RSAConference2018
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ZERO-SUM PARTITIONS OF PHOTON PERMUTATIONS

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Zero-Sum Partitions of PHOTON Permutations

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April, 2018

Introduction (1/2)

Hash functions are one of the most important primitives in symmetric-key cryptography.

Sponge functions are a way of building hash functions from a fixed permutation.

Modern cryptanalytic approaches target both hash function primitives and underlying ciphers or permutations.

Introduction (2/2)

PHOTON [GPP11] is a (lightweight) family of sponge-like hash proposed at CRYPTO 2011 and recently standardized by ISO.

W.r.t. the security claims made by the designers, *we show - for the first time - zero-sum partitions for (almost) all of those full 12-round (inner) permutation variants that use a 4-bit S-Box.*

Our results are theoretical in nature:

there is currently no reason to believe that the security of PHOTON as a hash function is endangered.

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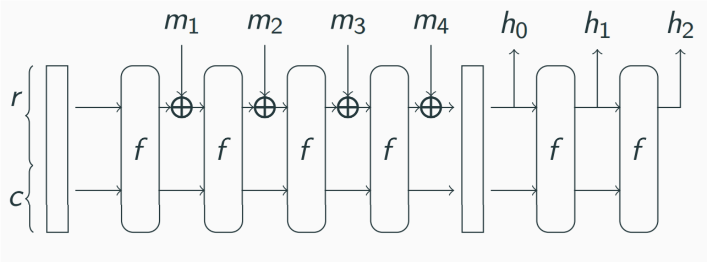
Part I

PHOTON

PHOTON [GPP11]

PHOTON is a (lightweight) family of sponge-like hash function



PHOTON Family

5 Variants of PHOTON denoted by PHOTON-$n/r/r'$:

- n is the bit-size of the hash output
- r and r' are the input and the output bit rate respectively
- c is the bit-size of the capacity part of the internal state
- $t = c + r$ is the internal state size

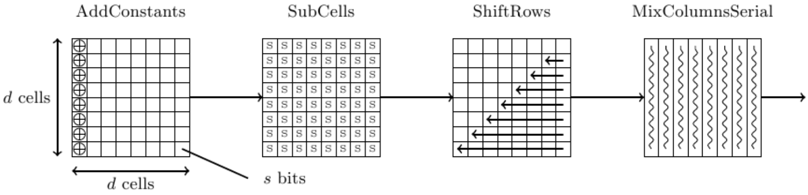
Table: Parameters of PHOTON-$n/r/r'$ with **4-bit S-Box**

Versions	t	n	c	r	r'	d
PHOTON-80/20/16	100	80	80	20	16	5
PHOTON-128/16/16	144	128	128	16	16	6
PHOTON-160/36/36	196	160	160	36	36	7
PHOTON-224/32/32	256	224	224	32	32	8

PHOTON Permutation

The internal state is viewed as a $d \times d$ matrix of 4-bit cells.

The internal Permutation of PHOTON iterates 12 times a round composed of 4 operations:



Part II

Zero-Sum

Zero-Sum

Let F be a function from $\mathbb{F}_{2^n}$ into $\mathbb{F}_{2^m}$.

A zero-sum for F of size K is a subset $\{x_1, \dots, x_K\} \subset \mathbb{F}_{2^n}$ of elements which sum to zero and for which the corresponding images by F also sum to zero, i.e.

$$\bigoplus_{i=1}^{K} x_i = \bigoplus_{i=1}^{K} F(x_i) = 0.$$

Given a function F and an affine subspace $\mathcal{A} \subset \mathbb{F}_{2^n}$ with dimension $(deg(F) + 1)$, then

$$\bigoplus_{x \in \mathcal{A}} F(x) = 0.$$

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Zero-Sum Partition

Let F be a function from $\mathbb{F}_{2^n}$ into $\mathbb{F}_{2^m}$.

A zero-sum partition for F of size $K = 2^k$ is a collection of 2^{n-k} disjoint sets $\{X_1, \ldots, X_{2^{n-k}}\}$ with the following properties

- $X_i = \{x_1, \ldots, x_{2^k}\} \subset \mathbb{F}_{2^n}$ for each i such that

$$\bigcup_i X_i = \mathbb{F}_{2^n};$$

- for each $X_i = \{x_1, \ldots, x_{2^k}\}$

$$\bigoplus_{i=1}^{2^k} x_i = \bigoplus_{i=1}^{2^k} F(x_i) = 0.$$

Zero-Sum: Inside-Out Approach (1/2)

Given a permutation P

$$P(\cdot) = R^{r+s}(\cdot) \equiv \underbrace{R \circ \cdots \circ R}_{r+s \text{ times}}(\cdot)$$

how to construct a zero-sum?

(1st) Consider affine subspaces $X = \{x^i\}_i$ and $Y = \{y^i\}_i$ such that

$$\bigoplus_i R^{-s}(y^i) = 0 \qquad \bigoplus_i R^{r}(x^i) = 0.$$

The previous equalities are satisfied if

$$dim(X) \geq deg(R^r) + 1 \quad \text{and} \quad dim(Y) \geq deg(R^{-s}) + 1$$

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Zero-Sum: Inside-Out Approach (2/2)

(2nd) Since one can work with the intermediate states, chooses texts in $X \oplus Y$:

- define the *plaintexts* p_i *as the* s*-round decryption of* $X \oplus Y$;
- define the corresponding *ciphertexts* c_i *as the* r*-round encryption of* $X \oplus Y$.

Note that:

$$X \oplus Y = \bigcup_{y \in Y} X \oplus y = \bigcup_{x \in X} Y \oplus x.$$

Result: *A zero-sum* $\{p_i\}_{i=1,\dots,K}$ *with the properties* $\bigoplus_{i=1}^{K} p_i = \bigoplus_{i=1}^{K} c_i = 0$ *is created for permutation* P.

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Part III

MILP Automatic Tool to search Zero-Sum based on Division Property

Division Property

Division Property: "generalization" of Integral Property

Definition [Tod15] *Let* $\mathbb{X} \subset (\mathbb{F}_{2^n})^m$, *and* $k^i \in \{0, 1, 2, ..., n\}$ *for* $i = 0, ..., m-1$. $\mathbb{X}$ *has the division property* $\mathcal{D}_{\mathbf{k}}^{n,m}$ *- where* $\mathbf{k} = (k^0, k^1, ..., k^{m-1})$ *- if*

$$\bigoplus_{x \in \mathbb{X}} x^{\mathbf{u}} = 0$$

for all $\mathbf{u}$ *such that*

$$\{\mathbf{u} = (u_0, u_1, \ldots, u_{m-1}) \in (\mathbb{F}_{2^n})^m \mid (wt(u_0), ..., wt(u_{m-1})) \not\succeq \mathbf{k}\}$$

(where $wt(\cdot)$ is the Hamming weight - $a \succeq b$ means that $a^i \geq b^i$ for all i)

- Construct input multiset with division property $\mathcal{D}_{\mathbf{k}_0}^{n,m}$
- Propagate the initial division property r rounds to get the output divsion property $\mathcal{D}_{\mathbf{k}_r}^{n,m}$
- Extract useful integral from $\mathcal{D}_{\mathbf{k}_r}^{n,m}$

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- Extract useful integral from $\mathcal{D}^{n,m}_{\mathbf{k}_r}$

Bit-based Division Property and Division Trail

Bit-Based Division Property [TM16]: *division property of each bit is treated independently*

Advantage detailed division property, longer distinguishers

Disadvantage time/memory complexity much *higher* than for division property (*upper* bounded by $O(2^n)$ where n is the block size)

$\Rightarrow$ works "only" for ciphers with small block size!

At Asiacrypt 2016, Xiang *et al.* [XZB+16] built an *automatic tool based on mixed integer linear programming (MILP) to study the division property of SPNs with bit-permutation linear layers*

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MILP Automatic Tool

A MILP model $\mathcal{M}$ consists of

- variables $\mathcal{M}.var$
- linear constraints $\mathcal{M}.con$
- objective function $\mathcal{M}.obj$.

Example:

$$\begin{aligned}
\mathcal{M}.obj &\leftarrow \text{maximize } x + y + 2z \\
\mathcal{M}.con &\leftarrow x + 2y + 3z \leq 4 \\
\mathcal{M}.con &\leftarrow x + y \geq 1 \\
\mathcal{M}.var &\leftarrow x, y, z \text{ as binary.}
\end{aligned}$$

The solution to the model $\mathcal{M}$ is 3, where $(x, y, z) = (1, 0, 1)$.

MILP - Division Trail

Division Trail [XZB+16] *Assume the input multiset to a block cipher has initial division property* $\mathbb{K}_0 \equiv \mathcal{D}^{n,m}_{\mathbf{k}_0}$*, and denote the division property after* i*-round through round function* $R(\cdot)$ *by* $\mathbb{K}_i \equiv \mathcal{D}^{n,m}_{\mathbf{k}_i}$*. We have the following trail of division property propagations:*

$$\mathbb{K}_0 \xrightarrow{R(\cdot)} \mathbb{K}_1 \xrightarrow{R(\cdot)} \cdots \xrightarrow{R(\cdot)} \mathbb{K}_r.$$

Thus, $(\mathbf{k}_0, \mathbf{k}_1, \cdots \mathbf{k}_r)$ *is an* r*-round division trail if* $\mathbf{k}_i$ *can propagate to* $\mathbf{k}_{i+1}$ *for all* $0 \leq i \leq r-1$.

Rule to determine the existence of Zero-sum:

Proposition

Assume $\mathbb{X}$ *is a multiset with division property* $\mathcal{D}^{n,m}_{\mathbf{k}}$*, then* $\mathbb{X}$ *does not have zero-sum property if and only if* $\mathbf{k}$ *contains all the* n *unit vectors.*

MILP - Aided Bit-based Division Property

It follows that we *only need to detect whether* $\mathbf{k}_r$ *contains all unit vectors*:

$\Rightarrow$ by previous Prop., the existence of any vector $\mathbf{v}$ s.t. $wt(\mathbf{v}) \geq 2$ implies that the state satisfies zero-sum property;

$\Rightarrow$ *if* $\mathbf{k}_{r+1}$ *contains all unit vectors*, the division property propagation should stop and *an* r*-round distinguisher can be derived*.

Denote $\mathbf{k}_0 \equiv (k_0^0, \cdots, k_{n-1}^0) \to \cdots \to \mathbf{k}_r \equiv (k_0^r, \cdots, k_{n-1}^r)$ an r-round bit-based division trail.

- The objective function is

$$Min: \; k_0^r + k_1^r + \cdots + k_{n-1}^r$$

$\Rightarrow$ we need *linear inequalities that describe all operations (XOR, S-Box, MC, Copy, ...)*

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Model S-Box

Given the PRESENT S-Box:

$$(a_{n-1}, \dots, a_1, a_0) \xrightarrow{\text{S-Box}} (b_{n-1}, \dots, b_1, b_0).$$

it can be described by *8 linear inequalities (which is 3 less w.r.t. [XZB+16])*

$$\begin{cases} -a_2 - a_1 + b_3 + b_1 + b_0 \geq -1 \\ -3a_3 - 3a_2 - 3a_1 + b_3 + 2b_2 + b_1 + 2b_0 \geq -5 \\ -2a_3 - a_2 - a_1 - 2a_0 + 5b_3 + 5b_2 + 5b_1 + 2b_0 \geq 0 \\ -a_0 - b_3 - b_2 + 2b_1 - b_0 \geq -2 \\ a_3 + a_2 + a_1 + a_0 - 2b_3 - 2b_2 + b_1 - 2b_0 \geq -1 \\ -a_0 + 2b_3 - b_2 - b_1 - b_0 \geq -2 \\ -a_0 - 2b_3 + b_2 - 2b_1 + b_0 \geq -3 \\ a_3 + a_2 + a_1 + 2a_0 - 2b_2 - 2b_1 - 2b_0 \geq -1 \end{cases}$$

How to Decrease the Algebraic Degree?

PRESENT S-Box:

$$(x_0, x_1, x_2, x_3) \mapsto \text{S-Box}(x_0, x_1, x_2, x_3) = (y_0, y_1, y_2, y_3)$$

where

$$\begin{aligned}
y_3 &= 1 \oplus x_0 \oplus x_1 \oplus x_3 \oplus x_1 x_2 \oplus x_0 \cdot (x_1 x_2 \oplus x_1 x_3 \oplus x_2 x_3)\\
y_2 &= 1 \oplus x_2 \oplus x_3 \oplus x_0 x_1 \oplus x_0 x_3 \oplus x_1 x_3 \oplus x_0 \cdot (x_1 x_3 \oplus x_2 x_3)\\
y_1 &= x_1 \oplus x_3 \oplus x_1 x_3 \oplus x_2 x_3 \oplus x_0 \cdot (x_1 x_2 \oplus x_1 x_3 \oplus x_2 x_3)\\
y_0 &= x_0 \oplus x_2 \oplus x_3 \oplus x_1 x_2
\end{aligned}$$

When x_0 is fixed as constant, the degree decreases from 3 to 2.

Number of Rounds of Zero-Sums by the MILP Division Property Tool

Dimension (in bit) of the space X and Y s.t.

$$\text{zero-sum} \xleftarrow{R^{-s}(\cdot)} Y \qquad X \xrightarrow{R^{r}(\cdot)} \text{zero-sum}$$

found by the MILP Division Property Tool (for PHOTON internal permutation) used to set up Zero-Sums

	P_{100}			P_{144}			P_{196}			P_{256}		
	Forward Direction											
#rounds	4	5	6	4	5	6	4	5	6	4	5	6
[Tod15]	12	20	72	12	24	84	12	24	84	12	28	92
Ours	**11**	20	72	**11**	**23**	84	**11**	24	84	**11**	**27**	92
	Backward Direction											
#rounds	3	4	5	3	4	5	3	4	5	3	4	5
Ours	**11**	**19***	**71***	**11**	**23**	**83***	**11**	**23***	**83***	**11**	**27**	**91***

* Partial balanced

Results from MILP automatic tools - Example

Given

$$\mathbb{B} \xleftarrow{R^{-s}} \begin{pmatrix} A & C & C & C & C & C \\ A & C & C & C & C & C \\ A & C & C & C & C & C \\ A & C & C & C & C & C \\ A & C & C & C & C & C \\ aaac & C & C & C & C & C \end{pmatrix}, \begin{pmatrix} A & C & C & C & C & C \\ C & A & C & C & C & C \\ C & C & A & C & C & C \\ C & C & C & A & C & C \\ C & C & C & C & A & C \\ C & C & C & C & C & aaac \end{pmatrix} \xrightarrow{R^r} \mathbb{B}$$

where $\mathbb{B}$ denotes (full/partial) balanced/zero-sum, then

$$\mathbb{B} \xleftarrow{R^{-s}} \begin{pmatrix} A & C & C & C & C & C \\ A & A & C & C & C & C \\ A & C & A & C & C & C \\ A & C & C & A & C & C \\ A & C & C & C & A & C \\ aaac & C & C & C & C & aaac \end{pmatrix} \xrightarrow{R^r} \mathbb{B}$$

Part IV

1-Round Extension: Subspace Trail Cryptanalysis

Observation on X and Y

Goal: using MILP automatic tools based on division property, find subspaces X and Y such that

$$\text{zero-sum} \xleftarrow{R^{-s}(\cdot)} Y \qquad X \xrightarrow{R^{r}(\cdot)} \text{zero-sum}$$

Note: such MILP tools can only found "zero-sum" for which the nibbles - of the input set $X \oplus Y$ - can be active/partial active or constant.

Other more generic cases are not considered, including the ones for which some particular (linear) relationships between the nibble hold.

Idea: use "subspace trail" to extend - for free - the results found by the MILP tools!

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Idea: use "subspace trail" to extend - for free - the results found by the MILP tools!

The Space Y - Backward Direction

The space Y found using MILP automatic tools corresponds to a "**column space**" in subspace trail notation [GRR16]

$$\mathcal{C}_i := \langle e_{0,i}, e_{1,i}, ..., e_{n-1,i} \rangle$$

E.g. if $n = 6$ and $i = 0$:

$$C_0 \equiv \begin{pmatrix} x_0 & 0 & 0 & 0 & 0 & 0 \\ x_1 & 0 & 0 & 0 & 0 & 0 \\ x_2 & 0 & 0 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 0 & 0 & 0 \\ x_4 & 0 & 0 & 0 & 0 & 0 \\ x_5 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

General case: given $I \subseteq \{0, 1, ..., n-1\}$ let

$$\mathcal{C}_I := \bigoplus_{i \in I} \mathcal{C}_i$$

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The Mixed Space $\mathcal{M}$ - Backward Direction

Let the "**mixed space**" defined as

$$\mathcal{M}_i := MC \circ SR(\mathcal{C}_i)$$

E.g. if $n = 6$ and $i = 0$:

$$M_0 \equiv \begin{pmatrix} x_0 & 2x_1 & 8x_2 & 5x_3 & 8x_4 & 2x_5 \\ 2x_0 & 12x_1 & 6x_2 & 2x_3 & x_4 & 5x_5 \\ 12x_0 & 13x_1 & 8x_2 & 8x_3 & 15x_4 & 9x_5 \\ 13x_0 & x_1 & 10x_2 & 3x_3 & 11x_4 & 5x_5 \\ x_0 & 8x_1 & 11x_2 & 14x_3 & 13x_4 & 15x_5 \\ 8x_0 & 8x_1 & 2x_2 & 3x_3 & 3x_4 & 2x_5 \end{pmatrix}$$

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Subspace Trail Results

For each $a \in \mathcal{C}_I^\perp$, there exists $b \in \mathcal{M}_I^\perp$ such that

$$R(\mathcal{C}_I \oplus a) = \mathcal{M}_I \oplus b.$$

If

$$\text{zero-sum} \xleftarrow{R^{-s}(\cdot)} \mathcal{C}_I \oplus a$$

then

$$\text{zero-sum} \xleftarrow{R^{-s}(\cdot)} \mathcal{C}_I \oplus a \xleftarrow{R^{-1}(\cdot)} \mathcal{M}_I \oplus b$$

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Subspace Trail Results - Add 1 round in the middle

MILP Tool: Given $Y \equiv \mathcal{C}_I \oplus a$ and X such that

$$\text{zero-sum} \xleftarrow{R^{-s}(\cdot)} \mathcal{C}_I \oplus a \qquad X \xrightarrow{R^{r}(\cdot)} \text{zero-sum}$$

then

$$\text{zero-sum} \xleftarrow{R^{-s}(\cdot)} (\mathcal{C}_I \oplus X) \oplus a \xrightarrow{R^{r}(\cdot)} \text{zero-sum}.$$

Subspace Trails: Since $\mathcal{C}_I \oplus a \xleftarrow{R^{-1}(\cdot)} \mathcal{M}_I \oplus a'$, it follows that

$$\text{zero-sum} \xleftarrow{R^{-(s+1)}(\cdot)} (\mathcal{M}_I \oplus X) \oplus a' \xrightarrow{R^{r}(\cdot)} \text{zero-sum}.$$

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Our Results

Variants PHOTON	**Security Claim**	**# Rounds**	**Cost Size N**	**Property**
-80/20/16	80	10	2^{40}	Balance
		11	2^{76}	Balance
-128/16/16	128	10	2^{47}	Balance
		11	2^{107}	Balance
		12	$\mathbf{2^{127}}$	**PBalance**
-160/36/36	160	10	2^{48}	Balance
		11	2^{108}	Balance
		12	$\mathbf{2^{159}}$	**PBalance**
-224/32/32	224	10	2^{55}	Balance
		11	2^{124}	Balance
		12	$\mathbf{2^{184}}$	**Balance**

where "PBalance" $\equiv$ Partial Balance (Input or/and Output bits)

(Similar results are given in the paper for less than 10 rounds)

Part V

Final Remarks

Final Remarks

Several Zero-Sums results in the literature, most prominently on Keccak [AM09,BC10]:

- *it seems hard to exploit zero-sum distinguishers to set up an attack on an hash function; however, the inner permutation of a sponge construction must look like a random permutation!*

  Note: Keccak team [BDP+] decided to increase the number of rounds of Keccak (from 18 to 24) in round 2 of the SHA-3 competition to prevent this distinguisher!

Final Remarks

- *zero-sum distinguishers are meaningful since they can not be set up for any arbitrary number of rounds*:

  zero-sum distinguishers proposed in this paper don't work if the number of rounds of PHOTON are increased from 12 to (e.g.) 16.

- there is currently no reason to believe that the security of PHOTON as a hash function is endangered.

Final Remarks

- *zero-sum distinguishers are meaningful since they can not be set up for any arbitrary number of rounds*:

  zero-sum distinguishers proposed in this paper don't work if the number of rounds of PHOTON are increased from 12 to (e.g.) 16.

- there is currently no reason to believe that the security of PHOTON as a hash function is endangered.

Thanks for your attention!

Questions?

Comments?

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RSAConference2018

San Francisco | April 16 – 20 | Moscone Center

NOW MATTERS

SESSION ID: CRYP-R04

IMPROVED SECURITY BOUND OF LIGHTMAC_PLUS AND ITS SINGLE-KEY VARIANT

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Improved Security Bound of LightMAC_Plus

Result 1

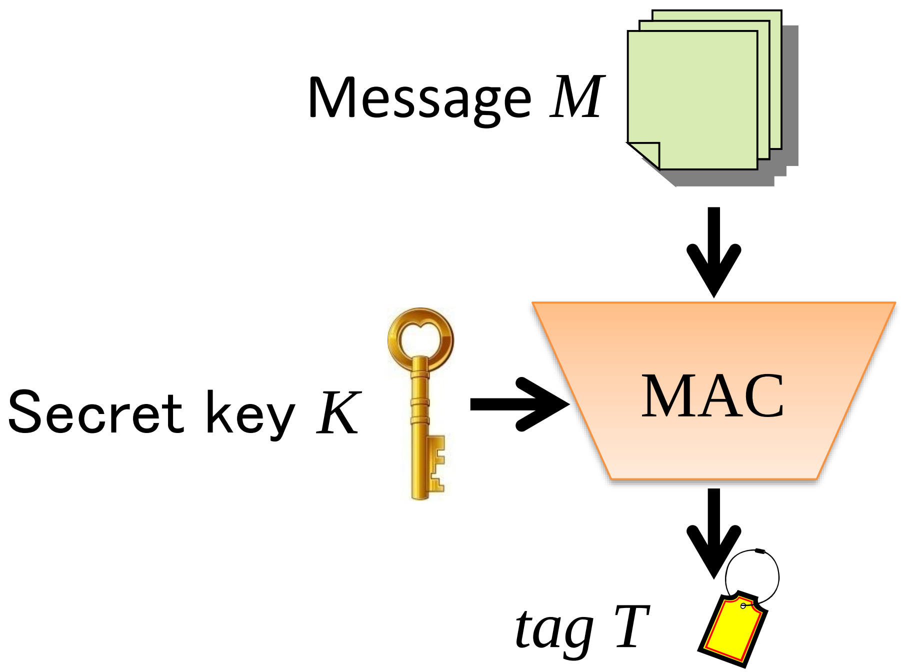
and Its Single-Key Variant

Result 2

Yusuke Naito

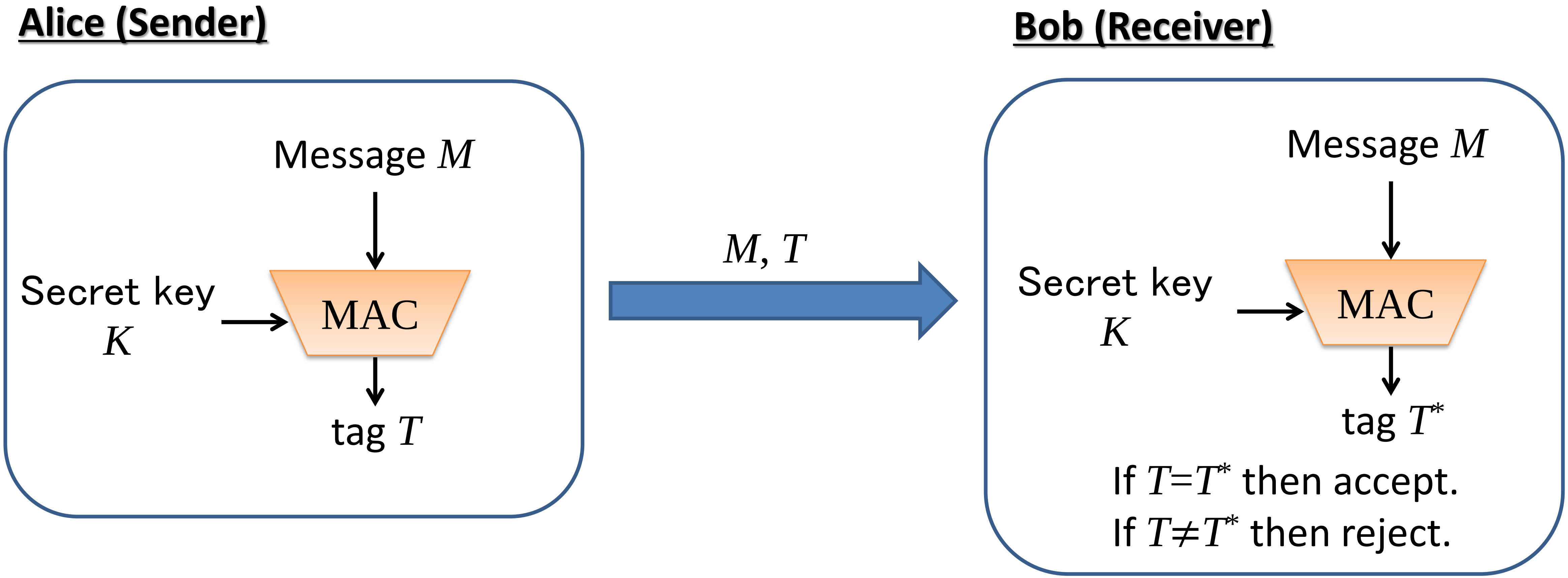
Mitsubishi Electric Corporation

Message Authentication Code (MAC)

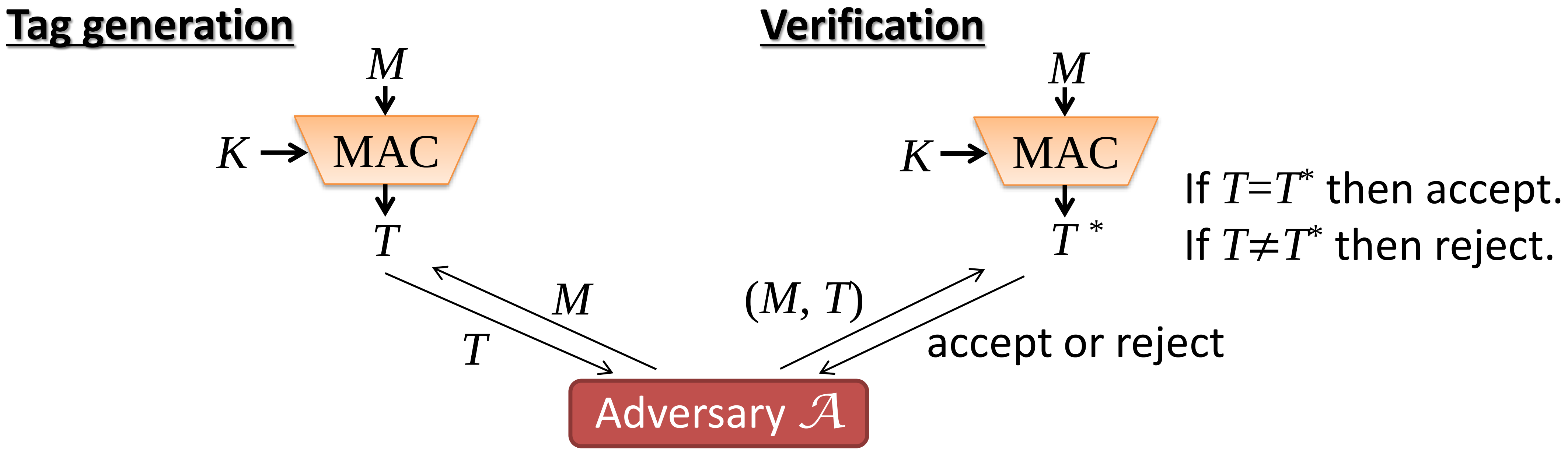


- Symmetric-key primitive.
- Used for integrity check.
- Input: a secret key and a variable-length message.
- Output: a fixed-length value, called tag.

Integrity Check Using MAC



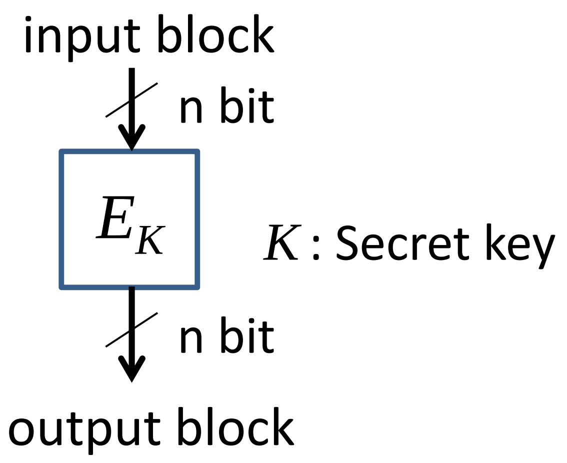
MAC Security (Unforgeability under CPA)



- Adversary $\mathcal{A}$ has access to
  - the tag generation algorithm (tagging queries)
  - the verification algorithm (verification queries).
- The goal of $\mathcal{A}$ is to forge a message and tag, i.e., make a (non-trivial) verification query s.t. accept is returned.
- Designing a MAC, Adv($\mathcal{A}$) = Pr[$\mathcal{A}$ forges] is evaluated.

MAC Design

- Underlying Primitives
  - Blockcipher, Tweakable Blockcipher, Permutation, ...
- Blockcipher
  - Standard: AES, Camellia, CLEFIA, PRESENT, ...
  - Family of permutations indexed by a key.
  - Security: Strong Pseudorandom Permutation
    (Ind. between E_K and a random permutation).
- Security proof of a blockcipher-based MAC
  - E_K is replaced with a random permutation.



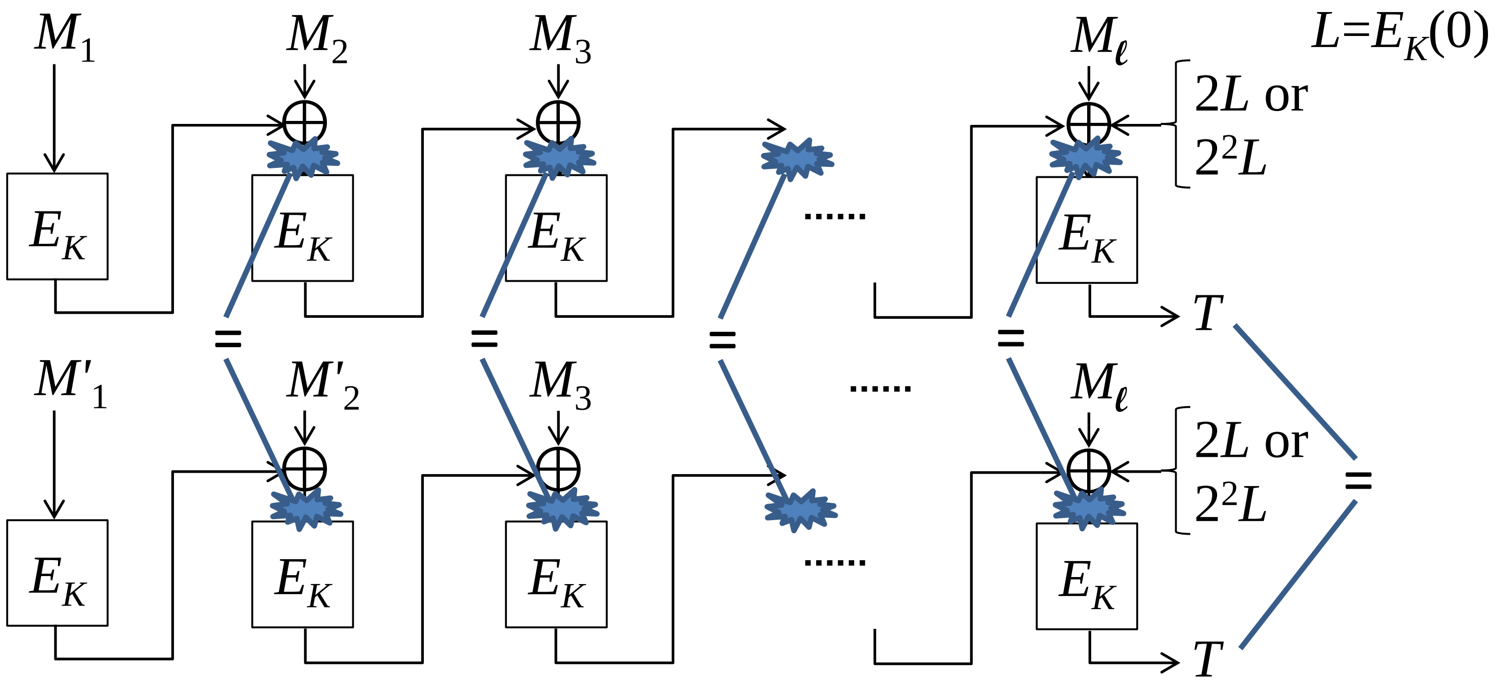
Birthday-Bound Secure MACs

- Many blockcipher-based MACs have been designed to have birthday-bound security.
  - Birthday-bound: $\mathrm{Adv}(\mathcal{A}) \leq O((\ell q)^2/2^n)$ (security up to $q=O(2^{n/2}/\ell)$).
    - ℓ: message length in (n-bit) blocks, i.e., # of blockcipher calls by a query.
    - q: # of (tagging or verification) queries.
  - Birthday-bound secure MACs: CMAC, PMAC, CBC-MAC (with prefix-free messages), ...
- Birthday-bound $O((\ell q)^2/2^n)$ security is not enough (e.g., Sweet32 at CCS 2016),
  - when large amounts of data are processed,
  - when a large number of connections need to be kept secure, or
  - when the block size n is small e.g., $n=64$.
- Designing a beyond-birthday-bound (BBB) secure MAC (i.e., having a better security bound) is an important topic.

How to Design a BBB-secure MAC

- The birthday bound $O((\ell q)^2/2^n)$ comes from collisions in E_K inputs/(n-bit) internal state.

e.g., CMAC:



How to Design a BBB-secure MAC

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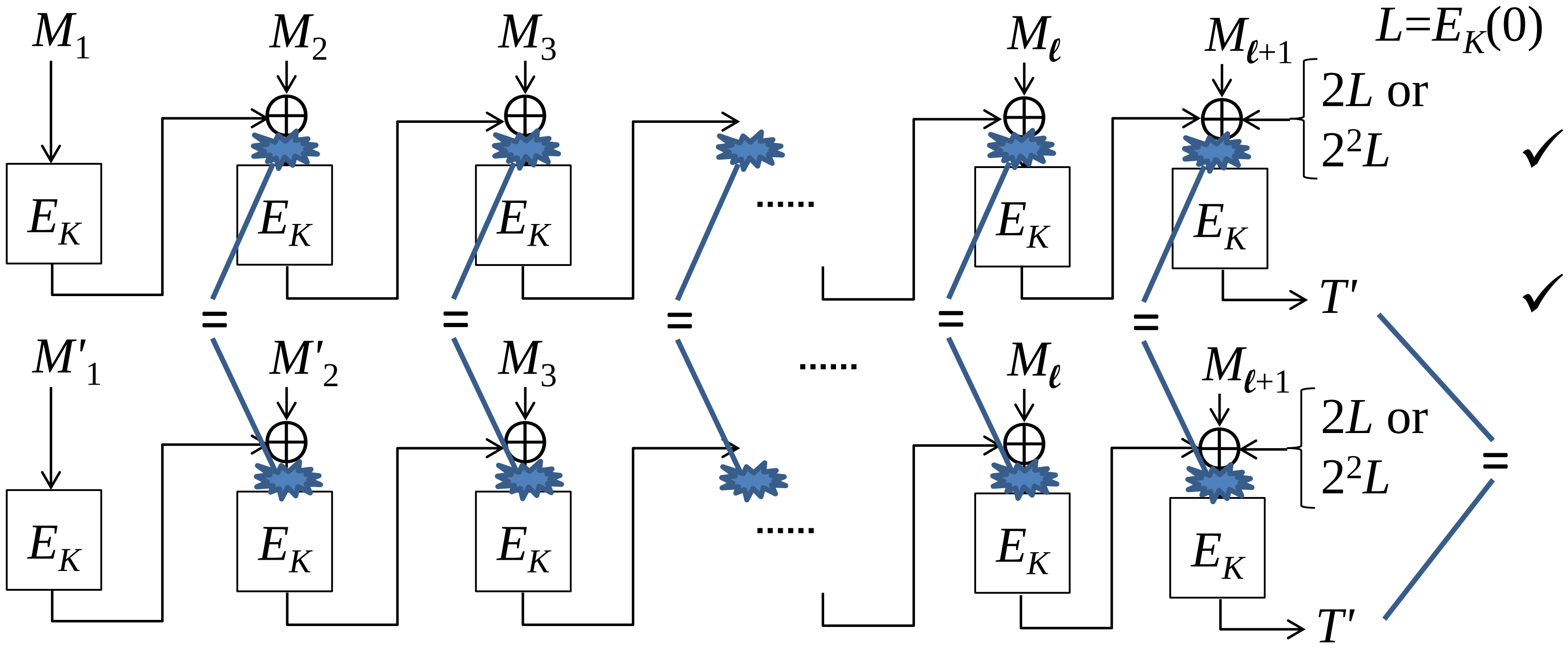
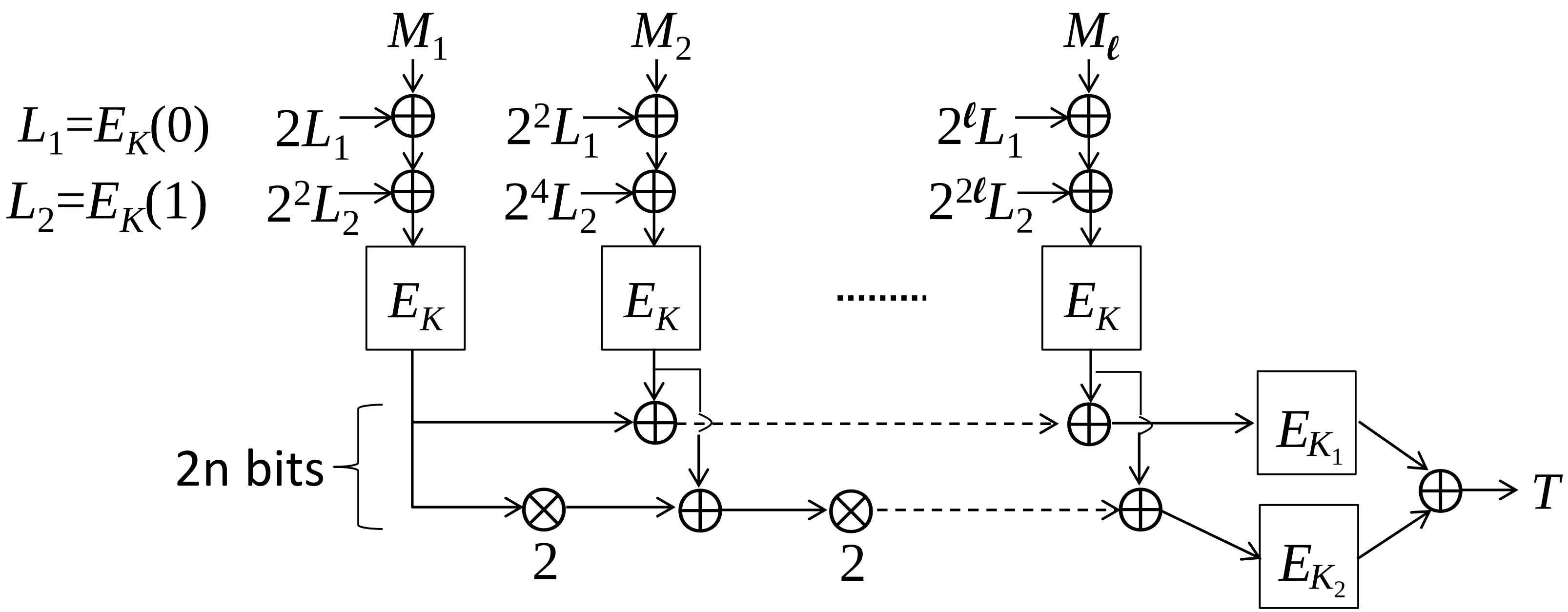


- ✓ An input collision for E_K triggers a forgery.
- ✓ Since there are ℓq inputs, by the birthday analysis, Collision Prob. = $O((\ell q)^2/2^n)$.

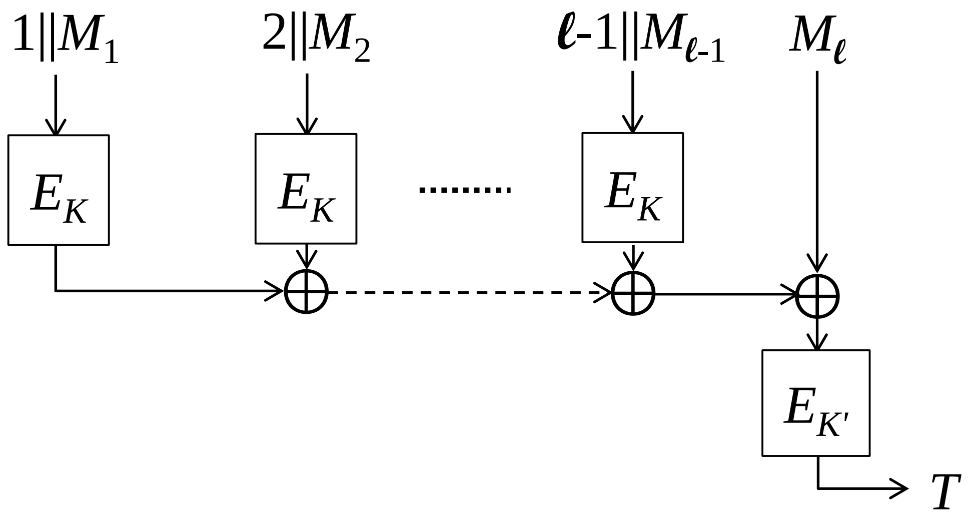
- In order to achieve BBB-security, we need to design a MAC so that the influences of collisions in E_K inputs / internal state are weakened.
- Existing BBB-secure MACs:
  e.g., PMAC_Plus, LightMAC, LightMAC_Plus, LightMAC_Plus2.

PMAC_Plus (Yasuda, CRYPTO 2011)



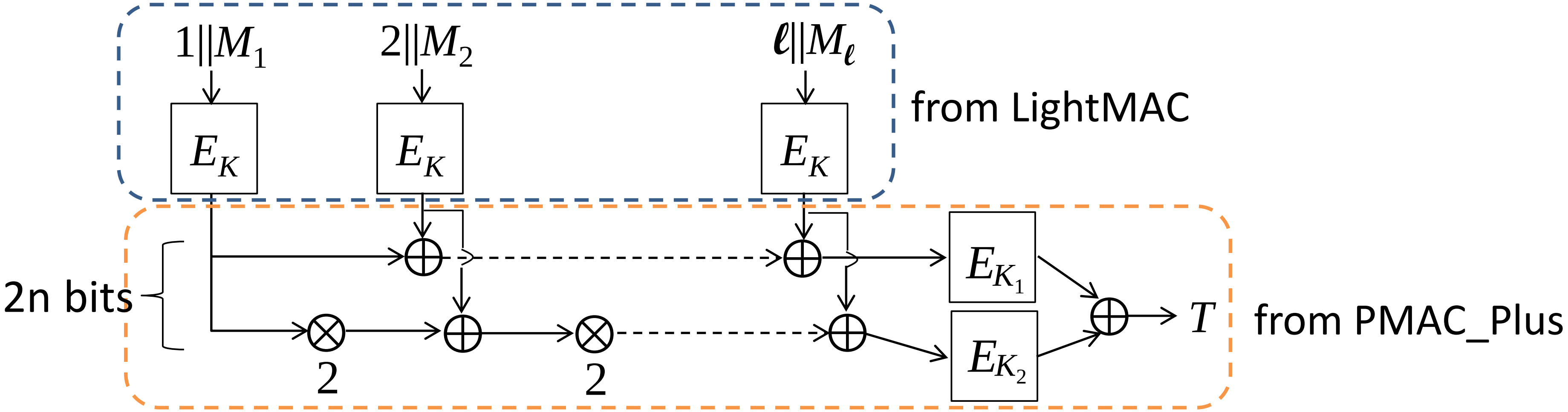
- In order to weaken the collision influences, PMAC_Plus employs
  - double secret masks
    -> the influence of a collision in E_K inputs is weakened,
  - double length (2n bit) internal state
    -> the collision prob. on the internal state is improved.
- Adv($\mathcal{A}$) $\leq O((\ell q)^3/2^{2n})$ (security up to $q=O(2^{2n/3}/\ell)$).
- Security level: $2^{n/2}/\ell$ -> $2^{2n/3}/\ell$.

LightMAC (Luykx et al., FSE 2016)



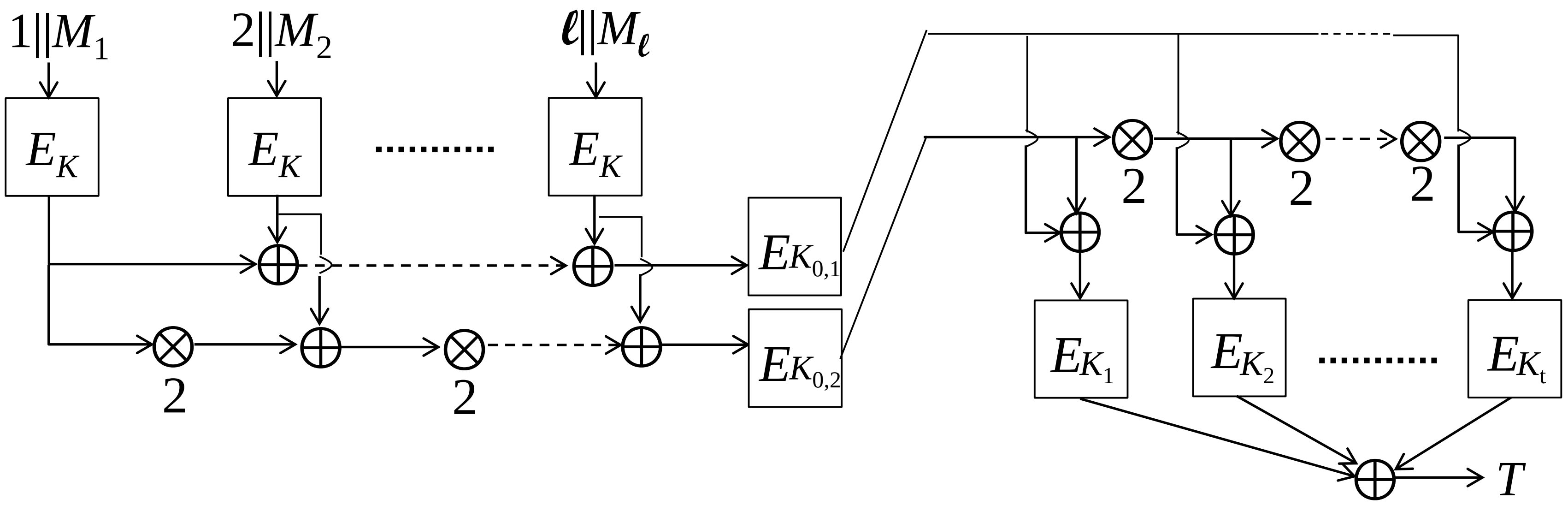
- LightMAC uses the counter-based construction:
  -> the input collision can be avoided for any ℓ,
  -> the message length ℓ can be removed from the security bound.
- By the birthday analysis for the n-bit internal state,
  Adv($\mathcal{A}$) $\leq O(q^2/2^n)$ (security up to $q=O(2^{n/2})$).
- Security level: $2^{n/2}/\ell$ -> $2^{n/2}$.

LightMAC_Plus (Naito, Asiacrypt 2017)



- Combination of LightMAC and PMAC_Plus
- From the LightMAC structure, the message length ℓ can be removed.
- From the PMAC_Plus structure, the collision prob. on the internal state is improved.
- Adv($\mathcal{A}$) $\leq O(q^3/2^{2n})$ (security up to $q=O(2^{2n/3})$).
- Security level: $2^{n/2}/\ell$ -> $2^{2n/3}/\ell$, $2^{n/2}$ -> $2^{2n/3}$.

LightMAC_Plus2 (Naito, Asiacrypt 2017)



- Has the better security bound than LightMAC_Plus.
- The finalization function is modified.
- Adv($\mathcal{A}$) $\leq O(q^{t+1}/2^{tn})$ for $t \leqq 7$ (security up to $q=O(2^{tn/(t+1)})$).
- Security level: $2^{n/2}/\ell$ -> $2^{2n/3}/\ell$, $2^{n/2}$ -> $2^{2n/3}$ -> $2^{tn/(t+1)}$.

Comparison and Question

Comparison

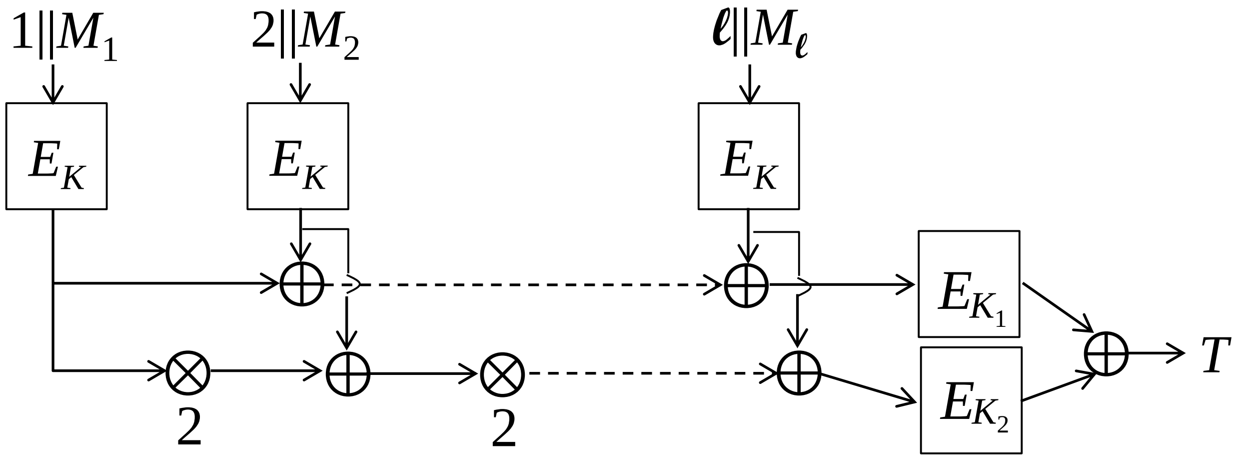
	Security Bound	Security Level	$\|M_i\|$	#BC in FF	Key Size
PMAC_Plus	$(\ell q)^3/2^{2n}$	$2^{2n/3}/\ell$	n	2	3
LightMAC	$q^2/2^n$	$2^{n/2}$	$n - c$	1	2
LightMAC_Plus	$q^3/2^{2n}$	$2^{2n/3}$	$n - c$	2	3
LightMAC_Plus2	$q^{(t+1)}/2^{tn}+q^2/2^{2n}$ $(t \leqq 7)$	$2^{tn/(t+1)}$	$n - c$	$t+2$	$t+3$
	$q^4/2^{3n}+q^2/2^{2n}$ $(t=3)$	$2^{3n/4}$	$n - c$	5	6
	$q^5/2^{4n}+q^2/2^{2n}$ $(t=4)$	$2^{4n/5}$	$n - c$	6	7
	⋮	⋮	⋮	⋮	⋮

- LightMAC_Plus2: Increasing the security level (i.e., increasing t), the efficiency (in the finalization function) is degraded and the key size is increased.

Question

- Can we obtain a highly secure MAC without degrading the efficiency or increasing the key size.

Our Result 1: Improved Security Bound of LightMAC_Plus



- Security bound: $O(q^3/2^{2n})$ -> $O(q_t^2 q_v/2^{2n})$.
  - q_t : # of tagging queries
  - q_v : # of verification queries
  - $q = q_t + q_v$
- If $q_t << q_v$ (e.g., a sender does not send a message frequently) or $q_v << q_t$ (e.g., # of forgery attempts is limited by a system) then LightMAC_Plus becomes a highly secure MAC without degrading the efficiency or increasing the key size.
- e.g., $q_v \leq 2^{n/2}$ -> security up to $2^{3n/4}$ queries; $q_v \leq 2^{n/3}$ -> security up to $2^{5n/6}$ queries; etc.

Comparison

Comparison

	Security Bound	Security Level	$\lvert M_i \rvert$	#BC in FF	Key Size
PMAC_Plus	$(\ell q)^3/2^{2n}$	$2^{2n/3}/\ell$	n	2	3
LightMAC	$q^2/2^n$	$2^{n/2}$	n - c	1	2
LightMAC_Plus	$q^3/2^{2n}$ $q_t^2 q_v/2^{2n}$ (Result 1)	$2^{2n/3}$	n - c	2	3
LightMAC_Plus2	$q^{(t+1)}/2^{tn}+q^2/2^{2n}$ ($t \leqq 7$)	$2^{tn/(t+1)}$	n - c	t+2	t+3
	$q^4/2^{3n}+q^2/2^{2n}$ (t=3)	$2^{3n/4}$	1	5	6
	$q^5/2^{4n}+q^2/2^{2n}$ (t=4)	$2^{4n/5}$	1	6	7
	⋮	⋮	⋮	⋮	⋮

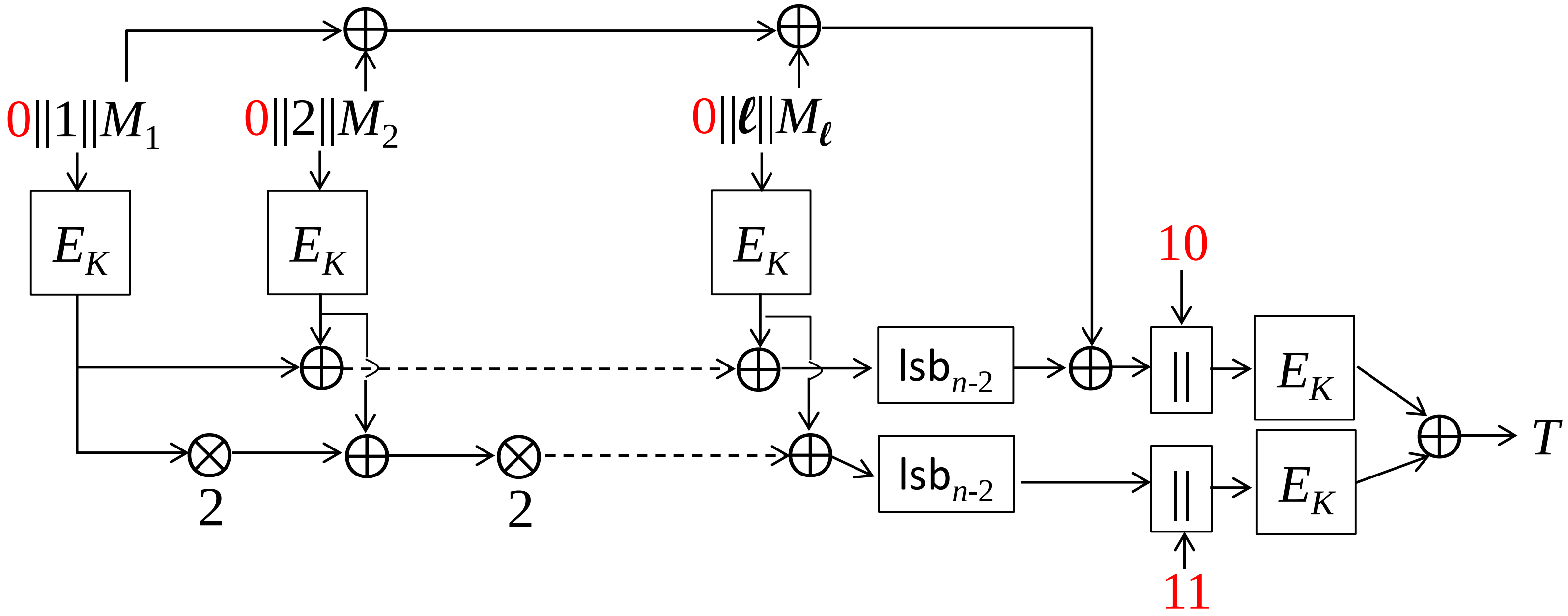
- LightMAC_Plus becomes a highly secure MAC without degrading the efficiency or increasing the key size if $q_t << q_v$ or $q_t >> q_v$ but uses 3 blockcipher keys.

Question

- Can we reduce the key size of LightMAC_Plus while keeping the BBB-security?

Our Result 2: Single-Key Variant of LightMAC_Plus

LightMAC_Plus1k



- In order to reduce the key size, the domain separation technique is used.
  - Hashing: 0
  - Finalization: 10, 11
- The keyed blockciphers with distinct inputs can be seen as distinct keyed blockciphers.
- In order to avoid a forgery with a collision in blockcipher outputs (due to 2-bit truncation), the ZMAC technique is used: XOR of message blocks are input to the internal state.
- Security bound: $O(q_t^2 q_v / 2^{2n})$.

Comparison

	Security Bound	Queries	$\lvert M_i \rvert$	#BC in FF	Key Size
PMAC_Plus	$\ell^3 q^3/2^{2n}$	$2^{2n/3}/\ell$	n	2	3
LightMAC	$q^2/2^n$	$2^{n/2}$	$n - c$	1	2
LightMAC_Plus	$q^3/2^{2n}$	$2^{2n/3}$	$n - c$	2	3
	$q_t^2 q_v/2^{2n}$ (Result 1)				
LightMAC_Plus1k	$q_t^2 q_v/2^{2n}$		$n - c$	2	1 (Result 2)
LightMAC_Plus2	$q^{(t+1)}/2^{tn}+q^2/2^{2n}$ ($t \leqq 7$)	$2^{tn/(t+1)}$	$n - c$	t+2	t+3
	$q^4/2^{3n}+q^2/2^{2n}$ (t=3)	$2^{3n/4}$	1	5	6
	$q^5/2^{4n}+q^2/2^{2n}$ (t=4)	$2^{4n/5}$	1	6	7
	⋮	⋮	⋮	⋮	⋮

Conclusion

for a greener tomorrow

- Result 1: Improved the security bound of LightMAC_Plus:
  - The security bound: $O(q^3/2^{2n})$ -> $O(q_t^2 q_v)/2^{2n}$.
  - If $q_t << q_v$ (e.g., a sender does not send a message frequently) or $q_v << q_t$ (e.g., # of forgery attempts is limited by a system) then LightMAC_Plus becomes a highly secure MAC without degrading the efficiency or increasing the key size.
- Result 2: Proposed LightMAC_Plus1k, the single key variant of LightMAC_Plus:
  - The key size: 3 -> 1.
  - The security bound: $O(q_t^2 q_v)/2^{2n}$.

Thank you for your attention!