DARK SPRING - A SIMPLE INTERPRETATION OF THE SUSSKIND- HOROWITZ-POLCHINSKY CORRESPONDENCE BETWEEN SCHWARZSCHILD BLACK HOLE AND STRINGS

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Abstract

In this work we suggest a simplified interpretation of Susskind-Horowitz-Polchinski correspondence between Schwarzschild black hole and strings. Firstly, similarly to naive, classical mechanical Laplace determination of the Schwarzschild radius, we suggest a simple, classical mechanical equation. It determines amplitude of such sufficiently strong classical elastic force that forbids escape of a Planck mass particle moving by speed of light from end of corresponding classical elastic spring, simply called dark spring. Also, by use of a formal identity between given elastic force and Schwarzschild gravitational "force", we introduce phenomenologically a simple quantization rule. It states that circumference (corresponding to elastic force amplitude equivalent formally to Schwarzschild radius) holds natural number of corresponding reduced Compton's wave length. (It is deeply analogous to Bohr's quantization postulate in Bohr's atomic theory interpreted by de Broglie relation.) Then, very simply (by simple algebraic equations only) and surprisingly, we obtain such dark spring characteristics corresponding to basic thermodynamical characteristics (Bekenstein-Hawking entropy and Hawking temperature) for corresponding Schwarzschild black hole. Finally, simple comparison between obtained dark spring characteristics and Susskind-Horowitz-Polchinski correspondence, a simple correspondence between strings and dark spring, i.e. classical linear harmonic oscillator follows. Square root of the elasticity coefficient of the dark spring corresponds to quotient of one string coupling g and Newtonian gravitational constant, or, classical elasticity coefficient of the dark spring corresponds to reciprocal (inverse) value of the square root of a string state excitation level N.

1 Introduction

Within physics two different types of the correct approximation of an exact theory are possible. First one represents homogeneous approximation within which all approximate concepts have equivalent limitation of accuracy in respect to exact theory. For this reason, within given, self-consistently closed, approximation, all approximation concepts can be connected simply or "continuously". An example of such approximation type represents the classical Newtonian dynamics and gravitation as the approximation of the Einstein general theory of relativity. Also, other example of such approximation type represents small perturbation theory of the quantum mechanics as the exact theory.

Second one approximation type represents heterogeneous or hybrid approximation within which different approximation concepts can have different accuracy limitation in respect to exact theory. Some concepts can be completely exact while some other concepts can be completely imprecise (even impossible!). For this reason within given hybrid approximation theory all its characteristic concepts cannot be connected simply, "continuously", but only "discretely". An example of such approximation type represents Bohr atomic theory, i.e. nave quantum theory in respect to quantum mechanics as the exact theory. As it is well-known within Bohr atomic theory electron energies are determined completely exactly, i.e. identically to quantum-mechanically predictions, while electron trajectories, used within Bohr atomic theory, do not exist at all within quantum mechanics. Other example of the hybrid approximation represents La Place classical mechanical determination of the dark star radius. As it is well-known [1], [2] it yields value exactly identical to Schwarzschild, general relativistic determination of the black hole radius, even if Laplace used classical expression for light particle kinetic energy completely inconsistent with general theory of relativity and even with wave theory of light.

In this work, according to some our previous results [3], [4], we shall suggest a simplified interpretation of Susskind-Horowitz-Polchinski correspondence between Schwarzschild black hole and strings [5], [6]. Firstly, similarly to nave, classical mechanical Laplace determination of the Schwarzschild radius [1], [2], we shall suggest a simple, classical mechanical equation. It determines amplitude of such sufficiently strong classical elastic force that forbids escape of a Planck mass particle moving by speed of light from end of corresponding classical elastic spring, simply called dark spring. Also, by use of a formal identity between given elastic force and Schwarzschild gravitational "force" and according to some our previous results [3], [4], we shall introduce phenomenologically a simple quantization rule. It states that circumference (corresponding to elastic force amplitude equivalent formally to Schwarzschild radius) holds natural number of corresponding reduced Compton's wave length. (It is deeply analogous to Bohr's quantization postulate in Bohr's atomic theory interpreted by de Broglie relation.) Then, very simply (by simple algebraic equations only) and surprisingly, we obtain such dark spring characteristics corresponding to basic thermodynamical characteristics (Bekenstein-Hawking entropy and Hawking temperature) for corresponding Schwarzschild black hole. Finally, simple comparison between obtained dark spring characteristics and Susskind-Horowitz-Polchinski correspondence, a simple correspondence between strings and dark spring, i.e. classical linear harmonic oscillator follows. Square root of the elasticity coefficient of the dark spring corresponds to quotient of one string coupling g and Newtonian gravitational constant, or, classical elasticity coefficient of the dark spring corresponds to reciprocal (inverse) value of the square root of a string state excitation level N.

As it is well-known [1], [2] Laplace determined by simple, classical mechanical method, the radius R of a dark star, i.e. a star with sufficiently large mass M so that even light cannot escape from the star surface. Surprisingly given radius is identical to the Schwarzschild radius of corresponding Schwarzschild black hole predicted accurately by general theory of relativity.

Formally generalizing given Laplace method, suppose that there is such classical mechanical elastic spring, simply called dark spring that generates attractive, sufficiently strong classical elastic force. It forbids escape of a Planck mass particle $m_P = (\frac{\hbar c}{G})^{\frac{1}{2}}$ (where G represents the Newtonian gravitational constant, $\hbar$ - reduced Planck constant and c - speed of light) moving by speed of light from end point of dark spring. Also, suppose that in this case amplitude of the elastic force is identical to radius R of corresponding Schwarzschild black hole with mass M. It implies equation

$$\frac{m_P c^2}{2} = \frac{kR^2}{2} = \frac{G m_P M}{R} \tag{1}$$

where k represents dark spring elasticity coefficient. It implies

$$R = \frac{2MG}{c^2} = \frac{m_P^{\frac{1}{2}} c}{k^{\frac{1}{2}}} \tag{2}$$

and

$$M = \frac{Rc^2}{2G} = \frac{kR^3}{2Gm_P} = \frac{m_P^{\frac{1}{2}} c^3}{2Gk^{\frac{1}{2}}}. \tag{3}$$

Differentiation of M over k yields

$$dM = -\frac{m_P^{\frac{1}{2}} c^3}{4Gk^{\frac{3}{2}}} dk \tag{4}$$

that implies

$$dE = d(Mc^2) = -\frac{m_P^{\frac{1}{2}} c^5}{4Gk^{\frac{3}{2}}} dk \tag{5}$$

Now we shall shortly repeat formal but very simple calculation (practically by simple algebraic equations only) of the basic black hole thermodynamical characteristics, Bekenstein-Hawking entropy and Hawking temperature, presented in [3], [4]. All this, as it will be shown, can refer on dark spring too.

Suppose that the mass of the black hole M is quantized and that given quantums satisfy the following quantization condition

$$m_n c R = n \frac{\hbar}{2\pi}, \qquad \text{for} \quad n = 1, 2, ... \tag{6}$$

that implies

$$2\pi R = n \frac{\hbar}{m_n c} = n \lambda_{rn} \qquad \text{for} \quad n = 1, 2, ... \tag{7}$$

where $\lambda_{rn} = \frac{\hbar}{m_n c}$ represents reduced Compton wave length corresponding to m_n for $n = 1, 2, .$

Last expression means, in fact, that the circumference of the black hole horizon holds n reduced Compton wavelength of the mass quantums with mass m_n for $n = 1, 2, .$ Obviously, (6), (7) correspond, in some degree, to remarkable Bohr postulate on the electron orbital momentum quantization and de Broglie wave interpretation of this postulate in the atomic physics.

Expression (6) implies

$$m_n = n \frac{\hbar}{2\pi c R} = n m_1 \qquad \text{for} \quad n = 1, 2, ... \tag{8}$$

where, according to (2), (3),

$$m_1 = \frac{\hbar}{2\pi c R} = \frac{\hbar c}{4\pi G M} = \frac{\hbar k^{\frac{1}{2}}}{2\pi m_P^{\frac{1}{2}} c^2} \tag{9}$$

represents the minimal, i.e. ground mass of the mass quantums.

Further, suppose that black hole, i.e. dark spring mass quantums do a statistical ensemble. In other words, suppose that there is a gravitational self-interaction of the black hole, i.e. dark spring which can be described statistically. Suppose that in the thermodynamical equilibrium almost all quantums occupy ground mass state. It implies that black hole, i.e. dark spring mass quantums represent the Bose-Einstein quantum systems, i.e. bosons. Then Bekenstein-Hawking entropy S can be phenomenologically, according to (3), (9), determined by

$$S = k_B \frac{M}{m_1} = k_B \frac{4\pi G M^2}{\hbar c} = k_B \frac{\pi m_P c^5}{G \hbar k} \tag{10}$$

where k_B represents the Boltzmann constant.

Differentiation of S (10) over M or k yields

$$dS = k_B \frac{8\pi G M}{\hbar c} dM = k_B \frac{8\pi G M}{\hbar c^3} dE = -k_B \frac{\pi m_P c^5}{G \hbar k^2} dk. \tag{11}$$

Then, according to the first thermodynamical law $dE = TdS$, (5) and (8) simply imply Hawking black hole, i.e. dark spring temperature

$$T = \frac{\hbar c^3}{k_B 8\pi G M} = \frac{\hbar k^{\frac{1}{2}}}{k_B 4\pi m_P^{\frac{1}{2}}}. \tag{12}$$

Now we shall discuss obtained results. First of all there is a serious question: holds formally introduced dark spring any real (within some approximation) physical sense.

Answer on given question can be obtained by comparison of the presented expressions for black hole characteristics with corresponding expressions in Susskind -Horowitz-Polchinski correspondence between Schwarzschild black hole and strings [5], [6]. It is sufficient to compare our and Susskind -Horowitz-Polchinski mass and entropy.

In the natural units system ($\hbar = c = k_B = 1$) and $m_B = G^{\frac{1}{2}}$ expressions (3), (10) turn out in

$$M = \frac{1}{2G^{\frac{5}{4}} k^{\frac{1}{2}}} \tag{13}$$

$$S = \frac{\pi}{G^{\frac{3}{2}}} \frac{1}{k}. \tag{14}$$

Corresponding Susskind-Horowitz-Polchinski expressions are

$$M = \frac{1}{(2G)^{\frac{1}{2}} N^{\frac{1}{2}}} \tag{15}$$

$$S = 2\pi N^{\frac{1}{2}} \tag{16}$$

where N represents corresponding string state excitation level that determines one string coupling by expression

$$g = \frac{1}{2^{\frac{1}{2}} N^{\frac{1}{4}}}. \tag{17}$$

Equivalence between (13) and (16) as well as between (14) and (17) yields

$$k = \frac{1}{2G^{\frac{3}{2}} N^{\frac{1}{2}}} = \frac{g^2}{G^{\frac{3}{2}}}. \tag{18}$$

In this way it is proved that dark spring elasticity coefficient is unambiguously determined by one of two string coupling. Also, other string coupling α', as it is known [5], [6] is practically identical to R^2. It means that dark spring amplitude R representing square root of other string coupling is unambiguously determined by this other coupling. For this reason it can be concluded that string corresponding to Schwarzschild black hole in Susskind-Horowitz-Polchinski sense can be simplifiedly effectively presented as a classical linear harmonic oscillator, i.e. mentioned dark spring.

In conclusion the following can be shortly repeated and pointed out. In this work, according to some our previous results, we suggest a simplified interpretation of Susskind-Horowitz-Polchinski correspondence between Schwarzschild black hole and strings. Firstly, similarly to nave, classical mechanical Laplace determination of the Schwarzschild radius, we suggest a simple, classical mechanical equation. It determines amplitude of such sufficiently strong classical elastic force that forbids escape of a Planck mass particle moving by speed of light from end of corresponding classical elastic spring, simply called dark spring. Also, by use of a formal identity between given elastic force and Schwarzschild gravitational "force" and according to some our previous results, we shall introduce phenomenologically a simple quantization rule. It states that circumference (corresponding to elastic force amplitude equivalent formally to Schwarzschild radius) holds natural number of corresponding reduced Compton's wave length. (It is deeply analogous to Bohr's quantization postulate in Bohr's atomic theory interpreted by de Broglie relation.) Then, very simply (by simple algebraic equations only) and surprisingly, we obtain such dark spring characteristics corresponding to basic thermodynamical characteristics (Bekenstein-Hawking entropy and Hawking temperature) for corresponding Schwarzschild black hole. Finally, simple comparison between obtained dark spring characteristics and Susskind-Horowitz-Polchinski correspondence, a simple correspondence between strings and dark spring, i.e. classical linear harmonic oscillator follows. Square root of the elasticity coefficient of the dark spring corresponds to quotient of one string coupling g and Newtonian gravitational constant, or, classical elasticity coefficient of the dark spring corresponds to reciprocal (inverse) value of the square root of a string state excitation level N.

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