Appendix A. Scalar vs. vector analysis of vector data

The purpose of this Appendix is to explain the discrepancy between the main manuscript's scalar and vector results (Figs. 1&2, respectively). To this end we constructed and analyzed two simulated datasets to consider alongside the main manuscript's results. We caution readers that these datasets were generated to highlight particular concepts of vector statistics.

Two simulated datasets are depicted in Fig.S1a & b below, and the numerical data, along with scalar and vector hypothesis testing results, are provided in Tables S1 & S2.

For Simulated Dataset 1 (Fig.S1a, Table S1), the two-sample Hotelling's T^2 (vector) test reaches significance (p=0.019) but scalar testing does not (r_x: p=0.162, r_y: p=0.129). Simulated Dataset 2 (Fig.S1b, Table S2) produces the opposite result: scalar tests reach significance[1] (r_x: p=0.039, r_y: p=0.042) but the vector test does not (p=0.058).

Scalar and vector result disagreement can be explained by two factors: (a) the resultant vector effect, and (b) covariance. The former refers to Pythagoras' theorem:

$$|\Delta\overline{\boldsymbol{r}}|^2 = \Delta\overline{r}_x^2 + \Delta\overline{r}_y^2 \tag{A.1}$$

where the magnitude of the resultant is clearly larger than the magnitude of either its components — except in the experimentally unlikely cases of $\Delta\overline{r}_x$=0 and/or $\Delta\overline{r}_y$=0. This implies that separate vector component analysis may underestimate group differences. The latter factor (covariance) refers to $\boldsymbol{W}$, the matrix representing the variance-within and and correlation-between vector components. A two-component covariance matrix, just like a two-component inertia matrix, defines the size and orientation of an ellipse (Fig.S1). If the vector components are correlated (i.e. the off-diagonal components of $\boldsymbol{W}$ are not zero), and the ellipse is not a circle (i.e. the main diagonal components are not equivalent) then variance is direction-dependent,

[1] The Dataset 2 scalar tests assume that just one scalar (either r_x or r_y) was extracted and tested. If both r_x and r_y were extracted and tested, then a correction for multiple tests would be necessary. A Šidák correction, for example, would require these p values to be less than 0.0253, and would thereby produce agreement between the scalar and vector results. However, since single vector components are often extracted and analyzed, without also considering the other vector components at that instant, here we consider the uncorrected case. The main point is that vector hypothesis testing can disagree with scalar hypothesis testing when one fails to consider interaction amongst vector components

and variance may be small or large in the direction of $\Delta\overline{\boldsymbol{r}}$.

This implies that scalar analysis of vector data is generally non-objective because a vector's components are coordinate system (CS)-dependent. Whereas $|\Delta\overline{\boldsymbol{r}}|$ and the variance in the direction of $\Delta\overline{\boldsymbol{r}}$ are invariant to CS changes, the components r_x and r_y and their variances change systematically with the CS. From Fig.3 (main manuscript) it is clear that even small CS rotations can reverse the results of scalar hypothesis testing.

Considering the specific simulated datasets: scalar analyses of Dataset 1 exhibit Type II statistical error because the vector data contain a relatively strong effect (p=0.019), but scalar analyses fail to detect this effect (p>0.129). On the other hand, scalar analyses of Dataset 2 exhibit Type I error because the vector variance/covariance is large enough to drown the resultant vector effect.

Considering next the main manuscript's experimental data, it becomes apparent why the r_y scalar analyses (Fig.1e) disagree with the vector analyses (Fig.2a): although the r_y effect appears to be strong in the scalar data (Fig.1c), this effect is actually quite small with respect to the vector variance/covariance (Fig.S1d). It is worth noting that this disagreement between r_y scalar analyses and vector analyses would remain even if a different statistical model had been adopted (Appendix D).

Last, even though the main manuscript's r_x scalar results (Fig.1d) agree with the vector results (Fig.2), it is worth noting that this is rather lucky, because the r_x and r_y components are highly correlated (Fig.S1c). Importantly, this correlation makes the r_x effect quite sensitive to small coordinate system rotations (Fig.3). Since vector analyses are invariant to such rotations, and since scalar analyses can easily produce Type I (Table S2) or Type II error (Table S1), it is clear that vector analyses offer a more objective approach to vector data analysis.

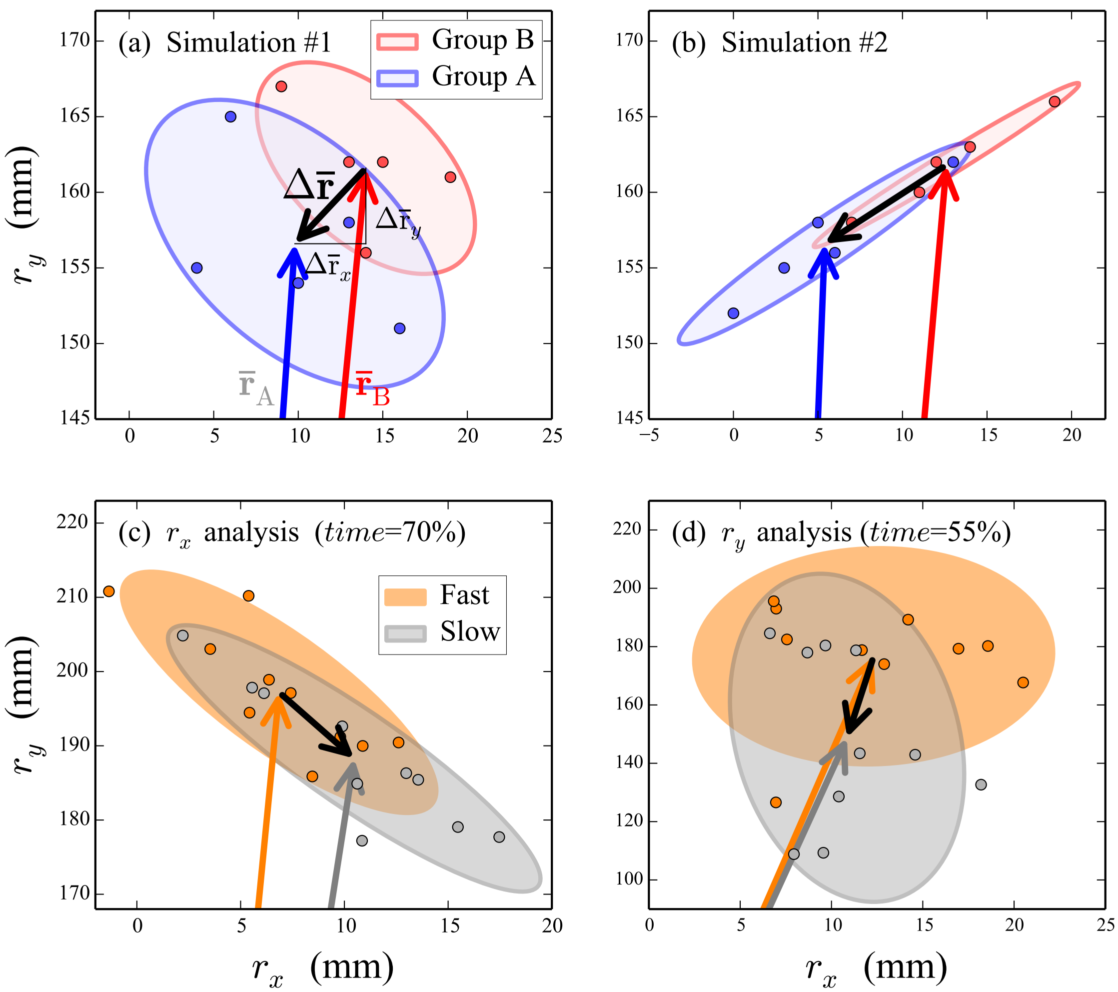

Figure S1: Graphical depiction of disagreement between scalar and vector analyses. Small circles depict individual responses. Thick arrows depict vector means and differences, and ellipses depict vector component covariance. Covariance ellipse radii are scaled to two principal axis standard deviations. (a) First simulated dataset (Table S1), exhibiting a vector effect, but no scalar component effects. (b) Second simulated dataset (Table S2), exhibiting scalar component effects, but no vector effect. (c) Experimental data at time=70%, where a vector effect (Fig.2) but no scalar r_x effect (Fig.1) were observed. (d) Experimental data at time=55%, where a scalar r_y effect but no vector effect was observed.

Table S1: First simulated COP dataset (units: mm), exhibiting a vector effect but no scalar component effects. (a) Observations $\boldsymbol{r}$=$[r_x,\ r_y]^\top$. (b)-(d) Scalar testing. (e)-(g) Vector testing.

		Group A	Group B	Inter-Group
	(a) COP data	$\boldsymbol{r}_{A1} = [13,\ 158]^\top$ $\boldsymbol{r}_{A2} = [6,\ 165]^\top$ $\boldsymbol{r}_{A3} = [16,\ 151]^\top$ $\boldsymbol{r}_{A4} = [10,\ 154]^\top$ $\boldsymbol{r}_{A5} = [4,\ 155]^\top$	$\boldsymbol{r}_{B1} = [9,\ 167]^\top$ $\boldsymbol{r}_{B2} = [15,\ 162]^\top$ $\boldsymbol{r}_{B3} = [13,\ 162]^\top$ $\boldsymbol{r}_{B4} = [19,\ 161]^\top$ $\boldsymbol{r}_{B5} = [14,\ 156]^\top$	
Univariate	(b) Means	$(\overline{r_x})_A = 9.8$ $(\overline{r_y})_A = 156.6$	$(\overline{r_x})_B = 14.0$ $(\overline{r_y})_B = 161.6$	$\Delta\overline{r_x} = 4.2$ $\Delta\overline{r_y} = 5.0$
	(c) St.dev.	$(s_x)_A = 4.9$ $(s_y)_A = 5.3$	$(s_x)_B = 3.6$ $(s_y)_B = 3.9$	$s_x = 4.3$ $s_y = 4.7$
	(d) t tests			t_x=1.540; p_x=0.162 t_y=1.693; p_y=0.129
Vector	(e) Means	$\overline{\boldsymbol{r}}_A = [9.8,\ 156.6]^\top$	$\overline{\boldsymbol{r}}_B = [14.0,\ 161.6]^\top$	$\Delta\overline{\boldsymbol{r}} = [4.2,\ 5.0]^\top$
	(f) Covariance	$\boldsymbol{W}_A = \begin{bmatrix} 24.2 & -13.4 \\ -13.4 & 28.3 \end{bmatrix}$	$\boldsymbol{W}_B = \begin{bmatrix} 13.0 & -7.5 \\ -7.5 & 15.3 \end{bmatrix}$	$\boldsymbol{W} = \begin{bmatrix} 18.6 & -10.4 \\ -10.4 & 21.8 \end{bmatrix}$
	(g) T^2 test			T^2=8.675; p=0.019

Table S2: Second simulated COP dataset (units: mm), exhibiting component scalar effects*, but not vector effect. *Note: these scalar tests assume that just one scalar (either r_x or r_y) was extracted and tested. If both r_x and r_y were extracted and tested, a correction for multiple tests would be necessary (see text).

		Group A	**Group B**	**Inter-Group**
	(a) COP data	$\boldsymbol{r}_{A1} = [6,\ 156]^\top$ $\boldsymbol{r}_{A2} = [13,\ 162]^\top$ $\boldsymbol{r}_{A3} = [0,\ 152]^\top$ $\boldsymbol{r}_{A4} = [3,\ 155]^\top$ $\boldsymbol{r}_{A5} = [5,\ 158]^\top$	$\boldsymbol{r}_{B1} = [19,\ 166]^\top$ $\boldsymbol{r}_{B2} = [12,\ 162]^\top$ $\boldsymbol{r}_{B3} = [14,\ 163]^\top$ $\boldsymbol{r}_{B4} = [11,\ 160]^\top$ $\boldsymbol{r}_{B5} = [7,\ 158]^\top$	
Univariate	(b) Means	$(\overline{r_x})_A = 5.4$ $(\overline{r_y})_A = 156.6$	$(\overline{r_x})_B = 12.6$ $(\overline{r_y})_B = 161.8$	$\Delta\overline{r_x} = 7.2$ $\Delta\overline{r_y} = 5.3$
	(c) St.dev.	$(s_x)_A = 4.8$ $(s_y)_A = 3.7$	$(s_x)_B = 4.4$ $(s_y)_B = 3.0$	$s_x = 4.6$ $s_y = 3.4$
	(d) t tests			t_x=2.467; p_x=0.039 t_y=2.425; p_y=0.042
Vector	(e) Means	$\overline{\boldsymbol{r}}_A = [5.4,\ 156.6]^\top$	$\overline{\boldsymbol{r}}_B = [12.6,\ 161.8]^\top$	$\Delta\overline{\boldsymbol{r}} = [7.2,\ 5.3]^\top$
	(f) Covariance	$\boldsymbol{W}_A = \begin{bmatrix} 24.2 & -13.4 \\ -13.4 & 28.3 \end{bmatrix}$	$\boldsymbol{W}_B = \begin{bmatrix} 13.0 & -7.5 \\ -7.5 & 15.3 \end{bmatrix}$	$\boldsymbol{W} = \begin{bmatrix} 21.3 & -15.2 \\ -15.2 & 11.5 \end{bmatrix}$
	(g) T^2 test			T^2=4.882; p=0.058