Appendix D Probability density functions (PDFs)

A PDF is a continuous function $f(x)$ which, when integrated over an interval $[x_0,\ x_1]$, specifies the probability that a random variable x adopts a value in that interval:

$$\mathrm{P}(x_0 < x < x_1) = \int_{x_0}^{x_1} f(x) dx \tag{D.1}$$

The probability that x adopts a specific value $\hat{x}$ is zero because there are an infinite number of other values it could adopt. The probability that x lies in the interval $[x_0,\ x_1]$ is at least zero and at most one. All PDFs additionally share the trivial constraint that x lies in the interval $[-\infty,\ \infty]$. These three constraints can be expressed as follows:

$$\begin{aligned} P(x = \hat{x}) &= 0 \\ 0 \leq P(x_0 < x < x_1) &\leq 1 \\ P(-\infty < x < \infty) &= 1 \end{aligned}$$

The key probability for classical hypothesis testing is the survival function — the probability that x exceeds (or 'survives') an arbitrary threshold u:

$$\mathrm{P}(x > u) = \int_{u}^{\infty} f(x) dx \tag{D.2}$$

When Eqn.D.2 is set to α, then u becomes a "critical threshold"; an experimentally observed value $\hat{x}$ which exceeds this threshold leads to null hypothesis rejection.

Random Field Theory (RFT) (Adler and Taylor, 2007) provides the foundation for generalizing Eqn.D.2 to the case of Gaussian nD continua. An important RFT probability is:

$$\mathrm{P}(x_{\max} > u) = \int_{u}^{\infty} f(x) dx \tag{D.3}$$

where $x_{\max}$ is the maximum continuum value. For classical hypothesis testing on 1D continua, setting Eqn.D.3 to α and solving for u yields the critical threshold for the null hypothesis rejection decision.