Set Theory

Mutually Exclusive = disjoint

$A_i \cap A_j = \emptyset \quad , i \neq j$

Collectively Exhaustive

$A_1 \cup A_2 \cup \cdots \cup A_n = S$

Partition

$A_1 \cdots A_n$ 이 Mutually Exclusive 하면서 Collectively Exhaustive 하면.

Outcome

any possible observation of experiment. → **Event**: set of outcomes of experiment.

Sample Space

finest-grain, mutually exclusive, collectively exhaustive set of ALL possible outcomes.

Probability Axioms

- For any event A, $P[A] \geq 0$
- $P[S] = 1$
- $A_1 \cdots A_n$ Mutually exclusive 하다면
  $P[A_1 \cup A_2 \cup \cdots \cup A_n] = P[A_1] + P[A_2] + \cdots + P[A_n]$
- $P[\emptyset] = 0$
- $P[A^c] = 1 - P[A]$
- 일반적인 경우, $P[A \cup B] = P[A] + P[B] - P[A \cap B]$
- $A \subset B$ 이면, $P[A] \leq P[B]$

Conditional Probability

- Probability of event A given B

$$P[A|B] = \frac{P[A\cap B]}{P[B]} = \frac{P[A]P[B|A]}{P[B]}$$

- Axioms
  - $P[A|B] \geq 0$
  - $P[A|A] = 1$
  - $P[A|B] = P[A_1|B] + P[A_2|B] + \cdots + P[A_n|B]$ $A_1 \sim A_n$은 Partition.

Total Probability

For any event A, partition $\{B_1, \cdots B_m\}$

$$\rightarrow P[A] = \sum_{i=1}^{m} P[A \cap B_i]$$

with $P[B_i] > 0$

$$\rightarrow P[A] = \sum_{i=1}^{m} P[B_i]P[A|B_i]$$

$$P[A|B] = \frac{P[A]P[B|A]}{P[B]}$$

Independence

- $P[A \cap B] = P[A] \cdot P[B]$, A and B is Independent.

- 여러개가 독립이려면, 두개씩 모든 쌍이 독립이어야 하고, → mutually independent
  $P[A_1 \cap \cdots \cap A_n] = P[A_1] \cdots P[A_n]$ 도 만족해야 함.

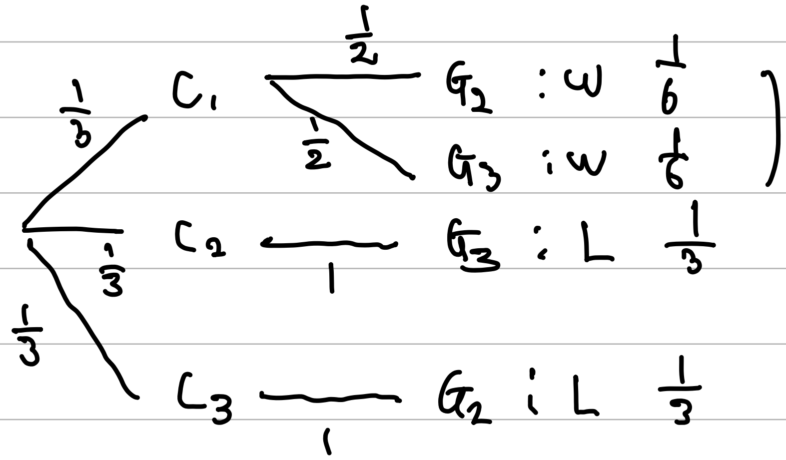
Tree diagrams

- Monty Hall

  C_i : 차가 i에 숨겨짐, G_i : 염소가 i에 있음을 보여줌.

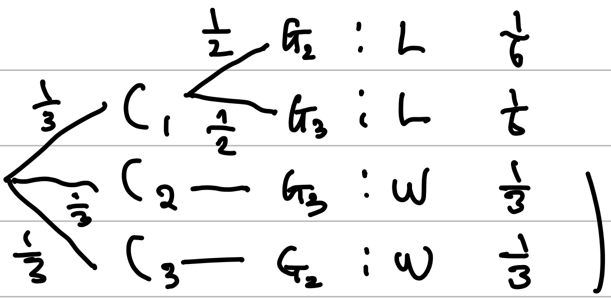
  W : 차를 찾음. L : 염소를 찾음.

<1번문을 선택, 공개후 바꾸지않음>



이길확률 $\frac{1}{3}$

<1번문을 선택, 공개후 바꿈>



이길 확률 $\frac{2}{3}$

Counting Methods.

- n choose k : $\binom{n}{k} = \begin{cases} \frac{n!}{k!(n-k)!}, & k=0,\dots,n \\ 0 & \text{otherwise.} \end{cases}$

- Multinomial Coefficient :

$$\binom{n}{n_0,\cdots,n_{m-1}} = \begin{cases} \frac{n!}{n_0!n_1!\cdots n_{m-1}!} & n=n_0+n_1+\cdots+n_{m-1} \\ & n_i \in \{0,1,\cdots n\}, i=0,\cdots,m-1 \\ 0 & \text{Otherwise.} \end{cases}$$

Independent Trials

- $P[E_{n_0,n_1}] = \binom{n}{n_1} p^{n_1} (1-p)^{n-n_1} = \binom{n}{n_0} (1-p)^{n_0} p^{n-n_0}$

- $P[E_{n_0,\cdots,n_{m-1}}] = \binom{n}{n_0,\cdots,n_{m-1}} p_0^{n_0} \cdots p_{m-1}^{m-1}$

↑

Multinomial Coefficient