Discrete Mathematics

Programming Assignment 2

Recursion

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4 Nov, 2019

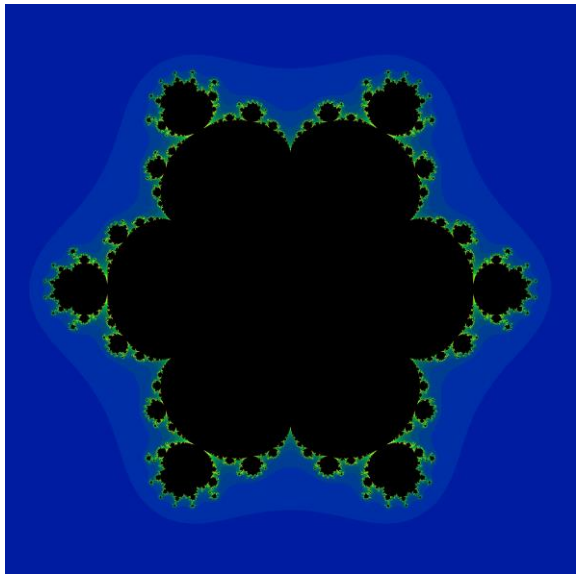
Assignment Overview

- Write a program for each of the following five tasks
  - Tasks 1-3. Fractals
  - Task 4. Triangulation
  - Task 5. CNF converter
- Submission (see more details at Slides XX—XX)
  - deadline: 15 Nov (Fri), 11:59 PM
  - deliverables
    - five programs (one for each task)
    - one write up for the five tasks
- Team: work with your team members to make one submission

Fractal

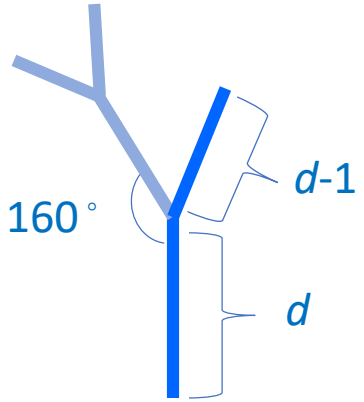
- A *fractal* is a geometric object whose structure is identical to the structures of its components.
  - a structural pattern is repeated recursively
- Many structures in nature are formed as fractals.

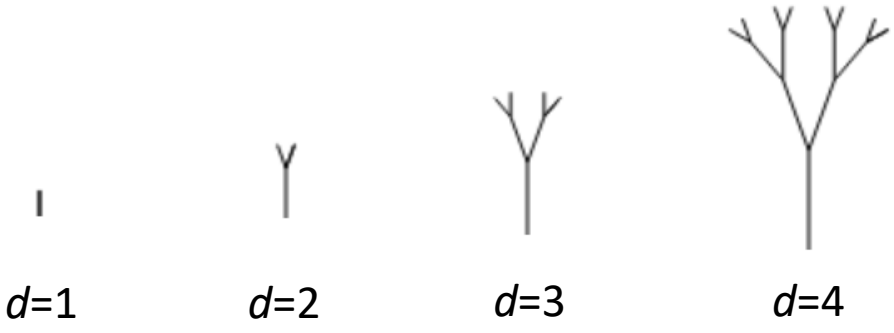


Tree Fractal (1/2)

- A tree of depth *d* has one stem and two branches.
  - The length of the stem is *d*
  - The angles between the stem and the two branches are -160° and +160°, respectively.
- Each branch is in a form of a tree of depth *d* – 1.





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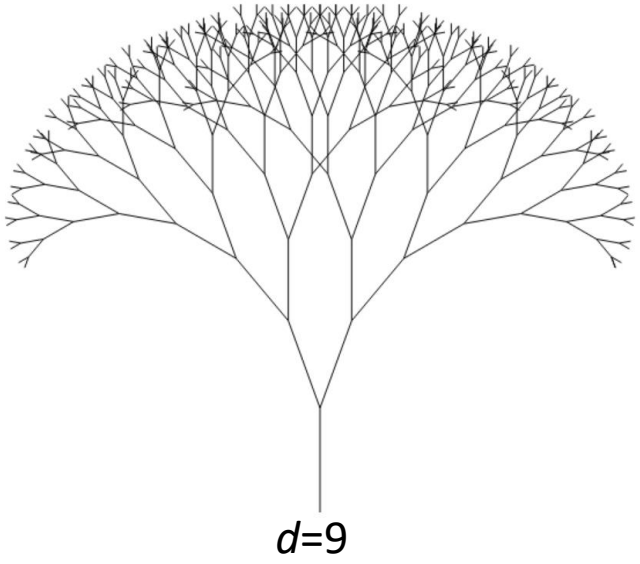


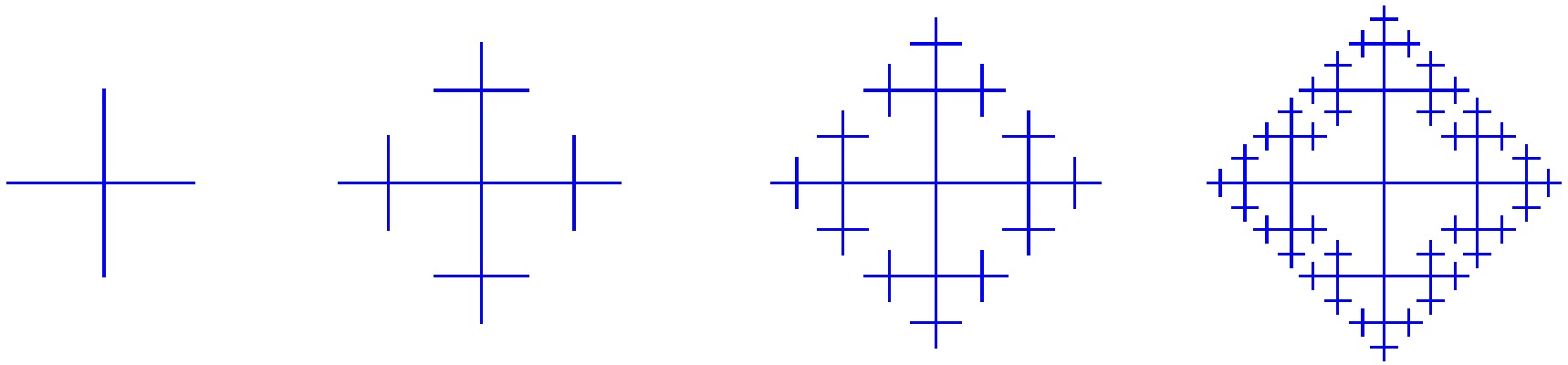
```html
<html> <body>
    <canvas id="canvas" width="600" height="500"></canvas>

    <script type="text/javascript">
      var elem = document.getElementById('canvas');
      var context = elem.getContext('2d');
      context.fillStyle = '#000';
      context.lineWidth = 1;
      var deg_to_rad = Math.PI / 180.0;
      var depth = 9;

      function drawLine(x1, y1, x2, y2, brightness){
        context.moveTo(x1, y1);
        context.lineTo(x2, y2);
      }

      function drawTree(x1, y1, angle, depth){
        if (depth !== 0){
          var x2 = x1 + (Math.cos(angle * deg_to_rad) * depth * 10.0);
          var y2 = y1 + (Math.sin(angle * deg_to_rad) * depth * 10.0);
          drawLine(x1, y1, x2, y2, depth);
          drawTree(x2, y2, angle - 20, depth - 1);
          drawTree(x2, y2, angle + 20, depth - 1);
        }
      }

      context.beginPath();
      drawTree(300, 500, -90, depth);
      context.closePath();
      context.stroke();
</script>
</body> </html>
```

Tree Fractal (2/2)

- Sample code
  https://rosettacode.org/wiki/Fractal_tree#JavaScript

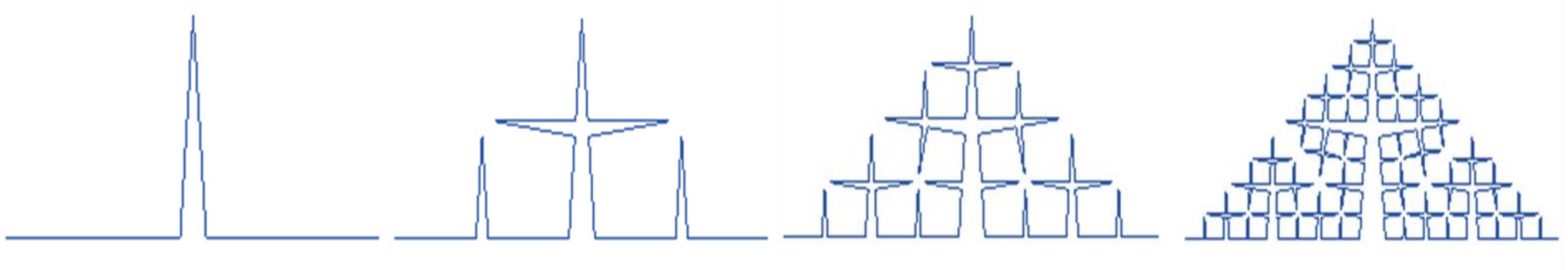
1. Fractal 1

- Define the depth and the parameters of a fractal of the given example, and write a JavaScript program that draws the following fractal for the given depth and parameters



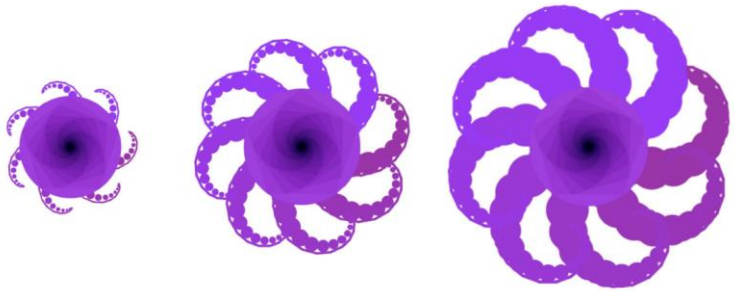
2. Fractal 2

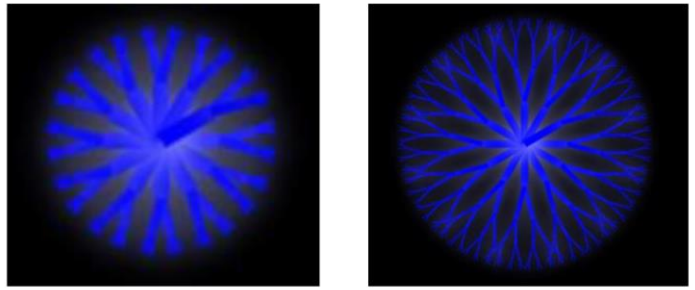
- Define the depth and the parameters of a fractal of the given example, and write a JavaScript program that draws the following fractal for the given depth and parameters



3. Fractal 3

- Suggest an interesting, creative, unique fractal by yourself and write a program to draw the fractal in JavaScript
  - the class will vote for submitted fractals
  - the number of votes for your fractal will be partly counted in evaluation
  - c.f. Fractal gallery of the 2018 class https://github.com/hongshin/DiscreteMath/blob/master/assignments/fractal2018.pdf





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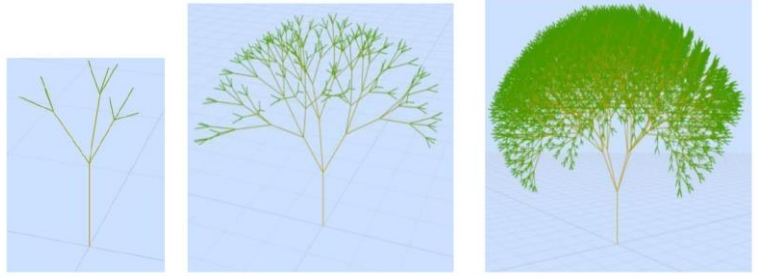

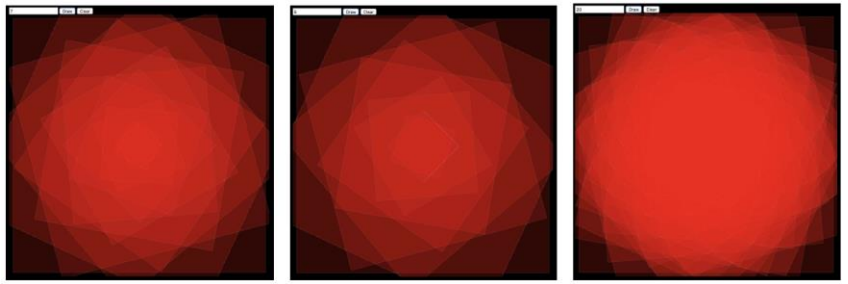

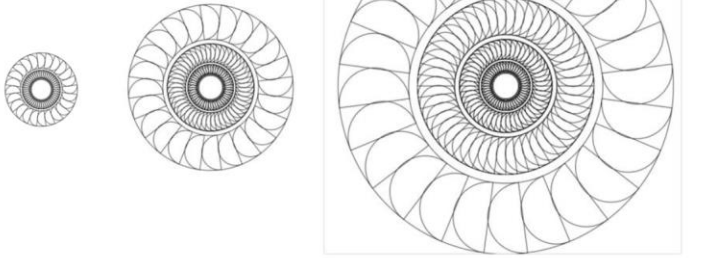

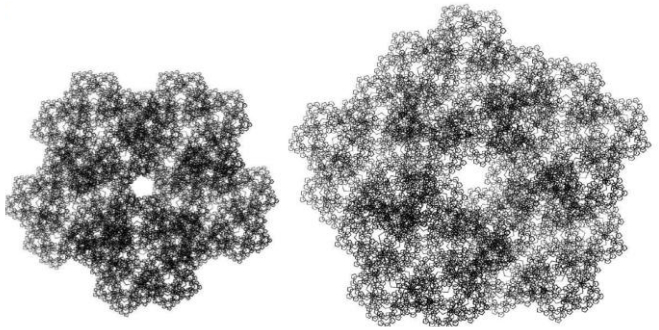

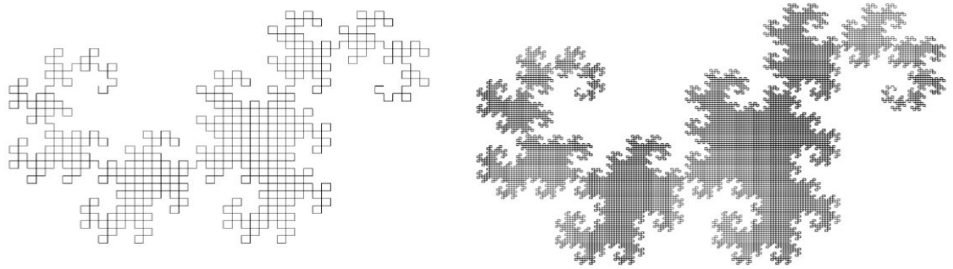

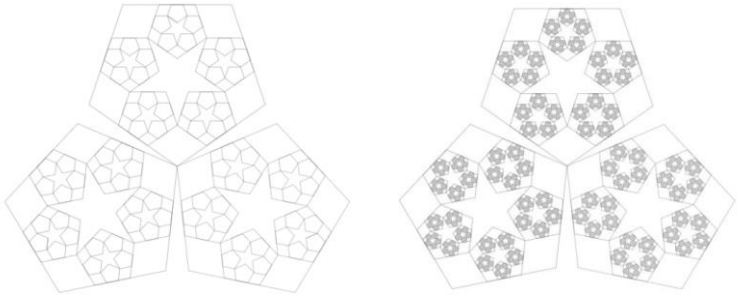

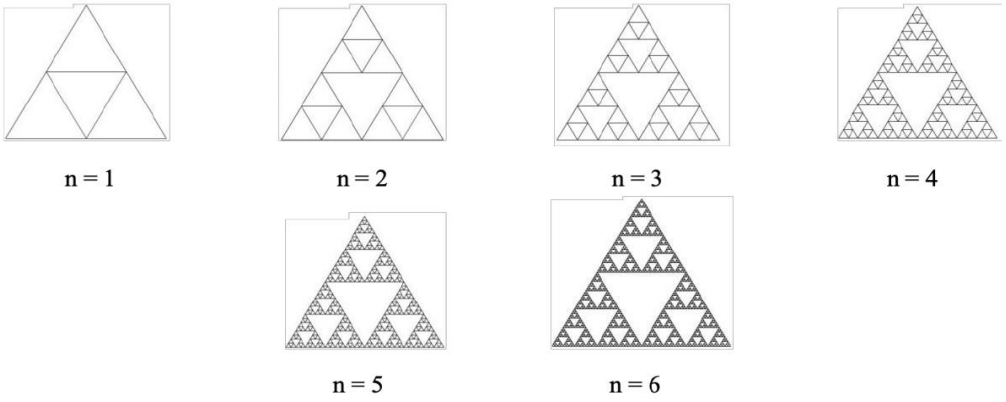

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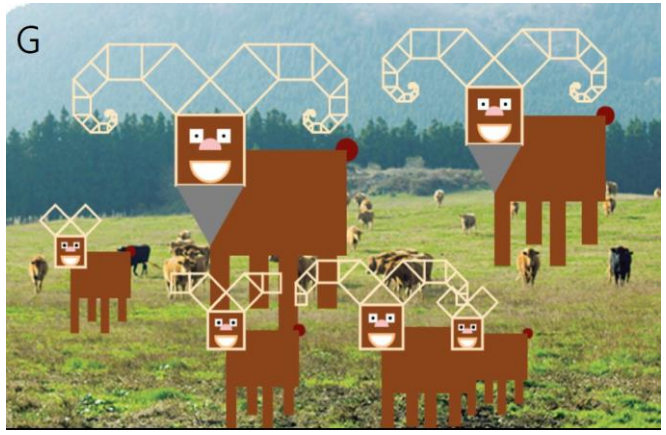
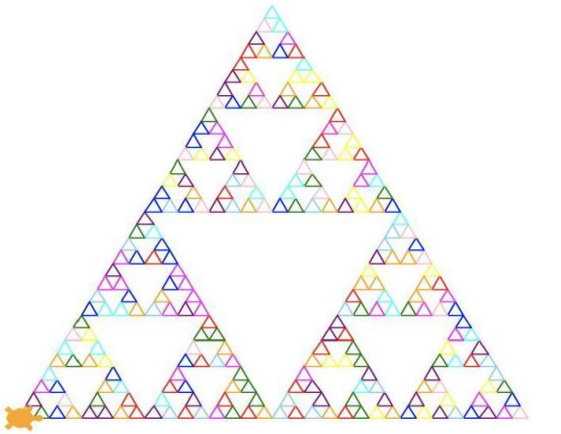
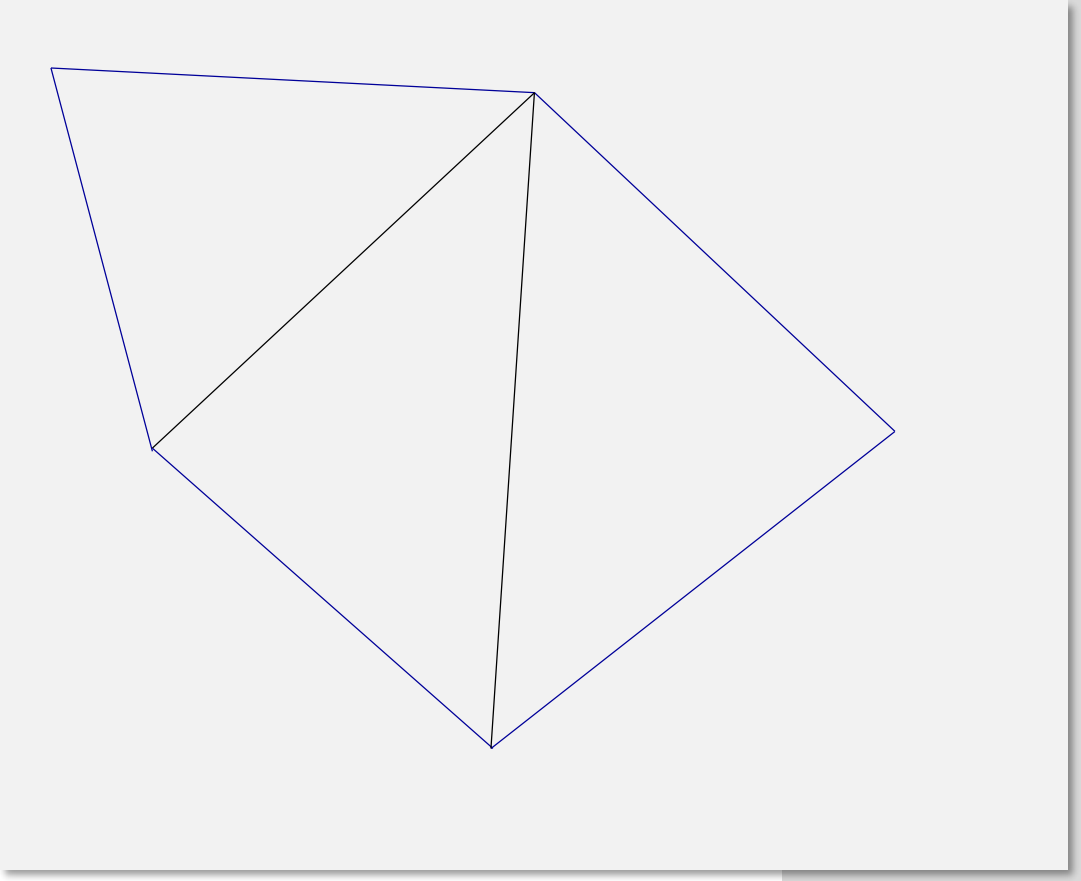




4. Triangulation

- Write a recursive program in JavaScript that receives a simple polygon and draws a triangulation result
  - a simple polygoin is given as the list of points in a 100x100 coordinate plane
  - each point is denoted as a pair of integers (x, y) for $1 \leq x \leq 100$ and $1 \leq y \leq 100$
  - two adjacent points make a side, and the first and the last points make a side
  - e.g., "(1,1) (40,10) (80, 40) (40,80), (5,40)"
  - draw nothing if the given list does not represent a simple polygon



5. CNF Converter (1/3)

- Write a C program that receives a propositonal formula and transforms it into a Conjunctive Normal Form (CNF)
  - the program uses recursion for the tranformation

- Conjunctive Normal Form (CNF)
  - Two-level propositional formulas in a form of
    $(a_{11} \vee a_{12} \vee \cdots \vee a_{1n_1}) \wedge (a_{21} \vee a_{22} \vee \cdots a_{1n_2}) \wedge \ldots (a_{m1} \vee a_{m2} \vee \ \ldots a_{mn_m})$
    where a_{ij} is p or $\neg p$ for a atomic propositional variable p
  - every propositional formula has an equivalent CNF form
    - e.g. $p \vee \neg(\neg(q \vee r) \vee s) \ \leftrightarrow \ \ (p \vee \neg s) \wedge (p \vee q \vee r)$

5. CNF Converter (2/3)

- A propositional formula $\phi \in P$ is specified by the following rules:
  - **a**$n \in P$ for $n \in \mathbb{N}$
  - **(and** $\phi_1\ \phi_2 \ldots \phi_m$**)** for $\phi_i \in P$
  - **(or** $\phi_1\ \phi_2 \ldots \phi_m$**)** for $\phi_i \in P$
  - **(not** ϕ**)** for $\phi \in P$
- A CNF is represented as a list of lines where each line represents a disjunictive clauses of atomic propositional variables and their negations
  - a positive integer n represents a propositional variable $\mathbf{a}n$
  - a negative integer $-n$ represents a negation of a propositional variable $\neg\mathbf{a}n$
- Example
  - Input: `(or a1 (not (or (not (or a2 a3)) a4)))`
  - Output

```
1 -4
1 2 3
```

- Your program must print out an error message if the given input does not follow the syntax

5. CNF Converter (3/3)

1. Define a tree-like data structure to represent a propositional formula
2. Write a recursive algorithm to translate a given text to a formula structure
3. Write a recursive algorithm that converts a given formula to a Negation Normal Form (NNF)
   - a NNF has a negation only at a leaf node (i.e., as $\neg a_n$)
4. Write a recursive algorithm that applies the De Morgan's law to transform a NNF into a CNF

Program Structure

- For Tasks 1 to 4, write each program as a HTML with JavaScript
  - Receive input from TextBox objects
  - Describe how to run the program in the same HTML page

- For Task 5, write a program as a C program running on UNIX/Linux
  - Input and output must be via the standard input and output, respetively
  - You must submit build scripts and README together with source code files
    - Buid script: Bash script, Makefile, Ant, Maven, etc.
    - REAME: instruction/manual on how to build and run your program

Submission

- Deadline: 15 Nov (Fri), 11:59 PM
  - no late submission will be accepted
  - one submission per team

- Each team should submit the five programs and one write-up (report) on the program designs and results
  - Programs: source code files, build script and README
  - Write up: must not exceed 5 pages (single-sided A4)

- Submit all deliverables via Hisnet homework submission repository

Evalution Criteria

- Write up (60 points)
  - Description (45 points)
    - check whether you found all constraints of a solution
    - check whether each constraint is correctly represented as a logic formula
    - check whether you demonstrate the correctness of your programs in a convincing way (e.g., by tests)
    - check whether all descriptions are clear and consistent
  - Discussion (15 points)
    - detailed analysis of results, interesting observations, lessons learned, suggestions, new ideas, etc.
- Tests (40 points)
  - Run each program with several inputs to see whether the results are correct

Notes

- Right after the deadline, an individual homework HW 2 that extends PA 2 will be given
  - You will have 5 days for HW 2
- Right after the deadline, peer evaluation of your team members will follow
- Your submissions may be open to the class and public
- Help desk by TAs will be offered
  - the schedule is TBA