Assignment 1

Neuromorphic Engineering
Bipin Rajendran

Due August 14, 2015 at 5PM

Note: In all the problems, be mindful of the units of various quantities and the sign conventions for currents and voltages.

Your submission must include:

1. A .pdf or .doc file clearly documenting your code, figures, and results.

2. Your MATLAB code. The code must be saved in plain text files that can be immediately run in MATLAB; include a file called main.m that runs the functions you implemented and generates the figures described in your assignment write-up. You will lose credit if your code is absent or cannot be run.

Store your write-up and code in a single directory named hw1_yourID (for example hw1_12D423222) and submit it in moodle.

Do not wait till the last minute to start the Assignment as you will require at least at least 8 hours to complete all the questions.

Late submission policy:

Before the solution key is uploaded in moodle: If your original score is S and you submitted the HW X hours after the deadline, your score will be $S \exp(-X/24)$.

After the solution key is uploaded in moodle: 0 credit.

Spiking Neuron Models

In this assignment, we will learn how to model the activity of spiking neurons starting with the simplest model. We will also implement the Hodgkin-Huxley neuron model and use it to determine an estimate of the energy cost associated with a spike.

Problem 1: Leaky Integrate and Fire Model
The dynamics of the membrane potential $V(t)$, in the LIF neuron model is given by the equation

$$C\frac{dV(t)}{dt} = -g_L(V(t) - E_L) + I_{app}(t) \tag{1}$$

C is the membrane capacitance, g_L is the leak conductance and E_L is the leak reversal potential. I_{app} is the externally applied current and is assumed to be positive for current flowing into the cell.

When $V \geq V_T$, a spike is issued and V is reset to E_L (We will write this as $V(t) \longrightarrow E_L$). (Assume that C= 300 pF, g$_L$ = 30 nS, V_T = 20 mV and E$_L$ = -70 mV).

(a) Assume that a constant current I_0 is applied to the neuron. Write an expression for the steady state value of the membrane potential. Hence, determine the minimum value of the steady state current, I_c necessary to initiate a spike. **2 points**

(b) In order to simulate the behavior of a set of N neurons, it is useful to define a $N \times 1$ column vector to the store the membrane potentials of the N neurons. Write a program to solve the equivalent difference equation numerically using Runge-Kutta second order method for a set of N neurons driven by external current input. (You should not use a FOR loop to calculate the potential of the N neurons, rather, the potential of all the neurons should be calculated in one step.)
Assume that the input to the program is a $N \times M$ column vector, representing the input current for the N neurons for M time-intervals where $M = T/\Delta t$. The output of your program should be a $N \times M$ matrix storing the values of the membrane potential for the N neurons, for the M time-intervals. **8 points**

(c) We would like to now use this framework to study the dynamics of LIF neurons. Assume that you have a population of 10 identical neurons, with each neuron receiving a constant current. Let the magnitude of the input current for the k^{th} neuron be given by the expression

$$I_{app,k} = (1 + k\alpha)I_c \tag{2}$$

where $\alpha = 0.1$. (In this example, the current does not vary with time, so, all the values across any row of the input current matrix is a constant). Plot the membrane potential for neurons $2, 4, 6$ and 8 from $t = 0$ to 500 ms. (Assume $\Delta t = 0.1$ ms and at $t = 0$, the neuron is in steady-state with $I_{app}(t) = 0$). **6 points**

(d) Plot the average time interval between spikes from (c) as a function of $I_{app,k}$. **4 points**

Problem 2: Izhikevich Model

The dynamics of the membrane potential $V(t)$, in the Izhikevich neuron model is given by the equations

$$C\frac{dV(t)}{dt} = k_z(V(t)-E_r)(V(t)-E_t) - U(t) + I_{app}(t) \qquad (3)$$

$$\frac{dU(t)}{dt} = a\left[b(V(t)-E_r) - U(t)\right] \qquad (4)$$

When $V(t) \geq v_{peak}$, $V(t) \longrightarrow c$ and $U(t) \longrightarrow U(t)+d$.

By varying the parameters $C, E_r, E_t, k_z, a, b, c$ and d, a variety of neuronal behaviors can be modeled.

	C(pF)	k_z(μS/V)	E_r(mV)	E_t(mV)	a (KHz)	b(nS)	c(mV)	d(pA)	v_{peak}(mV)
RS	100	0.7	-60	-40	0.03	-2	-50	100	35
IB	150	1.2	-75	-45	0.01	$+5$	-56	130	50
CH	50	1.5	-60	-40	0.03	$+1$	-40	150	25

(a) What are the steady state values of V and U for $I_{app} = 0$? **2 points**

(b) Write the equivalent difference equations for (3) and (4). **2 points**

(c) Write a program to solve the equivalent difference equation for a set of N Izhikevich model neurons using Runge-Kutta fourth order method. The neuron type should be a parameter in your function call, for each of the N neurons. You may use $\Delta t = 0.1$ ms and plot the response of the three neurons above from $t = 0$ to 500 ms, for $I_{app} = 400, 500, 600$ pA. **16 points**

Note: Try to re-run 2(c) with larger values of Δt. You will notice that the overall dynamics can change drasticially, especially for neuron CH. This is because of the inaccuracy in determining the exact time when $V(t)$ exceeds v_{peak}. For a good description of this issue and how to get around it, see: *Hybrid spiking models, Phil. Trans. R. Soc. A November 13, 2010 368 (1930) 5061-5070.*

We will ignore these issues here, and for the sake of displaying, you may chose to artificially set the value of membrane potential just before reset to v_{peak} in your code.

Problem 3: Adaptive Exponential Integrate-and-Fire Model

The dynamics of the membrane potential $V(t)$, in the AEF neuron model is given by the equations

$$C\frac{dV(t)}{dt} = -g_L(V(t) - E_L) + g_L\Delta_T \exp\left(\frac{V(t) - V_T}{\Delta_T}\right) - U(t) + I_{app}(t) \tag{5}$$

$$\tau_w\frac{dU(t)}{dt} = a\left[V(t) - E_L\right] - U(t) \tag{6}$$

When $V(t) \geq 0$, $V(t) \longrightarrow V_r$ and $U(t) \longrightarrow U(t) + b$.

As before, by varying the parameters a variety of neuronal behaviors can be modeled.

	C(pF)	g_L(ns)	E_L(mV)	V_T(mV)	Δ_T(mV)	a(nS)	τ_w(ms)	b(pA)	V_r(mV)
RS	200	10	-70	-50	2	2	30	0	-58
IB	130	18	-58	-50	2	4	150	120	-50
CH	200	10	-58	-50	2	2	120	100	-46

(a) Write the equivalent difference equations for (5) and (6). **2 points**

(b) What are the steady state values of V and U for $I_{app} = 0$? (Determine the answer numerically, such that the value of V is accurate within $\pm 1\mu$V). **6 points**

(c) Write a program to solve the equivalent difference equation for a set of N AEF model neurons using Euler method. The neuron type should be a parameter in your function call, for each of the N neurons. You may use $\Delta t = 0.1$ ms and plot the response of the three neurons above from $t = 0$ to 500 ms, for $I_{app} = 250, 350, 450$ pA. **12 points**

Problem 4: Spike energy based on Hodgkin-Huxley neuron model

The dynamics of the membrane potential $V(t)$, in the Hodgkin-Huxley neuron model is given by the equations

$$C\frac{dV(t)}{dt} = -i_{Na}(t) - i_K(t) - i_l(t) + I_{ext}(t) \tag{7}$$

where

$$\begin{aligned} i_{Na}(t) &= g_{Na}m^3h(V(t) - E_{Na}) && (8)\\ i_K(t) &= g_K n^4(V(t) - E_K) && (9)\\ i_l(t) &= g_l(V(t) - E_l) && (10) \end{aligned}$$

The variables n, m and h lie in the interval $[0, 1]$ and obey the equation

$$\frac{dx}{dt} = \alpha_x(t)(1 - x) - \beta_x(t)x \tag{11}$$

where

$$\alpha_n(t) = \frac{0.01(V(t) + 55)}{1 - \exp(-(V(t) + 55)/10)} \qquad \beta_n(t) = 0.125\exp(-(V(t) + 65)/80) \tag{12}$$

$$\alpha_m(t) = \frac{0.1(V(t) + 40)}{1 - \exp(-(V(t) + 40)/10)} \qquad \beta_m(t) = 4\exp(-0.0556(V(t) + 65)) \tag{13}$$

$$\alpha_h(t) = 0.07\exp(-0.05(V(t) + 65)) \qquad \beta_h(t) = \frac{1}{1 + \exp(-0.1(V(t) + 35))} \tag{14}$$

$V(t)$ is assumed to be measured in mV in the above expressions.

Assume that C=1μF/cm^2, $E_{Na} = 50$ mV, $E_K = -77$ mV and $E_l = -55$ mV and $g_{Na} = 120$ mS/cm^2, $g_K = 36$ mS/cm^2 and $g_l = 0.3$ mS/cm^2.

Solve the equivalent difference equations for (7) and (11) to determine the ion currents and the membrane potential for a step current waveform in the interval $0 < t \leq 5T$, described as follows: $I_{ext}(t) = I_0[H(t - T) - H(t - 4T)]$ where $H(x)$ is the Heaviside step function defined as $H(x) = 1$ if $x \geq 0$ and $H(x) = 0$ otherwise. You should set your initial conditions such that there are no spikes when there is no external input current to the neuron. $\Delta t = 0.01$ ms, $T = 100$ms.

(a) Explain the response of the neuron for $I_0 = \pm 1\mu$A/cm^2 and $I_0 \pm 10\mu$A/cm^2. **6 points**

(b) Plot the spike rate as a function of the input current for $0 < I_0 \leq 50\mu$A/cm^2. **6 points**

(c) The instantaneous power dissipated (per unit area) in the various ion channels can be approximated by the expression

$$P_x(t) = i_x(t)(V(t) - E_x) \tag{15}$$

Similarly, the power spent in charging/discharging the membrane capacitance (per unit area) can be approximated as

$$CV(t)\frac{dV(t)}{dt} = [-i_{Na}(t) - i_K(t) - i_l(t) + I_{ext}(t)]\, V(t) \tag{16}$$

Plot the relative magnitudes of the instantaneous power dissipated in the three ion channels and in the membrane capacitor as a function of time for one cycle of the action potential. **4 points**

(d) Numerically integrate the power in (b) to determine the total energy dissipated in one cycle of the action potential for a patch of the cell membrane with area of 1 μm^2 **4 points**.