Appendix E. Canonical correlation analysis (CCA)

CCA aims to quantify the amount of variance that a multivariate predictor (i.e. vector) $\boldsymbol{X}$ can explain in a multivariate response $\boldsymbol{Y}$. One type of CCA useful for hypothesis testing is to find the maximum possible correlation coefficient that can be obtained when the coordinate systems defining $\boldsymbol{X}$ and $\boldsymbol{Y}$ are permitted to mutually rotate.

Consider a response variable $\boldsymbol{Y}$ that describes three orthogonal force components F:

$$\boldsymbol{Y}_j = \begin{bmatrix} F_{1j} & F_{2j} & F_{3j} \end{bmatrix}^\top \tag{E.1}$$

where "1", "2" and "3" represent orthogonal axes and where j indexes a total of J responses. Next consider a predictor variable $\boldsymbol{X}$ that describes the rotations θ about two orthogonal axes at a given joint:

$$\boldsymbol{X}_j = \begin{bmatrix} \theta_{1j} & \theta_{2j} \end{bmatrix}^\top \tag{E.2}$$

where "1" and "2" indicate the two joint axes. The relevant null hypothesis is: $\boldsymbol{X}$ and $\boldsymbol{Y}$ are not linearly related.

To test this hypothesis one needs to assemble three covariance matrices. The first is a (3 $\times$ 3) response covariance matrix $\boldsymbol{W}_{YY}$ which describes variance within and the co-variation between the three force components (see Appendix D). The second is a (2 $\times$ 2) predictor covariance matrix $\boldsymbol{W}_{XX}$ which describes the variance and covariance of the two joint angles. The third is a (2 $\times$ 3) predictor-response covariance matrix $\boldsymbol{W}_{XY}$ which describes how each of the predictor variables co-varies with each of the response variables.

The predictor-response covariance matrix $\boldsymbol{W}_{XY}$ is relevant to the null hypothesis because it embodies the strength of linear correlation between $\boldsymbol{X}$ and $\boldsymbol{Y}$. For completion, in the example above $\boldsymbol{W}_{XY}$ has six elements, corresponding to:

1. The linear correlation between θ_1 and F_1
2. The linear correlation between θ_1 and F_2
3. The linear correlation between θ_1 and F_3
4. The linear correlation between θ_2 and F_1
5. The linear correlation between θ_2 and F_2
6. The linear correlation between θ_2 and F_3

Initially these correlations refer only to $\boldsymbol{X}$'s and $\boldsymbol{Y}$'s original coordinate systems. Since arbitrary coordinate systems can bias non-directed hypothesis testing (Appendix B), we must allow the coordinate systems to rotate in order to most objectively test our null hypothesis.

One CCA solution is to choose the $\boldsymbol{X}$ and $\boldsymbol{Y}$ coordinate systems that mutually maximize a single correlation coefficient. The logic is that all other coordinate systems underestimate correlation strength. In other words, as the coordinate systems rotate the elements of $\boldsymbol{W}_{XY}$ change, and one (not necessarily unique) coordinate system combination maximizes an element of $\boldsymbol{W}_{XY}$. CCA solves this problem efficiently using the maximum eigenvalue of the canonical correlation matrix (Eqn.7, main manuscript).

As an aside, we note that the K=2 model in the main manuscript is equivalent to a K=1 model (i.e. only a running speed regressor) because only one (diagonal) element of $\boldsymbol{W}_{XX}$ is non-zero. For generalizability the main manuscript treats CCA in its $K > 1$ form.