Anthropic decision theory for self-locating beliefs

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Abstract

This paper sets out to solve the Sleeping Beauty problem and various related anthropic (self-locating belief) problems, not through the calculation of anthropic probabilities, but through finding the correct decision to make. Given certain simple assumptions, it turns out to be possible to do so without knowing the underlying anthropic probabilities. Most common anthropic problems are underspecified from the decision perspective, and this can explain some of the differing intuitions in the subject: selfless and selfish agents, total and average utilitarians, will all reach different decisions in the same problem. These results are formalised into an anthropic decision theory, that is then used to solve many anthropic problems and paradoxes, such as the Presumptuous Philosopher, Adam and Eve, and Doomsday problems.

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1 Introduction

We cannot have been born on a planet unable to support life. This self-evident truism is an example of anthropic or self-locating OREF reasoning: we cannot see ourselves as 'outside observers' when looking at facts that are connected with our own existence. This means that simply by realising that we exist, we change the odds of certain things being true or false.

There have been several different ways of formalising the above intuition. The two most popular revolve around the 'Self-Sampling Assumption' and the 'Self-Indication Assumption' ([Bos02]). Both of these assumptions give a way of computing anthropic probabilities, answering questions such as 'Given that I exist and am human, what probability should I assign to there being billions (or trillions) of other humans?'. Unfortunately the two assumptions are incompatible with each other, and give different answers to standard anthropic problems. Nor do they tell you how to translate the probabilities they give you into actual decisions: many anthropic problems revolve around identical copies, who will presumably decide exactly the same thing as you. In order to figure out the consequences of your actions, you have to figure out if you bear responsibility for the behaviour of identical copies. Causal decision theory [Lew81] would claim that you have no such responsibility, while evidential decision theory [GH81] would claim the opposite. And depending on how you resolve this question, SIA and SSA may make you always follow the same behaviour, while still proclaiming different probabilities!

Hence this paper does not take any position on anthropic probabilities, and instead seeks to find the correct decision in anthropic problems. This seems an extraordinary ambition – how can the right decision be made, if the right

probabilities aren't known? It turns out that with a few broad assumptions, it is indeed possible to solve these problems, and thus get the correct decision.

This will be illustrated by careful analysis of one of the most famous anthropic problems, the Sleeping Beauty problem [Elg00]. It turns out that one of the reasons that people have had contradictory intuitions on the problem is that it is underspecified from the decision perspective. The correct decision differs depending on whether the Sleeping Beauties are assumed to be selfless or selfish, or total or average utilitarians.

Many anthropic problems can be similarly solved using these assumptions, and this is often enough. But it still leaves many questions unanswered, especially those involving non-identical agents. Thus this paper then presents an Anthropic Decision Theory, which gives the correct solution to all the Sleeping Beauty variants and generalises to other anthropic problems. The last part of the paper illustrates the application of anthropic decision theory to many famous anthropic problems and paradoxes. It provides a more intuitive resolution of the Presumptuous Philosopher, Adam and Eve [Bos01], and Doomsday problems [CM83].

2 The Sleeping Beauty Problem

The Sleeping Beauty [Elg00] problem is one of the most fundamental in anthropic probability, and any decent theory of anthropics needs to be able to deal with it. It is related to many similar problems, such as the absent-minded driver [AHP96], the Sailor's Child Problem [Nea06], the incubator and the presumptuous philosopher [Bos02]. From the perspective of this paper, all those problems are actually the same one, the only difference being in the selflessness of the agents.

In the standard setup, Sleeping Beauty is put to sleep on Sunday, and awoken again Monday morning, without being told what day it is. She is put to sleep again at the end of the day. A fair coin was tossed before the experiment began. If that coin showed heads, she is never reawakened. If the coin showed tails, she is fed a one-day amnesia potion (so that she does not remember being awake on Monday) and is reawakened on Tuesday, again without being told what day it is. At the end of Tuesday, she is left to sleep for ever (see Figure 1).

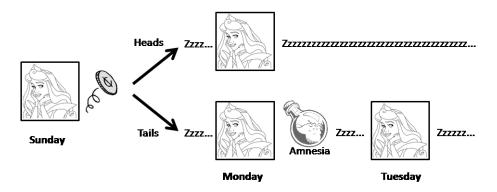


Figure 1: The Sleeping Beauty problem

The incubator variant of the problem [Bos02] has no initial Sleeping Beauty, just one or two copies of her *created* (in different, identical rooms), depending on the result of the coin flip (see Figure 2). As we shall see in Section 3.1.2, the fact there is no initial Sleeping Beauty means that extra assumptions will be needed to solve this incubator variant.

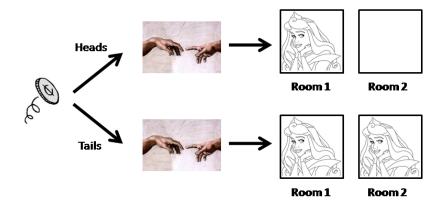


Figure 2: The incubator variant of the Sleeping Beauty problem

The question then is what probability a recently awoken or created Sleeping Beauty should give to the coin falling heads or tails and it being Monday or Tuesday when she is awakened (or whether she is in Room 1 or 2). There are two main schools of thought on the subject, making use of either the Self-Sampling Assumption, or the Self-Indication Assumption [Bos02].

2.1 The Self-Sampling Assumption

The self-sampling assumption (SSA) relies on the insight that Sleeping Beauty, before being put to sleep on Sunday, expects that she will be awakened in future. Thus her awakening grants her no extra information, and she should continue to give the same credence to the coin flip being heads as she did before, namely 1/2.

In the case where the coin is tails, there will be two copies of Sleeping Beauty, one on Monday and one on Tuesday, and she will not be able to tell, upon awakening, which copy she is. She should assume that both are equally likely. This leads to SSA:

Definition 2.1 (Self-Sampling Assumption). All other things being equal, an observer should reason as if they are randomly selected from the set of all actually existent observers (past, present and future) in their reference class.

There are some issues with the concept of 'reference class' [Bos02], but here it is enough to set the reference class to be the set of all other Sleeping Beauties woken up in the experiment.

Given this, the probability calculations become straightforward:

```
\begin{array}{lll} P_{\rm SSA}({\rm Heads}) & = & 1/2, \\ P_{\rm SSA}({\rm Tails}) & = & 1/2, \\ P_{\rm SSA}({\rm Monday}|{\rm Heads}) & = & 1, \\ P_{\rm SSA}({\rm Tuesday}|{\rm Heads}) & = & 0, \\ P_{\rm SSA}({\rm Monday}|{\rm Tails}) & = & 1/2, \\ P_{\rm SSA}({\rm Tuesday}|{\rm Tails}) & = & 1/2. \end{array}
```

By Bayes' theorem, these imply that:

```
P_{\text{SSA}}(\text{Monday}) = 3/4,
P_{\text{SSA}}(\text{Tuesday}) = 1/4.
```

2.2 The Self-Indication Assumption

There is another common way of doing anthropic probability, namely to use the self-indication assumption (SIA). This derives from the insight that being woken up on Monday after a heads, being woken up on Monday after a tails, and being woken up on Tuesday are all subjectively indistinguishable events, which each have a probability 1/2 of happening, therefore we should consider them equally probable. This is formalised as:

Definition 2.2 (Self-Indication Assumption). All other things being equal, an observer should reason as if they are randomly selected from the set of all possible observers.

Note that this definition of SIA is slightly different from that used in Bostrom [Bos02]; what we would call SIA he designated as the combined SIA+SSA. We shall stick with the definition above, however, as it is coming into general use. Note that there is no mention of reference classes, as one of the great advantages of SIA is that any reference class will do, as long as it contains the observers in question.

Given SIA, the three following observer situations are equiprobable (each has an 'objective' probability 1/2 of happening), and hence SIA gives them equal probabilities 1/3:

```
P_{\mathrm{SIA}}(\mathrm{Monday} \cap \mathrm{Heads}) = 1/3,

P_{\mathrm{SIA}}(\mathrm{Monday} \cap \mathrm{Tails}) = 1/3,

P_{\mathrm{SIA}}(\mathrm{Tuesday} \cap \mathrm{Tails}) = 1/3.
```

This allows us to compute the probabilities:

```
P_{\text{SIA}}(\text{Monday}) = 2/3,

P_{\text{SIA}}(\text{Tuesday}) = 1/3,

P_{\text{SIA}}(\text{Heads}) = 1/3,

P_{\text{SIA}}(\text{Tails}) = 2/3.
```

SIA and SSA are sometimes referred to as the thirder and halfer positions respectively, referring to the probability they give for Heads.

2.3 Long-run frequencies

It might be tempting to resolve the dispute between SIA and SSA by appealing to long-run frequencies. But it turns out the criteria for counting these long-run frequencies completely determines which assumption comes out on top.

This is easiest to see in the incubator variant of the Sleeping Beauty problem. Imagining running the incubator variant as a million separate experiments, each dependent on a separate coin flip. Each created copy is then asked to guess whether their coin flip came up heads or tails. If the winning criteria is how many of the copies guess right in total across all experiments, then SIA comes out ahead: the copies will all guess tails, and 2/3 of them will be right. But if instead each experiment simply reported 'all copies guessed right/all copies guessed wrong', then guessing heads or tails will give the same score: SSA gives the correct odds for this.

These two different ways of aggregating long-run frequencies are closely related to the difference between the selfish, selfless and total and average utilitarian variants discussed in the next section.

3 Solving Sleeping Beauty

The probabilities in anthropic situations depend on whether one accepts SSA or SIA. However, when confronted with anthropic decision problems, it is often possible to determine the correct decision, without reference to either assumption. To do so, one needs to make use of a few hopefully self-evident axioms.

First, in order to transform the Sleeping Beauty problem into a decision problem, assume that every time she is awoken, she is offered a coupon that pays out $\pounds 1$ if the coin fell tails. She must then decide at what cost she is willing to buy that coupon.

The very first axiom is that of temporal consistency. If your preferences are going to predictably change, then someone will be able to exploit this, by selling you something now that they will buy back for more later, or vice versa. This axiom is implicit in the independence axiom in the von Neumann-Morgenstern axioms of expected utility [vNM], where non-independent decisions show inconsistency after partially resolving one of the lotteries. It appears, under the name diachronic consistency, in David Wallace's work proving the Born rule in quantum mechanics. For our purposes, we will define it as:

Axiom 1 (Temporal consistency). If an agent at two different times has the same knowledge and preferences, then the past version will never give up anything of value in order to change the conditional decision of the future version.

Here 'conditional decision' means the decision, conditional on extra information the future copy may have. Hence you wouldn't want to determine your future copy's decision if they had seen a coin flip that you hadn't; but you would want to say 'if they've seen heads, then they should do this, but if they've seen tails, then they should do that'.

This axiom is appropriate for the standard Sleeping Beauty problem, but not for the incubator variant, where the different copies are not future or past versions of each other. To deal with that, we extend the axiom to: **Axiom 1** (Consistency). If two copies of an agent have the same knowledge and preferences, then the one version will never give up anything of value in order to change the conditional decision of the other version.

Note that while 'same preferences' is something we could expect to see for the same agent at different times, it is not something the case for copies, who are generally assumed to be selfish towards each other. Indeed, this whole issue of selflessness, altruism and selfishness will be of great import for the agent's behaviour, as we shall now see.

3.1 SIA-like situations

3.1.1 Selfless Sleeping Beauty

In order to specify the decision problem fully, we decree that Sleeping Beauty has an entirely selfless utility function. It need not be a *nice* utility function – she may be entirely devoted to hugging sick orphans, or her life's goal may be drowning cute kittens – but she is entirely selflessly committed to it, above any thoughts of herself. To simplify matters, we will further assume her utility is linear in cash (though cash for her is simply a tool to accomplish her selfless goals).

Before Sleeping Beauty is put to sleep the first time, she will follow the following reasoning:

"In the tails world, future copies of myself will be offered the same deal twice. Any profit they make will be dedicated to hugging orphans/drowning kittens, so from my perspective, profits (and losses) will be doubled in the tails world. If my future copies will buy the coupon for $\pounds x$, there would be an expected $\pounds 0.5(2 \times (-x+1) + 1 \times (-x+0)) = \pounds (1-3/2x)$ going towards my goal. Hence I would want my copies to buy whenever x < 2/3."

Then by the temporal consistency axiom, this is indeed what her future copies will do. Note that Sleeping Beauty is here showing a behaviour similar to the SIA probabilities – she is betting on 2:1 odds that she is in the world with two copies.

3.1.2 Selfless Sleeping Beauty without the initial Beauty

In some variants of the Sleeping Beauty problem (such as the sailor's child and the incubator), there is no initial Sleeping Beauty to make decisions for her future copies. Thus consistency is not enough to resolve the decision problem, even for a selfless Sleeping Beauty. To do so, we will need to make use of our second axiom:

Axiom 2 (Outside agent). If there exists a collection of identical agents (which may be the same agent at different times) with same preferences, then another copy of them with the same information would never give up anything of value to make them change their conditional decisions.

With this axiom, the situation reduces to the above selfless Sleeping Beauty, by simply adding in the initial Sleeping Beauty again as 'another copy'.

Some might feel that that axiom is too strong, that invariance under the creation or destruction of extra copies is something that cannot be simply assumed in anthropic reasoning. An equivalent axiom could be:

Axiom 2 (Total agent). If there exists a collection of identical agents (which may be the same agent at different times) with same preferences, then they will make their decisions as if they were a single agent simultaneously controlling all their (linked) conditional decisions.

This axiom is equivalent to the other, with the total agent taking the role of the outside agent. Since all the agents are identical, going through exactly the same reasoning process to reach the same decision, the total agent formulation may be more intuitive. They are, from a certain perspective, literally the same agent. This is the decision that the agents would reach if they could all coordinate with each other, if there were a way of doing this without them figuring out how many of them there were.

3.2 Altruistic total utilitarian Sleeping Beauty

Selfless agents (who follow non-personal preferences that don't include any other agents' personal preferences) can be distinguished from altruistic agents (who follow the aggregate of theirs and other agents' personal preferences). The selfless agent only cares about the monetary aspect of a bet in as much as the money can be used to further their pet outcome; the altruist cares about the monetary aspect of a bet in as much as the money provides personal utility to each winner.

However, an altruistic total utilitarian will have the same preferences over the possible outcomes in a Sleeping Beauty situation: the outcomes in the tails world is doubled, as any gain/loss happens twice, and the altruist adds up the effect of each gain/loss. Hence the altruistic total utilitarian Sleeping Beauty will make the same decisions as the selfless one.

3.2.1 Copy-altruistic total utilitarian Sleeping Beauty

The above argument does not require that Sleeping Beauty be entirely altruistic, only that she be altruistic towards all her copies. Thus she may have selfish personal preferences ("I prefer to have this chocolate bar, rather than letting Snow White get it"), as long as these are not towards her copies ("I'm indifferent as to whether I or Sleeping Beauty II gets the chocolate bar"). And then she will make the same decision in this problem as if she was entirely altruistic.

3.3 SSA-like situations

3.3.1 Altruistic average utilitarian Sleeping Beauty

In Section 2.3, we mentioned how different way of combining long run results (total correct versus average correct) led to SSA or SIA being considered correct. Similarly, average and total utilitarians will behave differently in the Sleeping Beauty problem.

In the incubator variant, consider the reasoning of an Outside/Total agent (as in axiom 2):

"If the various Sleeping Beauties decide to pay $\pounds x$ for the coupon, they will make $-\pounds x$ in the heads world. In the tails world, they will each make $\pounds(1-x)$ each, so an average of $\pounds(1-x)$. This give me an expected utility of $\pounds 0.5(-x+(1-x))=\pounds(0.5-x)$, so I would want them to buy the coupon for any price less than $\pounds 0.5$."

And this will then be the behaviour the agents will follow. Thus they would be behaving as if they were following SSA odds, and putting equal probability on the heads versus tails world.

For a version of this that makes senses for the classical Sleeping Beauty problem, one could imagine that she to be awakened a week after the experiment. Further imagine she would take her winnings and losses during the experiment in the form of chocolate, consumed immediately. Then because of the amnesia drug, she would only remember one instance of this in the tails world. Hence if she valued memory of pleasure, she would want to be average utilitarian towards the pleasures of her different versions, and would follow SSA odds.

3.3.2 Reference classes and copy-altruistic agents

Standard SSA has a problem with reference classes. For instance, the larger the reference class becomes, the more the results of SSA in small situations become similar to SIA. The above setup mimics the effect: if there is a very large population of outsider individuals that Sleeping Beauty is altruistic towards, then the gains to two extra copies will tend to add, rather than average: if Ω is large, then $2x/(2+\Omega)$ (averaged gain to two created agents each gaining x) is approximately twice $x/(1+\Omega)$ (averaged gain to one created agent gaining x), so she will behave more closely to SIA odds.

This issue is not present for *copy-altruistic* average utilitarian Sleeping Beauties, as she doesn't care about any outsiders.

3.3.3 Selfish Sleeping Beauty

In all of the above example, the goals of one Sleeping Beauty were always in accordance with the goals of her copies or the past and future versions of herself. But what happens when this fails? What happens when the different versions are entirely selfish towards each other? Very easy to understand in the incubator variant (the different created copies feel no mutual loyalty), it can also be understood in the standard Sleeping Beauty problem if she is a hedonist with a high discount rates.

Since the different copies do have different goals, the consistency axioms no longer apply. It seems that we cannot decide what the correct decision is in this case. There is, however, a tantalising similarity between this case and the altruistic average utilitarian Sleeping Beauty. The setups (including probabilities) are the same. By 'setup' we mean the different worlds, their probabilities, the number of agents in each world, and the decisions faced by these agents. Similarly, the possible 'linked' decisions are the same. See Section 4.2 for a proper definition of linked decisions; here it just means that all copies will have to make the same decision, so there is one universal 'buy coupon' or 'reject coupon'. And, given this linking, the utilities derived by the agents is the same for either outcome in the two cases.

To see this, consider the selfish situation. Each Sleeping Beauty will make a single decision, whether to buy the coupon at the price offered. Not buying the coupon nets her £0 in all worlds. Buying the coupon at price £x nets her $-\pounds x$ in the heads world, and £(1-x) in the tails world. The linking is present but has no impact on these selfish agents: they don't care what the other copies decide.

This is exactly the same for the altruistic average utilitarian Sleeping Beauties. In the heads world, buying the coupon at price $\pounds x$ nets her $-\pounds x$ worth of utility. In the tails world, it would net the current copy $\pounds(1-x)$ worth of individual utility. Since the copies are identical (linked decision), this would happen twice in the tails world, but since she only cares about the average, this grants both copies only $\pounds(1-x)$ worth of utility in total. The linking is present, and has an impact, but that impact is divided by the average utilitarianism of the copies.

Thus the two situations have the same setup, the same possible linked decisions and the same utility outcomes for each possible linked decision. It would seem there is nothing relevant to decision theory that distinguishes these two cases. This gives us the last axiom:

Axiom 3 (Isomorphic decisions). If two situations have the same setup, the same possible linked decisions and the same utility outcomes for each possible linked decision, and all agents are aware of these facts, then agents should make the same decisions in both situations.

This axiom immediately solves the selfish Sleeping Beauty problem, implying that agents there must behave as they do in the Altruistic average utilitarian Sleeping Beauty problem, namely paying up to $\pounds 0.50$ for the coupon. In this way, the selfish agents also behave as if they were following SSA probabilities, and believed that heads and tails were equally likely.

3.4 Summary of results

We have broadly four categories of agents, and they follow two different types of decisions (SIA-like and SSA-like). In the Sleeping Beauty problem (and in more general problems), the categories decompose as:

- 1. Selfless agents who will follow SIA-type odds.
- 2. (Copy-)Altruistic total utilitarians who will follow SIA-type odds.
- 3. (Copy-)Altruistic average utilitarians who will follow SSA-type odds.
- 4. Selfish agents who will follow SSA-type odds.

For the standard Sleeping Beauty problem, the first three decisions derived from consistency. The same result can be established for the incubator variants using the Outside/Total agent axioms. The selfish result, however, needs to make use of the Isomorphic decisions axiom.

4 Anthropic decision theory

In this section, we will see a decision procedure that is compatible with all the axioms above, and applies to more general anthropic and linked decision making problems.

4.1 Probabilities and decisions

SSA and SIA both give you methods for determining the probabilities in the Sleeping Beauty problem, but they don't tell you what decisions to make. Even apart from the selfless/selfish distinction above in Section 3, one has to worry about the total impact of the decision, when several agents are making the same decision. For example, consider a setup where all copies of Sleeping Beauty have to vote unanimously on whether to implement a particular policy. Since they are identical, they will vote the same way. One could use a principle of divided responsibility: when there are two copies, each one is responsible for a half of the ultimate decision (this could be termed the causal decision theory [Lew81] position). Or one could use a principle of total responsibility: each copy is responsible for the totality of the decision since if they voted otherwise, the decision would fail (this could be termed the evidential decision theory [GH81] position).

Note that SIA with divided responsibility would reach the same decisions as SSA with total responsibility – the doubling of odds in the heads world exactly compensated by the halving of responsibility. In this setting, the probabilities themselves become meaningless: the first agent may claim 'SIA!' while the second claim 'SSA!', but they will agree on every decision, making those simply slogans. If someone shouts 'Vive la revolution!' while the other shouts 'Vive le Roi!', then we would expect them to sometimes reach different decisions; if not, their claims are empty. They may believe that the 'true' probability is 1/3 or 1/2, but since this never shows up in a difference in action, it is hard to see what these beliefs actually mean.

For this reason this paper takes the actual decisions as the fundamental objects of interest, with the probabilities being merely tools, and having no separate objective meaning.

4.2 Linked decisions and Anthropic Decision Theory

Identical copies of Sleeping Beauty will make the same decision when faced with same situations (technically true until quantum and chaotic effects cause a divergence between them, but most decision processes will not be sensitive to random noise like this). Similarly, Sleeping Beauty and the random man on the street will make the same decision when confronted with a twenty pound note: they will pick it up. However, while we could say that the first situation is linked, the second is coincidental: were Sleeping Beauty to refrain from picking up the note, the man on the street would not so refrain, while her copy would.

The above statement brings up subtle issues of causality and counterfactuals, a area of dispute between causal [Lew81] and evidential decision theory [GH81]. To sidestep the issue entirely, let us recast the problem in programming terms, seeing the agent's decision process as a deterministic algorithm. If agent α is an agent that follows an automated decision algorithm A, then if A knows its

own source code (by quining [BM72] for instance), it might have a line saying something like:

Module M: If B is another algorithm, belonging to agent β , identical with A ('yourself'), assume A and B will have identical outputs on identical inputs, and base your decision on this.

This could lead, for example, to α and β cooperating in a symmetric Prisoner's Dilemma. And there is no problem with A 'believing' the above assumption, as it is entirely true: identical deterministic algorithms on the same input do produce the same outputs. With this in mind, we give an informal definition of a linked decision (either conditional or standard) as:

Definition 4.1 (Linked decisions). Agent α 's decisions are linked with agent β 's, if both can prove they will both make the same decision, even after taking into account the fact they know they are linked.

An example of agents that are *not* linked would be two agents α and β , running identical algorithms A and B on identical data, except that A has module M while B doesn't. Then A's module might correctly deduce that they will output the same decision, but only if A disregards the difference between them, i.e. module M itself. So A can 'know' they will output the same decision, but if it acts on that knowledge, it makes it incorrect. If A and B both had module M, then they could both act on the knowledge and it would remain correct. Generally, identical causal decision theorists [Lew81] will not be linked, while identical evidential decision theorists [GH81] will be.

The 'objective probabilities' of a world are the prior probabilities given (such as the 50-50 odds in the Sleeping Beauty problem), updated, as usual, by each agent applying Bayes theorem to the likelihood of them making the observations they have made. Given this definition, anthropic decision theory (ADT) can be simply stated as:

Anthropic Decision Theory (ADT) An agent should first find all the decisions linked with their own. Then they should maximise expected utility, acting as if they simultaneously controlled the outcomes of all linked decisions, and using the objective (non-anthropic) probabilities of the various worlds.

ADT is similar to SSA in that it makes use of reference classes. However, SSA needs to have the reference class information established separately before it can calculate probabilities, and different reference classes give very different results [Bos02]. In contrast, the reference class for ADT is part of the definition. It is not the class of identical or similar agents; instead, it is the class of linked decisions which (by definition) is the class of decisions that the agent can prove are linked. Hence the whole procedure is perfectly deterministic, and known for a given agent.

Now, module M is enough when the agents/algorithms are strictly identical, but fails when they differ slightly. For instance, imagine a variant of the selfless Sleeping Beauty problem where the two agents aren't exactly identical in tails world. There are two separate agents, α and α' , where they are both selflessly devoted to the same cause, but α' has some personal displeasure in engaging in trade – if she buys the coupon, she will suffer a single $-\pounds 0.05$ penalty for

doing so. Agent α is indifferent to the other agent suffering this penalty. In the tails world, there will be one of each type, while in the heads world, the single agent is equally likely to be of either (trade-neutral or trade-disliking) type. The objective probabilities are the same as before: any given Sleeping Beauty has 50% probability of being agent α in the tails world (because she is one of two) or the heads world (because each is equally probable). Thus updating on realising she is agent α or α' does not change her priors.

Then if the coupon is priced at £0.60, something quite interesting happens. If the agents do not believe they are linked, they will refuse the offer: their expected returns are 0.5(-0.6 + (1 - 0.6)) = -0.1 and -0.1 - 0.05 = -0.15 respectively. If however they believe their decisions are linked, they will calculate the expected return from buying the coupon as 0.5(-0.60 + 2(1 - 0.60)) = 0.1 and 0.1 - 0.05 = 0.05 respectively. Since these are positive, they will buy the coupon: meaning their assumption that they were linked was actually correct!

If the coupon is priced at £0.66, things change. If the two agents assume their decisions are linked, then they will calculate their expected return from buying the coupon as 0.5(-0.66+2(1-0.66))=0.01 and 0.01-0.05=-0.04 respectively. The first agent will buy, and the second will not – they were wrong to assume they were linked

A more general module that gives this kind of behaviour is:

Module N: Let H be the hypothesis that the decision of A ('myself') and those of algorithm B are linked. I will then compute what each of us will decide if we were both to accept H. If our ultimate decisions are indeed the same, and if the other agent also has a module N, then I will accept H.

The module N gives correct behaviour. It only triggers if the agents can prove that accepting H will ensure that H is true – and then N makes them accept H, hence making H true.

For the coupon priced at £0.60, it will correctly tell them they are linked, and they will both buy it. For the coupon priced at £0.66, it will not trigger, and both will refuse to buy it – though they reach the same decision, they will not have done so if they had assumed they were linked. For a coupon priced above £2/3, module N will correctly tell them are linked again, and they will both refuse to buy it.

4.2.1 Precommitments

Another important problem in decision theory is Parfit's hitchhiker [Par84] and similar situations. Here, a starving hitchhiker meets a driver who could drive him to the nearest town. The hitchhiker promises to pay the driver a reward upon arrival. Unfortunately, both the driver and hitchhiker are selfish and perfectly rational, and realise the hitchhiker will have no incentive to pay once he is driven to the town. So the driver motors away, leaving him to die. There is a lot of literature on this subject, which will not be summarised here; the main point being that it is advantageous for an agent to be able to make binding public precommitments. These can be seen as an extension of temporal consistency: once an agent has decided what to do, it commits its future copies to the same behaviour, to avoid the possibility of predictably losing utility to any agent that

can exploit its inconsistency. To allow ADT agents to succeed in situations like this, we define 'decisions' to include precommitments.

4.3 Axioms

Let's look at the various axioms laid down in Section 3, and show that ADT (as defined in the previous Section 4.2) obeys them all:

- 1. Consistency: If two copies of an agent have the same knowledge and preferences, then the one version will never give up anything of value in order to change the conditional decision of the other version.
- 2. Total agent: If there exists a collection of identical agents (which may be the same agent at different times) with same preferences, then they will make their decisions as if they were a single agent simultaneously controlling all their (linked) conditional decisions.
- 3. Isomorphic decisions: If two situations have the same setup, the same possible linked decisions and the same utility outcomes for each possible linked decision, and all agents are aware of these facts, then agents should make the same decisions in both situations.

To show that ADT obeys the consistency axiom, merely note that the decision of identical ADT agents with same preferences facing the same (conditional) decision will be the same, that both agents know this, and hence their decisions must be linked. ADT makes each agent act like a total agent, so that axiom must be satisfied. Finally the isomorphic decisions algorithm says, in effect, "if two situations differ only by information that is not relevant to ADT, then ADT will decide the same way in both".

Now since ADT obeys all these axioms, it must give the correct decision as described in the various Sleeping Beauty variants of Section 3.

It would be nice if these axioms were enough to define ADT, but that is not the case. They all require exact equality between the preferences of the agents in various situations, but ADT simply needs the linking of their decisions. As we saw in the previous Section 4.2, not perfectly identical agents can still coordinate. The important assumption underlying module N was that if the agents assumed they were linked, and then proceeded to maximise their utility on that assumption, they would indeed end up linked. This would be the case for agents that all agreed which was the best option among all the possible (linked) decisions. To cover these cases, we can extend the Isomorphic decisions axiom to:

Axiom 3 (Isomorphic best decision). If two situations have the same setup, the same possible linked decisions and the same best outcome for all agents among the possible linked decisions, and all agents are aware of these facts, then agents should make the same decisions in both situations.

These three axioms then establish ADT: the ADT solution is the one that all agents agree is the best linked decision available. Then we can transform this into situation where identical agents have identical utilities, but without changing the best linked decision, and the first two axioms are then enough to show the agents will indeed choose the ADT solution.

4.3.1 Probabilistic linking

Now, in the real world, rather than requiring a mathematical proof of linking, it would probably be more useful to talk of the probability of two decisions being linked. One might for instance be willing to cooperate with broadly similar people in a Prisoner's Dilemma. ADT should be adaptable to handle uncertainty over linkings of decisions the same way it handles all uncertainty – for instance it can model the selfish Sleeping Beauty problem as a 50-50 mixture of selfless Sleeping Beauty and Sleeping Anti-Beauty (see Section 5.4). That approach is beyond the scope of the current paper, however. Instead, the next section will delve into the applications of ADT to various common anthropic problems.

5 Applying Anthropic Decision Theory

5.1 The Presumptuous Philosopher

The Presumptuous Philosopher was introduced by Nick Bostrom [Bos02] as a way of pointing out the absurdities in SIA. In the setup, the universe either has a trillion observers, or a trillion trillion observers, and physics is indifferent as to which one is correct. Some physicists are preparing to do an experiment to determine the correct universe, until a presumptuous philosopher runs up to them, claiming that his SIA probability makes the larger one nearly certainly the correct one. In fact, he will accept bets at a trillion trillion to one odds that he is in the larger universe, repeatedly defying even strong experimental evidence with his SIA probability correction.

What does ADT have to say about this problem? Implicitly, when the problem is discussed, the philosopher is understood to be selfish towards any putative other copies of himself (similarly, Sleeping Beauty is often implicitly assumed to be selfless, which may explain the divergence of intuitions that people have on the two problems). Are there necessarily other similar copies? Well, in order to use SIA, the philosopher must believe that there is nothing blocking the creation of presumptuous philosophers in the larger universe; for if there was, the odds would shift away from the larger universe (in the extreme case when only one presumptuous philosopher is allowed in any universe, SIA finds them equiprobable). So the expected number of presumptuous philosophers in the larger universe is a trillion trillion times greater than the expected number in the small universe.

Now if the philosopher is indeed selfish towards his copies, then ADT reduces to SSA-type behaviour: the philosopher will correctly deduce that in the larger universe, the other trillion trillion philosophers or so will have their decision linked with his. However, he doesn't care about them: any benefit that accrue to them are not of his concern, and so if he correctly guesses that he resides in the larger universe, he will accrue a single benefit. Hence there will be no paradox: he will bet at 1:1 odds of residing in either the larger or the smaller universe.

If the philosopher is an altruistic total utilitarian, on the other hand, he will accept bets at odds of a trillion trillion to one of residing in the larger universe. But this no longer counter-intuitive (or at least, no more counter-intuitive than maximising expect utility with very small probabilities): the other

presumptuous philosophers will make the same bet, so in the larger universe, their total profit and loss will be multiplied by a trillion trillion. And since the philosopher is altruistic, the impact on his own utility is multiplied by a trillion trillion in the large universe, making his bets rational.

At this point, it might be fair to ask what would happen if some of the philosophers were altruistic while others were selfish. How would the two interact; would the selfless philosopher incorrectly believe his own decision was somehow 'benefiting' the selfish ones? Not at all. The decisions of the selfless and selfish philosophers are not linked: they both use ADT, but because they have very different utilities, they cannot prove that their decisions are linked. Which is fortunate, because they aren't.

5.2 Adam and Eve

The presumptuous philosopher thought experiment was designed to show up problems with SIA reasoning. Another thought experiment by the same author [Bos01] was designed to show problems with SSA reasoning.

In this thought experiment, Adam and Eve, the first two humans, have the opportunity to breed or not to breed. If they breed, they will produce trillions of trillions of descendants (see Figure 3). Under SSA odds, the probability of being Adam or Eve in a universe with trillions of trillion humans is tiny, while the corresponding probability in a universe with just two observers is one. Therefore Adam and Eve should conclude that whatever they do, it is tremendously unlikely that they will succeed in breeding. Nick Bostrom them proceeds to draw many amusing consequences from this 'future affecting' paradox, such as the couple forming the firm intention of having sex (and hence risking pregnancy) if an edible animal doesn't wander through the entrance of their cave in the next few minutes. There seems to be something very wrong with the reasoning, but it is a quite natural consequence of SSA.

What does ADT have to say on the problem? This depends on whether the decisions of Adam and Eve and their (potential) descendants are linked. There is prima facie no reason for this to be the case; in any case, how could potential future descendants make the decision as to whether Adam and Eve should breed? One way of imagining this is if each human is born fully rational, aware of the possible world they is born into, but in ignorance as to their identity and position in that world. Call this the ignorant rational baby stage. They can then make a decision as to what they would do, conditional upon discovering their identity. They may then decide or not to stick to their decision. Hence we can distinguish several scenarios:

- These agents have no 'ignorant rational baby stage', and do not take it into account.
- 2. These agents have an 'ignorant rational baby stage', but do not allow precommitments.
- 3. These agents have an 'ignorant rational baby stage', but do allow precommitments.
- 4. These agents have no 'ignorant rational baby stage', but require themselves to follow the hypothetical precommitments they would have made had they had such a stage.

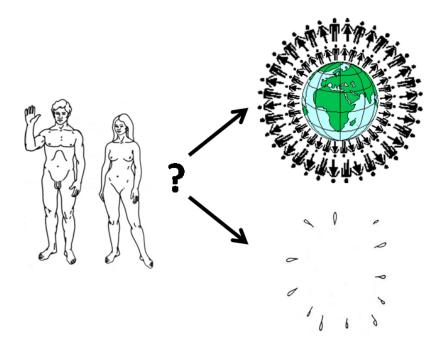


Figure 3: The Adam and Eve problem: what are the odds of descendants?

To make this into a decision problem, assume all agents are selfish, and know they will be confronted by a choice between a coupon C_1 that pays out £1 to Adam and Eve if they have no descendants and C_2 that pays out £1 to Adam and Eve if they have (trillions of trillions of) descendants. Assume Adam and Eve will have sex exactly once, and the (objective) chance of them having a successful pregnancy is 50%. Now each agent must decide on the relative values of the two coupons.

Obviously in situation 1, the decisions of Adam and Eve and their descendants are not linked, and ADT means that Adam and Eve will value C_1 compared with C_2 according to their objective estimates of having descendants, i.e. they will value then equally. There is no SSA-like paradox here. Their eventual descendants will also value the coupons as equally worthless, as they will never derive any value from them.

Now assume there is an 'ignorant rational baby stage'. During this stage, the decisions of all agents are linked, as they have the same information, the same (selfish) preferences, and they all know this. Each rational baby can then reason:

"If I am in the world with no descendants, then I am Adam or Eve, and the C_1 coupon is worth £1 to me (and C_2 is worthless). If, on the other hand, I am in the world with trillions of trillions of descendants, there is only two chances in $2 + 10^{24}$ of me being Adam or Eve, so I value the C_2 coupon at £2/(2+10²⁴) (and C_1 is worthless). These worlds are equiprobable. So I would value C_1 as being $1 + 0.5 \cdot 10^{24}$ times more valuable than C_2 ."

So the rational babies in situations 2 and 3 would take C_1 as much more valuable than C_2 , if the deal was proposed to them immediately. Since there are no precommitments in situation 2, once the rational babies discover who they actually are, they would revert to situation 1 and take them as equally valuable. If precommitments are allowed, then the rational babies would further reason:

"The strategy 'upon discovering I am Adam or Eve, take C_1 ' nets me an expected £1/2, while the strategy 'upon discovering I am Adam or Eve, take C_2 ' nets me an expected £1/(2+10²⁴), because it is very unlikely that I would actually discover that I am Adam and Eve. Hence the strategy 'upon discovering I am Adam or Eve, accept trades between C_1 and C_2 at $2:(2+10^{24})$ ratios' is neutral in expected utility, and so I will now precommit to accepting any trade at any ratios slightly better than this."

So in situation 3, even after discovering that they are Adam or Eve, they will continue to accept deals at ratios that would seem to imply that they believe in the SSA odds, i.e. that they are nearly certain to not have descendants. But it is a lot less of a paradox now; it simply arises because there was a time when they were uncertain as to what their actual position was, and the effects of this uncertainty were 'locked in' by their precommitment.

Situation 4 is very interesting. By construction, it reproduces the seemingly paradoxical behaviour, but here there was never a rational baby stage where the behaviour made sense. Why would any agent follow such a behaviour? Well, mainly because it allows trade between agents who might not otherwise be able to agree on a 'fair' distribution of goods. If all agents agree to the division that they would have wanted had they been ignorant of their identity (a 'rawlsian veil of ignorance' situation [Raw71]), then they can trade between each other without threats or bargaining in these simple cases.

If the agents are simply altruistic average utilitarians, then Adam and Eve would accept SSA odds in all four situations; things that benefit them specifically are weighted more highly in a universe with few people. So the varying behaviour above is a feature of selfishness, not of SSA-type behaviour, and it seems precommitments become very important in the selfish case. This certainly merits further study. Temporal consistency is a virtue, but does it extend to situations like this, where the agent makes binding decisions before knowing their identity? Certainly, if the agent had to make a decision immediately, and if there were anyone around to profit from temporal inconsistencies, the agent should remain consistent, which means following precommitments. However this is not entirely obvious that it should still be the case if there were no-one to exploit the inconsistency.

This is not, incidentally, a problem only of ADT – SIA has similar problem under the creation of 'irrelevant' selfless agents who don't yet know who they are, while SSA has problems under the creation of agents who don't yet know what reference class they are in.

5.3 The Doomsday argument

Closely related to the Adam and Eve paradox, though discovered first, is the Doomsday argument [CM83]. Based on SSA's preference for 'smaller' universes, it implies that there is a high probability of the human race becoming extinct

within a few generations – at the very least, a much higher probability than objective factors would imply.

Under SIA, the argument goes away ([Die07]), so it would seem that ADT must behave oddly: depending on the selfishness and selflessness of the agents, they would give different probabilities to the extinction of the human race. This is not the case, however. Recall that under ADT, decisions matter, not probabilities. And agents that are selfish or average utilitarians would not be directly concerned with the extinction of the human race, so would not act in bets to prevent this.

This is not a specious point – there are ways of constructing the doomsday argument in ADT, but they all rely on odd agents who are selfish with respect to their own generation but selfless with respect to the future survival of the human race. This lacks the potency of the original formulation: having somewhat odd agents behaving in a somewhat odd fashion is not very surprising. For the moment, until a better version is produced, we should simply say that the doomsday argument is not present in ADT.

5.4 Sleeping Anti-Beauty

Sleeping Anti-Beauty is a thought experiment similar to the Sleeping Beauty experiment, but with one important caveat: the two copies in the tails world hate each other. This works best if the two copies are duplicates, rather than the same person at different times. One could imagine, for instance, that a week after the experiment, all copies of Sleeping Beauty are awakened and made to fight to the death – maybe they live in a civilization that prohibits more than one copy of an agent from existing. The single copy in the heads world will be left unmolested, as she has nobody to fight.

That means that the Sleeping Beauties in the tail world are in a zero sum game; any gain for one is a loss for the other, and vice-versa. Actually, we need a few more assumptions for this to be true: the Sleeping Beauties have to be entirely selfish apart from their rivalry, and they do not get offered any goods for immediate consumption. Given all these assumptions, what does ADT have to say about their decisions?

As in Section 3, all existent copies of Sleeping Beauty are offered a coupon that pays out $\pounds 1$ if the coin fell tails, and asked how much she would be willing to give for that. ADT reasoning proceeds for each agent as follows:

"In the heads world, if I pay $\pounds x$, all that happens is that I lose $\pounds x$. In the tails, world, if I pay $\pounds x$, I gain $\pounds (1-x)$. However my other hated copy will make the same decision, and also gain $\pounds (1-x)$. This causes me the same amount of loss as the gain of $\pounds (1-x)$ does, so I gain nothing at all in the tails world, whatever $\pounds x$ is. So I value the coupon precisely at zero: I would not pay any amount to get it."

In this, and other similar decisions, Sleeping Beauty would act as if she had an absolute certainty of being in the heads world, offering infinity to one odds of this being the case, as she cannot realise any gains – or losses! – in the tails world.

It should be noted that selfish Sleeping Beauty can be correctly modelled by seeing it as a 50-50 mix of selfless Sleeping Beauty and Sleeping Anti-Beauty.

6 Conclusion

Anthropic decision theory is a new way of dealing with anthropic (self locating belief) problems, focused exclusively on finding the correct decision to make, rather than the correct probabilities to assign. It deals successfully with many classical anthropic puzzles, and resolves various paradoxes such as the Presumptuous Philosopher and the Adam and Eve problem, in ways that don't clash with intuition.

In many situations, ADT is moreover a consequence of simple axioms of consistency, total agents and isomorphism. In these problems, standard intuitions were all over the place, partially because people were not clearly distinguishing between probabilities and decisions. This lead to the problem being underspecified: since people were looking for probabilities, facts such as whether the agents were selfless or selfish, total or average utilitarians, were not stated and assumed not to matter – though they were often used in the implicit intuitive defence of various positions. But in making a decision, they matter tremendously: the difference between SIA-like and SSA-like behaviour is down to how copies value each other's goals.

References

- [AHP96] Robert J. Aumann, Sergiu Hart, and Motty Perry. Absent-minded driver. Proceedings of the 6th conference on Theoretical aspects of rationality and knowledge, 1996.
- [BM72] Paul Bratley and Jean Millo. Computer recreations: Self-reproducing automata. Software Practice and Experience, 2:397–400, 1972.
- [Bos01] Nick Bostrom. The doomsday argument, adam & eve, un++, and quantum joe. *Synthese*, 127(3), 2001.
- [Bos02] Nick Bostrom. Anthropic Bias: Observation Selection Effects in Science and Philosophy. Routledge: New York, 2002.
- [CM83] Brandon Carter and W. H. McCrea. The anthropic principle and its implications for biological evolution. *Philosophical Transactions of the* Royal Society of London, 310(1512):347–363, 1983.
- [Die07] Dennis Dieks. Reasoning about the future: Doom and beauty. Synthese, 156(3), 2007.
- [Elg00] A. Elga. Self-locating belief and the sleeping beauty problem. Analysis, $60:143-147,\ 2000.$
- [GH81] A. Gibbard and W. L. Harper. Counterfactuals and two kinds of expected utility. *Ifs: Conditionals, Beliefs, Decision, Chance, and Time*, pages 153–190, 1981.
- [Lew81] D. Lewis. Causal decision theory. Australasian Journal of Philosophy, 59(1):5–30, 1981.
- [Nea06] R. M. Neal. Puzzles of anthropic reasoning resolved using full nonindexical conditioning. 2006. Arxiv preprint math/0608592.

- $[Par84] \quad \text{Derek Parfit. } \textit{Reasons and Persons}. \ \text{Clarendon Press, Oxford, 1984}.$
- [Raw71] John Rawls. A Theory of Justice. Cambridge, Massachusetts: Belknap Press, 1971.
- $[vNM] \quad \mbox{ John von Neumann and Oskar Morgenstern. } \mbox{ Theory of Games and } \\ Economic \mbox{ Behavior}.$