1 Algorithm: Backpropagation to Input

The computation of $\nabla_x a^{[L]}(x)$ follows the chain rule and can be efficiently done using the backpropagation algorithm:

Algorithm 1 Compute $\nabla_x a^{[L]}(x)$

1: Perform a forward pass to compute $z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$ and $a^{[l]} = \sigma(z^{[l]})$ for all layers $l = 2, \ldots, L$.
2: Initialize $\delta^{[L]} = 1$ since $a^{[L]}$ is a scalar output.
3: **for** $l = L, L-1, \ldots, 2$ **do**
4: $\quad \delta^{[l-1]} = (W^{[l]})^T \big(\sigma'(z^{[l]}) \odot \delta^{[l]}\big)$
5: **end for**
6: Output (strict form): $\nabla_x a^{[L]}(x) = (W^{[2]})^T \big(\delta^{[2]} \odot \sigma'(z^{[2]})\big)$
7: Output (notation simplification): $\nabla_x a^{[L]}(x) \equiv \delta^{[1]}$
