ONLINE MASTERS IN **DATA SCIENCE**

DSC 257R - UNSUPERVISED LEARNING

ℓ_p NORMS

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Measuring distance in $\mathbb{R}^m$

Usual choice: **Euclidean distance:**

$$\|x - z\|_2 = \sqrt{\sum_{i=1}^{m} (x_i - z_i)^2}\,.$$

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For $p \geq 1$, here is ℓ_p **distance:**

$$\|x - z\|_p = \left(\sum_{i=1}^{m} |x_i - z_i|^p \right)^{1/p}$$

- $p = 2$: Euclidean distance
- ℓ_1 distance: $\|x - z\|_1 = \sum_{i=1}^{m} |x_i - z_i|$
- ℓ_∞ distance: $\|x - z\|_\infty = \max_i |x_i - z_i|$

Example 1

Consider the all-ones vector $(1, 1, \ldots, 1)$ in $\mathbb{R}^d$.
What are its ℓ_2, ℓ_1, and ℓ_∞ length?

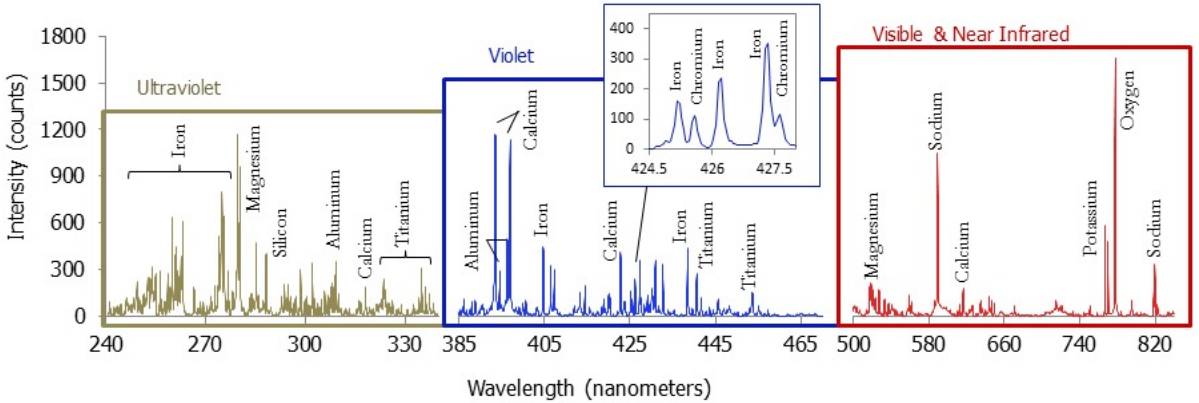
Example 2

In $\mathbb{R}^2$, draw all points with

1. ℓ_2 length 1
2. ℓ_1 length 1
3. ℓ_∞ length 1

Weighted ℓ_1 Norm

For ChemCam, what if we want to emphasize/de-emphasize certain elements?



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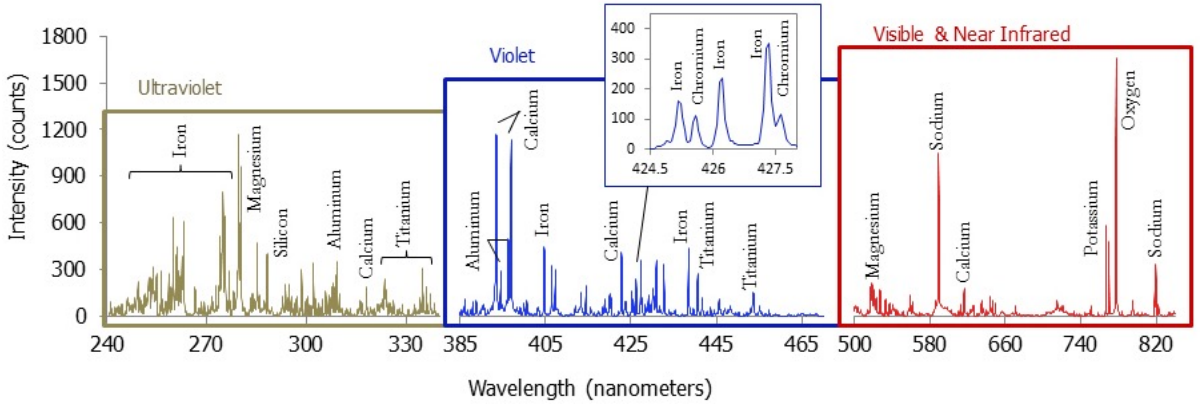

Weighted ℓ_1 norm between x and x':

$$\sum_{i=1}^{m} w_i |x_i - x_i'| .$$

Weighted ℓ_p Norm

How would you define a weighted ℓ_p norm?