7. Conditional Probability Model

- Conditional CDF

Event B가 P[B] > 0 일 때 $F_{X|B}(x) = P[X \le x \mid B]$

- Conditional PMF

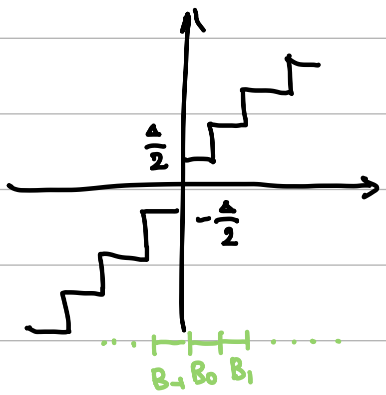
$$P_{X|B}(x) = P[X = x \mid B] = \frac{P[\{X = x\} \cap B]}{P[B]}$$

$$= \begin{cases} \frac{P_X(x)}{P[B]} & x \in B \\ 0 & \text{o.w.} \end{cases}$$

- Conditional PDF

$$f_{X|B}(x) = \frac{dF_{X|B}(x)}{dx} = \begin{cases} \frac{f_X(x)}{P[B]} & x \in B \\ 0 & \text{o.w.} \end{cases}$$

- X : uniform $(\frac{-r}{2}, \frac{r}{2})$



b-bits $n = 2^b$

b=3 이라면 $n = 2^3 = 8$

$\Delta = \frac{r}{n} = \frac{r}{8}$

$$f_X(x) = \begin{cases} \frac{1}{r} & -\frac{r}{2} \le x < \frac{r}{2} \\ 0 & \text{o.w.} \end{cases}$$

event $B_i = \{ i\Delta \leq X < (i+1)\Delta \}$

$$P[B_i] = \int_{i\Delta}^{(i+1)\Delta} \frac{1}{r}\, dx = \frac{\Delta}{r} = \frac{1}{n} = \frac{1}{8}$$

$$f_{X|B_i}(x) = \begin{cases} \frac{f_X(x)}{P[B_i]} & , x \in B_i \\ 0 & , \text{o.w.} \end{cases} = \begin{cases} \frac{1}{\Delta} & i\Delta \leq x < (i+1)\Delta \\ 0 & , \text{o.w.} \end{cases}$$

Total Probability

Partition $B_1, \cdots, B_n$ 에 대해서

- $P_X(x) = \sum_{i=1}^{m} P_{X|B_i}(x)\, P[B_i]$
- $f_X(x) = \sum_{i=1}^{m} f_{X|B_i}(x)\, P[B_i]$

Conditional Expected Value

- $E[X|B] = \sum_{x \in B} x\, P_{X|B}(x)$
- $E[X|B] = \int_{-\infty}^{\infty} x\, f_{X|B}(x)\, dx$

- for partition $B_1, \cdots, B_m$

$$E[X] = \sum_{i=1}^{m} E[X|B_i] P[B_i]$$

- $Y = g(x)$

$$E[Y|B] = \sum_{x \in B} g(x) P_{X|B}(x)$$

$$E[Y|B] = \int_{-\infty}^{\infty} g(x) f_{X|B}(x)\, dx$$

Conditional Variance

$$Var[X|B] = E[(X - \mu_{X|B})^2 | B] = E[X^2 | B] - \mu_{X|B}^2$$

Conditional Joint PMF

$$P_{X,Y|B}(x,y) = P[X=x, Y=y | B]$$

$$= \begin{cases} \dfrac{P_{X,Y}(x,y)}{P[B]} & (x,y) \in B \\ 0 & \text{o.w.} \end{cases}$$

Conditional Expected Value

$W = g(X, Y)$

$$E[W \mid B] = \sum_{x \in S_X} \sum_{y \in S_Y} g(x,y)\, P_{X,Y|B}(x,y)$$

$$E[W \mid B] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)\, f_{X,Y|B}(x,y)\, dx\, dy$$

Conditioning by RV

$$P_{X|Y}(x|y) = P[X = x \mid Y = y] = \frac{P_{X,Y}(x,y)}{P_Y(y)}$$

$$P_Y(y) \cdot P_{X|Y}(x|y) = P_{X,Y}(x,y)$$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$f_Y(y) \cdot f_{X|Y}(x|y) = f_{X,Y}(x,y)$$