History and Philosophy of Mathematic Reform

– Dr. William Lawvere, Inter-Disciplinary Conference, Windsor, February 10, 1996 –

For a long time calculus teachers have been demanding better textbooks. In recent months, after much fanfare, a few books claiming to respond to this demand have been launched. But many teachers are saying already that this "new" is very similar to the old that worked so poorly, except now with much more and elaborate and subtle attempts to persuade the student (and the teacher) that some kind of "understanding" is going on, even while all the more resolutely concealing the essential principles.

Also in recent months, a journal for the guidance of college math teachers carried a lead editorial aiming to refute the "myth of scientific literacy". Under the banner of this "scientific literacy" a major curriculum-restructuring over the past decades had been justified, which many scientists judged to be a forced watering-down of content, aiming at producing students who had no understanding of science but who would be able to speak and write about it all the same. However now we are told that most students should not achieve even this "literacy", but only a much-reduced goal of "awareness". A current example (from the journal of a teachers union) of what such awareness means is the ability to just recognize terms such as "chaos theory" and "paradigm shift" and on that basis submit to whatever post-modernist reorganization is being proposed.

The claim that we are now in a "postmodern" and "playful" age is being put forward, by university centers for pedagogy, as a basis for replacing the learning of mathematics with "an inquiry into our habits". In an attempt to enlist some professors for this transformation, the old ecoterrorist claims, that mathematical thinking supports military technology and destruction of the environment, and that we "are" our mathematics, have been resurrected.

Philosophical basis for these counter-reform "reforms" is also being offered in recent months. In U.S. and Canadian journals for college math teachers, it is revealed that mathematical ideas are social in character and that such social rituals constitute a third category of being, and we are assured that curriculum comes between the individual and the collective. At least two recent mathematics publications have praised the philosophers Kuhn, Lakatos, Feyerabend, and Popper, one saying that the philosophy of mathematics needs some great innovators in their mould, the other calling them exemplary "philosophers of Irrationalism ".

The relation in 1996 between teaching of mathematics and philosophy of

mathematics is only the latest stage in a long process. Three hundred years ago the Leibniz rule for the rate of change of a product, and Newton's theorem that the rate of change of the area under a curve is the height of the curve, were made explicit and publicized. This calculus was developed by Bernoulli and Euler, Cauchy and Maxwell into the universal instrument for designing engines, ships, electric power and communication systems, etc. Yet after three hundred years, most people, including most people who actually build those wonders, are still in no position to challenge these designs on their own grounds because knowledge of the instrument has been denied. Even more bizarre: the lack of comprehension is shared even by most people who have been through courses which the State has gone to considerable effort to provide.

There do exist professors who believe that most students are incapable and unwilling to learn any serious subject. But already around 1750 in Milan, Italy, Maria Agnesi wrote and printed a textbook, based on the premise that all Italian youth could and should learn calculus. Her enlightened vision is yet to be realized (in Italy or elsewhere). What happened?

After the French revolution had introduced the decimalized system, it was necessary to recalculate the trigonometric and logarithmic tables used in construction, navigation, etc. A few years ago the following true story about this was widely publicized by a giant multinational computer corporation: An accomplished engineer called Prony was appointed to organize the large task of calculation. Borrowing from Adam Smith, he divided the personnel into three levels: Level A consisted of a few mathematicians who were able to invent appropriate formulas, Level B of a somewhat larger group of people who were able to convert the formulas into algorithms, but a much larger group C of men actually carried out the algorithms by adding and multiplying. The point which the computer corporation found worthy of resurrecting two hundred years later was this: "It was found that the work went more smoothly if those in group C knew no mathematics."

In Britain in the 1830's millions of people were in motion, demanding democracy. Among the measures for quelling the demands of this Chartist movement, the Privy Council created for the first time a system of state-supported schools and teacher-training and inspection, to "introduce order and discipline into the working-class population when older methods of wielding authority had broken down." (*Encyclopedia Brittanica*) The education provided by these new schools is consciously two- tiered: for example one of the defining documents requires that "Arithmetic is the Logic of the poor". In the school systems modeled on that, it is often apparent that while teachers are struggling to teach enough, administrative expenditures and regulations have the larger aim of insuring that we don't teach too much.

In the period before 1848 there was optimism for the possibility of general enlightenment. For example, the Danish physicist Oersted, who discovered an

important principle relating electricity and magnetism, set up an institution to make it known to all. The German mathematician Grassmann, who in 1844 published a new theory and method in geometry which now is becoming widely used by physicists, was actually a high-school teacher who insisted that his new dialectical philosophy was at least as important, since it was directed explicitly at assisting students to learn and understand. However, in the 1870's, when one of his followers published a book showing in detail how Grassmann's methods could be used to teach not only geometry, but also to introduce calculus in high-school, he received a very scathing review and condemnation for suggesting such an upset in the Prussian order of things; the author of the review, Felix Klein, was later the official representative of that Empire at the World's Fair held in conjunction with the opening of Rockefeller's University of Chicago.

Following the Privy Council's lead there has been developed a body of technique, a sort of prize-fighter's technique for occasionally appearing to give in to the demands for reform while actually thereby directing our energies to serve an opposite aim. For example, forty years after Grassmann's death the Prussian establishment decided to make him "a great German soul" but he was portrayed in pragmatist journals as a philosophical idealist. In 1908 Lenin defended Grassmannn's materialist philosophy from this unwarranted distortion, and also remarked, in connection with some proposals to introduce higher mathematics into the schools, that it was surely not being done in order to deepen and broaden the knowledge of science, but rather to provide a basis for the promotion of idealist philosophy.

Indeed, the popularizers of pragmatism and the organizers of collegiate math teaching were closely associated for many years, and the leading circles of philosophy, such as the Gifford Lectures in Scotland and the Silliman Lectures in Yale, began to systematically misuse mathematics and especially their audience's ignorance of mathematics. The Prime Minister of the British empire (who was nicknamed "bloody Balfour" for his suppression of the Irish and later would be famous for his declaration in support of Zionism) who wrote several books on philosophy, was also known for his *Education Act* which reorganized the high schools. Balfour stated in one of his Gifford lectures "I wish I were a mathematician".

Indeed being known as a mathematician became a road to historical recognition as a philosopher. For example Bertrand Russell's opinions on everything became sought-for and he even eventually received the Nobel Prize, partly because of his notoriety as a mathematician; through clever wordplay he devised a new branch of philosophy known as "foundations of mathematics" whose only role is to give mathematics permission to exist, and which must be written in symbols different from the usual mathematical ones. The latter ruse he had learned from Peano, whose followers had proudly produced a high-school text written entirely in symbols, in order to dispel any false idea that with "numbers", "lines", or "space" we are really referring to

anything; amazingly, this work was advertised as a clarification of Grassmann.

Perhaps the best-known 20th century figure who consciously guided his actions by the pragmatic philosophy was Mussolini; but probably as important was John Dewey, whose teachings and organizations had a tremendous influence on education throughout the world. He was occasionally quite clear about the direction of his reform; for example, in China in 1919 he gave a course at a college for teachers, in which he enunciated his infamous principle:

"Teach the child, not the science"!

Of course conscientious teachers through the centuries have done both: the acquisition of some portion of the accumulated knowledge of humanity (science) is the purpose of the child's presence in school, but the teacher endeavors to guide this acquisition with due regard for each child's particular situation. Why then Dewey's prohibition of the teaching of the knowledge? In China he compared the alleged "authoritarianism" of science with the recently-overthrown imperial regime, and since then the broad-brush charge of "authoritarianism" has been used thousands of times as a pretext to eliminate from school systems the teaching of the deductive aspect in geometry, of the grammatical parts of speech, of the diagramming of sentences, etc. Indeed, now many college teachers of mathematics analyze that a large part of the difficulties which are had by students fresh from high-school are not due to mathematics itself, but due to their first real encounter with the requirement that ordinary language be used in a precise way. Dewey's powerful principle of "teaching the child, not the science" has many corollaries, such as the anti-child theory that "learning is fun", and ultimately the logic whereby jokes take the place of reasoning; certainly the principle includes the injunction often addressed to pupils: "Say it in your own words". This injunction is very attractive to teachers, who know that understanding requires a conscious act by the individual. However, the whole atmosphere of the school often mandates that "in your own words" should mean "as imprecisely as possible" thus destroying the acquisition of concepts in any usable form.

Already in the early 1900's the tradition, that school should be devoted to learning the accumulated human knowledge, was being eroded in another way: The steel city of Gary, Indiana was built whole at Rockefeller's demand, the factories, workers' homes, sidewalks, and school system. To minimize the free time of the sons and daughters of the workers, extra-curricular activities at the schools were declared essential to the development of "the child". This Dewey-endorsed school system was studied by administrators from all over the world who journeyed to observe it. The whole Dewey program was styled "progressive" education, illustrating through the use of this term the fundamental tenet of pragmatist epistemology: truth is what you can get away with.

In 1915 the U.S. mathematical organization split into two, one devoted primarily to the promotion of research, and the other supposedly devoted to the promotion of college teaching. The latter maintained and deepened its ties with pragmatic philosophy in 1921 when, at a meeting at Wellesly College, the widow of the leading publicizer of pragmatism, Paul Carus (whose stated aim was to promote religion on the basis of recent science), gave several thousand dollars to finance a series of monographs. At the same meeting the president of the organization gave his address entitled: "The religion of a mathematician", consisting of principles such as "since we know infinitesimals, we must also know our own insignificance; since we believe in infinity, we must also believe in an almighty; since we can imagine the fourth dimension, we can also imagine heaven, etc." This policy of the organization has never been repudiated, and its publications, which aim to give guidance to college teachers, have over the years refined to a precise art a writing style similar to that of the *Scientific American*, i.e. it is presumed that the readers will not advance from a lower to a higher level, and hence under the guise of "popularization", all concepts are made sufficiently imprecise so as to be unusable by anybody. The deliverer of that presidential address was the author of one of the few texts on the history of mathematics then available in English.

Mathematics itself is often said to have made more advances in the 20th century than in all previous centuries. The advances include not only the solution and formulation of difficult problems with geometrical and other content, but also (indispensably to that) the development of unifying concepts which are of great simplifying and clarifying value. An opportunity seemed to present itself around 1960 to disseminate that simplifying and clarifying value to a vastly larger number of students. The occasion, as I understand it, was the following: The U.S. ruling circles, fresh from rejoicing that their friend Khrushchev had succeeded in overthrowing the socialist system and was transforming it into a pseudo-socialist system, suddenly realized that they were thereby also faced with a rival superpower. This implied a certain shift of the boundary between the B and C Levels on the Prony scale, a readjustment of the line between "arithmetic" and "logic" on the Privy Council's anti-Chartist plan: more students would have to learn more math and science in order to counter the Sputnik threat. Whatever the precise details of the background, the opportunity was presented around 1960 to have university researchers directing summer schools for eager high-school teachers, to have writing teams producing new text books for pupils and teachers, etc. The challenge was taken up enthusiastically by many professionals in the spirit of letting those concepts, which had proved to be so enlightening for them, also serve to enlighten everyone. Of course such an undertaking requires several years of pupil feedback and text revision (and new mathematical research!) in order to become successful. But that stage was never reached because the movement was discredited; the enthusiastic professionals had underestimated the preparation of the opposition. By an

artful confusion of the meanings of words like "foundation", the foundationalist trend (which had become entrenched since the days when Bertrand Russell started at the London School of Economics - LSE) insisted that the texts must be written in their idiosyncratic notation. And the professional schools of pedagogy (housed since Dewey in ivory towers remote from the actual scientific departments) took leadership of the movement from the bewildered scientists, insuring its destruction.

It would seem that an obvious way to improve math teaching would be to give more examples and more applications. This is correct, of course, but to give that as a demand and stop there was again to underestimate what we are up against. The 1970's and 80's saw the publication of many "applied" calculus texts in which explicit principles were subordinated to problems watered-down and distorted beyond usability from various fields. But as many professors in those various fields understand, math is theory. What a student needs of math in a field of application such as chemistry, business management, etc. is to know math as well as possible in order that the applied concepts be approachable with little math-related mystery and in order that mastery of appropriate new methods can be partly self-guided. Isolated, particular methods learned mechanically and then forgotten, and especially, half-baked attempts to teach an alleged application instead of the explicit principles of calculus, can only negatively affect the students' ability to apply math.

The demand for better calculus text books in the English-speaking world thus began, for the above and many other reasons, to become more threatening. The initial response of the publishers (that they would never change their policy of offering the next year an exact copy of the competing text which had made most profit the previous year, only with more colors) was met by deserved contempt. Some of the more recent offerings are the result of multimillion dollar government intervention.

Contrary to the portrait of the professor who views teaching and research as inimical to each other, many see them as mutually supportive.

Many of the ideas, which have led to long and fruitful development by researchers, actually arose from attempts to explain matters more clearly to students. For example, attempts in the 1960's to provide a clearer, simpler, yet rigorous base for understanding calculus led to a new trend of research in the foundations of topology, logic, and analysis within which many innovative papers and over a dozen books have now been produced. On the other hand, research leads from time to time to new synthesizing concepts, which clarify matters enormously for the researchers, who then struggle to find ways to spread this clarification to students. For example research into the mathematical foundations of continuum thermomechanics and the constitutive relations of materials led to new, more direct, ways of dealing with infinitesimals, function spaces, and extensive quantities, which are now

being taught to undergraduates in some places.

Research and teaching are of course different aspects of endeavor, but as long as they are still alive, they have an orientation in common, a commitment to tirelessly combating the absence of knowledge.

History shows that the teachers, yearning for a greater opportunity to participate in the creation and dissemination of enlightenment, will not be satisfied by waiting for this or that establishment entity to provide it. Not only would the fulfillment of these needs remain forever a mere policy objective; our enthusiasm would continue to be used as the engine for the spread of still more pseudo-knowledge and pessimism. The problem can be taken up for solution, without million-dollar grants, both by devising teaching materials which reflect the actual historical development of a given field (neither repeating some entrenched hundred-year old false summation, nor succumbing to the ultrarevolutionary post-modernist degeneration), as well as by making explicit the philosophy which emerges from actual research developments of the recent decades. Collective effort is necessary, however, to concentrate such materials and to disseminate them so as to serve the needs of society as a whole.

(Hardial Bains Resource Centre Archives)

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