ONLINE MASTERS IN **DATA SCIENCE**

DSC 257R - UNSUPERVISED LEARNING

KEY PROPERTIES OF THE MEAN & VARIANCE

SANJOY DASGUPTA, PROFESSOR

UC San Diego

COMPUTER SCIENCE & ENGINEERING

HALICIOĞLU DATA SCIENCE INSTITUTE

Linear Functions of a Single Random Variable

- If you double a set of numbers, how are their mean and variance affected?

- If you increase a set of numbers by 1, how much do their mean and variance change?

- Let X be any random variable.
  For some constants, $a, b,$ define a new random variable $V = aX + b$.
  Express $\mathbb{E}(V)$ and $var(V)$ in terms of $\mathbb{E}(X)$ and $var(X)$.

Linearity of Expectation

A powerful and extremely useful property:

Linearity of expectations: For any random variable $X_1, \ldots, X_m$,

$$\mathbb{E}(X_1 + X_2 + \cdots + X_m) = \mathbb{E}(X_1) + \mathbb{E}(X_2) \cdots + \mathbb{E}(X_m).$$

Linearity of Variance

We've seen that $\mathbb{E}(X + Y) = \mathbb{E}(X) + \mathbb{E}(Y)$.
Is this also true of variance, i.e., is $var(X + Y) = var(X) + var(Y)$?

- In general, **no**. Give a counterexample.

- But it is true if X and Y are **independent**.