

# Part 3

SPM & SnPM

# STATISTICAL <sup>non</sup>PARAMETRIC MAPPING



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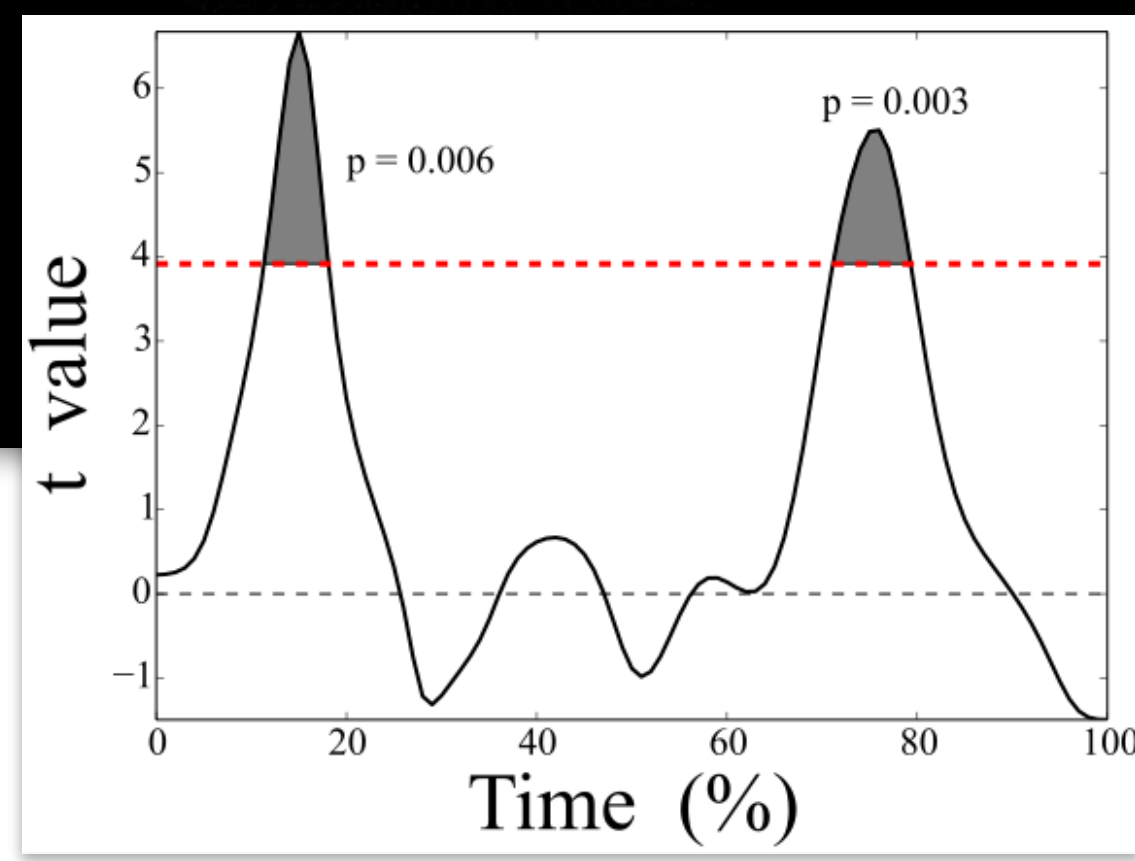
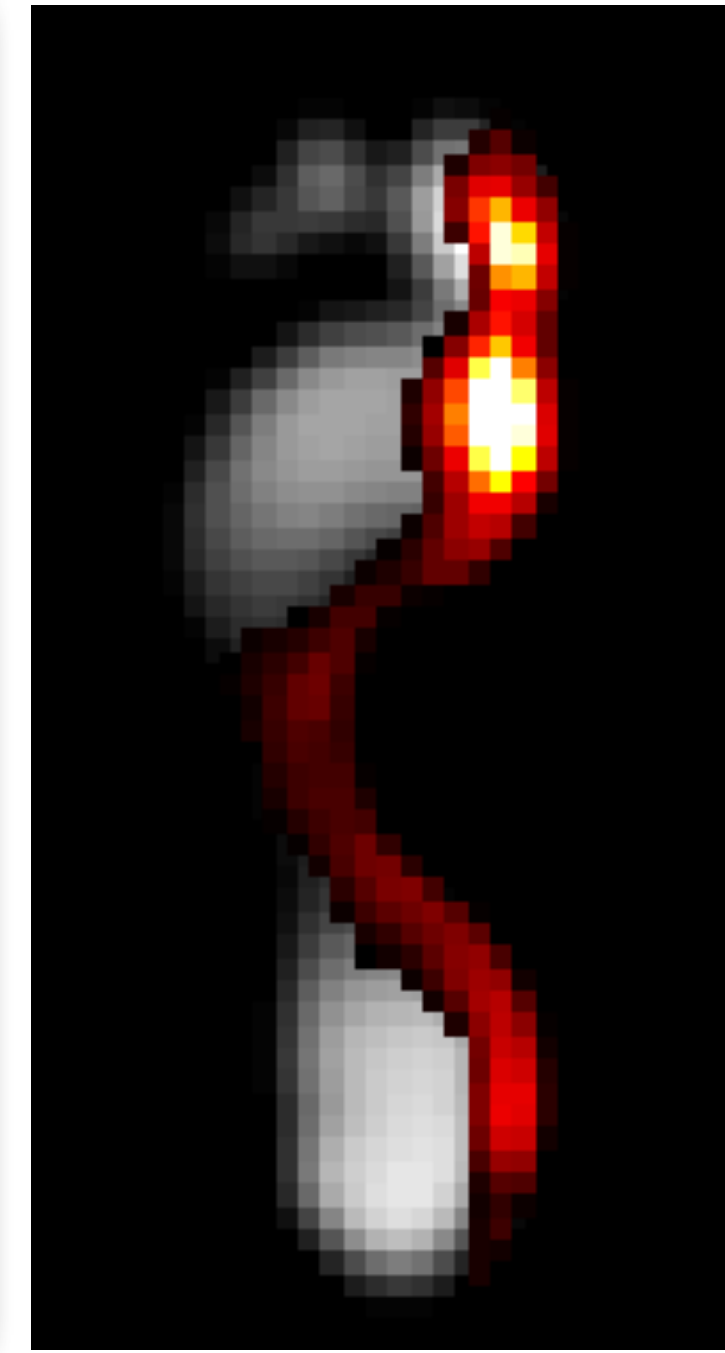
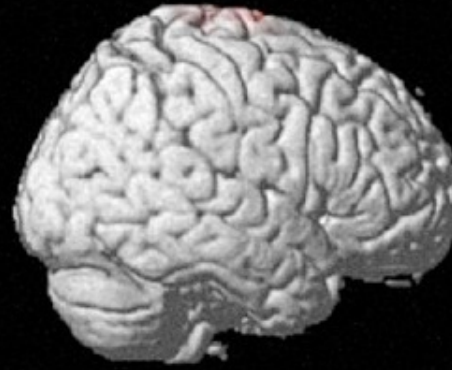
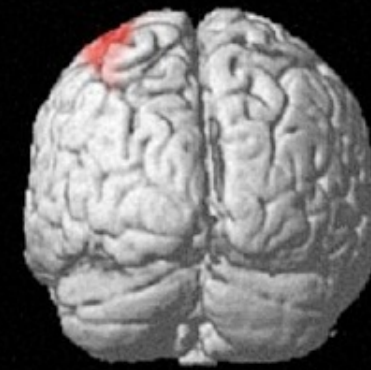
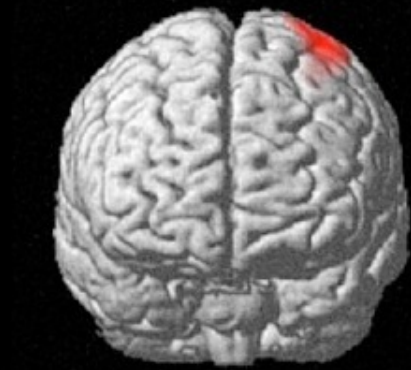
Mark Robinson

Institute for Sport and  
Exercise Sciences

# Overview

- Historical context
- Theory
- Example t tests (parametric & non-parametric)
- Generalizability (regression, ANOVA, etc.)

# Historical context



# A brief history of SPM

1976 Adler & Hasofer, Annals of Prob.

1990 Friston et al. J Cerebral Blood Flow

1995 Friston et al. Human Brain Mapping

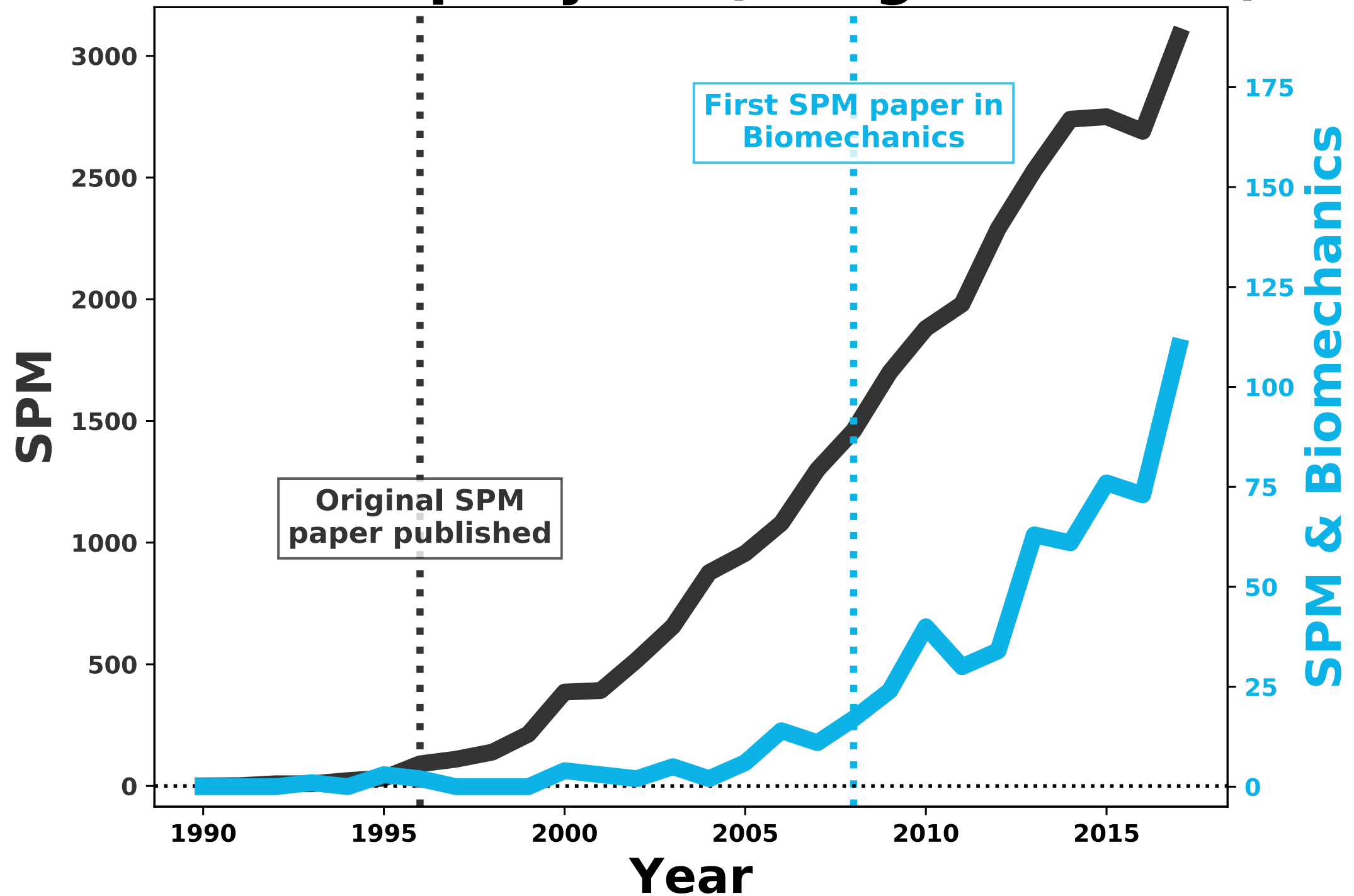
9140 citations  
H-index: 211  
i10-index: 811

2004 Worsley et al. NeuroImage

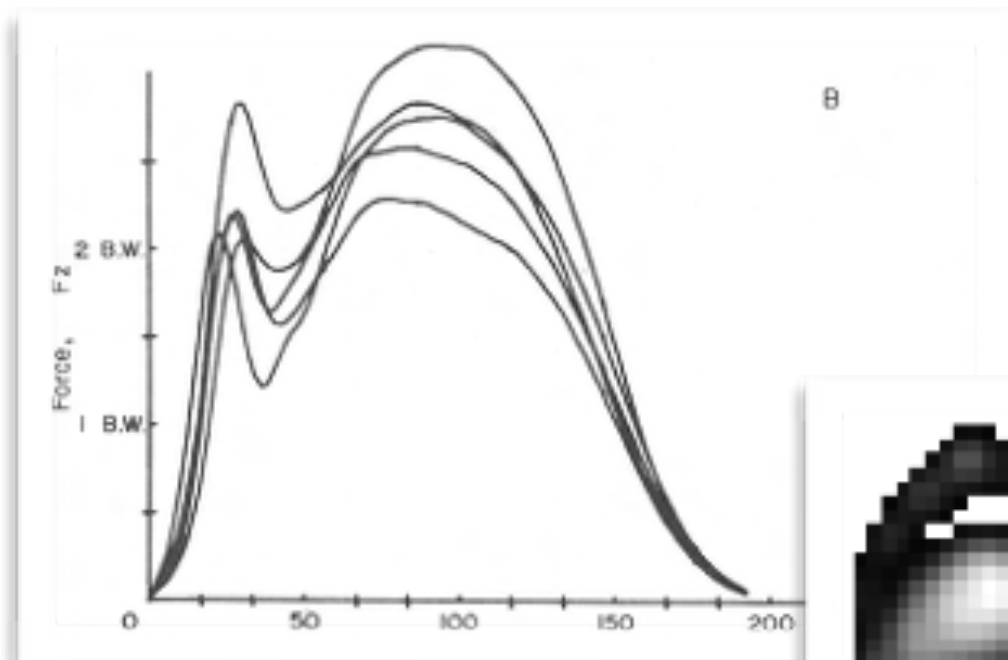
2008 Pataky et al. J Biomech

2009 Li et al. Bone 44: 596-602

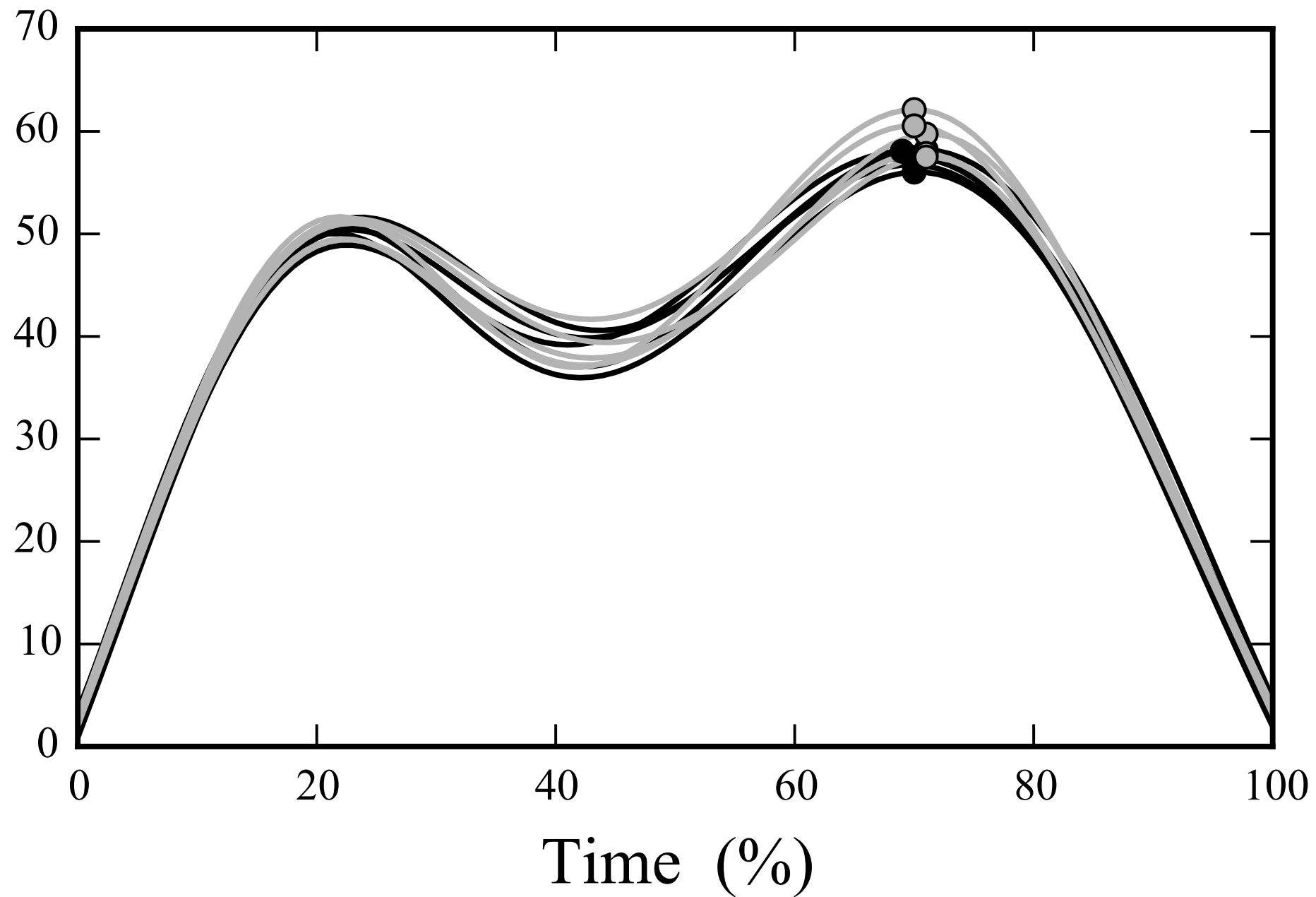
# Citations per year (Google Scholar)



# $n$ -D continua







**By definition:**  
**Random effects occur in random places**

Applied  
Randomness

0D

Scalar

Vector

Gaussian

Multivariate  
Gaussian

t tests  
regression  
ANOVA

T<sup>2</sup> tests  
CCA  
MANOVA

*n*D

Scalar

Vector

**Random Field  
Theory**

**SPM**

# Theory

Demo



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# *Journal of Statistical Software*

July 2016, Volume 71, Issue 7.

doi: 10.18637/jss.v071.i07

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## **rft1d: Smooth One-Dimensional Random Field Upcrossing Probabilities in Python**

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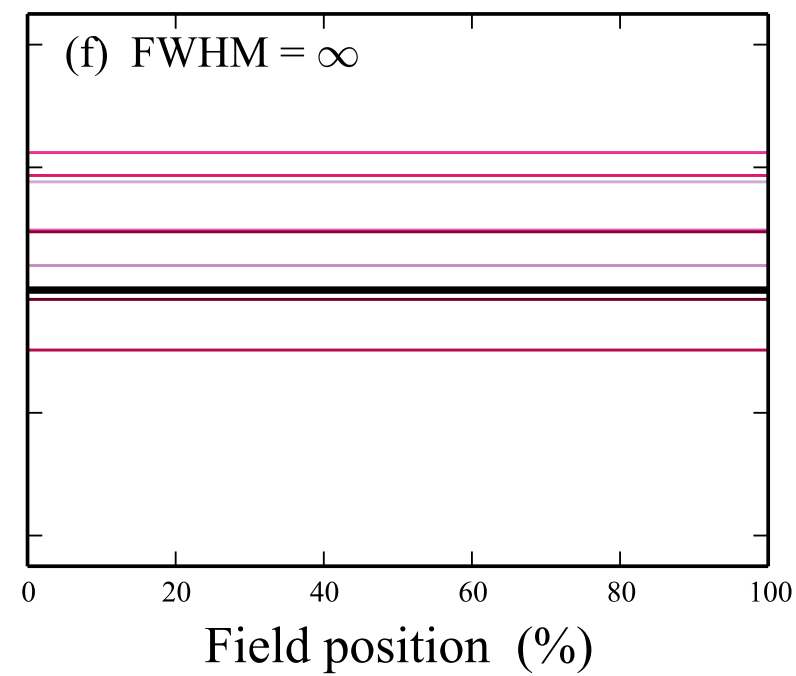
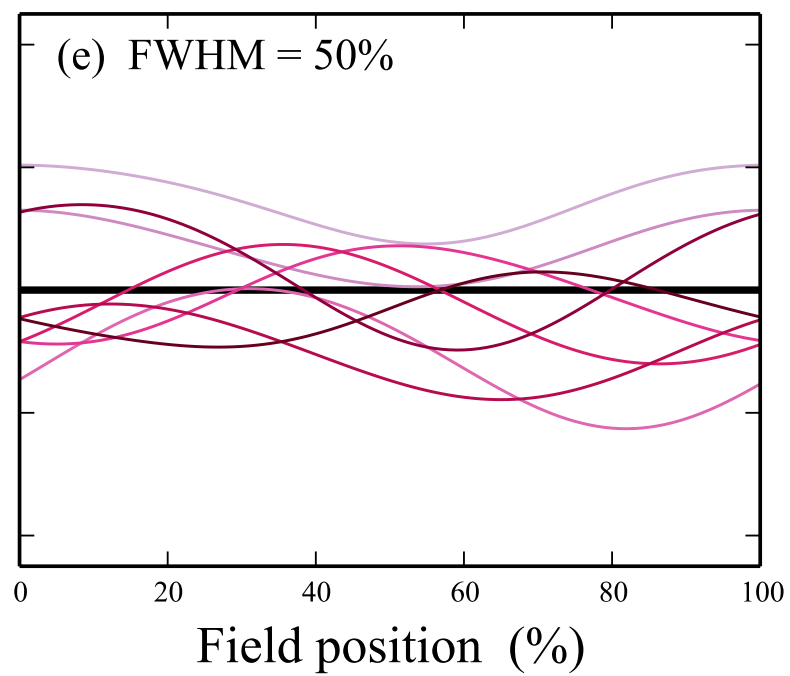
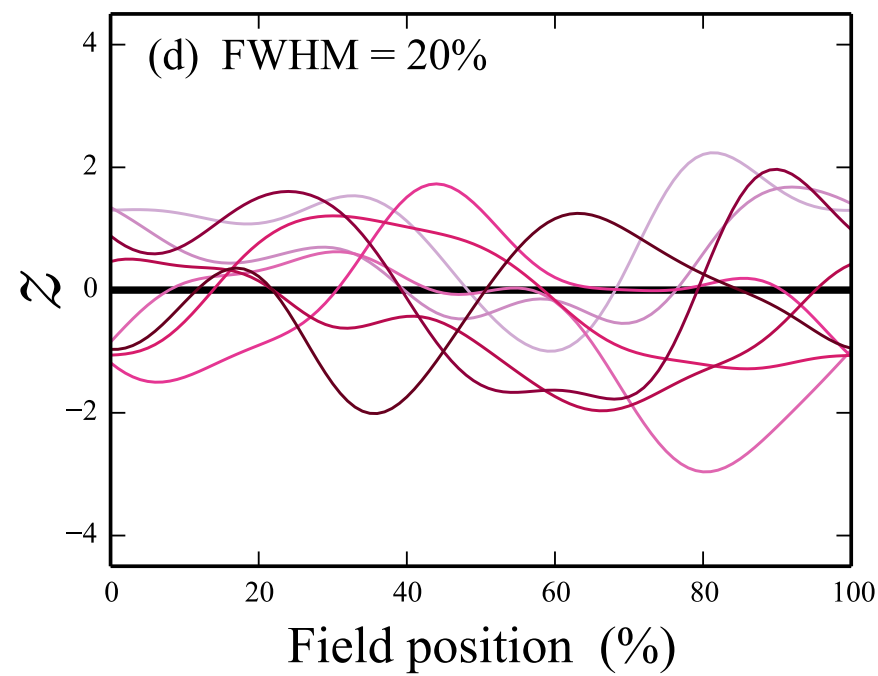
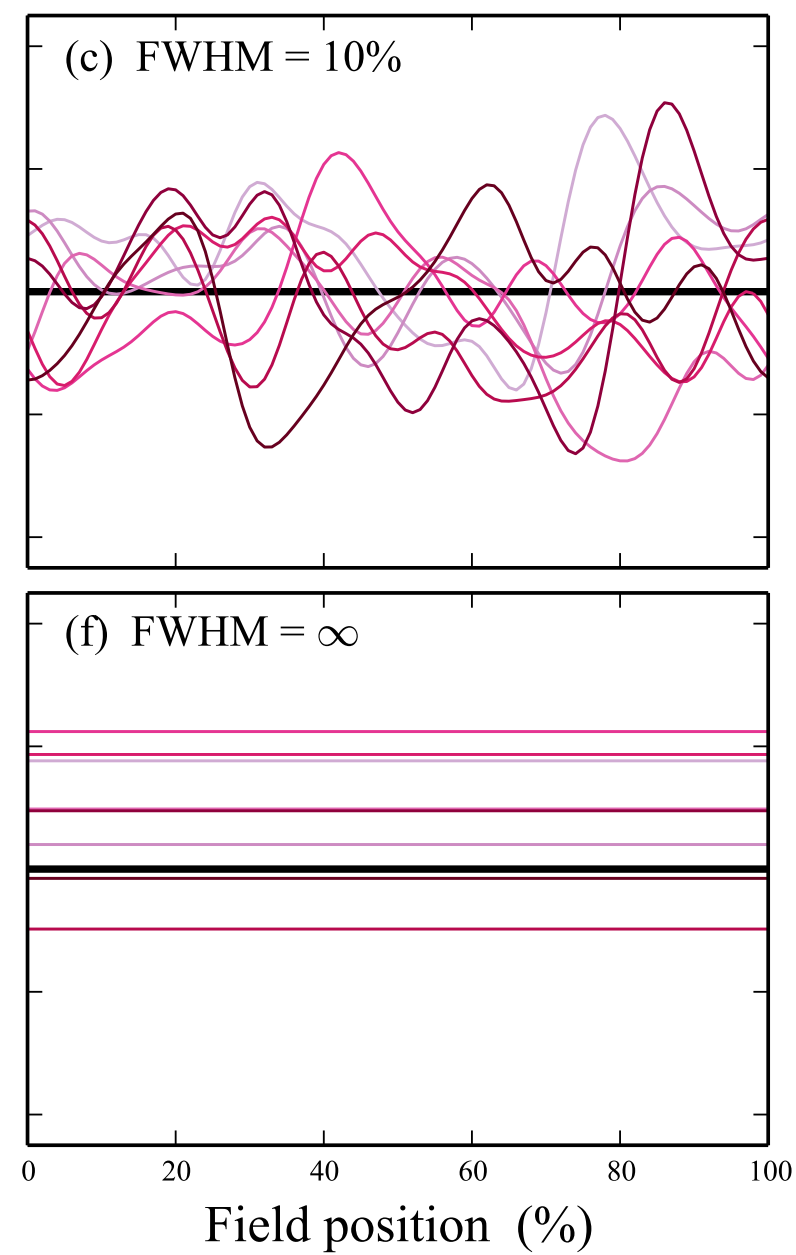
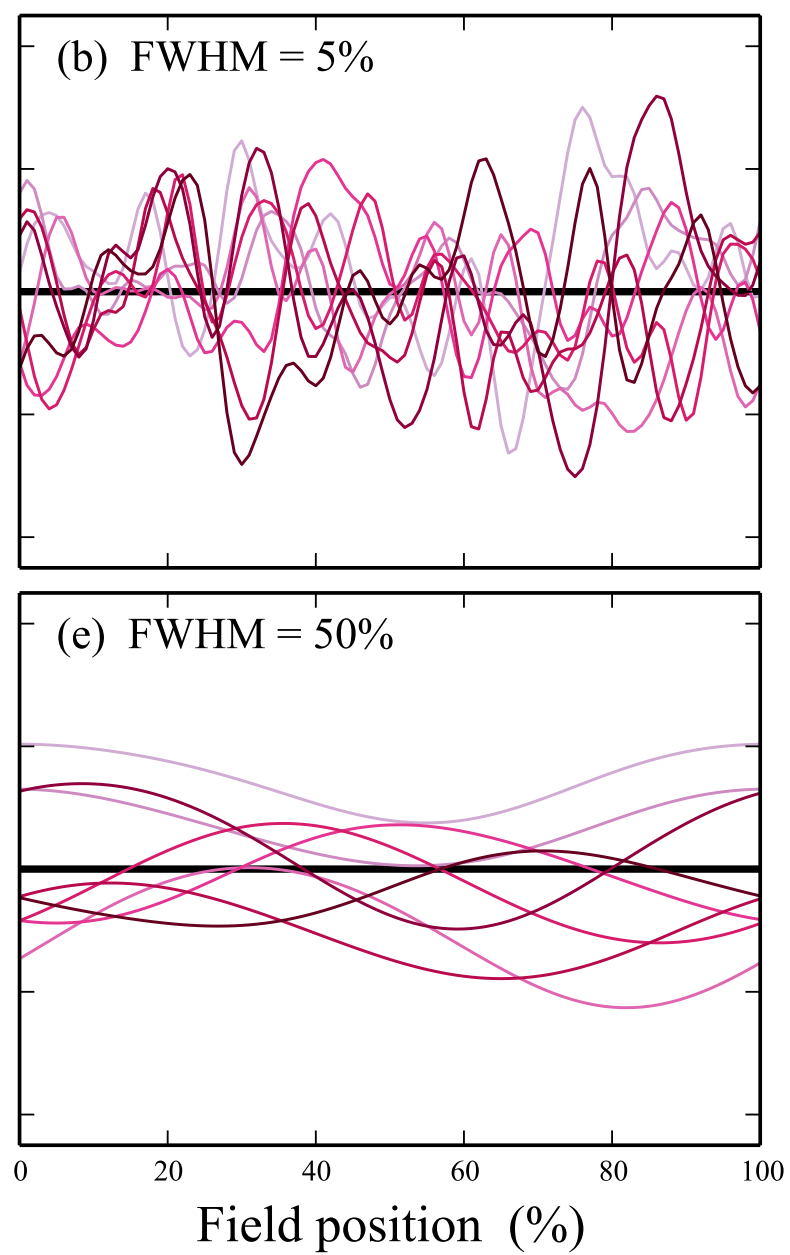
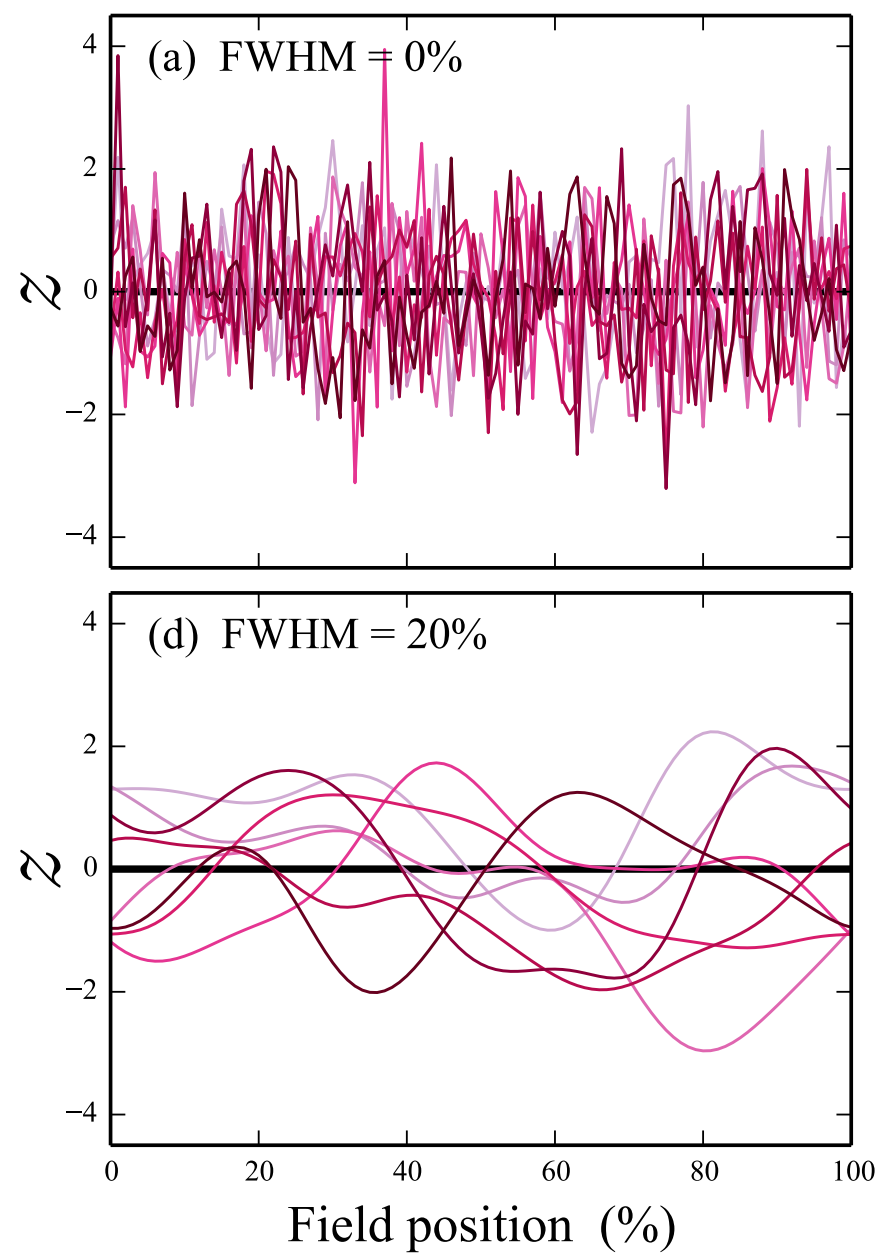
### **Abstract**

Through topological expectations regarding smooth, thresholded  $n$ -dimensional Gaussian continua, random field theory (RFT) describes probabilities associated with both the field-wide maximum and threshold-surviving upcrossing geometry. A key application of RFT is a correction for multiple comparisons which affords field-level hypothesis testing for both univariate and multivariate fields. For unbroken isotropic fields just one parameter in addition to the mean and variance is required: the ratio of a field's size to its smoothness. Ironically the simplest manifestation of RFT (1D unbroken fields) has rarely surfaced in the literature, even during its foundational development in the late 1970s. This Python package implements 1D RFT primarily for exploring and validating RFT expectations, but also describes how it can be applied to yield statistical inferences regarding sets of experimental 1D fields.

*Keywords:* random field theory, Gaussian random fields, multivariate analysis, time series, continuum analysis.

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rft1d



rft1d

# Analytical Distributions

- $Z$

- $t$

- $F$

- $\chi^2$

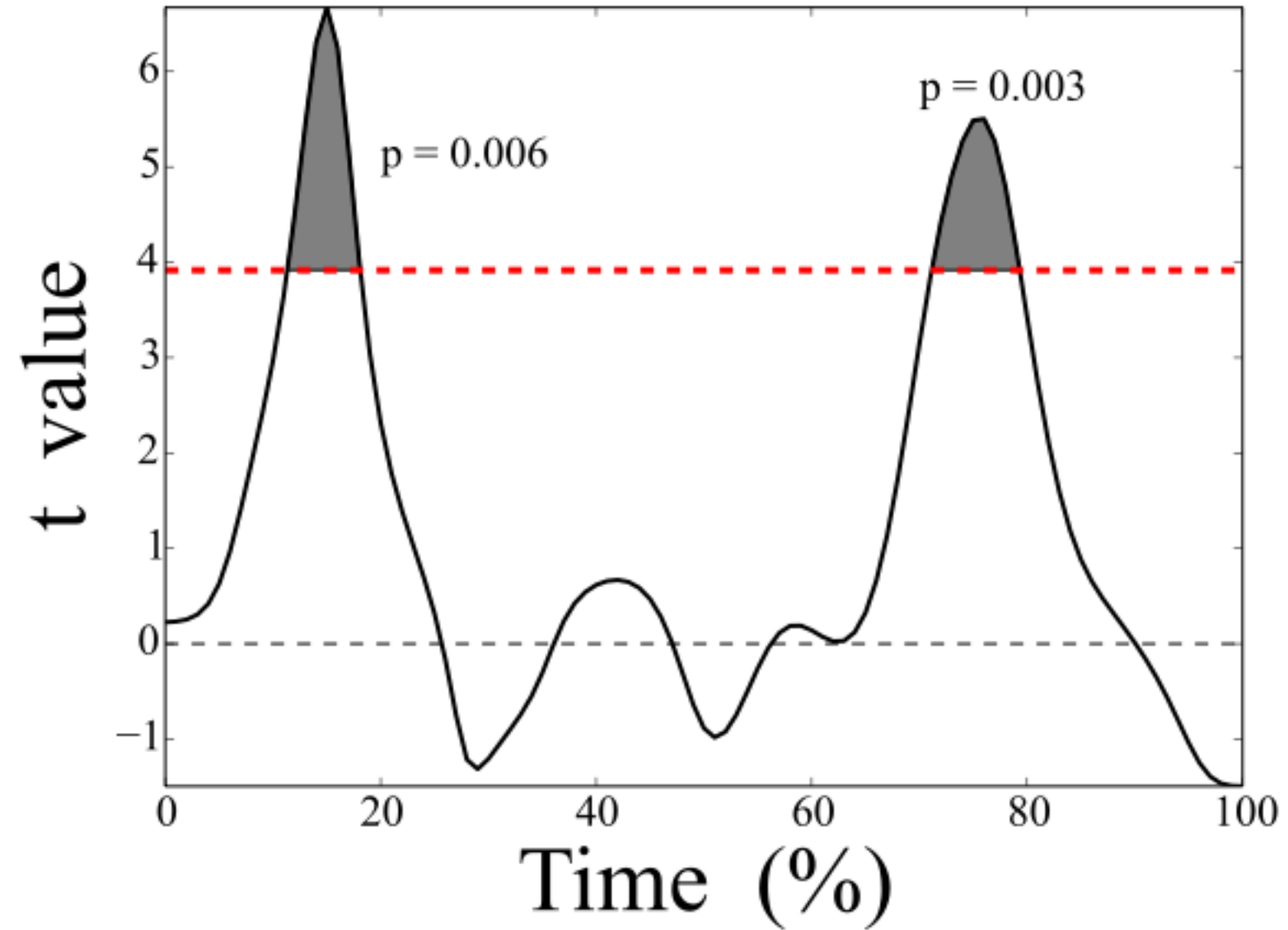
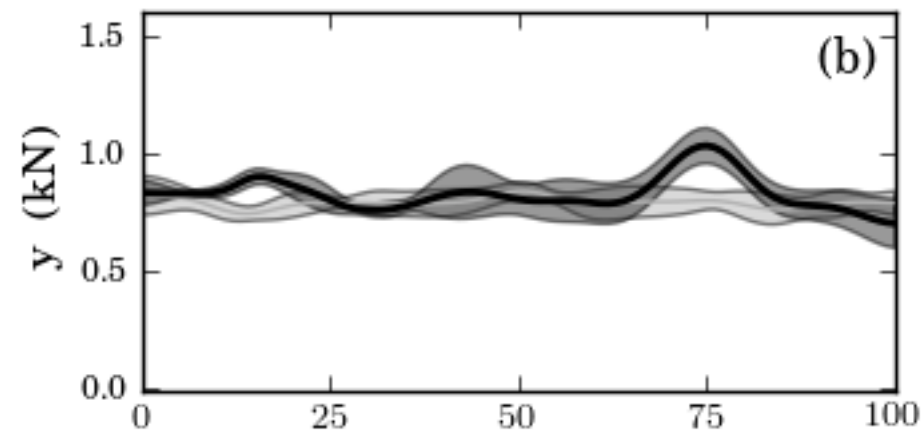
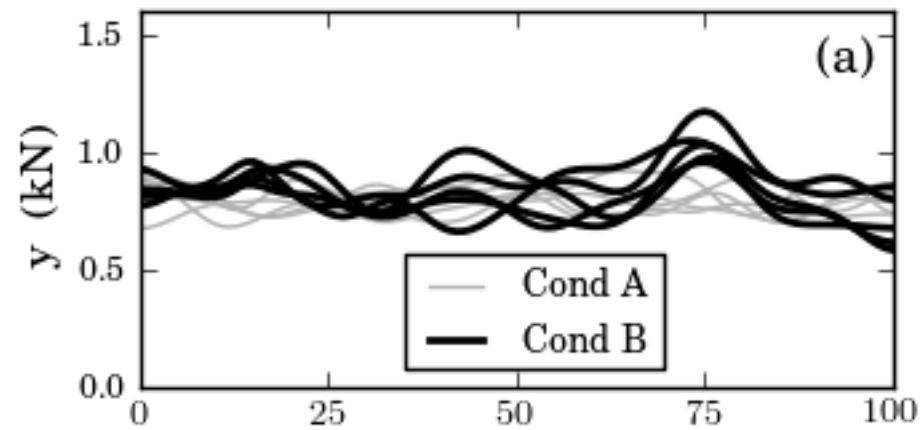
- $T^2$

# Example t tests

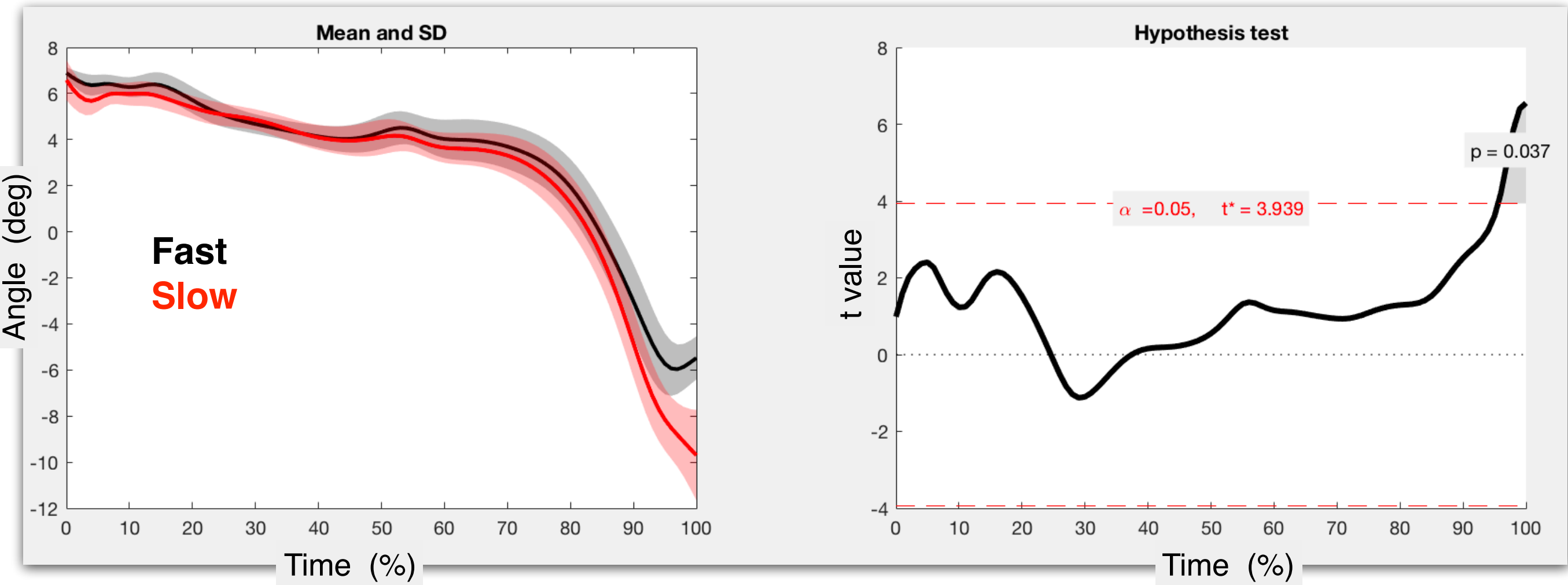
(parametric & non-parametric)



# Simulated dataset



# Plantar arch angle dataset



# SnPM

Group A

Group B

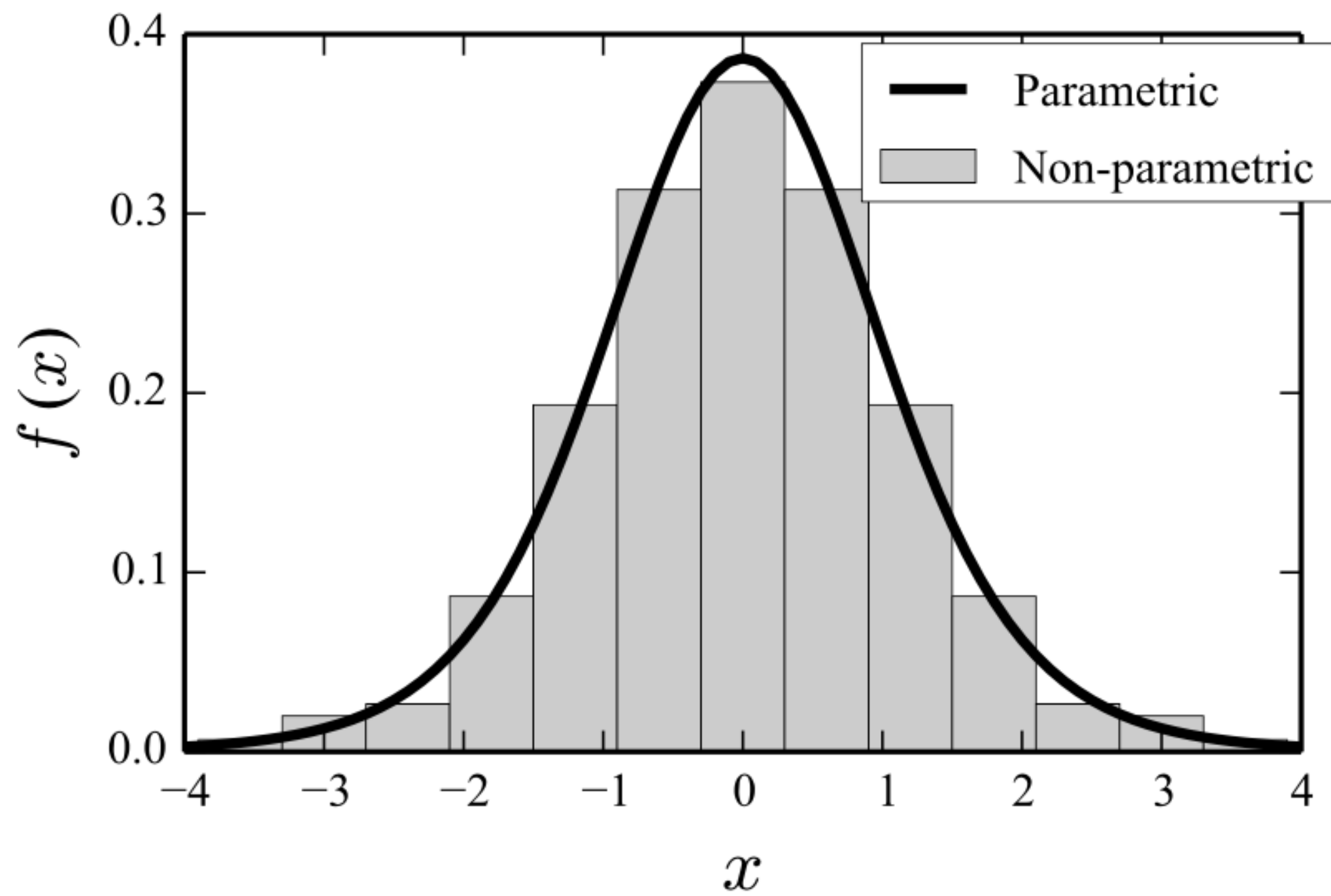
1.14 1.21 1.25 1.43 1.57 1.37 1.52 1.61 1.74 1.54

Data	1.14	1.21	1.25	1.43	1.57	1.37	1.52	1.61	1.74	1.54	t value
Labels 1	A	A	A	A	A	B	B	B	B	B	2.378
Labels 2	B	A	A	A	A	A	B	B	B	B	1.208
Labels 3	A	B	A	B	A	A	B	B	A	B	-0.310
	...	...	...	...	...	...	...	...	...	...	
Labels N	B	B	B	B	B	A	A	A	A	A	-2.378

t = 2.378

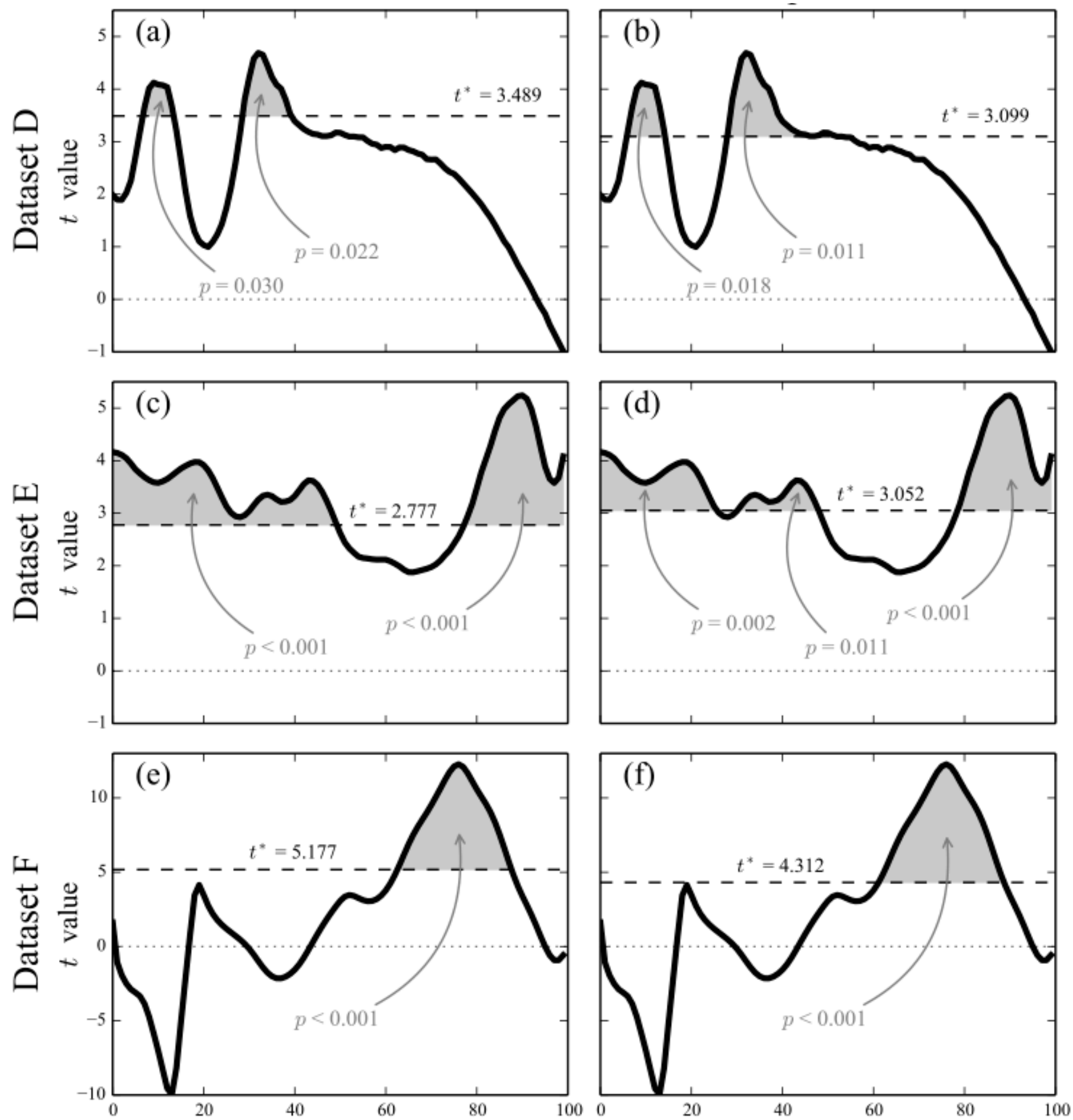
Parametric result: (v=8) p=0.022

Non-parametric result: p=0.028



# SPM

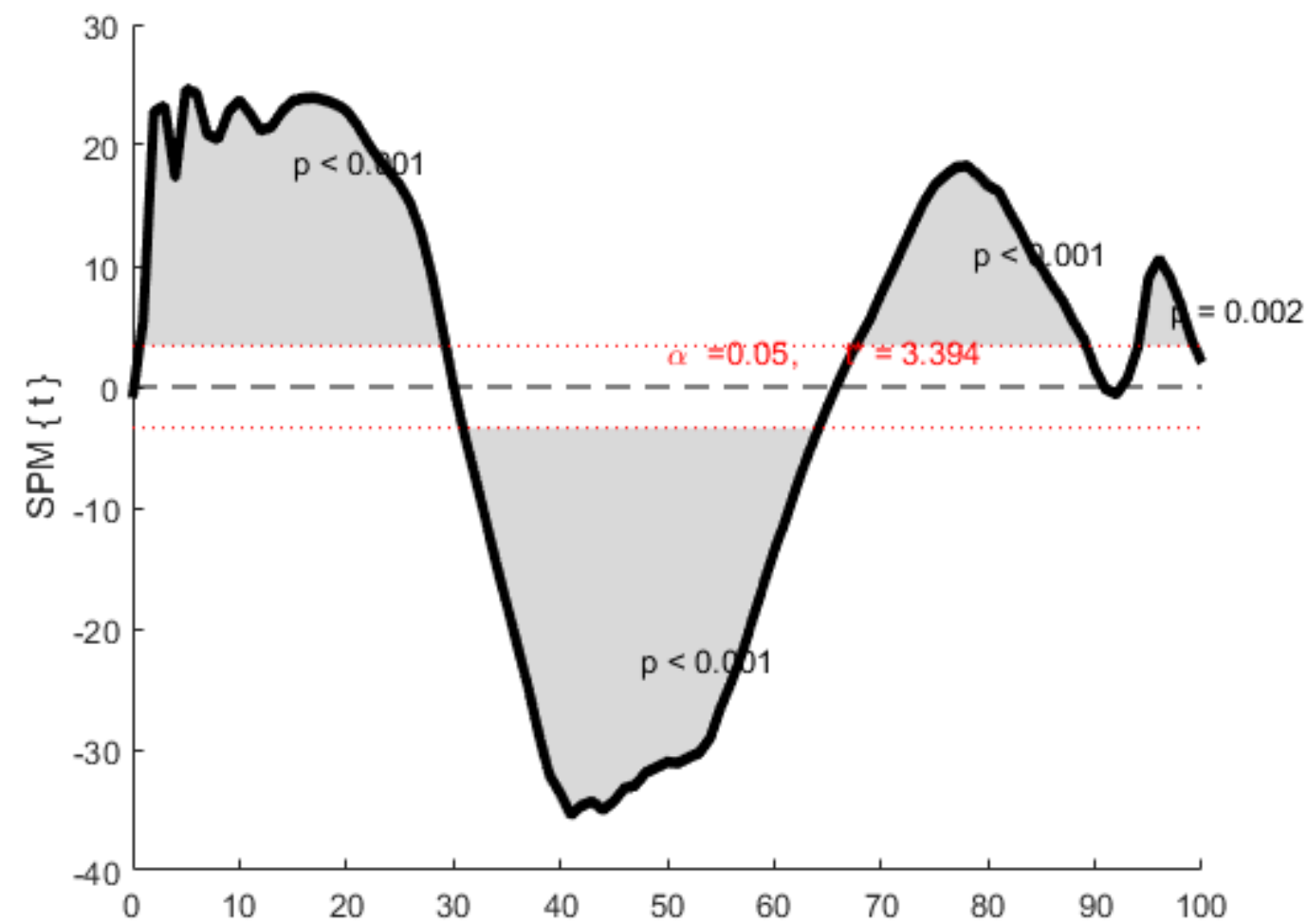
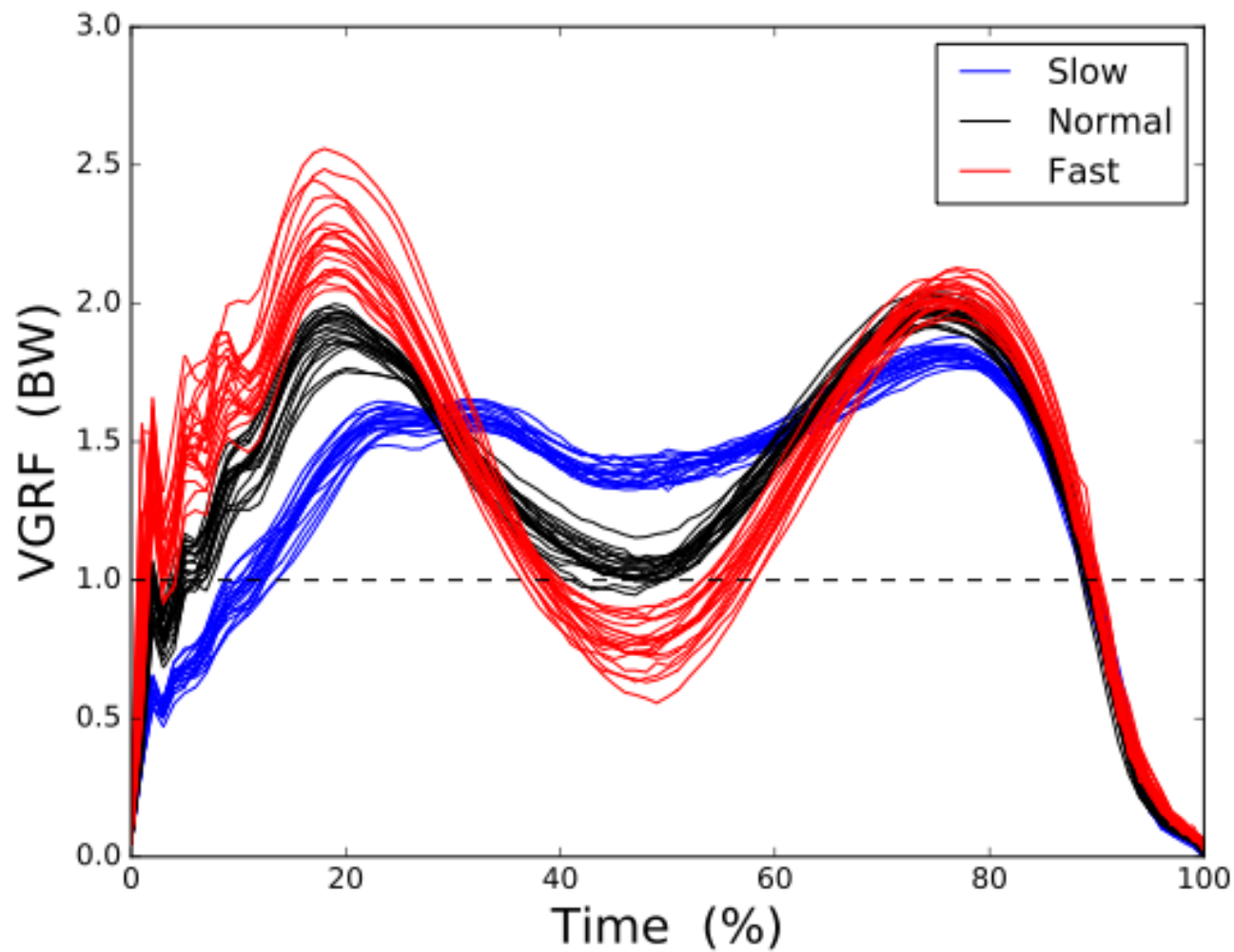
# S<sub>n</sub>PM

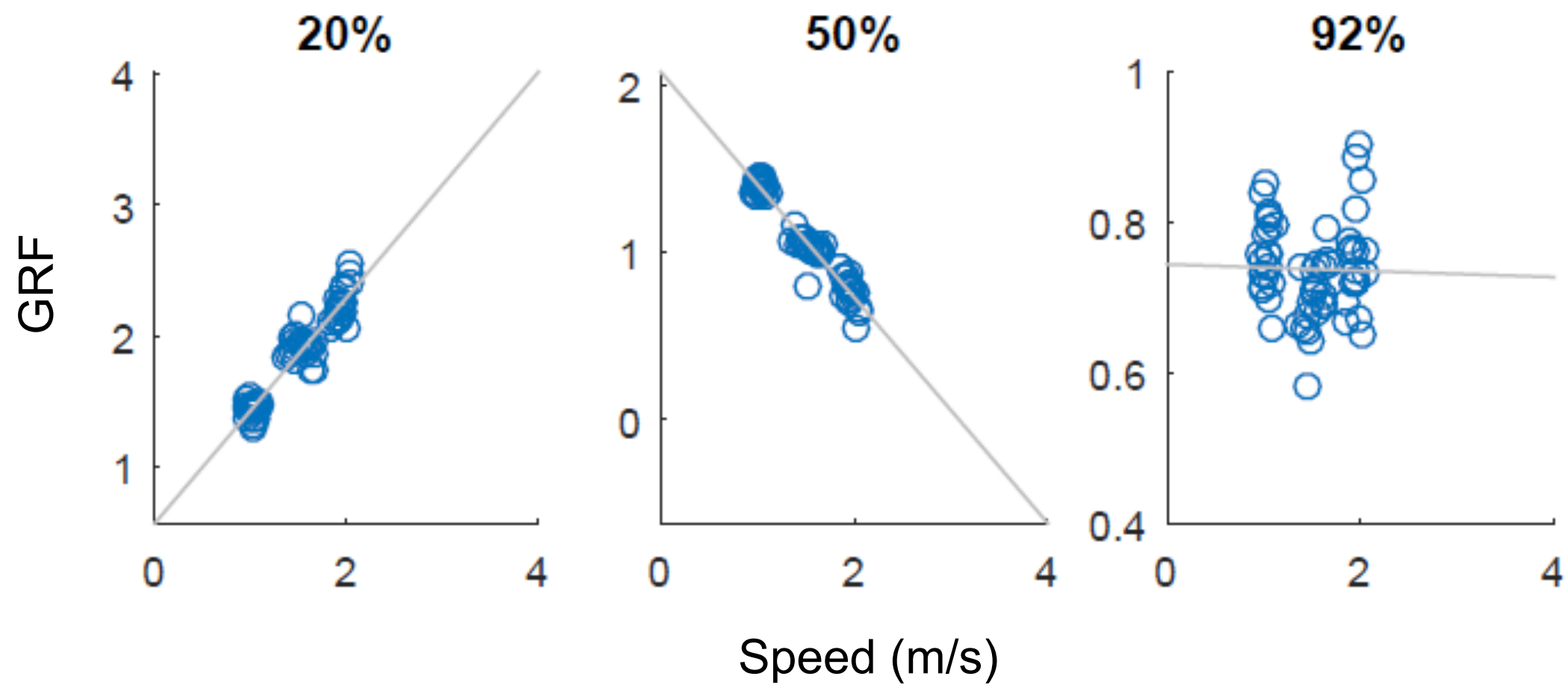
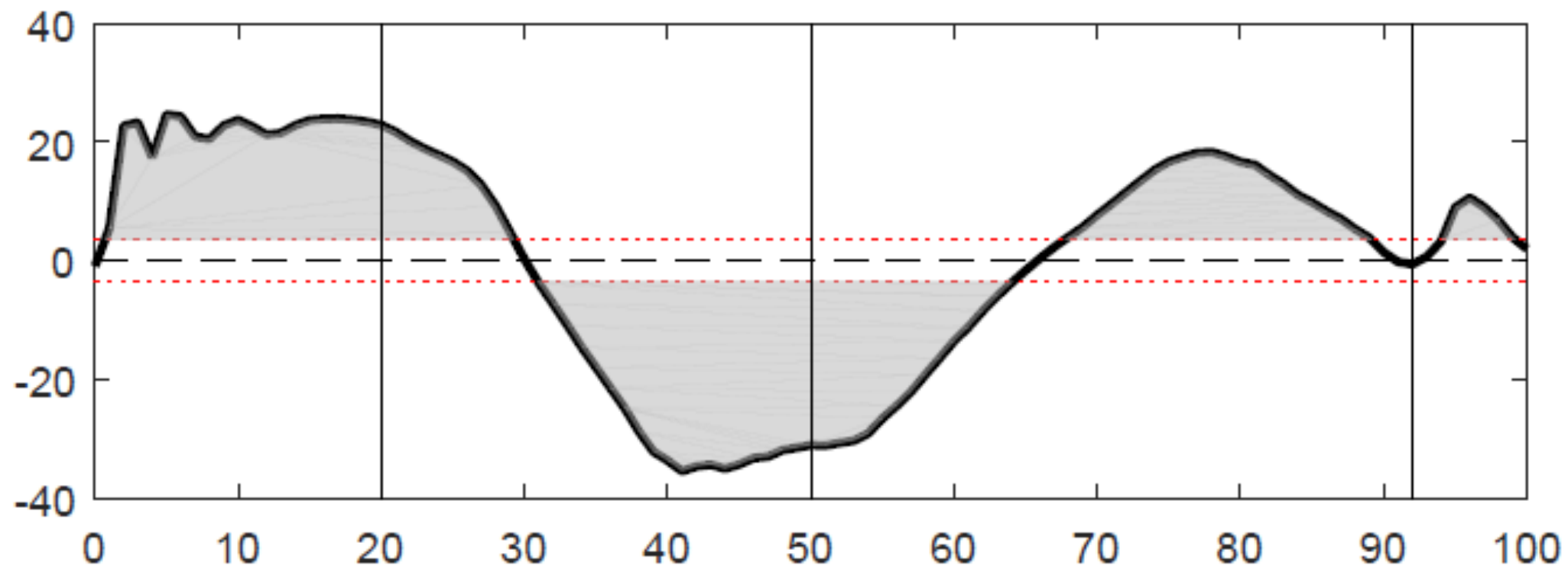


# Generalizability

(regression, ANOVA & beyond)

# Example regression

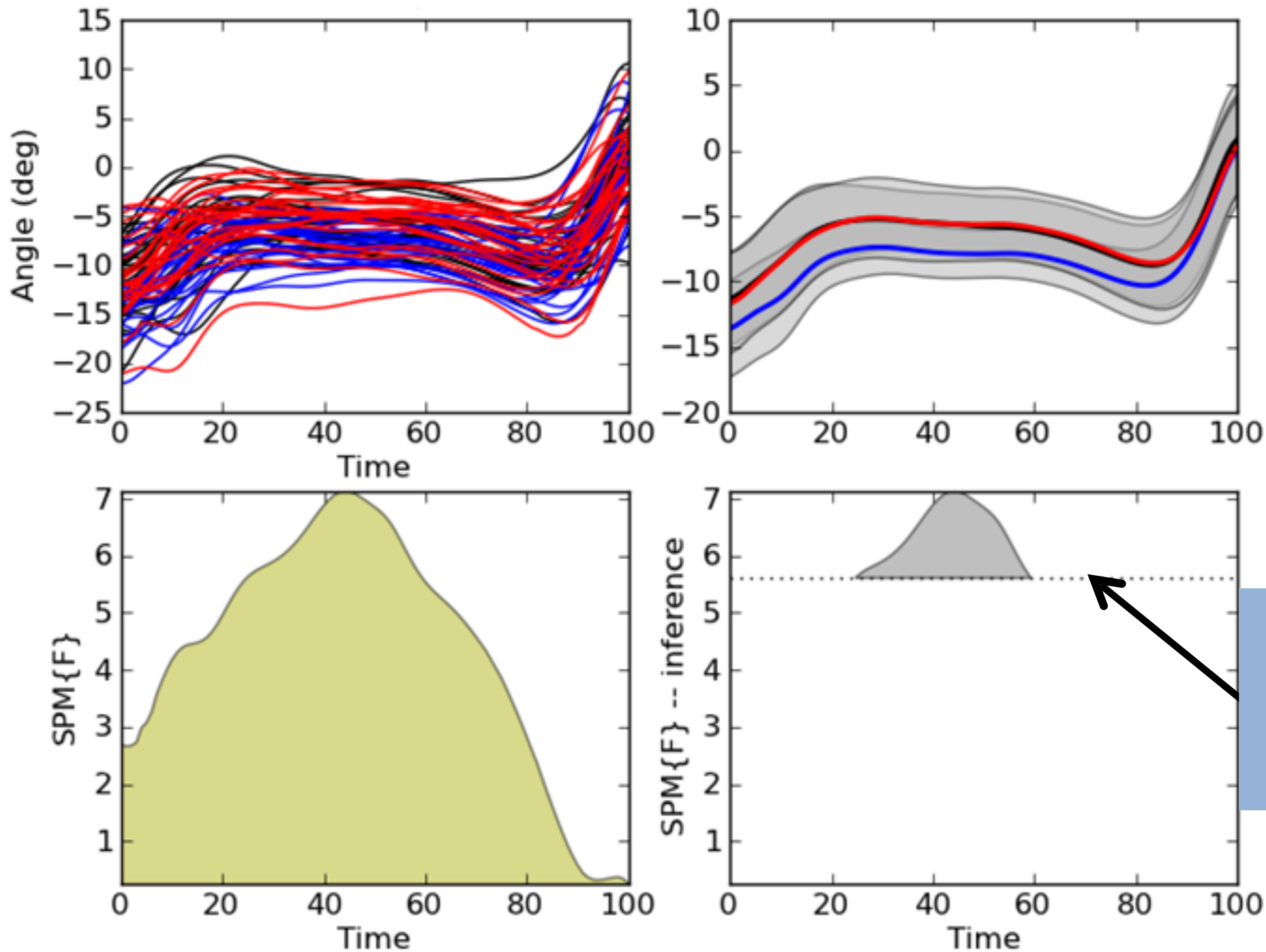






# One-way ANOVA

`spm1d.stats.anova1(CAI, COP, CON);`



3 groups  
foot kinematics



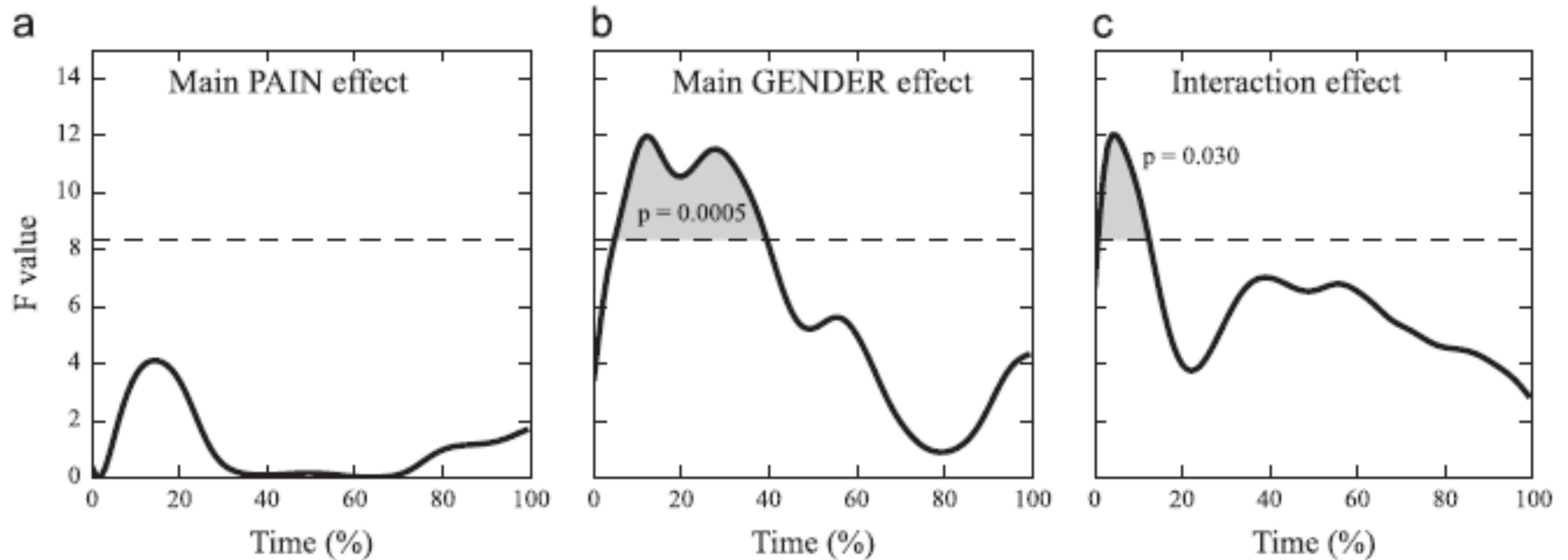
Critical F  
threshold 6.746  
(at  $\alpha = 0.05$ )

De Ridder et al. (2013). MSSE

# Example two-way (2 x 2) ANOVA

Besier et al. (2009) J Biomech

Pataky et al. (2015) J Biomech



Interaction:  
Pain effect depends on  
gender



## Univariate

- t tests
- Regression
- ANOVA
  - 1-, 2-, 3-way
  - repeated measures

## Multivariate

- Hotelling's  $T^2$
- CCA
- MANOVA
  - 1-way

... and normality tests

# Summary

## **SPM** process

1. Compute test statistic (t, F,  $\chi^2$ ) at all continuum points
2. Check if the **RFT** threshold is exceeded

# Summary

Random field theory (**RFT**) generalizes 0D Gaussian randomness to  $n$ D Gaussian randomness

**SPM** uses **RFT** to generalize 0D tests to  $n$ D tests

**S****n****PM** results converge to **RFT** distributions when the data are normal