converge (iii) If the spectral sequence converged uniformly things became nice.

Olav Arnfinn Laudal

If $\mathbb{G} = (G, \varepsilon, \delta)$ is a comonad in a category $\mathcal{C}$ and $E : \mathcal{C} \to \mathcal{A}$ is a functor into an abelian category, homology functors $H_n(X, E)$ relative to $\mathbb{G}$ are defined as the homology objects of the complex associated with

$$\cdots \to EG^{n+1}X \to \cdots \to EG^2X \to EGX \to 0$$

in $\mathcal{A}$ with boundary $\partial_n = \Sigma(-1)^i\, G^i \varepsilon G^{n-i} X$ for $0 \le i \le n$. Axioms for these functors, such as the exact homology sequence relative to a sequence $0 \to E' \to E \to E'' \to 0$ exact in the functor category $(\mathcal{C}, \mathcal{A})$ are considered. X is $\mathbb{G}$-projective if and only if a sequence $X \to GX \to X$ equal to the identity exists. Two comonads $\mathbb{G}$ and $\mathbb{K}$ give the same homology if [$\mathbb{G}$-projective $\Longleftrightarrow$ $\mathbb{K}$-projective]. Certain computations on the relation of this cohomology theory to the one defined by André are given.

Michael Barr

$f : \mathfrak{A} \to \mathfrak{S}, \ \eta : \mathfrak{A}^{op} \to \mathfrak{S}$

$\int^a f\, d\eta = \varinjlim [(H, f) \xrightarrow{\eta} \mathfrak{S}]$

Presentations of right complete categories (preliminary)

If $\mathbb{C}$ small, $\mathfrak{S}^{\mathbb{C}^{op}}$ is the free right complete cat on $\mathbb{C}$. An attempt is made to extend this to a monad ("triple") in Cat by taking the "small part" of $\mathfrak{A} \rightsquigarrow$ Rt conts$(\mathfrak{S}^{\mathfrak{A}}, \mathfrak{S})$ & to present categories of algebras & sheaves by $\mathbb{R} \rightrightarrows \mathfrak{S}^{\mathbb{C}^{op}} \xrightarrow{\text{"coeq"}} \mathfrak{X}$. This should be extended to right complete left exact cats w. distr. law w. "algebras" = topos. Compact & complete atomic cats are defined similarly using "higher order theories" on $\mathfrak{S}^{\mathbb{C}^{op}}$ as power set!

F. Wm Lawvere

Lawvere 23 juli 66

Mainly conjectures.

$\mathfrak{X}$ right complete, $\mathbb{C}$ small

$$\begin{tikzcd} \mathbb{C} \arrow[r] \arrow[dr, "A"'] & \mathbb{S}^{\mathbb{C}^{opp}} \arrow[d, dashed, "\exists !"] \\ & \mathfrak{X} \end{tikzcd}$$

$\exists !$ right continuous (unique up to natural equivalence).

So $\mathbb{S}^{\mathbb{C}^{opp}}$ is the free right complete category

Large theories in the cat of cat's. : every possible limit will be an algebraic operation

$\mathbb{S}^{\mathbb{C}^{opp}}$ is in some sense the power set. ; so one can use it for higher order theories. Grothendieck topology is ordinary topology, but with ~~a~~ new power set functor.

Why power set? Think of the closed cat. of ordered set

Then the analogue of $\mathbb{S}^{\mathbb{C}^{opp}}$ is

$$2^{\Lambda^{op}}$$

(2 analogue of $\mathbb{S}$) ; power set if Λ discrete

„Triple" on ordered sets. (not really).

$$\mathbb{C} \xrightarrow{f} \mathbb{C}' \qquad \mathbb{S}^{\mathbb{C}^{opp}} \underset{\square}{\overset{adj}{\rightleftarrows}} \mathbb{S}^{\mathbb{C}'^{opp}}$$

adj instead of $\square$ makes it covariant. (Corresponds to covariant power set with direct image).

$\mathbb{S} \circlearrowleft P$ (Power set with direct image, is a monad).

The algebras: right complete lattices. (the morphisms preserve arbitrary sup's). Free alg's: Sets with relations.

This suggests the following: The Kleisli is just putting in more functions. Take e.g. P_0 (set of finite subsets).

E.g. cat of cat.

$\mathbb{C} \longrightarrow \mathbb{D}$. But a generalized functor is

Cocontinuous$(\mathcal{S}^{\mathbb{C}^{opp}}, \mathcal{S}^{\mathbb{D}^{opp}})$

$$\cong \mathcal{S}^{\mathbb{C} \times \mathbb{D}^{opp}}$$

Objects in here are simply pairings (sense of Cohn). Composition is a generalized matrix multiplication

This has been looked at in ring theory. Morita-theory: these things are just bimodules. Freyd showed the same to hold for theories. — Or take top. spaces and sheaves.

Every monad in sets had an algebraic part. The composition $P \circ P_0$ carries a monad structure; it describes Rcled lattices; (distributive law $x \wedge \bigvee y_i = \bigvee x \wedge y_i$ required).

Close connection with Hausdorff^{opp} and Rcled

$$\mathcal{H}^{opp} \underset{top}{\rightleftarrows} \text{Rcled}$$

top

This was to prepare the way for a similar monad in the cat. of cat's, to get topos.

Given two cats, both relative over some common closed cat, and an object S common to both.

$$\begin{tikzcd} \mathcal{R}^{op} \arrow[dr, shift left, "\mathcal{R}(-,S)"'] & \\ & \text{Cat} \arrow[ul, shift left, "\text{Cat}(-,S)"'] \end{tikzcd}$$

will automatically be adjoint. The monads should have some special properties („analytical")

$$\begin{tikzcd} \mathrm{OrdVect} \arrow[dr, shift right, "\mathrm{Ord}\mathcal{V}(-,R)"'] & \\ & \mathrm{Top} \arrow[ul, shift right, "\mathrm{Top}(-,R)"'] \end{tikzcd}$$

Gives the integration theory monad on Top.

The previous monad is

$$\mathfrak{X} \rightsquigarrow R(\mathfrak{S}^{\mathfrak{X}}, \mathfrak{S})$$

I don't know what the algebras are. If $\mathfrak{X}$ has small hom sets, I have the Yoneda mapping

$$\mathfrak{X}^{opp} \longrightarrow \mathfrak{S}^{\mathfrak{X}} \quad ;$$

restricting along this, I obtain a functor

$$R(\mathfrak{S}^{\mathfrak{X}}, \mathfrak{S}) \longrightarrow \mathfrak{S}^{\mathfrak{X}^{opp}}$$

The same procedure for the measure case gives

$$\mathcal{O}(C(X,R), R) \to \mathrm{Meas}(X,R)$$

The Riesz repr. theorem gives conditions when this is an equivalence.

Is there a Riesz theorem for

$$R(\mathfrak{S}^{\mathfrak{X}}, \mathfrak{S}) \longrightarrow \mathfrak{S}^{\mathfrak{X}^{opp}}$$

$$\int f\, dm = \varinjlim (-, \mathfrak{S}) \xrightarrow{m} \mathfrak{S})$$

(the canonical limit).

One can speak of the small part of a monad in the cat of cats

$$\tilde{T}\mathfrak{X} = \bigcup_{\substack{\mathfrak{C} \to \mathfrak{X} \\ \text{small}}} R(\mathfrak{S}^{\mathfrak{C}}, \mathfrak{X}) \subseteq T\mathfrak{X}$$

$\hat{T}$ is a monad

I conjecture the alg's ~~of~~ over this is right complete categories (with chosen limit functors).

$$\begin{tikzcd} \mathrm{Dir}(\mathfrak{X}) \arrow[r] \arrow[dr, "\text{factors}"'] & \mathbb{S}^{\mathfrak{X}} \\ & \mathbb{S}^{\mathfrak{X}^{op}} \arrow[u, "\text{conjugation (Isbell)}"'] \arrow[r, dashed, "\xi"'] & \mathfrak{X} \end{tikzcd}$$

An algebra ξ . . .

<u>Presentations</u>. Given a pair of small cats

$$\mathbb{R} \rightrightarrows \mathbb{S}^{\mathbb{C}^{opp}} \xrightarrow[\text{coeq in } \mathbb{R}]{} \mathfrak{X}$$

Grothendieck topology

For small cat's: take the sheaves ~~with~~ for the canonical topology. - is the triple.

The triple in the cat of cat's has the following property, that

$$\mathbb{S}^{(\mathbb{S}^{\mathfrak{X}^{op}})^{op}} \longrightarrow \mathbb{S}^{\mathfrak{X}^{op}} \qquad \text{then}$$

$\mu_{\mathfrak{X}} \dashv \eta_{\mathfrak{X}} T$; so the μ is determined by η

$$\begin{tikzcd} & \mathbb{S}^{opp} \arrow[dl, phantom] \\ \mathbb{S}^{opp} \arrow[r, "2^{()}"'] & \mathbb{S} \arrow[u, "2^{()}"'] \arrow[loop right] \end{tikzcd}$$

we get complete atomic Boolean alg's as alg's.