Appendix D. Estimating 1D residual smoothness

This Appendix summarizes the smoothness estimation procedures of Kiebel et al. (1999). While that paper describes the procedure for 3D data, the procedure is conceptually identical for 1D data. The 1D domain could be time, space or any other continuous variable, but for writing convenience we shall consider only 1D temporal trajectories.

The ultimate goal is to estimate the temporal smoothness of experimentally observed 1D residuals (Fig.1d, main manuscript) using a single scalar parameter: the FWHM (Appendix A). That single FWHM value represents the breadth of a Gaussian kernel which, when convolved with uncorrelated Gaussian time series (Appendix B) would yield random trajectories which have the same smoothness as the observed residuals. The procedure is depicted in Fig.D1 and is described in detail below.

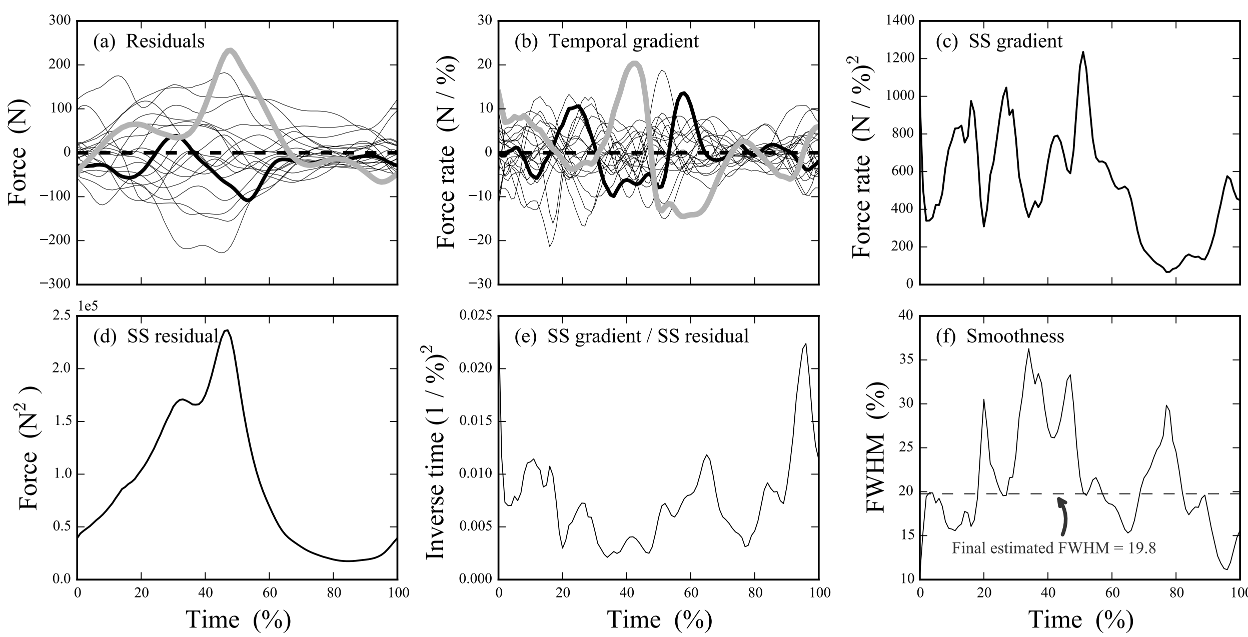

Figure D1: Overview of the FWHM estimation procedure of Kiebel et al. (1999). The residual data (a) are from the cycling normal pedal force of Kautz et al. (1991). SS = sum-of-squares.

Imagine that there are J residual trajectories which are each sampled at Q discrete time points and that the jth residual trajectory is denoted "$r_j(q)$". The first step is to compute the temporal gradient at each point q for each of the J trajectories (Fig.D1b):

$$r'_j(q) \equiv \frac{dr_j(q)}{dq} \tag{D.1}$$

The gradients can be estimated most easily using the differences between adjacent samples (i.e. $r_j(q+1) - r_j(q)$), but could also be done using alternative procedures. Practically, differences in gradient estimation procedures will likely have negligible effects on the ultimately estimated FWHM value. Regardless of the procedure, gradient estimation yields a total of J gradient trajectories.

Next, the true gradient magnitude at point q is estimated as the sum-of-squares of the observed gradients (Fig.D1c):

$$SS[r'](q) = \sum_{j=1}^{J} \left(r'_j(q) \right)^2 \tag{D.2}$$

In order to normalize across datasets and experiments the sum-of-squared residual values is also needed (Fig.D1d):

$$SS[r](q) = \sum_{j=1}^{J} \left(r_j(q) \right)^2 \tag{D.3}$$

The estimated gradients are then normalized by the residual magnitudes (Fig.D1e):

$$\lambda(q) = \frac{SS[\lambda](q)}{SS[r](q)} \tag{D.4}$$

Last, the FWHM trajectory (Fig.D1f) is given as:

$$\mathrm{FWHM}(q) = \sqrt{\frac{4 \log 2}{\lambda(q)}} \tag{D.5}$$

where an unbiased estimate of the true FWHM is simply the mean of the FWHM trajectory:

$$\hat{\text{FWHM}} = \frac{1}{Q} \sum_{q=1}^{Q} \text{FWHM}(q) \tag{D.6}$$

Note that estimated FWHM values are generally different at each point q in the 1D field (Fig.D1f). Smoothness which is non-constant across the field is termed 'anisotropic'. In order to deal with this issue let's consider 'apparent' vs. 'real' anisotropy. If the anisotropy is merely apparent, then Eqn.D.6 is valid. To understand why, consider a 0D random variable x which comes from a population with constant variance. Although the population variance is constant, random samples of x will yield different sample variances. Nevertheless, sample variance is an unbiased estimate of the true population variance. Similarly, even when the true FWHM is constant, randomly sampled 1D data will yield sample FWHM estimates which vary not only from sample to sample but also from point to point in the 1D field. Thus the mean FWHM value is an unbiased estimate of the true population FWHM when the anisotropy is merely apparent. Note also this FWHM estimation procedure has been validated for 1D data elsewhere (Pataky, 2015).

'True anisotropy' is a theoretically less trivial problem which exists when different field regions actually do have different population-level smoothnesses. As an example, consider impacts in running ground reaction forces: the initial impact phase is generally associated with higher signal frequencies than the midstance and push-off phases, so in this situation the true population FHWM is likely different in the different phases. There is fortunately an easy solution to the problem (Worsley et al. 1999). If there are Q points in the field, one simply computes the field length Q' for which smoothness is isotropic. Since the procedures are validated in Worsley et al. (1999), and since this anisotropy correction has no effect on the main paper's conclusions regarding 0D vs. 1D false positives, we leave the issue of anisotropic smoothness for future projects where assuming isotropic smoothness may have less trivial effects on analyses' results.