ITP20002-01 Discrete Mathematics

Logic and Proofs

31 Aug, 2018

Chapter 1. Logic and Proofs

- Propositional logic (1.1, 1.2)
- Logical equivalence and satisfiability (1.3)
- Predicate logic (1.4, 1.5)
- Inference (1.6, 1.7)
- Proof basics (1.8, 1.9)

Logic

- Logic is a way to state arguments and to reason with arguments, clearly and correctly

- A logic system has the syntactic and the semantic aspects
  - syntax: symbolic structure of arguments
  - semantics: a relation between symbolic structures and meaning

Proposition

- A proposition is a declarative sentence that is either true or false
  - 1 + 1 = 2
  - *Vancouver is the capital of Canada*
  - ~~1 + 2 + 3~~
  - ~~x + 1 = 2~~

- The negation of *p* for a proposition *p,* denoted as $\neg p$, is the proposition that is true only when *p* is false.

- A compound proposition is formed from existing propositions using logical operators
  - logical operators: negation, disjunction, conjunction, exclusive-or, implication, etc.
  - propositional variable: a variable that represents a proposition

Conditional Statement

- A conditional statement (or implication) $p \rightarrow q$ for propositions p and q is the proposition that is false when p is true and q is false, and $p \rightarrow q$ is true otherwise
  - if you do not take midterm, then you get F
  - if you are in the Handong campus, you are in Pohang
  - if Juan has a smartphone, then 2 + 3 = 5
  - (2 + 3 = 4) → (1 + 2 = 4)

- The converse of $p \rightarrow q$ is $q \rightarrow p$.
- The inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$.
- The contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$.

Propositional Satisfiability

- A compound proposition p is **satisfiable** if there is an assignment of truth values to the propositional variables that makes p true
  - Such assignment is called as a solution

- A compound proposition p is **unsatisfiable** if p is not satisfiable
  - A unsatisfiable proposition is called as contradiction

- A compound proposition p is **valid** if p is true for all assignments
  - A valid proposition is called as tautology
  - E.g., if x = y, then x = y
  - E.g., *I just want to live while I am alive - Bon Jovi*