Assignment 1: Spiking Neuron Models

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1 Leaky Integrate and Fire Model

(a) Expression for steady state value of membrane potential on application of an external current I_0:

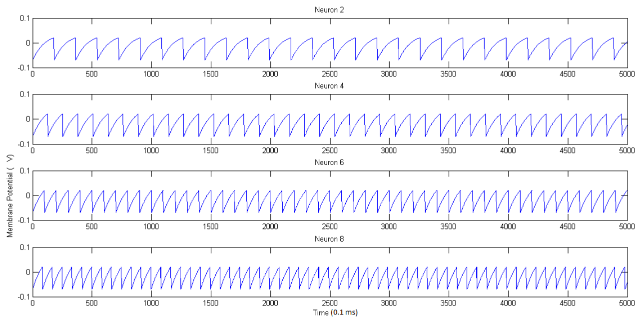
$$V_\infty = \frac{I_0}{g_L} + E_L \tag{1}$$

The minimum value of steady state current, I_c, so that a spike is initiated is when V_∞ goes to V_T:

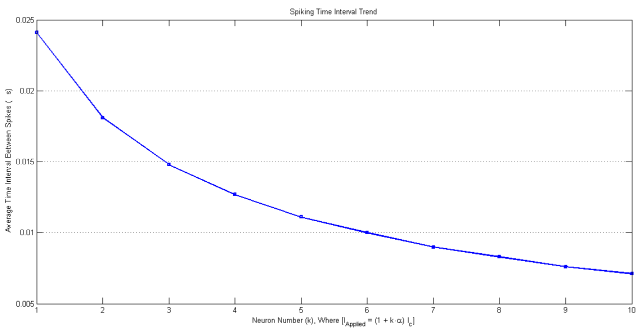
$$\begin{align} I_c &= g_L(V_T - E_L) \tag{2}\\ &= 30nS(20mV - (-70mV)) \tag{3}\\ &= 2.7nA \tag{4}\end{align}$$

(b) See code in file '$\backslash LIF \backslash LIF.m'$

(c)



(d) The trend of average time interval between spikes with linearly increasing current:



2 Izhikevich Model

(a) Expressions for the steady state values of $V(t)$ and $U(t)$ are:

$$V_{I_{app}=0,\infty} = \frac{b}{k_z} + E_t \tag{5}$$

$$U_{I_{app}=0,\infty} = b(\frac{b}{k_z} + E_t - E_r) \tag{6}$$

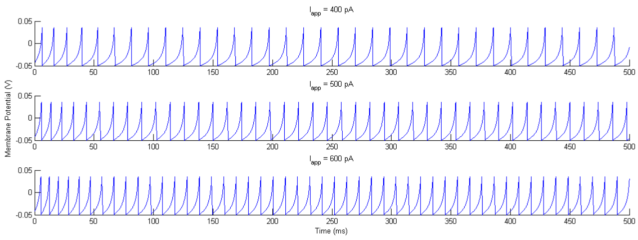
(b) The equivalent difference equations are:

$$V_{n+1} = V_n + \frac{1}{C}[k_z(V_n - E_r)(V_n - E_t) - U_n + I_{app}]\triangle t \tag{7}$$

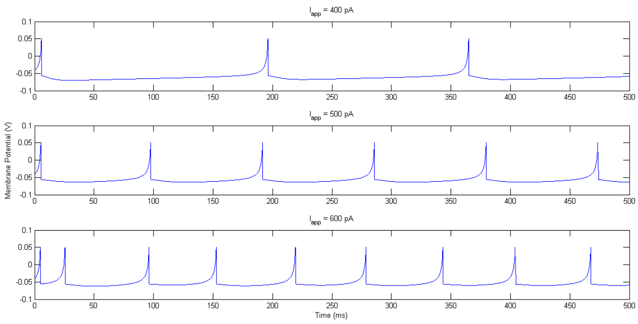
$$U_{n+1} = U_n + [a(b(V_n - E_r) - U_n)]\triangle t \tag{8}$$

(c) See code in file ‘$\backslash Izhikevich \backslash Izhikevich.m'$.

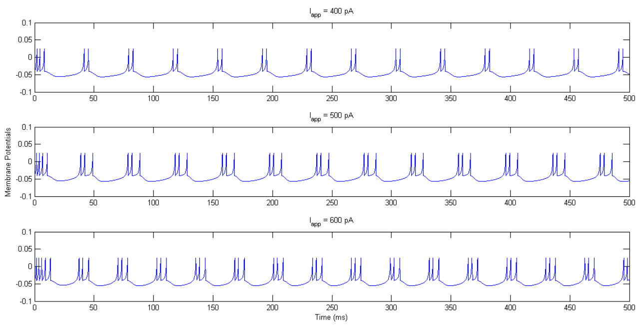
The trend for the RS neuron:



For the IB neuron:



For the CH neuron:



3 Adaptive Exponential Integrate-and-Fire Model

(a) The equivalent difference equations are:

$$V_{n+1} = V_n + \frac{1}{C}[g_L(\triangle_T(\exp(\frac{V_n - V_T}{\triangle_T})) - (V_n - E_L)) - U_n + I_{app}]\triangle t \tag{9}$$

$$U_{n+1} = U_n + \frac{1}{\tau_\omega}[a(V_n - E_L) - U_n]]\triangle t \tag{10}$$

(b) Setting $\frac{dV}{dt} = 0$ and $I_{app} = 0$ gives the equation:

$$g_L(\triangle_T(\exp(\frac{V_\infty - V_T}{\triangle_T})) - (V_\infty - E_L)) = U \tag{11}$$

Since everything on the LHS is constant, the RHS , U, must also be constant. Thus. we can replace U with U_∞ and also set $\frac{dU}{dt} = 0$ to get:

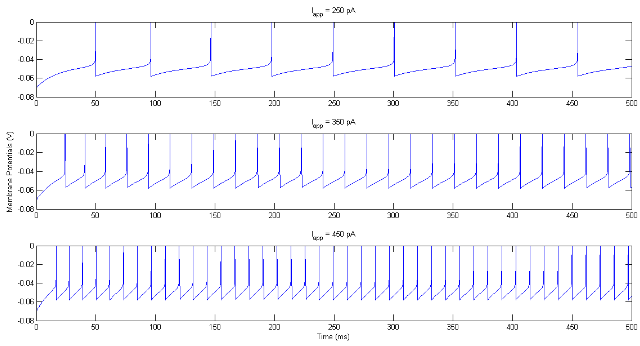
$$a(V_\infty - E_L) = U_\infty \tag{12}$$

Substituting for U in the first equation gives a fixed-point non-linear equation that can be solved using iterative numerical procedures:

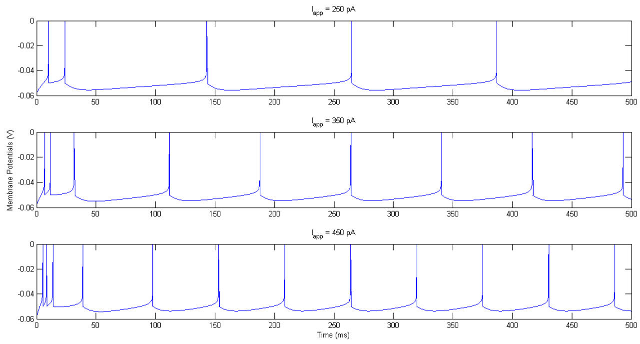
$$\frac{g_L \triangle_T}{g_L + a}(\exp(\frac{V_\infty - V_T}{\triangle_T})) + E_L = V_\infty \tag{13}$$

From V_∞, it is an easy matter to get U_∞ using equation (12). See the file '$\backslash AEF \backslash fixedPoint.m'$ for the code that solves this non-linear equation (finds the fixed point of the function of V_∞ on the lHS) and finds the values to initialise the variables V and U with in the simulation.

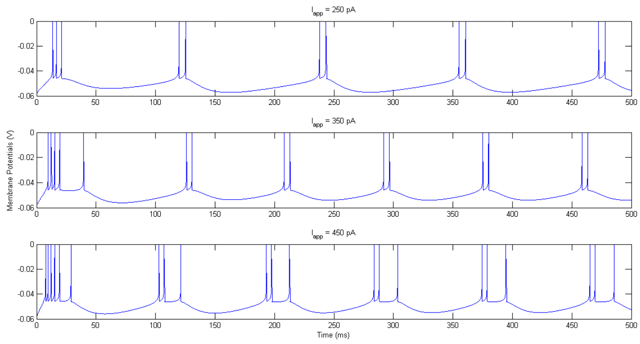
(c)
The trend for the RS neuron:



For the IB neuron:



For the CH neuron:



4 Hodgkin-Huxley Model

(a)