NM08

- polynomial interpolation

$n+1$ distinct points 가 주어지면

$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$ 을 구함.

↓ 대입하면

$n+1$개의 식과 $n+1$개의 unknown a 가 있음.

$Va = y$, $V = [x_i^j]$, $a = [a_0 \cdots a_n]^T$, $y = [y_0 \cdots y_n]^T$

↓ vandermonde matrix

$$\begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{bmatrix}$$

가우시안 소거법

$$\begin{bmatrix} & \diagdown & \\ 0 & & \end{bmatrix}$$

$\diagdown$ 만 곱하면 det

$$\det(V) = \prod_{0 \le i < j \le N} (x_j - x_i)$$

- lagrange interpolation

$$P_n(x) = \sum_{i=0}^{n} f(x_i) L_i(x) \qquad L_j(x_i) = \begin{cases} 1 & i=j \\ 0 & i \ne j \end{cases}$$

$$L_i(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)}$$

$$\therefore \; L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \left[\frac{x - x_j}{x_i - x_j} \right] \qquad i = 0, \cdots, n$$

- newton's interpolation

divided difference table