Mészáros effect

In cosmology, the development of initial perturbations which eventually give rise to structures such as galaxies, clusters of galaxies, etc. can be described in the fluid approximation by perturbing the Friedmann equations. The relative density constrast $\delta \equiv \Delta\rho/\langle\rho\rangle$ (where ρ is the matter-energy density and $\Delta\rho$ is the small excess in any given region) obeys the perturbation equation

$$\ddot{\delta} + 2\left(\frac{\dot{a}}{a}\right)\dot{\delta} + \left(v_s^2 k^2 - 4\pi G\rho\right)\delta = 0. \tag{1}$$

e.g. [1, 2, 3], where a is the expansion scale factor of the Universe, $v_s = (\partial P/\partial\rho)^{1/2}$ is the sound speed, k is wavenumber, G is Newton's constant. For perturbations inside the horizon this leads to well-known results for the Jeans mass and growth times for non-relativistic collisionless perturbations (cold "matter" $P = 0$) and for relativistic collisional perturbations (e.g. a "radiation gas" $P = \rho c^2/3$). In the early Universe the $P = 0$ case corresponds to what is known today as cold dark matter, and the radiation case corresponds to photons and neutrinos. After radiation-matter decoupling $z < z_{dec} \sim 10^3$ the perturbation growth is given by the $P = 0$ matter-dominated solution, while before the matter-radiation equilibrium epoch $z < z_{eq} \sim 10^4$ the growth is given by the radiation-dominated solution.

However, in a combined picture of collisionless matter in a radiation background one has a mode of perturbations inside the horizon where the collisionless (non-relativistic) matter component of density ρ_m is perturbed relative to the relativistic radiation component of density ρ_r (which for $z > z_{dec}$ oscillates and on average can be considerd as unperturbed, just following the Universe's expansion). This was first considered in [4], leading to the perturbation equation

$$\ddot{\delta} + 2\left(\frac{\dot{a}}{a}\right)\dot{\delta} - \left(4\pi G\rho_m\right)\delta = 0, \tag{2}$$

where now $(\dot{a}/a)^2 = 8\pi G(\rho_m + \rho_r)/3$, and $k = 0$ is appropriate for early times. Changing variables to $y = \rho_m/\rho_r \equiv (a/a_{eq})$ and using Friedman's equation leads to

$$\delta'' + \frac{2+3y}{2y(1+y)}\delta' - \frac{3}{2y(1+y)} = 0. \tag{3}$$

This equation has closed analytical solutions, the growing mode of which can be seen to be

$$\delta \propto 1 + \frac{3}{2}y\ . \tag{4}$$

That is, for $a < a_{eq}$ the (cold) matter perturbation remain "frozen", $\delta =$ constant, while for $a > a_{eq}$ the matter perturbation grows linearly with $y = a/a_{eq}$. This is referred to as the Mészáros effect (or equation); e.g. Google these keywords, or see references above (also [6, 7, 8, 9, 10],..). This effect is important for the initial perturbations in the cold dark matter to achieve non-linearity, leading to the observed galaxies and clusters at the present epochs; it adds extra growth time between z_{eq} and z_{dec}.

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