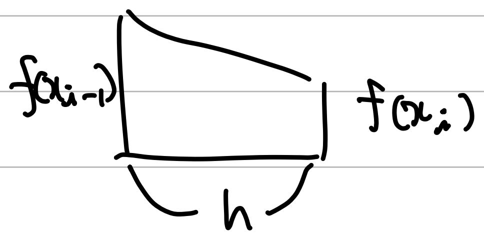
NM 13

- trapezoidal rule (사다리꼴)

uniform interval $[a, b] \rightarrow h = \frac{b-a}{n}$ 총 $n+1$개의 점



$\rightarrow \frac{h}{2}\left(f(x_{i-1}) + f(x_i)\right) = A_i$

전체넓이 $T_n = A_1 + \cdots + A_n = \frac{h}{2}\left(f(x_0)+f(x_1)\right) + \frac{h}{2}\left(f(x_1)+f(x_2)\right) + \cdots$

$= \frac{h}{2}\left(f(x_0)+f(x_n)\right) + h\sum_{i=1}^{n-1} f(x_i)$

Composite trapezoidal

$|E_i| \le \frac{h^3}{12} M$ 전체

$\frac{O(h^3)}{\text{하나}} \rightarrow \frac{O(h^2)}{\text{전체}}$ ∴ (1차일때) exact

$|E_T| \le n\frac{h^3}{12} M$

$= (b-a)\frac{h^2}{12} M$

$M = f''(x)$의 최댓값

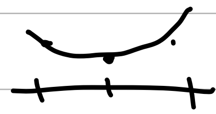
- 예시 $n=1$, $\int_1^3 (2x+1)\,dx$ $h = \frac{3-1}{1} = 2$

$T_n = \frac{h}{2}\left(f(x_0)+f(x_n)\right) + h\sum_{i=1}^{n-1} f(x_i)$

$= \frac{2}{2}\left(f(1)+f(3)\right) = 3+7 = 10$

Simpson's rule

n이 짝수이어야 함.



$$S_i = \frac{h}{3}\left(f(x_{i+1}) + 4f(x_i) + f(x_{i-1})\right)$$

$$h = \frac{b-a}{2}$$

$$|E_i| \le \frac{h^5}{90} M \qquad O(h^5)$$

M = $\max(f''''(x))$

composite simpson's rule

$$|E_i| \le \frac{n}{2}\frac{h^5}{90} M = (b-a)\frac{h^4}{180} M$$

$$S_n = \frac{h}{3}\Big(\underbrace{f(0)}_{\text{시작점 1개}} + \underbrace{2\sum_{i=1}^{m-1} f(x_{2i})}_{\text{짝수점 2배}} + \underbrace{4\sum_{i=1}^{m} f(x_{2i-1})}_{\text{홀수 4배}} + \underbrace{f(x_{2m})}_{\text{끝점 1개}}\Big)$$

$$m = \frac{n}{2}$$

$O(h^4)$ //

(3차식 까지 exact)

예제

$n=2$ $\quad \int_1^2 (4x^3 - 2x + 3)\,dx$ $\qquad 32-4+3$

$h = \frac{2-1}{2} = \frac{1}{2}$, $\quad m=1$ $\qquad (1, \frac{3}{2}, 2)$

$$S_n = \frac{h}{3}\left(f(1) + 4f(\tfrac{3}{2}) + f(2)\right) = \frac{1}{6}\left(5 + 4\times\frac{27}{2} + 31\right)$$

$$= \frac{90}{6} = 15$$

Simpson's $\frac{3}{8}$ rule , 점 4개 사용

$$h = \frac{b-a}{3} \qquad \int_a^b f(x)\,dx \approx \frac{3}{8}h\left(f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right)$$