DSC 215 - PROBABILITY AND STATISTICS FOR DATA SCIENCE

SUMMARIZING NUMERICAL DATA



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Summarizing Numerical Data

We will:

Visualize data using scatterplots and histograms, and

- Succinctly summarize and visualize data using statistics:
 - Mean, median, mode
 - Variances
 - Quartiles

Scatterplots for Paired Data

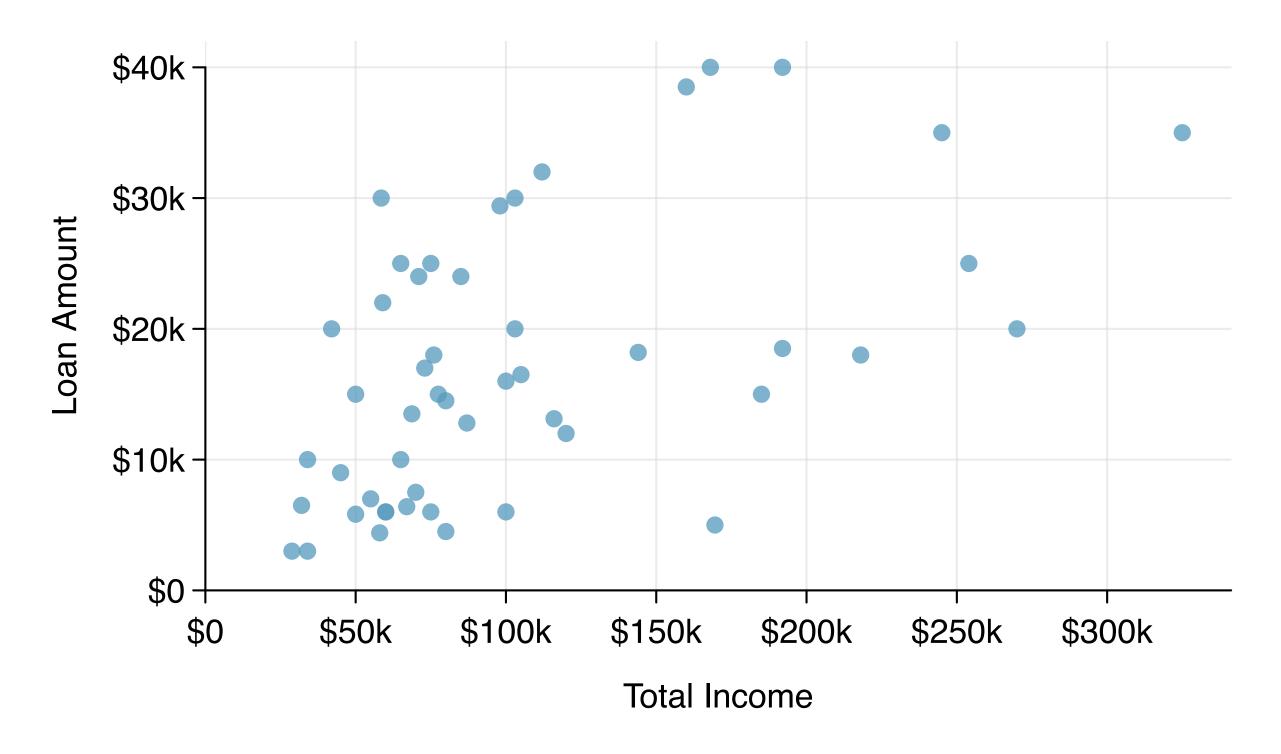


Figure from Open-intro Statistics textbook, Chapter 1.

• In a scatterplot each point represents a single case.

Scatterplots

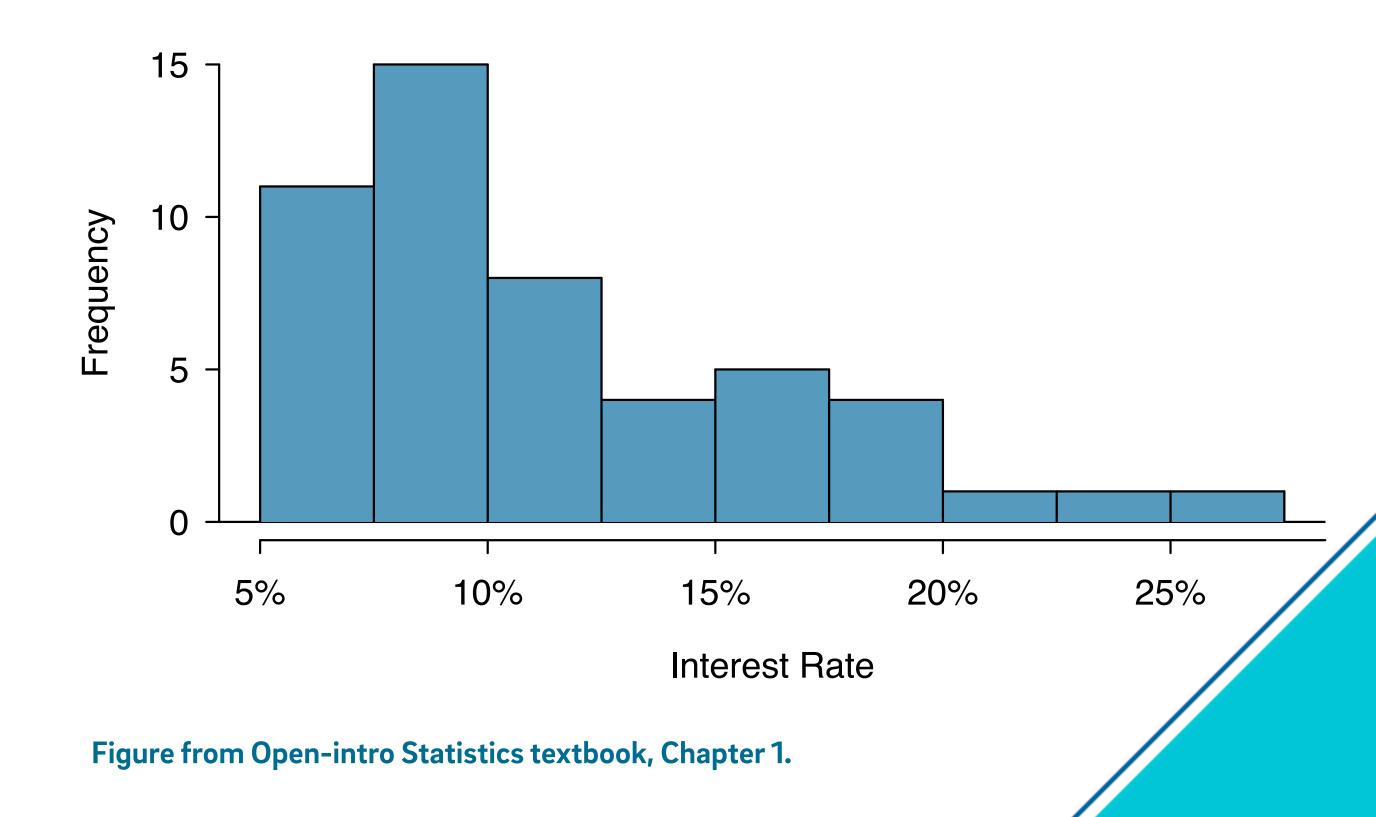
- help visualize the relationship between two numerical variables.
- help spot associations between variables, and identify whether relations are simple or complex.

Histograms for Single Variables

Interest Rate	5.0% - 7.5%	7.5% - 10.0%	10.0% - $12.5%$	12.5% - 15.0%	• • •	25.0% - 27.5%
Count	11	15	8	4	• • •	1

- When we are interested in a single numerical variable's distribution, we can use a **histogram** as a visual aid.
- We think of each numerical value as belonging to a **bin**, and we count the number of cases falling in that bin.
- Histograms provide a view of the data density: higher bars

 data in the bin is more common



Histograms

- Histogram suggests that most loans have rates under 15%. Few have rates above 20%.
- When data trail off to the right this way, we say it has a longer **right tail.** The shape is said to be **right skewed.**
- When data trail off to the left and has a longer left tail, the shape is said to be left skewed.
- Data sets that show roughly equal trailing off in both directions are called **symmetric**.

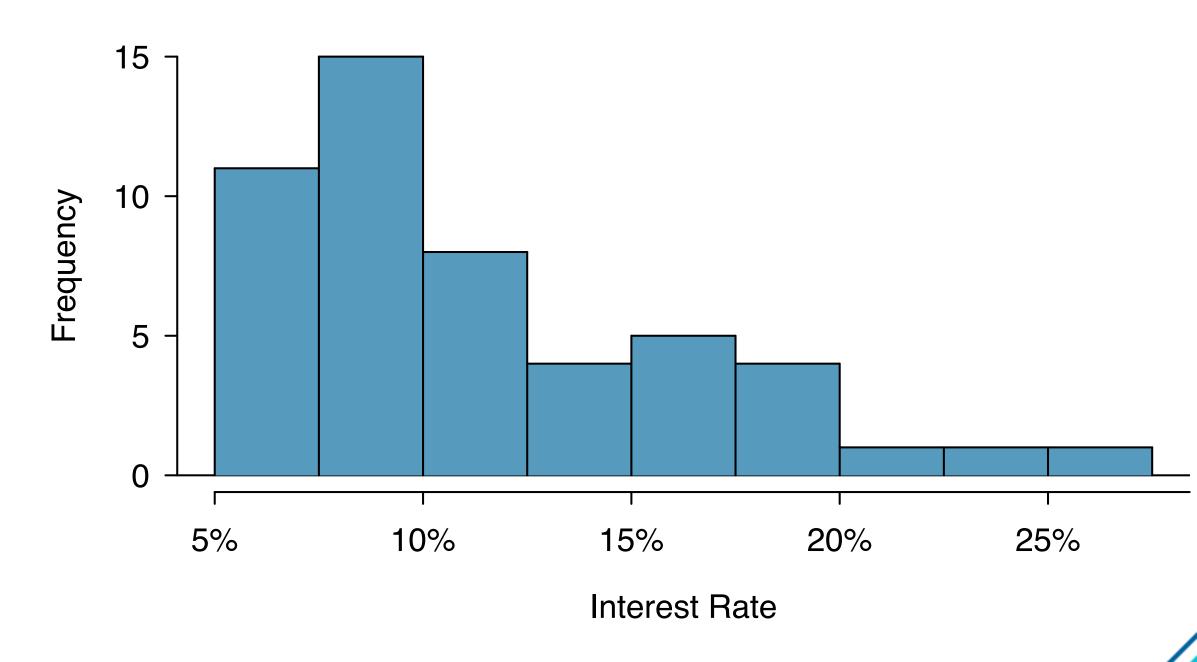


Figure from Open-intro Statistics textbook, Chapter 1.

Sample Mean (or Average)

- The mean (or average), is a common way to measure the center of a distribution of data.
- To compute the mean of a variable, we add up all the case values and divide by the number of observations, say, *n*:

$$\bar{x} := \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

• Example: to compute the mean interest rate, we add all the interest rates and divide by the number of observations

$$\bar{x} := \frac{10.9 + 9.92 + \dots + 6.08}{50} = 11.57 \%$$

Sample Mean (or Average)

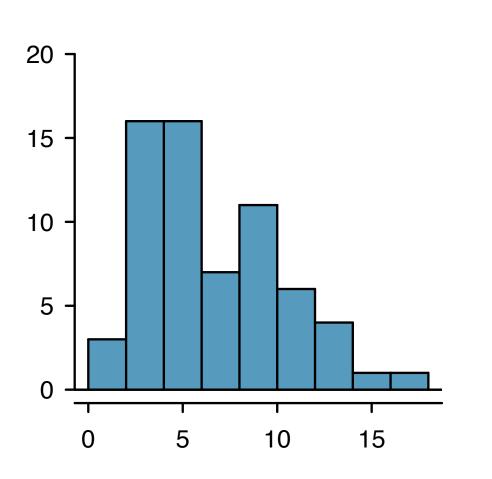
 Knowing the mean of a set of data can be useful because it allows us to make comparisons.

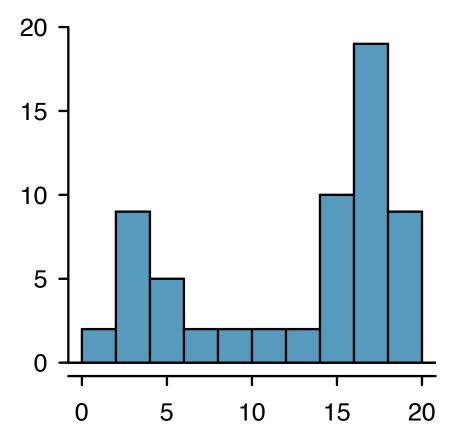
For example:

- We can compare average income of people with college degrees to the average income of people without college degrees.
- We can compare average increase/decrease in blood pressure of people taking a certain drug to the average increase/decrease in blood pressure of people taking a placebo.

Mode

- The mode of a data set, strictly speaking, is the value that occurs the most.
- Instead, we think of it as a value (or bin) with a prominent peak in the distribution





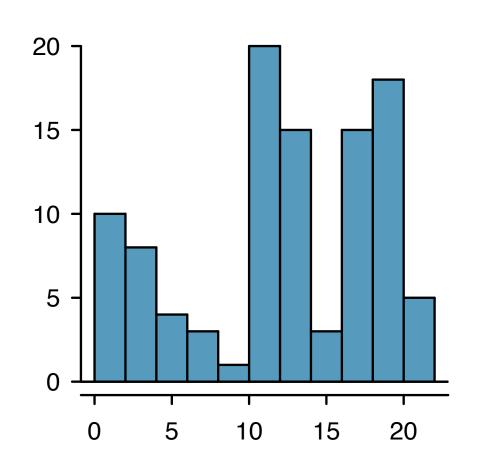


Figure from Open-intro Statistics textbook, Chapter 1.

- Figure shows histograms that have one, two, or three prominent peaks.
- Such distributions are called unimodal, bimodal, and multimodal, respectively.

Variance and Standard Deviation

- The mean, roughly speaking, describes the center of a data set.
- Variability is also of interest: How much does data deviate from the mean?
- Deviation of one data point = distance from the mean = $x_i x$
- Sample variance:

$$s^{2} := \frac{1}{n-1} \sum_{i=1}^{n} (x_{i} - \bar{x})^{2}$$

- Standard deviation is the square root of variance, denoted s.
- Useful in considering how far the data are distributed away from the mean.

Mean and Standard Deviation

- Different data can have very similar (even identical) means and standard deviations.
- The distributions on the right all have mean 0, and standard deviation 1.

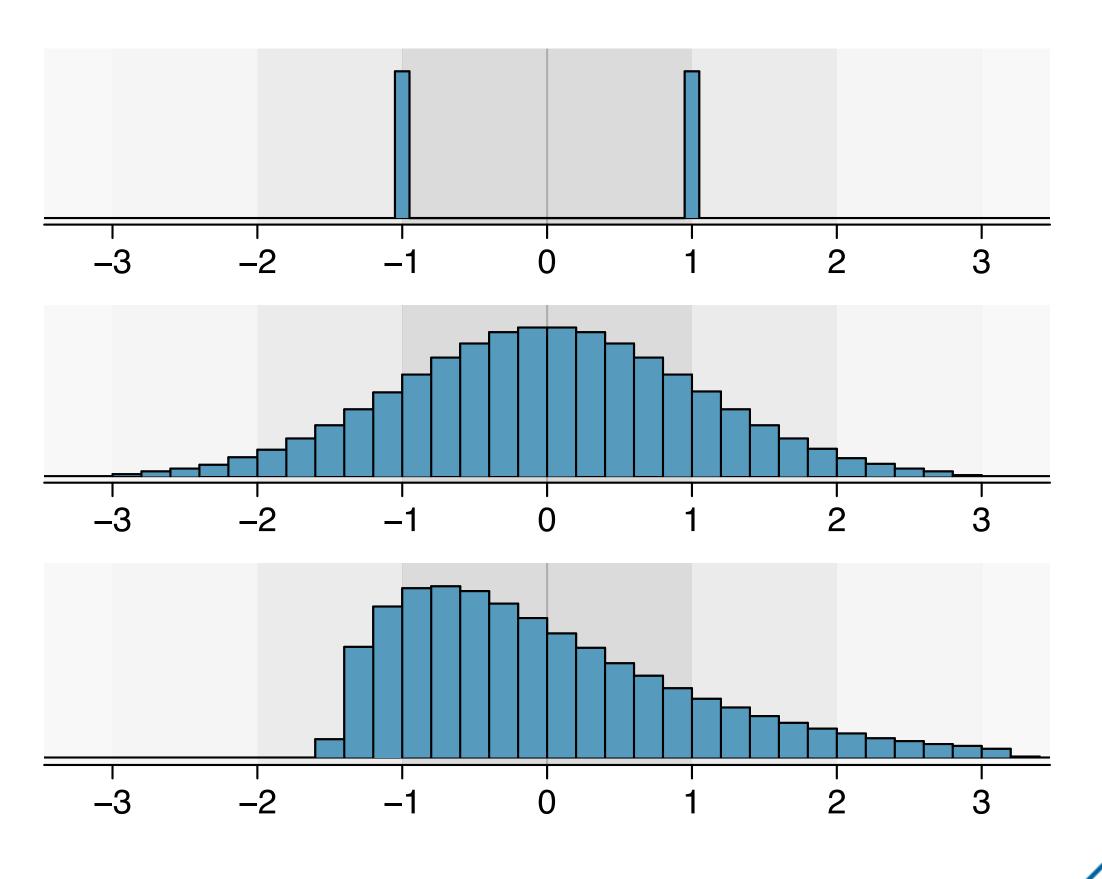


Figure from Open-intro Statistics textbook, Chapter 2.

The Median

- The median of a data set of size n is the number "in the middle" so $\sim 50\%$ of the data lie above it, and 50% lie below it.
- More precisely, if we sort the data (in increasing or decreasing order, the median is
 - the number in the middle, if n is odd. So 5 is the median of $\{1,7,5\}$
 - the average of the two numbers in the middle, if n is even. So 5.5 is the median of $\{1,7,5,6\}$.
- The median is a robust statistic: extreme observations have little effect on it.
- To see this: consider the mean net worth of 1000 individuals, and the median.
 - How does each change if Bill Gates was in the sample?

Quartiles and the Interquartile Range (IQR)

- Quartiles, like the median, are robust statistics.
- The first quartile (Q_1) is the middle number between minimum and median of a data set. 25% of the data lies below it.
- The second quartile (Q_2) is simply the median. 50% of the data lies below it.
- The **third quartile** (Q_3) is the middle number between median and maximum of a data set. 75 % of the data lies below it.
- Interquartile range (IQR): $IQR = Q_3 Q$

Box Plots

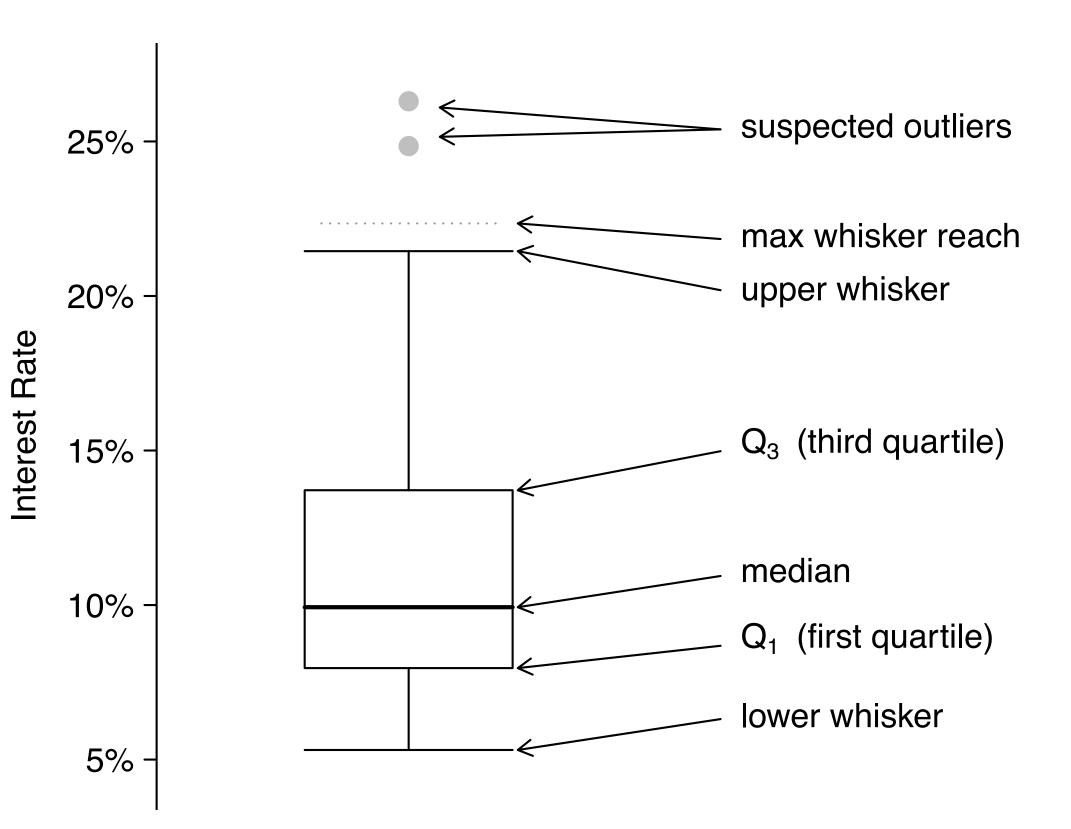
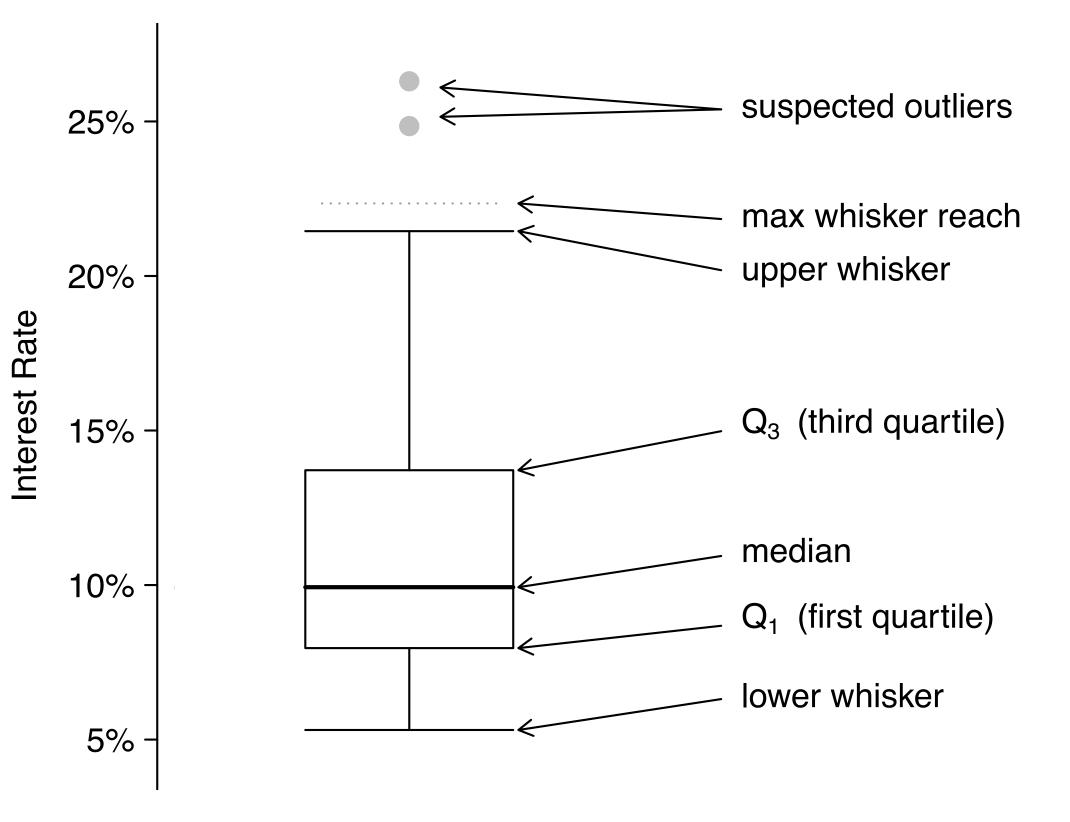


Figure from Open-intro Statistics textbook, Chapter 2.

- A box plot summarizes the data using 5 statistics:
- The median
 - Q_1 and Q_3
 - Upper whisker: marks largest data point below $Q_3 + 1.5 \times IQR \leftarrow \max$ whisker reach
 - Lower whisker: marks smallest data point above $Q_1 1.5 \times IQR \leftarrow \text{min whisker reach}$
- If no data lies below the lower whisker (or above the upper whisker), no need to show that reach.
- It often also plots suspected outliers:
 observations lying beyond the whiskers
 (unusually distant observations).

Outliers



• An **outlier** is an observation that appears extreme relative to the rest of the data.

- Examining data for outliers is important in:
 - Identifying strong skew in the distribution.
 - Identifying possible data collection or data entry errors.
 - Providing insight into interesting properties of the data.

Figure from Open-intro Statistics textbook, Chapter 2.