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**DSB Classes 9-10, February 9, 2018**

- **Dimensionality Reduction. Clustering and Segmentation**

# Structure of the course

- SESSIONS 1-2 (AO): Data analytics process; from Excel to R
  - Tutorial 1: Getting comfortable with R
- SESSIONS 3-4 (AO): Time Series Models
- SESSIONS 5-6 (AO): Intro to classification, logistic regression and machine learning
  - Tutorial 2: Midterm R help / classification
- SESSIONS 7-8 (SZ): Advanced Classification; From .R to Notebooks; Dimensionality reduction
- **SESSIONS 9-10 (SZ): Dimensionality Reduction; Clustering and Segmentation**
  - Tutorial 3: Q&A on R for three main modules
- SESSIONS 11-12 (SZ): Catch-up and wrap-up; Guest speaker
  - Tutorials 4, 5: Hands-on help on projects
- SESSIONS 13-14 (AO+SZ): Project presentations

# Plan for the day

## Learning objectives

- Derived attributes and dimensionality reduction
  - Generate (a small number of) new manageable/interpretable attributes that capture most of the information in the data
- Clustering and segmentation
  - Group observations in a few segments so that data within any segment are similar while data across segments are different
- Work on business solution template for market segmentation (Assignment 3) for the Boats (A) case

# Derived Attributes and Dimensionality Reduction

- What is dimensionality reduction?
  - Generate (a small number of) new attributes that are (linear) combinations of the original ones, and capture most of the information in the original data
  - Often used as the first step in data analytics
- Why do dimensionality reduction?
  - Computational and statistical reasons: with thousands of features, very expensive and hard to estimate a good model
  - Managerial reason: the new attributes are interpretable and actionable
- The key idea of dimensionality reduction
  - Transform the original variables into a smaller set of **factors**
  - Understand and interpret the factors
  - Use the factors for subsequent analysis

# Dimensionality Reduction: Key Questions

1. How many factors do we need?
2. How would you name the factors? What do they mean?
3. How interpretable and actionable are the factors we found?

# (A) Process for Dimensionality Reduction

1. Confirm the data is metric
2. Scale the data
3. Check correlations
4. Choose number of factors
5. Interpret the factors
6. Save factor scores

# Applying Dimensionality Reduction

## Evaluation of MBA Applications

Variables available:

1. GPA
2. GMAT score
3. Scholarships, fellowships won
4. Evidence of communications skills
5. Prior job experience
6. Organizational experience
7. Other extra curricular achievements

Which variables are correlated? What do these groups of variables capture?

# Step 1: Confirm data is metric

	Variables	GPA	GMAT	Fellow	Comm	Job.Ex	Organze	Extra
1	1	3	580	2	3.5	5	3.8	4
2	2	3.2	570	2	3.8	6	3.8	3.8
3	3	3.7	690	3	3.3	3	3.2	3.6
4	4	3.9	760	3	3.8	5	3.9	3.2
5	5	2.8	480	2	3.2	6	3.8	3.8
6	6	3.4	520	2.5	2.6	2	2.5	2.4
7	7	3.6	670	3	3.7	4	3.5	2.9
8	8	3.6	760	3	3.9	5	3.3	3.2



# Step 2: Scale the data

Before standardization

	Variables	min	X25.percent	median	mean	X75.percent	max	std
1	GPA	2.5	2.8	3.45	3.31	3.62	3.9	0.47
2	GMAT	380	480	575	583.5	682.5	760	119.44
3	Fellow	1	2	2.8	2.45	3	3.8	0.91
4	Comm	2	3.18	3.4	3.34	3.73	3.9	0.49
5	Job.Ex	2	3	5	4.25	5.25	6	1.52
6	Organze	1	3.05	3.4	3.2	3.8	3.9	0.73
7	Extra	2.4	2.88	3.4	3.3	3.8	4	0.52

# Step 2: Scale the data

Standardization....

```
ProjectDatafactor_scaled=apply(ProjectDataFactor,2, function(r) { #''2'' applies the function over columns
  if (sd(r)!=0) {
    res=(r-mean(r))/sd(r)
  } else {
    res=0*r; res
  }
})
```

# Step 2: Scale the data

After standardization

	Variables	min	X25.percent	median	mean	X75.percent	max	std
1	GPA	-1.72	-1.08	0.31	0	0.68	1.27	1
2	GMAT	-1.7	-0.87	-0.07	0	0.83	1.48	1
3	Fellow	-1.6	-0.5	0.39	0	0.61	1.49	1
4	Comm	-2.73	-0.33	0.13	0	0.8	1.16	1
5	Job.Ex	-1.48	-0.82	0.49	0	0.66	1.15	1
6	Organze	-2.99	-0.2	0.27	0	0.82	0.95	1
7	Extra	-1.75	-0.83	0.19	0	0.97	1.36	1

# Step 3: Check correlations

	GPA	GMAT	Fellow	Comm	Job.Ex	Organze	Extra
GPA	1.00	0.90	0.92	0.56	0.15	-0.03	0.01
GMAT	0.90	1.00	0.86	0.78	0.33	0.19	0.16
Fellow	0.92	0.86	1.00	0.59	0.18	0.01	0.02
Comm	0.56	0.78	0.59	1.00	0.60	0.47	0.39
Job.Ex	0.15	0.33	0.18	0.60	1.00	0.80	0.77
Organze	-0.03	0.19	0.01	0.47	0.80	1.00	0.61
Extra	0.01	0.16	0.02	0.39	0.77	0.61	1.00

# Step 3: Check correlations

	GPA	GMAT	Fellow	Comm	Job.Ex	Organze	Extra
GPA	1.00	0.90	0.92	0.56	0.15	-0.03	0.01
GMAT	0.90	1.00	0.86	0.78	0.33	0.19	0.16
Fellow	0.92	0.86	1.00	0.59	0.18	0.01	0.02
Comm	0.56	0.78	0.59	1.00	0.60	0.47	0.39
Job.Ex	0.15	0.33	0.18	0.60	1.00	0.80	0.77
Organze	-0.03	0.19	0.01	0.47	0.80	1.00	0.61
Extra	0.01	0.16	0.02	0.39	0.77	0.61	1.00

# Step 4: Choose the number of factors

We use Principal Component Analysis

Package: psych

```
UnRotated_Results<-principal(ProjectDataFactor, nfactors=ncol(ProjectDataFactor),  
                             rotate="none", score=TRUE)
```

- Factors are linear combinations of the original raw attributes...
- ...so that they capture as much of the variability in the data as possible
- Factors are uncorrelated, and as many as the variables
- Each factor has an associated “eigenvalue” – which corresponds to the amount of variance captured by that factor
- First factor has the highest eigenvalue and explains most of the variance, then the second, ..., and so on

# Step 4: Choose the number of factors

Package: FactoMineR

```
Variance_Explained_Table_results<-PCA(ProjectDataFactor, graph=FALSE)
```

```
Variance_Explained_Table<-Variance_Explained_Table_results$eig
```

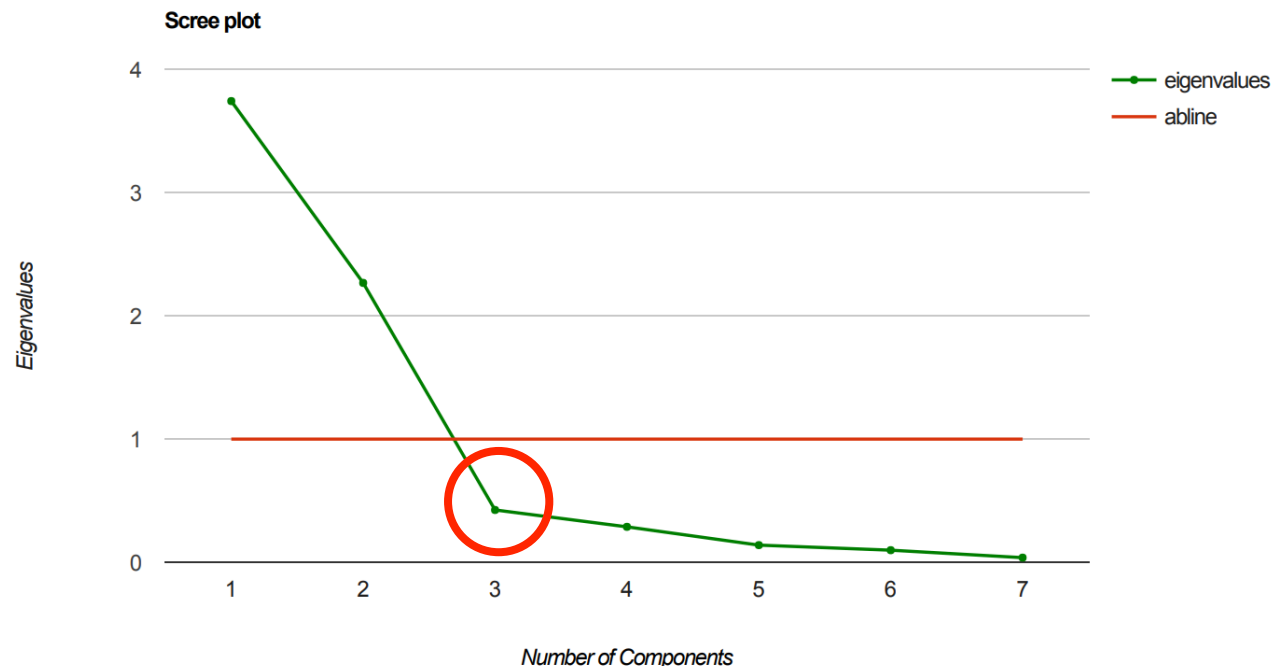
	Eigenvalue	Pct of explained variance	Cumulative pct of explained variance
Component 1	3.74	53.48	53.48
Component 2	2.27	32.40	85.88
Component 3	0.42	6.07	91.95
Component 4	0.29	4.11	96.06
Component 5	0.14	1.99	98.05
Component 6	0.10	1.41	99.46
Component 7	0.04	0.54	100.00

```
> Variance_Explained_Table[1,1]/sum(Variance_Explained_Table[,1]) ??  
[1] 0.5347987
```

# Step 4: Choose the number of factors

We want to capture as much of the variance as possible, with as few factors as possible. How to choose the factors? Three criteria to use:

- Select all factors with eigenvalue  $> 1$
- Select factors with highest eigenvalues up to exceeding a threshold (e.g. 65%) in cumulative % of explained variance
- Select factors up to the “elbow” of the scree plot





# Step 5: Interpret the factors

To interpret the factors, we want them to use only a few, non-overlapping original attributes

- Factor “rotations” transform the estimated factors into new ones that satisfy that, while capturing the same information

# Step 5: Interpret the factors

Package: psych

```
Rotated_Results<-principal(ProjectDataFactor, nfactors=max(factors_selected),  
                           rotate="varimax", score=TRUE)
```

```
Rotated_Factors<-round(Rotated_Results$loadings,2)
```

	Component 1	Component 2
GPA	0.96	-0.05
GMAT	0.95	0.19
Fellow	0.95	-0.01
Comm	0.70	0.54
Job.Ex	0.19	0.93
Organze	0.01	0.89
Extra	0.01	0.86

**To better visualize and interpret: suppress loadings with small values**

```
Rotated_Factors_thres <- Rotated_Factors
```

```
Rotated_Factors_thres[abs(Rotated_Factors_thres) < 0.5]<-NA
```

	Component 1	Component 2
GPA	0.96	
GMAT	0.95	
Fellow	0.95	
Comm	0.70	0.54
Job.Ex		0.93
Organze		0.89
Extra		0.86

# Step 5: Interpret the factors

What factor loads “look good”? Three technical quality criteria:

1. For each factor (column) only a few loadings are large (in absolute value)
2. For each raw attribute (row) only a few loadings are large (in absolute value)
3. Any pair of factors (columns) should have different "patterns" of loading

# Step 6: Save factor scores

Replace the original data with a new dataset where each observation is described using the derived factors

- For each row, estimate the **factor scores**: how the observation “scores” for each of the selected factors

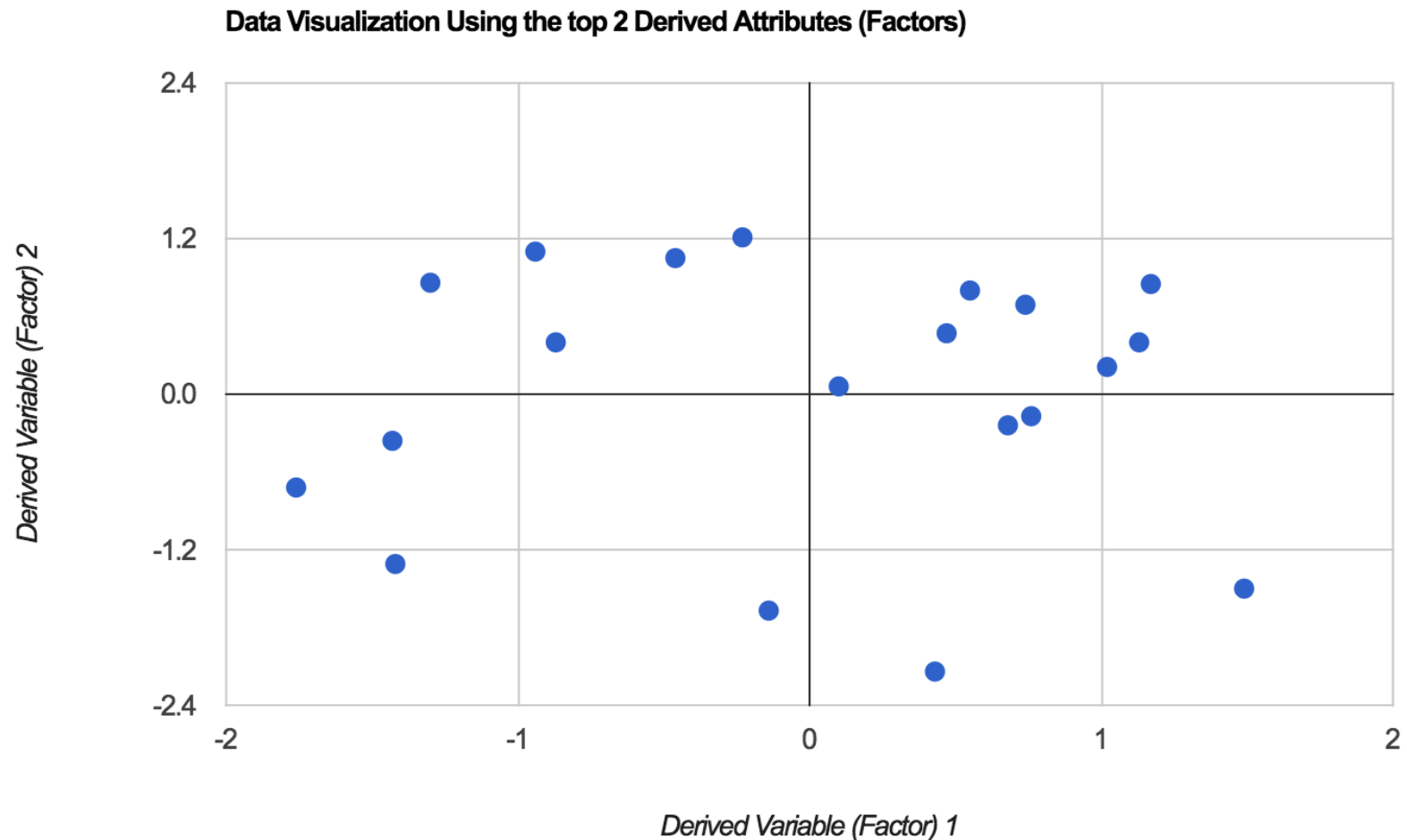
Package: psych

```
NEW_ProjectData <- round(Rotated_Results$scores[,1:factors_selected],2)
```

	Derived Variable (Factor) 1	Derived Variable (Factor) 2
observation 01	-0.46	1.05
observation 02	-0.23	1.21
observation 03	0.68	-0.24
observation 04	1.13	0.40
observation 05	-0.94	1.10
observation 06	-0.14	-1.67
observation 07	0.76	-0.17
observation 08	1.02	0.21
observation 09	-1.76	-0.72
observation 10	0.43	-2.14

# Step 6: Save factor scores

Then continue the analysis (e.g., make decision, or do clustering, etc.)  
with the new attributes



# Clustering and Segmentation

- What is clustering and segmentation?
  - Processes and tools to organize data in a few segments, with data being as similar as possible within each segment, and as different as possible across segments
- Applications
  - Market segmentation
  - Co-moving asset classes
  - Geo-demographic segmentation
  - Recommender systems
  - Text mining

# (A) Process for Clustering

1. Confirm the data is metric
2. Scale the data
3. Select segmentation variables
4. Define similarity measure
5. Visualize pair-wise distances
6. Method and number of segments
7. Profile and interpret the segments
8. Robustness analysis

# Step 3. Select segmentation variables

Critically important decision for the solution

- Requires lots of contextual knowledge and creativity

## **Segmentation attributes vs. profiling attributes**

For market research:

- Use attitudinal data for segmentation, so as to segment customers based on attitudes/needs
  - If ran dimensionality reduction before: segmentation attributes can be the original attributes with the highest absolute factor loading for each factor
- Use demographic and behavioral data for profiling the clusters found



# Step 4. Define similarity measure

Important: need to understand what makes two observations “similar” or “different”

There are infinitely many rigorous mathematical definitions of distance between two observations

Euclidean distance:

$$\|x - z\|_2 = \sqrt{(x_1 - z_1)^2 + \dots + (x_p - z_p)^2}$$

Manhattan distance:

$$\|x - z\|_1 = |x_1 - z_1| + \dots + |x_p - z_p|$$

# Step 4. Define similarity measure

Using Euclidean distance:

	Obs.01	Obs.02	Obs.03	Obs.04	Obs.05	Obs.06	Obs.07	Obs.08	Obs.09	Obs.10
Obs.01	0									
Obs.02	4	0								
Obs.03	4	3	0							
Obs.04	4	4	4	0						
Obs.05	4	4	5	4	0					
Obs.06	4	3	3	4	4	0				
Obs.07	6	5	6	6	4	5	0			
Obs.08	4	3	4	4	4	4	5	0		
Obs.09	5	4	5	4	3	4	4	3	0	
Obs.10	8	6	7	7	8	5	7	7	7	0

# Step 4. Define similarity measure

Can also define distance manually

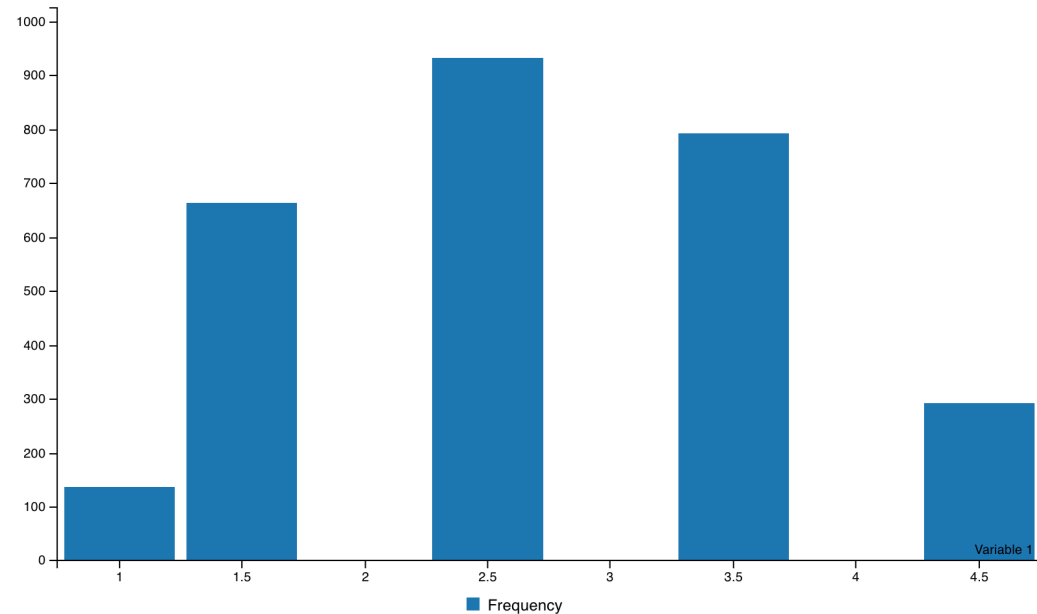
- Let's say that the management team believes that two customers are similar for an attitude if they do not differ in their ratings for that attitude by more than 2 points
- We can manually assign a distance of 1 for every question for which two customers gave an answer that differs by more than 2 points, and 0 otherwise

```
My_Distance_function<-function(x,y){ # x, y are vectors (answers of customers)
  sum(abs(x - y) > 2)
}
```

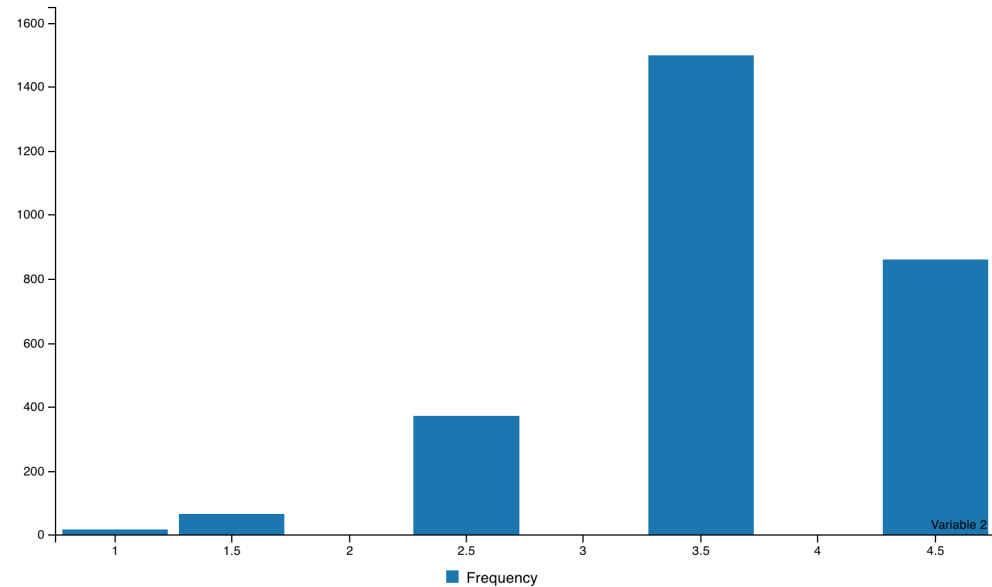
# Step 5. Visualize pairwise distances

Visualize individual attributes...

Q1.27: Boating is the number one thing I do in my spare time

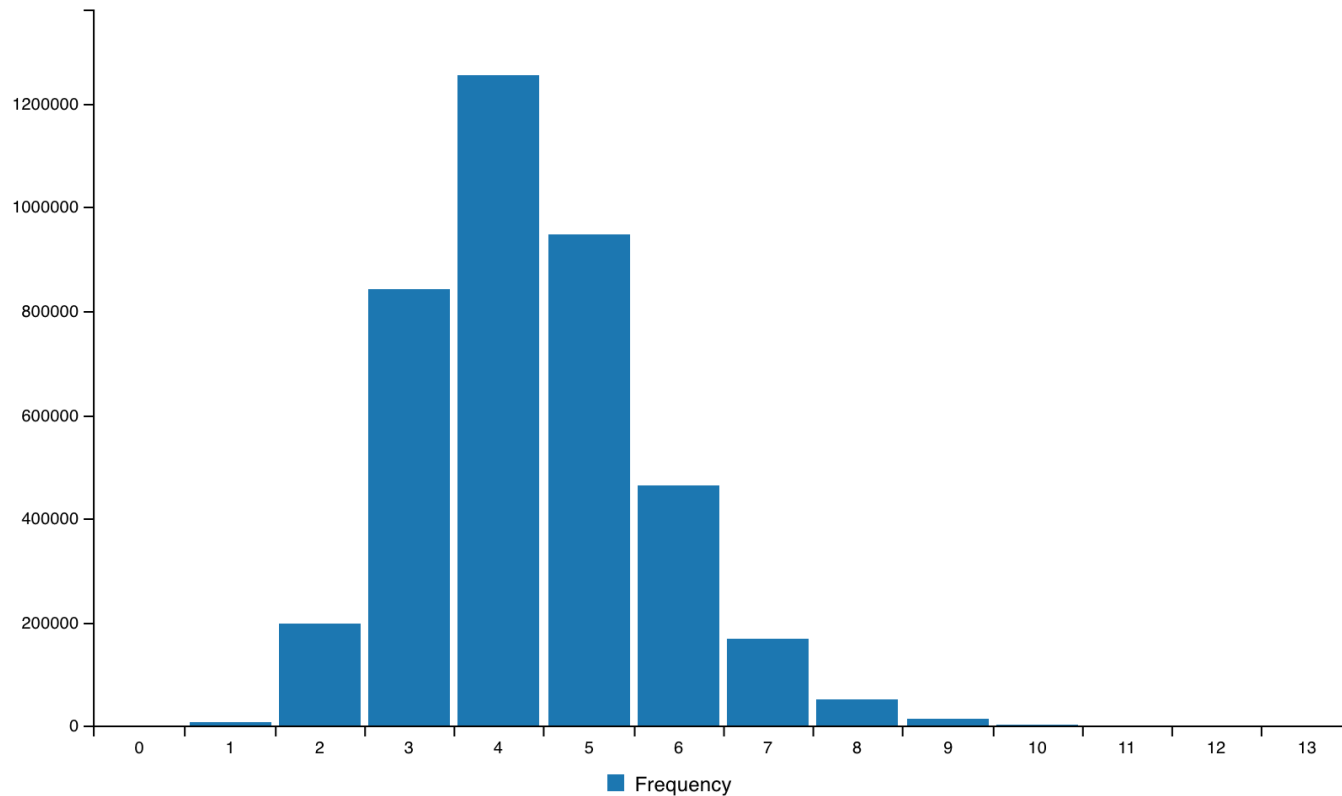


Q1.24: Boating gives me an outlet to socialize with family and/or friends



# Step 5. Visualize pairwise distances

... and pairwise distances



# Step 6. Method and number of segments

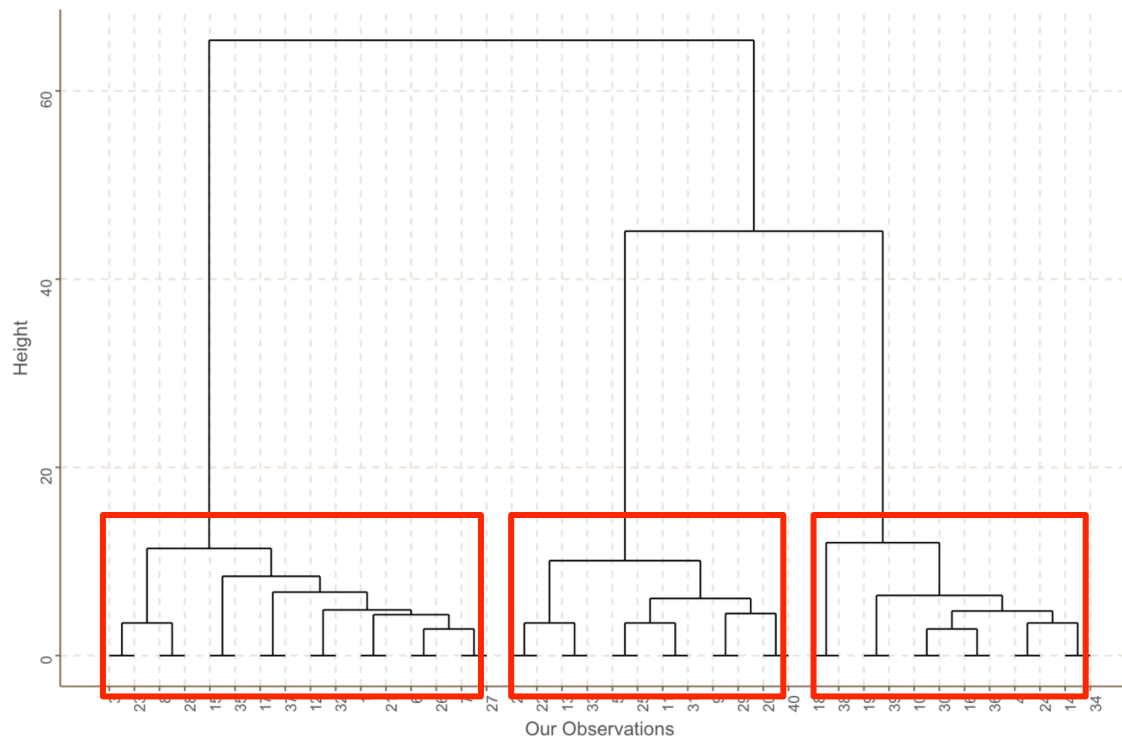
Many clustering methods. In practice, we want to use various approaches and select the solution that is robust, interpretable, actionable.

- Hierarchical clustering
- K-means

We can plug-and-play this “black box” in our analysis – with care

# Step 6. Method and number of segments

## Hierarchical Clustering



“Dendrogram”

- Observations that are the closest to each other are grouped together
- Start with pairs
- Merge smaller groups into larger ones
- Eventually all our data are merged into one segment
- Heights of the branches of the tree indicate how different are the clusters merged at that level of the tree
- Then cut the tree so as to create the desired number of clusters

# Step 6. Method and number of segments

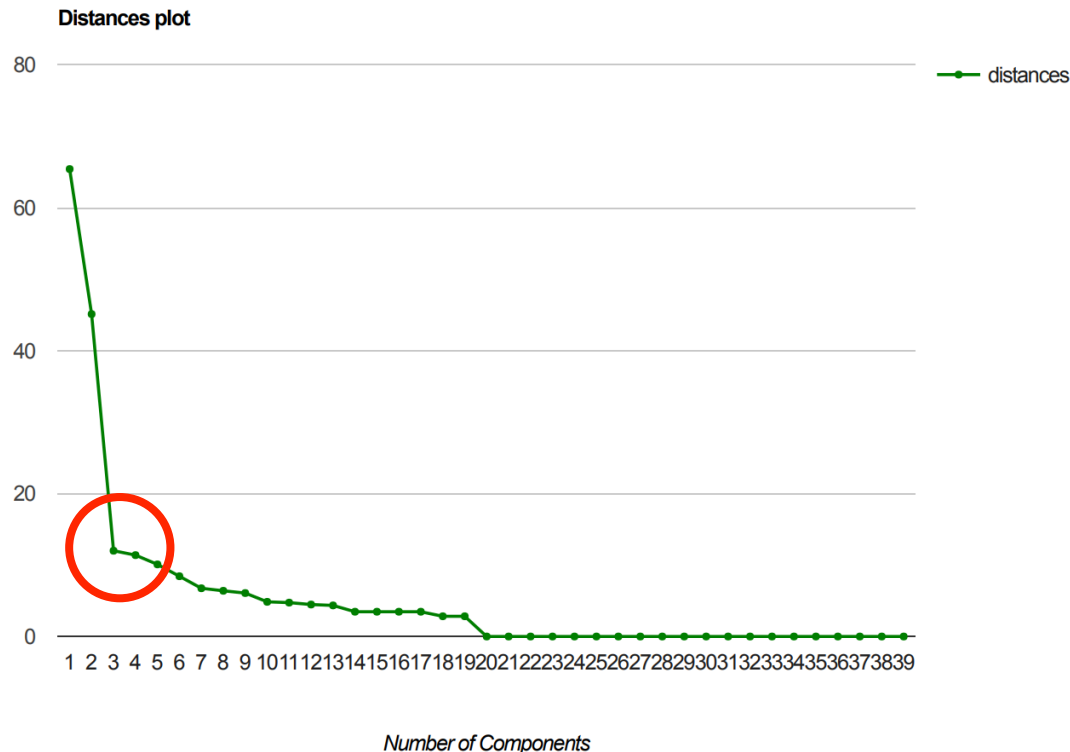
## Hierarchical Clustering

```
ProjectData_segment <- ProjectData[,segmentation_attributes_used]
Hierarchical_Cluster_distances <- dist(ProjectData_segment, method="euclidean")
Hierarchical_Cluster <- hclust(Hierarchical_Cluster_distances, method="ward.D")
# Display dendrogram
iplot.dendrogram(Hierarchical_Cluster)
```



# Step 6. Method and number of segments

## Hierarchical clustering: Choosing the number of clusters



- Rule of thumb: set number of clusters as the “elbow” of the plot
- In practice: start with above rule, then explore different numbers of clusters
- Select final solution using also interpretability

# Step 6. Method and number of segments

Hierarchical clustering on Boats data



To retrieve segment membership:

```
cluster_memberships_hclust <- as.vector(cutree(HierarchicalCluster,  
                                              k=numb_clusters_used))
```

# need to input a number of clusters for cutting the tree, not earlier

Observation Number	Cluster_Membership
1	1
2	2
3	1
4	3
5	4
6	1
7	4
8	2
9	4
10	3

# Step 6. Method and number of segments

**K-means clustering** aims to partition the observations into k sets so as to minimize the sum of within-cluster variances

- In each iteration, every observation is assigned to the nearest mean
- K-means does not necessarily lead to the same solution every time you run it

```
kmeans_clusters <- kmeans(ProjectData_segment, centers = numb_clusters_used, iter.max = 2000,  
                           algorithm="Lloyd")
```

# need to input a number of clusters as soon as clustering method is called

To retrieve segment membership:

```
kmeans_clusters$cluster
```

Observation Number	Cluster_Membership
1	5
2	5
3	5
4	5
5	5
6	5
7	5
8	5
9	5
10	3

Different methods may put observations in different clusters

# Step 7. Profile and interpret the segments

What are the resulting segments? We need to be able to understand and interpret the clustering solution

- Profile the segments using the profiling attributes

Average values within each segment and in total population

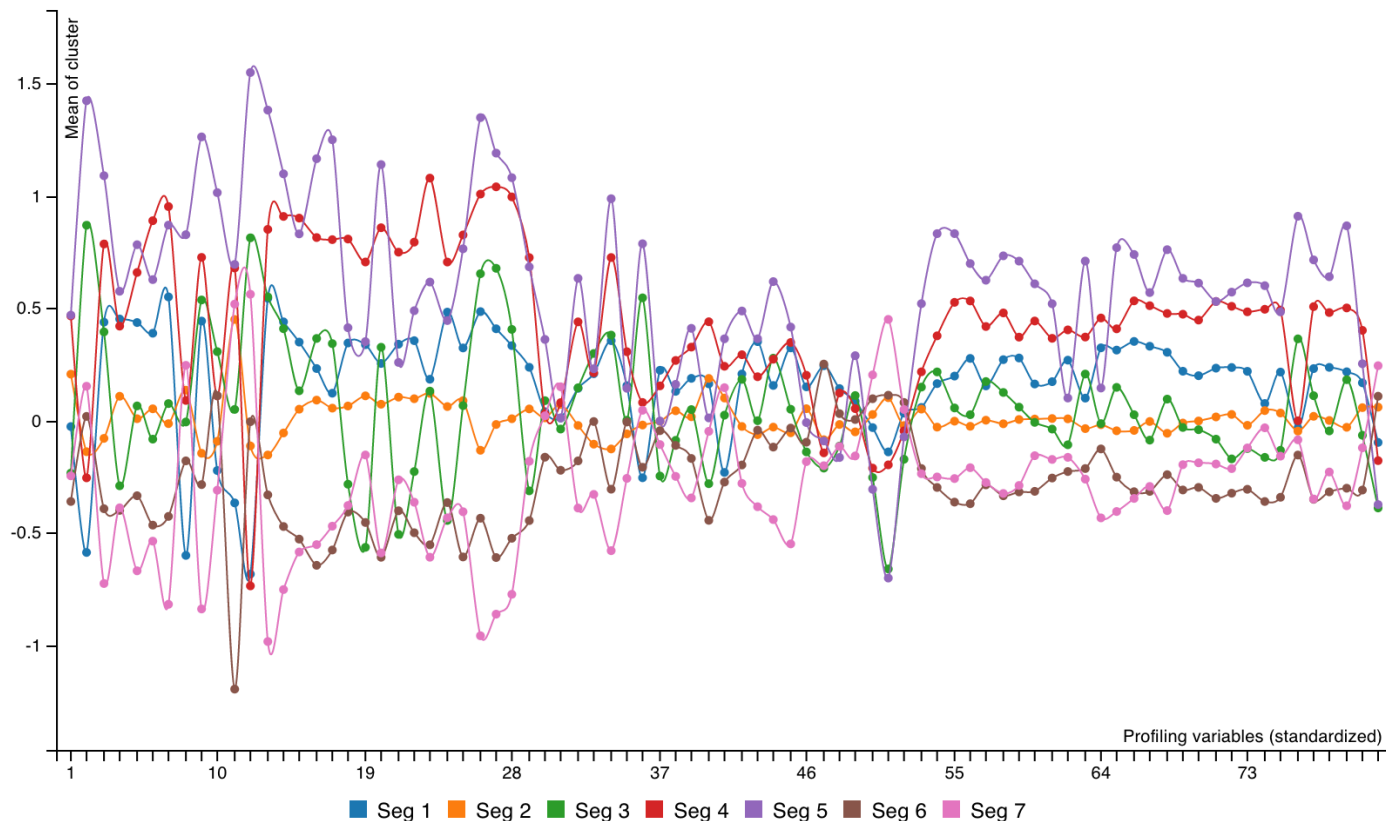
	Population	Seg.1	Seg.2	Seg.3	Seg.4	Seg.5	Seg.6	Seg.7
Q1.1	4.03	4.01	4.20	3.84	4.41	4.41	3.73	3.83
Q1.2	2.89	2.29	2.74	3.77	2.63	4.33	2.90	3.04
Q1.3	3.12	3.56	3.03	3.52	3.92	4.23	2.71	2.37

$\text{avg}(\text{segment})/\text{avg}(\text{population}) - 1$

	Seg.1	Seg.2	Seg.3	Seg.4	Seg.5	Seg.6	Seg.7
Q1.1	0.01-	0.04	0.05-	0.09	0.10	0.07-	0.05-
Q1.2	0.21-	0.05-	0.30	0.09-	0.50	0.01	0.05
Q1.3	0.14	0.03-	0.13	0.26	0.36	0.13-	0.24-

# Step 7. Profile and interpret the segments

Snake plots for each cluster: means of (standardized) profiling variables



# Step 8. Robustness analysis

The segments found should be relatively robust to changes in the clustering methodology

- Large changes indicate that segmentation is not valid

Two basic tests for statistical robustness and stability of interpretation:

1. How much overlap is there between the clusters found using different approaches?
2. How similar are the profiles of the segments found?

Also try different

- subsets of the original data
- variations of the original segmentation attributes
- different distance metrics
- different numbers of clusters

# Assignment 3 & Break-out Rooms

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- Assignment: Parts 1 and 2 of MarketSegmentationProcessInClass
- BORs: 319-329, 361A, 361E

# Summary of Sessions 9-10

- Derived attributes and dimensionality reduction
  - Principal Component Analysis, how to choose number of factors
  - Then continue analysis on the new attributes
- Clustering and segmentation
  - Create groups of similar observations
  - Hierarchical clustering, K-means clustering
- Template for market segmentation (Assignment 3) for the Boats (A) case



# Next...

- Tutorial 3 [Fri Feb 9, 7.15 pm, Amphi 102]
  - Walk through code chunks of market segmentation process
  - Help with Assignment 3
- Sessions 11-12 [Tue Feb 13, Amphi 105]
  - Catch up/wrap up
  - Guest speaker: advanced analytics leader in BCG's Financial Institutions and Insurance practices
- Assignment 3. Complete the market segmentation process + answers to questions of Parts 1 and 2
- Proposal for final project (due Feb 14)
- Final project (due Feb 20)

The background is a green-tinted collage of various business school scenes. It includes a large crowd of students in a lecture hall, a modern glass-fronted building with the INSEAD logo, a group of students in a meeting, and silhouettes of people in a dynamic pose.

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