A. La Rosa Lecture Notes

INTRODUCTION TO QUANTUM MECHANICS

PART- I The TRANSITION from CLASSICAL to QUANTUM PHYSICS

APPENDIX -1 (Chapter-3: The Origins of Quantum Physics)

A. Symmetry of the physics law at the micro-scale

Let's put the concepts above within the context of a microscopic description.

A.1 Principle of detailed balance

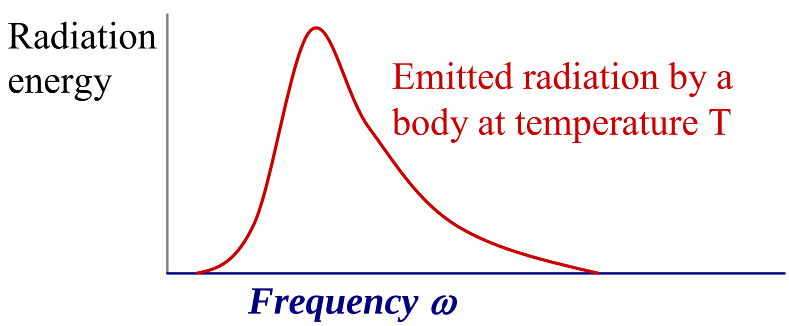


Fig. 1

- Since radiation is composed of many different frequencies, the equilibrium condition $R_{emitted} = R_{absorbed}$ leaves open few possibilities under which it could be satisfied.

  For example, it is conceivable that,
  in one frequency range the body radiates more power than it absorbs, while

in another frequency range it radiates less power than it absorbs, but
in such a way that the overall total energy balance preserves.

- Using very general arguments a stronger principle can be established:

  In equilibrium the power radiated and absorbed by a body must be equal for any particular element of area of the body for any particular direction of polarization, and for any frequency range. (1)

  This is the principle of detailed balance.[1]

A.2 Invariance of the physics law under time reversal

- The basic justification of the principle of detailed balance rest in the fundamental laws of microscopic physics, i.e. the Schrodinger equation, Maxwell equation, etc. These microscopic laws are invariant under reversal of the time (change of variable from *t* to -*t*.)

  ➢ For example, when describing a collision among particles, a time reversal implies a corresponding reversal of all the velocities; thus one obtains the "reverse" collision. (2)

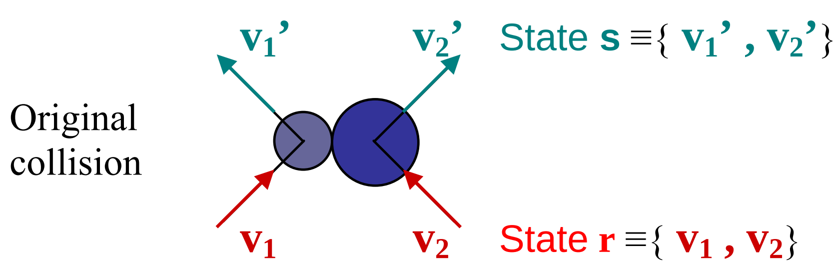
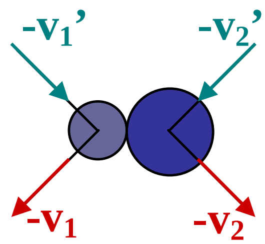


Fig. 2 Direct collision

Let's call w_{rs}=Probability that the initial state **r** results into a state **s** after the collision. Symbolically,

$$w_{rs} = \text{Prob}(r \rightarrow s) \qquad (3)$$

Reverse collision



State **s*** $\equiv\{-\mathbf{v}_1', -\mathbf{v}_2'\}$

State **r*** $\equiv\{-\mathbf{v}_1, -\mathbf{v}_2\}$

Fig. 3 Reverse collision

For the reverse collision we would have a transition from the state **s*** (obtained reversing the velocities of state **s**) to the state **r*** (obtained reversing the velocities of state **r**.)

$$W_{s^*r^*} = \mathrm{Prob}(\, s^* \rightarrow r^*) \qquad (4)$$

The invariance of the microscopic laws implies,

$$W_{rs} = W_{s^*r^*} \qquad (5)$$

- Another example considers the emission process of a photon with a wave vector $\boldsymbol{k}$. The reverse process obtained by reversing the sign of the time t is the absorption of a photon of wave vector $-\boldsymbol{k}$. The microscopic reversibility (4) indicates that these two processes occur with equal probability. We use this concept in the following section.

B. Kirchhoff's law

The principle of detailed balance requires a more specific notation.

$\mathcal{P}_e(\boldsymbol{k})\ d\omega\ d\Omega$ = emitted power per unit area of the body, composed of electromagnetic radiation of frequency between ω and ω+dω, and directed into a solid angle dΩ about the direction $\boldsymbol{k}$ (6)

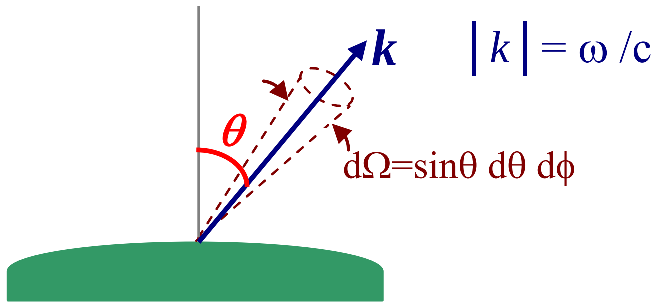


Fig. 4 Radiation of wavevector around **k** emitted by a surface at temperature T

G. R. Kirchhoff proved in 1859 by using general thermodynamic arguments that, at any given electromagnetic wavelength,

the ratio of the emissive power $\mathcal{P}_e$ to the absorption coefficient a is the same for all bodies at the same temperature

$$\left.\frac{\mathcal{P}_e}{a}\right|_1 = \left.\frac{\mathcal{P}_e}{a}\right|_2 = \left.\frac{\mathcal{P}_e}{a}\right|_3 = etc \qquad (7)$$

Justification:

Consider a body at temperature T, surrounded by an atmosphere of electromagnetic radiation. Let's specialize in the radiation of angular frequency ω [or, equivalently, in radiation of wavelength λ=c/(w/2π).] In terms of the wave vector $\boldsymbol{k}$ of magnitude k$\equiv$2π/λ, we have,

Fig. 5 Incident and emission radiation processes.

Direct process:

A radiation power $\mathcal{P}_i(\boldsymbol{k})\ d\omega\ d\Omega$ is incident on a unit area

A fraction $a(\boldsymbol{k})$ of this energy is absorbed, $a(\boldsymbol{k})\,\mathcal{P}_i(\boldsymbol{k})\,d\omega\,d\Omega$,

the rest $\sum_{\theta'}\mathcal{R}_{\theta'}$ being reflected in (eventually) many directions.

The reverse process, which occurs with the same probability (as indicated above), will resemble

Incident waves of energy $\sum_{\theta'}\mathcal{R}^{*}_{\theta'}$; plus

emission of energy from the sample

both terms making a total of $\mathcal{P}_e(-\boldsymbol{k})\,d\omega\,d\Omega$

Since the energy associated to $\sum_{\theta}\mathcal{R}_{\theta}$ and $\sum_{\theta}\mathcal{R}^{*}_{\theta}$ should be the same (the inversion of $\boldsymbol{k}$ does not change the energy of the states), the application of the principle of detailed balance requires that

$$\mathcal{P}_e(-\boldsymbol{k}) = a(\boldsymbol{k})\,\mathcal{P}_i(\boldsymbol{k}) \tag{8}$$

Or, equivalently

$$\frac{\mathcal{P}_e(-\boldsymbol{k})}{a(\boldsymbol{k})} = \underbrace{\mathcal{P}_i(\boldsymbol{k})} \tag{9}$$

Notice, this term is independent of the nature of the body.

Notice in the above expression that the term on the right

is independent of the form of the body, or any other intrinsic material properties of the body;

it will depend only on the necessary conditions to establish equilibrium, all of which is determined by the equilibrium temperature T.

Kirchhoff concluded then that $\mathcal{P}_e(-\boldsymbol{k})/a(\boldsymbol{k})$ is the same for all bodies at the same temperature, including the blackbody radiator.

$$\left.\frac{\mathcal{P}_e}{a}\right|_1 = \left.\frac{\mathcal{P}_e}{a}\right|_2 = \mathcal{P}_e\big|_{blackbody} \tag{10}$$

C. Cavity's aperture as an approximation to a blackbody radiator

A blackbody radiator is an idealization; however, a small orifice in a cavity closely approximates one.

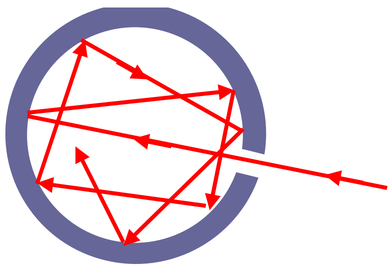


Fig. 6 *Blackbody radiator.*

- Any radiation incident from the outside and passing through the hole will almost completely be absorbed in multiple reflections inside the cavity; that is, the hole has an effective absorption coefficient close to unity ($a=1$)
- Since the cavity is in thermal equilibrium,
  the radiation inside and that escaping from the small opening will be equal (e=1)

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[1] F. Reif, "Fundamentals ofStatistical and Thermal Physics," McGraw Hill, Section 9.15.