Physical realization of a Qubit

Introduction to Quantum Computing

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Outline

What have we heard so far?

- The definition of a qubit
- State space, $\mathbb{C} \times \mathbb{C}$
- Vector space
- Superposition
- Measurement
- and many others...

What shall we do now?

- We will attempt to give a physical context to the various concepts discussed in the previous lecture.
- To do that we shall discuss a little bit of physics.
- Is a discussion of physics really necessary to learn computation?

Information, Computation and Physics

- The classical bit may exist in one of two states, these states are labelled as “0” and “1”. This is the most fundamental unit of information.
- A physical realization of the bit is required for performing computation. In a classical computer, bits are realised in the states of a register.
- By the same token, a physical realization of a qubit is required for performing quantum computation.

Light waves

- Light is described by an electromagnetic wave, consisting of electric and magnetic field components.
- The electric field of a plane polarized light is given by the following function:

$$\vec{\mathbf{E}} = \hat{\mathbf{n}}\, E_0 \sin(\omega t - kz)$$

  here, the light wave is propagating (moving) along the $\hat{\mathbf{z}}$ direction.
- $\hat{\mathbf{n}}$ is a unit vector in the $x - y$ plane.

Light particles

- At around 1900, it was established that light waves consist of particles called photons.
- The electromagnetic wave can now be visualised as a stream of a very large number of photons.
- The electromagnetic wave is now associated with each photon.
- Therefore, when we refer to a property of the light waves, they also become properties of the photons.

Polarization of light waves

- The polarization of an electromagnetic wave propagating along the z-axis is in the $x - y$ plane.
- Any vector in the $x - y$ plane can be represented in terms of its x and y components.

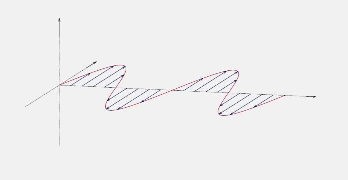

Figure 1: Horizontally Polarized Wave

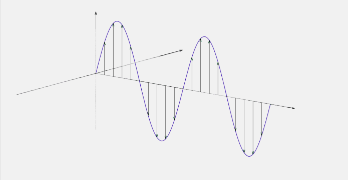

Figure 2: Vertically Polarized Wave

A little bit of Vector Algebra

- The linearly independent vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ to represent vertical and horizontal polarizations respectively.

- A polarization vector inclined at an angle θ to the vertical is represented as follows:

$$\cos\theta \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sin\theta \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{1}$$

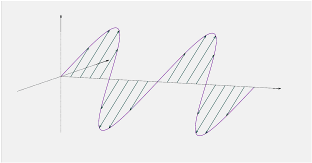

Figure 3: An obliquely polarized wave

Fun with polarizers

- Polarizers are optical filters that allow light waves of a particular polarization to pass through them.
- A vertically aligned polarizer will block all horizontally polarized light from passing through and vice versa.
- But what will happen when an obliquely polarized light wave is incident on a vertically (or horizontally) aligned polarizer?
- The following video answers the above question through demonstrations:
  `https://www.youtube.com/watch?v=6N3bJ7Uxpp0`

A classical result

- When a light wave with polarization as defined in equation (1) is incident on a vertically aligned polarizer, only a fraction of this wave emerges.
- The fraction of the intensity that is transmitted is given by $|\cos\theta|^2$. In the case of a horizontally aligned polarizer, the same fraction is $|\sin\theta|^2$.
- In general, the fraction of the intensity that is transmitted is given by $|\cos\alpha|^2$ where α is the angle between the polarization vector and the direction along which the polarizer is aligned.
- It should be remembered that the polarization of the transmitted light wave is along the alignment of the polarizer.

The Quantum Perspective

- One of the earliest conclusions of quantum theory was that light waves exhibited particle like behaviour and these particles were named photons.
- Light waves discussed thus far may be visualised as a stream of a very large number of photons.
- The reduction of intensity when a light wave passes through a polarizer implies that only a fraction of the incident photons are transmitted. This fraction as mentioned before is $|\cos\alpha|^2$.
- It is once again emphasized that the polarization of the photons emerging is along the direction in which the polarizer is aligned.

The Photon as a Qubit

- Based on the ideas discussed so far, it is possible to state the following about single photons incident on a polarizer.
- It is in general not possible to state with certainty if an incident photon will be transmitted by a polarizer.
- A photon polarized along the direction of a polarizer will certainly be transmitted. A photon with a polarization that is orthogonal to a polarizer's alignment will certainly not be transmitted.
- The probability of a photon being transmitted by a polarizer is once again given by $|\cos \alpha|^2$.
- The aforementioned facts have all been verified through experiments.

Concluding Remarks

- The photon is a particle that can exist simultaneously in both (orthogonal) polarization states.
- The polarization of the photon may be represented as a linear combination of these orthogonal states.
- The outcome of an experiment to estimate the polarization of a photon can only be interpreted statistically. The experiment also leaves the state of the photon changed.
- Such photon based qubits are used extensively in Quantum Information and Communication, most notably in Quantum Key Distribution (QKD).

But what of the complex numbers?

- The photon states described so far have all been real linear combinations of the vertical and the horizontal polarization states.

- Complex linear combination of these states are used to define polarization states such as circular and elliptical polarization.

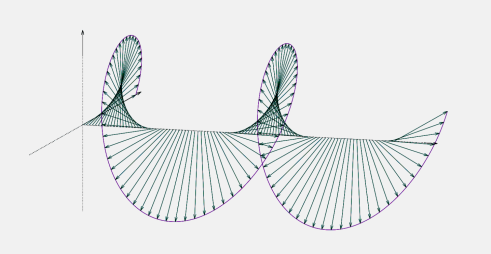

Figure 4: A circularly polarized wave