NM10

— Linear least squares

$$E(a,b) \text{ 최소화 } = \sum_{i=1}^{n} \left(f(x_i) - y_i \right)^2$$

$$= \sum_{i=1}^{n} \left(a x_i + b - y_i \right)^2$$

$$\frac{\partial E(a,b)}{\partial a} = 0 , \quad \frac{\partial E(a,b)}{\partial b} = 0$$ 일 때 최소.

$$\sum_{i=1}^{n} 2 (a x_i + b - y_i) x_i = 0 \qquad \sum_{i=1}^{n} 2 (a x_i + b - y_i) = 0$$

$$2 \left((a+b-1) \cdot 1 + (2a+b-1) \cdot 2 + (3a+b-2) \cdot 3 + (4a+b-2) \cdot 4 + (5a+b-4) \cdot 5 \right) = 0$$

$$2 \left(\overset{56}{\cancel{(1+4+9+16+25)}} a + \overset{15}{\cancel{(1+2+3+4+5)}} b - \overset{37}{\cancel{(1+2+6+8+20)}} \right) = 0$$

↓ 연립 방정

$$p = \sum_{i=0}^{n} x_i \quad q = \sum_{i=0}^{n} y_i \quad r = \sum_{i=0}^{n} x_i y_i \quad s = \sum_{i=0}^{n} x_i^2$$

$$\begin{pmatrix} s & p \\ p & n+1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} r \\ q \end{pmatrix}$$

$n+1$: 점 개수

Cramer's rule 사용하여 풀기.

$$D = \det\begin{pmatrix} S & p \\ p & n+1 \end{pmatrix} = (n+1)S - p^2$$

$$a = \frac{1}{D}\det\begin{pmatrix} r & p \\ q & n+1 \end{pmatrix} = \frac{(n+1)r - pq}{D}$$

$$b = \frac{1}{D}\det\begin{pmatrix} S & r \\ p & q \end{pmatrix} = \frac{qS - rp}{D}$$

— Non linear

exponential

$$f(x) = ae^{bx} \rightarrow \underbrace{\ln f(x)}_{F(x)} = \underbrace{\ln a}_{\alpha} + \underbrace{b}_{\beta}x$$

점 (x_i, y_i) 대신 $(x_i, \ln y_i)$ 이용

hyperbolic

$$f(x) = a + \frac{b}{x}$$ $X = \frac{1}{x}$ 대치

$$F(x) = bX + a$$ 점 (x_i, y_i) 대신 $(\frac{1}{x_i}, y_i)$ 대입

— multivariable least squares

optimal $x^* = \min \| b - Ax \|_2^2 = \sqrt{\Sigma(b-Ax)^2}^{\,2}$

잔차 $r = b - Ax$

$r=0 \rightarrow \quad 0 = A^T r = A^T(b - Ax^*) = A^T b - A^T A x^*$

$\therefore A^T A x^* = A^T b \qquad x^* = \underbrace{(A^T A)^{-1} A^T}_{A^+ \text{ (pseudoinverse)}} b$

$\mathrm{cond}(A) = \|A\| \cdot \|A^{-1}\| = \|A\| \cdot \|A^+\|$

$$x^* = A^+ \cdot b$$

또는

$$A^T A x^* = A^T b$$ 사용

↑

구하는것