

# RSA<sup>®</sup>Conference2018

San Francisco | April 16 – 20 | Moscone Center

SESSION ID: CRYPT-08

## WHY JOHNNY THE DEVELOPER CAN'T WORK WITH PUBLIC KEY CERTIFICATES

### Martin Ukrop

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Red Hat research cooperation

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Centre for Research on Cryptography  
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#RSAC

Simple and Solvable Error

# File does not exist.



File `/etc/ssl/certs/certificate1.pem`  
does not exist.

Clicking OK will solve everything and create  
a secure solution with no bugs or vulnerabilities.

Cancel

OK

Annoying Security Control

**Something happened** and you need to click OK to get on with doing things.



Security error:  
Permitted subtree violation

Certificate mismatch security identification  
administrator communication intercept liliputian  
snotweasel foxtrot omegaforce.

[Show more technical crap](#)

Cancel

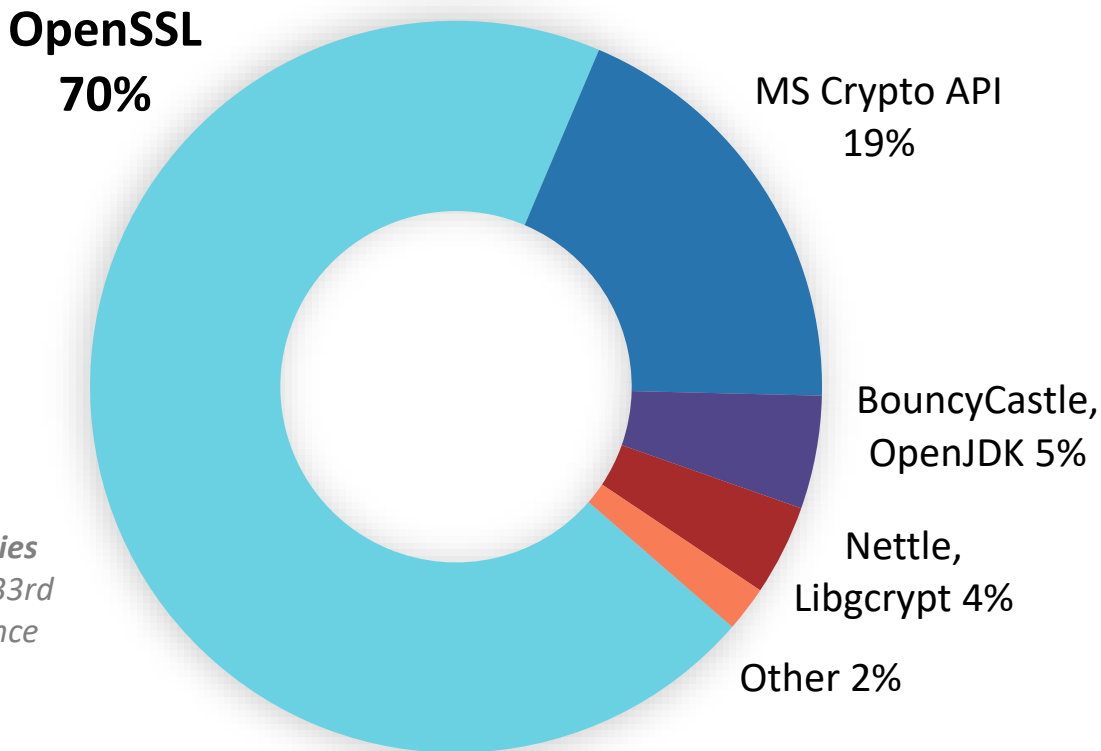
OK

# Is it really that bad?



- Let's find out!
- What is the most used tool for generating certificates?

Source: Matus Nemec, Dusan Klinec, Petr Svenda, Peter Sekan and Vashek Matyas: *"Measuring Popularity of Cryptographic Libraries in Internet-Wide Scans"*. In *Proceedings of the 33rd Annual Computer Security Applications Conference (ACSAC'2017)*, ACM, 2017.



# Q: Have you ever used OpenSSL?

```
[xukrop@styx ~]$ openssl version
```

```
OpenSSL 1.1.0g-fips  2 Nov 2017
```

```
[xukrop@styx ~]$
```

```
[xukrop@styx ~]$ openssl x509 -help
```

```
Usage: x509 [options]
```

```
Valid options are:
```

-help	Display this summary
-inform format	Input format - default PEM (one of DER, NET or PEM)
-in infile	Input file - default stdin
-outform format	Output format - default PEM (one of DER, NET or PEM)
-out outfile	Output file - default stdout
-keyform PEM DER	Private key format - default PEM
-passin val	Private key password/pass-phrase source
-serial	Print serial number value
-subject_hash	Print subject hash value
-issuer_hash	Print issuer hash value
-hash	Synonym for -subject_hash
-subject	Print subject DN
-issuer	Print issuer DN
-email	Print email address(es)
-startdate	Set notBefore field



# Empirical experiment in usability



- 87 participants (👤) of **DEVCONF.cz**  
(developer conference by Red Hat Czech)

*Task:*

*You are a software tester.*

*Use command line OpenSSL (v1.0.2g)*

- 1. ... to **issue** a self-signed certificate.*
- 2. ... to **validate** 4 given certificates.*

# Conference research booth at **DEVCONF.cz**





# **“Interacting with OpenSSL *voluntarily?*”**

**“Sorry, not even for research.”**

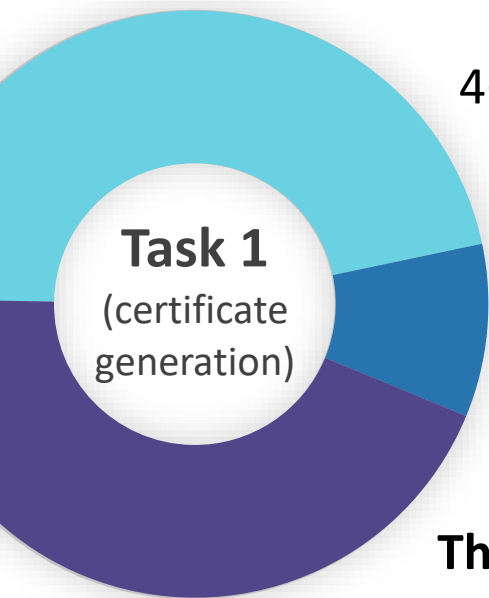




**“We all know OpenSSL sucks.”**

“But finally, there is someone  
collecting empirical evidence.”

# Task success



Succeeded  
46% (39/87)

Did not succeed  
10% (8/87)

Thought that they  
succeeded but did not  
44% (37/87)

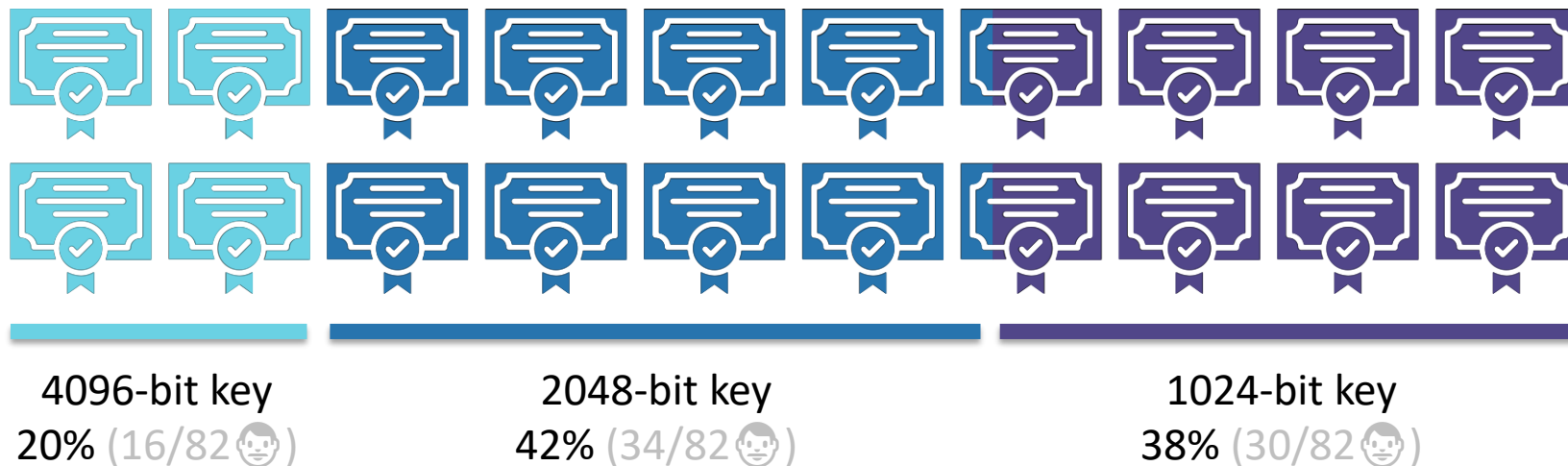
Incorrect  
10% (7/72)

Success  
19% (14/72)

Ignoring OS cert store  
71% (51/72)

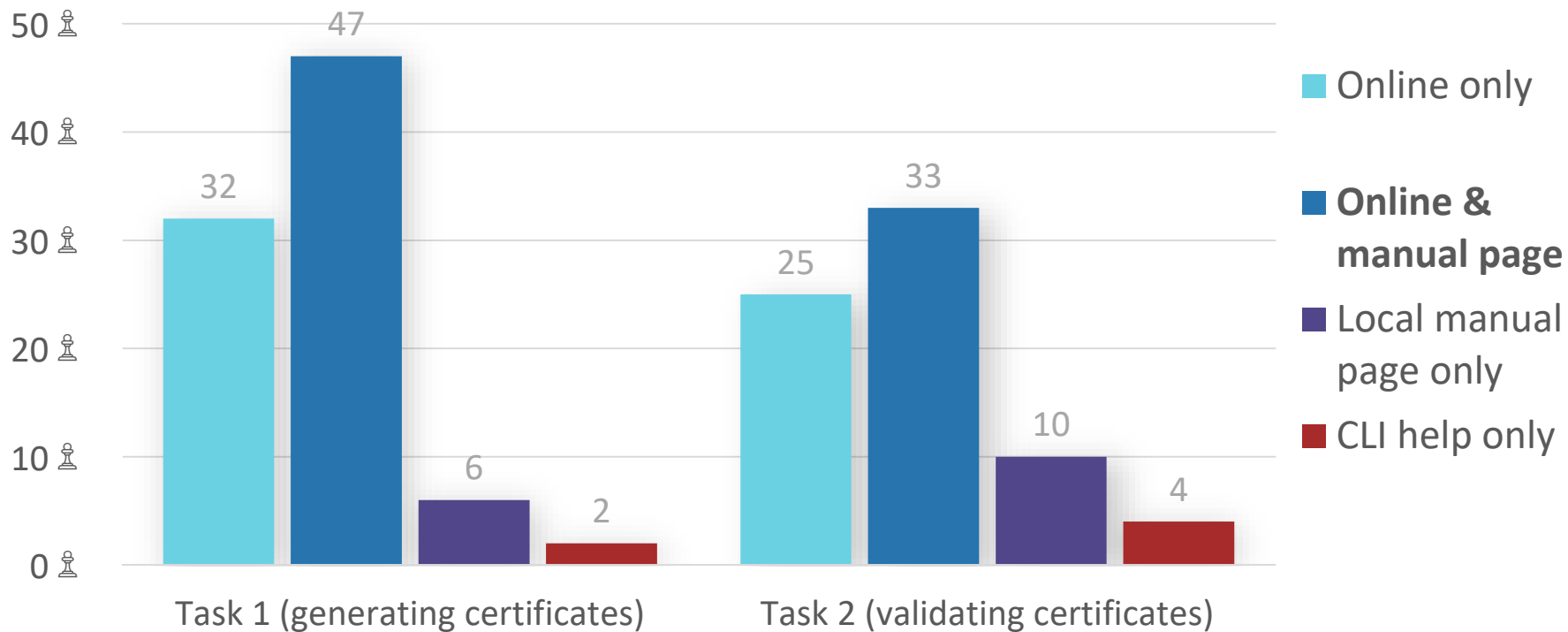


# Created certificates



- Organization = “Internet Widgits Pty Ltd.”: 42% certificates (27/65)
- cca 260 000 such certificates online (Censys.io dataset 2018-02-28)

# Used resources





# Stack Exchange: 73% people (58/79 ♟ )

[Questions](#)[Developer Jobs](#)[Tags](#)[Users](#)

## How to create a self-signed certificate with openssl?



744



504

I'm adding https support to an embedded linux device. I have tried to generate a self-signed certificate with these steps:

```
openssl req -new > cert.csr
openssl rsa -in privkey.pem -out key.pem
openssl x509 -in cert.csr -out cert.pem -req -signkey key.pem -days 1001
cat key.pem>>cert.pem
```

This works, but I get some errors with, for example, google chrome:

This is probably not the site you are looking for!  
The site's security certificate is not trusted!

Am I missing something? Is this the correct way to build a self-signed certificate?

[ssl](#)[openssl](#)[certificate](#)[ssl-certificate](#)[x509certificate](#)

# WISC knowledge base: 40% people (29/72 ♟ )

University of Wisconsin KnowledgeBase

DoIT UW MyUW PEOPLE



**Middleware**  
KnowledgeBase

Search the KB...

All Topics ▼

**SEARCH**

Advanced

## Quick Links ▼

[DoIT Help Desk](#)

[Main KB](#)

[Middleware @  
DoIT](#)

[NetID Login  
Service Docs](#)

## Verifying that a Certificate is issued by a CA

How to use OpenSSL on the command line to verify that a certificate was issued by a specific CA, given that CA's certificate

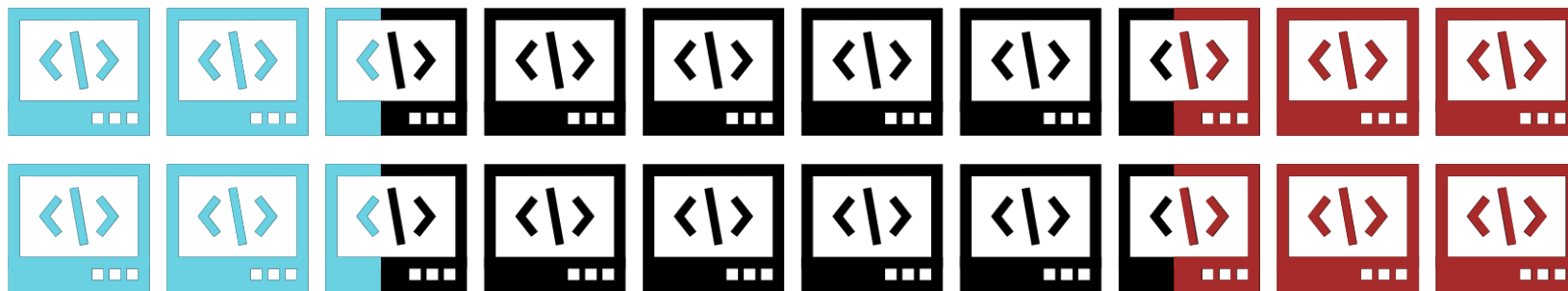
```
$ openssl verify -verbose -CAfile cacert.pem server.crt  
server.crt: OK
```

If you get any other message, the certificate was not issued by that CA.

## See Also:

- [How to turn a X509 Certificate in to a Certificate Signing Request](#)
- [Verifying that a Private Key Matches a Certificate](#)

# Web pages used








- Describing security implications: 23% web pages
- Explaining individual parameters: 27% web pages
- Changes after copy-paste: 9% people (8/87 🧑)

Q: How to display manual page for this?



*openssl verify -CAfile ca.pem cert.pem*

1. man openssl	?%	(?/53 
2. man openssl verify	28%	(15/53 
3. man openssl.verify	2%	(1/53 
4. man openssl-verify	8%	(4/53 
5. man verify	100%	(53/53 



# OpenSSL manual page

OPENSSL(1)

OpenSSL

OPENSSL(1)

## NAME

`openssl` - OpenSSL command line tool

## SYNOPSIS

`openssl` command [ command\_opts ] [ command\_args ]

`openssl list` [ **standard-commands** | **digest-commands** | **cipher-commands** |  
**cipher-algorithms** | **digest-algorithms** | **public-key-algorithms** ]

`openssl no-XXX` [ arbitrary options ]

## DESCRIPTION

OpenSSL is a cryptography toolkit implementing the Secure Sockets Layer (SSL v2/v3) and Transport Layer Security (TLS v1) network protocols and related cryptography standards required by them.

The **openssl** program is a command line tool for using the various cryptography functions of OpenSSL's **crypto** library from the shell. It can be used for

# Has the world moved on?



## Our work

- *man openssl verify* now works
- Fixed URLs for online documentation
- Research into better error messages

## OpenSSL team

- High-level *help* command
- *-help* argument for each command
- Improved defaults (key lengths, ...)

OpenSSL usability is poor.

(But better than other tools.)

Improvement is possible!



## Takeaways II.



People may not know  
they failed if the tool  
*does not tell them.*

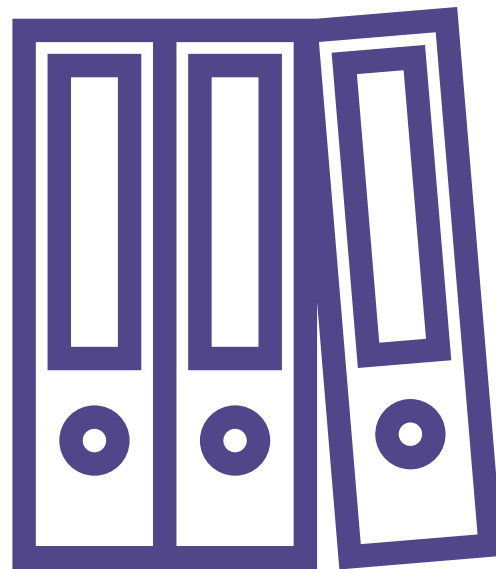






Documentation  
(manuals, tutorials, Q&A forums, ...)  
matters a lot.

Stack Overflow  
is a seriously used resource.



# What should you do next?



## USING security products?

- Ask your developers what they find unusable.
- Investigate past vulnerabilities: Caused by tool unusability?
- Report usability issues back to developers.

## DEVELOPING security products?

- In your project, strive for good “developer experience” (DX).
- Ask users on usability feedback.
- Organize a usability lab study to improve your product.

**RSA**Conference2018



#RSAC

# CARE FOR YOUR DEVELOPERS. DEVELOPER EXPERIENCE (DX) MATTERS.

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Masaryk University, Brno, Czech Republic



Centre for Research on  
Cryptography and Security

# RSA<sup>®</sup>Conference2018

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Cryptographers' Track (CT-RSA)



#RSAC

## Improved Factorization of $N = p^r q^s$

*Jean-Sébastien Coron,*

*Rina Zeitoun*

*Speaker: Mehdi Tibouchi*



UNIVERSITÉ DU  
LUXEMBOURG





# Agenda



- 1 State-of-the-art / Motivations
- 2 Reminder on Coppersmith / BDH Methods
- 3 [CFRZ16] Method to factorize  $N = p^r q^s$
- 4 An Improvement to Factorize  $N = p^r q^s$



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# RSA Cryptosystem / Signature



## RSA Key Generation

- Generate two large primes  $p$  and  $q$
- Compute  $N = p \times q$
- Select  $(e, d)$  such that  $ed \equiv 1 \pmod{\phi(N)}$

## Encryption/Decryption Process

$$C \equiv m^e \pmod{N} \quad \longrightarrow \quad m \equiv C^d \pmod{N}$$

Factorize  $N = p \times q \Rightarrow$  Break RSA

# State-of-the-art: Modulus $N = p^r q$



## Decryption with Modulus $N = p^r q$ [Takagi98]

- For equivalent security, use a smaller prime  $p$   
👉 **Decryption becomes faster**

[Takagi98] *Fast RSA-type cryptosystem modulo  $p^k q$* . Takagi, 1998.

# State-of-the-art: Modulus $N = p^r q$



## Decryption with Modulus $N = p^r q$ [Takagi98]

- For equivalent security, use a smaller prime  $p$   
    👉 **Decryption becomes faster**

## Vulnerability of $N = p^r q$ [BDH99]

- **BDH method:** Factoring  $N = p^r q$  for Large  $r$   
    👉 **Condition:**  $r \simeq \log q$

[Takagi98] *Fast RSA-type cryptosystem modulo  $p^k q$* . Takagi, 1998.

[BDH99] *Factoring  $N = p^r q$  for Large  $r$* . Boneh, Durfee, Howgrave-Graham, 1999.



# What About Modulus $N = p^r q^s$ ?



## Decryption with Modulus $N = p^r q^s$ [LKYL00]

- Decryption is even faster
  - 👉 For an 8192-bit  $N = p^2 q^3$ : decryption is 15 times faster

[LKYL00] *A Generalized Takagi-Cryptosystem with a Modulus of the Form  $p^r q^s$* . Lim, Kim, Yie, Lee, 2000.

# What About Modulus $N = p^r q^s$ ?



## Decryption with Modulus $N = p^r q^s$ [LKYL00]

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## Problem: Factorization of $N = p^r q^s$

- Factorization of  $N = p^r q^s$  in polynomial time
  - 👉 Left as an open problem in [BDH99]

[LKYL00] *A Generalized Takagi-Cryptosystem with a Modulus of the Form  $p^r q^s$* . Lim, Kim, Yie, Lee, 2000.

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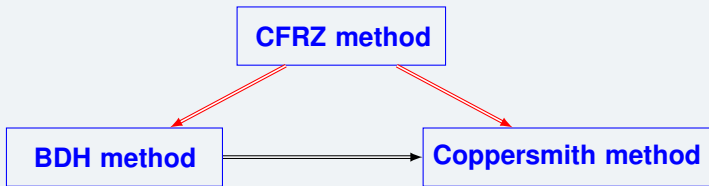
# [CFRZ16] Result



Polynomial time factorization of  $N = p^r q^s$  with  $r > s$  if:

$$r \simeq \log^3 \max(p, q)$$

Based on Lattice Attacks



[CFRZ16] *Factoring  $N = p^r q^s$  for Large  $r$  and  $s$* . Coron, Faugère, Renault, Zeitoun, CT-RSA 2016.

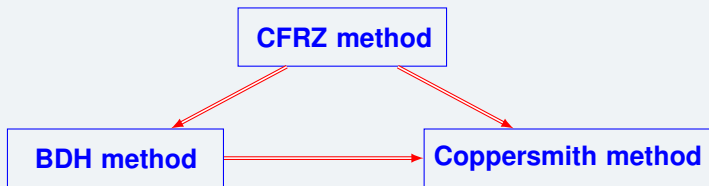
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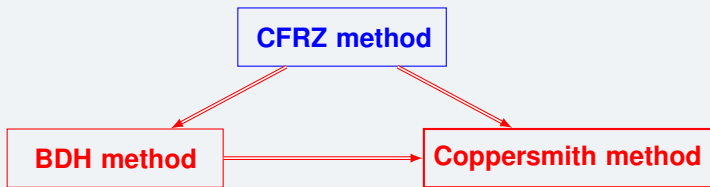
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# Outline



- 1 State-of-the-art / Motivations
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# Coppersmith's Theorem



## The Problem (Univariate Modular Case)

- **Input:**
  - A polynomial  $f(x) = x^d + a_{d-1}x^{d-1} + \dots + a_1x + a_0$
  - $N$  an integer of unknown factorization
- **Find:**
  - All integers  $x_0$  such that  $f(x_0) \equiv 0 \pmod{N}$

# Coppersmith's Theorem



## The Problem (Univariate Modular Case)

- **Input:**
  - A polynomial  $f(x) = x^\delta + a_{d-1}x^{\delta-1} + \dots + a_1x + a_0$
  - $N$  an integer of unknown factorization
- **Find:**
  - All integers  $x_0$  such that  $f(x_0) \equiv 0 \pmod{N}$

## Coppersmith's Theorem for the Univariate Modular case

- The solutions  $x_0$  can be found in polynomial time in  $\log(N)$  if

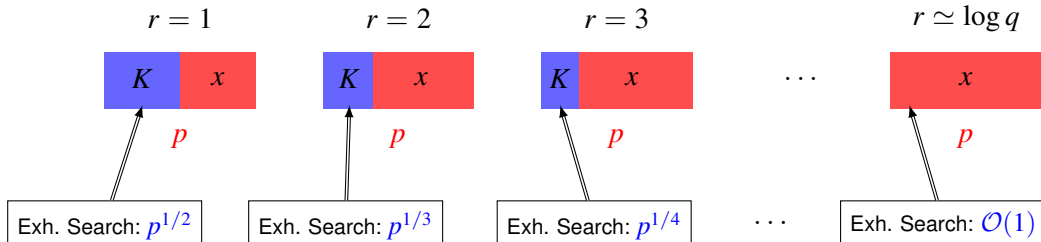
$$|x_0| < N^{1/\delta}$$

# BDH Method to factorize $N = p^r q$



## Extension of Coppersmith method for unknown modulus

- Write  $p = K \cdot t + x$
- Solve  $f(x) = (K \cdot t + x)^r \equiv 0 \pmod{p^r}$



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# [CFRZ16] Result



Polynomial time factorization of  $N = p^r q^s$  with  $r > s$  if:

$$r = \Omega(\log^3 \max(p, q))$$

Decompose  $r$  and  $s$ , Apply BDH or Coppersmith

$$N = p^r q^s = (p^\alpha q^\beta)^u p^a q^b = P^u Q$$

[CFRZ16] *Factoring  $N = p^r q^s$  for Large  $r$  and  $s$* . Coron, Faugère, Renault, Zeitoun, CT-RSA 2016.



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$\Rightarrow$  Apply BDH or Coppersmith method depending on  $a$  and  $b$

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$\Rightarrow$  Apply BDH or Coppersmith method depending on  $a$  and  $b$

$$\text{👉 } u = \Omega(\log Q) \quad \Rightarrow \quad r = \Omega(\log^3 \max(p, q))$$

[CFRZ16] *Factoring  $N = p^r q^s$  for Large  $r$  and  $s$* . Coron, Faugère, Renault, Zeitoun, CT-RSA 2016.

# [CFRZ16] Method to Factorize $N = p^r q^s$



First Step: Decompose  $r$  and  $s$

$$\begin{cases} r = \boxed{u} \cdot \boxed{\alpha} + \boxed{a} \\ s = \boxed{u} \cdot \boxed{\beta} + \boxed{b} \end{cases}$$

large      small      small

# [CFRZ16] Method to Factorize $N = p^r q^s$



First Step: Decompose  $r$  and  $s$

$$\begin{cases} r = \boxed{u} \cdot \boxed{\alpha} + \boxed{a} \\ s = \boxed{u} \cdot \boxed{\beta} + \boxed{b} \end{cases} \quad \text{Use LLL}$$

large   small   small

# [CFRZ16] Method to Factorize $N = p^r q^s$



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large   small   small

Rewrite  $N$  from new decomposition

$$N = p^r q^s = p^{u \cdot \alpha + a} q^{u \cdot \beta + b} = p^{u \cdot \alpha} q^{u \cdot \beta} p^a q^b = (p^\alpha q^\beta)^u p^a q^b = P^u Q$$

# [CFRZ16] Method to Factorize $N = p^r q^s$



First Step: Decompose  $r$  and  $s$

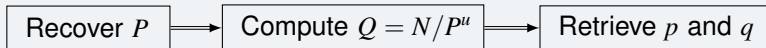
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large   small   small

Rewrite  $N$  from new decomposition

$$N = p^r q^s = p^{u \cdot \alpha + a} q^{u \cdot \beta + b} = p^{u \cdot \alpha} q^{u \cdot \beta} p^a q^b = (p^\alpha q^\beta)^u p^a q^b = P^u Q$$

Last Step: recover  $p$  and  $q$





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# Our Result



Polynomial time factorization of  $N = p^r q^s$  with  $r > s$  if:

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# Our Result



Polynomial time factorization of  $N = p^r q^s$  with  $r > s$  if:

$$r = \Omega(\log q)$$

The idea: Decompose  $N^\alpha$  instead of  $N$

**[CFRZ16]:**

- Write  $N = P^u Q$
- Apply BDH/Copp. on  $N$
- Condition:  $u = \Omega(\log Q)$

**This paper:**

- Write  $N^\alpha = P^r q$
- Apply BDH on  $N^\alpha$
- Condition:  $r = \Omega(\log q)$

# Our Improvement to Factorize $N = p^r q^s$



First Step: Find  $\alpha$  and  $\beta$

Since  $\gcd(r, s) = 1$ , there exist  $\alpha, \beta \in \mathbb{N}$  such that:

$$\alpha \cdot s - \beta \cdot r = 1$$

# Our Improvement to Factorize $N = p^r q^s$



First Step: Find  $\alpha$  and  $\beta$

Since  $\gcd(r, s) = 1$ , there exist  $\alpha, \beta \in \mathbb{N}$  such that:

$$\alpha \cdot s - \beta \cdot r = 1$$

Rewrite  $N^\alpha$  from new decomposition

$$N^\alpha = (p^r q^s)^\alpha = p^{\alpha r} q^{\alpha s} = p^{\alpha r} q^{\beta r + 1} = (p^\alpha q^\beta)^r q = P^r q$$

# Our Improvement to Factorize $N = p^r q^s$



First Step: Find  $\alpha$  and  $\beta$

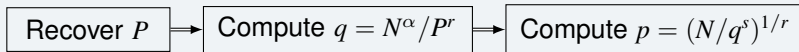
Since  $\gcd(r, s) = 1$ , there exist  $\alpha, \beta \in \mathbb{N}$  such that:

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Rewrite  $N^\alpha$  from new decomposition

$$N^\alpha = (p^r q^s)^\alpha = p^{\alpha r} q^{\alpha s} = p^{\alpha r} q^{\beta r + 1} = (p^\alpha q^\beta)^r q = P^r q$$

Last Step: Apply BDH and recover  $p$  and  $q$





# Complexity comparison:



Time complexities for factoring  $N = p^r q^s$ , where  $\log p \approx \log q$

Condition on $N = p^r q^s$	[CFRZ16]	New Method
$r = \Omega(\log q)$		$\mathcal{O}(\log^{12.5} N)$
$r = \Omega(\log^3 q)$	$\mathcal{O}(\log^8 N)$	$\mathcal{O}(\log^{14.25} N)$

# Generalization for $k$ prime factors



Factoring  $N = \prod_{i=1}^k p_i^{r_i}$  for large  $r_i$

- **Condition** to find a non-trivial factor of  $N$  in time polynomial (with  $r = \max\{r_i\}$  and  $p = \max\{p_i\}$ ):

$$r = \Omega(\log^{\theta_k} p)$$

$$\text{where } \theta_k = 2(k-1) \left( 1 + \sum_{i=1}^{k-2} \prod_{j=i}^{k-2} j \right) + 1$$

k	2	3	4	5	6
$\theta_k$ in [CFRZ16]	3	17	61	257	1301
New $\theta_k$	1	9	31	129	651

# Experiments for $N = p^r q^s$ with 128-bit primes $p$ and $q$



	Method	Decomposition	Bits given	Dim.	LLL <sub>f</sub>	LLL <sub>c</sub>	Est. time
<b><math>N = p^5 q^3</math></b>	[CFRZ16]	$N = (p^2 q)^3 p^{-1}$	57	52	17s	3.5s	$1.6 \cdot 10^{10}$ years
	<b>New</b>	$N^2 = (p^2 q)^5 q$	<b>46</b>	<b>78</b>	<b>0.3h</b>	<b>29s</b>	<b><math>6.5 \cdot 10^7</math> years</b>
<b><math>N = p^7 q^4</math></b>	[CFRZ16]	$N = (p^2 q)^4 p^{-1}$	51	57	45s	2.4s	$1.7 \cdot 10^8$ years
	<b>New</b>	$N^2 = (p^2 q)^7 q$	<b>43</b>	<b>92</b>	<b>1.9h</b>	<b>291s</b>	<b><math>8.1 \cdot 10^7</math> years</b>
<b><math>N = p^8 q^3</math></b>	[CFRZ16]	$N = (p^2 q)^4 q^{-1}$	51	61	86s	4.2s	$3 \cdot 10^8$ years
	<b>New</b>	$N^3 = (p^3 q)^8 q$	<b>57</b>	<b>95</b>	<b>6h</b>	<b>320s</b>	<b><math>1.4 \cdot 10^{12}</math> years</b>
<b><math>N = p^9 q^5</math></b>	[CFRZ16]	$N = (p^2 q)^5 p^{-1}$	48	61	113s	4.2s	$3.7 \cdot 10^7$ years
	<b>New</b>	$N^2 = (p^2 q)^9 q$	<b>43</b>	<b>108</b>	<b>3.9h</b>	<b>801s</b>	<b><math>2.2 \cdot 10^8</math> years</b>

👉 Still unpractical but asymptotically polynomial-time for large  $r$

# Conclusion and Remarks



## Conclusion

- Improvement over [CFRZ16] method to factorize  $N = p^r q^s$  for large  $r$
- Generalization for moduli  $N = \prod p_i^{r_i}$

# Conclusion and Remarks



## Conclusion

- Improvement over [CFRZ16] method to factorize  $N = p^r q^s$  for large  $r$
- Generalization for moduli  $N = \prod p_i^{r_i}$

## Remarks

- Much simpler than [CFRZ16]
  - Coppersmith's method is not used anymore
  - Bézout instead of LLL used to find good decomposition of  $r$  and  $s$
- For 128-bit primes, unpractical compared to ECM factorisation
- But ECM scales exponentially while our method is polynomial
  - For large  $p$  and  $q$ , our algorithm must outpace ECM