Amstelhaege calculations.

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December 10, 2017

Here we make some calculations that provide insight into the case *Amstelhaege* for the course *Heuristics*, as desribed on `http://heuristieken.nl/wiki/index.php?title=Amstelhaege`.

Estimated upperbound for the state space.

We want to obtain a worst case estimation of the number of possible different configurations of houses in the case *Amstelhaege*. We do this for the 20-houses variant. The value of a house depends on the free space, which is measured in whole meters. Therefore we can work on a discrete grid of points for the possible positions. Houses could be rotated with many angles, but we only take two possibilities into account here. Not rotated and rotated 90 degrees.

We start with a single family house. The area for the case is 180m × 160m. We have to take the dimensions of the house into account, as well as the required minimal free space. In Figure 1 is a schematic explanation of the situation. We find that there are

$$(180 - 2 - (8 + 2)) \times (160 - 2 - (8 + 2)) = 24864$$

positions where we can place the first single family house. There are 12 single family houses that need to be placed. This means there are

$$\prod_{i=0}^{11} (24864 - i)$$

possible placements for the single family houses.

Similarly a bungalow will have

$$(180 - 3 - (8 + 3)) \times (160 - 3 - (10 + 3)) = 24048$$

possibilities to be placed. Although the single family houses all confiscated at least $8 \times 8 = 64$ points on the grid, we can not conclude that all these points become unavailable for bungalows, because the single family houses may be positioned all

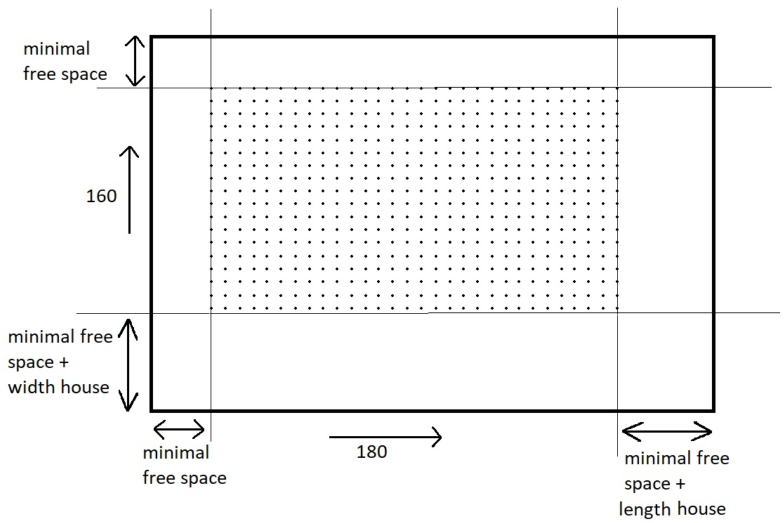

Figure 1: A sketch for the possible positions of the top left corner of a house.

the way to the edges (in the margin at the bottom or right in Figure 1).
The bungalows add

$$2 \times \prod_{i=0}^{4} (24108 - i)$$

possible placements. Where the factor 2 comes from the fact the bungalows can be rotated 90 degrees.

The maisons all have

$$(180 - 6 - (11 + 6)) \times (160 - 6 - (10 + 6)) = 21666$$

possibilities to be placed, and add

$$2 \times \prod_{i=0}^{2} (21666 - i)$$

possible placements.

Symmetry causes us to count every configuration 4 times, thus we need to correct that. Eventually this results in

$$\left(\prod_{i=0}^{11} (24864 - i) \times 2 \times \prod_{i=0}^{4} (24048 - i) \times 2 \times \prod_{i=0}^{2} (21666 - i) \right) \times \frac{1}{4}$$

possible placements of all 20 houses. This is
$(55679968251086038769975375593619488198241253131673600 \times$
$80392254875146585324 80 \times$
$10168949232960) \approx 4.55 \times 10^{87}$.

The minimum scores.

The minimum scores are obtained when all houses have less then 1 meter extra free space. For the 20 houses variant this means we have

$$12 \times 285000 \times 5 \times 399000 \times 3 \times 610000 = 7245000.$$

The 40 and 60 houses variants have respectively 14490000 and 21735000 as minimum score.

Estimated upper bounds for the scores.

We calculate the 20 houses variant. We have an area of 28800m^2 on which
$12 \times (8m \times 8m) + 5 \times (10m \times 7.5m) + 3 \times (10.5m \times 11m) = 1489.5m^2$
is taken by the houses themselves. Suppose for simplicity the space that is left is optimally used and maximally increases the free space of every house. This is clearly an oversimplification of the case and will cause the estimation to be far too big.

A single family house uses less then $12^2 - 8^2 = 80m^2$ for their required free space. Thus we have at least
$28800m^2 - 1489.5m^2 - 80m^2 = 27230.5m^2$
extra free space for a single family house. This is about 165m for both length and width, which gives this house a value of
$285000 + (0.03 * 285000) * 165 = 1695750$.
Similar calculations show that the values of bungalows and maisons could be has high as respectively 3032400 and 6539200. Theoratically the score could be up to $12 \times 1695750 + 5 \times 3032400 + 3 \times 6539200 = 55128600$. This is more than 40 million over our highest score. If we want to lower this estimated upperbound for the score, we need to make the calculations much more complicated. This would develop into solving the case.