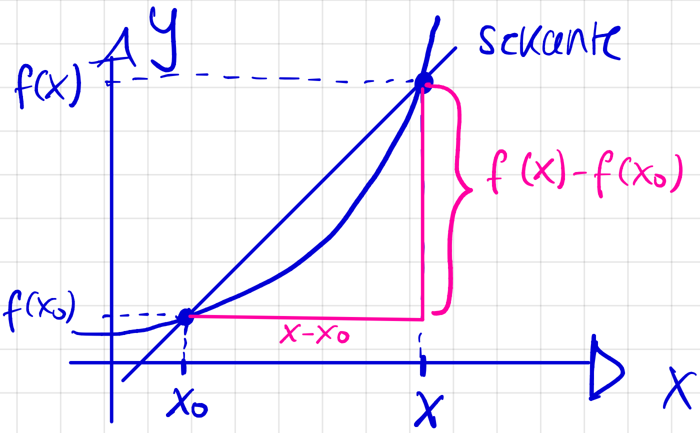
AB

Kettenregel – Beweis

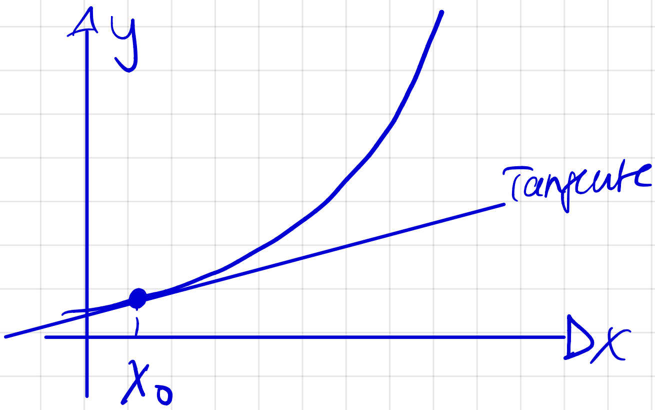
Wiederholung:

Es gilt $f'(x_0) = \lim\limits_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0}$



$$m(x) = \frac{f(x) - f(x_0)}{x - x_0}$$

(mittlere Änderungsrate)



$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

(momentane ÄR)

oder

für $x \to x_0$ gilt $\dfrac{f(x) - f(x_0)}{x - x_0} \to f'(x_0)$

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Ableitung von Verkettungen – Beweis

$f(x) = u(v(x))$

$$m(x) = \frac{f(x) - f(x_0)}{x - x_0} = \frac{u(v(x)) - u(v(x_0))}{x - x_0}$$

$$= \frac{u(v(x)) - u(v(x_0))}{v(x) - v(x_0)} \cdot \frac{v(x) - v(x_0)}{x - x_0}$$

Es gilt:

$$\lim_{x \to x_0} \frac{v(x) - v(x_0)}{x - x_0} = v'(x_0)$$

$$\lim_{x \to x_0} \frac{u(v(x)) - u(v(x_0))}{v(x) - v(x_0)} = u'(v(x_0))$$

Also:

$$\lim_{x \to x_0} m(x) = u'(v(x_0)) \cdot v'(x_0)$$

■

Kettenregel: $f(x) = u(v(x))$ u und v diff'bare Funktionen

$$f'(x) = u'(v(x)) \cdot v'(x)$$

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