

# RSA<sup>®</sup>Conference2018

San Francisco | April 16 – 20 | Moscone Center

SESSION ID: Digital Signatures

## REASSESSING SECURITY OF RANDOMIZABLE SIGNATURES

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Joint work with [David Pointcheval](#)



#RSAC



# Agenda

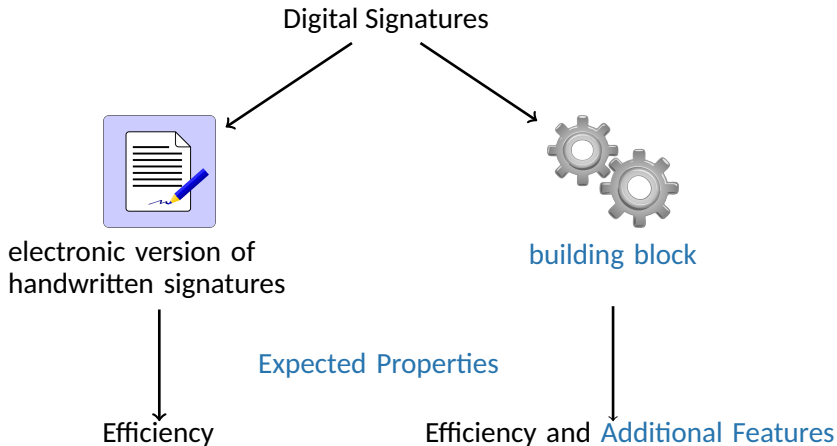


- Context
- Randomizable Signatures
- Our Contribution
- Conclusion



# Context

# Digital Signatures



# Example: Anonymous Authentication

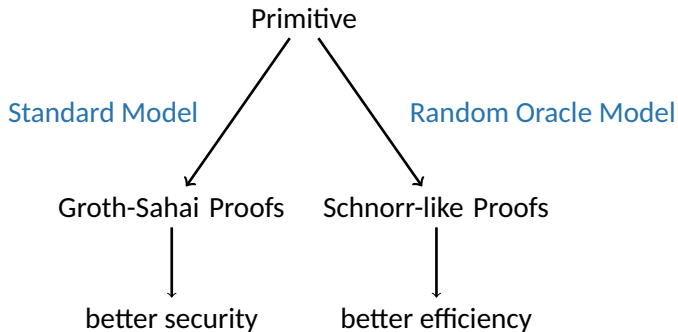


- Secure Devices (TPM, Intel SGX,...) may need to **authenticate**
- **Standard** Digital Signature is **unsuitable**:  
devices could be traced, which raises **legal issues**
- Such devices need (and use) **anonymous authentication** mechanisms

# Zero-Knowledge Proofs



Anonymous authentication usually combines digital signatures with ZK proofs:



- Digital signature must interact smoothly with such proofs
- For practical uses, constructions in the ROM are unavoidable



- Complexity of ZK proofs **increases with the number of elements to hide**  
⇒ this number must be reduced as much as possible
- **Randomizability** allows to derive **unlinkable versions**  $\sigma'$  from a signature  $\sigma$
- $\sigma'$  can be shown  
⇒ significantly improves efficiency



# Randomizable Signature



# Camenisch-Lysyanskaya Signatures



- CL signatures achieve **randomizability** in a **bilinear setting**
  - $\sigma = (\sigma_1, \sigma_2, \sigma_3)$  valid on  $m \Rightarrow \forall t \in \mathbb{Z}_p, \sigma' = (\sigma_1^t, \sigma_2^t, \sigma_3^t)$  valid on  $m$
  - $\sigma$  and  $\sigma'$  are unlinkable under the DDH assumption
- **Bilinear Groups:**  $\mathbb{G}_1, \mathbb{G}_2$  and  $\mathbb{G}_T$  of prime order  $p$  along with a map  $e$  such that
  - $\forall (g, \tilde{g}) \in \mathbb{G}_1 \times \mathbb{G}_2$  and  $a, b \in \mathbb{Z}_p$   $e(g^a, \tilde{g}^b) = e(g, \tilde{g})^{a \cdot b}$
  - $e(g, \tilde{g}) = 1_{\mathbb{G}_T} \implies g = 1_{\mathbb{G}_1}$  or  $\tilde{g} = 1_{\mathbb{G}_2}$
- Popular setting for privacy-preserving protocols:
  - Group Signature
  - Electronic Cash
  - ...

# Example of Group Signature



- **Join:** Alice gets a signature  $\sigma \leftarrow (\sigma_1, \sigma_2, \sigma_3)$  on her secret key  $sk \in \mathbb{Z}_p$
- To **anonymously** prove membership in the group, Alice
  - 1 randomize  $\sigma$ :  $t \xleftarrow{\$} \mathbb{Z}_p, \sigma' \leftarrow (\sigma_1^t, \sigma_2^t, \sigma_3^t)$
  - 2 sends  $\sigma'$  and proves that it is valid on the secret  $sk$ .
- Only  $sk$  needs to be hidden:  
 $\Rightarrow$  leads to very efficient protocols

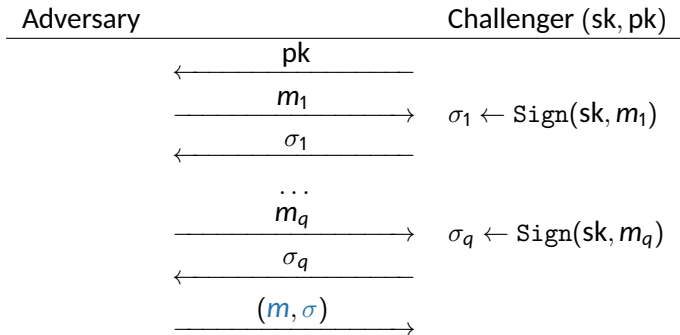
# Pointcheval-Sanders Signatures



- Complexity of CL-signatures increases with the number  $r$  of elements to sign
- PS signatures offer the same features with improved performances

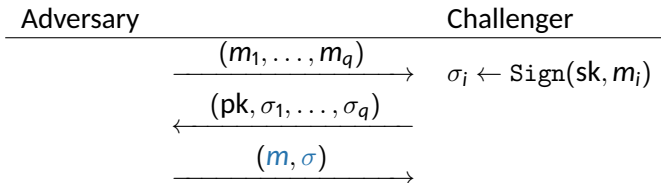
	CL	PS
Size	$(1 + 2r) \mathbb{G}_1$	$2 \mathbb{G}_1$
Sign Cost	$1 R_{\mathbb{G}_1} + 2r \mathbb{G}_1$	$1 R_{\mathbb{G}_1} + 1 \mathbb{G}_1$
Verif Cost	$4r P + r \mathbb{G}_2$	$2 P + r \mathbb{G}_2$
Randomizable	✓	✓

- The standard security notion for signatures is **EUF-CMA security**:



- The adversary succeeds if  $\sigma$  is valid on  $m$  and  $m \neq m_i$

- The weaker EUF-wCMA security notion can be enough:



- The adversary succeeds if  $\sigma$  is valid on  $m$  and  $m \neq m_i$
- The messages are no longer adaptively chosen

# Limits of Randomizable Signatures



- Randomizability of CL and PS signatures comes at a cost:  
security relies on the interactive LRSW and PS assumptions
- These assumptions essentially state the EUF-CMA security
- The lack of precise security assessment is an obstacle to a widespread deployment of these signatures
- Non-randomizable alternatives can be preferred for efficiency reasons

# Boneh-Boyen Signature



- BB signatures is a popular (non-randomizable) alternative for privacy-preserving primitives
- EUF-CMA security relies on *q*-SDH assumption:

given  $(g, g^x, \dots, g^{x^q})$ , it is hard to output  $(w, g^{\frac{1}{x+w}})$  with  $w \in \mathbb{Z}_p^*$

- *q*-SDH assumption is *better accepted* than interactive assumptions:
  - it is *easier to assess*
  - it is not directly related to EUF-CMA security



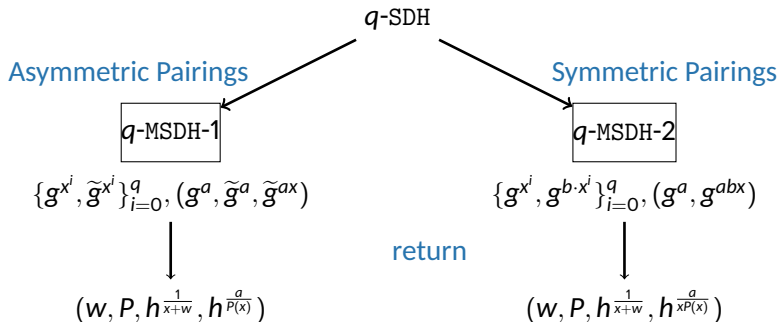
# Our Contribution



# $q$ -MSDH Assumptions



$\mathbb{G}_1 = \langle g \rangle$  and  $\mathbb{G}_2 = \langle \tilde{g} \rangle$



- $h \in \mathbb{G}_1^*$ ,  $w \in \mathbb{Z}_p$  such that  $P(w) \neq 0$ .
- We prove that they underlie **EUF-wCMA security of PS and CL signatures**

# $q$ -MSDH Assumptions



- We prove that **both assumptions hold in the generic group model**
- Intuition for  $q$ -MSDH-1:

$$\{g^{x^i}, \tilde{g}^{x^i}\}_{i=0}^q, (g^a, \tilde{g}^a, \tilde{g}^{ax}) \rightarrow (w, P, h^{\frac{1}{x+w}}, h^{\frac{a}{P(x)}})$$

- randomizability implies that  **$h$  can be any element of  $\mathbb{G}_1^*$**
- $h^{\frac{a}{P(x)}}$  can be computed from  $g^a$  only if  $h = g^{\lambda P(x)}$  with  $\lambda \in \mathbb{Z}_p$
- In such a case  $h^{\frac{1}{x+w}} = g^{\frac{\lambda P(x)}{x+w}}$  cannot be computed from  $\{g^{x^i}\}_{i=0}^q$  since  $(x+w) \nmid P(x)$
- The **same reasoning holds for  $q$ -MSDH-2:**

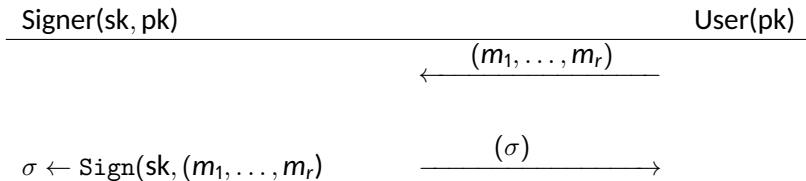


- These assumptions only underlie EUF-wCMA security of CL or PS signatures
- This security notion can be enough in some contexts if the certified secret value is generated collaboratively
- Example: in the Join procedure of a group signature scheme
  - Alice generates  $sk_1$  and proves knowledge of it
  - the group manager selects and sends  $sk_2$  along with a certificate on  $sk_1 + sk_2$
  - Alice sets  $sk$  as  $sk_1 + sk_2$

# Achieving EUF-CMA Security



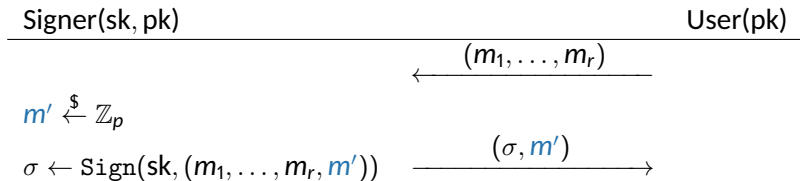
- CL and PS signatures can sign several messages



# Achieving EUF-CMA Security



- CL and PS signatures can sign several messages



- Signing an additional message  $m'$  is enough to achieve EUF-CMA security under these assumptions
- Slight increase of the complexity

# Keeping Randomizability



- The additional message  $m'$  adds an element to the signature
- $m'$  cannot be randomized
- In the ROM, we can set  $m' = H(m_1, \dots, m_r)$ 
  - ⇒ CL and PS features are kept
- No need to check that  $m' = H(m_1, \dots, m_r)$  in the verification process
  - ⇒ compatibility with ZK proofs is ensured
- Most protocols based on CL and PS signatures already use the ROM



# Conclusion

# Conclusion



- We reassessed security of CL and PS signatures and showed that:
  - they are EUF-wCMA secure under variants of  $q$ -SDH assumptions
  - they are EUF-CMA secure under the same assumptions assuming slight modifications
- We prove that these assumptions hold in the generic groups model
- CL or PS signatures can be used without jeopardizing security
  - ⇒ no need to choose between security and randomizability





thank you

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SESSION ID: CRYPT-R12

## DIFFERENTIAL ATTACKS ON DETERMINISTIC SIGNATURES

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*Joint work with Christopher Ambrose, Joppe W. Bos, Björn Fay, Manfred Lochter, and  
Bruce Murray*



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# Elliptic Curve Signatures



1985 ElGamal introduces first  
DLog-based signatures

1985 Koblitz and Miller propose elliptic  
curve cryptography

(?) 1991 Kravitz designs a variant of  
ElGamal signature scheme (DSA)

1993 NIST adopts DSA as FIPS 186

2000 NIST includes **ECDSA** in FIPS 186-2

2012 Bernstein, Duif, Lange, Schwabe,  
and Yang publish **EdDSA**



American National Standard  
for Financial Services

ANS X9.62–2005

Public Key Cryptography for the Financial  
Services Industry

The Elliptic Curve Digital Signature Algorithm  
(ECDSA)



Accredited Standards Committee X9, Inc.  
Financial Industry Standards

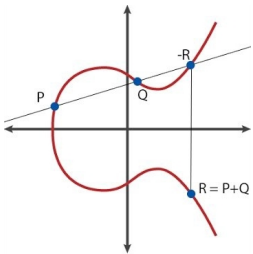
Date Approved: November 16, 2005

American National Standards Institute

### Key generation

public key  $\langle G \rangle$  of prime order  $n$

private key  $d \xleftarrow{\$} [1, n - 1]$



### Signature

Input: Message  $m \Rightarrow e = \mathcal{H}(m)$

- 1  $k \xleftarrow{\$} [1, n - 1]$
- 2  $r \leftarrow x([k]G) \bmod n \quad \triangleright r \neq 0$
- 3  $s \leftarrow k^{-1}(e + dr) \bmod n \quad \triangleright s \neq 0$

Output: Signature on  $m$  is  $(r, s)$



random integer  $k$  cannot be re-used!!

ECDSA is sensitive to PRNG's quality

- **nonce  $k$  cannot be re-used**
- worse, prediction of a number of bits of  $k$  allows the recovery of private key  $d$

Given signatures  $(r_1, s_1)$  and  $(r_2, s_2)$  on 2 different messages  $m_1$  and  $m_2$ :

$$\begin{cases} s_1 \leftarrow k_1^{-1}(e_1 + dr_1) \bmod n \\ s_2 \leftarrow k_2^{-1}(e_2 + dr_2) \bmod n \end{cases}$$

If  $k_1 = k_2$  then  $d = \dots$



Dec 2010: Fail0verflow recovers ECDSA private key used to sign code for PS3

## Solution

Generate signatures in a completely **deterministic** way





# Deterministic ECDSA



## ECDSA

**Input:** Message  $m \Rightarrow e = \mathcal{H}(m)$

- 1  $k \xleftarrow{\$} [1, n - 1]$
- 2  $r \leftarrow x([k]G) \bmod n$
- 3  $s \leftarrow k^{-1}(e + dr) \bmod n$

**Output:** Signature on  $m$  is  $(r, s)$

## Deterministic ECDSA

**Input:** Message  $m \Rightarrow e = \mathcal{H}(m)$

- 1  $u \leftarrow \text{GenU}(d, e)$
- 2  $r \leftarrow x([u]G) \bmod n$
- 3  $s \leftarrow u^{-1}(e + dr) \bmod n$

**Output:** Signature on  $m$  is  $(r, s)$

This is the approach used in **EdDSA**



Edwards-curve DSA

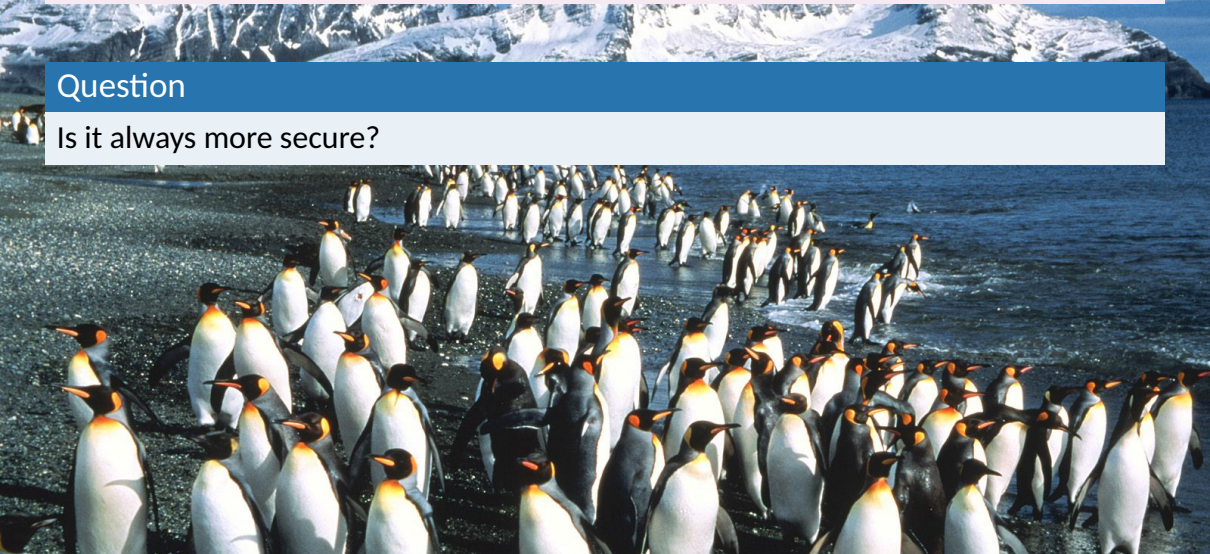
Bernstein, Duif, Lange, Schwabe, and Yang  
JCEN, 2012

## Solution

Generate signatures in a completely **deterministic** way

## Question

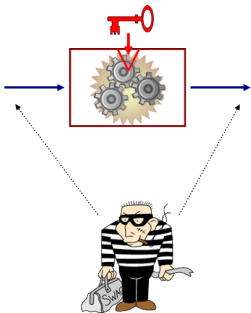
Is it always more secure?





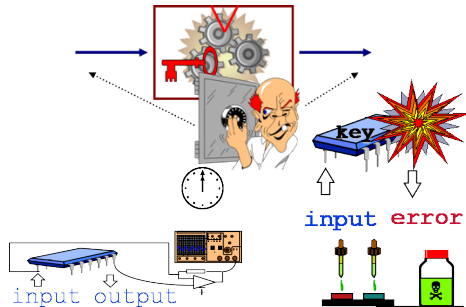
## Black-box attacks

- Attacker has only access to I/O



## Physical attacks

- Attacker can monitor the execution and inject faults



# Edwards-Curve DSA (EdDSA)



## Key generation

**public key**  $\langle B \rangle$  of prime order  $\ell$  and  $\underline{A}$   
where  $A = sB$

**private key**  $k \xleftarrow{\$} [1, \ell]$

$$\rightsquigarrow \mathcal{H}_2(k) = (h_0, h_1, \dots, h_{2b-1})$$

$$\rightsquigarrow s = 2^c \cdot (1, h_{n-1}, \dots, h_c)_2$$

## Signature

**Input:** Message  $m$

- ①  $m' \leftarrow \mathcal{H}'(m)$   $\triangleright$  prehash function  $\mathcal{H}'$
- ②  $r \leftarrow \mathcal{H}(h_b, \dots, h_{2b-1}, m') \bmod \ell$
- ③  $R \leftarrow rB$
- ④  $t \leftarrow \mathcal{H}(\underline{R}, \underline{A}, m')$
- ⑤  $S \leftarrow r + ts \bmod \ell$

**Output:** Signature on  $m$  is  $(\underline{R}, \underline{S})$

# Attacks against EdDSA



Fault attack on base-point  $B$  during import:  $B \rightsquigarrow \tilde{B}$



Secret  $s$  can be recovered since

$$S - \tilde{S} \equiv (t - \tilde{t})s \pmod{\ell}$$

and  $t, \tilde{t}$  can be computed

## Signature

Input: Message  $m$

- 1  $m' \leftarrow \mathcal{H}'(m)$
- 2  $r \leftarrow \mathcal{H}(h_b, \dots, h_{2b-1}, m') \bmod \ell$
- 3  $\tilde{R} \leftarrow r\tilde{B}$
- 4  $\tilde{t} \leftarrow \mathcal{H}(\tilde{R}, \underline{A}, m')$
- 5  $\tilde{S} \leftarrow r + \tilde{t}s \bmod \ell$

Output: Signature on  $m$  is  $(\tilde{R}, \tilde{S})$

# Our Results



where	attack	type	number of faults
Import point $B$	fault	uncontrolled	$\geq 1$
Import point $A$	fault	controlled	$\geq 1$
Hash computation of $r$	fault	controlled	$\geq 1$
Hash computation of $r$ with fixed (unknown) output	{ fault	uncontrolled	$\geq 1$ }
Scalar multiplication $rB$	fault	uncontrolled	$\geq 1$
Hash computation of $t$	fault	controlled	$\geq 1$
Hash computation of $t$ with fixed (unknown) output	{ fault	controlled	$\geq 2$ }
Computation of $S$	fault	controlled	$\geq 1$
Hash computation of $r$	DPA/DEMA	-	-

# Countermeasures (1)



## Fully compliant countermeasures

- Check the validity of targeted points
  - Use redundancy (e.g., double computation)
  - Harden the hash computation
- ▷ do not cover all our attacks!
- ▷ against the side-channel attack

↪ Significant impact on performance



# Countermeasures (2)



## Not fully compliant countermeasures

- **Adaptive solution**  $\rightsquigarrow$  Include random noise in the computation of  $r$

$$r \leftarrow \mathcal{H}(\underbrace{\kappa}_{\text{random noise}}, \underbrace{h_b, \dots, h_{2b-1}}_{\text{secret input}}, \underbrace{m'}_{\text{prehashed message } m'}) \bmod \ell$$

(or unknown counter if no random source is available)



# Summary



- Removing randomness in signature generation does not necessarily eliminate all attack vectors: 8 fault attacks and 1 side-channel attack
- Countermeasures fully compliant with the current specification of EdDSA seem to have a significant performance impact
- Deviating from the specification and introducing high-quality randomness [where this is possible] allows the construction of cheap countermeasures
  - and does not affect the key generation and signature verification

We hope this work serves as valuable input when the community and the various standardization bodies start to define new cryptographic digital signature algorithms

# Comments/Questions?

