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Particle energy and Hawking temperature

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ABSTRACT

Some authors have recently found that the tunneling approach gives a different Hawking temperature for a Schwarzschild black hole in a different coordinate system. In this Letter, we find that to work out the Hawking temperature in a different coordinate system by the tunneling approach, we must use the correct definition of the energy of the radiating particles. By using a new definition of the particle energy, we obtain the correct Hawking temperature for a Schwarzschild black hole in two dynamic coordinate systems, the Kruskal–Szekers and dynamic Lemaitre coordinate systems.

1. Introduction

In recent years, a semi-classical method for controlling Hawking radiation as a tunneling effect has been developed and has garnered much interest [1–32]. Angheben et al. [1] and Padmanabhan et al. [9–11] used the complex path analysis that was developed by Mann et al. [12,13]. In this method, the semiclassical propagator $K(r_2,t_2;r_1,t_1)$ in $(1+1)$-dimensional Schwarzschild spacetime is $K(r_2,t_2;r_1,t_1) = N\exp(\frac{i}{\hbar}I(r_2,t_2;r_1,t_1))$, where I is the classical action of the trajectory to leading order in $\hbar$ for a massless particle to propagate from (t_1,r_1) to (t_2,r_2) and is constructed by using the Hamilton–Jacobi frame. N is a suitable normalization constant. We can separate variables $I = -Et + W(r)$ due to the symmetries of the spacetime, where E is the energy of particle. This action acquires a singularity at the event horizon in analogy with the quantum tunneling process in quantum mechanics. In semiclassical quantum mechanics, this singularity is regularized by specifying a suitable complex contour [8]. In the case of a black hole [9], we should take the contour to be an infinitesimal semicircle above the pole $r = r_H$ for outgoing particles ($\partial_r I > 0$) on the left of the horizon and ingoing particles ($\partial_r I < 0$) on the right; similarly, for the ingoing particles on the left and outgoing particles on the right of the horizon (corresponding to the time reversed situation), the contour is below the pole. After integrating around the pole, we find that the action $I(r_2,t_2;r_1,t_1)$ is complex, so the probability $\Gamma \propto e^{-2\,\mathrm{Im}\,I}$ and the probability of the emission of particles are not the same as the probability of absorption, the ratio is $\Gamma[\text{emission}] = e^{-8\pi ME}\Gamma[\text{absorption}]$. This result shows that it is more likely for a particular region to gain particles than lose them. Further, the exponential dependence on the energy allows one to give a 'thermal' interpretation to this result. In a system with a temperature T_H, the absorption and the emission probabilities are related by

$$\Gamma[\text{emission}] = e^{-E/T_H}\Gamma[\text{absorption}]. \tag{1.1}$$

The above relation can be interpreted to be equivalent to a thermal distribution of particles in analogy with that observed in any system interacting with black body radiation. Then, the standard Hawking temperature is recovered.

However, in the case of a black hole, the action for ingoing particles should be real, so we employ a normalization condition on the action $I' = I(r_2,t_2;r_1,t_1) + K$, where K can be a complex constant that ensures that the imaginary of action for ingoing particles is equal to zero. Thus, the probabilities are

$$\Gamma[\text{emission}] \propto e^{-2\,\mathrm{Im}[I_+ + K]}, \qquad \Gamma[\text{absorption}] \propto e^{-2\,\mathrm{Im}[I_- + K]} = 1, \tag{1.2}$$

the ratio is

$$\Gamma[\text{emission}] = e^{-2[\mathrm{Im}\,I_+ - \mathrm{Im}\,I_-]}\Gamma[\text{absorption}], \tag{1.3}$$

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then

$$e^{-E/T_H} = e^{-2[\mathrm{Im}\, I_+ - \mathrm{Im}\, I_-]}, \tag{1.4}$$

where $I_\pm$ are the square roots of the relativistic Hamilton–Jacobi equation (4.3) corresponding to outgoing and ingoing particles.

Using this method, some authors recently found that the tunneling approach gives a different Hawking temperature of the Schwarzschild black hole using different coordinates. These coordinates are all stationary metrics; but what about non-stationary metrics? If one employs non-stationary metrics, e.g. the Kruskal–Szekers coordinates or dynamic Lemaitre coordinates, some other amazing facts come to light. For example, in Ref. [11], the authors pointed out that "In the case of Kruskal coordinate [sic], which is the maximal extension of Schwarzschild spacetime, it is easy to show that the semiclassical action when expressed in terms of Kruskal coordinates *does not contain the singularity*. (The HJ equation (of a massless particle) when expressed in terms of the Kruskal coordinates (V, U, θ, φ) is of the form $(\partial S_0/\partial V)^2 - (\partial S_0/\partial U)^2 = 0$ (for S-wave, i.e. $l = 0$). The solution of the equation can be easily obtained and is given by $S_0(V_2, U_2; V_1, U_1) = S_0(2, 1) = -p_V(V_2 - V_1) \pm p_U(U_2 - U_1)$.)" That is to say, the Hawking temperature cannot be recovered! As for dynamic Lemaitre coordinates, the authors [11] used the transformations $U = 3(R^* - \tau^*)/4M$ and $V = 3(R^* + \tau^*)/4M$, which already change the primitivity of this coordinate system. It is easy to see that these transformations will gives the line element

$$ds^2 = \frac{4}{9}M^2\left[\left(1 - U^{-2/3}\right)\left(dV^2 + dU^2\right) - 2\left(1 + U^{-2/3}\right)dV\,dU\right] + 4M^2U^{4/3}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{1.5}$$

Obviously, in the outer region $R^* - \tau^* > 4M/3$, the metrics are $g_{VV} = g_{UU} < 0$, whereas in the inner region $R^* - \tau^* < 4M/3$, the metrics are $g_{VV} = g_{UU} > 0$. That is, the time-like/space-like character of V or U are reversed again when they cross the horizon. In Ref. [13], the authors studied the Dirac particle radiation in the Kruskal coordinates by mathematically setting $\partial_\chi = N(X\partial_T + T\partial_X)$ and $\partial_\chi I = -E$. Yet, its physical meaning was not specified.

How can we obtain the correct Hawking temperature of a black hole in different coordinates? We learn from the formulism (1.1) that if the energy of the particles E is incorrect, we cannot find the correct Hawking temperature. Therefore, first of all, we should clarify the energy of the radiating particles in different coordinates. In this manuscript, we will study the problem carefully.

The Letter is organized as follows. In Section 2, the different coordinate representations for the Schwarzschild black hole are presented. In Section 3, the expression of the particle energy is presented. In Section 4, the Hawking temperature of the Schwarzschild black hole from scalar particle tunneling in Kruskal–Szekers coordinates is investigated. In Section 5, the Hawking temperature of the Schwarzschild black hole from scalar particle tunneling in dynamic Lemaitre coordinates is studied. The last section is devoted to a summary.

2. Coordinate representations for a Schwarzschild black hole

In standard coordinates, the line element of Schwarzschild black hole is

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{1}{1 - \frac{2M}{r}}dr^2 + r^2\,d\theta^2 + r^2\sin^2\theta\, d\varphi^2, \tag{2.1}$$

with an event horizon $r_H = 2M$. We introduce two different coordinate representations for the static black hole below.

2.1. Kruskal–Szekers coordinate representation

The Kruskal–Szekers coordinate transformation is

$$\begin{aligned}
&\text{when } r > 2M, \quad \tau = \sqrt{\frac{r}{2M} - 1}\,e^{r/4M}\sinh\left(\frac{t}{4M}\right), \qquad R = \sqrt{\frac{r}{2M} - 1}\,e^{r/4M}\cosh\left(\frac{t}{4M}\right),\\
&\text{when } r < 2M, \quad \tau = \sqrt{1 - \frac{r}{2M}}\,e^{r/4M}\cosh\left(\frac{t}{4M}\right), \qquad R = \sqrt{1 - \frac{r}{2M}}\,e^{r/4M}\sinh\left(\frac{t}{4M}\right).
\end{aligned} \tag{2.2}$$

The line element (2.1) in four-dimensional spacetime becomes

$$ds^2 = \frac{32M^3}{r}e^{-\frac{r}{2M}}\left(-d\tau^2 + dR^2\right) + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.3}$$

where $R = \tau$ at the event horizon $r_H = 2M$. It is easy to see its metric $g_{\mu\nu}$ is the function of τ, R and θ, so it is a dynamic coordinate system. In this coordinate, its coordinate singularity has been removed. τ is a time and R is a space coordinate inside and outside the horizon.

2.2. Dynamic Lemaitre coordinate representation

The dynamic Lemaitre coordinate representation is [33]

$$ds^2 = -d\tau^{*2} + \left[\frac{3}{4M}(R^* - \tau^*)\right]^{-2/3}dR^{*2} + (2M)^2\left[\frac{3}{4M}(R^* - \tau^*)\right]^{4/3}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.4}$$

where $R^* - \tau^* = 4M/3$ at the event horizon $r_H = 2M$. In this coordinate system, the coordinate singularity has also been removed. The proper time is equal to coordinate time, the R^*-axis is a spatial axis and the τ^*-axis is a temporal one not only inside but also outside the event horizon. The geodesic of the particles is continuous at the horizon.

3. The definition of the energy of radiating particles

According to quantum fields in curved spacetime, it is well known that one can define the particle energy as long as the spacetime has temporal-translational invariance and this energy is conserved. In standard coordinates, line element (2.1) obviously has temporal-translational invariance, so the particle energy is $E = -\partial_t I$, where I is the particle action, which can be found via separating variables $I = -Et + I'(\vec{x})$. However, the line elements (2.3) and (2.4) do not have this temporal-translational Invariance; therefore, $\partial_\tau I$ is not a constant of the motion. However, the particle energy should be a conserved quantity, so the key problem is how to find the expression of the particle energy in different coordinate systems.

As mentioned in [34], for the particles moving along a geodesic, the scalar product between the time-like Killing vector and the particle four-momentum $p^\mu = mdx^\mu/d\lambda$ is a constant, i.e.

$$\xi_\mu p^\mu = \text{constant}. \tag{3.1}$$

Furthermore, this quantity $\xi_\mu p^\mu$ is not only a conserved quantity along the geodesic, but also an invariant quantity in different coordinates. In a word, this quantity is a good one and we can use it to define the particle energy in different coordinate representations.

Consider the following Lagrangian of the massive radiating particle.

$$L = \frac{1}{2} m g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}, \tag{3.2}$$

where λ is an affine parameter defined along the geodesic. Constructing the action function $I = \int L\, d\lambda$, the possible physical process demands that, for variations $\delta I = 0$, one can obtain the Euler–Lagrangian equation

$$\frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{x}^\mu}\right) - \frac{\partial L}{\partial x^\mu} = 0, \tag{3.3}$$

where the overdot represents derivation with respect to the affine parameter λ. From the Euler–Lagrangian equation (3.3), the respective conjugate momentum p_μ is

$$p_\mu = \frac{\partial L}{\partial \dot{x}^\mu} = \int \frac{\partial L}{\partial x^\mu} d\lambda = \frac{\partial}{\partial x^\mu} \int L\, d\lambda = \partial_\mu I, \tag{3.4}$$

and the constant $\xi_\mu p^\mu = \xi^\mu p_\mu = \xi^\mu \partial_\mu I$. In the standard coordinate representation (2.1), the time-like Killing vector is $\tilde{\xi}^\mu = (1, 0, 0, 0)$, and the metric tensor is independent of the time coordinate t. Using the Lagrangian nomenclature, t is a cyclic coordinate, p_t is the conjugate momentum, and the particle energy E is the projection of four-momentum on the time-like tetrad. Thus, $E = -p_t = -\partial_t I$. For this case, $E = -\partial_t I = -\tilde{\xi}^\mu \tilde{\partial}_\mu I = -\tilde{\xi}^\mu \tilde{p}_\mu$, hence this constant can be defined as the particle energy, i.e.

$$E = -\xi^\mu p_\mu. \tag{3.5}$$

When the particles travel from the exterior region to interior region, the Killing vector changes its character into space-like, but the numerical value of $\xi^\mu p_\mu$ is still conserved [35].

In the Kruskal–Szekers coordinate system, using transformation (2.2), the Killing vector is

$$\xi^\mu = \frac{\partial x^\mu}{\partial \tilde{x}^\nu} \tilde{\xi}^\nu = \left(\frac{R}{4M}, \frac{\tau}{4M}, 0, 0\right). \tag{3.6}$$

Then, the energy of test particle is

$$E = -\xi^\mu p_\mu = -\xi^\mu \partial_\mu I = -\left(\frac{R}{4M}\partial_\tau + \frac{\tau}{4M}\partial_R\right) I. \tag{3.7}$$

In the dynamic Lemaitre coordinates, the Killing vector is

$$\xi^\mu = \frac{\partial x^\mu}{\partial \tilde{x}^\nu} \tilde{\xi}^\nu = (1, 1, 0, 0), \tag{3.8}$$

and then the particle energy is

$$E = -\xi^\mu p_\mu = -\xi^\mu \partial_\mu I = -(\partial_{\tau^*} + \partial_{R^*}) I. \tag{3.9}$$

The significance of this definition is that it specifies our conception of the particle energy in different coordinates. We will see that the Hawking temperature can be obtained using this expression of the energy.

4. Temperature of Schwarzschild black hole in the Kruskal–Szekers coordinate representation

In this section, we use the energy definition above in the study of the Hawking temperature of the Schwarzschild spacetime by employing the Kruskal–Szekers coordinates (2.3).

Applying the WKB approximation

$$\phi(\tau, R, \theta, \varphi) = \exp\left[\frac{i}{\hbar} I(\tau, R, \theta, \varphi) + I_1(\tau, R, \theta, \varphi) + \mathcal{O}(\hbar)\right] \tag{4.1}$$

to the Klein–Gordon equation

$$\frac{1}{\sqrt{-g}} \partial_\mu\left[\sqrt{-g} g^{\mu\nu} \partial_\nu \phi\right] - \frac{m^2}{\hbar^2} \phi = 0, \tag{4.2}$$

then, to leading order in $\hbar$, we obtain the following relativistic Hamilton–Jacobi equation

$$g^{\mu\nu}\partial_\mu I \partial_\nu I + m^2 = 0. \tag{4.3}$$

Now the action I is the Hamiltonian principal function with canonical momentum $p_\mu = \partial_\mu I$, and the Hamiltonian $H = \frac{1}{2m} g^{\mu\nu} p_\mu p_\nu = \frac{1}{2m} g^{\mu\nu}(\tau, R, \theta) p_\mu p_\nu$, so the time τ is not the coordinate that can be disregarded. The momentum $p_\tau = \partial_\tau I$ is not a constant, and we cannot separate τ from R in the action I as in Refs. [28,29]. In this coordinate representation, there exists a solution of the form

$$I = I_0(\tau, R) + J(\theta, \varphi) + K. \tag{4.4}$$

Inserting Eq. (4.4) and the metric (2.3) into the Hamilton–Jacobi equation (4.3), we obtain a partial differential equation

$$\frac{r}{32M^3} e^{\frac{r}{2M}} \left[-(\partial_\tau I_0)^2 + (\partial_R I_0)^2\right] + g^{ij} J_i J_j + m^2 = 0. \tag{4.5}$$

Using Eq. (3.7), substituting $\partial_\tau I_0 = -\frac{4M}{R}(E + \frac{\tau}{4M}\partial_R I_0)$ into Eq. (4.5), we obtain

$$(\partial_R I_0)_\pm = \frac{4ME\tau \pm R\sqrt{16M^2E^2 - [R^2 - \tau^2]\frac{32M^3}{r} e^{-\frac{r}{2M}} (g^{ij} J_i J_j + m^2)}}{R^2 - \tau^2}, \tag{4.6}$$

where $i, j = \theta, \varphi$; $J_i = \partial_i I$. One solution of Eq. (4.6) corresponds to the scalar particles moving away from the black hole (i.e. "+" outgoing), and the other solution corresponds to particles moving toward the black hole (i.e. "−" incoming). To find the relation between the total differential coefficient dI_0 and the partial differential coefficients $\partial_\tau I_0$ or $\partial_R I_0$, we need to know $\partial_\tau I_0$. From Eq. (3.7) and (4.6) we obtain

$$(\partial_\tau I_0)_\pm = -\frac{4MER \pm \tau\sqrt{16M^2E^2 - [R^2 - \tau^2]\frac{32M^3}{r} e^{-\frac{r}{2M}} (g^{ij} J_i J_j + m^2)}}{R^2 - \tau^2}. \tag{4.7}$$

It is easy to prove

$$\partial_R(\partial_\tau I_0) = \partial_\tau(\partial_R I_0), \qquad dI_0 = \partial_R I_0\, dR + \partial_\tau I_0\, d\tau, \tag{4.8}$$

so the definite integration of I_0 is

$$I_0 = \int \partial_R I_0\, dR + \partial_\tau I_0\, d\tau = \int \partial_R I_0 \left(dR - \frac{\tau}{R} d\tau\right) - \int \frac{4ME}{R} d\tau = \frac{1}{2}\int \frac{\partial_R I_0}{R} d\left(R^2 - \tau^2\right) - \int \frac{4ME}{R} d\tau. \tag{4.9}$$

Imaginary parts of the action can only come from the pole at the horizon, so the second integration of Eq. (4.9) is real, which shows that there is no temporal contribution in the Kruskal–Szekers coordinate system. Integrating around the pole $R = \tau$ at the horizon leads to

$$\begin{aligned} &(\mathrm{Im}\, I_0)_\pm = \mathrm{Im}\left[\frac{1}{2}\int \frac{4ME\frac{\tau}{R} \pm \sqrt{16M^2E^2 - (R^2 - \tau^2)\frac{32M^3}{r} e^{-\frac{r}{2M}} (g^{ij} J_i J_j + m^2)}}{R^2 - \tau^2} d\left(R^2 - \tau^2\right)\right], \\ &(\mathrm{Im}\, I_0)_+ = 4\pi ME, \qquad (\mathrm{Im}\, I_0)_- = 0. \end{aligned} \tag{4.10}$$

The probability of tunneling particles is

$$\frac{\Gamma[\text{emission}]}{\Gamma[\text{absorption}]} = \exp\left[-2(\mathrm{Im}\, I_+ - \mathrm{Im}\, I_-)\right] = \exp[-8\pi ME]. \tag{4.11}$$

Then, we obtain the Hawking temperature

$$T_H = \frac{1}{8\pi M}, \tag{4.12}$$

which shows that the temperature of Schwarzschild black hole is the same as that found in previous work using standard coordinates [11].

5. Hawking temperature of the Schwarzschild black hole in the dynamic Lemaitre coordinate representation

The definition of particle energy can also be used in the dynamic Lemaitre coordinate representation. In this section, we study the temperature of the Schwarzschild black hole in dynamic Lemaitre coordinates (2.4) due to scalar particle tunneling.

We also cannot separate the time coordinate τ^* from the radial coordinate R^* in the form of the particle's action, so there exists a solution in the form

$$I = I_0(\tau^*, R^*) + J(\theta, \varphi) + K. \tag{5.1}$$

Inserting the metric (2.4) and Eq. (5.1) into the Hamilton–Jacobi equation (4.3), we obtain

$$-(\partial_{\tau^*} I_0)^2 + \left[\frac{3}{4M}(R^* - \tau^*)\right]^{2/3} (\partial_{R^*} I_0)^2 + g^{ij} J_i J_j + m^2 = 0. \tag{5.2}$$

Substituting Eq. (3.9) into Eq. (5.2), we obtain

$$(\partial_{R^*} I_0)_\pm = \frac{E \pm \sqrt{[\frac{3}{4M}(R^* - \tau^*)]^{2/3} E^2 - \{[\frac{3}{4M}(R^* - \tau^*)]^{2/3} - 1\}(g^{ij} J_i J_j + m^2)}}{[\frac{3}{4M}(R^* - \tau^*)]^{2/3} - 1}. \tag{5.3}$$

One solution of Eq. (5.3) corresponds to the scalar particles moving away from the black hole (i.e. "+" outgoing) and the other solution corresponds to particles moving toward the black hole (i.e. "−" incoming). In order to seek the relation between the total differential dI_0 and partial differential $\partial_{R^*} I_0, \partial_{\tau^*} I_0$, we need to find $\partial_{\tau^*} I_0$. From Eq. (3.9) and (5.3) we obtain

$$(\partial_{\tau^*} I_0)_\pm = -\left\{E + \frac{E \pm \sqrt{[\frac{3}{4M}(R^* - \tau^*)]^{2/3}E^2 - \{[\frac{3}{4M}(R^* - \tau^*)]^{2/3} - 1\}(g^{ij}J_iJ_j + m^2)}}{[\frac{3}{4M}(R^* - \tau^*)]^{2/3} - 1}\right\}. \tag{5.4}$$

It is easy to prove

$$\partial_{R^*}(\partial_{\tau^*} I_0) = \partial_{\tau^*}(\partial_{R^*} I_0), \qquad dI_0 = \partial_{R^*} I_0\, dR^* + \partial_{\tau^*} I_0\, d\tau^*, \tag{5.5}$$

therefore the definite integration of I_0 is

$$I_0 = \int_{(R_0^*,\tau_0^*)}^{(R_1^*,\tau_1^*)} [\partial_{R^*} I_0\, dR^* + \partial_{\tau^*} I_0\, d\tau^*] = \int_{(R_0^*,\tau_0^*)}^{(R_1^*,\tau_1^*)} \big[\partial_{R^*} I_0\, dR^* + (-E - \partial_{R^*} I_0)\, d\tau^*\big] = \int_{(R_0^*-\tau_0^*)}^{(R_1^*-\tau_1^*)} \partial_{R^*} I_0\, d(R^* - \tau^*) - \int_{\tau_0^*}^{\tau_1^*} E\, d\tau^*, \tag{5.6}$$

where the point (R_0^*, τ_0^*) is inside the event horizon $\tau^* = R^* - \frac{4M}{3}$, and the point (R_1^*, τ_1^*) is outside the horizon. Imaginary parts of the action can only come from the pole at the horizon, so that the second integration of (5.6) is real, which tells us that there is no temporal contribution in the dynamic Lemaitre coordinate system. Substituting Eq. (5.3) into (5.6), then integrating around the pole $R^* - \tau^* = 4M/3$ at the horizon leads to

$$(\operatorname{Im} I_0)_\pm = \operatorname{Im}\left[\int \frac{E \pm \sqrt{[\frac{3}{4M}(R^* - \tau^*)]^{2/3}E^2 - \{[\frac{3}{4M}(R^* - \tau^*)]^{2/3} - 1\}(g^{ij}J_iJ_j + m^2)}}{[\frac{3}{4M}(R^* - \tau^*)]^{2/3} - 1} d(R^* - \tau^*)\right],$$
$$(\operatorname{Im} I_0)_+ = 4\pi ME, \ (\operatorname{Im} I_0)_- = 0. \tag{5.7}$$

The probability of a particle tunneling from inside to outside the horizon is

$$\frac{\Gamma[\text{emission}]}{\Gamma[\text{absorption}]} = \exp\big[-2(\operatorname{Im} I_+ - \operatorname{Im} I_-)\big] = \exp[-8\pi ME]. \tag{5.8}$$

We also obtain the correct Hawking temperature.

From the above discussions, it is easy to see that, with the definition of the radiating particle energy (3.5), the problem of the Hawking radiation in Kruskal–Szekers and dynamic Lemaitre coordinates is solved, and the Hawking temperature is invariant.

6. Summary

To study the Hawking radiation of a black hole in different coordinates, we learn from the formulism (1.1) that the key step is to define the energy of the radiating particles in different coordinates. By means of the Euler–Lagrangian equation and using the fact that $\xi_\mu p^\mu$ is a constant in coordinate transformations, we present an expression of the energy of the radiating particles: $E = -\xi^\mu p_\mu$.

As examples, we study the Hawking temperature of the Schwarzschild black hole in the Kruskal–Szekers and dynamic Lemaitre coordinates using the definition of the energy of the particles. In these two coordinates, there are no coordinate singularities at the event horizon, and there is no inversion between time and space across the event horizon. We find that the Hawking temperature is invariant under these two dynamic coordinate representations.

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References

[1] M. Angheben, M. Nadalini, L. Vanzo, S. Zerbini, J. High Energy Phys. 0505 (2005) 014.
[2] C.K. Ding, J.L. Jing, Class. Quantum Grav. 25 (2008) 145015.
[3] C.K. Ding, J.L. Jing, Gen. Relativ. Gravit. 41 (2009), in press.
[4] M.K. Parikh, F. Wilczek, Phys. Rev. Lett. 85 (2000) 5042.
[5] P. Kraus, F. Wilczek, Mod. Phys. Lett. A 9 (1994) 3713.
[6] P. Kraus, F. Wilczek, Nucl. Phys. B 433 (1995) 403.
[7] P. Kraus, F. Wilczek, Nucl. Phys. B 437 (1995) 231.
[8] L.D. Landau, E.M. Lifshitz, Quantum Mechanics (Non-relativistic Theory), Course of Theoretical Physics vol. 2, Pergamon, New York, 1975.
[9] K. Srinivasan, T. Padmanabhan, Phys. Rev. D 60 (1999) 024007.
[10] S. Shankaranarayanan, K. Srinivasan, T. Padmanabhan, Mod. Phys. Lett. A 16 (2001) 571.
[11] S. Shankaranarayanan, T. Padmanabhan, K. Srinivasan, Class. Quantum Grav. 19 (2002) 2671.
[12] R. Kerner, R.B. Mann, Phys. Rev. D 73 (2006) 104010.
[13] R. Kerner, R.B. Mann, Class. Quantum Grav. 25 (2008) 095014.
[14] R. Kerner, R.B. Mann, Phys. Lett. B 665 (2008) 277.
[15] E.T. Akhmedov, V. Akhmedova, D. Singleton, Phys. Lett. B 642 (2006) 124.
[16] E.T. Akhmedov, V. Akhmedova, D. Singleton, T. Pilling, Int. J. Mod. Phys. A 22 (2007) 1705.
[17] B.D. Chowdhury, Pramana 70 (2008) 593.

[18] M.K. Parikh, Phys. Lett. B 546 (2002) 189.
[19] M.K. Parikh, Int. J. Mod. Phys. D 13 (2004) 2351.
[20] M.K. Parikh, arXiv:hep-th/0402166.
[21] M. Arzano, A. Medved, E. Vagenas, J. High Energy Phys. 0509 (2005) 037.
[22] Q.-Q. Jiang, S.-Q. Wu, X. Cai, Phys. Rev. D 73 (2006) 064003.
[23] J. Zhang, Z. Zhao, Phys. Lett. B 638 (2006) 110.
[24] S.-Q. Wu, Q.-Q. Jiang, J. High Energy Phys. 0603 (2006) 079.
[25] A.J.M. Medved, E. Vagenas, Mod. Phys. Lett. A 20 (2005) 2449.
[26] B. Chatterjee, A. Ghosh, P. Mitra, Phys. Lett. B 661 (2008) 307.
[27] P. Mitra, Phys. Lett. B 648 (2007) 240.
[28] V. Akhmedova, T. Pilling, A. de Gill, D. Singleton, Phys. Lett. B 666 (2008) 269.
[29] E.T. Akhmedov, T. Pilling, D. Singleton, Int. J. Mod. Phys. D 17 (2009) 2453.
[30] S.P. Kim, J. High Energy Phys. 0711 (2007) 048.
[31] D.Y. Chen, Q.Q. Jiang, X.T. Zu, Class. Quantum Grav. 25 (2008) 205022.
[32] S. Stotyn, K. Schleich, D. Witt, Class. Quantum Grav. 26 (2009) 065010.
[33] Y.A.B. Zeldovich, I.D. Novikov, Rilativistic Astrophysics I, University of Chicago Press, Chicago, 1971.
[34] H.C. Ohanian, R. Ruffini, Gravitation and Spacetime, 2nd ed., W.W. Norton & Company, Inc., 1994.
[35] R. Doran, F.S.N. Lobo, P. Crawford, Found. Phys. 38 (2008) 160.