12 OPTICAL METROLOGY*

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12.1 GLOSSARY

B	baseline length
f	focal length
f	signal frequency
f_b	back focal length
I	irradiance
N	average group refractive index
R	radius of curvature of an optical surface
r	radius of curvature of a spherometer ball
R	range
α	attenuation coefficient
λ	wavelength of light
Λ	synthetic wavelength
τ	delay time

In the optical shop, the measuring process has the purpose to obtain a comparison of physical variables using optical means. In this chapter we describe the most common procedures for the measurements of length, angle, curvature, and focal length of lenses and mirrors. The reader may obtain some more details about these procedures in the book *Optical Shop Testing* by D. Malacara,[1] or in Chap. 32, "Interferometers," in Vol. I.

*Note: Figures 27, 30 and 31 are from *Optical Shop Testing*, 3d ed., edited by D. Malacara. (Reprinted with permission John Wiley and Sons, Inc. New York, 2007.)

12.2 INTRODUCTION AND DEFINITIONS

Lens parameter measuring in the optical shop has been a permanent problem in respect to the unit's agreement. Usually, rather than using SI units for length specifications, wavelength units is still a common reference for precision length measurements. Although the meter is obtained practically from the wavelength of a krypton source, helium-neon lasers are a common reference wavelength. The generally accepted measurement system of units is the International System or Sistéme International (SI).

Time is the fundamental standard and is defined as follows: "The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom." Formerly, the meter was a fundamental standard defined as 1,650,763.73 wavelengths in vacuum of the orange-red spectral line from the $2p_{10}$ and $5d_5$ levels of the krypton 86 atom. Shortly after the invention of the laser, it was proposed to use a laser line as a length standard.[2] In 1986, the speed of light was defined as 299,792, 458 m/s, thus the meter is now a derived unit defined as "the distance traveled by light during 1/299,792,458 of a second." The advantages of this definition, compared with the former meter definition, lie in the fact that it uses a relativistic constant, not subjected to physical influences, and is accessible and invariant. To avoid a previous definition of a time standard, the meter could be defined as "The length equal to 9,192,631,770/299,792,458 wavelengths in vacuum of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom."[3,4] Modern description of the time and distance standards are described by Cundiff et al.[5]

The measurement process is prone to errors. They may be systematic or stochastic. A systematic error occurs in a poorly calibrated instrument. An instrument low in systematic error is said to be accurate. Accuracy is a measure of the amount of systematic errors. The accuracy is improved by adequate tracing to a primary standard. Stochastic errors appear due to random noise and other time-dependent fluctuations that affect the instrument. Stochastic errors may be reduced by taking several measurements and averaging them. A measurement from an instrument is said to be reproducible when the magnitude of stochastic errors is low. Reproducibility is a term used to define the repeatability for the measurements of an instrument. In measurements, a method for data acquisition and analysis has to be developed. Techniques for experimentation, planning, and data reduction can be found in the references.[6,7]

12.3 LENGTH AND STRAIGHTNESS MEASUREMENTS

Length measurements may be performed by optical methods, since the definition of the meter is in terms of a light wavelength. Most of the length measurements are, in fact, comparisons to a secondary standard. Optical length measurements are made by comparisons to an external or internal scale a light time of flight, or by interferometric fringe counting.

Stadia and Range Finders

A *stadia* is an optical device used to determine distances. The principle of the measurement is a bar of known length W set at one end of the distance to be measured (Fig. 1). At the other end, a prism superimposes two images, one coming directly and the other after a reflection from a mirror, which are then observed through a telescope. At this point, the image of one end of the bar is brought in coincidence with that of the other end by rotating the mirror an angle θ. The mirror rotator is calibrated in such a way that for a given bar length W, a range R can be read directly in a dial, according to the equation:[8]

$$R = \frac{W}{\theta} \tag{1}$$

where θ is small and expressed in radians.

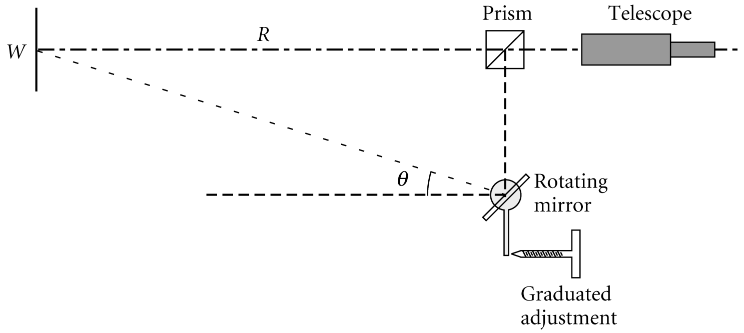

FIGURE 1 A stadia range meter. (*From Patrick.*[8])

Another stadia method uses a graduated bar and a calibrated reticle in the telescope. For a known bar length W imaged on a telescope with focal length f, the bar on the focal plane will have a size i, and the range can be calculated approximately by:[9]

$$R = \left(\frac{f}{i}\right) W \tag{2}$$

Most surveying instruments have stadia markings usually in a 1:100 ratio, to measure distances from the so-called anallatic point, typically the instrument's center. A theodolite may be used for range measurements using the subtense bar method. In this method, a bar of known length is placed at the distance to be measured from the theodolite. By measuring the angle from one end to another end of the bar, the range can be easily measured.

A range finder is different from the stadia, in that the reference line is self-contained within the instrument. A general layout for a range finder is shown in Fig. 2. Two pentaprisms are separated a known baseline B; two telescopes form an image, each through a coincidence prism. The images from the same reference point are superimposed in a split field formed by the coincidence prism. In one branch, a range compensator is adjusted to permit an effective coincidence from the reference images.

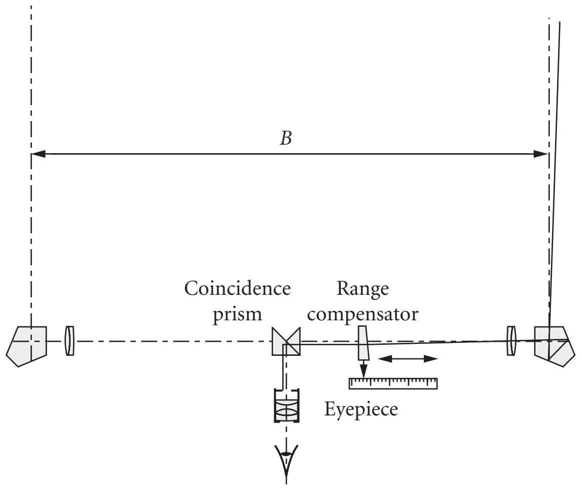

FIGURE 2 A range finder.

Assuming a baseline B and a range R, for small angles (large distances compared with the baseline), the range is

$$R = \frac{B}{\theta} \tag{3}$$

For an error $\Delta\theta$ in the angle measurement, the corresponding error ΔR in the range determination would be

$$\Delta R = -B\theta^{-2}\Delta\theta \tag{4}$$

and by substituting in Eq. (3),

$$\Delta R = -\frac{R^2}{B}\Delta\theta \tag{5}$$

From this last equation, it can be seen that the range error increases with the square of the range. Also, it is inversely proportional to the baseline. The angle error $\Delta\theta$ is a function of the eye's angular acuity, about 10 arcsec or 0.00005 rad.[10]

Pentaprisms permit a precise 90° deflection, independent of the alignment, and are like mirror systems with an angle of 45° between them. The focal length for the two telescopes in a range finder must be closely matched to avoid any difference in magnification. There are several versions of the range compensator. In one design, a sliding prism makes a variable deviation angle (Fig. 3*a*); in another system (Fig. 3*b*), a sliding prism displaces the image, without deviating it. A deviation can be also made with a rotating glass block (Fig. 3*c*). The Risley prisms (Fig. 3*d*) are a pair of counter rotating prisms located on the entrance pupil for one of the arms. Since the light beam is collimated, there is no astigmatism contribution from the prisms.[10] A unit magnification Galilean telescope (Fig. 3*e*) is made with two weak lenses. A sliding lens is used to form a variable wedge to deviate the image path.[8]

Time-Based and Optical Radar

Distance measurements can also be done by the time-of-flight method. An application of the laser, obvious at the time of its advent, is the measurement of range. Distance determination by precise timing is known as optical radar. Optical radar has been used to measure the distance to the moon.

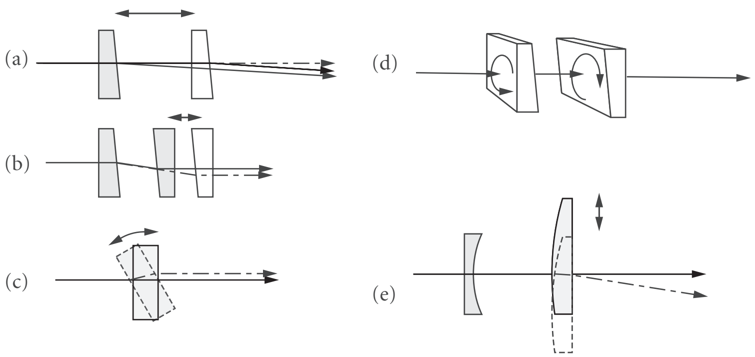

FIGURE 3 Range compensators for range finders (*a*) and (*b*) sliding prisms; (*c*) rotating glass block; (*d*) counterrotating prisms; and (*e*) sliding lens.

Since a small timing is involved, optical radar is applicable for distances from about 10 km. For distances from about a meter up to 50 km, modulated beams are used.

Laser radars measure the time of flight for a pulsed laser. Since accuracy depends on the temporal response of the electronic and detection system, optical radars are limited to distances larger than 1 km. Whenever possible, a cat's eye retro reflector is set at the range distance, making possible the use of a low power-pulsed laser. High-power lasers can be used over small distances without the need of a reflector. Accuracies of the order of 10^{-6} can be obtained.[11]

The beam modulation method requires a high-frequency signal, about 10 to 30 MHz (modulating wavelength between 30 and 10 m) to modulate a laser beam carrier. The amplitude modulation is usually applied, but polarization modulation may also be used. With beam modulation distance measurements, the phase for the returning beam is compared with that of the output beam. The following description is for an amplitude modulation distance meter, although the same applies for polarizing modulation. Assuming a sinusoidally modulated beam, the returning beam can be described by

$$I_R = \alpha I_0[1 + A\sin\omega(t-\tau)] \tag{6}$$

where α is the attenuation coefficient for the propagation media, I_o is the output intensity for the exit beam; ω is 2π times the modulated beam frequency, and τ is the delay time. By measuring the relative phase between the outgoing and the returning beam, the quantity ω is measured. In most electronic systems, the delay time τ is measured, so, the length in multiples of the modulating wavelength is

$$L = \frac{c}{2N_g}\tau \tag{7}$$

where N_g is the average group refractive index for the modulating frequency.[11]

Since the measured length is a multiple of the modulating wavelength, one is limited in range to one-half of the modulating wavelength. To measure with acceptable precision, and at the same time measure over large distances, several modulating frequencies are used, sometimes at multiples of ten to one another. The purpose is to obtain a synthetic wavelength Λ obtained from the mixing of two wavelengths λ_1, λ_2 as follows:[12,13]

$$\Lambda = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} \tag{8}$$

To increment the range, several wavelengths are used to have several synthetic wavelengths. A frequency comb can increase the accuracy up to 8 nm in a range of 800 mm.[14] The use of a femtosecond mode locked laser produces the desired frequency comb.[15]

The traveling time is measured by comparing the phase of the modulating signal for the exiting and the returning beams. This phase comparison is sometimes made using a null method. In this method, a delay is introduced in the exiting signal, until it has the same phase as the returning beam, as shown in Fig. 4. Since the introduced delay is known, the light traveling time is thus determined.

Another method uses the down conversion of the frequency of both signals. These signals, with frequency f, are mixed with an electrical signal with frequency f_o, in order to obtain a signal with lower frequency f_L, in the range of a few kHz. The phase difference between the two low-frequency signals is the same as that between the two original signals. The lowering of the frequencies permits us to use conventional electronic methods to determine the phase difference between the two signals. A broad study on range finders has been done by Stitch.[16]

Interferometric Measurement of Small Distances

Interferometric methods may be used to measure small distances with a high degree of accuracy. This is usually done with a Michelson interferometer by comparing the thickness of the lens or glass

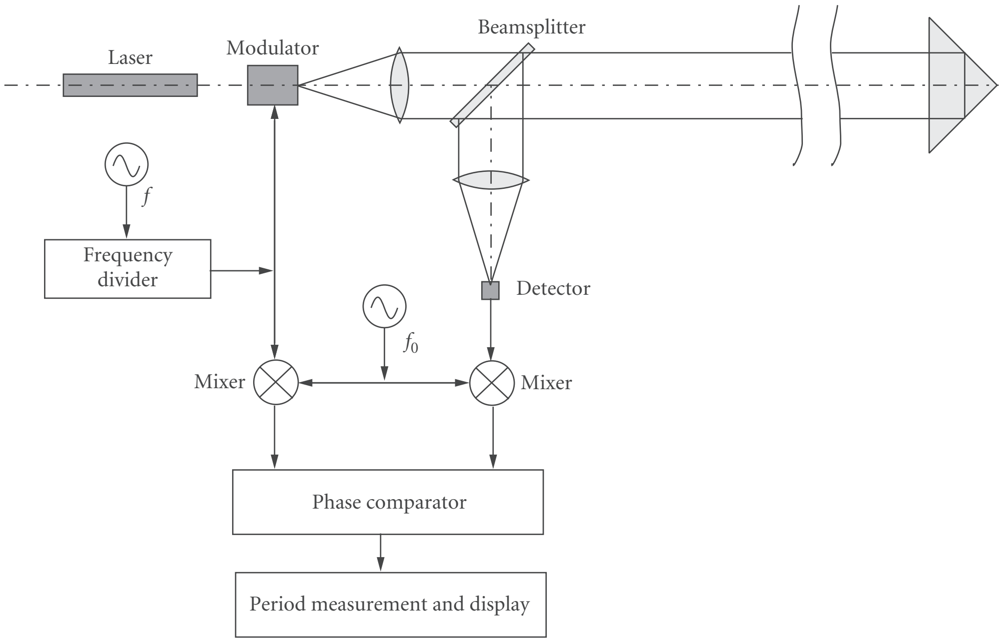

FIGURE 4 A wave modulation distance meter.

plate with that of a calibrated reference glass plate. Both plates must have approximately the same thickness and should be made with the same material.[17] The two mirrors of a dispersion-compensated Michelson interferometer are replaced by the glass plate to be measured and by a reference plane-parallel plate of the same material as the lens. (The next step is to adjust the interferometer, to produce white-light Newton rings with the front surfaces of the lens and the plate.) Then, the plate is translated along the arm of the interferometer until the rear surface produces white-light rings. The displacement is the optical thickness *Nt* of the measured plate.

Interferometric Measurement of Medium Distances

Long and medium distances may also be measured by interferometric methods.[18,19,20] Basically, the method counts the fringes in an interferometer while increasing or decreasing the optical path difference. The low temporal coherence or monochromaticity of most light sources limits this procedure to short distances. Lasers, however, have a much longer coherence length, due to their higher monochromaticity. Using lasers, it has been possible to make interferometric distance measurements over several meters.

In these interferometers, three things should be considered during their design. The first is that the laser light illuminating the interferometer should not be reflected back to the laser because that would cause instabilities in the laser, resulting in large irradiance fluctuations. As the optical path difference is changed by moving one of the mirrors, an irradiance detector in the pattern will detect a sinusoidally varying signal, but the direction of this change cannot be determined. Therefore, the second thing to be considered is that there should be a way to determine if the fringe count is going up or down; that is, if the distance is increasing or decreasing. There are two basic approaches to satisfy this last requirement. One is by producing two sinusoidal signals in phase quadrature (phase difference of 90° between them). The direction of motion of the moving prism may be sensed by determining the phase of which signal leads or lags the phase of the other signal. This information is used to make the fringe counter increase or decrease. The alternative method uses a laser with two different frequencies. Finally, the third thing to consider in the interferometer design is that the number of fringes across its aperture should remain low and constant while moving the reflector in one of the two interferometer arms. This last condition is easily satisfied by using retroreflectors instead of mirrors. Then the two interfering wavefronts will always be almost flat and parallel. A typical retroreflector is a cube corner prism of reasonable quality to keep the number of fringes over the aperture low.

One method of producing two signals in quadrature is to have only a small number of fringes over the interferogram aperture. One possible method is by deliberately using an imperfect retroreflector. Then, the two desired signals are two small slits parallel to the fringes and separated by one-fourth of the distance between the fringes.[11] To avoid illuminating back the laser, the light from this laser should be linearly polarized. Then, a $\lambda/4$ phase plate is inserted in front of the beam, with its slow axis set at 45° to the interferometer plane to transform it into a circularly polarized beam.

Another method used to produce the two signals in quadrature phase is to take the signals from the two interference patterns that are produced in the interferometer. If the beam splitters are dielectric (no energy losses), the interference patterns will be complements of each other and, thus, the signals will be 90° apart. By introducing appropriate phase shifts in the beam splitter using metal coatings, the phase difference between the two patterns may be made 90°, as desired.[21] This method was used by Rowley,[22] as illustrated in Fig. 5. In order to separate the two patterns from the incident light beam, a large beam splitter and large retroreflectors are used. This configuration has the advantage that the laser beam is not reflected back. This is called a nonreacting interferometer configuration.

One more method, illustrated in Fig. 6, is the nonreacting interferometer designed at the Perkin-Elmer Corporation by Minkowitz and Vanir.[23] A circularly polarized light beam, produced by a linearly polarized laser and a $\lambda/4$ phase plate, illuminates the interferometer. This beam is divided by a beam splitter into two beams going to both arms of the interferometer. Upon reflection by the retro reflector, one of the beams changes its state of polarization from right to left circularly polarized. The two beams with opposite circular polarization are recombined at the beam splitter, thus producing linearly polarized light. The angle of the plane of polarization is determined by the phase difference between the two beams. The plane of polarization rotates 360° if the optical path difference

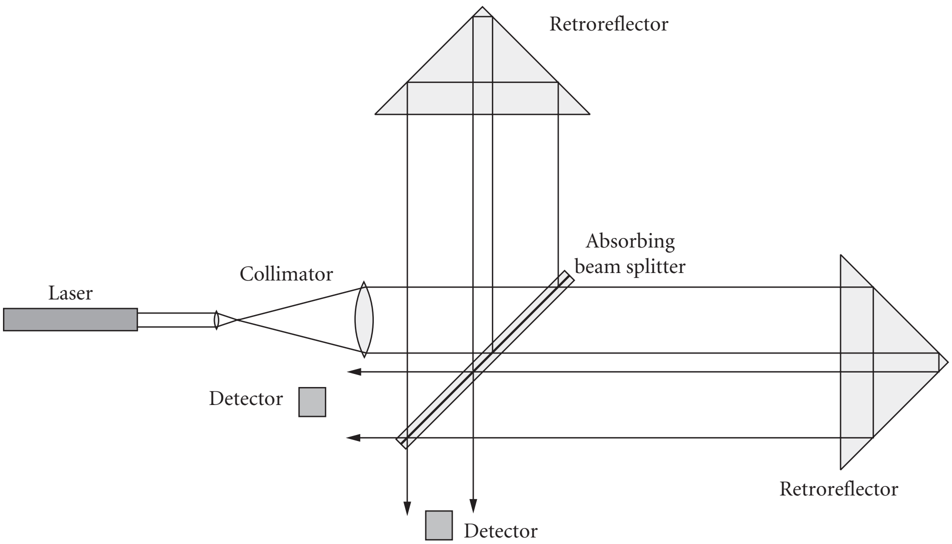

FIGURE 5 Two-interference pattern distance-measuring interferometer.

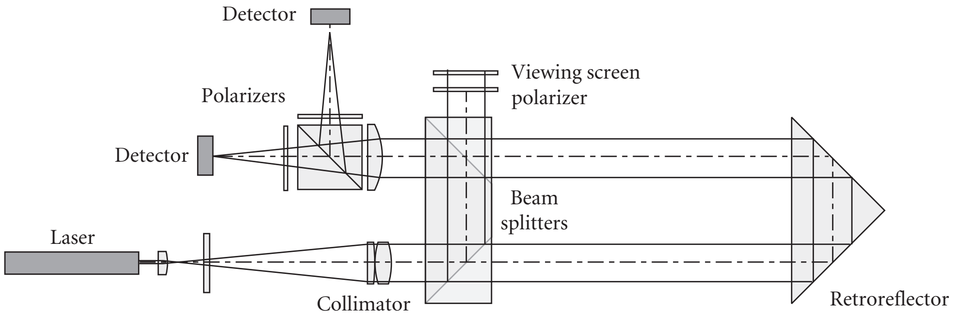

FIGURE 6 Minkowitz distance-measuring interferometer.

is changed by $\lambda/2$, the direction of rotation being given by the direction of the displacement. This linearly polarized beam is divided into two beams by a beam splitter. On each of the exiting beams, a linear polarizer is placed, one at an angle of +45° and the other at an angle of −45°. Then the two beams are in quadrature to each other.

In still another method, shown in Fig. 7, a beam of light, linearly polarized at 45° (or circularly polarized), is divided at a beam splitter, the *p* and *s* components. Then, both beams are converted to circular polarization with a $\lambda/4$ phase plate in front of each of them, with their axis at 45°. Upon reflection on the retro-reflectors, the handedness of the polarization is reversed. Thus, the linearly polarized beams exiting from the phase plates on the return to the beam splitter will have a plane of polarization orthogonal to that of the incoming beams. It is easily seen that no light returns to the laser. Here, the nonreacting configuration is not necessary but it may be used for additional protection. After recombination on the beam splitter, two orthogonal polarizations are present. Each plane of polarization contains information about the phase from one arm only so that no interference between the two beams has occurred. The two desired signals in quadrature are then generated by producing two identical beams with a nonpolarizing beam splitter with a polarizer on each exiting beam with their axes at +45° and −45° with respect to the vertical plane. The desired phase difference between the two beams is obtained by introducing a

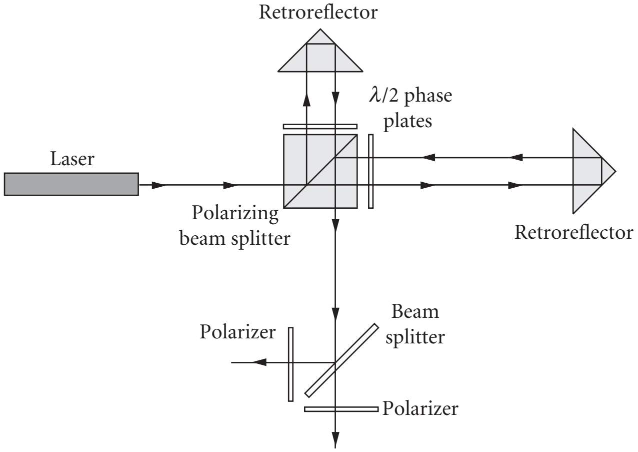

FIGURE 7 Brunning distance-measuring interferometer.

$\lambda/4$ phase plate after the beam splitter but before one of the polarizers with its slow axis vertical or horizontal. These two polarizers may be slightly rotated to make the two irradiances equal, at the same time preserving the 90° angle between their axes. If the prism is shifted a distance x, the fringe count is

$$\Delta_{\text{count}} = \pm\left[\frac{2x}{\lambda}\right] \tag{9}$$

where [ ] denotes the integer part of the argument.

These interferometers are problematic in that any change in the irradiance may be easily interpreted as a fringe monitoring the light source. A more serious problem is the requirement that the static interference pattern be free of, or with very few, fringes. Fringes may appear because of multiple reflections or turbulence.

A completely different method uses two frequencies. It was developed by the Hewlett Packard Co.[24,25] and is illustrated in Fig. 8. The light source is a frequency-stabilized He-Ne laser whose light beam is Zeeman split into two frequencies f_1 and f_2 by application of an axial magnetic field. The frequency difference is several megahertz and both beams have circular polarization, but with opposite sense. A $\lambda/4$ phase plate transforms the signals f_1 and f_2 into two orthogonal linearly polarized beams, one in the horizontal and the other in the vertical plane. A sample of this mixed signal is deviated by a beam splitter and detected at photo detector A, by using a polarizer at 45°. The amplitude modulation of this signal, with frequency $f_1 - f_2$, is detected and passed to a counter. Then, the two orthogonally polarized beams with frequencies f_1 and f_2 are separated at a polarizing beam splitter. Each is transformed into a circularly polarized beam by means of $\lambda/4$ phase plates. After reflection by the prisms, the handedness of these polarizations is changed. Then they go through the same phase plates where they are converted again to orthogonal linearly polarized beams. There is no light reflecting back to the laser. After recombination at the beam splitter, a polaroid at 45° will take the components of both beams in this plane. This signal is detected at the photo detector B. As with the other signal, the modulation with frequency $f_1 - f_2 + \Delta f$ is extracted from the carrier and sent to

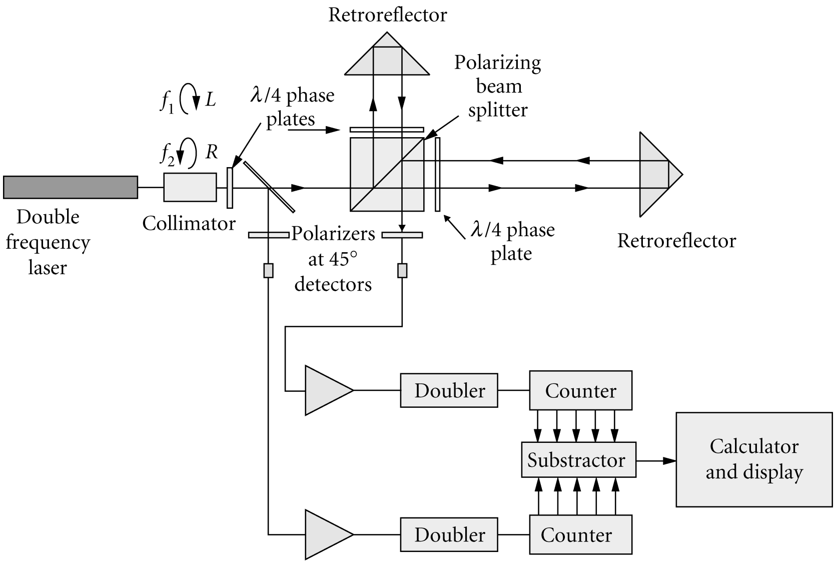

FIGURE 8 Hewlett Packard double-frequency distance-measuring interferometer.

another counter. The shift Δf in the frequency of this signal comes from a Doppler shift due to the movement of one of the retroreflectors,

$$\Delta f = \frac{2}{\lambda_2}\frac{dx}{dt} \tag{10}$$

where dx/dt is the cube corner prism velocity and λ_1 is the wavelength corresponding to the frequency f_1. The difference between the results of the two counters is produced by the displacement of the retroreflector. If the prism moves a distance x, the number of pulses detected is given by

$$\Delta_{\text{count}} = \pm\left[\frac{2x}{\lambda_2}\right] \tag{11}$$

The advantage of this method compared with the first is that fringe counting is not subject to drift. These signals may be processed to obtain a better signal-to-noise ratio and higher resolution.[25]

Straightness Measurements

Light propagation is assumed to be rectilinear in a homogeneous medium. This permits the use of a propagating light beam as a straightness reference. Besides the homogeneous medium, it is necessary to get a truly narrow pencil of light to improve accuracy. Laser light is an obvious application because of the high degree of spatial coherence. Beam divergence is usually less than 1 mrad for a He-Ne laser. One method uses a position-sensing detector to measure the centroid of the light spot despite its shape. In front of the laser, McLeod[26] used an axicon as an aligning device. When a collimated light beam is incident on an axicon, it produces a bright spot on a circular field. An axicon can give as much as 0. 01 arcsec.

Another method to measure the deviation from an ideal reference line uses an autocollimator. A light beam leaving an autocollimator is reflected by a mirror. The surface slope is measured at the mirror. By knowing the distance to the mirror, one can determine the surface's profile by integration, as in a curvature measurement (see "Optical Methods for Measuring Curvature," later in this chapter). A method can be designed for measuring flatness for tables on lathe beds.[27]

12.4 ANGLE MEASUREMENTS

Angle measurements, as well as distance measurements, require different levels of accuracy. For cutting glass, the required accuracy can be as high as several degrees, while for test plates, an error of less than a second of arc may be required. For each case, different measurement methods are developed.

Mechanical Methods

The easiest way to measure angles with medium accuracy is by means of mechanical nonoptical methods. These are

Sine Plate Essentially it is a table with one end higher than the other by a fixed amount, as shown in Fig. 9. Accuracy close to 30 arcmin may be obtained.

Goniometer This is a precision spectrometer. It has a fixed collimator and a moving telescope pointing to the center of a divided circle. Accuracies close to 20 arcsec may be obtained.

Bevel Gauge Another nonoptical method is by the use of a bevel gauge. This is made of two straight bars hinged at their edges by a pivot, as shown in Fig. 10. This device may be used to measure angle

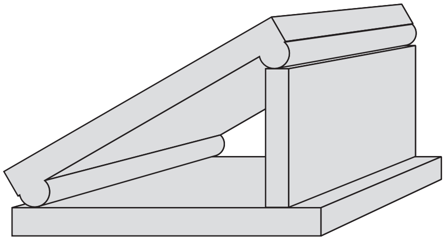

FIGURE 9 Sine plate.

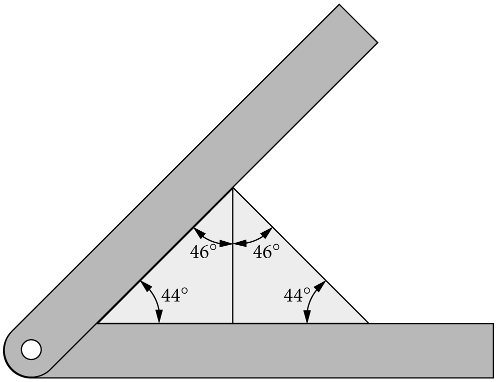

FIGURE 10 Bevel gauge.

prisms whose angle accuracies are from 45 to about 20 arcsec.[28] For example, if the measured prism has a 50-mm hypotenuse, a space of 5 μm at one end represents an angle of 0.0001 rad or 20 arcsec.

Numerically Controlled Machines (CNC) The advent of digitally controlled machines has brought to the optical shop machines to work prisms and angles with a high level of accuracy. Most machines can be adjusted within 0.5 arcmin of error.

Autocollimators

As shown in Fig. 11, an autocollimator is essentially a telescope focused at infinity with an illuminated reticle located at the focal plane. A complete description of autocollimators is found in Hume.[29] A flat reflecting surface, perpendicular to the exiting light beam, forms an image of the reticle on the same plane as the original reticle. Then, both the reticle and its image are observed through the eyepiece. When the reflecting surface is not exactly perpendicular to the exiting light beam, the reticle image is laterally displaced in the focal plane with respect to the object reticle. The magnitude of this of displacement d is

$$d = 2\alpha f \tag{12}$$

where α is the tilt angle for the mirror in radians and f is the focal length of the telescope.

Autocollimator objective lenses are usually corrected doublets. Sometimes a negative lens is included to form a telephoto lens to increase the effective focal length while maintaining compactness. The collimating lens adjustment is critical for the final accuracy. Talbot interferometry can be used for a precise focus adjustment.[30]

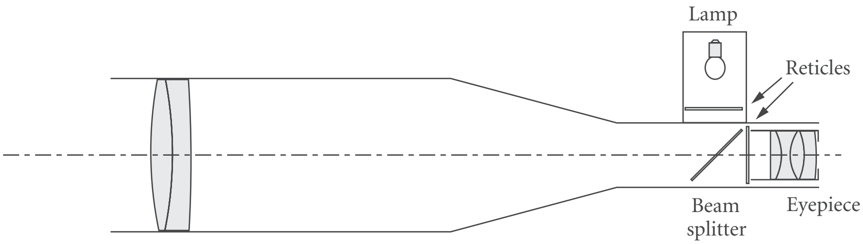

FIGURE 11 An autocollimator.

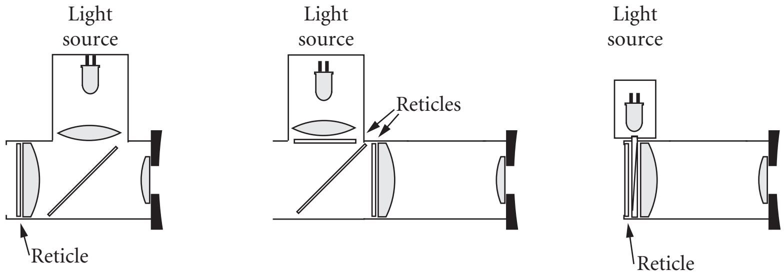

FIGURE 12 Illuminated eyepieces for autocollimators and microscopes. (*a*) Gauss; (*b*) bright line; and (*c*) Abbe.

The focal plane is observed through an eyepiece. Several types of illuminated reticles and eyepieces have been developed. Figure 12[31] illustrates some illuminated eyepieces, in all of which the reticle is calibrated to measure the displacement. Gauss and Abbe illuminators show a dark reticle on a bright field. A bright field[32] may be more appropriate for low reflectance surfaces. Rank[33] modified a Gauss eyepiece to produce a dark field. In other systems, a drum micrometer displaces a reticle to position it at the image plane of the first reticle. To increase sensitivity some systems, called microoptic autocollimators, use a microscope to observe the image.

Direct-reading autocollimators have a field of view of about 1°. Precision in an autocollimator is limited by the method for measuring the centroid of the image. In a diffraction-limited visual system, the diffraction image size sets the limit of precision. In a precision electronic measuring system, the accuracy of the centroid measurement is limited by the electronic detector, independent of the diffraction image itself, and can exceed the diffraction limit. In some photoelectric systems, the precision is improved by more than an order of magnitude.

Autocollimators are used for angle measurements in prisms and glass polygons. But they also have other applications; for example, to evaluate the parallelism between faces in optical flats or to manufacture divided circles.[34] By integrating measured slope values with an autocollimator, flatness deviations for a machine tool for an optical bed can also be evaluated.[27]

The reflecting surface in autocollimation measurements must be kept close to the objective in order to make the alignment easier and to be sure that all of the reflected beam enters the system. The reflecting surface must be of high quality. A curved surface is equivalent to introducing another lens in the system with a change in the effective focal length.[27]

Several accessories for autocollimators have been designed. For single-axis angle measurement, a pentaprism is used. An optical square permits angle measurements for surfaces at right angles. Perpendicularity is measured with a pentaprism and a mirror, as shown in Fig. 13. A handy horizontal reference can be produced with an oil pool, but great care must be taken with the surface stability.

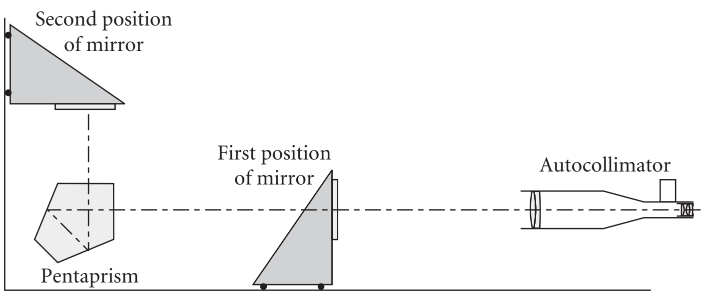

FIGURE 13 Perpendicularity measurement with an autocollimator.

Theodolites

Theodolites are surveying instruments to measure vertical and horizontal angles. A telescope with reticle is the reference to direct the instrument axis. In some theodolites, the telescope has an inverting optical system to produce an erect image. The reticle is composed of a crosswire and has a couple of parallel horizontal lines, called stadia lines, used for range measurements (see "Stadia and Range Finders" earlier in this chapter). The telescope has two focus adjustments: one to sharply focus the reticle according to the personal setting for each observer, and the other to focus the objective on the reticle. This later focus adjustment is performed by moving the whole eyepiece.

The theodolite telescope has a tree-screw base altitude-azimuth mounting on a base made horizontal with a spirit level. Divided circles attached to both telescope axes measure vertical and horizontal angles. In older instruments, an accuracy of 20 arcmin was standard. Modern instruments are accurate to within 20 arcsec, the most expensive of which can reach an accuracy of 1 arcsec. To remove errors derived from eccentric scales as well as orthogonality errors, both axes are rotated 180° and the measurement repeated. Older instruments had a provision for reading at opposite points of the scale. Scales for theodolites can be graduated in sexagesimal degrees or may use a centesimal system that divides a circle into 400 grades.

Some of the accessories available for theodolites include

1. An illuminated reticle that can be used as an autocollimator when directed to a remote retroreflector. The observer adjusts the angles until both the reflected and the instrument's reticle are superposed. This increases the pointing accuracy.
2. A solar filter, which can be attached to the eyepiece or objective side of the telescope. This is used mainly for geographic determination in surveying.
3. An electronic range meter, which is superposed to the theodolite to measure the distance. Additionally, some instruments have electronic position encoders that allow a computer to be used as an immediate data-gathering and reducing device.
4. A transverse micrometer for measuring angular separation in the image plane.

Accuracy in a theodolite depends on several factors in its construction. Several of these errors can be removed by a careful measuring routine. Some of the systematic or accuracy limiting errors are

1. Perpendicularity—deviation between vertical and horizontal scales. This error can be nulled by plunging and rotating the telescope, then averaging.
2. Concentricity deviation of scales. When scales are not concentric, they are read at opposite ends to reduce this error. Further accuracy can be obtained by rotating the instrument 90°, 180°, and 270° and averaging measurements.

Level

Levels are surveying instruments for measuring the deviation from a horizontal reference line. A level is a telescope with an attached spirit level. The angle between the telescope axis and the one defined by the spirit level must be minimized. It can be adjusted by a series of measurements to a pair of surveying staves (Fig. 14). Once the bubble is centered in the tube, two measurements are taken on

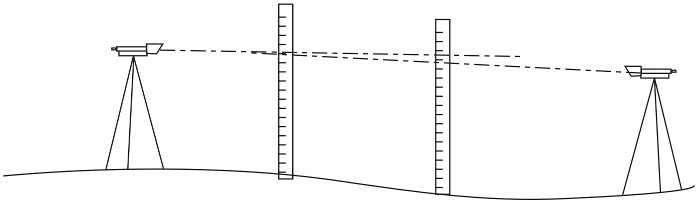

FIGURE 14 Level adjustment. (*After Kingslake.*[35])

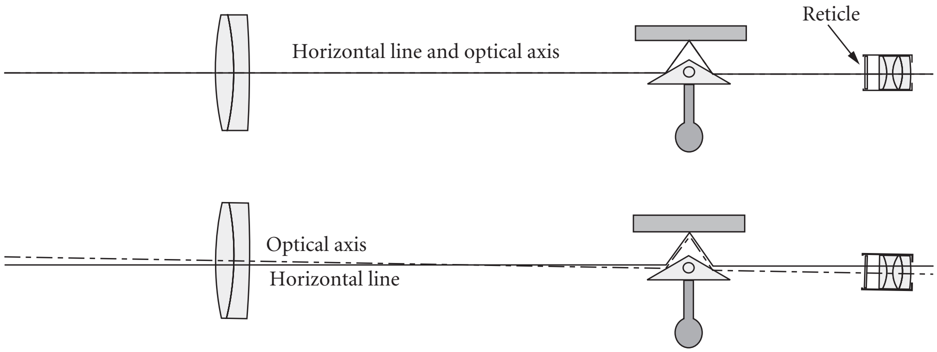

FIGURE 15 The autoset level.

each side of the staves.[35] The level differences between the two staves must be equal if the telescope axis is parallel with the level horizontal axis.

The autoset level (Fig. 15) uses a suspended prism and a fixed mirror on the telescope tube. The moving prism maintains the line aimed at the horizon and passing through the center of the reticle, despite the tube orientation, as long as it is within about 15 arcmin. Typical precision for this automatic level can go up to 1 arcsec.[27]

Interferometric Measurements

Interferometric methods find their main applications in measuring very small wedge angles in glass slabs[36,37] and in parallelism evaluation[38] by means of the Fizeau or Haidinger interferometers.[39]

Interferometric measurements of large angles may also be performed. In one method, a collimated laser beam is reflected from the surfaces by a rotating glass slab. The resulting fringes can be considered as coming from a Murty lateral shear interferometer.[40] This device can be used as a secondary standard to produce angles from 0° to 360° with accuracy within a second of arc. Further analysis of this method has been done by Tentori and Celaya.[41] In another system, a Michelson interferometer is used with an electronic counter to measure over a range of ±5° with a resolution of 10°.[42,43] An interferometric optical sine bar for angles in the milliseconds of arc was built by Chapman.[44]

Angle Measurements in Prisms

A problem frequently encountered in the manufacture of prisms is the precise measurement of angle. In most cases, prism angles are 90°, 45°, and 30°. These angles are easily measured by comparison with a standard but it is not always necessary.

An important aspect of measuring angles in a prism is to determine if the prism is free of pyramidal error. Consider a prism with angles A, B, and C (Fig. 16*a*). Let OA be perpendicular to plane ABC. If line AP is perpendicular to segment BC, then the angle AOP is a measurement of the pyramidal error. In a prism with pyramidal error, the angles between the faces, as measured in planes perpendicular to the edges between these faces, add up to over 180°. To simply detect pyramidal error in a prism, Johnson[45] and Martin[46] suggest that both the refracted and the reflected images from a straight line be examined (Fig. 16*b*). When pyramidal error is present, the line appears to be broken. A remote target could be graduated to measure directly in minutes of arc. A sensitivity of up to 3 arcmin may be obtained.

During the milling process in the production of a prism, a glass blank is mounted in a jig collinear with a master prism (Fig. 17). An autocollimator aimed at the master prism accurately sets the

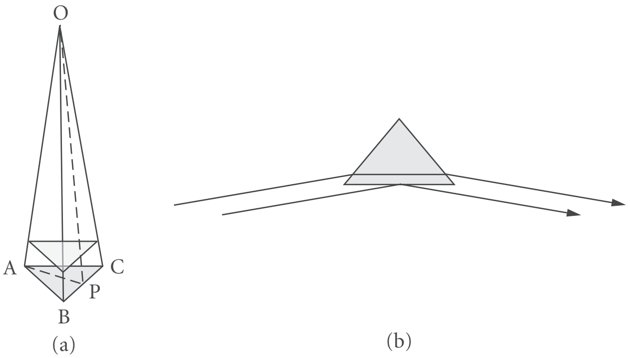

FIGURE 16 Pyramidal error in a prism (*a*) nature of the error and (*b*) test of the error.

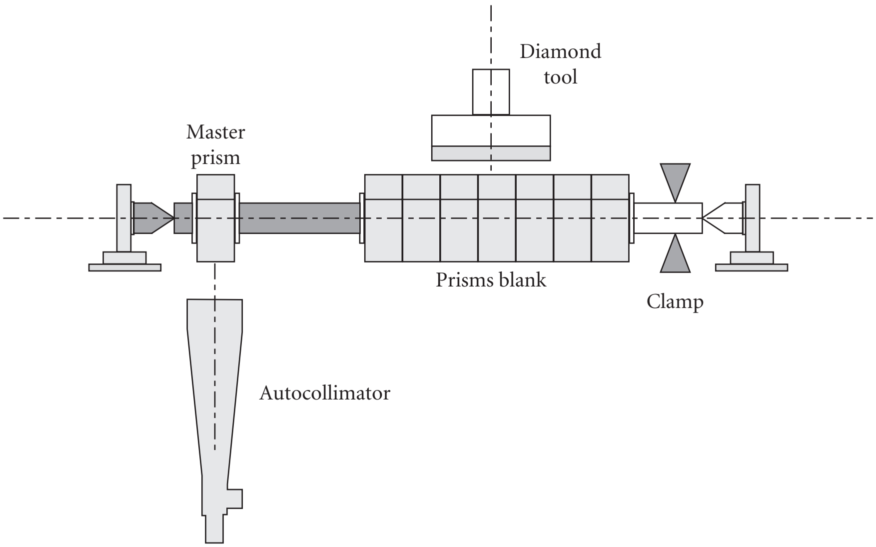

FIGURE 17 Milling prisms for replication.

position for each prism face.[47,48] With a carefully set diamond lap, pyramidal error is minimized. In a short run, angles can be checked with a bevel gauge. Visual tests for a prism in a bevel gauge can measure an error smaller than a minute of arc.[31]

A 90° angle in a prism can be measured by internal reflection, as shown in Fig. 18*a*. At the autocollimator image plane, two images are seen with an angular separation of $2N\alpha$, where α is the magnitude of the prism angle error, and its sign is unknown. Since the hypotenuse face has to be polished and the glass must be homogeneous, the measurement of the external angle with respect to a reference flat is preferred (Fig. 18*b*). In this case, the sign of the angle error is determined by a change in the angle by tilting the prism. If the external angle is decreased and the images separate further, then the external angle is less than 90°. Conversely, if the images separate by tilting in such way that the external angle increases, then the external angle is larger than 90°.

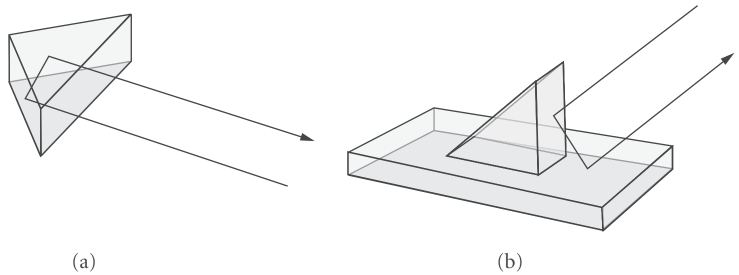

FIGURE 18 Right angle measurement in prisms: (*a*) internal measurement and (*b*) external measurement.

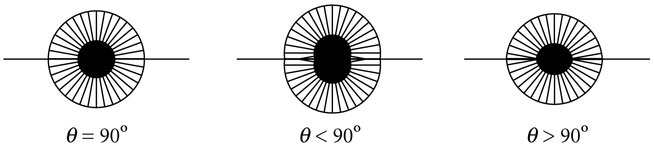

FIGURE 19 Retroreflected images of the observer's pupil in a 90° prism.

To determine the sign of the error, several other methods have been proposed. DeVany[49] suggested that when looking at the double image from the autocollimator, the image should be defocused inward. If the images tend to separate, then the angle in the prism is greater than 90°. Conversely, an outward defocusing will move the images closer to each other for an angle greater than 90°. Another way to eliminate the sign of the error in the angle is by introducing, between the autocollimator and the prism, a glass plate with a small wedge whose orientation is known. The wedge should cover only one-half of the prism aperture. Ratajczyk and Bodner[50] suggested a different method using polarized light.

Right-angle prisms can be measured using an autocollimator with acceptable precision.[51] With some practice, perfect cubes with angles more accurate than 2 arcsec can be obtained.[49] An extremely simple test for the 90° angle in prisms[45] is performed by looking to the retroreflected image of the observer's pupil without any instrument. The shape of the image of the pupil determines the error, as shown in Fig. 19. The sensitivity of this test is not very great and may be used only as a coarse qualitative test. As shown by Malacara and Flores,[52] a small improvement in the sensitivity of this test may be obtained if a screen with a small hole is placed in front of the eye, as in Fig. 20*a*. A cross centered on the small hole is painted on the front face of the screen. The observed images are as shown in the same Fig. 20*b*. As opposed to the collimator test, there is no uncertainty in the sign of the error in the tests just described, since the observed plane is located where the two prism surfaces intersect. An improvement described by Malacara and Flores,[52] combining these simple tests with an autocollimator, is obtained with the instrument in Fig. 21. In this system, the line defining the intersection between the two surfaces is out of focus and barely visible while the reticle is in perfect focus at the eyepiece.

Corner cube prisms are a real challenge to manufacture, since besides the large precision required in the angles, all surfaces should be exempt of any curvature. The dihedral angle in pentaprisms is tested usually with an interferometer. An error in the prism alignment results in an error in angle determination.[53] A simple geometric method for angle measurement in corner cube reflector has been described by Rao.[54] Also, the calculations for the electric field in a corner cube are performed by Scholl.[55] These calculations include the effect by nonhomogeneities, angle of incidence, and errors in the surface finish.

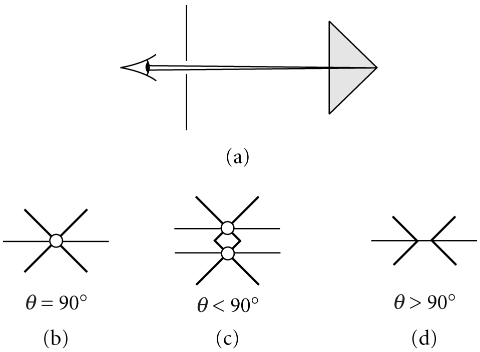

FIGURE 20 Testing a right-angle prism: (*a*) screen in front of the eye and (*b*) to (*d*) its observed images.

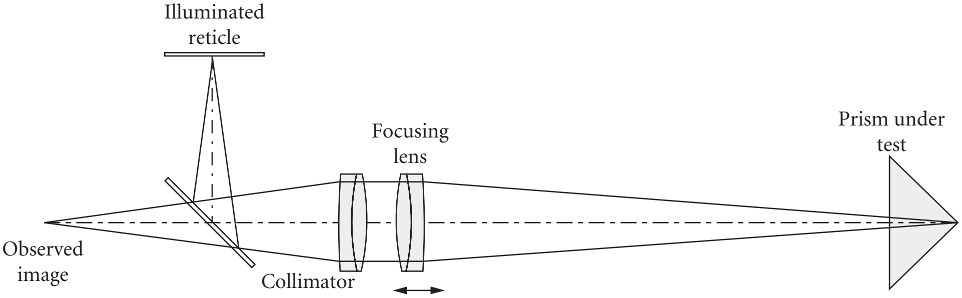

FIGURE 21 Modified autocollimator for testing the right angle in prisms without sign uncertainty in measured error.

12.5 CURVATURE AND FOCAL LENGTH MEASUREMENTS

The curvature of a spherical optical surface or the local curvature of an aspherical surface may be measured by means of mechanical or optical methods. Some methods measure the sagitta, some the surface slope, and some others directly locate the position of the center of curvature.

Mechanical Methods for Measuring Curvature

Templates The simplest and most common way to measure the radius of curvature is by comparing it with metal templates with known radii of curvature until a good fit is obtained. The template is held in contact with the optical surface with a bright light source behind the template and the optical surface. If the surface is polished, gaps between the template and the surface may be detected to an accuracy of one wavelength. If the opening is very narrow, the light passing through the gap becomes blue due to diffraction.

Test Plates This method uses a glass test plate with a curvature opposite to that of the glass surface to be measured. The accuracy is much higher than in the template method, but the surface must be polished.

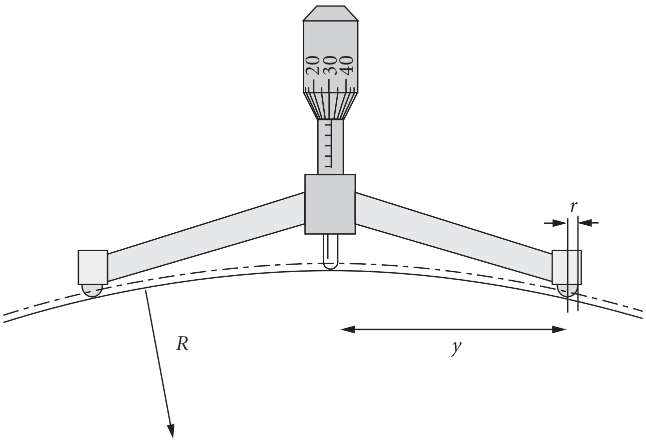

FIGURE 22 Three-leg spherometer.

Spherometers This is probably the most popular mechanical device used to measure radii of curvature. It consists of three equally spaced feet with a central moving plunger. The value of the radius of curvature is calculated after measuring the sagitta, as shown in Fig. 22. The spherometer must first be calibrated by placing it on top of a flat surface. Then it is placed on the surface to be measured. The difference in the position of the central plunger for these two measurements is the sagitta of the spherical surface being measured. Frequently, a steel ball is placed at the end of the legs as well as at the end of the plunger to avoid the possibility of scratching the surface with sharp points. In this case, if the measured sagitta is z, the radius of curvature R of the surface is given by

$$R = \frac{z}{2} + \frac{y^2}{2z} \pm r \tag{13}$$

where r is the radius of curvature of the balls. The plus sign is used for concave surfaces and the minus sign for convex surfaces. The precision of this instrument in the measurement of the radius of curvature for a given uncertainty in the measured sagitta may be obtained by differentiating Eq. (13)

$$\frac{dR}{dz} = \frac{1}{2} - \frac{y^2}{2z^2} \tag{14}$$

obtaining

$$\Delta R = \frac{\Delta z}{2}\left(1 - \frac{y^2}{z^2}\right) \tag{15}$$

This accuracy assumes that the spherometer is perfectly built and that its dimensional parameters y and r are well known. The uncertainty comes only from human or instrumental errors in the measurement of the sagitta. Noble[31] has evaluated the repeatability for a spherometer with $y = 50$ mm and an uncertainty in the sagitta reading equal to 5 μm, and has reported the results in Table 1 where it can be seen that the precision is better than 2 percent. An extensive analysis of the precision and accuracy of several types of spherometers is given by Jurek.[56]

A ring may be used instead of the three legs in a mechanical spherometer. A concave surface contacts the external edge of the cup, and a convex surface is contacted by the internal edge of the ring. Thus, Eq. (5) may be used if a different value of y is used for concave and convex surfaces, and r is taken as zero. Frequently in spherometers of this type, the cups are interchangeable in order to have different diameters for different surface diameters and radii of curvature. The main advantage of the

TABLE 1 Spheremeter precision*

Radius of Sphere R (mm)	Sagitta Z (mm)	Fractional Precision ΔR (mm)	Precision $\Delta R/R$
10,000	0.125	−400	−0.040
5,000	0.250	−100	−0.020
2,000	0.625	−16	−0.008
1,000	1.251	−4	−0.004
500	2.506	−1	−0.002
200	6.351	−0.15	−0.0008

*$y = 50$ mm; $\Delta z = 5$ μm.
Source: From Noble.[31]

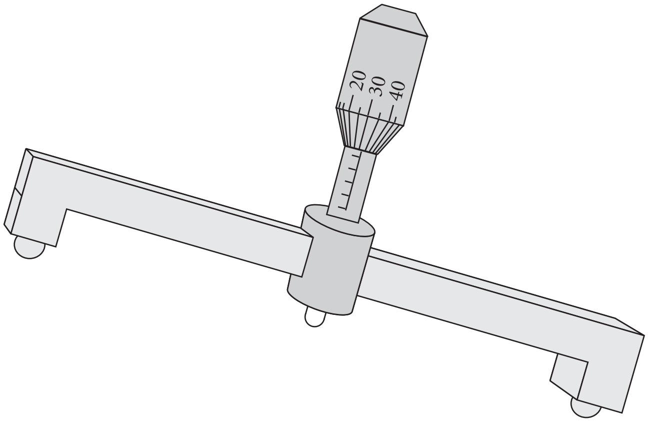

FIGURE 23 Bar spherometer.

use of a ring instead of three legs is that an astigmatic deformation of the surface is easily detected, although it cannot be measured.

A spherometer that permits the evaluation of astigmatism is the bar spherometer, shown in Fig. 23. It can measure the curvature along any diameter. A commercial version of a small bar spherometer for the specific application in optometric work is the Geneva gauge, where the scale is directly calibrated in diopters assuming that the refractive index of the glass is 1.53.

Automatic spherometers use a differential transformer as a transducer to measure the plunger displacement. This transformer is coupled to an electronic circuit and produces a voltage that is linear with respect to the plunger displacement. This voltage is fed to a microprocessor which calculates the radius of curvature or power in any desired units and displays it.

Optical Methods for Measuring Curvature

Foucault Test Probably the oldest and easiest method to measure the radius of curvature of a concave surface is the knife-edge test. In this method, the center of curvature is first located by means of the knife edge. Then, the distance from the center of curvature to the optical surface is measured.

Autocollimator The radius of curvature may also be determined through measurements of the slopes of the optical surface with an autocollimator as described by Horne.[57] A pentaprism producing a 90° deflection of a light beam independent of small errors in its orientation is used in this technique,

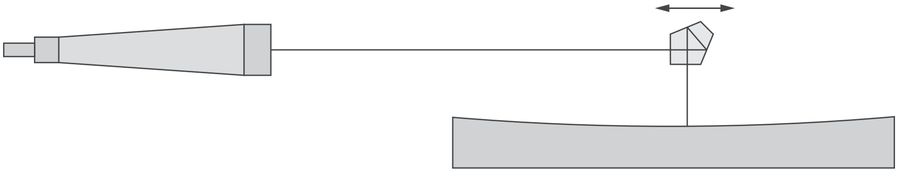

FIGURE 24 Autocollimator and pentaprism used to determine radius of curvature by measuring surface slopes.

as illustrated in Fig. 24, where the pentaprism travels over the optical surface to be measured along one diameter. First the light on the reticle of the autocollimator is centered on the vertex of the surface being examined. Then the pentaprism is moved toward the edge of the surface in order to measure any slope variations. From these slope measurements, the radius of curvature may be calculated. This method is useful only for large radii of curvature for either concave or convex surfaces.

Confocal Cavity Technique Gerchman and Hunter[58,59] have described the so-called optical cavity technique that permits the interferometric measurement of very long radii of curvature with an accuracy of 0.1 percent. The cavity of a Fizeau interferometer is formed, as illustrated in Fig. 25. This is a confocal cavity of nth order, where n is the number of times the path is folded. The radius of curvature is equal to approximately $2n$ times the cavity length Z.

Traveling Microscope This instrument is used to measure the radius of curvature of small concave optical surfaces with short radius of curvature. As illustrated in Fig. 26, a point light source is produced at the front focus of a microscope objective. This light source illuminates the concave optical surface to be measured near its center of curvature. Then this concave surface forms an image which is also close to its center of curvature. This image is observed with the same microscope used to illuminate the surface. During this procedure, the microscope is focused both at the center of curvature and at the surface to be measured. A sharp image of the light source is observed at both places. The radius of curvature is the distance between these two positions for the microscope.

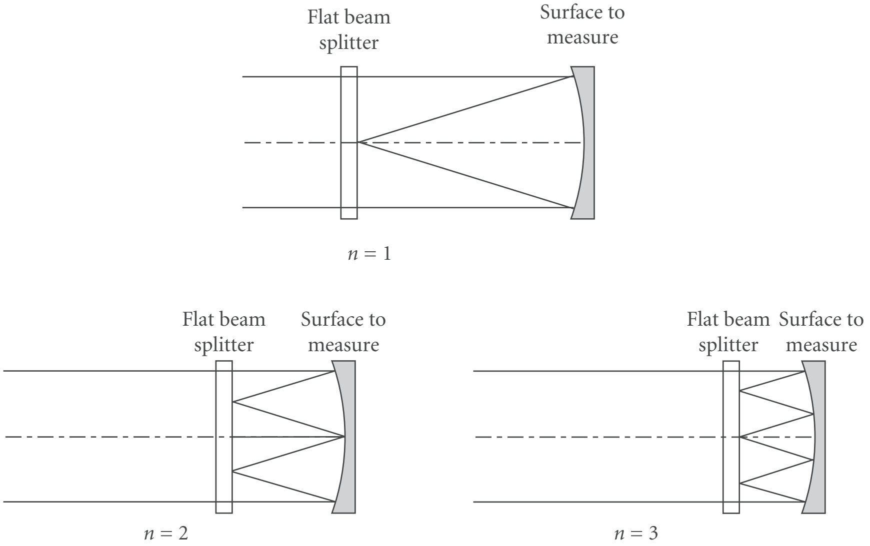

FIGURE 25 Confocal cavity arrangements used to measure radius of curvature.

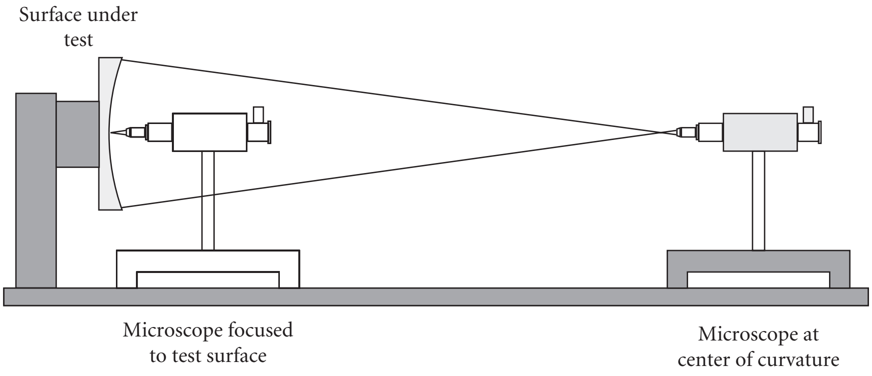

FIGURE 26 Traveling microscope to measure radii of curvature.

This distance traveled by the microscope may be measured on a vernier scale, obtaining a precision of about 0.1 mm. If a bar micrometer is used, the precision may be increased by an order of magnitude. In this case, two small convex buttons are required: one fixed to the microscope carriage and the other to the stationary part of the bench. They must face each other when the microscope carriage is close to the optical bench fixed component.

Carnell and Welford[32] describe a method that requires only one measurement. The microscope is focused only at the center of curvature. Then the radius of curvature is measured by inserting a bar micrometer with one end touching the vertex of the optical surface. The other end is adjusted until it is observed to be in focus on the microscope. Accuracies of a few microns are obtained with this method.

In order to focus the microscope properly, the image of an illuminated reticle must fall, after reflection, back on itself, as in the Gauss eyepiece shown in Fig. 12. The reticle and its image appear as dark lines in a bright field. The focusing accuracy may be increased with a dark field. Carnell and Welford obtained a dark field with two reticles, as in Fig. 12, one illuminated with bright lines and the other with dark lines.

A convex surface may also be measured with this method if a well-corrected lens with a conjugate longer than the radius of curvature of the surface under test is used. Another alternative for measuring convex surfaces is by inserting an optical device with prisms in front of the microscope, as described by Jurek.[56]

Some practical aspects of the traveling microscope are examined by Rank,[33] who obtained a dark field at focus with an Abbe eyepiece which introduces the illumination with a small prism. This method has been implemented using a laser light source by O'Shea and Tilstra.[60]

Additional optical methods to measure the radius of curvature of a spherical surface have been described. Evans[61,62,63] determines the radius by measuring the lateral displacements on a screen of a laser beam reflected on the optical surface when this optical surface is laterally displaced. Cornejo-Rodriguez and Cordero-Dávila,[64] Klingsporn,[65] and Diaz-Uribe et al.[66] rotate the surface about its center of curvature on a nodal bench.

Focal Length Measurements

There are two focal lengths in an optical system: the back focal length and the effective focal length. The back focal length is the distance from the last surface of the system to the focus. The effective focal length is the distance from the principal plane to the focus. The back focal length is easily measured, following the same procedure used for measuring the radius of curvature, using a microscope and the lens bench. On the other hand, the effective focal length requires the previous location of the principal plane.

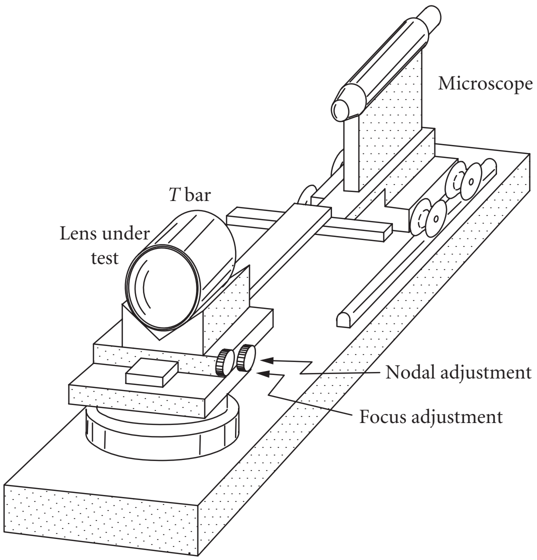

FIGURE 27 Nodal slide bench. (*From Malacara.*[1])

Nodal Slide Bench In an optical system in air, the principal points (intersection of the principal plane and the optical axis) coincide with the nodal points. Thus, to locate this point we may use the well-known property that small rotations of the lens about an axis perpendicular to the optical axis and passing through the nodal point do not produce any lateral shift of the image. The instrument used to perform this procedure, shown in Fig. 27, is called an optical nodal slide bench.[67] This bench has a provision for slowly moving the lens under test longitudinally in order to find the nodal point.

The bench is illuminated with a collimated light source and the image produced by the lens under test is examined with a microscope. The lens is then displaced slightly about a vertical axis as it is being displaced longitudinally. This procedure is stopped until a point is found in which the image does not move laterally while rotating the lens. This axis of rotation is the nodal point. Then, the distance from the nodal point to the image is the effective focal length.

Focimeters A focimeter is an instrument designed to measure the focal length of lenses in a simple manner. The optical scheme for the classical version of this instrument is shown in Fig. 28. A light source illuminates a reticle and a convergent lens, with focal length *f*, displaced at a distance *x* from the reticle. The lens to be measured is placed at a distance *d* from the convergent lens. The magnitude

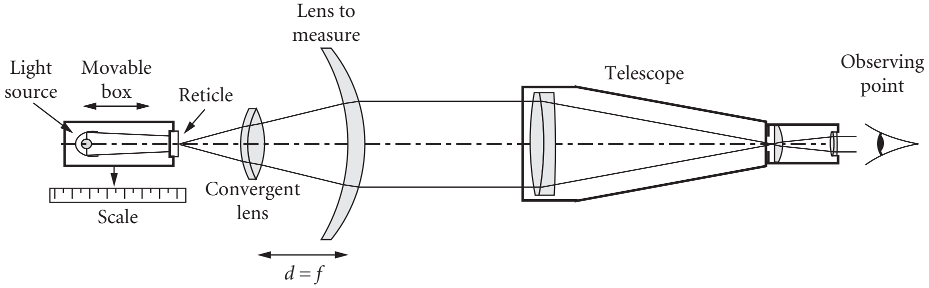

FIGURE 28 Focimeter schematics.

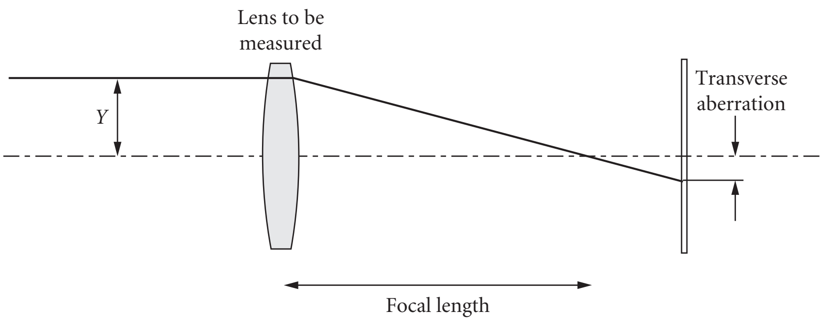

FIGURE 29 Focal length determination by transverse aberration measurements.

of x is variable and is adjusted until the light beam going out from the lens under test becomes collimated. This collimation is verified by means of a small telescope in front of this lens focused at infinity. The values of d and the focal length f are set to be equal. Then, the back focal length f_b of the lens under test is given by

$$\frac{1}{f_b} = \frac{1}{d} - \frac{x}{d^2} = P_v \tag{16}$$

where its inverse P_v is the vertex power. As can be seen, the power of the lens being measured is linear with respect to the distance x. There are many variations of this instrument. Some modern focimeters measure the lateral deviation of a light ray from the optical axis (transverse aberration), as in Fig. 29, when a defocus is introduced.[61,62,63] This method is mainly used in some modern automatic focimeters for optometric applications. To measure the transverse aberration, a position-sensing detector is frequently used. The power error of a focimeter can be obtained by derivation of Eq. (16)

$$\delta P_v = -\frac{\delta x}{f_c^z} \tag{17}$$

Thus, the power error is a linear function of the target position error and decreases with the square of the collimating lens focal distance.[70]

Other Focal Length Measurements

Moiré Deflectometry Moiré deflectometry method for focal length determination sends a collimated beam over a pair of Ronchi rulings (Fig. 30) depending on the convergence or divergence of the beam, the resulting moiré pattern rotates according to the convergence (Fig. 31). The rotation has an angle α that is related to the focal distance by[71,72]

$$f \approx \frac{d}{\theta \tan\alpha} \tag{18}$$

d being the ruling's pitch, θ is the angle between the ruling's lines, and α is the rotation angle of the moiré pattern.

Talbot Autoimages Talbot autoimages method for focal length determination is performed by sending a coherent beam of light into a Ronchi ruling. An image of the grating will be produced periodically and evenly spaced along the light beam. Every Talbot autoimage is an object for the lens

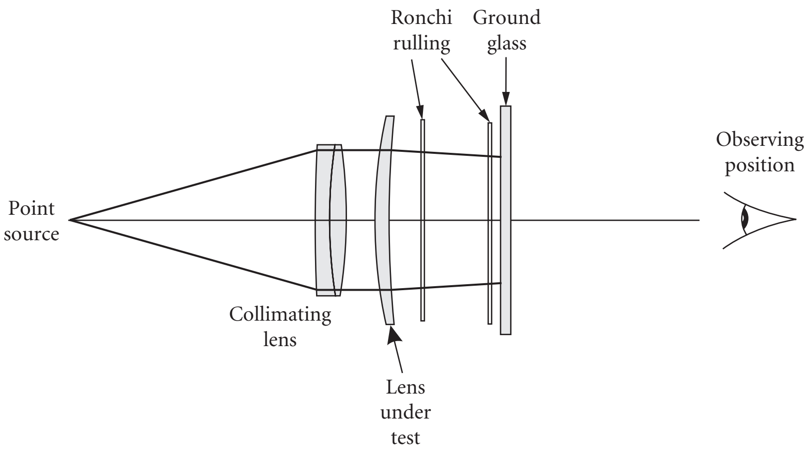

FIGURE 30 Moiré deflectometry lens power measurement. (*From Malacara.*[1])

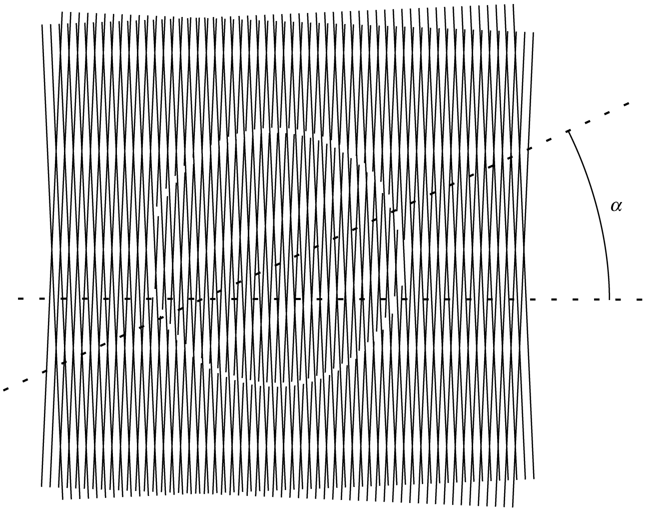

FIGURE 31 Moiré pattern as produced in a moiré deflectometer. (*From Malacara.*[1])

under test and produces a set of autoimages at the image side of the lens. To determine the lens focal length, another ruling coincident to the autoimages at the image plane is used.[73,74]

Fourier Transforms The Fourier-transforming property of a lens can be used in the focal-length determination. Horner[75] measured the diffraction pattern at the focal plane produced by a slit. For this method, the light beam does not have to be perfectly collimated.

Microlenses Micro-lenses applications require new methods for small focal length determination. The propagated Gaussian beam of a laser can be analyzed.[76,77] In this method, a lens is placed al the laser beam waist, then the propagating beam is measured to determine the focal length.

Fiber Optics A clever method used to automatically find the position of the focus has been described by Howland and Proll.[78] They used optical fibers to illuminate the lens in an autocollimating configuration, and the location of the image was also determined using optical fibers.

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