Hint 1-b

I've put together some tips on the paper. Please have a look at this!

Dr. Ryoko

IBM Quantum Challenge 2020

Hint for Learning Exercise I-B

Practical method

You can easily experiment by rewriting the code used in cell 22. Let's extend the size of the database register, the oracle(use any number up to $2^7 - 1$), and the diffusion. If you want to search for numbers that include 0, such as 0101111, use the X gate on the bits with 0s(in the example, 0th and 2nd) before and after the mct gate (extended CX gate) to flip and return them.

Theoretical aspects

For those who want to think a little more theoretically, here are some hint formulas. When the iteration is repeated t times, the probability of getting answer is as follows.

$$|\langle w|(U_s U_w)^t|\Psi_0\rangle|^2 = \sin^2((2t+1)\theta)$$

Therefore, the probability of finding a solution is high when $(2t+1)\theta$ is close to a right angle($\frac{\pi}{2}$). The following approximation can be made for a sufficiently large N.

$$\theta \sim \tan\theta$$

Then we got,

$$\tan\theta = \frac{1}{\sqrt{N-1}}$$

Also, we can approximate as shown for large N.

$$\frac{1}{\sqrt{N-1}} \sim \frac{1}{\sqrt{N}}$$

Therefore, we obtain the following constraints of t for a sufficiently large N.

$$(2t+1)\frac{1}{\sqrt{N}} = \frac{\pi}{2}$$

Transforming the formula gives:

$$t = \frac{1}{2}(\frac{\pi}{2}\sqrt{N} - 1) = \frac{\pi}{4}\sqrt{N} - \frac{1}{2}$$

By implementing the algorithm paying attention to the number of iterations in this way, a clearer solution can be obtained.