ONLINE MASTERS IN **DATA SCIENCE**

DSC 255 - MACHINE LEARNING FUNDAMENTALS

MAXIMIZING THE MARGIN OF A LINEAR CLASSIFIER

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Recall: The Perceptron Algorithm

The Perceptron Algorithm

- Initialize $w = 0$ and $b = 0$
- Keep cycling through the training data (x, y):
  - If $y(w \cdot x + b) \leq 0$ (i.e., point misclassified):
    - $w = w + yx$
    - $b = b + y$

Perceptron: Convergence

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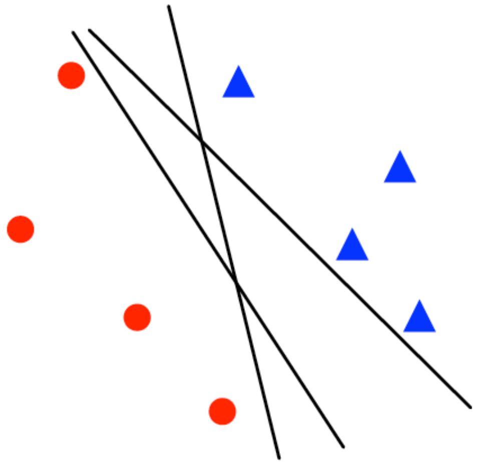

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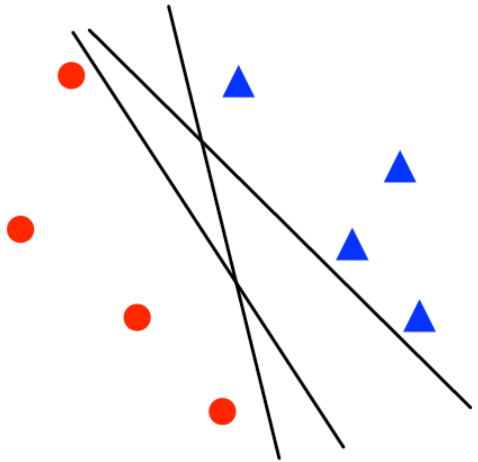

Is there a better, more systematic choice of separator?

The Learning Problem

Given: training data $\left(x^{(1)}, y^{(1)}\right), \ldots, \left(x^{(n)}, y^{(n)}\right) \in \mathbb{R}^d \times \{-1, +1\}$

Find: $w \in \mathbb{R}^d$ and $b \in \mathbb{R}$ such that $y^{(i)}(w \cdot x^{(i)} + b) > 0$ for all i.

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By scaling w, b, can equivalently ask for

$$y^{(i)}(w \cdot x^{(i)} + b) \geq 1 \text{ for all } i$$

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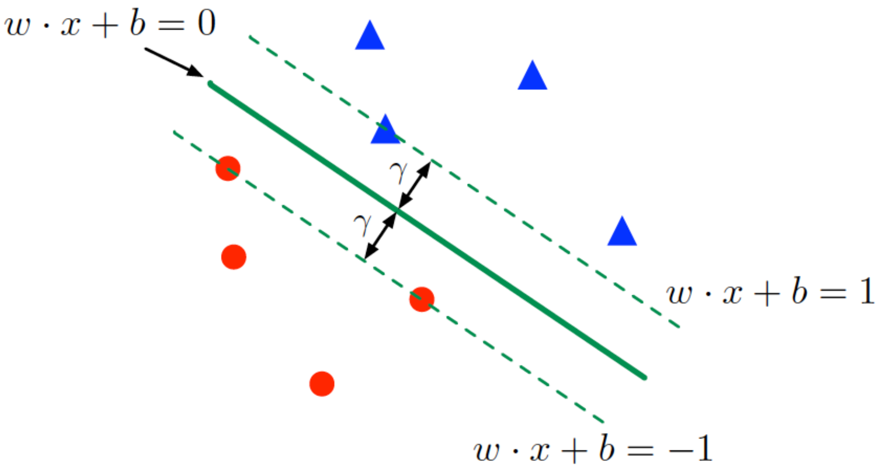

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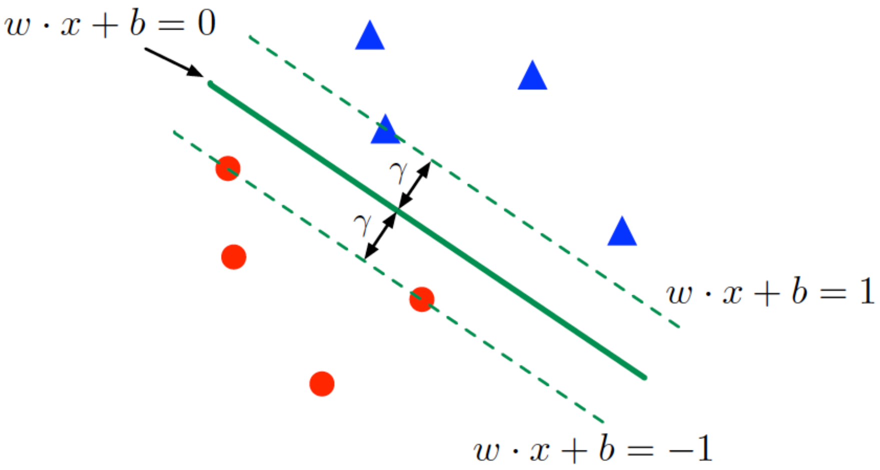

Maximize the **margin** γ.   Can show $\gamma = 1/\|w\|$.

Maximum-Margin Linear Classifier

- *Given:* $\left(x^{(1)}, y^{(1)}\right), \dots, \left(x^{(n)}, y^{(n)}\right) \in \mathbb{R}^d \times \{-1, +1\}$

$$\boxed{\begin{array}{c} \min_{w \in \mathbb{R}^d, b \in \mathbb{R}} \|w\|^2 \\ \\ y^{(i)}(w \cdot x^{(i)} + b) \geq 1 \quad \text{for all } i = 1, 2, \dots, n \end{array}}$$

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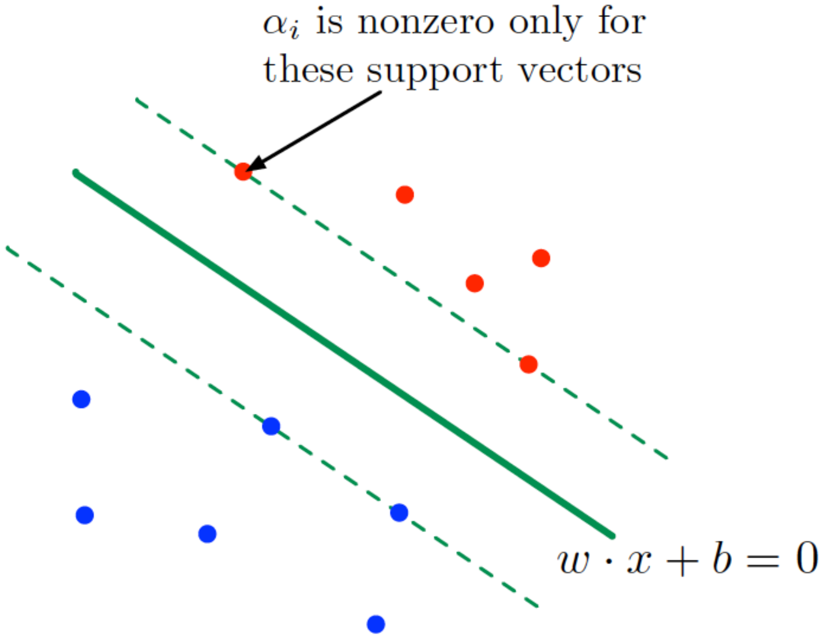
- This linear classifier is sometimes called the **(hard-margin) support vector machine.**

Support Vectors

- The solution $w = \sum_{i=1}^{n} \alpha_i y^{(i)} x^{(i)}$ is a function of just the **support vectors**: training points exactly on the margin, i.e., $y^{(i)}\left(w \cdot x^{(i)} + b\right) = 1$



Small Example: Iris Data Set

Fisher's **iris** data



150 data points from three classes:

- iris setosa
- iris versicolor
- iris virginica

Four measurements: petal width/length, sepal width/length

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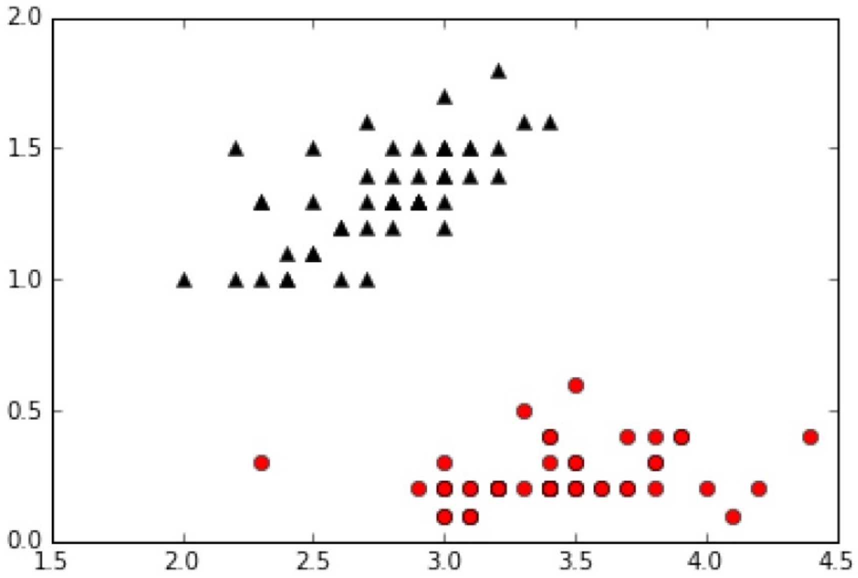
Two features: sepal width, petal width.
Two classes: setosa (red circles), versicolor (black triangles)



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