how to read and do proofs

chapter 1 : the truth of it all

1.1 The Objectives of this Book

- **proof** is a **method of communicating** a **mathematical truth** to another person.
- one of the objectives of this book is to explain various **proof tecniques** that are used in *all* proof.
- second objective is to learn how to **communicate your own proofs** of **known mathematical truths**.
- doing these things requires a good amount of **creativity**, **intuition**, and **experiece**.

What is a Proof?

- a **proof** is a convincing argument expressed in the language of mathematics that a statement is true.
- in mathematics, a **statement** is a sentence that either **true** or **false**.
- a statement like *"A implies B"* has three separate statements inside it.

1. statement A which is **hypthesis**
2. statement B which is **conclusion**
3. statement "A implies B"

- to prove *"A implies B"* is true, you must exactly know what it means for such a statement to be true.

1. A is true and B is true.   2. A is true and B is false.
3. A is false and B is true.   4. A is false and B is false.

- to determine when this statement "A implies B" is false, ask yourself which of the four foregoing cases is false.

- the **first step** in doing a proof is identify the **hypothesis** A and the **conclusion** B. This is easy to do when the implication is written in the form "if A, then B" but unfortunately, implications are not always written in this specific form.

- in such cases, everything that you are assuming to be true is the **hypothesis A**; everything that you are trying to prove is true is the **conclusion B**.

- examples with appropriate **hypothesis** and **conclusion**.

Example 1: The sum of the first n positive integers is $n(n+1)/2$.

> **Hypothesis:** n is a positive integer. (Note that this is implied for the statement to make sense.)
> **Conclusion:** The sum of the first n positive integers is $n(n+1)/2$.

Example 2: The quadratic equation $ax^2 + bx + c = 0$ has two real roots provided that $b^2 - 4ac > 0$, where $a \neq 0$, b, and c are given real numbers.

> **Hypothesis:** a, b, and c are real numbers with $a \neq 0$ and $b^2 - 4ac > 0$.
> **Conclusion:** The quadratic equation $ax^2 + bx + c = 0$ has two real roots.

Example 3: Two lines tangent to the endpoints of the diameter of a circle are parallel.

> **Hypothesis:** L_1 and L_2 are two lines that are tangent to the endpoints of the diameter of a circle.
> **Conclusion:** L_1 and L_2 are parallel.

Example 4: There is a real number x such that $x = 2^{-x}$.

> **Hypothesis:** None, other than your previous knowledge of mathematics.
> **Conclusion:** There is a real number x such that $x = 2^{-x}$.

- before starting a proof, **always be clear** what you are assuming - that is, the **hypothesis A** - and what you are trying to show - that is, the **conclusion B**.