Appendix B. Convolution and random 1D Gaussian fields

This Appendix describes one procedure for generating smooth 1D Gaussian fields. Consider the functions $f(q)$ and $g(q')$ which are defined on one-dimensional (1D) domains q and q', respectively (Fig.B1a,b). Convolution is a procedure which slides $g(q')$ over $f(q)$ (Fig.B1c) to yield an 'overlapping area' function $h(q)$ (Fig.B1d). Formally convolution is expressed as:

$$f(q) * g(q) = h(q) \tag{B.1}$$

Convolving $f(q)$ with a Gaussian kernel — also called "Gaussian filtering" — yields a similar but smoother result (Fig.B2). In the context of the main paper the functions $f(q)$, $g(q')$ and $h(q)$ represent the experimental data, a smoothing kernel, and the smoothed data, respectively.

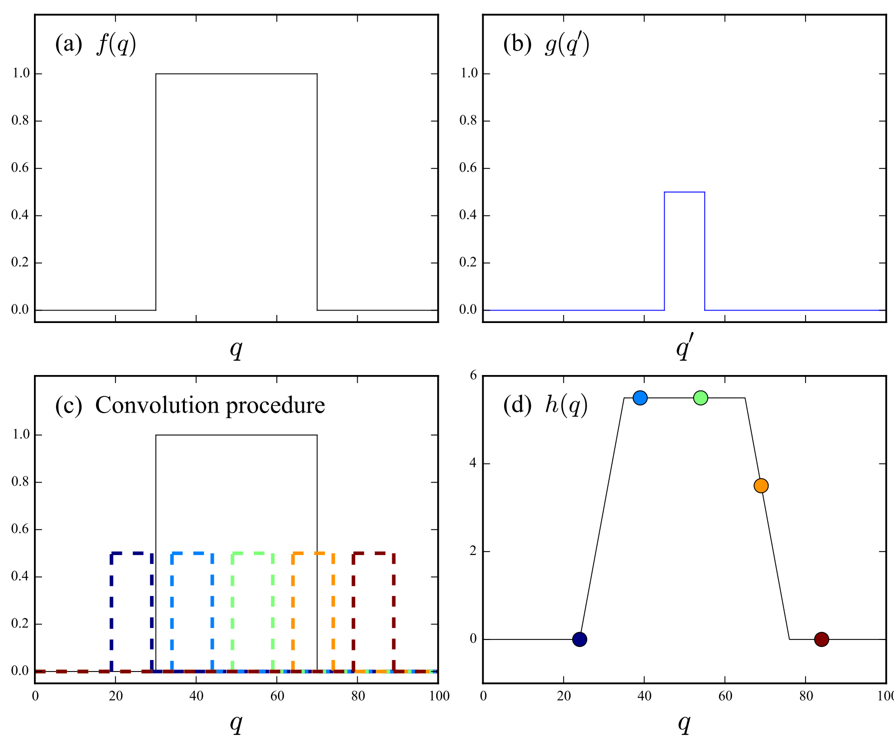

Figure B1: Convolution of two square waves. (a) Stationary function. (b) Moving function. (c) Depiction of $g(q')$ moving across $f(q)$. (d) Convolution result: colored circles depict the overlapping area between $f(q)$ and $g(q')$ when the right edge of $g(q')$ reaches the position q.

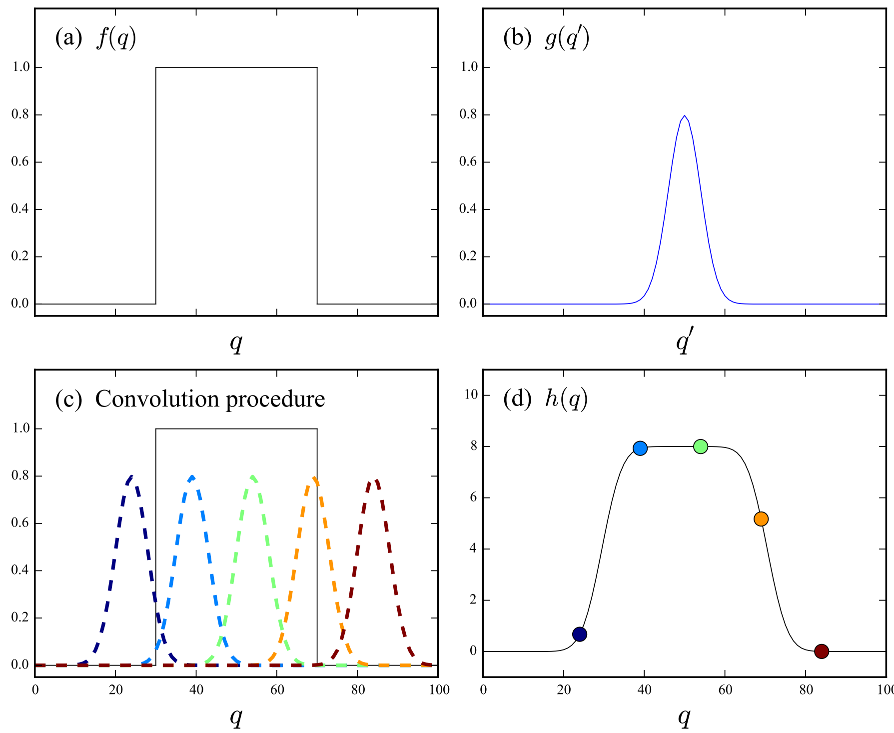

Figure B2: Convolution of a square wave with a Gaussian pulse.

When the data $f(q)$ are more like an experimental time series but consist of completely uncorrelated random Gaussian values (Fig.B3a) then convolving with a Gaussian kernel (Fig.B3b,c) yields a smooth Gaussian random field (Adler & Taylor, 2007) (Fig.B3d). The broader the smoothing kernel, the smoother the resulting random field (Fig.B4). Kernel breadth is parameterized by its full-width-at-half-maximum (FWHM) (Appendix A) and the FWHM parameter is central to 1D probability results (Friston et al. 2007).

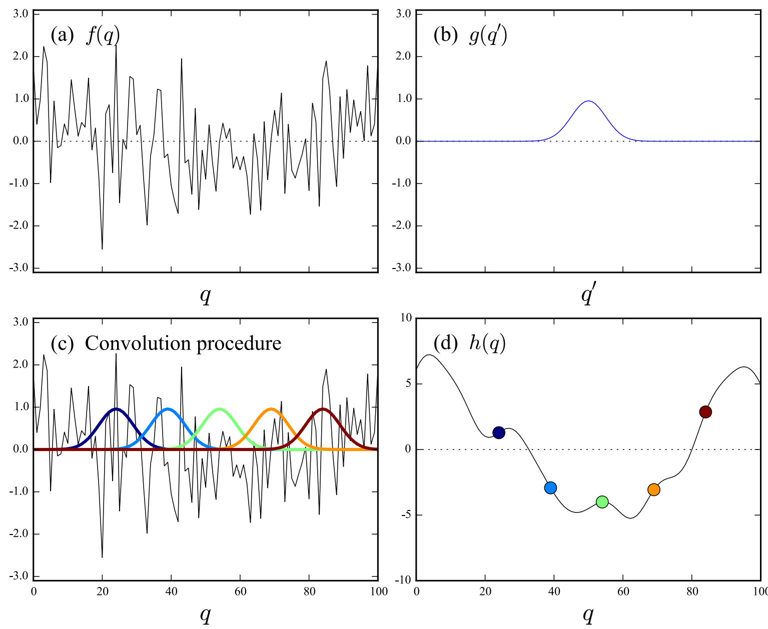

Figure B3: Convolution of uncorrelated Gaussian data (a) with a Gaussian kernel (b–c) yields a Gaussian random field (d).

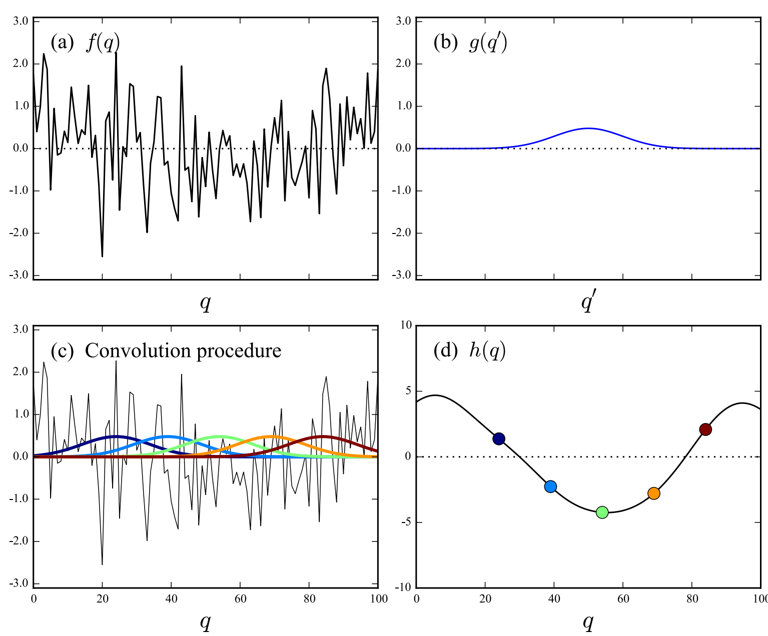

Figure B4: Identical to Fig.B3, but with a broader kernel (b) which yields a smoother random field (d).