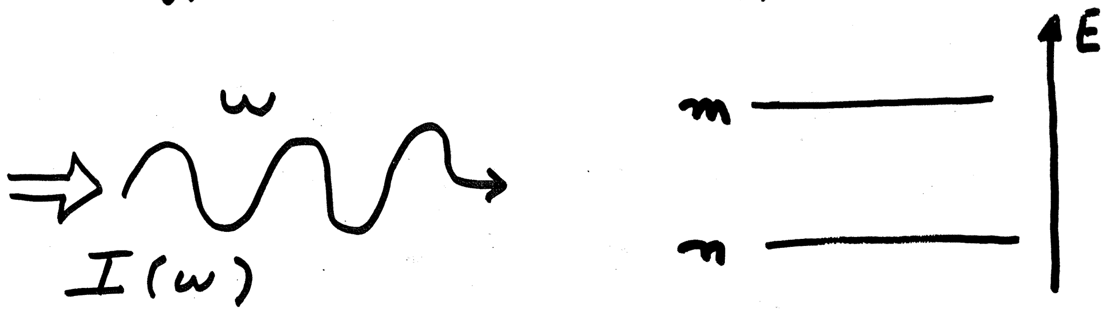
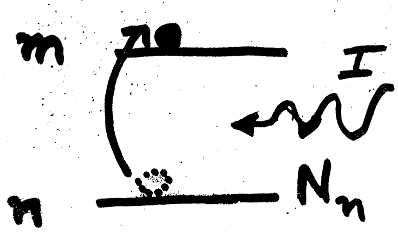
C.2 LIGHT-MATTER INTERACTION
Einstein's Law of Radiation

quantized energy levels E_m and E_n.



When the incident light has the right frequency $\hbar\omega = E_m - E_n$, three processes can occur:

<u>ABSORPTION</u>



An atom absorbes a photon thus making a transition from state n to state m

The probability of this transition would depend on

- the light intensity spectral density $I(\omega)$ in J/m^2
- the nature of the states m and n

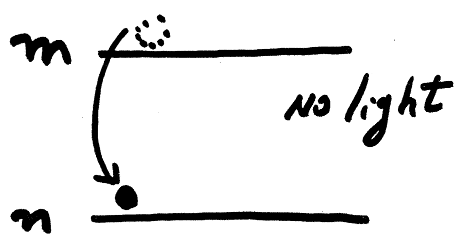
Let's assume the probability of this transition P_{nm} to occur per second is proportional to $I(\omega)$

$$P_{nm} = B_{nm} I(\omega) \qquad \text{Absorption}$$

EMISSION

Einstein suggested there should be two types of emission processes

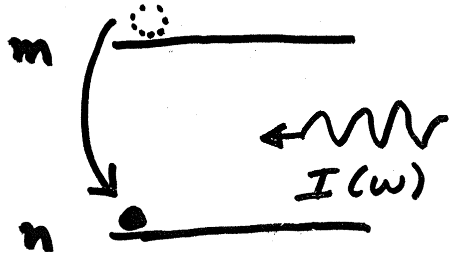
Spontaneous emission



Even when there is no light present, there is certain probability A_{mn} per second that the atom will transit from the excite state m to the lower state n

We further assume that A_{mn} is the same whether light is present or not

Stimulated emission



the emission probability is further influenced by the presence of light

Einstein assumed this probability per second to be proportional to light intensity spectral density $I(\omega)$

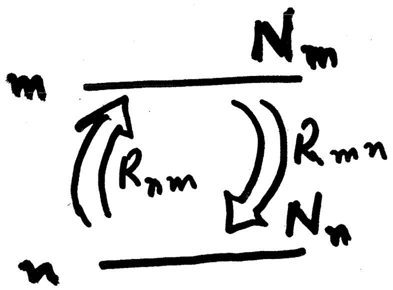
stimulated emission probability per second $= \underbrace{B_{mn}}_{\text{const of proportionality}} I(\omega)$

Accordingly, the total emission probability per second would be

$$P_{mn} = A_{mn} + B_{mn} I(\omega)$$

Equilibrium conditions

At temperature T, N_n atoms will be in the state n, and N_m in the state m.



$$R_{n\to m} = N_n B_{nm} I(\omega)$$

rate at which atoms transit from n to m

$$R_{m\to n} = N_m \left(A_{mn} + B_{mn} I(\omega) \right)$$

At equilibrium, these two rates should be equal (so the number of atoms in each energy level remains constant.)

$$N_n B_{nm} I(\omega) = N_m \left(A_{mn} + B_{mn} I(\omega) \right)$$

$\Rightarrow$

$$I(\omega) = \frac{A_{mn}}{\frac{N_n}{N_m} B_{nm} - B_{mn}}$$

Since $N_n \propto e^{-\frac{1}{kT} E_n}$

$$N_m \propto e^{-\frac{1}{kT} E_m}$$

$\Rightarrow$

$$\frac{N_n}{N_m} = e^{-\frac{1}{kT}(E_n - E_m)} = e^{\frac{1}{kT}(E_m - E_n)}$$

But the frequency ω that we are considering above is the one matching $\hbar\omega = E_m - E_n$.

thus,

$$\frac{N_n}{N_m} = e^{\frac{\hbar\omega}{kT}}$$

$$I(\omega) = \frac{A_{mn}}{B_{nm} e^{\frac{\hbar\omega}{kT}} - B_{mn}}$$

But Planck's formula tell us what $I(\omega)$ we should have under equilibrium conditions

$$I(\omega) = \frac{\hbar \omega^3}{\pi^2 c^2} \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}$$

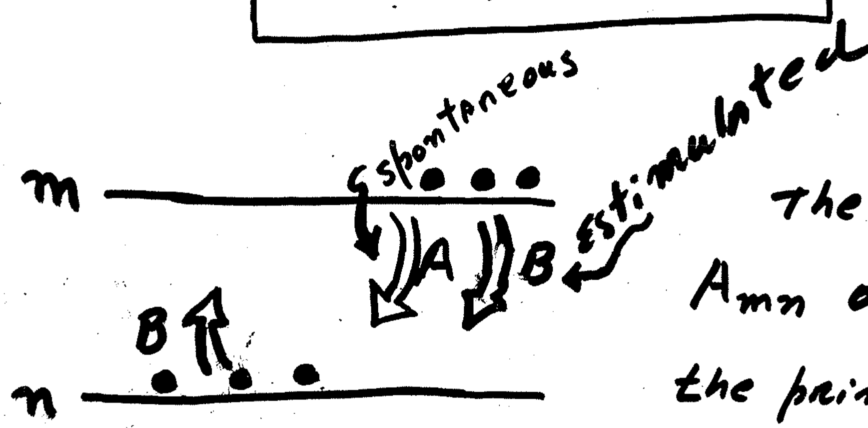
this implies,

$$\boxed{B_{mn} = B_{nm}}$$

Induced emission and absorption probabilities are equal.

and

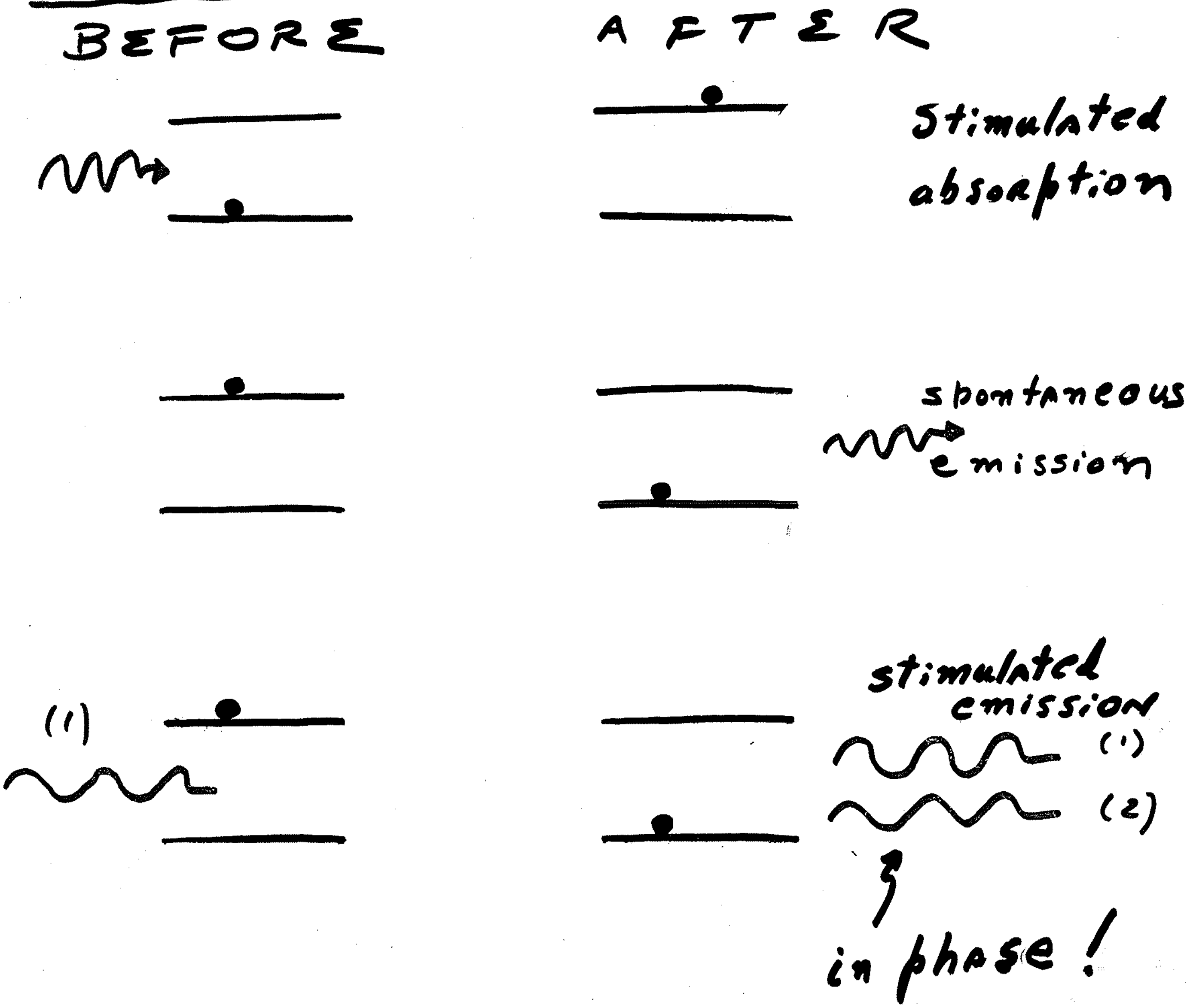
$$\boxed{A_{mn} = \frac{\hbar \omega^3}{\pi^2 c^2} B_{nm}}$$



The cubic dependence A_{mn} on ω accounts for the principal difficulty in achieving <u>laser action</u> at x-ray frequencies

At these high frequencies spontaneous emission occurs so rapidily that a sustained stimulated emission is difficulty to achieve. At lower frequencies (visible) this difficulty is, fortunately, no great.

Stimulated Emission (LASER)



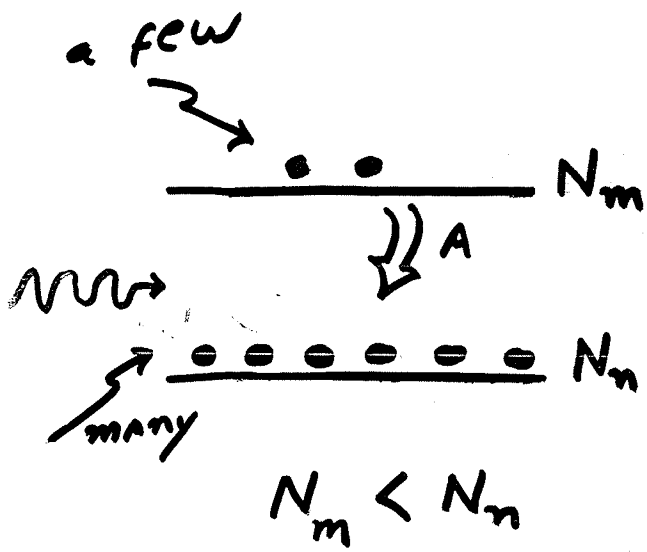
- The most striking feature in the stimulated emission process is that the photon resulting from the transition is <u>in phase</u> with the incident photon that provokes the transition.
- Stimulated emission is a constructive interference process.

<u>CAN WE have a sustained stimulated emission process ?</u>

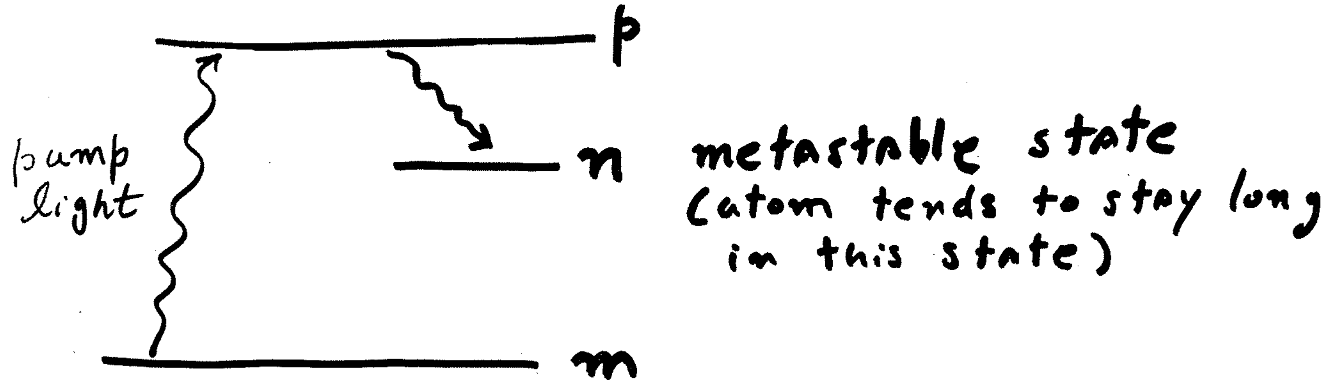
Two factors play against,

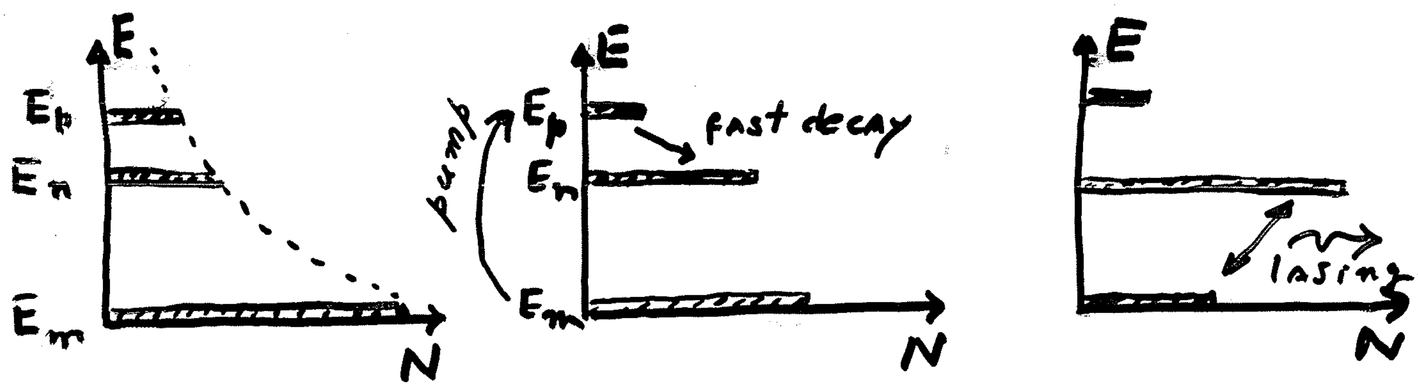
1) At equilibrium, there are only a few atoms in the excited state

2) Out of the few excited atoms, the spontaneous emission process lowers even more the number of atoms available for stimulated emission.



We can not avoid the spontaneous emission. So, scientists had to figure out how to revert the condition $N_m < N_n$, which implies to consider situations out of equilibrium then. One strategy is shown in the figure below: OPTICAL PUMPING



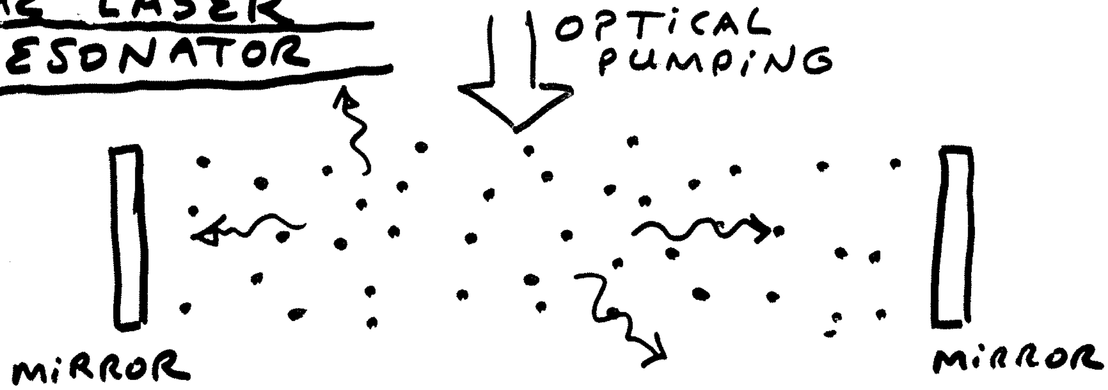


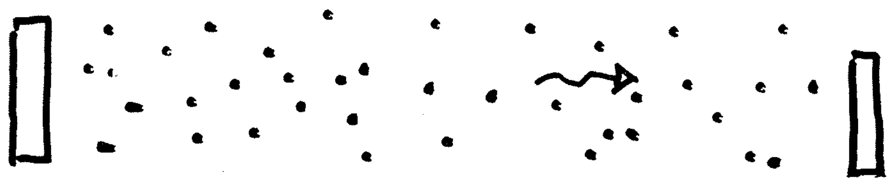
No pumping

Moderate pumping

Intense pumping

THE LASER RESONATOR





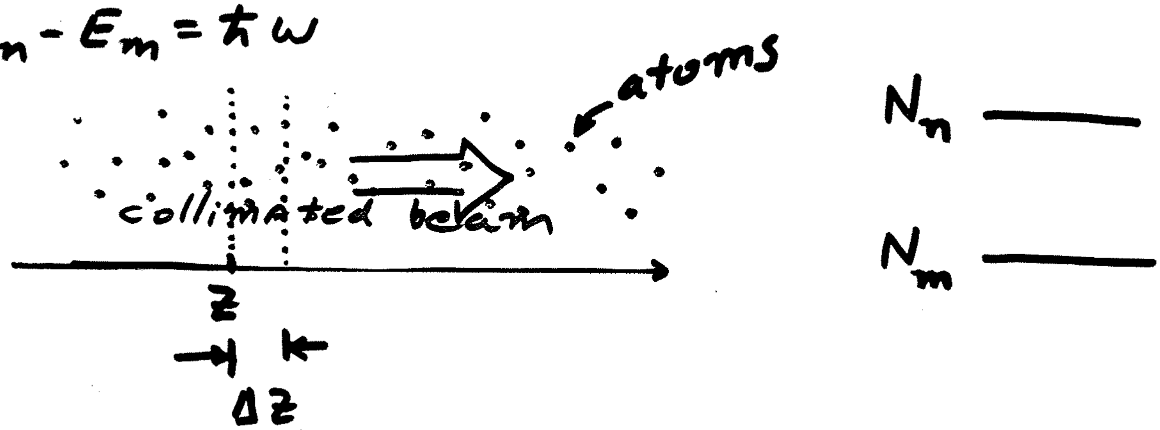
For perfectly parallel mirrors, the horizontally collimated beam intensity is reinforced through a) many back and forth reflections, b) the accompaning stimulated emission they produced.

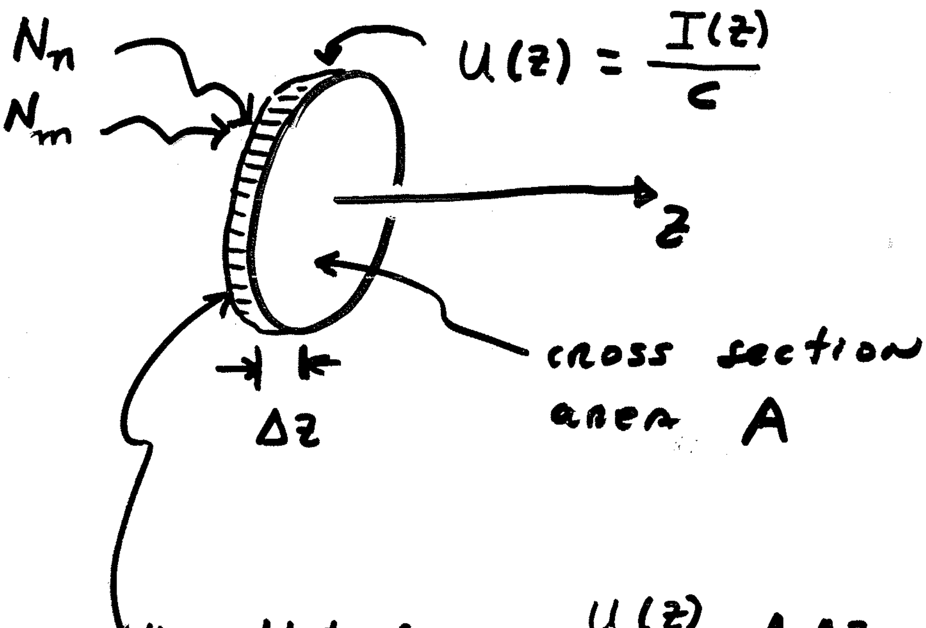
The latter occurs provided the population inversion is maintained.

ABSORPTION COEFFICIENT and POPULATION INVERSION

Let's consider the variation in intensity of a collimated beam (photons of energy $\hbar\omega$) advancing in a medium containing atoms that have energy levels E_m and E_n such that

$$E_n - E_m = \hbar\omega$$





$$U(z) = \frac{I(z)}{c}$$

U: energy per unit volume

$\dfrac{U}{\hbar\omega}$ = photons per unit volume $= \eta$

\# of photons in that volume $= \dfrac{U(z)}{\hbar\omega} A\,\Delta z = \eta(z) A\,\Delta z$

$$N_n = \mathcal{N}_n \, A \, \Delta z$$

$$N_m = \mathcal{N}_m \, A \, \Delta z$$

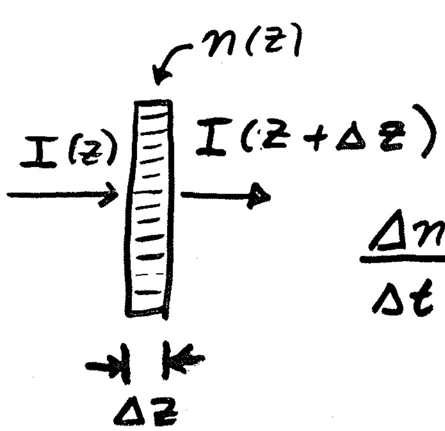
We assume there is a constant and uniform density of excited states $\mathcal{N}_n$, $\mathcal{N}_m$ (which will change, but only after the beam has passed)

$$N_m \underbrace{B_{mn}}_{\text{Probability per second an atom will be absorbed}} I(x) = \text{\# of photons lost per second by the collimated beam.}$$

$$N_n B_{nm} I(x) = \text{\# of photons gained per second by the collimated beam.}$$

$$(N_n - N_m) B_{nm} I(x) = \frac{\Delta n(z)}{\Delta t} \quad \text{Net gain of photons in the collimated beam} \qquad (1)$$

The net gain of photons can also be expressed in terms of the net flux traversing the volume $A \Delta z$:



$$\frac{\Delta n}{\Delta t} = \frac{I(z+\Delta z)}{\hbar \omega} A - \frac{I(z)}{\hbar \omega} A$$

$$\frac{\Delta n}{\Delta t} = \frac{A}{\hbar\omega}\left[I(z+\Delta z) - I(z)\right]$$

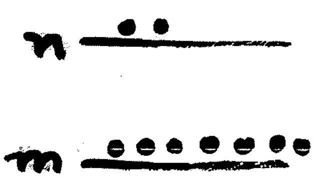
$$= \frac{A}{\hbar\omega}\,\frac{\Delta I}{\Delta z}\,\Delta z \qquad (2)$$

Equating (1) and (2)

$$\underbrace{(N_n - N_m)}_{\mathcal{N}_n A \Delta z} B_{nm}\, I(x) = \frac{dI}{dx}\,\frac{1}{\hbar\omega}\,A\Delta z$$

$$\Rightarrow \quad \frac{dI}{dx} = \hbar\omega(\mathcal{N}_n - \mathcal{N}_m)\, B_{nm}\, I(x)$$

If the gas of atoms is in equilibrium, $\mathcal{N}_n < \mathcal{N}_m$, so $\frac{dI}{dx}$ decreases.



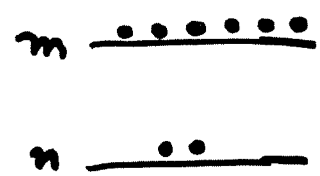
$$\frac{dI}{dz} = -\underbrace{(\mathcal{N}_m - \mathcal{N}_n)B_{nm}\hbar\omega}_{\alpha}\; I(z)$$

$$I(z) = I(0)\,e^{-\alpha z} \qquad \alpha = (\mathcal{N}_m - \mathcal{N}_n)B_{nm}\hbar\omega$$

Absorption coefficient

If, however, we created a condition where

$$\mathcal{N}_m > \mathcal{N}_n$$



the α would become negative, and the intensity I would grow with distance

$$I(z) = I(0)\, e^{\beta z}$$

$$\beta = (\mathcal{N}_n - \mathcal{N}_m)$$

called "small-signal gain coefficient