ONLINE MASTERS IN **DATA SCIENCE**

DSC 255 - MACHINE LEARNING FUNDAMENTALS

THE PERCEPTRON ALGORITHM

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Recall: Linear Separators for Binary Classification

Binary classification problem: data $x \in \mathbb{R}^d$ and labels $y \in \{-1, +1\}$

Linear Classifier:

- Parameters: $w \in \mathbb{R}^d$ and $b \in \mathbb{R}$
- Decision boundary $w \cdot x + b = 0$
- On point x, predict label $sign(w \cdot x + b)$

If the true label on point x is y:

- Classifier correct if $y = sign(w \cdot x + b)$
- Equivalent: if $y(w \cdot x + b) > 0$

A Loss Function for Classification

What is the **loss** of our linear classifier (given by w, b) on a point (x, y)?

One idea for a loss function:

- If $y(w \cdot x + b) > 0$: correct, no loss
- If $y(w \cdot x + b) < 0$: loss $-y(w \cdot x + b)$

A Simple Learning Algorithm

Fit a linear classifier w, b to the training set using **stochastic gradient descent.**

- Update w, b based on just one data point (x, y) at a time
- If $y(w \cdot x + b) > 0$: zero loss, no update
- If $y(w \cdot x + b) \leq 0$: loss is $-y(w \cdot x + b)$

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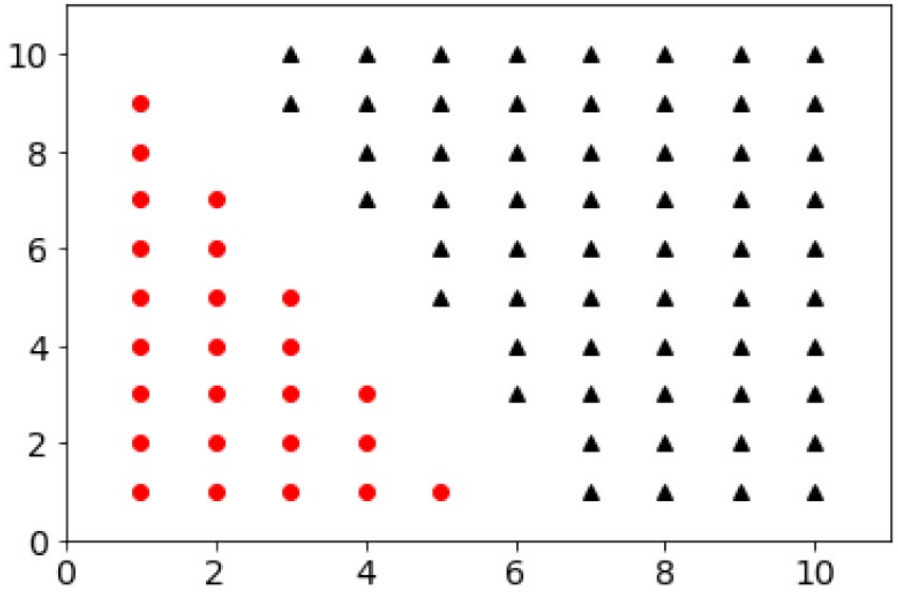
The Perceptron Algorithm

- Initialize $w = 0$ and $b = 0$
- Keep cycling through the training data (x, y):
  - If $y(w \cdot x + b) \leq 0$ (i.e., point misclassified):
    - $w = w + yx$
    - $b = b + y$

The Perceptron in Action

85 data points, linearly separable.



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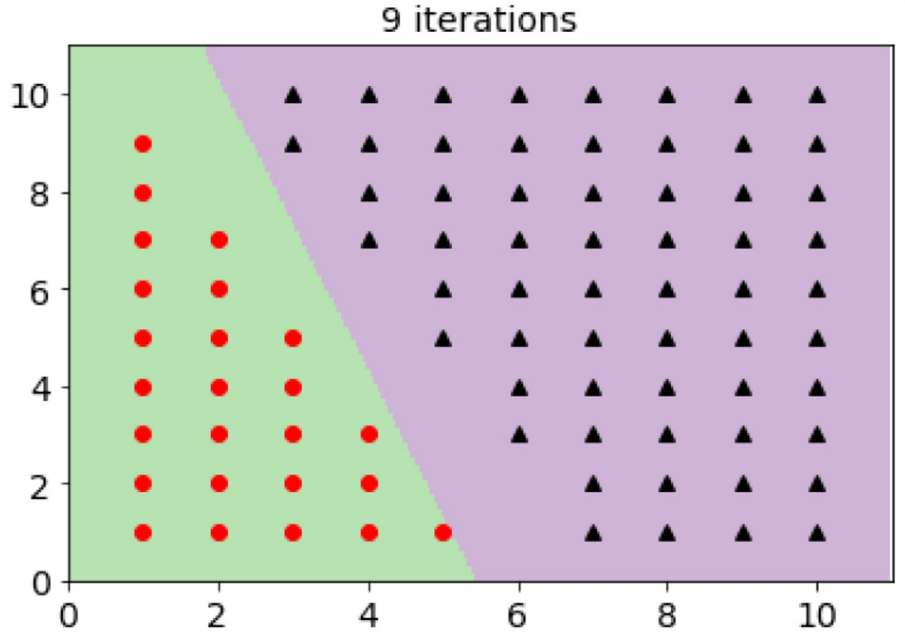

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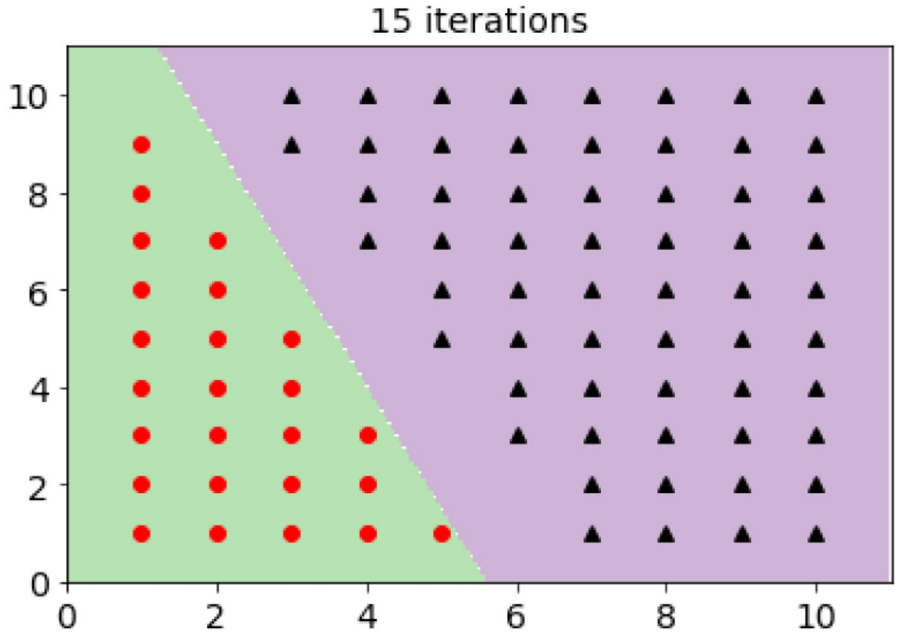

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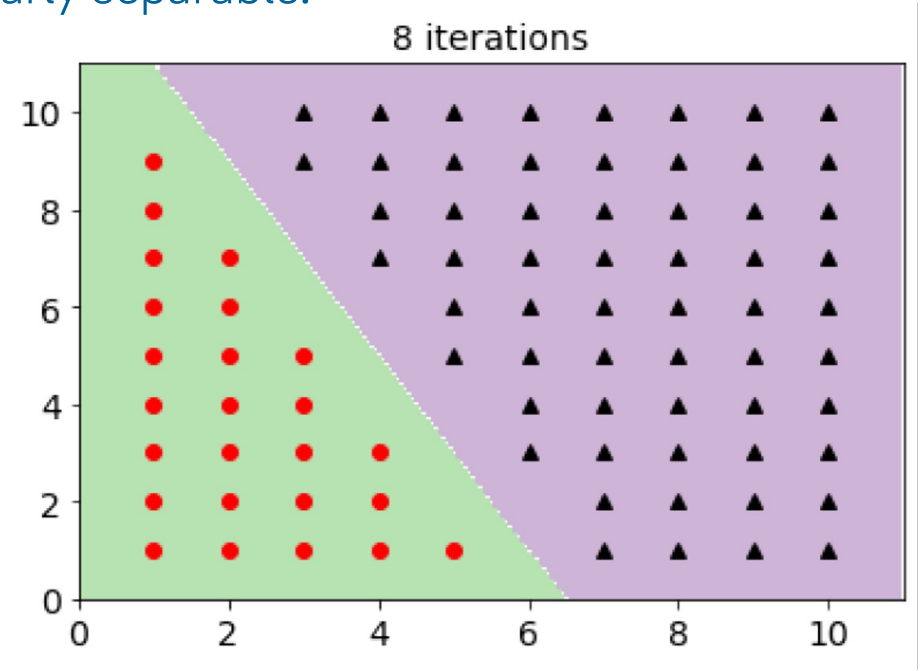

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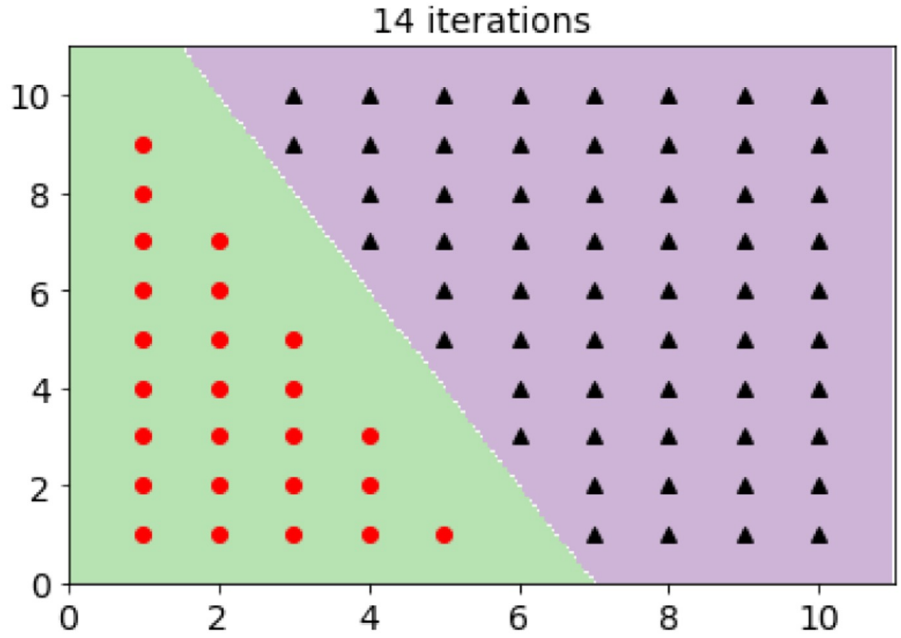

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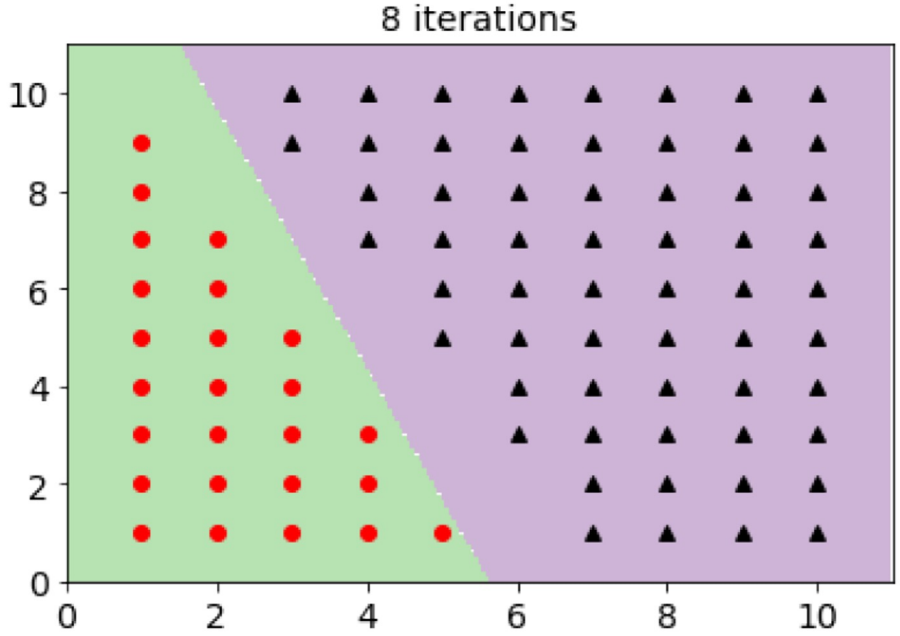

Perceptron: Convergence

If the training data is linearly separable:

- The Perceptron algorithm will and a linear classifier with zero training error.
- It will converge within a finite number of steps.
- The number of iterations can be bounded in terms of the *margin*: roughly, a measure of the space between the two classes.