Quantum neural networks

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Abstract. This chapter outlines the research, development and perspectives of quantum neural networks – a burgeoning new field which integrates classical neurocomputing with quantum computation [1]. It is argued that the study of quantum neural networks may give us both new undestanding of brain function as well as unprecedented possibilities in creating new systems for information processing, including solving classically intractable problems, associative memory with exponential capacity and possibly overcoming the limitations posed by the Church-Turing thesis.

Keywords. Quantum neural networks, associative memory, entanglement, many universes interpretation

Why quantum neural networks?

There are two main reasons to discuss quantum neural networks. One has its origin in arguments for the essential role which quantum processes play in the living brain. For example, Roger Penrose has argued that a new physics binding quantum phenomena with general relativity can explain such mental abilities as *understanding, awareness* and *consciousness* [2]. However, this approach advocates the study of intracellular structures, such as microtubules rather than that of the networks of neurons themselves [3]. A second motivation is the possibility that the field of classical artificial neural networks can be generalized to the quantum domain by eclectic combination of that field with the promising new field of quantum computing [4]. Both considerations suggest new understanding of mind and brain function as well as new unprecedented abilities in information processing. Here we consider quantum neural networks as the next natural step in the evolution of neurocomputing systems, focusing our attention on artificial rather than biological systems. We outline different approaches to the realization of quantum distributed processing and argue that, as in the case of quantum computing [5], Everett's many universes interpretation of quantum mechanics [6] can be used as a general framework for producing quantum analogs of well-known classical artificial neural networks. We also outline some perspectives on quantum neurocomputers in the next century.

Neural networks: toward quantum analogs

There are many different approaches to what we can call *quantum neural networks*. Many researchers use their own analogies in establishing a connection between quantum mechanics and neural networks. The main concepts of these two fields may be considered as follows [7-8]:

Table 1. Main concepts of quantum mechanics and neural networks

Quantum mechanics	Neural Networks
wave function	neuron
Superposition (coherence)	interconnections (weights)
Measurement (decoherence)	evolution to attractor
Entanglement	learning rule
unitary transformations	gain function (transformation)

One should be careful *not* to consider corresponding concepts in the two columns as analogical – in the table above their order is arbitrary. Indeed, the establishment of such correspondences is a major challenge in the development of a model of quantum neural networks.

To date, quantum ideas have been proposed for the effective realization of classical – rather than neural – computation. The concept of quantum computation may arguably be traced back to the pioneering work of Richard Feynman [1], who examined the role quantum effects would play in the development of future hardware. As hardware speeds continue to increase, hardware scales correspondingly continue to decrease and at some point in the not too distant future, Feynman realized, gates and wires may consist of only a few atoms, and quantum effects will then play a major role in hardware implementation[1]. Feynman concluded that such quantum devices can have

[1] It is somewhat remarkable that in 1982 just as Richard Feynman published his first paper on quantum computation, John Hopfield proposed his model of neural content-addressable memory [9], which attracted many physicists to the field of artificial neural networks.

significant advantages over classical computational mediums. In 1985 David Deutsch formalized the foundations of quantum computation [5].

Some Quantum Concepts

Quantum computation is based upon physical principles from the theory of quantum mechanics (QM), which is in many ways counterintuitive. Yet it has provided us with perhaps the most accurate physical theory (in terms of predicting experimental results) ever devised by science. The theory is well-established and is covered in its basic form by many textbooks (see for example [10]). Several necessary ideas that form the basis for the study of quantum computation are briefly reviewed here.

Linear superposition is closely related to the familiar mathematical principle of linear combination of vectors. Quantum systems are described by a wave function ψ that exists in a Hilbert space. The Hilbert space has a set of states, $|\phi_i\rangle$, that form a basis, and the system is described by a quantum state $|\psi\rangle$,

$$|\psi\rangle = \sum_i c_i |\phi_i\rangle$$

$|\psi\rangle$ is said to be in a linear superposition of the basis states $|\phi_i\rangle$, and in the general case, the coefficients c_i may be complex. Use is made here of the Dirac bracket notation, where the ket $|\cdot\rangle$ is analogous to a column vector, and the bra $\langle\cdot|$ is analogous to the complex conjugate transpose of the ket. In quantum mechanics the Hilbert space and its basis have a physical interpretation, and this leads directly to perhaps the most counterintuitive aspect of the theory. The counter intuition is this -- at the microscopic or quantum level, the state of the system is described by the wave function ψ, that is, as a linear superposition of all basis states (i.e. in some sense the system is in all basis states at once). However, at the macroscopic or classical level the system can be in only a single basis state. For example, at the quantum level an electron can be in a superposition of many different energies; however, in the classical realm this obviously cannot be.

Coherence and decoherence are closely related to the idea of linear superposition. A quantum system is said to be coherent if it is in a linear superposition of its basis states. A result of quantum mechanics is that if a system that is in a linear superposition of states interacts in any way with its environment, the superposition is destroyed. This loss of coherence is called decoherence and is governed by the wave function ψ. The coefficients c_i are called probability amplitudes, and $|c_i|^2$ gives the probability of $|\psi\rangle$ collapsing into state $|\phi_i\rangle$ if it decoheres. Note that the wave function ψ describes a real physical system that must collapse to exactly one basis state. Therefore, the probabilities governed by the amplitudes c_i must sum to unity. This necessary constraint is expressed as the unitarity condition

$$\sum_i |c_i|^2 = 1$$

In the Dirac notation, the probability that a quantum state $|\psi\rangle$ will collapse into an eigenstate $|\phi_i\rangle$ is written $|\langle\phi_i|\psi\rangle|^2$ and is analogous to the dot product (projection) of two vectors. Consider, for example, a discrete physical variable called spin. The simplest spin system is a two-state system, called a spin-1/2 system, whose basis states are usually represented as $|\uparrow\rangle$ (spin up) and $|\downarrow\rangle$ (spin down). In this simple system the wave function ψ is a distribution over two values (up and down) and a coherent state $|\psi\rangle$ is a linear superposition of $|\uparrow\rangle$ and $|\downarrow\rangle$. One such state might be

$$|\psi\rangle = \frac{2}{\sqrt{5}}|\uparrow\rangle + \frac{1}{\sqrt{5}}|\downarrow\rangle$$

As long as the system maintains its quantum coherence it cannot be said to be either spin up or spin down. It is in some sense both at once. Classically, of course, it must be one or the other, and when this system decoheres the result is, for example, the $|\uparrow\rangle$ state with probability

$$|\langle\uparrow|\psi\rangle|^2 = \left(\frac{2}{\sqrt{5}}\right)^2 = 0.8$$

A simple two-state quantum system, such as the spin-1/2 system just introduced, is used as the basic unit of quantum computation. Such a system is referred to as a quantum bit or *qubit*, and renaming the two states $|0\rangle$ and $|1\rangle$ it is easy to see why this is so.

Operators on a Hilbert space describe how one wave function is changed into another. Here they will be denoted by a capital letter with a hat, such as $\hat{A}$, and they may be represented as matrices acting on vectors. Using operators, an eigenvalue equation can be written $\hat{A}|\phi_i\rangle = a_i|\phi_i\rangle$, where a_i is the eigenvalue. The solutions $|\phi_i\rangle$ to such an equation are called eigenstates and can be used to construct the basis of a Hilbert space as discussed previously. In the quantum formalism, all properties are represented as operators whose eigenstates are the basis for the Hilbert space associated with that property and whose eigenvalues are the quantum allowed values for that property. It is important to note that operators in quantum mechanics must be linear operators and further that they must be unitary so that $\hat{A}^\dagger\hat{A} = \hat{A}\hat{A}^\dagger = \hat{I}$, $\hat{I}$ is the identity operator, and $\hat{A}^\dagger$ is the complex conjugate transpose, or adjoint, of $\hat{A}$.

Interference is a familiar wave phenomenon. Wave peaks that are in phase interfere constructively (magnify each other's amplitude) while those that are out of phase interfere destructively (decrease or eliminate each other's amplitude). This is a phenomenon common to all kinds of wave mechanics from water waves to optics. The well-known double slit experiment demonstrates empirically that at the quantum level interference also applies to the probability waves of quantum mechanics. As a simple example, suppose that the wave function described above is represented in vector form as

$$|\psi\rangle = \frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\end{pmatrix}$$

and suppose that it is operated upon by an operator $\hat{O}$ described by the following matrix,

$$\hat{O} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$$

The result is

$$\hat{O}|\psi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\end{pmatrix} = \frac{1}{\sqrt{10}}\begin{pmatrix}3\\1\end{pmatrix}$$

and therefore now

$$|\psi\rangle = \frac{3}{\sqrt{10}}|\uparrow\rangle + \frac{1}{\sqrt{10}}|\downarrow\rangle$$

Notice that the amplitude of the $|\uparrow\rangle$ state has increased while the amplitude of the $|\downarrow\rangle$ state has decreased. This is due to the wave function interfering with itself through the action of the operator -- the different parts of the wave function interfere constructively or destructively according to their relative phases just like any other kind of wave.

Entanglement is the potential for quantum states to exhibit correlations that cannot be accounted for classically. From a computational standpoint, entanglement seems intuitive enough -- it is simply the fact that correlations can exist between different qubits -- for example if one qubit is in the $|1\rangle$ state, another will be in the $|1\rangle$ state. However, from a physical standpoint, entanglement is little understood. The questions of what exactly it is and how it works are still not resolved. What makes it so powerful (and so little understood) is the fact that since quantum states exist as superpositions, these correlations exist in superposition as well. When the superposition is destroyed, the proper correlation is somehow communicated between the qubits, and it is this "communication" that is the crux of entanglement. Mathematically, entanglement may be described using the *density matrix* formalism. The density matrix ρ_ψ of a quantum state $|\psi\rangle$ is defined as

$$\rho_\psi = |\psi\rangle\langle\psi|$$

For example, the quantum state

$$|\xi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$$

appears in vector form as

$$|\xi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$$

and it may also be represented as the density matrix

$$\rho_\xi = |\xi\rangle\langle\xi| = \frac{1}{2}\begin{pmatrix}1&1&0&0\\1&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

while the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

is represented as

$$\rho_\psi = |\psi\rangle\langle\psi| = \frac{1}{2}\begin{pmatrix}1&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&1\end{pmatrix}$$

and the state

$$|\zeta\rangle = \frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|01\rangle + \frac{1}{\sqrt{3}}|11\rangle$$

is represented as

$$\rho_\zeta = |\zeta\rangle\langle\zeta| = \frac{1}{3}\begin{pmatrix}1&1&0&1\\1&1&0&1\\0&0&0&0\\1&1&0&1\end{pmatrix}$$

where the matrices and vectors are indexed by the state labels 00, ..., 11. Now, notice that ρ_ξ can be factorized as

$$\rho_\xi = \frac{1}{\sqrt{2}}\left(\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\otimes\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\right)$$

where $\otimes$ is the normal tensor product. On the other hand, ρ_ψ can not be factorized. States that cannot be factorized are said to be *entangled*, while those that can be factorized are not. Notice that ρ_ζ can be partially factorized two different ways, one of which is

$$\rho_\zeta = \frac{1}{\sqrt{3}}\left(\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\otimes\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\otimes\begin{pmatrix}1 & 1 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\end{pmatrix}\right)$$

(the other contains the factorization of ρ_ξ and a different remainder); however, in both cases the factorization is not complete. Therefore, ρ_ζ is also entangled, but not to the same degree as ρ_ψ (because ρ_ζ can be partially factorized but ρ_ψ cannot). Thus there are different degrees of entanglement and much work has been done on better understanding and quantifying it [11-12]. It is interesting to note from a computational standpoint that quantum states that are superpositions of only basis states that are maximally far apart in terms of Hamming distance are those states with the greatest entanglement. For example, ρ_ψ is a superposition of only the states 00 and 11, which have a maximum Hamming spread, and therefore ρ_ψ is maximally entangled. Finally, it should be mentioned that while interference is a quantum property that has a classical cousin, entanglement is a completely quantum phenomenon for which there is no classical analog.

Interpretations of quantum theory

It is important to note that much of the power of classical artificial neural networks is due to their massively parallel, distributed processing of information and also due to the nonlinearity of the transformation performed by the network nodes (neurons). On the other hand, quantum mechanics offers the possibility of an even more powerful *quantum parallelism* which is expressed in the principle of superposition. This principle provides quantum computing an advantage in processing huge data sets. Though quantum computing implies parallel processing of all possible configurations of the state of a register composed of *N* qubits, only one result can be read after the decoherence of the quantum superposition into one of its basis states. However, entanglement provides the possibility of measuring the states of all qubits in a register whose values are interdependent. Though the mathematics of quantum mechanics is fairly well understood and accepted, the physical reality of what the theory *means* is much debated and there exist different interpretations of quantum mechanics, including:

- Copenhagen interpretation [7];
- Feynman path-integral formalism [13];
- Many universes (many-world) interpretation of Everett [6], etc.

The choice of interpretation is important in establishing different analogies between quantum physics and neurocomputing.

The field of neural networks contains several important basic ideas, which include the concept of a processing element (*neuron*), the *transformation* performed by this element (in general, input summation and nonlinear mapping of the result into an output value), the *interconnection structure* between neurons, the network *dynamics*, and the *learning rule* which governs the modification of interconnection strengths. A major dichotomization of neural networks can be realized by considering whether they are trained in a supervised or unsupervised manner. An example of the latter is the *Hopfield model* of content-addressable memory using the concept of attractor states [9].

We shall argue below that it is adequate to choose such a Hopfield network as a reference point for the consideration of neural models in general. In fact, the Hopfield model itself was proposed during a previous "invasion" of physics into the theory of artificial neural networks in 1982. What Hopfield discovered was an analogy between networks with symmetrical bonds and *spin glasses*.

While quantum mechanics is a linear theory, neurocomputing is very dependent upon nonlinear approaches to data processing. At first glance, this appears to complicate the establishment of a correspondence between the two fields. However there are different ways to overcome this difficulty.

As mentioned earlier, evolutionary operators in quantum mechanics must be unitary, and certain aspects of any quantum computation must be considered as evolutionary. For example, storing patterns in a quantum system demands evolutionary processes since the system must maintain a coherent superposition that represents the stored patterns. On the other hand, other aspects of quantum computation preclude unitarity (and thus linearity) altogether. In particular, decoherence is a non-unitary process.

In the *Copenhagen interpretation*, non-unitary operators do exist in quantum mechanics and in nature. For example, any observation of a quantum system can be thought of as an operator that is neither evolutionary nor unitary. In fact, the Copenhagen school of thought suggests that this non-evolutionary behavior of quantum systems is just as critical to our understanding of quantum mechanics as is their evolutionary behavior. Now, since recalling a pattern from a quantum system would require the decoherence and collapse of the system at some point (at the very latest when the system is observed), it can be argued that pattern recall may be considered as a non-unitary process. In which case, the use of unitary operators becomes unnecessary. Since the decoherence and collapse of a quantum wave function is non-unitary and since pattern recall in a quantum system requires decoherence and collapse at some point, why not make explicit use of this non-unitarity, in the pattern recall process? This decoherence of a quantum state can be considered as the analog of the evolution of a neural network state to an attractor basin. This analogy has been mentioned in the work of Perus [14][2].

As a second approach to reconciling the linearity of quantum mechanics with the nonlinearity inherent in artificial neural networks, consider the *Feynman interpretation* of quantum mechanics, which is based on the use of path integrals. The probability of an event is expressed by the formula,

$$|\psi(t)\rangle = \sum_{all\ paths} e^{-\frac{i}{\hbar}\int_0^t [\frac{m\dot{x}^2(\tau)}{2} - V(x(\tau)]d\tau}$$

Here nonlinearity can be due both to the nonlinear form of the potential V(x) and also to the operation of the exponent. This fact has been used in approaches to modelling quantum neural networks by Elizabeth Behrman and coworkers [16-17] and Ben Goertzel [18] (some analogies used for the development of quantum neural networks are summarized in Table 2). Behrman et al. first developed a *temporal* model of a quantum neural network which utilizes a quantum dot molecule coupled to a substrate lattice through optical phonons [16]. In this model temporal evolution of the system resembles the equations for virtual neurons and the timeline discretization points for the Feynman path integral serve as these virtual neurons. The concept of neurons used here is rather artificial, and in fact the number of neurons depends on the parameters of the temporal discretization scheme, rather than on the number of quantum particles involved. Recently, this group working at Wichita State University proposed a *spatial* model for a quantum neural network based on the use of a spatial array of quantum dot molecules. It was shown that any logical gate, including a purely quantum one – phase shift – can be performed using these systems [17]. Note that another approach to quantum neural networks used by Ron Chrisley from the University of Sussex [19-20], considers the positions of slits in an interference experiment (similar to Young's double-slit experiment) as representing neuron state values while the positions of other slits encode the values of the network weights. Obviously, there is a high diversity of possible approaches to the construction of a model of quantum neural networks.

But we shall try to argue that as in the case of quantum computing the most consistent way to obtain a general model seems to be *Everett's many universes interpretation* of quantum mechanics. Everett's approach suggests that decoherence or collapse of the wave function is an *illusion*, and that actually the wave function obeys the Schrödinger equation at all times. Rather than causing the wave function to collapse, the effect of the measurement is to split the observer into a number of copies, each copy observing just one of the possible results of a measurement, unaware of the other possible outcomes. It follows that

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[2] However, in some sense, the formalism described by him is much more similar to the concept of the synergetical computer proposed by Hermann Haken [15].

there exist many, mutually unobservable but equally real universes, each corresponding to a single possible outcome of the measurement [21].

Using this metatheory of quantum mechanics as a starting point, we can combine the field of artificial neural networks with that of quantum computation in a natural way. For our purposes it is sufficient to consider the application of neural approaches, in their simplest forms, to pattern recognition. We shall then see how a concept of quantum neural networks naturally emerges from the theory of neurocomputing.

Table 2. Quantum analogies used for different concepts of artificial neural networks

Model	Neuron	Connections	Transformation	Network	Dynamics
Perus	quantum	Green function	linear	temporal	collapse as convergence to attractor
Chrisley	classical (slit position)	classical (slit position)	nonlinear through superposition	multilayer	non-superpositional
Behrman et al.	time slice, quantum	interactions through phonons	nonlinear through potential energy and exponent function	temporal and spatial	Feynman path integral
Goertzel	classical	quantum	nonlinear	classical	Feynman path-integral
Menneer and Narayanan	classical	classical	nonlinear	single-item networks in many universes	classical
Ventura	qubit	entanglement	-	single-item modules in many universes	unitary and non-unitary transformations

How pattern recognition leads us to quantum neural networks

One simple approach to pattern recognition can be termed a template-based method, in which examples of different pattern classes are stored separately as multiple templates. A presented stimulus can then be recognized (classified) according to the class of the template most similar to the input stimulus. To be efficient this process should be performed in parallel since the number of stored templates can be prohibitive to sequential processing. In general, this scheme is characterized by rather low performance due to a lack of generalization and also due to the need to guarantee invariant recognition.

Neural networks provide the ability to use only *one* system to store *multiple data* belonging to different classes and to classify the presented stimulus in a parallel, distributed manner [22]. Thus, the problem of parallelism is naturally solved in this approach. Further, the capability of approximating arbitrarily complex functions makes neural networks very effective for creating classification systems.

It is often desirable to use multimodular systems consisting of so-called single-class neural networks [23-25]. In this scheme, a network is trained using only examples of patterns belonging to a single class, and a different network is trained for each class to be recognized. Classification is performed by presenting the input stimulus to each of the different modules, comparing their outputs and using some criterion to choose a winning module. The problem of parallelism arises again in this approach (though, not nearly as acutely as in the case of a template-based method), but it has many advantages associated with the usefulness of spurious memories for generalization [26]. Various types of neural systems can be used as the basis for such a multimodular recognition scheme, including auto-associative perceptrons, but what is especially pertinent to this discussion is the fact that Hopfield networks seem to be especially good candidates for this role.

It is well known that any state of the Hopfield network is either a stable attractor or evolves to some such attractor. It is usual to interpret attractors as memorized patterns, or sometimes as spurious memories, while non-stable states can be considered as corrupted versions of memories containing enough partial information to retrieve the memorized pattern stored as the nearest stable state. Numerous studies have been performed to investigate the properties of content-addressable memories which can be implemented by the Hopfield model and its various derivatives [27]. The main drawback of such memories is their limited capacity. However, using a probabilistic interpretation of the network state energy – the functional which governs state dynamics – it can be argued that the Hopfield network is best suited for the extraction of the locally most plausible version of a single prototype, for which all stored patterns can be considered as corrupted versions [28]. This approach can be also thought of as implementing the use of distributed templates in the sense that all representatives of a given class are compared with an external stimulus in a parallel, distributed manner. What is even more interesting, this approach opens the door to the development of quantum neural networks by suggesting a further generalization of the idea of class-specific neural networks. Namely, we can generalize the idea to its extreme, considering a system of separate networks each trained with *only a single pattern*! This, in turn, brings us naturally to the many universes approach to quantum mechanics.

Many universes approach

In memorizing a set of patterns, why not use a set of many Hopfield networks, each of which stores a single pattern? In the classical Hopfield model we typically use only one network to store many patterns, and we sum all pattern correlations in order to build the network's Hebbian interconnections as follows.

$$T_{ij} = \sum_{s=1}^{P} \sigma_i^s \sigma_j^s \ , \quad T_{ii} = 0 \quad i, j = 1, \ldots, N$$

This summation causes multiple problems if we want to consider a network as a passive memorization system. The interference of the different patterns leads to a loss of the stability for some memories (producing, instead, a spurious memory) and, as a result, to a rather restricted memory capacity, which grows at best only linearly with the number of neurons [27].

If, on the other hand, we simply generate multiple Hopfield networks which store only one pattern each, we lose any parallelism in processing the information. But what about a quantum approach? Imagine, that we can store all patterns as the quantum superposition.

$$|\psi\rangle = \sum_{s=1}^{P} |\sigma_1^2 \sigma_2^s \ldots \sigma_N^s\rangle$$

In this case, each of the patterns can be considered as existing in a separate universe. Moreover, the interaction of such a superposition with the environment is performed in parallel, and further, this parallelism has a quantum nature. It has in fact been shown in, given a set of patterns, how such a superposition may be created [29], and each of the basis states in the superposition will play the role of a single memory state independent of the number of them that exist in the superposition. In theory, then, a *quantum associative memory can have exponential memory capacity!* (See [30].) It should also be mentioned here that although spurious states can arise in such a quantum memory, these spurious state are not the result of an interference of memories as in the classical case but instead arise for a completely different reason in the retrieval phase and therefore do not directly influence stored patterns.

Let us imagine that instead of the various memory states existing in parallel universes, we have single-memory, Hopfield-type networks existing in these universes. In the classical Hopfield network, the existence of symmetric, Hebbian connections guarantees the stability of a unique stored pattern; similarly, in a quantum analog of the Hopfield network the integrity of a stored pattern (basis state) is due to entanglement [31]. This property characterizes multi-particle systems and is the basis of all known quantum algorithms. Now we can consider quantum associative memory as a realization of the extreme condition of using many Hopfield networks, each storing a single pattern in parallel quantum universes!

Continuing this line of reasoning, we can further imagine more complex neural structures existing in such parallel worlds. Such an idea has been explored by Menneer and Narayanan, who consider a set of multilayer perceptrons, each trained on only one pattern that are combined into a quantum network whose weights are superpositions of the weights of all perceptrons existing in parallel universes [32].

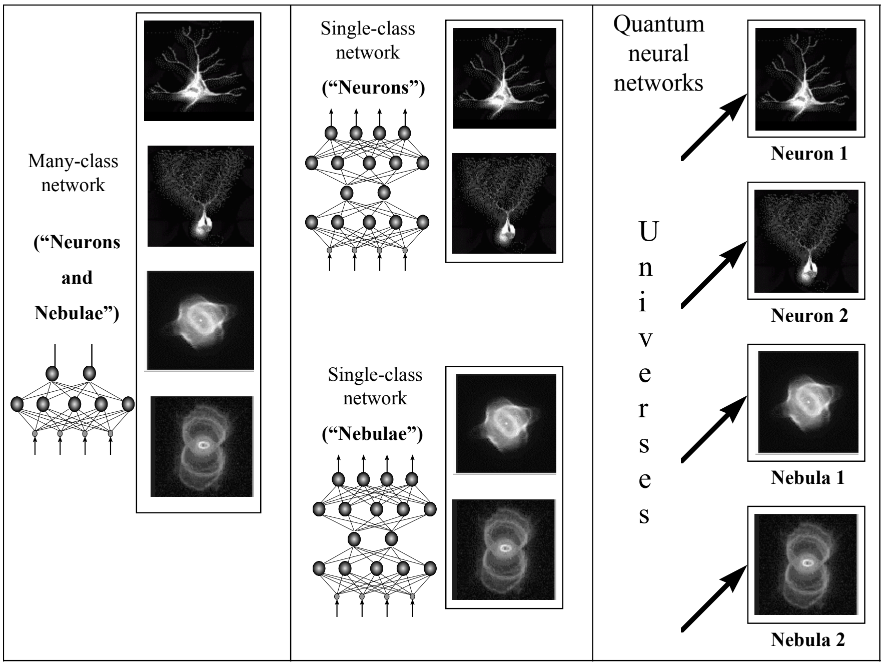

Fig. 1. Many-class networks are trained using the examples from different classes (here "Neurons" and "Nebulae" together) – left; A set of modular single-class neural networks use for training only the objects belonging to one class (two networks for two classes: "Neurons" and "Nebulae" separately) – center; Quantum neural networks may be trained using only pattern each! (four networks for four examples in many universes) – right.

Moreover, they also consider the many universes approach to quantum neural networks as methodologically correct and cognitively plausible. Indeed, fast learning of the networks in separate universes avoids the objection to neural network models being adequate accounts of mind because multiple presentations of patterns is implausible for human learning [32].

Quantum associative memory

One of the most promising approaches to quantum neurocomputing is the quantum associative memory, of which one approach is described in [33-35]. The task of pattern association can be broken down into two major components: memorization and recall. The memorization step consists of storing patterns in the memory while the recall step entails pattern completion or pattern association based on partial and/or noisy input.

Memorization

An efficient quantum algorithm for constructing a coherent state over n qubits to represent a set of m patterns is presented in [29]. The algorithm is implemented using a polynomial number (in the length and number of patterns) of elementary operations on one, two, or three qubits. The key operator in this process is

$$\hat{S}^p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{\frac{p-1}{p}} & \frac{-1}{\sqrt{p}} \\ 0 & 0 & \frac{1}{\sqrt{p}} & \sqrt{\frac{p-1}{p}} \end{bmatrix}$$

where $m \geq p \geq 1$. This is actually a set of operators that are conditional transforms – there is a different $\hat{S}^p$ operator associated with each pattern to be stored. The algorithm also makes use of various versions of some standard quantum computational operators such as the Controlled-Not and Fredkin gates. Now given a set P of m binary patterns of length n, the quantum algorithm for storing the patterns requires a set of $2n+1$ qubits, the first n of which actually store the patterns and can be thought of as n neurons in a quantum associative memory. The remaining $n+1$ qubits are ancillary qubits used for bookkeeping and are restored to the state $|\bar{0}\rangle$ after every storage iteration. Each iteration through the algorithm makes use of a different $\hat{S}^p$ operator and results in another pattern being incorporated into the quantum system. The result is a coherent superposition of states that correspond to the patterns, with the amplitudes of the states in the superposition all being equal. The algorithm requires $O(mn)$ steps to encode the m patterns as a quantum superposition over n quantum neurons. This is optimal in the sense that just reading each instance once cannot be done any faster than $O(mn)$.

Recall – completion

The recall capability of the quantum associative memory can be implemented using the quantum search algorithm due to Grover [36]. This algorithm has been traditionally considered as implementing a search for an item in an unsorted (quantum) database of N items, and it performs this task in O($\sqrt{N}$) time, a feat that is impossible classically. In the quantum computational setting, finding an item in the database means measuring the system and having the system collapse to the basis state which corresponds to the item in the database for which we are searching. Now, we can equally well consider the algorithm as accomplishing the task of pattern completion in a quantum associative memory. The basic idea of Grover's algorithm is to invert the phase of the desired basis state and then to invert all the basis states about the average amplitude of all the states. Repetition of this process produces an increase in the amplitude of the desired basis state to near unity followed by a corresponding decrease in the amplitude of the desired state back to its original magnitude. The process has a period of $\frac{\pi}{4}\sqrt{N}$ and thus after O($\sqrt{N}$) operations, the system may be observed in the desired state with near certainty. Define

$$\hat{I}_{\phi} = \text{identity matrix except for } \; i_{\phi\phi} = \text{-1}$$

which inverts the phase of the basis state ϕ,

$$\hat{W} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

which is often called the Walsh transform, and

$$\hat{G} = -\hat{W}\hat{I}_{\bar{0}}\hat{W}$$

which effects the inversion about average. Now to perform the search on a quantum database of size N, begin with the system in the $\left|\bar{0}\right\rangle$ state and apply the $\hat{W}$ operator. This initializes all the possible states to have the same amplitude. Finally, apply the operator $\hat{G}\hat{I}_{\tau}$ (recall that operators are applied right to left), where τ is the state being sought, $\frac{\pi}{4}\sqrt{N}$ times and observe the system.

Combining the algorithms

A quantum associative memory can now be implemented by combining the two algorithms just discussed. Define $\hat{P}$ as an operator that implements the algorithm for memorizing patterns. Then the operation of the memory can be described as follows. Memorizing a set of patterns is simply

$$|\psi\rangle = \hat{P}|\overline{0}\rangle$$

with $|\psi\rangle$ being a quantum superposition of appropriate basis states, one for each pattern. Now, suppose we know n-k bits of a pattern and wish to recall the entire pattern. We can use a modification of Grover's algorithm to complete the pattern, producing one of the stored patterns that matches on the n-k bits that we know. Thus, with $2n$+1 neurons (qubits) the quantum associative memory can store up to 2^n patterns in O(mn) time and requires O($\sqrt{2^n}$) time to recall a pattern. This last bound is somewhat slower than desirable and may be improved with a non-unitary recall mechanism. In fact, Grover's search algorithm has been proven to be optimal in the number of steps required when unitarity is required. Thus, we have another motivation for non-unitary processes in quantum neural computation.

Recall – association

Of course, in general, a quantum memory should not only be able to complete patterns but also to correct them. In other words, given a noisy stimulus, the memory should produce the pattern most similar to that input. This can be accomplished with further modification of the basic quantum memory model we have been discussing. This modification involves the use of distributed queries and is presented in detail in [37]. Briefly, a distributed query is a distribution of the form

$$|b^p\rangle = \sum_{x=0}^{2^d-1} b_x^p |x\rangle$$

over the amplitudes of all possible states in the memory . The index p marks one of these states, $|p\rangle$, which is the center of the distribution (real-valued amplitudes are distributed such that the maximal value occurs at this center, and the amplitudes of the other basis states decrease monotonically with Hamming distance from the center state). This leads to the introduction of spurious memories into the recall process; however, counter to intuition the presence of these spurious memories may actually facilitate memory recall [37]. Table 3 summarizes the analogies used in developing a quantum associative memory.

Table 3. Corresponding concepts from the domains of classical neural networks and quantum associative memory

Classical neural networks		Quantum associative memory	
Neuronal State	$x_i \in \{0,1\}$	Qubit	$\|x\rangle = a\|0\rangle + b\|1\rangle$
Connections	$\{w_{ij}\}_{ij=1}^{p-1}$	Entanglement	$\|x_0 x_1 \ldots x_{p-1}\rangle$

Learning rule	$\sum_{s=1}^{p} x_i^s x_j^s$	Superposition of entangled states	$\sum_{s=1}^{p} a_s \mid x_0^s \ldots x_{p-1}^s \rangle$
Winner search	$n = \max_i \arg(f_i)$	Unitary transformation	$U: \psi \to \psi'$
Output result	n	Decoherence	$\sum_{s=1} a_s \mid x^s \rangle \Rightarrow \mid x^k \rangle$

It should be noted that the "neuron" in the first row of the Table 3 is strictly artificial and should not be considered as a model of its biological analog. Really, as stated by Penrose "...it is hard to see how one could usefully consider a quantum superposition consisting of one neuron firing, and simultaneously not firing" [2]. There are many other arguments against attributing any biological meaning to this scheme, so we should consider it only in the context of the development of *artificial* quantum associative memory.

Implementation of QNN

How can quantum neural networks be implemented as real physical devices? First, let us mention briefly some of the difficulties we might face in the development of a physical realization of quantum neural networks.

Coherence. One of the most difficult problems in the development of any quantum computational system is the maintainence the system's coherence until the computation is complete [38]. This loss of coherence (decoherence) is due to the interaction of the quantum system with its environment. In quantum cryptography this problem may be resolved using error-correcting codes [38]. What about quantum neural networks? It has been suggested that if fact these systems may be implemented before ordinary quantum computers will be realized because of significantly lower demands on the number of qubits necessary to represent network nodes and also because of the relatively low number of state transformations required during data processing in order to perform useful computation [35, 39]. Another approach to the problem of decoherence in quantum parallel distributed processing proposed by Chrisley excludes the use of superpositional states at all and suggests the use of quantum systems for implementing standard neural paradigms, i.e. multilayer neural systems trained with backpropagation learning [20]. This model, however, takes no advantage of the use of quantum parallelism. A more promising approach to the implementation of quantum associative memory based on the use of Grover's algorithm is provided by bulk spin resonance computation (see below).

Connections. The high density of interconnections between processing elements is a major difficulty in the implementation of small-scale integration of computational systems. In ordinary neurocomputers these connections are made via wires. In (non-superpositional) quantum neurocomputers they are made via forces. In the quantum associative memory model discussed here, these connections are due to the entanglement of qubits.

Physical systems. Now we can outline what kind of physical systems might be used to develop real quantum neural networks and how these systems address the problems listed above.

- **Nuclear Magnetic Resonance.** A promising approach to the implementation of quantum associative memory based on the use of Grover's algorithm is provided by *bulk spin resonance computation*. This technique can be performed using Nuclear Magnetic Resonance systems for which coherence times on the order of thousands of seconds have been observed. Experimental verification of such an implementation has been done by Gershenfeld and Chuang [40] (among others), who used NMR techniques and a solution of chloroform ($CHCl_3$) molecules for the implementation of Grover's search on a system consisting of two qubits – the first qubit is decribed by the spin of the nucleus of the isotope C^{13}, while second one is described by the spin of the proton (hydrogen nucleus). Rather interestingly, this approach to quantum computation utilizes not a single quantum system but rather the statistical average of many copies of such a system (a collection of molecules). It is precisely for this reason that the maintenance of system coherence times is considerably greater than for true quantum implementations. Further, this technology is relatively mature, and in fact coherent computation on seven qubits using NMR has recently been demonstrated by Knill, et al. [41]. This technology is most promising in the short term, and good progress in this direction is possible in the early 21st century.

- **Quantum dots.** These quantum systems basically consist of a single electron trapped inside a cage of atoms. These electrons can be influenced by short laser pulses. Limitations to these systems which must be overcome include 1) short decoherence times due to the fact that the existence of the electron in its excited state lasts about a microsecond, and the required duration of a laser pulse is around a nanosecond; 2) the necessity of developing a technology to build computers from quantum dots of very small scale (10 atoms across); 3) the necessity of developing special lasers capable of selectively influencing different groups of quantum dots with different wavelengths of light. The use of quantum dots as the basis for the implementation of QNN is being investigated by Behrman and co-workers [16-17].

- **Other systems.** There are many other physical systems which are now being considered as possible candidates for the implementation of quantum computers (and therefore possibly quantum neurocomputers). These include various schemes of *cavity QED* (quantum electrodynamics of atoms in optical cavities), *ion traps*, *SQUID*s (superconducting quantum interference devices), etc. Each has its own advantages and shortcomings with regard to decoherence times, speed, possibility of miniaturization, etc. More information about these technologies can be found in [4, 31].

Can QNN outperform classical neural networks?

It is now known that quantum computing gives us unprecedented possibilities in solving problems beyond the abilities of classical computers. For example Shor's algorithm gives a polynomial solution (on a quantum computer) for the problem of prime factorization, which is believed to be classically intractable [42]. Also, as previously mentioned, Grover's algorithm provides super-classical performance in searching an unsorted database.

What of quantum neural networks? Will they give us some advantages unattainable by either traditional von Neumann computation or classical artificial neural networks? Compared to the latter, quantum neural networks will probably have the following advantages:

- exponential memory capacity [30];
- higher performance for lower number of hidden neurons [39];
- faster learning [32];
- elimination of catastrophic forgetting due to the absence of pattern interference [32];
- single layer network solution of linearly inseparable problems [32];
- absence of wires [17];
- processing speed (10^{10} bits/s) [17];
- small scale (10^{11} neurons/mm^3) [17];
- higher stability and reliability [39];

These potential advantages of quantum neural networks are indeed compelling motivation for their development. However, the more remote future possibilities of QNN may be even more exciting.

Frontiers of QNN

It is generally believed that the right hemisphere is responsible for spatial orientation, intuition, semantics etc., while the left hemisphere is responsible for temporal processing, logical thinking and syntax. Given this view, it is very natural to consider that neurocomputers can be thought of as imitating our right brain function while von Neumann computers can be thought as as mimicing the functionality of our left brain. Penrose characterizes these two types of computation as *bottom-up* and *top-down* respectively. Nevertheless, he argues that higher brain functions such as consciousness cannot be modelled using just these types of computation. The ideas discussed in this chapter introduce the possibility of combining the unique computational abilities of classical neural networks and quantum computation, thus producing a computational paradigm of incredible potential. However, we make no effort here to relate any of these concepts to biological systems; in fact, much of what we have discussed is most likely very different from biological neural information processing. Therefore it seems unlikely that quantum neural networks, at least in the context discussed here, could be considered a candidate for the basis of consciousness. However, Perus has suggested that neural networks can be a "macroscopic replica of quantum processing structures". If so, they "could be an interface between the macro-world of man's environment and the micro-world of his non-local consciousness" [43]. Thus, it is not out of the realm of possibility that future models of quantum neural networks may afterall provide significant insight into the workings of the mind and brain.

There are some proponents for the idea that QNN may be developed that have abilities beyond the restrictions imposed by the Church-Turing thesis. Simply put, according to this thesis, all existing computers are equivalent in computational power to the Universal Turing Machine. Moreover, all *algorithmic* processes we can perform in our mind can be realized on this machine and *vice versa*. No existing neurocomputers, nor any quantum computers theorized to date can escape the bounds imposed by the Church-Turing thesis. But what about quantum neural networks? Dan Cutting has posed the query, "*Would quantum neural networks be subject to the decidability constraints of the Church-Turing thesis*?" [39]. For existing models of QNN the answer seems surely to be "no", but some speculative physical systems (wormholes, for example) are discussed as possible candidates for the basis of QNN that could exceed these bounds [39]. This is a very intriguing question, and it is a challenge for the future to try to develop a theory of quantum neural networks that will give us completely new computational abilities for tackling problems that cannot now be solved even in principle. In the process we shall certainly be examining the concept of computation in a very different light and in so doing will be likely to make discoveries that to this point have been overlooked.

Acknowledgements

We are grateful to Professor Nikola Kasabov for his invitation to prepare this chapter. We also acknowledge useful discussions with Mitja Perus, Tony Martinez, Ron Chrisley, Dan Cutting, Elizabeth Behrman, and Subhash Kak on various aspects of quantum neural computation.

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