Discrete Random Variable

- X is DRV when range of X, S_X is countable set.

  $S_X = \{x_1, x_2, \cdots\}$

- DRV X 의 PMF : $P_X(x) = P[X=x]$

- for any x, $P_X(x) \geq 0$

- $\sum_{x \in S_X} P_X(x) = 1$

- for event $B \subset S_X$, $P[B] = \sum_{x \in B} P_X(x)$

CDF

- $F_X(x) = P[X \leq x]$

- $F_X(-\infty) = 0$, $F_X(\infty) = 1$

- 모든 $x' \geq x$ 에 대해서 $F_X(x') \geq F_X(x)$

- $F_X(x_i) = F_X(x)$ 라면 $x_i \leq x < x_{i+1}$

- 모든 $b \geq a$ 에 대해서, $P[a < X \leq b] = F_X(b) - F_X(a)$

Expected Value

- $E[X] = \mu_X = \sum_{x \in S_X} x \cdot P_X(x)$

- $E[aX+b] = aE[X]+b$

- $E[g(X)] = \sum_{x \in S_X} g(x) \cdot P_X(x)$

Variance and Standard Deviation

- $Var[X] = E[(X-\mu_X)^2] = E[X^2] - E[X]^2$

- $\sigma_X = \sqrt{Var[X]}$

- for RV X, nth moment is $E[X^n]$
  nth central moment is $E[(X-\mu_X)^n]$

- $Var[aX+b] = a^2 Var[X]$

Families of DRV

Bernoulli

Bernoulli (p) RV X → 성공 혹은 실패, 둘 중 하나

$$P_X(x) = \begin{cases} 1-p & x=0 \\ p & x=1 \\ 0 & \text{otherwise.} \end{cases} \quad (0<p<1)$$

$E[X] = p$

$Var[X] = (1-p)\,p$

Geometric

Geometric (p) RV X → 처음으로 성공할 때 까지 시행한 횟수

$$P_X(x) = \begin{cases} (1-p)^{x-1}\,p & , x = 1, 2, \cdots \\ 0 & \text{otherwise.} \end{cases} \quad (0<p<1)$$

$E[X] = \frac{1}{p}$

$Var[X] = \frac{1-p}{p^2}$

Binomial

Binomial (n, p) RV X → n번중에서 x번 성공할 확률

$$P_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$$ ($0<p<1$, n은 1보다 큰 정수)

$E[X] = np$

$Var[X] = np(1-p)$

Pascal

Pascal (k, p) RV X → k번 성공하기 위해 시도한 횟수.

$$P_X(x) = \binom{x-1}{k-1} p^k (1-p)^{x-k}$$ ($0<p<1$, k는 1보다 큰 정수)

$E[X] = \frac{k}{p}$

$Var[X] = \frac{k(1-p)}{p^2}$

Discreate Uniform

Discreate Uniform (k, l) RV X

$$P_X(x) = \begin{cases} \dfrac{1}{(l-k+1)} & x = k, k+1, k+2, \cdots, l \\ 0 & \text{otherwise.} \end{cases}$$

($k < l$ 인 정수)

$$E[X] = \frac{k+l}{2}$$

$$Var[X] = \frac{(l-k)(l-k+2)}{12}$$

Poisson

Poisson (α) RV X → 일정 기간동안 일어난 횟수, α는 평균횟수

$$P_X(x) = \begin{cases} \dfrac{\alpha^x e^{-\alpha}}{x!} & x = 1, 2, \cdots \\ 0 & \text{otherwise} \end{cases}$$

$(\alpha > 0)$

$$E[X] = \alpha$$

$$Var[X] = \alpha$$

Continuous Random Variables

- RV X is continuous iif S_X consists of intervals.
- $x \in S_X$, $P[X=x] = 0$.
- RV X is continuous if CDF $F_X(x)$ is continous.
- $f_X(x) = \dfrac{dF_X(x)}{dx}$ (즉, CDF의 기울기)
- $f_X(x) \geq 0$ for all x
- $F_X(x) = \int_{-\infty}^{x} f_X(u)\,du$
- $\int_{-\infty}^{\infty} f_X(x)\,dx = 1.$
- $P[x_1 < X \leq x_2] = \int_{x_1}^{x_2} f_X(x)\,dx$

Expected Values

- $E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$

  $E[g(X)] = \int_{-\infty}^{\infty} g(x) \cdot f_X(x)\,dx$

Families of CRV

Uniform

uniform (a,b) RV X

PDF $f_X(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise.} \end{cases}$ $(b>a)$

CDF $F_X(x) = \begin{cases} 0 & x<a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & x>b \end{cases}$

$E[X] = \frac{b+a}{2}$

$Var[X] = \frac{(b-a)^2}{12}$

$\Downarrow$

$K = \lceil X \rceil$, K is Discrete Uniform $(a+1, b)$ RV.

Exponential

exponential (λ) RV X

PDF $f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$ $(\lambda > 0)$

CDF $F_X(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$

$E[X] = \frac{1}{\lambda}$

$Var[X] = \frac{1}{\lambda^2}$

$\Downarrow$

$K = \lceil X \rceil$, K is geometric (p) RV, $p = 1 - e^{-\lambda}$

Erlang

Erlang (n, λ) RV X

PDF $f_X(x) = \begin{cases} \dfrac{\lambda^n x^{n-1} e^{-\lambda x}}{(n-1)!} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$

($\lambda > 0$, n은 1보다 큰 정수)

CDF $F_X(x) = 1 - \sum_{n=0}^{k-1} \frac{1}{n!} e^{-\lambda x} (\lambda x)^n$

$E[X] = \frac{n}{\lambda}$

$Var[X] = \frac{n}{\lambda^2}$