

ONLINE MASTERS IN **DATA SCIENCE**

DSC 255 - MACHINE LEARNING FUNDAMENTALS

# DECISION TREE BASICS

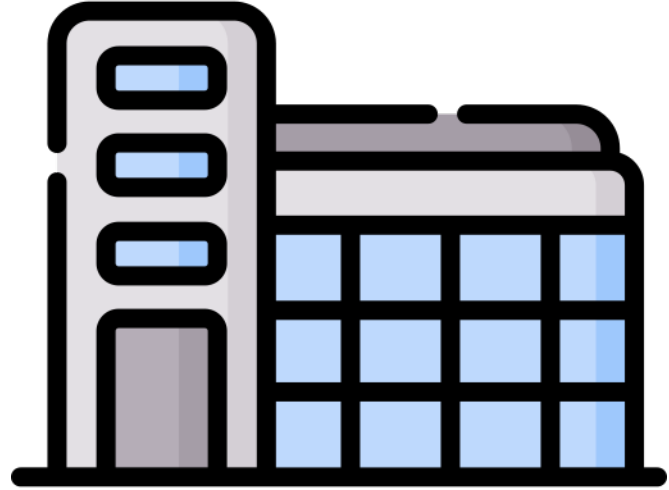
SANJOY DASGUPTA, PROFESSOR

UC San Diego

COMPUTER SCIENCE & ENGINEERING  
HALICIOĞLU DATA SCIENCE INSTITUTE

## Decision Trees

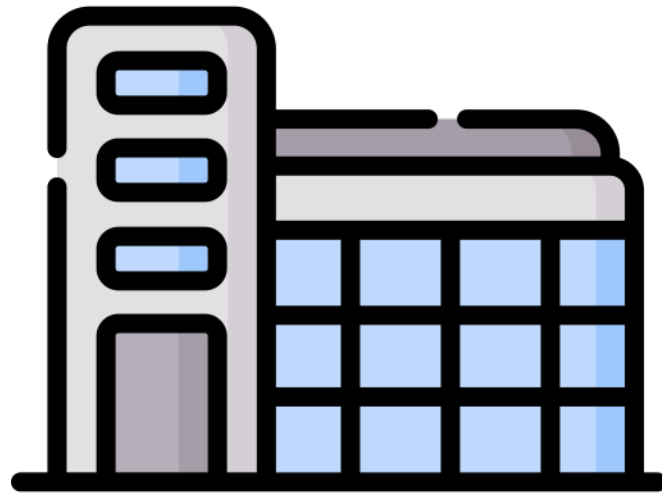
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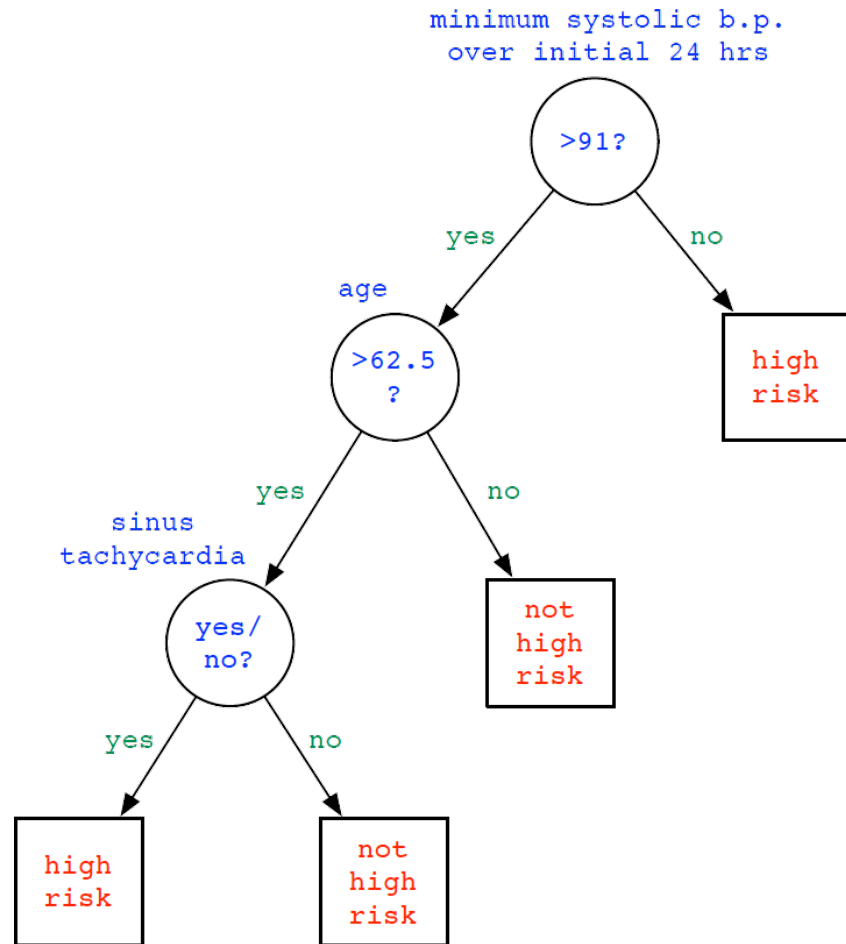
Data set:  
215 patients.  
37 (=20%) died.  
19 features.



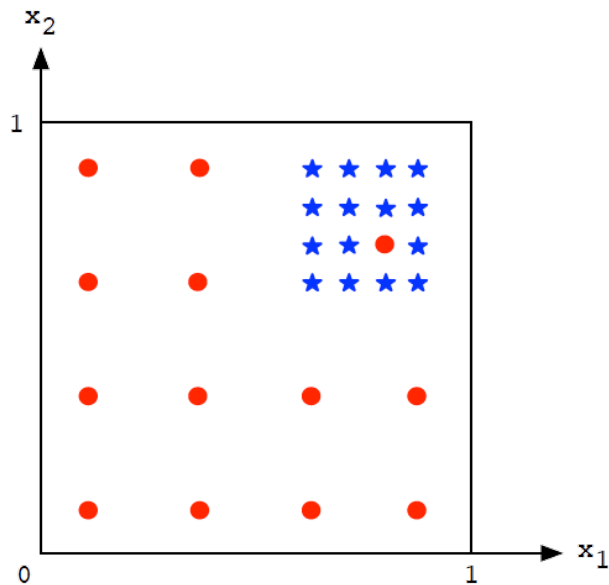
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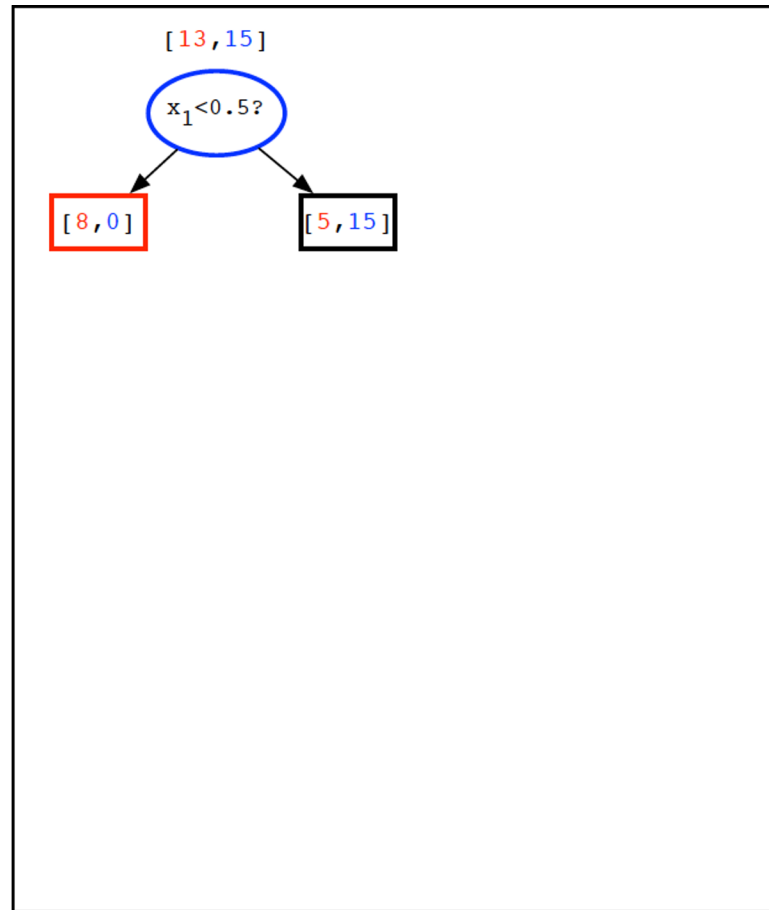
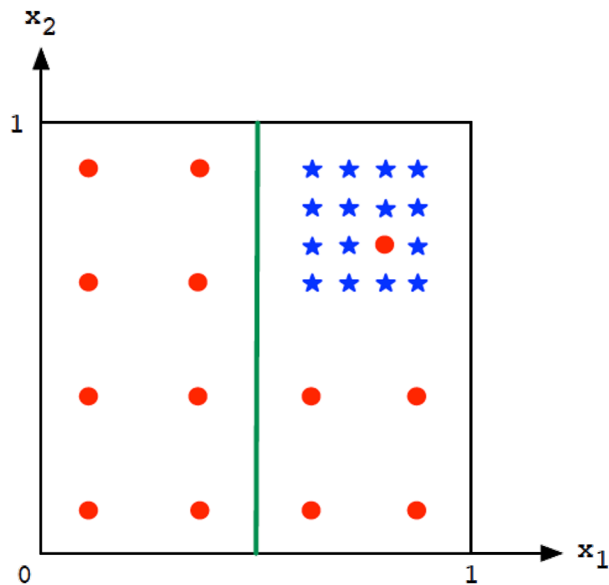


## Example: Building a Decision Tree

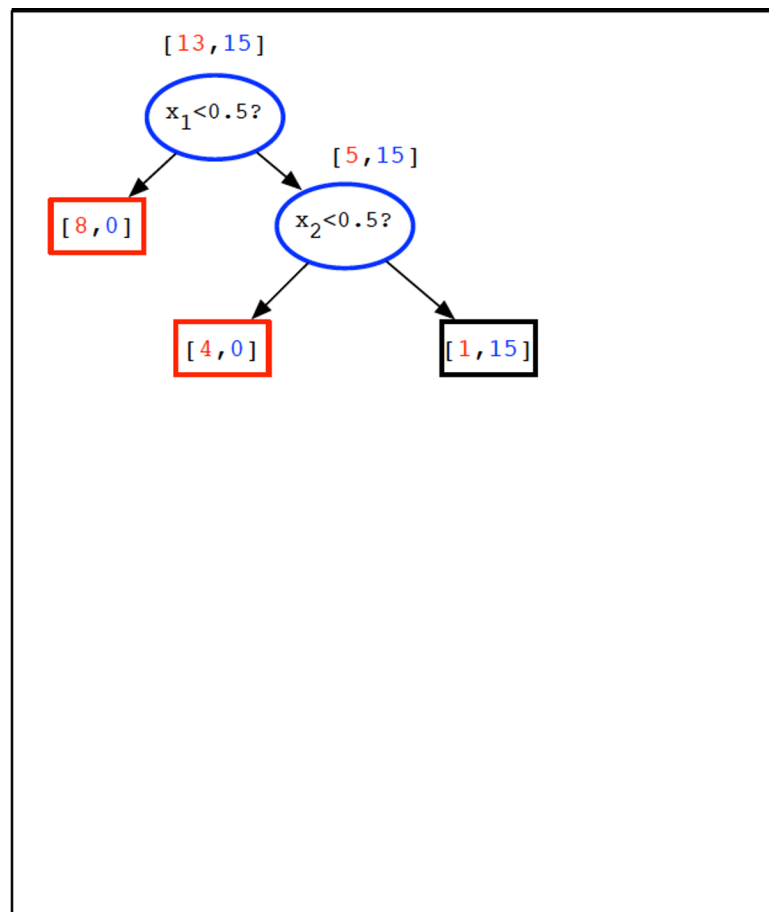
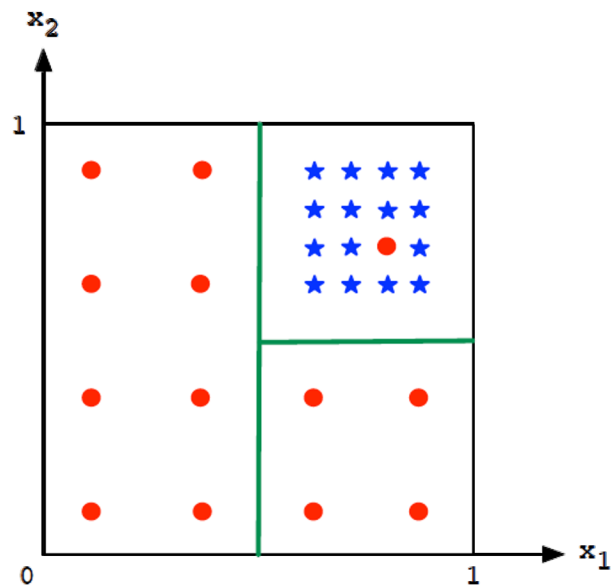


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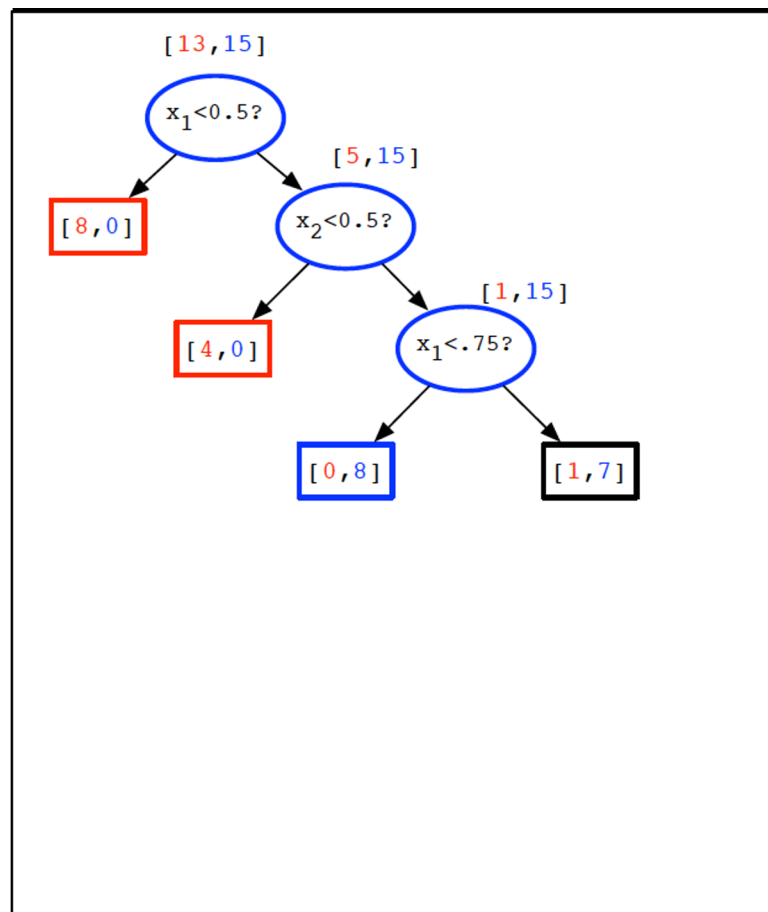
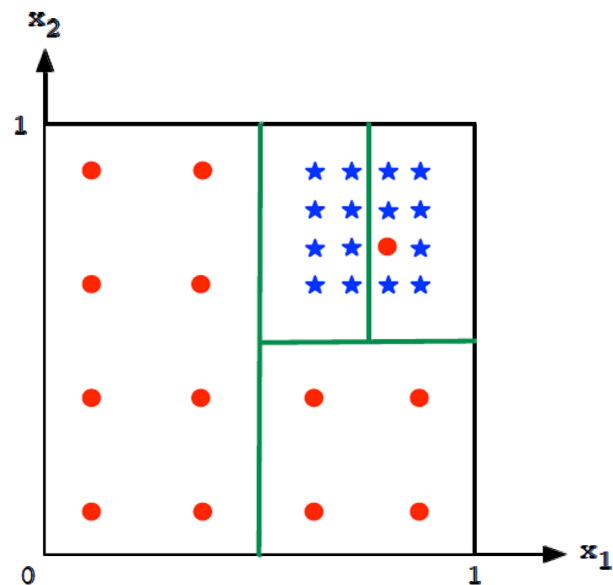
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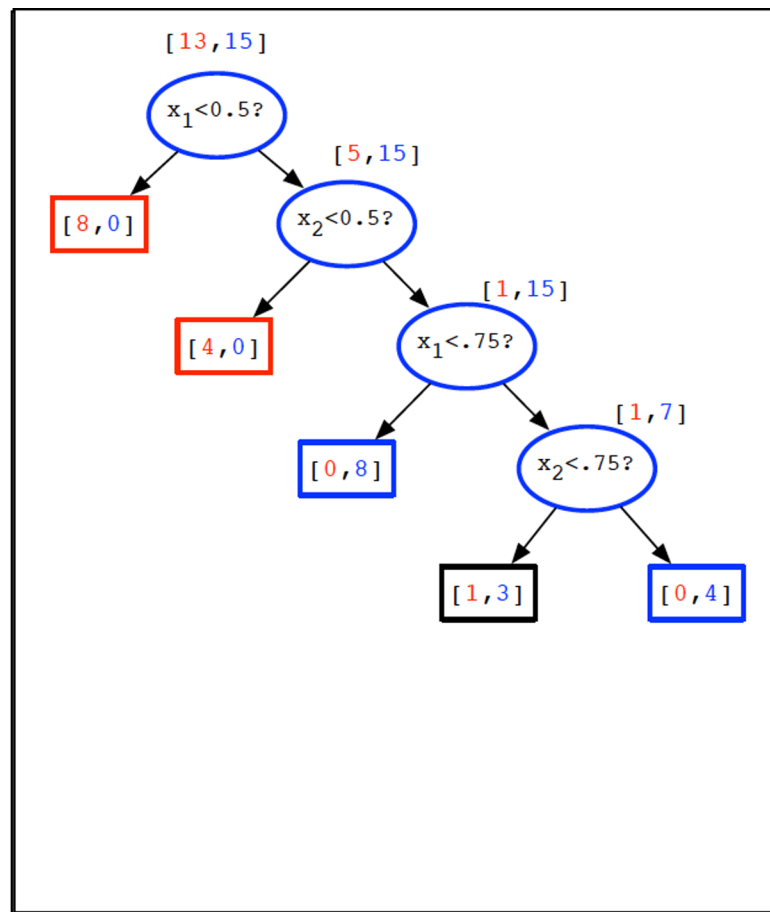
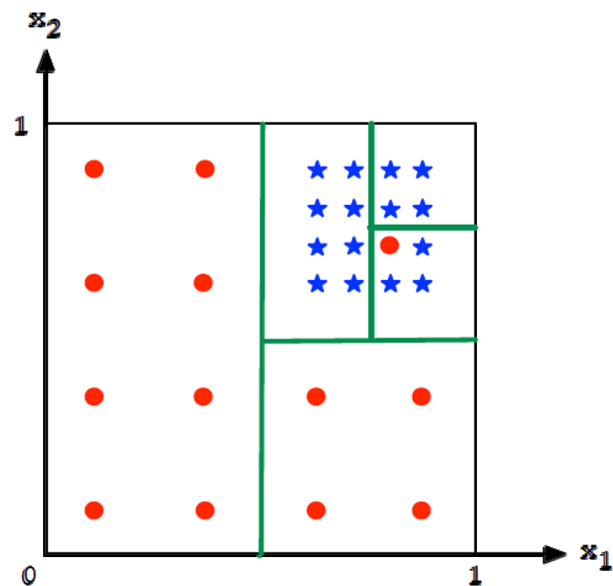


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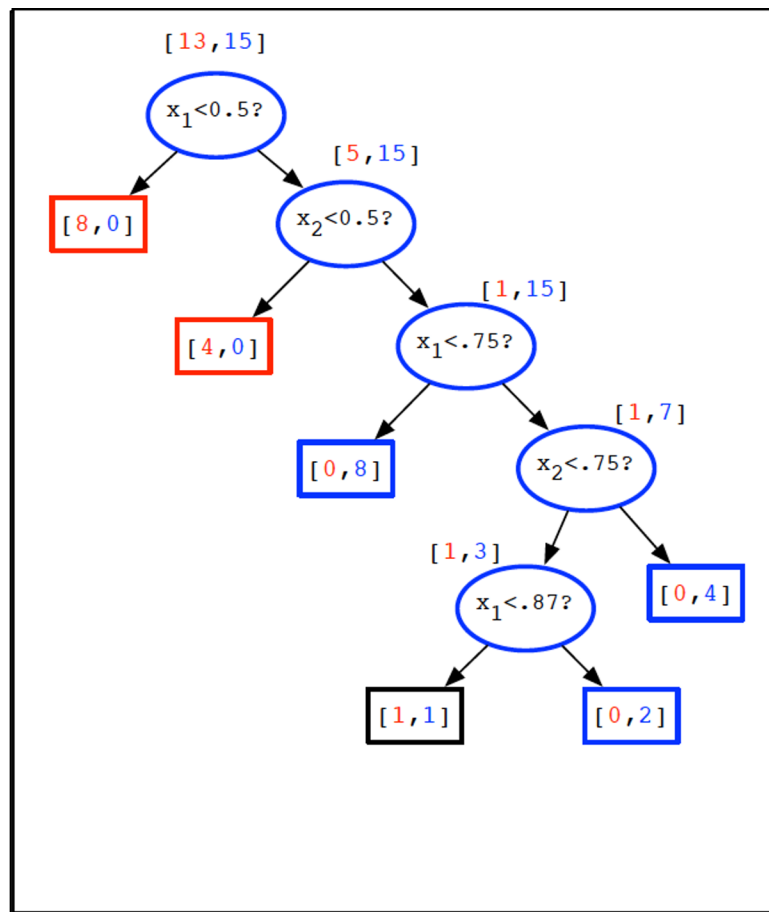
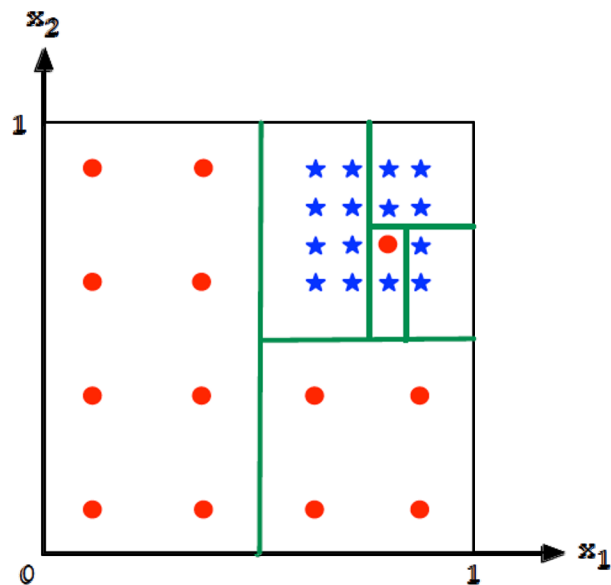




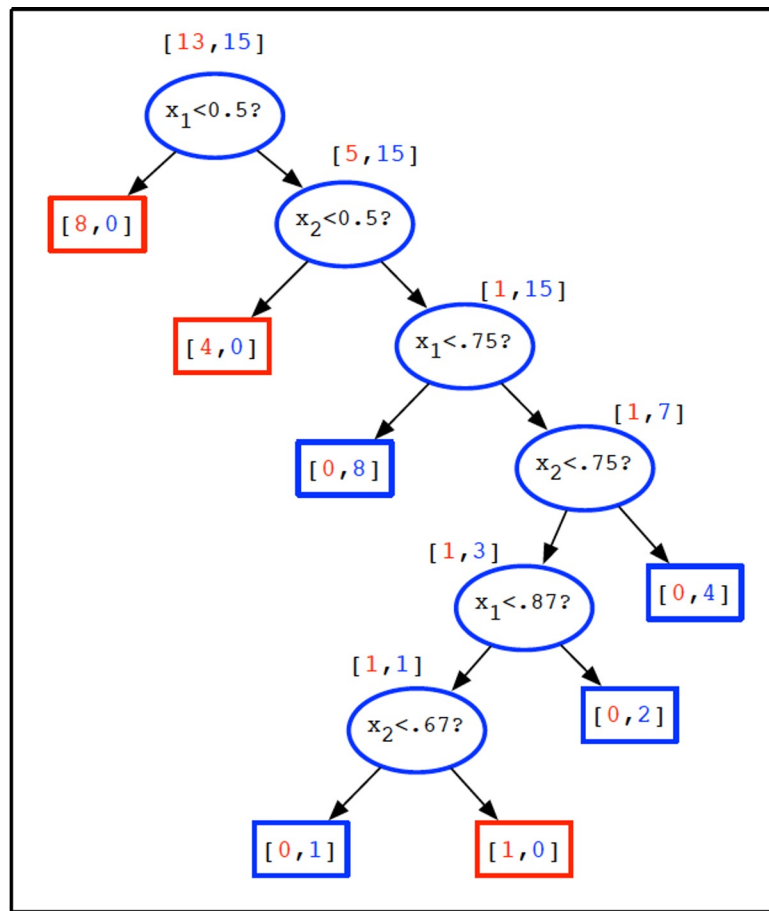
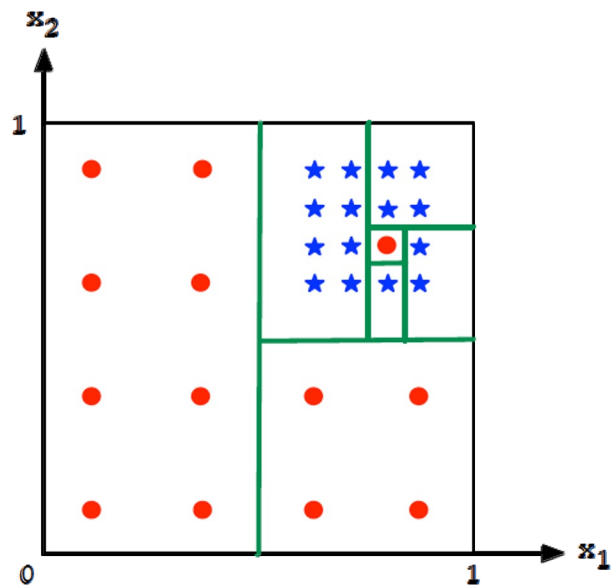
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## Building a Decision Tree

Greedy algorithm: build tree top-down.

- Start with a single node containing all data points
- Repeat:
  - Look at all current leaves and all possible splits
  - Choose the split that most decreases the uncertainty in prediction

We need a measure of **uncertainty in prediction**.

## Uncertainty in Prediction

Say there are two labels:

- + label  $p$  fraction of the points
- label  $(1 - p)$  fraction of the points

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1 Misclassification rate

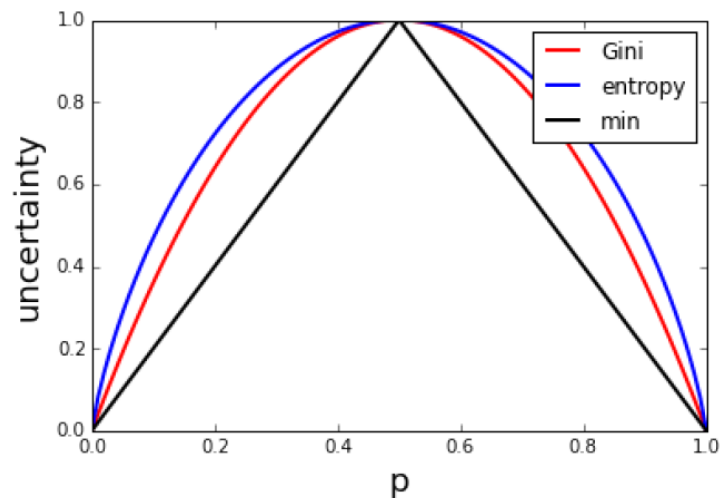
$$\min\{p, 1 - p\}$$

2 Gini index

$$2p(1 - p)$$

3 Entropy

$$p \log \frac{1}{p} + (1 - p) \log \frac{1}{1 - p}$$



## Uncertainty: $K$ Class

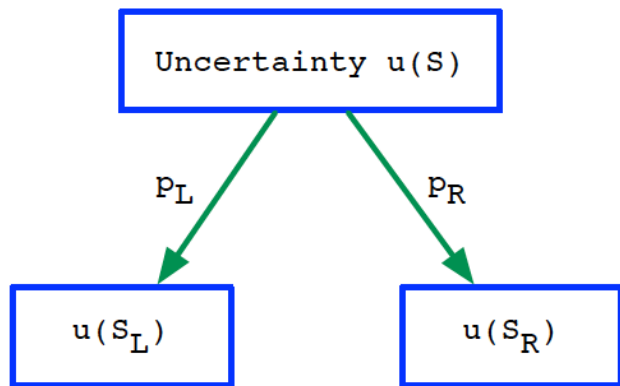
Suppose there are  $k$  classes, with probabilities  $p_1, p_2, \dots, p_k$ .

	$k = 2$	General $k$
Misclassification rate	$\min\{p, 1 - p\}$	$1 - \max_i p_i = 1 - \ p\ _\infty$
Gini Index	$2p(1 - p)$	$\sum_{i \neq j} p_i p_j = 1 - \ p\ ^2$
Entropy	$p \log \frac{1}{p} + (1 - p) \log \frac{1}{1 - p}$	$\sum_i p_i \log \frac{1}{p_i}$

## Benefit of a Split

Let  $u(S)$  be the uncertainty score for a set of labeled points  $S$ .

Consider a particular split:



Of the points in  $S$ :

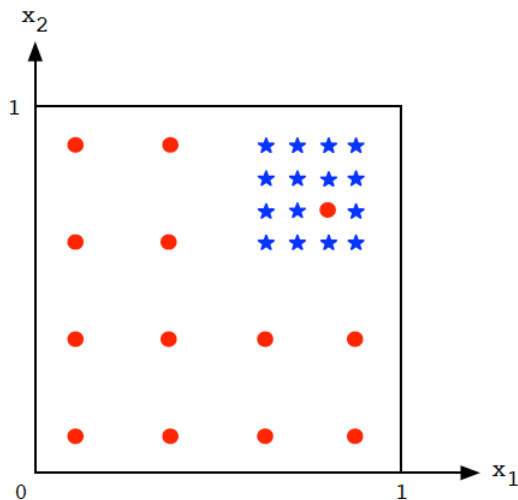
- $p_L$  fraction go to  $S_L$
- $p_R$  fraction go to  $S_R$

Benefit of split = reduction in uncertainty:

$$\left( u(S) - \underbrace{(p_L u(S_L) + p_R u(S_R))}_{\text{expected uncertainty after split}} \right) \times |S|$$

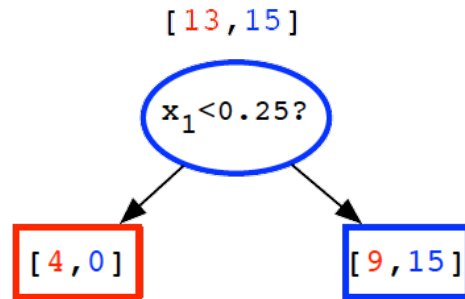


## Benefit of a Split: Example

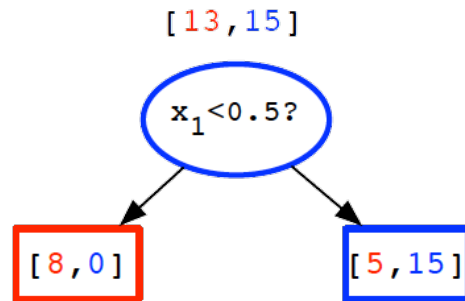


Initial Gini uncertainty:

$$2 \times \frac{13}{28} \times \frac{15}{28}$$



$$p_L u_L + p_R u_R = \frac{4}{28} \cdot 0 + \frac{24}{28} \cdot 2 \cdot \frac{9}{24} \cdot \frac{15}{24} = \frac{45}{112}$$



$$p_L u_L + p_R u_R = \frac{8}{28} \cdot 0 + \frac{20}{28} \cdot 2 \cdot \frac{5}{20} \cdot \frac{15}{20} = \frac{30}{112}$$

## Building a Decision Tree: Summary

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