ONLINE MASTERS IN **DATA SCIENCE**

DSC 257R - UNSUPERVISED LEARNING

METRICS

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Metric Spaces

Let $\mathcal{X}$ be the space in which data lie.

A distance function $d : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ is a **metric** if it satisfies these properties:

- $d(x, y) \geq 0$ (nonnegativity)
- $d(x, y) = 0$ if and only if $x = y$
- $d(x, y) = d(y, x)$ (symmetry)
- $d(x, z) \leq d(x, y) + d(y, z)$ (triangle inequality)

Example 1

$\mathcal{X} = \mathbb{R}^m$ and $d(x, y) = \|x - y\|_p$

Check:

- $d(x, y) \geq 0$ (nonnegativity)
- $d(x, y) = 0$ if and only if $x = y$
- $d(x, y) = d(y, x)$ (symmetry)
- $d(x, z) \leq d(x, y) + d(y, z)$ (triangle inequality)

Example 2

$\mathcal{X}$ = {strings over some alphabet} and d = edit distance

Check:

- $d(x,y) \geq 0$ (nonnegativity)
- $d(x,y) = 0$ if and only if $x = y$
- $d(x,y) = d(y,x)$ (symmetry)
- $d(x,z) \leq d(x,y) + d(y,z)$ (triangle inequality)

A Non-metric Distance Function

Let p, q be probability distributions on some set of $\mathcal{X}$.

The **Kullback-Leibler divergence** or **relative entropy** between p, q is:

$$d(p, q) = \sum_{x \in \chi} p(x) \log \frac{p(x)}{q(x)}.$$

Other Classes of Distance Functions

- Bregman divergences
- F-divergences
- $\vdots$