Tutorial 1

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The Stanford 'Aha' Sessions

- "This was an experimental project where we'd have three or four cameras in a basement studio and we would film classes of about an hour," says Knuth. "We got a bunch of our brightest students and gave them extremely difficult problems. You could literally see the 'Aha' taking place. People can watch the problem-solving process as it occurred."
- http://scpd.stanford.edu/knuth/index.jsp
- Co-operative learning improves ability to tackle difficult, open-ended problems
- American Journal of Physics – July 1992 – Volume 60, Issue 7, pp. 627

Instructions

- Work/discuss with partners in your group.
- If you have questions, raise your hand.
- Write down your ideas, thought process, any assumptions you are making
- Draw diagrams, graphs, pseudo-code etc.
- Before coding, think of the many ways you could approach the problem, write what you expect from the simulation before doing it.
- I want to know what methods did not work.
- Grading is based not on final results, but your thought process and your ideas to solve/approach the problem (your scribble notes).
- Document (roughly) all your thoughts, as it will help us in grading.
- You have 45 mins to work on this, each group should save all the codes you have written and give it to the TA.

- During World War II, the city of South London was hit by 535 bombs. For the sake of analysis, the total area of the city was divided into 576 squares, each with area 1/4 sq. km.
- The statistics of the actual hits/square is below:

Number of hits/sq	Number of squares
0	229
1	211
2	93
3	35
4	7
5	1

- Looking at this statistics, can you tell if the Germans were targeting specific areas of if they were bombing the city without any prior information?
- Can you test your hypothesis by a computer simulation ?

Actual Statistics

Source: http://www.bombsight.org/

Throwing balls into buckets

m identical balls are thrown one-by-one into n identical buckets.
$P_k(m)$ denotes the fraction of buckets that contains k balls $(0 \leq k \leq m)$.
Success, $S = 1$ if ball in bucket 1, else $S = 0$.

n buckets

$$p = \frac{1}{n}$$

$$\begin{aligned}
\mu &= \frac{S_1 + S_2 + \ldots S_m}{m} \\
&= \frac{(1 \times pm) + (0 \times (1-p)m)}{m} \\
&= p \times 1 + 0 \times (1-p) = p
\end{aligned}$$

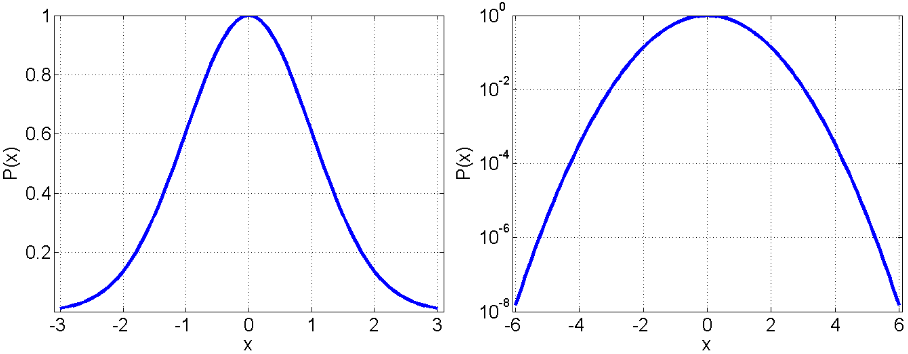
$$\begin{aligned}
\sigma^2 &= \frac{(S_1 - \mu)^2 + (S_2 - \mu)^2 + \ldots (S_m - \mu)^2}{m} \\
&= \frac{((1-p)^2 pm) + ((0-p)^2 (1-p)m)}{m} \\
&= p(1-p)^2 + (1-p)p^2 \\
&= p(1-p)
\end{aligned}$$

n buckets

$$
\begin{aligned}
P_k(m) &= \frac{m!}{k!(m-k)!}\left[p^k(1-p)^{m-k}\right] \\
&\longrightarrow \mathcal{N}\left[mp, mp(1-p)\right] \\
&= \mathcal{N}\left[m\mu, m\sigma^2\right] \\
\mathcal{N}\left[\mu, \sigma^2\right] &= \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(k-\mu)^2}{2\sigma^2}\right]
\end{aligned}
$$

Central Limit Theorem: The mean of a sufficiently large number of independent and identically distributed random variables, each with finite mean and variance, will be approximately normally distributed.

Gaussian (Normal) distribution

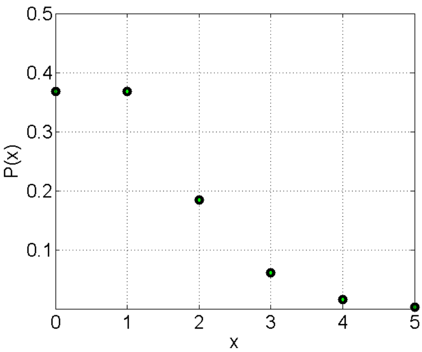


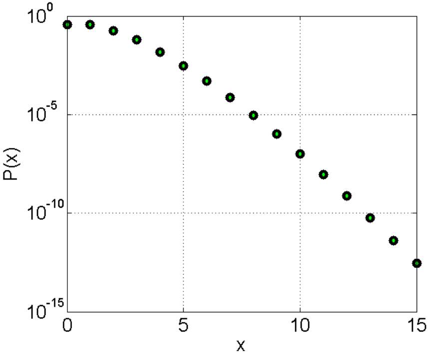
$$\mu = 0, \sigma = 1$$

Define $\lambda = m/n$.

$$\begin{aligned}
P_k(m) &= \frac{m!}{k!(m-k)!}\left(\frac{1}{n}\right)^k\left(1-\frac{1}{n}\right)^{m-k} \\
&= \frac{m!}{k!(m-k)!}\left(\frac{\lambda}{m}\right)^k\left(1-\frac{\lambda}{m}\right)^{m-k} \\
&= \left[\left(1-\frac{\lambda}{m}\right)^m\frac{\lambda^k}{k!}\right]\left[\frac{m!}{(m-k)!}\left(\frac{1}{m}\right)^k\left(1-\frac{\lambda}{m}\right)^{-k}\right] \\
&= \left[\left(1-\frac{\lambda}{m}\right)^m\frac{\lambda^k}{k!}\right]\left[\frac{m!}{(m-k)!(m-\lambda)^k}\right] \\
&= \left[\left(1-\frac{\lambda}{m}\right)^m\frac{\lambda^k}{k!}\right]\left[\frac{m}{m-\lambda}\right]\left[\frac{m-1}{m-\lambda}\right]\cdots\left[\frac{m-k+1}{m-\lambda}\right] \\
&\longrightarrow \left[\frac{e^{-\lambda}\lambda^k}{k!}\right]
\end{aligned}$$

Poisson distribution





$$\lambda = 1$$

A ball tossing experiment - Practice problem

n buckets

- Assume we have 4 buckets.
- Identical balls are thrown into these buckets, one at a time. Each ball lands up in one of the 4 buckets.
- Write a MATLAB code to simulate the throwing of $m = 2$ balls into these buckets.
- Repeat this experiment 10^6 times.
- Determine $P_k(m)$, the fraction of buckets that contains k balls? $(0 \leq k \leq m)$
- Repeat for $m = 64$ balls with (a) $n = 4$ buckets and (b) $n = 64$ buckets.
- Plot $P_k(m)$ as a function of k. Also plot the lines for the two distributions.

Poisson stimulus for biological neurons - HW problem

We need to create a Poisson stimulus to mimic signals that appear at the input of biological neurons. Assume that the stimulus have an average arrival rate of $\lambda = 1/5\text{ms}$. Usually, a time-step of $\delta t = 0.1\text{ms}$ is sufficient to maintain the resolution necessary for accurate biological simulations.

One way to model such inputs is to assume that at each time-step of duration δt, the stimulus can appear with a probability $= \lambda \delta t$.

Create a stimulus stream in the interval $t = [0, 1000\text{ms}]$.

How many times does the stimulus appear in your stream? How many random numbers did you use to arrive at your signal?