ONLINE MASTERS IN **DATA SCIENCE**

DSC 255 - MACHINE LEARNING FUNDAMENTALS

ℓ_p NORMS

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Measuring distance in $\mathbb{R}^m$

Usual choice: **Euclidean distance:**

$$\| x - z \|_2 = \sqrt{\sum_{i=1}^{m} (x_i - z_i)^2}$$

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For $p \geq 1$, here is ℓ_p **distance:**

$$\| x - z \|_p = \left(\sum_{i=1}^{m} |x_i - z_i|^p \right)^{1/p}$$

- $p = 2$: Euclidean distance
- ℓ_1 distance: $\| x - z \|_1 = \sum_{i=1}^{m} |x_i - z_i|$
- ℓ_∞ distance: $\| x - z \|_\infty = max_i \, |x_i - z_i|$

Example 1

Consider the all-ones vector $(1, 1, \ldots, 1)$ in $\mathbb{R}^d$.

What are its ℓ_2, ℓ_1 and ℓ_∞ length?

Example 2

In $\mathbb{R}^2$, draw all points with

1. ℓ_2 length 1
2. ℓ_1 length 1
3. ℓ_∞ length 1