Module 10 Solutions

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1. A student is interested in the times it takes (in minutes) to get to campus using three different routes. She would like to conduct a hypothesis test to see whether the mean times are equal. The data she collected is shown in the table below:

Statistics	Route 1	Route 2	Route 3
	30	27	16
	32	29	41
	27	28	22
	35	36	31
$\bar{x}$			

(a) What are the null and alternative hypotheses?

Solution: $H_0 : \mu_1 = \mu_2 = \mu_3$

H_A : At least one mean is different.

Here, μ_i represents the mean times it takes to get to campus using route i, where $i \in \{1, 2, 3\}$.

(b) How many degrees of freedom does the Mean Squared Error(MSE) have? What about the degrees of freedom of the Mean Squared between Groups(MSG)? Fill in the table for the unknown values.

Solution: $df_{MSG} = k - 1 = 3 - 1 = 2$; $df_{MSE} = n - k = 12 - 3 = 9$

Statistics	Route 1	Route 2	Route 3
	30	27	16
	32	29	41
	27	28	22
	35	36	31
$\bar{x}_i$	31	30	27.5

(c) Suppose we know that all the conditions are met, compute the F-ratio.

Solution: It can be computed that $\bar{x} = \frac{31+30+27.5}{3} = 29.5$

$$\begin{aligned} MSG &= \frac{1}{k-1}\sum_{i=1}^{k} n_i\left(\bar{x}_i - \bar{x}\right)^2 \\ &= \frac{1}{2}[4(31-29.5)^2 + 4(30-29.5)^2 + 4(27.5-29.5)^2] \\ &= 13 \end{aligned}$$

$$\begin{aligned} MSE &= \frac{1}{n-k}\left(\sum_{i=1}^{n}(x_i-\bar{x})^2 - \sum_{i=1}^{k} n_i\left(\bar{x}_i-\bar{x}\right)^2\right) \\ &= \frac{1}{9}[((30-29.5)^2+(32-29.5)^2+(27-29.5)^2+\dots) - (4(31-29.5)^2+4(30-29.5)^2+4(27.5-29.5)^2)] \\ &= \frac{1}{9}[467-26] \\ &= 49 \end{aligned}$$

$$F = \frac{MSG}{MSE} = \frac{13}{49} \approx 0.265$$

(d) What is the p-value? What would you conclude at significance level $\alpha = 0.05$?

Solution: We use R to compute: $p - \text{value} = P(F \geq 0.265) = \text{pf}(0.265, 2, 9, \text{lower.tail} = \text{FALSE}) \approx 0.7730$ Since $0.7730 > \alpha = 0.05$, we fail to reject the null hypothesis and do not have enough evidence against the null.

2. Random players from four basketball teams were chosen and their heights are taken (in centimeters) in the table below. We would like to conduct a hypothesis test to see whether the mean heights across teams are the same.

Statistics	Team 1	Team 2	Team 3	Team 4
	190	198	210	200
	194	200	190	193
	208	206	196	192
	198	194	191	205
$\bar{x}_i$				

(a) What are the null and alternative hypotheses?

Solution: $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_A : At least one mean is different.

Here, μ_i represents the mean heights of team i, where $i \in \{1, 2, 3, 4\}$.

(b) How many degrees of freedom does the Mean Squared Error(MSE) have? What about the degrees of freedom of the Mean Squared between Groups(MSG)? Fill in the table for the unknown values.

Solution: $df_{MSG} = k - 1 = 4 - 1 = 3$; $df_{MSE} = n - k = 16 - 4 = 12$

Statistics	Team 1	Team 2	Team 3	Team 4
	190	198	210	200
	194	200	190	193
	208	206	196	192
	198	194	191	205
$\bar{x}_i$	197.5	199.5	196.75	197.5

(c) Suppose we know that all the conditions are met, compute the F-ratio.

Solution: It can be computed that $\bar{x} = \frac{197.5+199.5+195.75+197.5}{4} = 197.56$

$$\begin{aligned} MSG &= \frac{1}{k-1}\sum_{i=1}^{k} n_i\left(\bar{x}_i - \bar{x}\right)^2 \\ &= \frac{1}{3}[4(197.5-197.56)^2 + 4(199.5-197.56)^2 + 4(196.75-197.56)^2 + 4(197.5-197.56)^2] \\ &= 5.903 \end{aligned}$$

$$\begin{aligned} MSE &= \frac{1}{n-k}\left(\sum_{i=1}^{n}(x_i-\bar{x})^2 - \sum_{i=1}^{k} n_i\left(\bar{x}_i-\bar{x}\right)^2\right) \\ &= \frac{1}{12}[((190-197.56)^2 + (194-197.56)^2 + (208-197.56)^2 + \dots) - 17.71] \\ &= \frac{1}{12}[178.82 - 17.71] \\ &\approx 13.426 \end{aligned}$$

$$F = \frac{MSG}{MSE} = \frac{5.899}{13.426} \approx 0.44$$

(d) What is the p-value? What would you conclude at significance level $\alpha = 0.10$?

Solution: We use R to compute: $p-\text{value} = P(F \geq 0.440) = \text{pf}(0.440, 3, 12, \text{lower.tail} = \text{FALSE}) \approx 0.7286$ Since $0.7730 > \alpha = 0.10$, we fail to reject the null hypothesis and do not have enough evidence against the null.