NM 14

— Romberg

$$n = 2^k, \quad h = \frac{b-a}{2^k}$$

$$R_{n,0} = \frac{b-a}{2^{k+1}}\left(f(a)+f(b)\right) + \frac{b-a}{2^k}\sum_{i=1}^{2^k-1} f\left(a+\frac{b-a}{2^k}i\right)$$

$$R_{0,0} = \frac{b-a}{2}\left(f(a)+f(b)\right)$$ 점 2개

$$R_{1,0} = \frac{R_{0,0}}{2} + \frac{b-a}{2} f\left(a+\frac{b-a}{2}\cdot 1\right)$$ 점 3개

(그냥 사다리꼴 사용해도 OK)

$$R_{2,0} = \frac{R_{1,0}}{2} + \frac{b-a}{4}\left(f\left(a+\frac{b-a}{4}\cdot 1\right) + f\left(a+\frac{b-a}{4}\cdot 3\right)\right)$$

$$R_{n,m} = \frac{4^m R_{n,m-1} - R_{n-1,m-1}}{4^m - 1} = R_{n,m-1} + \frac{1}{4^m-1}\left(R_{n,m-1} - R_{n-1,m-1}\right)$$

$R_{n,m}$는 에러 $O(h^{2(m+1)})$

~ gaussian quadrature

표준구간 $[-1, 1]$ 변환 $x = \frac{b-a}{2}z + \frac{a+b}{2}$

$$= a + \frac{b-a}{2}(z+1)$$

$$\therefore z = \frac{2x-a-b}{b-a}$$

$$\int_a^b f(x)dx = \frac{b-a}{2}\int_{-1}^{1} g(z)\,dz \approx \int_{-1}^{1} p(z)\,dz = \sum_{i=0}^{N} w_i\, g(z_i)$$

① $N=1$ (2개 노드)

$$\int_{-1}^{1} p(z)\,dz = w_0\, g(z_0) + w_1\, g(z_1)$$

차수 $2N+1$ 까지 exact

$g(z)=1$	$w_0\cdot 1 + w_1\cdot 1 = \int_{-1}^{1} 1\,dz = 2$	
z	$w_0\cdot z_0 + w_1\cdot z_1 = \int_{-1}^{1} z\,dz = 0$	→ 연결
z^2	$w_0\cdot z_0^2 + w_1\cdot z_1^2 = \int_{-1}^{1} z^2\,dz = \frac{2}{3}$	$w_0 = 1$, $w_1 = 1$
z^3	$w_0\, z_0^3 + w_1\cdot z_1^3 = \int_{-1}^{1} z^3\,dz = 0$	$z_0 = -\frac{\sqrt{3}}{3}$, $z_1 = \frac{\sqrt{3}}{3}$

$$\int_{-1}^{1} p(z)\,dz = g\left(-\frac{\sqrt{3}}{3}\right) + g\left(\frac{\sqrt{3}}{3}\right)$$

포인트 수 $-1 = N$, $2N+1$ 까지 exact

또는 $2(\text{포인트 수})-1$

정확도

composite trapezoidal $O(h^2)$ → 1차까지 exact

〃 Simpsons $O(h^4)$ 3차까지

Romberg $R_{n,m}$ $O(h^{2(m+1)})$ → $2m+1$ 까지

Gaussian Quadrature n-point 면 → $2n-1$ 까지