Gap area

gate2014-1 verbal-ability passage-reading normal

Answer

9.11.9 Passage Reading: GATE2014-2-GA-6 top https://gateoverflow.in/1943

The old city of Koenigsberg, which had a German majority population before World War 2, is now called Kaliningrad. After the events of the war, Kaliningrad is now a Russian territory and has a predominantly Russian population. It is bordered by the Baltic Sea on the north and the countries of Poland to the south and west and Lithuania to the east respectively. Which of the statements below can be inferred from this passage?

A. Kaliningrad was historically Russian in its ethnic make up
B. Kaliningrad is a part of Russia despite it not being contiguous with the rest of Russia
C. Koenigsberg was renamed Kaliningrad, as that was its original Russian name
D. Poland and Lithuania are on the route from Kaliningrad to the rest of Russia

gate2014-2 verbal-ability passage-reading normal

Answer

9.11.10 Passage Reading: GATE2014-2-GA-7 top https://gateoverflow.in/1944

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Number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in North India is twice the number diagnosed last year. Municipal authorities have concluded that measures to control the mosquito population have failed in this region.

Which one of the following statements, if true, does not contradict this conclusion?

A. A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent
B. More cases of dengue are now reported because of an increase in the Municipal Office's administrative efficiency
C. Many more cases of dengue are being diagnosed this year since the introduction of a new and effective diagnostic test
D. The number of people with malarial fever (also contracted from mosquito bites) has increased this year

gate2014-2 verbal-ability passage-reading normal

Answer

9.11.11 Passage Reading: GATE2014-3-GA-6 top https://gateoverflow.in/2029

A dance programme is scheduled for 10.00 a.m. Some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be accompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is

A. Students should come at 9.00 a.m. and parents should come at 10.00 a.m.\
B. Participating students should come at 9.00 a.m. accompanied by a parent, and other parents and students should come by 10.00 a.m.
C. Students who are not participating should come by 10.00 a.m. and they should not bring their parents. Participating students should come at 9.00 a.m.
D. Participating students should come before 9.00 a.m. Parents who accompany them should come at 9.00 a.m. All others should come at 10.00 a.m.

gate2014-3 verbal-ability passage-reading easy

Answer

9.11.12 Passage Reading: GATE2014-3-GA-7 top https://gateoverflow.in/2031

By the beginning of the 20th century, several hypotheses were being proposed, suggesting a paradigm shift in our understanding of the universe. However, the clinching evidence was provided by experimental measurements of the position of a star which was directly behind our sun.

Which of the following inference(s) may be drawn from the above passage?

i. Our understanding of the universe changes based on the positions of stars
ii. Paradigm shifts usually occur at the beginning of centuries
iii. Stars are important objects in the universe
iv. Experimental evidence was important in confirming this paradigm shift

A. i, ii and iv
B. iii only
C. i and iv
D. iv only

gate2014-3 verbal-ability passage-reading easy

Answer

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9.11.13 Passage Reading: GATE2014-AE-3 top

https://gateoverflow.in/40302

Rajan was not happy that Sajan decided to do the project on his own. On observing his unhappiness, Sajan explained to Rajan that he preferred to work independently.

Which one of the statements below is logically valid and can be inferred from the above sentences?

A. Rajan has decided to work only in a group.
B. Rajan and Sajan were formed into a group against their wishes.
C. Sajan had decided to give in to Rajan's request to work with him.
D. Rajan had believed that Sajan and he would be working together.

gate-2014-ae passage-reading logical-reasoning

Answer

9.11.14 Passage Reading: GATE2015-3-GA-9 top

https://gateoverflow.in/8388

Most experts feel that in spite of possessing all the technical skills required to be a batsman of the highest order, he is unlikely to be so due to lack of requisite temperament. He was guilty of throwing away his wicket several time after working hard to lay a strong foundation. His critics pointed out that until he addressed his problem, success at the highest level will continue to elude him.

Which of the statement(s) below is/are logically valid and can be inferred from the above passage?

i. He was already a successful batsman at the highest level.
ii. He was to improve his temperament in order to become a great batsman.
iii. He failed to make many of his good starts count.
iv. Improving his technical skills will guarantee success.

A. iii and iv
B. ii and iii
C. i, ii and iii
D. ii only

gate2015-3 verbal-ability normal passage-reading

Answer

9.11.15 Passage Reading: GATE2016-2-GA-07 top

https://gateoverflow.in/39533

Computers were invented for performing only high-end useful computations. However, it is no understatement that they have taken over our world today. The internet, for example, is ubiquitous. Many believe that the internet itself is an unintended consequence of the original invention. With the advent of mobile computing on our phones, a whole new dimension is now enabled. One is left wondering if all these developments are good or, more importantly, required.

Which of the statement(s) below is/are logically valid and can be inferred from the above paragraph?

(i) The author believes that computers are not good for us.

(ii) Mobile computers and the internet are both intended inventions.

A. (i) only
B. (ii) only
C. Both (i) and (ii)
D. Neither (i) nor (ii)

gate2016-2 verbal-ability passage-reading normal

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Answer

9.11.16 Passage Reading: GATE2016-Session-3-GA-7 top

https://gateoverflow.in/108717

Social science disciplines were in existence in an amorphous form until the colonial period when they were institutionalized. In varying degrees, they were intended to further the colonial interest. In the time of globalization and the economic rise of postcolonial countries like India, conventional ways of Knowledge production have become obsolete.

Which of the following can be logically inferred from the above statements?

(i) Social science disciplines have become obsolete.

(ii) Social science disciplines had a pre-colonial origin.

(iii) Social science disciplines always promote colonialism.

(iv) Social science must maintain disciplinary boundaries.

A. (ii) only
B. (i) and (iii) only.
C. (ii) and (iv) only.
D. (iii) and (iv) only.

gate2016session3aptitude logical-reasoning passage-reading

Answer

9.11.17 Passage Reading: GATE2016-Session-4-GA-7 top

https://gateoverflow.in/110844

The overwhelming number of people infected with rabies in India has been flagged by the World Health Organization as a source of concern. It is estimated that inoculating 70% of pets and stray dogs against rabies can lead to a significant reduction in the number of people infected with rabies.

Which of the following can be logically inferred from the above sentences?

A. The number of people in India infected with rabies is high.
B. The number of people in other parts of the world who are infected with rabies is low.
C. Rabies can be eradicated in India by vaccinating 70% of stray dogs.
D. Stray dogs are the main source of rabies worldwide.

gate2016session4aptitude verbal-ability passage-reading

Answer

9.11.18 Passage Reading: GATE2016-Session-8-GA-4 top

https://gateoverflow.in/111283

R2D2 is a robot. R2D2 can repair aeroplanes. No other robot can repair aeroplanes. Which of the following can be logically inferred from the above statements?

A. R2D2 is a robot which can only repair aeroplanes.
B. R2D2 is the only robot which can repair aeroplanes.
C. R2D2 is a robot which can repair only aeroplanes.
D. Only R2D2 is a robot.

gate2016session8aptitude passage-reading logical-reasoning

Answer

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9.11.19 Passage Reading: GATE2016-Session-8-GA-7 top

https://gateoverflow.in/111294

A poll of students appearing for masters in engineering indicated that 60% of the students believed that mechanical engineering is a profession unsuitable for women. A research study on women with masters or higher degrees in mechanical engineering found that 99% of such women were successful in their professions.

Which of the following can be logically inferred from the above paragraph?

A. Many students have misconceptions regarding various engineering disciplines.
B. Men with advanced degrees in mechanical engineering believe women are well suited to be mechanical engineers.
C. Mechanical engineering is a profession well suited for women with masters or higher degrees in mechanical engineering.
D. The number of women pursuing higher degrees in mechanical engineering is small.

gate2016session8aptitude logical-reasoning passage-reading

Answer

9.11.20 Passage Reading: GATE2016-Session-8-GA-8 top

https://gateoverflow.in/111300

Sourya committee had proposed the establishment of Sourya Institutes of Technology (SITs) in line with Indian Institutes of Technology (IITs) to cater to the technological and industrial needs of a developing country.

Which of the following can be logically inferred from the above sentence?

Based on the proposal,

i. In the initial years, SIT students will get degrees from IIT.
ii. SITs will have a distinct national objective.
iii. SIT like institutions can only be established in consultation with IIT.
iv. SITs will serve technological needs of a developing country.

A. (iii) and (iv) only.
B. (i) and (iv) only.
C. (ii) and (iv) only.
D. (ii) and (iii) only.

gate2016session8aptitude logical-reasoning passage-reading

Answer

9.11.21 Passage Reading: GATE2017-1-GA-6 top

https://gateoverflow.in/118409

"The hold of the nationalist imagination on our colonial past is such that anything inadequately or improperly nationalist is just not history."

Which of the following statements best reflects the author's opinion?

A. Nationalists are highly imaginative.
B. History is viewed through the filter of nationalism.
C. Our colonial past never happened.
D. Nationalism has to be both adequately and properly imagined.

gate2017-1 verbal-ability passage-reading

Answer

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Answers: Passage Reading

9.11.1 Passage Reading: GATE-2012-AE-10 top

https://gateoverflow.in/40221

A : Did not question that progress was a fact. **True** They took progress for granted.

B: Did not approve of Biology : **False** . No mention regarding this in paragraph. It's mention that it was not inspired from biology and that's a different thing.

C: Framed the laws of progress : **False** They attempted to discover its laws. Frame and discover are different things.

D: Clearly false.

2 votes -- Mojo Jojo (4k points)

9.11.2 Passage Reading: GATE2010-63 top

https://gateoverflow.in/2371

Selected Answer

D. People in military establishments like to use chemical agents in war.

10 votes -- Kathleen Bankson (59.4k points)

9.11.3 Passage Reading: GATE2011-61 top

https://gateoverflow.in/2171

Selected Answer

(C) what the leading causes of death are

5 votes -- Kathleen Bankson (59.4k points)

9.11.4 Passage Reading: GATE2011-GG-GA-10 top

https://gateoverflow.in/40211

Selected Answer

(B)

This is the best possible explanation.

3 votes -- Sourav Mishra (सौरव मिश्रा) (1.9k points)

9.11.5 Passage Reading: GATE2012-AR-10 top

https://gateoverflow.in/40231

Selected Answer

answer must be D according to me because if i check options A is wrong bcz in the passage they did not mention who knows or who does not know about the existence of document so A will be wrong. B totally irrelevant from passage and C is wrong bcz after reading the passage you got an idea media is waiting once they got the information about existence of document then they will expose them .. then remaining is **D which is related to passage so D will be answer according to me**

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4 votes -- rajan (5k points)

9.11.6 Passage Reading: GATE2012-CY-GA-6 top

https://gateoverflow.in/40237

Selected Answer

(A)

This is the best possible option.

4 votes -- Sourav Mishra (सौरव मिश्रा) (1.9k points)

9.11.7 Passage Reading: GATE2013-63 top

https://gateoverflow.in/1567

Selected Answer

D is my answer. He gave up hope and wanted to commit suicide until he saw the spider in his struggles so no struggle or difficulty is worth giving up hope. Continue on!

13 votes -- Kathleen Bankson (59.4k points)

9.11.8 Passage Reading: GATE2014-1-GA-6 top

https://gateoverflow.in/774

Selected Answer

The answer is C. The primary statement is about the Palghat Gap being low lying which is mentioned in the first sentence. The second part mentions the results of that causing lots of rain and unusual temperatures in the other areas. (Tamil Nadu and Kerala)

11 votes -- Kathleen Bankson (59.4k points)

9.11.9 Passage Reading: GATE2014-2-GA-6 top

https://gateoverflow.in/1943

Selected Answer

option A)

> The old city of Koenigsberg, which had a German majority population before World War 2, is now called Kaliningrad

This line clearly shows that Kaliningrad was historically German

option C) Don't just assume anything. No where it is mentioned why was the past name changed and what is the significance of the new name or it's origin.

option D) Again no information about path which is b/w Russia and this kaliningrad. Why only this path why can't be Baltic sea?

option B) It is clearly mentioned that this Kaliningrad is surrounded by different countries like Lithuania and Poland so that means it is apart from rest of Russia.

Only option B can be inferred.

1 votes -- Rupendra Choudhary (11.5k points)

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9.11.10 Passage Reading: GATE2014-2-GA-7 top https://gateoverflow.in/1944

Selected Answer

D is closer to possibility and doesn't atleast contradict directly with any information.

A. The reason is provided for increase in dengue. It straightaway contradicts the given para.

B. Municipal Office's administrative efficiency has definitely not increased. To increase administrative efficiency is to improve the capability of institution to produce more desirable results. Apparently, it's not the case here. Contradiction

C. "new and effective diagnostic test (to detect dengue). It is clearly mentioned that the reason behind the increase in dengue cases is failure of control measures adopted by Municipal Authorities to control mosquito population (and not any flawed diagnostic test).

D. It doesn't contradict directly with any of the information given in paragraph. In fact there is a high probability this might happen as malaria and dengue both are caused by mosquitoes. And from that fact that mosquito population control measures has failed, we can say malaria might increase. (If it was mentioned that dengue increased because mosquitoes carrying dengue viruses has increased, then it would not be appropriate to conclude increase in malaria)

2 votes -- Gaurav (5.4k points)

9.11.11 Passage Reading: GATE2014-3-GA-6 top https://gateoverflow.in/2029

Selected Answer

It will be B.

8 votes -- Gate Keeda (19.6k points)

9.11.12 Passage Reading: GATE2014-3-GA-7 top https://gateoverflow.in/2031

Selected Answer

A paradigm shift means, a fundamental change in approach or underlying assumptions.
And a change in paradigm happens only when we have an experimented evidence. It is crucial to have an evidence.
In this para the evidence was provided by the experimental measurements of the position of a star which was directly behind our sun.

Option D) suits well for the given para.

6 votes -- Srinath Jayachandran (3.7k points)

9.11.13 Passage Reading: GATE2014-AE-3 top https://gateoverflow.in/40302

Selected Answer

option A discarded: Sajan preferred to work independently

option B discarded: No expectation so no unhappiness

option C discarded: Sajan preferred to work independently

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option D **correct**: unhappy as he believed to be working together

3 votes -- Prince Kumar (227 points)

9.11.14 Passage Reading: GATE2015-3-GA-9 top

https://gateoverflow.in/8388

Selected Answer

"possessing all the technical skills" - iv is false

"throwing away his wicket several time after working hard to lay a strong foundation" - iii is true

"he is unlikely to be so due to lack of requisite temperament" - ii is true

"success at the highest level will continue to elude him" - i is false

So, B.

11 votes -- Arjun Suresh (348k points)

9.11.15 Passage Reading: GATE2016-2-GA-07 top

https://gateoverflow.in/39533

Selected Answer

"Many believes that the internet itself is unintended consequence of the original invention. So (ii) statement does not follow from the passage."

The author has no where said that the computers are bad, authoring is talking about the way computers are being used today and the author questions this way. So, (i) statement does not follow.

Hence, option (d) is the answer.

10 votes -- Ashish Deshmukh (1.5k points)

9.11.16 Passage Reading: GATE2016-Session-3-GA-7 top

https://gateoverflow.in/108717

Selected Answer

Statement III is wrong as their is no mention in passage about promotion of colonialism **always** , only mention their interest to increase it further.
"conventional ways of Knowledge production have become obsolete." this line supports statement (i) . but option B is false due to statement III .

and Statement IV are false clearly, not related to the passage. That makes option C and D false.

"Social science disciplines were in existence in an amorphous form(means without any clear form) until the colonial period " this statement directly say Social science disciplines had a pre-colonial origin. which makes option A correct.

so answer is option A , statement (ii) .

1 votes -- Bikram (67k points)

9.11.17 Passage Reading: GATE2016-Session-4-GA-7 top

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Selected Answer

https://gateoverflow.in/110844

The number of people in India infected with rabies is high
option A

3 votes -- Sudhanshu Srivastava (19.6k points)

9.11.18 Passage Reading: GATE2016-Session-8-GA-4 top

Selected Answer

https://gateoverflow.in/111283

Option A and C, say same thing - R2D2 can repair only Aeroplanes — no other things it can repair. There is no mention in the passage about other tasks that R2D2 can perform. This makes options A and C false.

But the passage says that R2D2 is the only robot which can repair Aeroplanes.

No other robot can repair aeroplanes.

So, option B can be directly inferred from the passage.

Also this passage does not state that there others robots does not exist. This makes statement D false.

2 votes -- Bikram (67k points)

9.11.19 Passage Reading: GATE2016-Session-8-GA-7 top

Selected Answer

https://gateoverflow.in/111294

This sentence in passage "A research study on women with masters or higher degrees in mechanical engineering found that 99% of such women were successful in their professions." directly supports option C.

Option D and option A are false as there is no relation with passage.

Option B says about men with advanced degree but what is stated in passage is about a group of students, so this option is false.

Only option C is correct.

1 votes -- Bikram (67k points)

9.11.20 Passage Reading: GATE2016-Session-8-GA-8 top

Selected Answer

https://gateoverflow.in/111300

Statement (i) - nothing is mentioned in the passage from where SIT students will get degree. So, it is wrong statement.

Statement (ii) - " to cater to the technological and industrial needs of a developing country." this line supports statement (ii). It means Sourya has distinct national objectives.

Statement (iii) - Nothing is mention about consultation with IIT so this makes statement (iii) false.

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Statement (iv) - Again this statement "to cater to the technological and industrial needs of a developing country." directly supports statement (iv).

So, only statements (ii) and (iv) are correct. Option C correct .

2 votes -- Bikram (67k points)

9.11.21 Passage Reading: GATE2017-1-GA-6 top https://gateoverflow.in/118409

Selected Answer

It says that we try to overlook facts of history which is not in favour of our nationalism interest.

So, B. history is viewed through nationalism.

6 votes -- Aboveallplayer (18.5k points)

9.12 Phrasal Verbs(1) top

9.12.1 Phrasal Verbs: GATE2016-Session-8-GA-1 top https://gateoverflow.in/111274

The chairman requested the aggrieved shareholders to _____ him.

A. Bare with
B. Bore with
C. Bear with
D. Bare

gate2016session8aptitude verbal-ability english-grammar phrasal-verbs

Answer

Answers: Phrasal Verbs

9.12.1 Phrasal Verbs: GATE2016-Session-8-GA-1 top https://gateoverflow.in/111274

Selected Answer

A. **Bare with -** invitation to undress,thus not appropriate here.
B. **Bore with** - past tense of bear with
e.g., He bore with me for twenty minutes as I searched for the information he wanted, but then he could stand it no longer and he hung up the phone
C. **Bear with** - request for patience or tolerance.appropriate in this context and hence correct answer.
D. **Bare** - Bare is an adjective and is defined as lacking the usual or appropriate covering or clothing. This is not suitable in this context.

3 votes -- Arnabi Bej (7.9k points)

9.13 Statements Follow(1) top

9.13.1 Statements Follow: GATE2015-1-GA-8 top https://gateoverflow.in/8012

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The given statement is followed by some courses of action. Assuming the statement to be true, decide the correct option.

Statement:

There has been a significant drop in the water level in the lakes supplying water to the city.

Course of action:

I. The water supply authority should impose a partial cut in supply to tackle the situation.
II. The government should appeal to all the residents through mass media for minimal use of water.
III. The government should ban the water supply in lower areas.

A. Statements I and II follow.
B. Statements I and III follow.
C. Statements II and III follow.
D. All the statements follow.

gate2015-1 verbal-ability normal statements-follow

Answer

Answers: Statements Follow

9.13.1 Statements Follow: GATE2015-1-GA-8 top

https://gateoverflow.in/8012

Selected Answer

Statements I and II are correct measures. Option A,

6 votes -- Arjun Suresh (348k points)

9.14 Synonym(4) top

9.14.1 Synonym: GATE2011-GG-GA_2 top

https://gateoverflow.in/40203

Choose the word from the options given below that is most nearly opposite in meaning to the given word:

Polemical

A. imitative
B. conciliatory
C. truthful
D. ideological

gate2011_gg verbal-ability synonym

Answer

9.14.2 Synonym: GATE2012-AR-1 top

https://gateoverflow.in/40222

Which one of the following options is the closest in meaning to the word given below?

Pacify

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A. Excite
B. Soothe
C. Deplete
D. Tire

gate2012-ar verbal-ability synonym

Answer

9.14.3 Synonym: GATE2012-CY-GA-3 top

https://gateoverflow.in/40234

Which one of the following options is the closest in meaning to the word given below?

Latitude

A. Eligibility
B. Freedom
C. Coercion
D. Meticulousness

gate2012-cy verbal-ability synonym

Answer

9.14.4 Synonym: GATE2017-2-GA-1 top

https://gateoverflow.in/118415

Choose the option with words that are not synonyms.

A. aversion, dislike
B. luminous, radiant
C. plunder, loot
D. yielding, resistant

gate2017-2 verbal-ability synonym

Answer

Answers: Synonym

9.14.1 Synonym: GATE2011-GG-GA_2 top

https://gateoverflow.in/40203

(B)

Polemical means a strong verbal or written attack on someone or something.

Conciliatory means making someone less angry or hostile, hence the most suitable antonym for Polemical.

2 votes -- Sourav Mishra (सौरव मिश्रा) (1.9k points)

9.14.2 Synonym: GATE2012-AR-1 top

https://gateoverflow.in/40222

Selected Answer

Answer is [B].

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Pacify means to calm down.

A] Excite : to arouse

B] soothe : to calm down, to slow down

C] deplete : to evacuate.

D] tire : to annoy.

3 votes -- Desert_Warrior (8.5k points)

9.14.3 Synonym: GATE2012-CY-GA-3 top

https://gateoverflow.in/40234

Selected Answer

Latitude means scope for freedom of action or thought.
Option B.

1 votes -- Monanshi Jain (9.3k points)

9.14.4 Synonym: GATE2017-2-GA-1 top

https://gateoverflow.in/118415

Selected Answer

Yield means to give in and resistance means to not give in.
Rest are all synonyms.

Hence, (D) is correct.

9 votes -- Kloseup (601 points)

9.15 Tenses(3) top

9.15.1 Tenses: GATE2013-59 top

https://gateoverflow.in/1563

Were you a bird, you _____ in the sky.

A. would fly
B. shall fly
C. should fly
D. shall have flown

gate2013 verbal-ability tenses normal

Answer

9.15.2 Tenses: GATE2014-2-GA-2 top

https://gateoverflow.in/1939

Who _____ was coming to see us this evening?

A. you said
B. did you say
C. did you say that

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D. had you said

gate2014-2 verbal-ability tenses normal

Answer

9.15.3 Tenses: GATE2017-1-GA-1 top

https://gateoverflow.in/118403

After Rajendra Chola returned from his voyage to Indonesia, he _____ to visit the temple in Thanjavur.

A. was wishing
B. is wishing
C. wished
D. had wished

gate2017-1 verbal-ability tenses english-grammar normal

Answer

Answers: Tenses

9.15.1 Tenses: GATE2013-59 top

https://gateoverflow.in/1563

Selected Answer

A. would fly

9 votes -- Kathleen Bankson (59.4k points)

9.15.2 Tenses: GATE2014-2-GA-2 top

https://gateoverflow.in/1939

Answer should be B.

5 votes -- Poshita Shrivastava (437 points)

9.15.3 Tenses: GATE2017-1-GA-1 top

https://gateoverflow.in/118403

Selected Answer

According to the rule : **when the main clause is in the past or past perfect tense, the subordinate clause must be in the past or past perfect tense**.

so **option (C)** is correct.

4 votes -- Akash Verma (2.6k points)

9.16 Verbal Inference(2) top

9.16.1 Verbal Inference: GATE2012-61 top

https://gateoverflow.in/2209

Wanted Temporary, Part-time persons for the post of Field Interviewer to conduct personal interviews to collect and collate economic data. Requirements: High School-pass, must be available for Day, Evening and Saturday work. Transportation paid, expenses

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reimbursed.

Which one of the following is the best inference from the above advertisement?

A. Gender-discriminatory
B. Xenophobic
C. Not designed to make the post attractive
D. Not gender-discriminatory

gate2012 verbal-ability verbal-inference normal

Answer

9.16.2 Verbal Inference: GATE2014-1-GA-7 top https://gateoverflow.in/775

Geneticists say that they are very close to confirming the genetic roots of psychiatric illnesses such as depression and schizophrenia, and consequently, that doctors will be able to eradicate these diseases through early identification and gene therapy.

On which of the following assumptions does the statement above rely?

Select one:

A. Strategies are now available for eliminating psychiatric illnesses
B. Certain psychiatric illnesses have a genetic basis
C. All human diseases can be traced back to genes and how they are expressed
D. In the future, genetics will become the only relevant field for identifying psychiatric illnesses

gate2014-1 verbal-ability verbal-inference normal

Answer

Answers: Verbal Inference

9.16.1 Verbal Inference: GATE2012-61 top https://gateoverflow.in/2209

Selected Answer

D. Not gender_discriminatory. The post mentions "persons" meaning any gender.

Xenophobic means having or showing an intense or irrational dislike or fear of people from other countries, so does not apply.

C and A do not apply.

11 votes -- Kathleen Bankson (59.4k points)

9.16.2 Verbal Inference: GATE2014-1-GA-7 top https://gateoverflow.in/775

Selected Answer

B is correct, The first sentence mentions two specific illnesses, (depression and schizophrenia). B) is the only one that mentions certain illnesses.

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A) states strategies are now available. The statement says they are very close so its not yet available.

C) states that ALL human diseases can be traced back. The statement only mentions two specific illnesses.

D) the statement does not mention at all that it is the only relevant field.

13 votes -- Kathleen Bankson (59.4k points)

9.17 Verbal Reasoning(19) top

9.17.1 Verbal Reasoning: GATE-2013-AE-GA-3 top https://gateoverflow.in/40244

Mahatama Gandhi was known for his humility as

A. he played an important role in humiliating exit of British from India.
B. he worked for humanitarian causes.
C. he displayed modesty in his interactions.
D. he was a fine human being

gate2013-ae verbal-ability verbal-reasoning

Answer

9.17.2 Verbal Reasoning: GATE2013-EE-5 top https://gateoverflow.in/40292

Statement: You can always give me a ring whenever you need.

Which one of the following is the best inference from the above statement?

A. Because I have a nice caller tune.
B. Because I have a better telephone facility.
C. Because a friend in need is a friend indeed.
D. Because you need not pay towards the telephone bills when you give me a ring.

gate2013-ee verbal-reasoning verbal-ability

Answer

9.17.3 Verbal Reasoning: GATE2013-ee-7 top https://gateoverflow.in/40294

Statement: There were different streams of freedom movements in colonial India carried out by the moderates, liberals, radicals, socialists, and so on.

Which one of the following is the best inference from the above statement?

A. The emergence of nationalism in colonial India led to our Independence.
B. Nationalism in India emerged in the context of colonialism.
C. Nationalism in India is homogeneous.
D. Nationalism in India is heterogeneous

gate2013-ee passage-reading verbal-ability verbal-reasoning

Answer

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9.17.4 Verbal Reasoning: GATE2014-AG-GA3 top https://gateoverflow.in/41667

Which of the following options is the closest in meaning to the sentence below?

"As a woman, I have no country."

A. Women have no country.
B. Women are not citizens of any country.
C. Women's solidarity knows no national boundaries.
D. Women of all countries have equal legal rights.

gate2014-ag general-aptitude verbal-ability verbal-reasoning normal

Answer

9.17.5 Verbal Reasoning: GATE2014-AG-GA7 top https://gateoverflow.in/41671

Moving into a world of big data will require us to change our thinking about the merits of exactitude. To apply the conventional mindset of measurement to the digital, connected world of the twenty-first century is to miss a crucial point. As mentioned earlier, the obsession with exactness is an artefact of the information-deprived analog era. When data was sparse, every data point was critical, and thus great care was taken to avoid letting any point bias the analysis. From "BIG DATA" Viktor Mayer-Schonberger and Kenneth Cukier. The main point of the paragraph is:

A. The twenty-first century is a digital world
B. Big data is obsessed with exactness
C. Exactitude is not critical in dealing with big data
D. Sparse data leads to a bias in the analysis

gate2014-ag verbal-ability verbal-reasoning passage-reading normal

Answer

9.17.6 Verbal Reasoning: GATE2014-EC01-GA2 top https://gateoverflow.in/41491

Read the statements:

All women are entrepreneurs.

Some women are doctors.

Which of the following conclusions can be logically inferred from the above statements?

A. All women are doctors
B. All doctors are entrepreneurs
C. All entrepreneurs are women
D. Some entrepreneurs are doctors

gate2014-ec01 verbal-ability mathematical-logic verbal-reasoning easy

Answer

9.17.7 Verbal Reasoning: GATE2014-EC01-GA6 top https://gateoverflow.in/41495

Find the odd one from the following group:

W,E,K,O I,Q,W,A F,N,T,X N,V,B,D

A. W,E,K,O

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B. I,Q,W,A
C. F,N,T,X
D. N,V,B,D

gate2014-ec01 verbal-ability verbal-reasoning normal

Answer

9.17.8 Verbal Reasoning: GATE2014-EC02-GA1 top https://gateoverflow.in/41507

Choose the most appropriate word from the options given below to complete the following sentence.

Communication and interpersonal skills are _____ important in their own ways.

A. Each
B. Both
C. All
D. Either

gate2014-ec02 verbal-ability verbal-reasoning most-appropriate-word normal

Answer

9.17.9 Verbal Reasoning: GATE2014-EC03-GA1 top https://gateoverflow.in/41140

"India is a country of rich heritage and cultural diversity." Which one of the following facts best supports the claim made in the above sentence?

A. India is a union of 28 states and 7 union territories.
B. India has a population of over 1.1 billion.
C. India is home to 22 official languages and thousands of dialects.
D. The Indian cricket team draws players from over ten states.

gate2014-ec03 verbal-reasoning verbal-ability

Answer

9.17.10 Verbal Reasoning: GATE2014-EC04-GA7 top https://gateoverflow.in/41469

If '$KCLFTSB$' stands for 'best of luck' and '$SHSWDG$' stands for 'good wishes', which of the following indicates 'ace the exam'?

A. $MCHTX$
B. $MXHTC$
C. $XMHCT$
D. $XMHTC$

gate2014-ec04 verbal-ability verbal-reasoning normal

Answer

9.17.11 Verbal Reasoning: GATE2015-3-GA-6 top https://gateoverflow.in/8306

Alexander turned his attention towards India, since he had conquered Persia.

Which one of the statements below is logically valid and can be inferred from the above sentence?

A. Alexander would not have turned his attention towards India had he not conquered Persia.
B. Alexander was not ready to rest on his laurels, and wanted to march to India.

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C. Alexander was not completely in control of his army and could command it to move towards India.
D. Since Alexander's kingdom extended to Indian borders after the conquest of Persia, he was keen to move further.

gate2015-3 verbal-ability normal verbal-reasoning

Answer

9.17.12 Verbal Reasoning: GATE2016-1-GA07 top

https://gateoverflow.in/39613

Indian currency notes show the denomination indicated in at least seventeen languages. If this is not an indication of the nation's diversity, nothing else is.

Which of the following can be logically inferred from the above sentences?

A. India is a country of exactly seventeen languages.
B. Linguistic pluralism is the only indicator of a nation's diversity.
C. Indian currency notes have sufficient space for all the Indian languages.
D. Linguistic pluralism is strong evidence of India's diversity.

gate2016-1 verbal-ability verbal-reasoning normal

Answer

9.17.13 Verbal Reasoning: GATE2016-Session-1-GA-3 top

https://gateoverflow.in/108069

Despite the new medicine's _____ in treating diabetes, it is not _____widely.

A. effectiveness --- prescribed
B. availability --- used
C. prescription --- available
D. acceptance --- proscribed

gate2016session1aptitude verbal-ability verbal-reasoning

Answer

9.17.14 Verbal Reasoning: GATE2016-Session-1-GA-7 top

https://gateoverflow.in/108067

In a world filled with uncertainty, he was glad to have many good friends. He had always assisted them in times of need and was confident that they would reciprocate. However, the events of the last week proved him wrong.

Which of the following inference(s) is/are logically valid and can be inferred from the above passage?

i. His friends were always asking him to help them.
ii. He felt that when in need of help, his friends would let him down.
iii. He was sure that his friends would help him when in need.
iv. His friends did not help him last week.

A. (i) and (ii)
B. (iii) and (iv)
C. (iii) only
D. (iv) only

gate2016session1aptitude passage-reading verbal-reasoning

Answer

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9.17.15 Verbal Reasoning: GATE2016-Session-1-GA-8 top

https://gateoverflow.in/108089

Leela is older than her cousin Pavithra. Pavithra's brother Shiva is older than Leela. When Pavithra and Shiva are visiting Leela, all three like to play chess. Pavithra wins more often than Leela does.

Which one of the following statements must be TRUE based on the above?

A. When Shiva plays chess with Leela and Pavithra, he often loses.
B. Leela is the oldest of the three.
C. Shiva is a better chess player than Pavithra.
D. Pavithra is the youngest of the three.

gate2016session1aptitude verbal-ability passage-reading verbal-reasoning

Answer

9.17.16 Verbal Reasoning: GATE2016-Session-2-GA-3 top

https://gateoverflow.in/108259

Choose the statement(s) where the underlined word is used correctly:

i. A prone is a dried plum.
ii. He was lying prone on the floor.
iii. People who eat a lot of fat are prone to heart disease.

A. (i) and (iii) only
B. (iii) only
C. (i) and (ii) only
D. (ii) and (iii) only

gate2016session2aptitude verbal-reasoning verbal-ability

Answer

9.17.17 Verbal Reasoning: GATE2016-Session-2-GA-7 top

https://gateoverflow.in/108299

A smart city integrates all modes of transport, uses clean energy and promotes sustainable use of resources. It also uses technology to ensure safety and security of the city, something which critics argue, will lead to a surveillance state. Which of the following can be logically inferred from the above paragraph?

i. All smart cities encourage the formation of surveillance states.
ii. Surveillance is an integral part of a smart city.
iii. Sustainability and surveillance go hand in hand in a smart city.
iv. There is a perception that smart cities promote surveillance.

A. (i) and (iv) only
B. (ii) and (iii) only
C. (iv) only
D. (i) only

gate2016session2aptitude passage-reading verbal-reasoning

Answer

9.17.18 Verbal Reasoning: GATE2016-Session-7-GA-3 top

https://gateoverflow.in/110881

Choose the most appropriate set of words from the options given below to complete the following sentence.

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_____, _____ is a will, _____ is a way.

A. Wear, there, their
B. Were, their, there
C. Where, there, there
D. Where, their, their

gate2016session7aptitude verbal-reasoning verbal-ability

Answer

9.17.19 Verbal Reasoning: GATE2017-2-GA-6 top https://gateoverflow.in/118420

"We lived in a culture that denied any merit to literary works, considering them important only when they were handmaidens to something seemingly more urgent – namely ideology. This was a country where all gestures, even the most private, were interpreted in political terms."

The author's belief that ideology is not as important as literature is revealed by the word:

A. 'culture'
B. 'seemingly'
C. 'urgent'
D. 'political'

gate2017-2 passage-reading verbal-reasoning

Answer

Answers: Verbal Reasoning

9.17.1 Verbal Reasoning: GATE-2013-AE-GA-3 top https://gateoverflow.in/40244

Answer C: He displayed modesty in his interactions.

hu·mil·i·ty
(h)yōō'milədē/

noun

1. a modest or low view of one's own importance; humbleness.
   synonyms: modesty, humbleness, meekness, diffidence,unassertiveness; **More**

2 votes -- Kathleen Bankson (59.4k points)

9.17.2 Verbal Reasoning: GATE2013-EE-5 top https://gateoverflow.in/40292

Selected Answer

C) seems to be an appropriate option.

4 votes -- Monanshi Jain (9.3k points)

9.17.3 Verbal Reasoning: GATE2013-ee-7 top https://gateoverflow.in/40294

Selected Answer

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Option D. Hetero means different.

It is clearly mentioned that the movement comprised of moderates, liberals, radicals, socialists and so on.

3 votes -- Mojo Jojo (4k points)

9.17.4 Verbal Reasoning: GATE2014-AG-GA3 top https://gateoverflow.in/41667

Selected Answer

As a women I have no country

What this statement means? Does it mean that women are not accepted in any country? Definitely no. Rather it means that a women does not consider people from other countries any different. i.e., we are not talking about nationalist women here rather we are talking about women who considers those from other countries the same as from their own country (of course it does not make them anti-national as some believe). If interested you can read more here.

Now, when a statement is given and asked for the meaning we cannot consider any external facts into consideration.

A. Wrong as this statement means that no country is welcome to women as against the meaning conveyed by the given sentence
B. Wrong - the given statement says nothing about citizenship
C. Correct - expresses the same meaning as given sentence
D. Wrong - this may or may not be universally true, but in no way expresses the same meaning as given statement

2 votes -- Arjun Suresh (348k points)

9.17.5 Verbal Reasoning: GATE2014-AG-GA7 top https://gateoverflow.in/41671

Selected Answer

Option C :

Exactitude is not critical in dealing with big data

4 votes -- Pratik Agrawal (165 points)

9.17.6 Verbal Reasoning: GATE2014-EC01-GA2 top https://gateoverflow.in/41491

Selected Answer

Option D is correct.

"Some Entrepreneurs are Doctor".

PS: In the first statement if we replace "All" by "Some", then none of the options is true.

11 votes -- Muktinath Vishwakarma (35.3k points)

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9.17.7 Verbal Reasoning: GATE2014-EC01-GA6 top https://gateoverflow.in/41495

Selected Answer

Ans is (D)

the no of alphabet between B,D in N,V,B,D is 1 while in other 3 sets, it is 4. Alphabet sequence in all the sets are obtained by adding 8,6,4 respectively to the previous one in sequence.

5 votes -- srestha (86.6k points)

9.17.8 Verbal Reasoning: GATE2014-EC02-GA1 top https://gateoverflow.in/41507

Selected Answer

Answer will be (B)

6 votes -- srestha (86.6k points)

9.17.9 Verbal Reasoning: GATE2014-EC03-GA1 top https://gateoverflow.in/41140

Selected Answer

Answer C) India is home to 22 official languages and thousands of dialects.
It talks about 22 languages and 1000s of dialects, which clearly show existence of rich heritage and cultural diversity.
Other options A,B and D only show there are many states, which does not necessarily imply the rich heritage and diversity,

5 votes -- Abhilash Panicker (9.4k points)

9.17.10 Verbal Reasoning: GATE2014-EC04-GA7 top https://gateoverflow.in/41469

Selected Answer

Best of luck - KCLFTSB
If we read it backwards BSTFLCK

Good wishes SHSWDG
If we read it backwards GDWSHS

Similarly, for
Ace the exam
It should be
MXHTC
which when read bacward CTHXM

Answer B) MXHTC

Edit: added by Sonam Vyas
Just remove the vowels and read from backward to get the answer :)

4 votes -- Abhilash Panicker (9.4k points)

9.17.11 Verbal Reasoning: GATE2015-3-GA-6 top https://gateoverflow.in/8306

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Selected Answer

Answer should be A as other options required more information

Let P be "Alexander turned his attention towards India " and Q be "he had conquered Persia"

$P \text{ since } Q \equiv (\neg Q \implies \neg P)$

$\neg P$ is "Alexander would not have turned his attention towards India"

$\neg Q$ is "he had not conquered Persia"

23 votes -- Srijay Deshpande (389 points)

9.17.12 Verbal Reasoning: GATE2016-1-GA07 top

https://gateoverflow.in/39613

Selected Answer

Answer is (D) Linguistic pluralism is strong evidence of India's diversity.

19 votes -- sushma nayak (171 points)

9.17.13 Verbal Reasoning: GATE2016-Session-1-GA-3 top

https://gateoverflow.in/108069

Selected Answer

Despite the new medicine's **effectiveness** in treating diabetes, it is not **prescribed** widely.
option A

4 votes -- Sudhanshu Srivastava (19.6k points)

9.17.14 Verbal Reasoning: GATE2016-Session-1-GA-7 top

https://gateoverflow.in/108087

Selected Answer

(iii) He was sure that his friends would help him when in need.

(iv) His friends did not help him last week.

Only these 2 statements can be inferred. O**ption B**.

3 votes -- Sudhanshu Srivastava (19.6k points)

9.17.15 Verbal Reasoning: GATE2016-Session-1-GA-8 top

https://gateoverflow.in/108089

Selected Answer

$\text{Shiva} > \text{Leela} > \text{Pavithra}$

Pavithra is the youngest. Option D.

3 votes -- Sudhanshu Srivastava (19.6k points)

9.17.16 Verbal Reasoning: GATE2016-Session-2-GA-3 top

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Selected Answer

prone means :

1. likely or liable to suffer from, which is option (iii)
2. lying flat, especially face downwards. which is option (ii)

And a *prune* is a dried plum of any cultivar. Prune means a plum preserved by drying and having a black, wrinkled appearance .So statement (i) is wrong.

Hence correct answer is option D.

2 votes -- Bikram (67k points)

9.17.17 Verbal Reasoning: GATE2016-Session-2-GA-7 top

https://gateoverflow.in/108299

Selected Answer

"It also uses technology to ensure safety and security of the city, something which critics argue, will lead to a surveillance state."

critics argue because of perception that smart city promotes surveillance that makes statement (iv) correct.

There is nothing mention in passage about encouragement about formation of surveillance states so that makes statement (i) false.

And, "will lead to a surveillance state tis makes " this line makes statement (ii) false.

and there is nothing mention about " both Sustainability and surveillance" in passage so statement (iii) is wrong too.

Hence only statement (iv) is correct which is option C .

2 votes -- Bikram (67k points)

9.17.18 Verbal Reasoning: GATE2016-Session-7-GA-3 top

https://gateoverflow.in/110881

Selected Answer

Options A and B clearly wrong as **Wear** and **Were** cannot start the given statement.

Among options C and D, **Their** means belongs to some people but meaning of the given sentence says it should be **There** means to that position .

That makes option C correct.

Where there is a will there is a way means If one really wants to do something, he always find a way to do it.

2 votes -- Bikram (67k points)

9.17.19 Verbal Reasoning: GATE2017-2-GA-6 top

https://gateoverflow.in/118420

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Selected Answer

Seemingly means external appearance as **distinguished from** true character — means it is not actually what it looks like.

"considering them important only when they were handmaidens (means assisting or helping) to something seemingly more urgent" means it looks like urgent but in real it is not so urgent.

So, ideology is not as important as literature, is revealed by the word "seemingly".

6 votes -- Bikram (67k points)

9.18 Word Pairs(6) top

9.18.1 Word Pairs: GATE-2013-AE-GA-5 top

https://gateoverflow.in/40246

Select the pair that best expresses a relationship similar to that expressed in the pair:

water: pipe::

A. cart: road
B. electricity: wire
C. sea: beach
D. music: instrument

gate2013-ae verbal-ability word-pairs

Answer

9.18.2 Word Pairs: GATE2010-60 top

https://gateoverflow.in/2368

The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

Unemployed : Worker

A. fallow : land
B. unaware : sleeper
C. wit : jester
D. renovated : house

gate2010 verbal-ability word-pairs normal

Answer

9.18.3 Word Pairs: GATE2013-57 top

https://gateoverflow.in/1560

Complete the sentence:

Universalism is to particularism as diffuseness is to _____.

A. specificity
B. neutrality
C. generality
D. adaptation

gate2013 verbal-ability normal word-pairs

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Answer

9.18.4 Word Pairs: GATE2013-CE-5 top

https://gateoverflow.in/40272

Select the pair that best expresses a relationship similar to that expressed in the pair:

Medicine: Health

A. Science: Experiment
B. Wealth: Peace
C. Education: Knowledge
D. Money: Happiness

gate2013-ce word-pairs verbal-ability

Answer

9.18.5 Word Pairs: GATE2015-1-GA-5 top

https://gateoverflow.in/8008

Which one of the following combinations is incorrect?

A. Acquiescence - Submission
B. Wheedle - Roundabout
C. Flippancy - Lightness
D. Profligate - Extravagant

gate2015-1 verbal-ability difficult word-pairs

Answer

9.18.6 Word Pairs: GATE2015-3-GA-4 top

https://gateoverflow.in/8302

Select the pair of best expresses a relationship similar to that expressed in the pair:

Children : Pediatrician

A. Adult : Orthopaedist
B. Females : Gynaecologist
C. Kidney : Nephrologist
D. Skin : Dermatologist

gate2015-3 verbal-ability easy word-pairs

Answer

Answers: Word Pairs

9.18.1 Word Pairs: GATE-2013-AE-GA-5 top

https://gateoverflow.in/40246

Selected Answer

Answer B: Water runs through a pipe like electricity runs through a wire.

5 votes -- Kathleen Bankson (59.4k points)

9.18.2 Word Pairs: GATE2010-60 top

https://gateoverflow.in/2368

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Selected Answer

A. Fallow: Land

Fallow is **land** that is uncultivated

Unemployed is a **worker** without a job

15 votes -- Kathleen Bankson (59.4k points)

9.18.3 Word Pairs: GATE2013-57 top

https://gateoverflow.in/1560

Selected Answer

A. Specificity. This is asking for opposites.

Specificity	Diffuseness
Direct, to the point, purposeful in relating	Indirect, circuitous, seemingly "aimless" forms of relating
Precise, blunt, definitive and transparent	Evasive, tactful, ambiguous, even opaque
Principles and consistent moral stands independent of the person being addressed	Highly situational morality depending upon the person and context encountered

11 votes -- Kathleen Bankson (59.4k points)

9.18.4 Word Pairs: GATE2013-CE-5 top

https://gateoverflow.in/40272

Selected Answer

C.

Medicine improves health; education improves knowledge

8 votes -- Rahul Singla (249 points)

9.18.5 Word Pairs: GATE2015-1-GA-5 top

https://gateoverflow.in/8008

Selected Answer

B Wheedle - Roundabout

9 votes -- Kathleen Bankson (59.4k points)

9.18.6 Word Pairs: GATE2015-3-GA-4 top

https://gateoverflow.in/8302

Selected Answer

Option B

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Orthopaedist: An orthopaedic surgeon, a doctor who corrects congenital or functional abnormalities of the bones with surgery, casting, and bracing.

Gynaecologist a physician or surgeon qualified to practice in functions and diseases specific to women and girls, especially those affecting the reproductive system.

Nephrologist is a medical doctor who specializes in kidney care and treating diseases of the kidneys.

Dermatologist is a medical practitioner qualified to diagnose and treat skin disorders.

Kidney and skin are parts of a body. Females is the only option which represents **group of people** like children which is the correct option.

11 votes -- Anoop Sonkar (5k points)

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