

Appendix B Functional data analysis and random field theory

Functional data analysis (FDA) (Ramsay and Silverman 2005) emerged in the 1990s as a tool for statistically analyzing one-dimensional continua or “functions”. By regarding experimentally sampled continua as continuous functions, FDA shows that experimental data can be well-approximated by a set of mathematically precise basis functions, including for example: splines and Fourier series. Representing the data in this manner opens up a wide range of analysis possibilities for describing continua, covariance between continua, etc. Although FDA was initially developed primarily as an exploratory tool of 1D continuum variance, over the years it has expanded to a wide array of statistical uses including classical hypothesis testing in arbitrary experimental designs through a variety of inference techniques.

Random field theory (RFT) (Adler and Taylor 2007) was initially developed in the 1970s to extend the (0D) Gaussian distribution to n -dimensional continua with arbitrary geometrical bounds. RFT shows, for example, how smooth 1D Gaussian continua exhibit particular geometric features (like maximum continuum height) with known probability. Statistical Parametric Mapping (SPM) emerged in the 1990s to apply RFT to experimentally measured continua (Friston et al. 2007). In the case of unbroken 1D continua, SPM estimates just one parameter more than is estimated for common 0D analyses — the ratio of continuum length to smoothness — then uses RFT to make probabilistic conclusions, like the probability that smooth 1D Gaussian data will yield a t continuum which reaches a height of 3.0 in a two-sample experiment. Directly related to classical hypothesis testing, SPM can use RFT to compute the critical height t^* above which only $\alpha\%$ of t continua would reach if those t continua were produced by smooth 1D Gaussian continua in an infinite number of identical experiments.

From a classical hypothesis testing perspective for 1D data, there is thus only one difference between FDA and RFT. Whereas FDA inference procedures are widely flexible, with a varying number of parameters, RFT inference is based on a single parameter: the continuum length-to-smoothness ratio. Since they can both describe random 1D continua, they may be regarded as equivalent for the purposes of the present paper. The main manuscript focusses on separate issues: 0D vs. 1D, parametric vs. non-parametric, and confidence interval vs. hypothesis testing procedures. While we could have used FDA address these issues, we opted for RFT simply because we find RFT easier to describe.