ONLINE MASTERS IN **DATA SCIENCE**

DSC 215 - PROBABILITY AND STATISTICS FOR DATA SCIENCE

EXPECTATION, VARIANCE AND STANDARD DEVIATION

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Expectation (Expect Values) - Discrete RVs

- Let X be a discrete random variable, taking values in some set S, and suppose X has probability mass function (pmf) $p_X(x)$.
  Then we define the **expectation (or expected value) of X as**

$$\mathbb{E}(X) = \sum_{x \in S} x p_X(x)$$

- **Example**: If X is the random variable associated with the rolling of a fair die, then

$$\mathbb{E}(X) = \sum_{i=1}^{6} i \times \frac{1}{6} = \frac{1}{6} + \ldots + \frac{6}{6} = 3.5$$

Expectation (Expect Values) - Discrete RVs

- More generally, if X is a discrete random variable, and $g(X)$ is a real valued function of X, then we define

$$\mathbb{E}(g(X)) = \sum_{x \in S} g(x) \times p_X(x)$$

- **Example**: If X is the random variable associated with the rolling of a fair die, then

$$\mathbb{E}(X^2) = \sum_{i=1}^{6} i^2 \times \frac{1}{6} = \frac{1^2}{6} + \ldots + \frac{6^2}{6} \approx 15.1667$$

Expectation (Expect Values) - Continuous RVs

- Let X be a continuous random variable whose pdf is $f_X(x)$. Then we define the **expectation (or expected value) of X as**

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x\, f_X(x)dx$$

- **Example:**
  - Let X be a uniform random variable, so its pdf is $f_X(x) = \begin{cases} 1, & \text{if} \quad x \in [0,1] \\ 0, & \text{otherwise.} \end{cases}$
  - Then $\mathbb{E}(X) = \int_{-\infty}^{\infty} x\, f_X(x)dx = \int_0^1 xdx = 1/2.$

Expectation (Expect Values) - Continuous RVs

- By the same token, if g is a real valued function of a continuous random variable X whose pdf is $f_X(x)$. Then we define

$$\mathbb{E}(g(X)) = \int_{-\infty}^{\infty} g(x)\, f_X(x)dx$$

- **Example**:
  - Let X be a uniform random variable whose pdf is $f_X(x) = \begin{cases} 1, & \text{if} \quad x \in [0,1] \\ 0, & \text{otherwise.} \end{cases}$
  - Then $\mathbb{E}(X^2 + 1) = \int_{-\infty}^{\infty} (x^2 + 1)\, f_X(x)dx = \int_0^1 (x^2 + 1)dx = 4/3.$

Linearity of Expectation

- By its definition as an integral (continuous rv's) or a sum (discrete rv's), the **expectation is linear.**
- Thus, if X and Y are random variables, while g, h are functions and a, b are numbers, we have
  - $\mathbb{E}(a \; g(X) + b \; h(Y)) = a\mathbb{E}(g(X)) + b\mathbb{E}(h(Y))$
- **Example**:
  - Suppose the expected sales price of an apple is $\$1$, while the expected sales price of an orange is $\$2$.
  - Then the expected sale price of 2 apples and 3 oranges is

    $2 \times 1 + 3 \times 2 = \$8.$

Variance and Standard Deviation

- Previously we saw the variance and standard deviation for a data set.
- Similarly, **variance and standard deviation** can be used to describe the variability of a random variable.
- For a r.v. X with expected value $\mu = \mathbb{E}(X)$, **the variance of X is defined as**

$$\sigma^2 = \mathbb{E}\big((x-\mu)^2\big).$$

- Often we denote the variance of a random variable X, by $Var(X)$.
- **The standard deviation of X**, labeled σ, is the square root of the variance.

Variance and Standard Deviation

- **Example**: Let X be the random variable associated with the rolling of a fair die, then

$$\sigma^2 = \mathbb{E}\left((X-\mu)^2\right) = \sum_{i=1}^{6} (i-3.5)^2 \times \frac{1}{6} = 105/36$$

and $$\sigma = \sqrt{105/36} \approx 1.7078.$$

- **Example**: Let X be the random variable with pdf $f_X(x) = \begin{cases} 1, & \text{if } \quad x \in [0,1] \\ 0, & \text{otherwise.} \end{cases}$

Then $$\sigma^2 = \mathbb{E}\left((X-\mu)^2\right) = \int_0^1 (x-\frac{1}{2})^2 dx = \frac{1}{12}$$

and $$\sigma = \sqrt{1/12}.$$

Properties of Variance

- The variance satisfies $\sigma^2 = \mathbb{E}\big((X-\mu)^2\big) = \mathbb{E}(X^2) - \mu^2$.
- We saw that the expectation of a sum is essentially the sum of expectations, i.e.:

  $$\mathbb{E}(a\ g(X) + b\ h(Y)) = a\mathbb{E}(g(X)) + b\mathbb{E}(h(Y))$$

- **What can we say about the variance**: $\mathrm{Var}(a\ g(X) + b\ h(Y))$?
- From the definition of variance, and ***provided X and Y are independent***:

  $$\mathrm{Var}(a\ g(X) + b\ h(Y)) = a^2\mathrm{Var}(g(X)) + b^2\mathrm{Var}(h(Y))$$