Module 6 Solutions

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1. Consider the following situations for which we would like to conduct rigorous hypothesis testing. Write the corresponding null and alternative hypotheses:

(a) The dropout rate for undergraduate college students in the U.S. in 2018 was 40%. We are interested in whether the percentage has changed.

Solution: Let p be the proportion of undergraduate college students who drop out. We test $H_0 : p = 0.4$ against $H_A : p \neq 0.4$.

(b) A company is testing a new drink and they will introduce it to the market if the percentage of people who like the flavor exceeds 65%.

Solution: Let p be the proportion of people that favor this newly developed drink. We test $H_0 : p = 0.65$ against $H_A : p > 0.65$.

2. A hypothesis test is conducted to investigate whether the proportion of adults that own a cell phone is more than that from two decades ago, the p-value is computed as 0.015 at significance level 0.05. Do you think they should reject or fail to reject the null hypothesis? Is it reasonable to conclude that more adults have phones now? Explain.

Solution: Since the p-value is less than $\alpha = 0.05$, we reject the null hypothesis that the proportion is unchanged, and have strong evidence to conclude that more adults have phones now.

3. A hypothesis test based on a 95% confidence interval is conducted to investigate whether the proportion of residents in a small town who support a certain law is 68% or not. A random sample of 200 residents were chosen and it is found that 140 of them do support the law.

(a) Check the conditions required in order to proceed with the test.

Solution: This sample is taken randomly. The S/F condition: $n\hat{p} = 140 \geq 10; n(1-\hat{p}) = 60 \geq 10$.

(b) Compute the 95% confidence interval, interpret it in the context, and make a conclusion of whether you should reject or fail to reject the null hypothesis.

Solution: $SE = \sqrt{\frac{0.7\times0.3}{200}} = 0.032$; $I = (\hat{p} - z^* \times SE, \hat{p} + z^* \times SE) = (0.7 - 1.96 \times 0.032, 0.7 + 1.96 \times 0.032) = (0.636, 0.764)$. We are 95% confident that the proportion of all residents in the small town who support a piece of law is between 63.6% and 76.4%. Since 68% is in this interval, the data does not provide enough evidence to reject the null hypothesis(at 95% confidence level).

4. Some students in a statistics class would like to know whether more than 80% of people in the U.S. are right handed. The students took a random sample of 100 people and found out that 87 claimed they are right-handed. The students then conducted a hypothesis test below. Identify all of the mistakes (if any) in each of the following steps. Hint: they made a lot of mistakes.

- First, they identified the null and alternative hypotheses as $H_0 : \hat{p} = 0.8$ and $H_A : \hat{p} < 0.8$
- Then, the checked the conditions for using the CLT approximation: Random sample; $n\hat{p} = 87 \geq 10$.
- Then, they calculated $\frac{87}{100} = 0.87$; $SE = \sqrt{\frac{0.87\times0.13}{100}} = 0.034$
  $z = \frac{0.87-0.8}{0.034} = 2.06$
  p-value: $P(Z > 2.06) = 0.020$
- Finally, they concluded that we should reject the null hypothesis at significance level $\alpha = 0.05$, and we have strong evidence that more than 80% people are right handed in the U.S..

Solution: 1. It should be p in the hypotheses, not $\hat{p}$. In the alternative hypothesis, it should be $>$ instead of $<$.
2. Conditions missed: $n(1 - \hat{p}) = 13 \geq 10$.
3. For the standard error, it should be $SE = \sqrt{\frac{0.8\times0.2}{100}} = 0.04$
Consequently, the z-score should be calculated as: $z = \frac{0.87-0.8}{0.04} = 1.75$ and the p-value should be calculated as: $P(Z > 1.75) = 0.04$
4. Their conclusion is valid based on their calculation.

5. A jury is trying to decide whether a suspect P is guilty or not guilty. When would Type I and Type II error occur?

Solution: In this problem, H_0 : P is not guilty; H_A : P is guilty. Then Type I error occurs when the jury judges P guilty while P is actually not guilty. Type II error occurs when the jury judges P not guilty when P is guilty.

6. A hypothesis test is conducted based on $H_0 : \mu = 39; H_A : \mu > 39$. The null hypothesis was rejected. Later μ is found to be 38.9. Do you think a Type I error occurred, a Type II error occurred, or neither?

Solution: The null hypothesis is rejected when it is actually true. A Type I error occurred.