Solution 1

Solution 1 (a)

Step 1: Identify characteristics of the dataspace

We are given 10 dimensional vectors where each element can be any real number ($x_i \in \mathbb{R}$):

$\therefore$ we can express the dataspace χ as: $\chi = \mathbb{R}^{10}$

Solution 1 (b)

Step 1: Identify characteristics of the dataspace

We are given 3 dimensional vectors where each element is zero or one ($x_i \in [0, 1]$):

$\therefore$ we can express the dataspace χ as: $\chi = [0, 1]^3$

Solution 2

Solution 2 (a)

Step 1: Define Euclidean distance (ℓ_2)

$$\ell_2 = \|p - q\|_2 = \sqrt{\sum_{i=1}^{n}(p_i - q_i)^2}$$

Step 2: Compute ℓ_2

Let $p = 1$ and $q = 10$

$$\begin{aligned}
\ell_2 &= \sqrt{\sum_{i=1}^{n}(p_i - q_i)^2} \\
\ell_2 &= \sqrt{\sum_{i=1}^{1}(1 - 10)^2} \\
\ell_2 &= \sqrt{(-9)^2} \\
\ell_2 &= 9
\end{aligned}$$

$\therefore \ell_2 = 9$

Solution 2 (b)

Step 1: Define Euclidean distance (ℓ_2)

$$\ell_2 = \|p - q\|_2 = \sqrt{\sum_{i=1}^{n}(p_i - q_i)^2}$$

Step 2: Compute ℓ_2

Let $p = \begin{bmatrix} -1 \\ 12 \end{bmatrix}, q = \begin{bmatrix} 6 \\ -12 \end{bmatrix}$

$$\begin{aligned}
\ell_2 &= \sqrt{\sum_{i=1}^{n}(p_i - q_i)^2} \\
\ell_2 &= \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2} \\
\ell_2 &= \sqrt{(-1 - 6)^2 + (12 - (-12))^2} \\
\ell_2 &= \sqrt{(-7)^2 + (24)^2} \\
\ell_2 &= \sqrt{625} \\
\ell_2 &= 25
\end{aligned}$$

$\therefore \ell_2 = 25$

Solution 2

Solution 2 (c)

Step 1: Define Euclidean distance (ℓ_2)

$$\ell_2 = \|p - q\|_2 = \sqrt{\sum_{i=1}^{n}(p_i - q_i)^2}$$

Step 2: Compute ℓ_2

Let $p = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}, q = \begin{bmatrix} 5 \\ 2 \\ 11 \end{bmatrix}$

$$\begin{aligned}
\ell_2 &= \sqrt{\sum_{i=1}^{n}(p_i - q_i)^2} \\
\ell_2 &= \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2 + (p_3 - q_3)^2} \\
\ell_2 &= \sqrt{(1-5)^2 + (5-2)^2 + (-1-11)^2} \\
\ell_2 &= \sqrt{(-4)^2 + (3)^2 + (-12)^2} \\
\ell_2 &= \sqrt{169} \\
\ell_2 &= 13
\end{aligned}$$

$\therefore \ell_2 = 13$

Solution 3

Solution 3 (a)

Step 1: Normalize the vector x

Let $x = \begin{bmatrix} 10 \\ 15 \\ 25 \end{bmatrix}$

$$\sum_{i=1}^{3} x_i = x_1 + x_2 + x_3 = 10 + 15 + 25 = 50$$

Now, divide each entry by the total sum:

$$p = \frac{1}{50} \cdot x = \frac{1}{50} \begin{bmatrix} 10 \\ 15 \\ 25 \end{bmatrix} = \begin{bmatrix} 10/50 \\ 15/50 \\ 25/50 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.5 \end{bmatrix}$$

$\therefore$ the result (p) of scaling vertor x is the following:

$$p = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.5 \end{bmatrix}$$

Solution 3 (b)

Step 1: Define dimension of the probability simplex

The dimension of vector p is 3 and $k = n - 1$ where k is the dimension of the probability simplex

$\therefore$ vector p lies in the probability simplex(Δ_2) for $k = 2$